

UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO



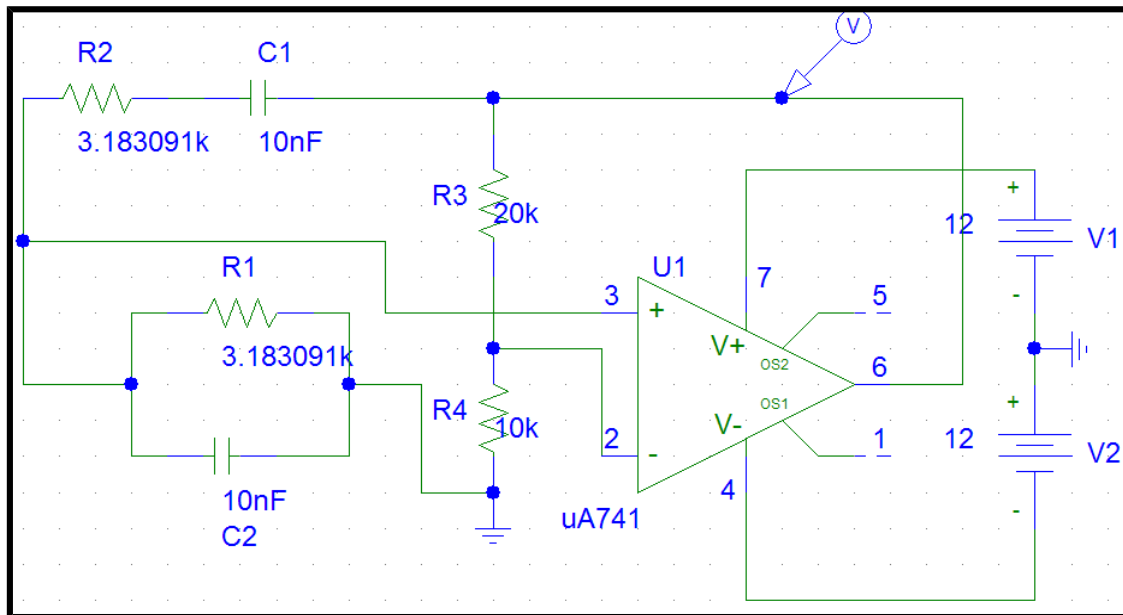
05/05/2006, Ciudad Universitaria, México

SIMULACIÓN DE CIRCUITO GENERADOR DE ONDA TRIANGULAR

Especificaciones: oscilador de **5KHz, 10 Hz, 100KHz, 1MHz.**

	F	C	R=1/(WC)
5KHz	5.00E+03	1.0000E-09	3.183E+04
	5.00E+03	1.0000E-08	3.183E+03
	5.00E+03	1.0000E-06	3.183E+01
10 Hz	1.00E+01	1.00E-09	1.592E+07
	1.00E+01	1.00E-06	1.592E+04
	1.00E+01	2.20E-06	7.234E+03
100KHz	1.00E+05	1.00E-09	1.592E+03
	1.00E+05	1.00E-06	1.592E+00
	1.00E+05	2.20E-06	7.234E-01
1MHz	1.00E+09	1.00E-12	1.591546E+05

1nF	1.00E-09
10nF	1.00E-08
1uF	1.00E-06
2.2uF	2.20E-06
10uF	1.00E-05
47uF	4.70E-05
1pF	1.00E-12
1nF	1.00E-09

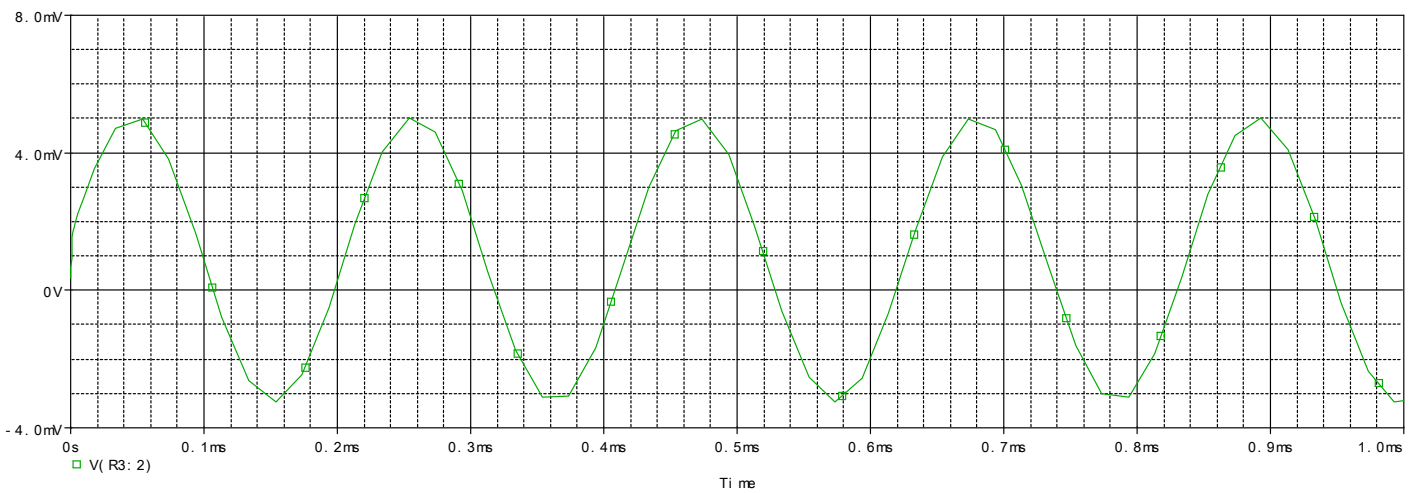


$$K=(1+R3/R4)=3$$

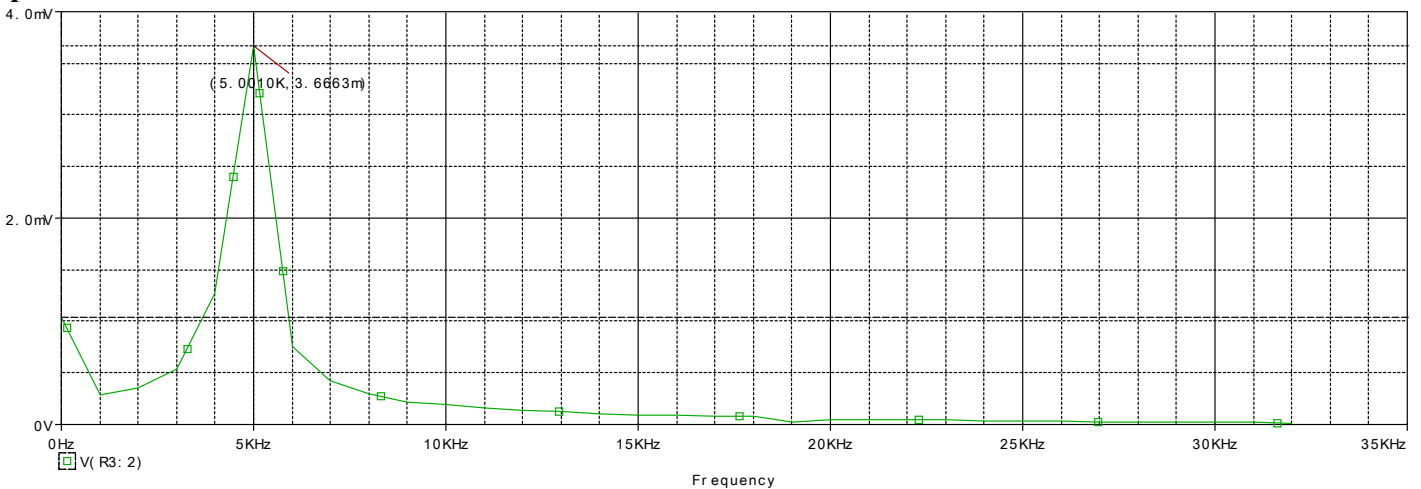
$$R3=2*R4$$

$$R4=10k \rightarrow R3=20k$$

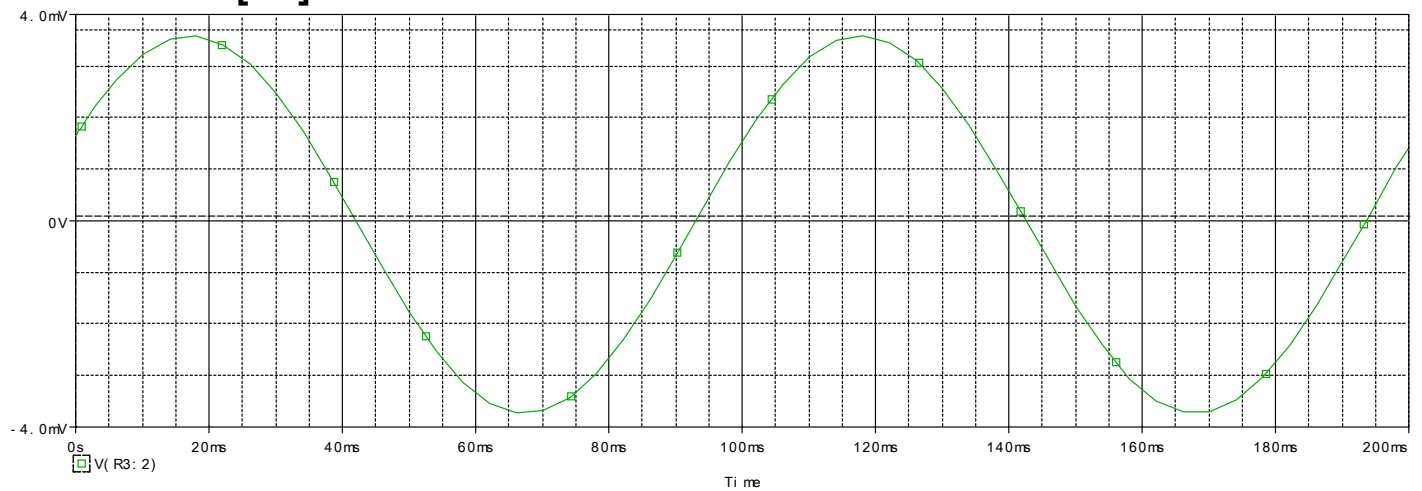
Simulación 5 [kHz] → oscilador diseñado en clase



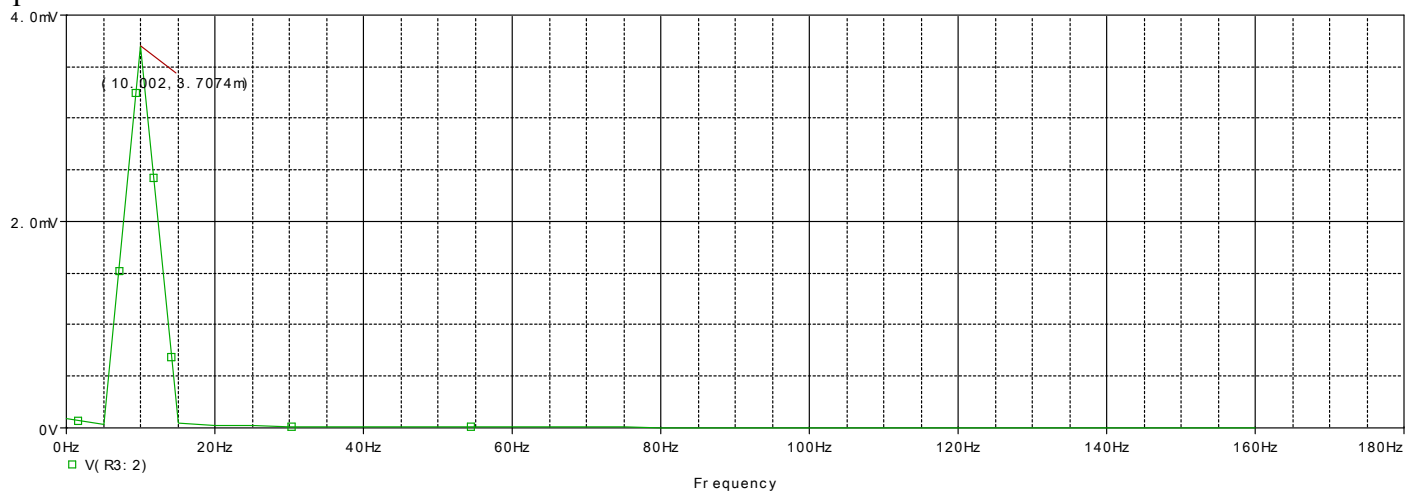
Espectro en frecuencias



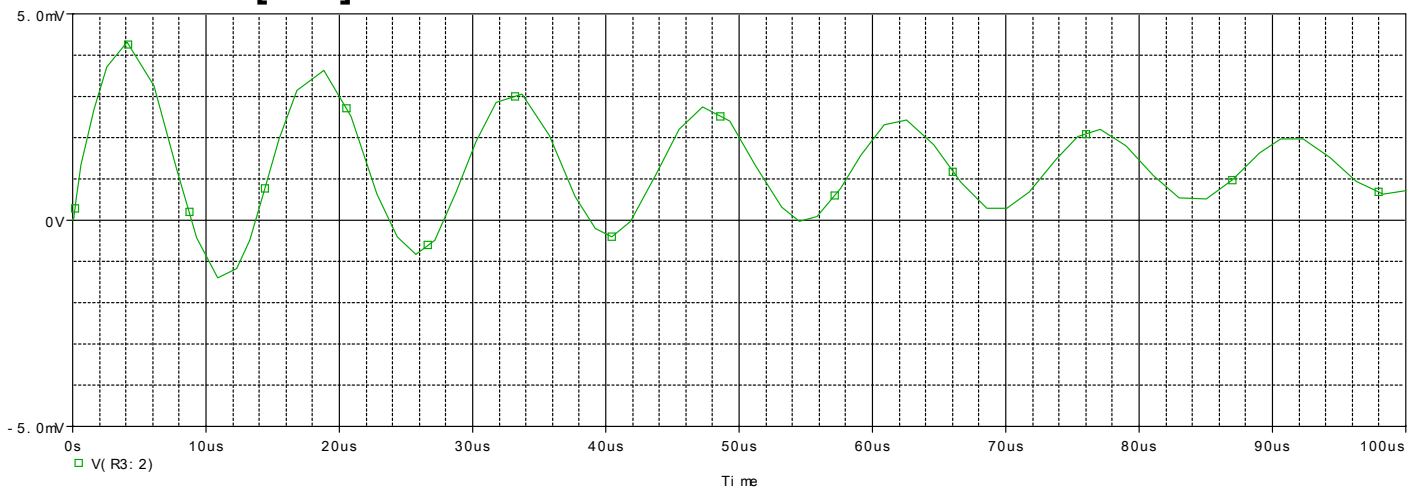
Simulación 10 [Hz]



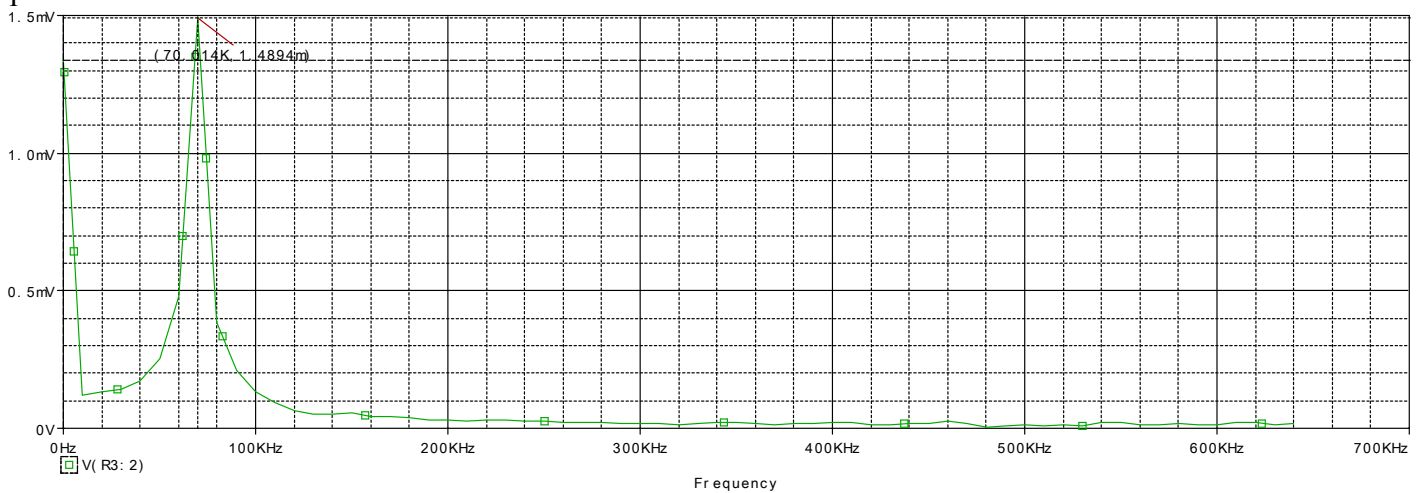
Espectro en frecuencias



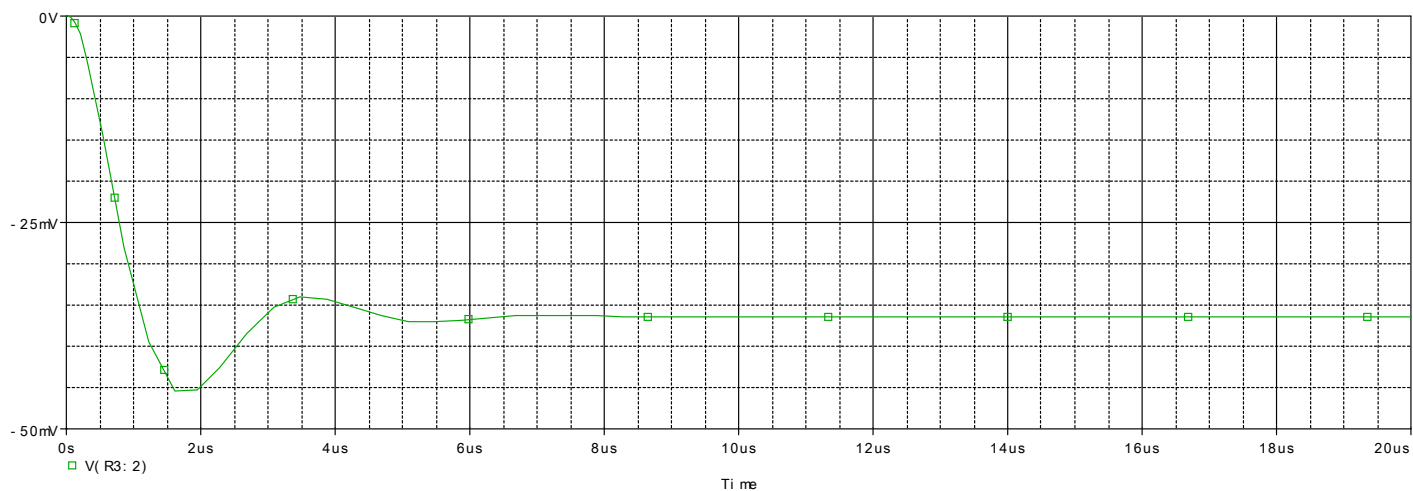
Simulación 100 [KHz]



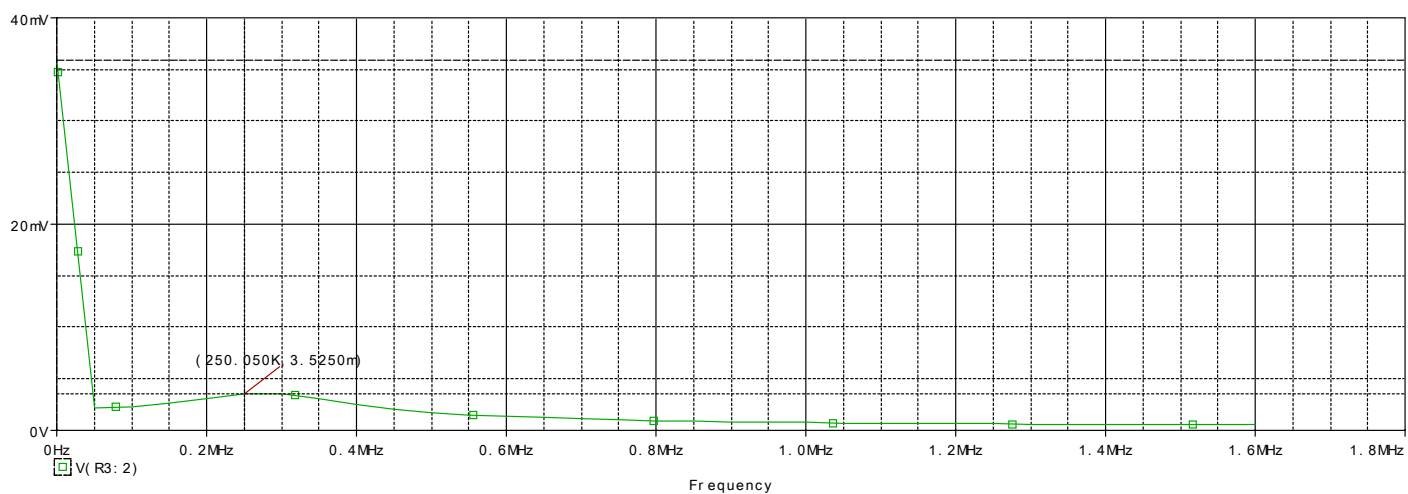
Espectro en frecuencias



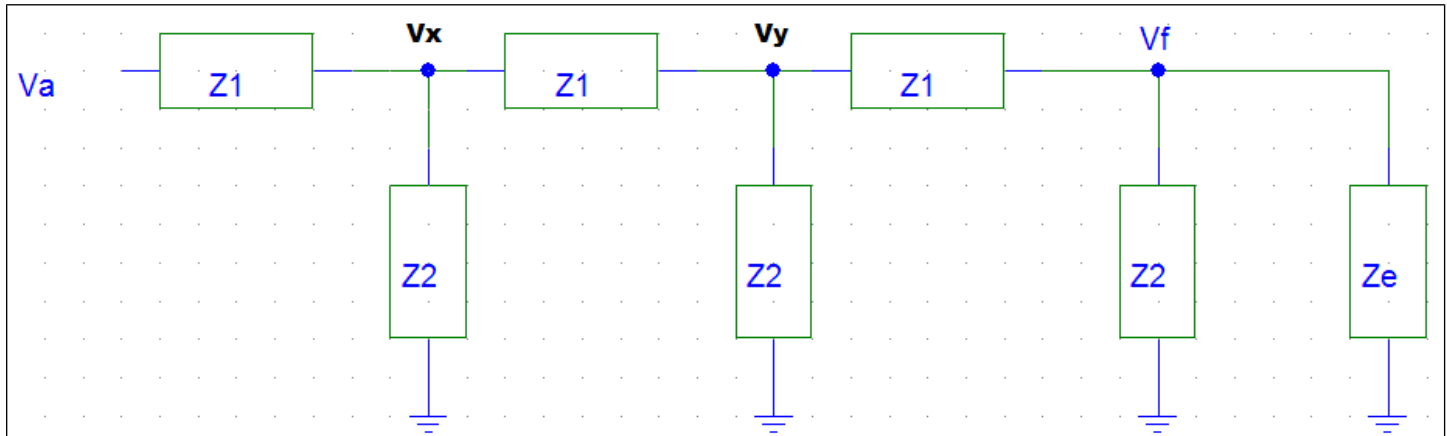
Simulación 1 [MHz]



Espectro en frecuencias



Resolver:



$$Z_{eq} = Z_2 \parallel Z_e = \frac{Z_e Z_2}{Z_e + Z_2} \text{-----(1)}$$

$$\frac{v_a - v_x}{Z_1} = \frac{v_x - 0}{Z_2} + \frac{v_x - v_y}{Z_1} \text{-----(2)}$$

$$\frac{v_x - v_y}{Z_1} = \frac{v_y - 0}{Z_2} + \frac{v_y - v_f}{Z_1} \text{-----(3)}$$

$$\frac{v_y - v_f}{Z_1} = \frac{v_f - 0}{Z_{eq}} \text{-----(4)}$$

Reacomodando de (2):

$$\begin{aligned} \frac{v_a}{Z_1} - \frac{v_x}{Z_1} &= \frac{v_x}{Z_2} + \frac{v_x}{Z_1} - \frac{v_y}{Z_1} \\ \frac{v_a}{Z_1} - v_x \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_1} \right] + \frac{v_y}{Z_1} &= 0 \text{-----(A)} \end{aligned}$$

Reacomodando de (3):

$$\begin{aligned} \frac{v_x}{Z_1} - \frac{v_y}{Z_1} &= \frac{v_y}{Z_2} + \frac{v_y}{Z_1} - \frac{v_f}{Z_1} \\ \frac{v_x}{Z_1} - v_y \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_1} \right] + \frac{v_f}{Z_1} &= 0 \\ 0 + \frac{v_x}{Z_1} - v_y \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_1} \right] + \frac{v_f}{Z_1} &= 0 \text{-----(B)} \end{aligned}$$

Reacomodando de (4):

$$0 + 0 + \frac{v_y}{Z_1} - \frac{v_f}{Z_1} - \frac{v_f}{Z_{eq}} = 0$$

$$0 + 0 + \frac{v_y}{Z_1} - v_f \left(\frac{1}{Z_1} + \frac{1}{Z_{eq}} \right) = 0 \text{------(C)}$$

Tenemos un sistema de ecuaciones:

$$\frac{v_a}{Z_1} - v_x \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_1} \right] + \frac{v_y}{Z_1} = 0$$

$$0 + \frac{v_x}{Z_1} - v_y \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_1} \right] + \frac{v_f}{Z_1} = 0$$

$$0 + 0 + \frac{v_y}{Z_1} - v_f \left(\frac{1}{Z_1} + \frac{1}{Z_{eq}} \right) = 0$$

Resolver para Vx y Vy, para después despejar Va y Vf

$$-v_x \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_1} \right] + \frac{v_y}{Z_1} = -\frac{v_a}{Z_1}$$

$$\frac{v_x}{Z_1} - v_y \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_1} \right] = -\frac{v_f}{Z_1}$$

Obtenemos la matriz

$$\begin{vmatrix} -\left(\frac{2}{Z_1} + \frac{1}{Z_2}\right) & \frac{1}{Z_1} \\ \frac{1}{Z_1} & -\left(\frac{2}{Z_1} + \frac{1}{Z_2}\right) \end{vmatrix} \begin{vmatrix} v_x \\ v_y \end{vmatrix} = \begin{vmatrix} -\frac{v_a}{Z_1} \\ -\frac{v_f}{Z_1} \end{vmatrix}$$

$$\begin{vmatrix} -\left(\frac{2}{Z_1} + \frac{1}{Z_2}\right) & \frac{1}{Z_1} \\ \frac{1}{Z_1} & -\left(\frac{2}{Z_1} + \frac{1}{Z_2}\right) \end{vmatrix} \begin{vmatrix} v_x \\ v_y \end{vmatrix} = \begin{vmatrix} -\frac{v_a}{Z_1} \\ -\frac{v_f}{Z_1} \end{vmatrix}$$

$$v_x = \frac{\left(\frac{2}{Z_1} + \frac{1}{Z_2}\right)\left(\frac{2}{Z_1} + \frac{1}{Z_2}\right) - \frac{1}{Z_1} \frac{1}{Z_1}}{-\frac{v_a}{Z_1}}$$

$$v_y = \frac{\left(\frac{2}{Z_1} + \frac{1}{Z_2}\right)\left(\frac{2}{Z_1} + \frac{1}{Z_2}\right) - \frac{1}{Z_1} \frac{1}{Z_1}}{-\frac{v_f}{Z_1}}$$

De (4)

$$\frac{v_y}{Z_1} - \frac{v_f}{Z_1} = \frac{v_f}{Z_{eq}}$$

$$\frac{v_y}{Z_1} = \frac{v_f}{Z_{eq}} + \frac{v_f}{Z_1}$$

$$\frac{v_y}{Z_1} = v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)$$

$$v_y = v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \text{-----(5)}$$

(5) en (3)

$$\frac{v_x - v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)}{Z_1} = \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - 0}{Z_2} + \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - v_f}{Z_1}$$

$$\frac{v_x - v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)}{Z_1} = \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)}{Z_2} + \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - v_f}{Z_1}$$

$$\frac{v_x}{Z_1} - v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) = v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1}$$

Despejando Vx

$$v_x = Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right] \text{-----(6)}$$

(6) en dos

$$\frac{v_a - v_x}{Z_1} = \frac{v_x - 0}{Z_2} + \frac{v_x - v_y}{Z_1}$$

$$v_y = v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)$$

$$\frac{v_a}{Z_1} - \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} = \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_2}$$

$$+ \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} - \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)}{Z_1}$$

$$\frac{v_a}{Z_1} - \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} = \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_2}$$

$$+ \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} - \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)}{Z_1}$$

$$\frac{v_a}{Z_1} = \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_2}$$

$$+ \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} - \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)}{Z_1} + \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1}$$

factorizando vf

$$\frac{v_a}{Z_1} = v_f \left\{ \frac{Z_1 \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_2} \right.$$

$$\left. + \frac{Z_1 \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} - \frac{Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)}{Z_1} + \frac{Z_1 \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} \right\}$$

$$\frac{v_a}{Z_1} = v_f \left\{ \frac{Z_1 \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_2} \right.$$

$$\left. + \frac{Z_1 \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} - \frac{Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)}{Z_1} + \frac{Z_1 \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} \right\}$$

$$\frac{1}{Z_1} \left\{ \frac{Z_1 \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_2} \right.$$

$$\left. + \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right] - \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right] \right\} = \frac{v_f}{v_a}$$

Simplificando

$$\frac{1}{Z_1 \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right] \left\{ \frac{Z_1}{Z_2} + 1 - \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + 1 \right\}} = \frac{v_f}{v_a}$$

$$\frac{1}{Z_1 \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right] \left\{ \frac{Z_1}{Z_2} + 1 - \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + 1 \right\}} = \frac{v_f}{v_a}$$

$$\frac{1}{\left[\frac{Z_1}{Z_2} \left(\frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_1} \right) + \left(\frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_1} \right) - \frac{Z_1}{Z_1} + \left(\frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_1} \right) \right] \left\{ \frac{Z_1}{Z_2} + 1 - \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + 1 \right\}} = \frac{v_f}{v_a}$$

$$\frac{1}{\left[\left(\frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_1} \right) \left\{ \frac{Z_1}{Z_2} + 1 + 1 \right\} - \frac{Z_1}{Z_1} \left\{ \frac{Z_1}{Z_2} + 1 - \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + 1 \right\} \right]} = \frac{v_f}{v_a}$$

$$\frac{1}{\left[\left(\frac{Z_1}{Z_{eq}} + 1 \right) \left[\frac{Z_1}{Z_2} + 2 \right] - 1 \right] \left\{ \left[\frac{Z_1}{Z_2} + 2 \right] - \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right\}} = \frac{v_f}{v_a}$$

$$\frac{1}{\left\{ \left(\frac{Z_1}{Z_{eq}} + 1 \right) \left[\frac{Z_1}{Z_2} + 2 \right] \left[\frac{Z_1}{Z_2} + 2 \right] - \left(\frac{Z_1}{Z_{eq}} + 1 \right) \left[\frac{Z_1}{Z_2} + 2 \right] \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \left(\frac{Z_1}{Z_{eq}} + 1 \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right\}} = \frac{v_f}{v_a}$$

$$\frac{1}{\left\{ \left(\frac{Z_1}{Z_{eq}} + 1 \right) \left[\frac{Z_1}{Z_2} + 2 \right] \left[\frac{Z_1}{Z_2} + 2 \right] - \left(\frac{Z_1}{Z_{eq}} + 1 \right) \left[\frac{Z_1}{Z_2} + 2 \right] \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \left(\frac{Z_1}{Z_{eq}} + 1 \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right\}} = \frac{v_f}{v_a}$$

$$\frac{1}{\left(\frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} + 2 \frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_2} + 2 \right) \left[\frac{Z_1}{Z_2} + 2 \right] - \left(\frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} + \frac{Z_1}{Z_2} + 2 \right) \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \left(\frac{Z_1}{Z_{eq}} + 1 \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)} = \frac{v_f}{v_a}$$

$$\frac{1}{\frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} \frac{Z_1}{Z_2} + 2 \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} + 2 \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} + 4 \frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_2} \frac{Z_1}{Z_2} + 2 \frac{Z_1}{Z_2} + 2 \frac{Z_1}{Z_2} + 4 - \left(\frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} \frac{1}{Z_{eq}} + \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} \frac{1}{Z_1} + \frac{Z_1}{Z_2} \frac{1}{Z_{eq}} + \frac{Z_1}{Z_2} \frac{1}{Z_1} + \frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \left(\frac{Z_1}{Z_{eq}} + 1 \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)}$$

$$\frac{1}{\frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} \frac{Z_1}{Z_2} + 2 \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} + 2 \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} + 4 \frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_2} \frac{Z_1}{Z_2} + 2 \frac{Z_1}{Z_2} + 2 \frac{Z_1}{Z_2} + 4 - \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} \frac{1}{Z_{eq}} - \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} \frac{1}{Z_1} - \frac{Z_1}{Z_2} \frac{1}{Z_{eq}} - \frac{Z_1}{Z_2} \frac{1}{Z_1} - \frac{1}{Z_{eq}} - \frac{1}{Z_1} - \frac{1}{Z_{eq}} - \frac{1}{Z_1}}$$

$$\frac{1}{\frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} \frac{Z_1}{Z_2} + 4 \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} + 3 \frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_2} \frac{Z_1}{Z_2} + 4 \frac{Z_1}{Z_2} + 4 - \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} \frac{1}{Z_{eq}} - \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} \frac{1}{Z_1} - \frac{Z_1}{Z_2} \frac{1}{Z_{eq}} - \frac{Z_1}{Z_2} \frac{1}{Z_1} - 1} = \frac{v_f}{v_a}$$