UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO



05/05/2006, Ciudad Universitaria, México

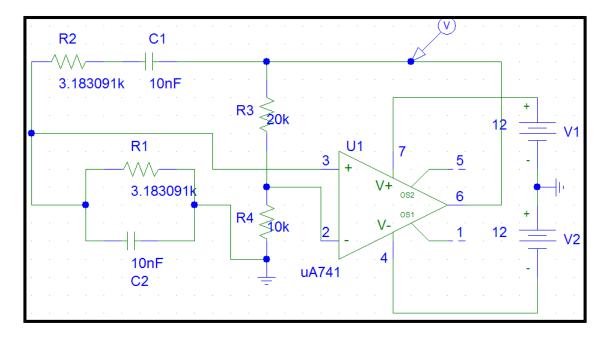
UNAM, Facultad de Ingeniería Autor: Santiago Cruz Carlos

8-may-06 Titulo: Simulación

SIMULACIÓN DE CIRCUITO GENERADOR DE ONDA TRIANGULAR

	F	С	R=1/(WC)
	5.00E+03	1.0000E-09	3.183E+04
5KHz	5.00E+03	1.0000E-08	3.183E+03
	5.00E+03	1.0000E-06	3.183E+01
	1.00E+01	1.00E-09	1.592E+07
10 Hz	1.00E+01	1.00E-06	1.592E+04
	1.00E+01	2.20E-06	7.234E+03
	1.00E+05	1.00E-09	1.592E+03
100KHz	1.00E+05	1.00E-06	1.592E+00
	1.00E+05	2.20E-06	7.234E-01
1MHz	×		1.591546E+0
	1.00E+09	1.00E-12	5

1nF	1.00E-09	
10nF	1.00E-08	
1uF	1.00E-06	
2.2uF	2.20E-06	
10uF	1.00E-05	
47uF	4.70E-05	
1pF	1.00E-12	
1nF	1.00E-09	

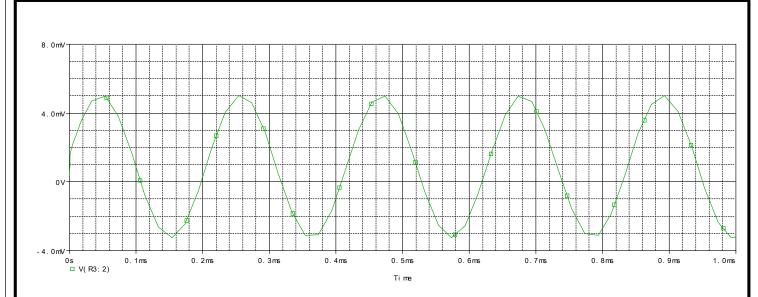


K=(1+R3/R4)=3

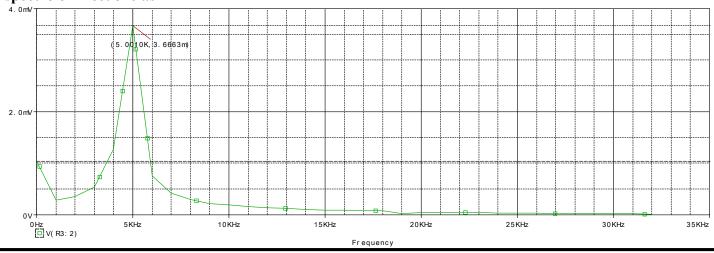
R3=2*R4

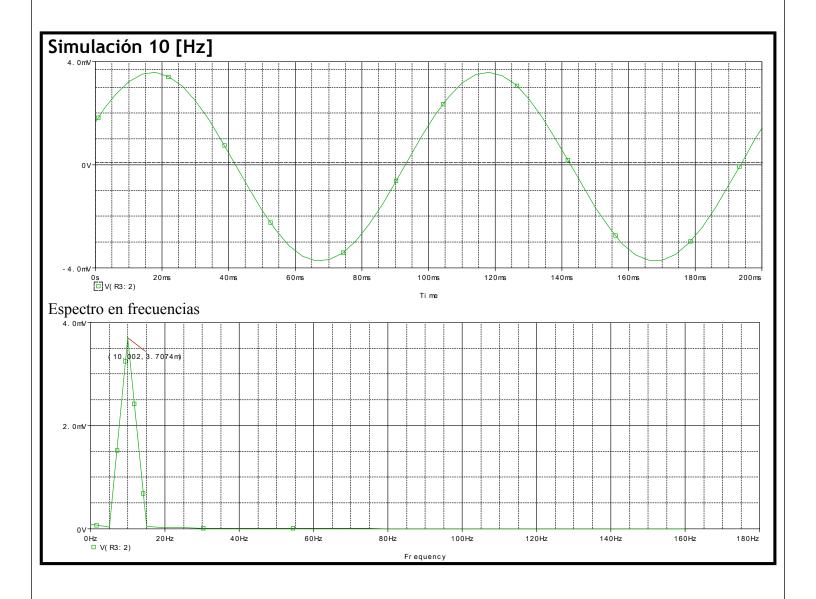
R4=10k→ R3=20k

Simulación 5 [kHz] → oscilador diseñado en clase



Espectro en frecuencias





0.5 mV

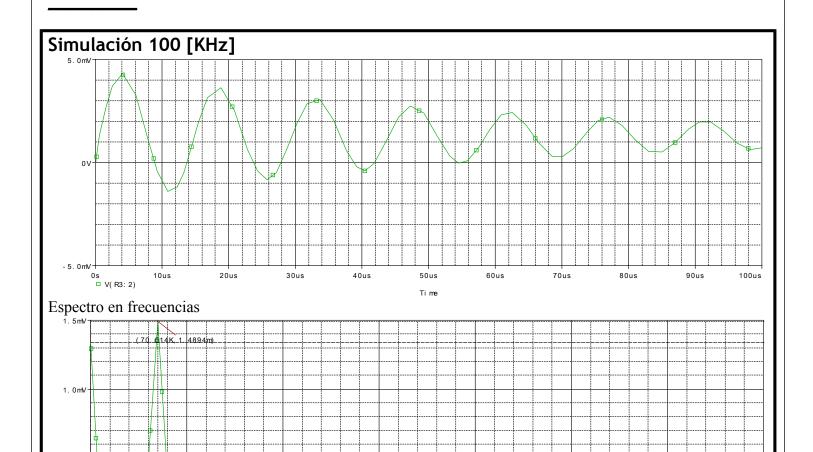
100KHz

200KHz

700KHz

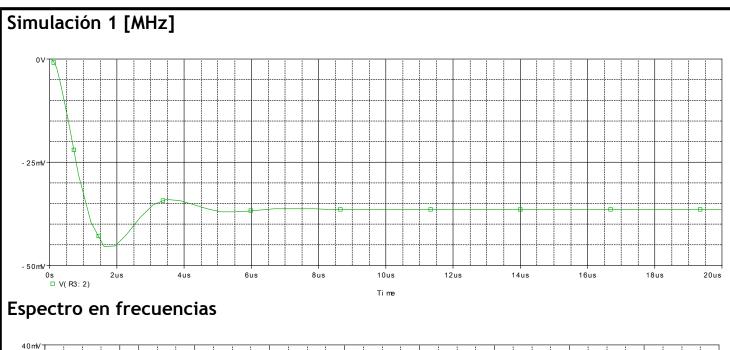
600KHz

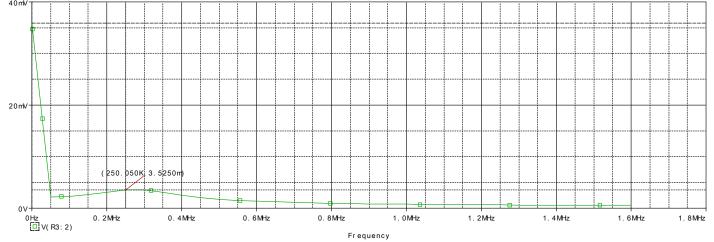
500KHz



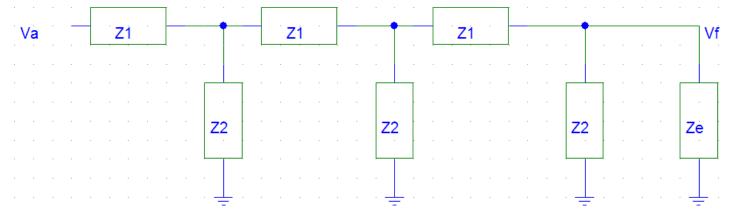
300KHz

Frequency





Resolver:



$$Z_{eq} = Z_2 \parallel Z_e = \frac{Z_e Z_2}{Z_e + Z_2}$$
 ----(1)

$$\frac{v_a - v_x}{Z_1} = \frac{v_x - 0}{Z_2} + \frac{v_x - v_y}{Z_1} - \dots (2)$$

$$\frac{v_x - v_y}{Z_1} = \frac{v_y - 0}{Z_2} + \frac{v_y - v_f}{Z_1} - \dots - (3)$$

De (4)

$$\frac{v_{y}}{Z_{1}} - \frac{v_{f}}{Z_{1}} = \frac{v_{f}}{Z_{eq}}$$

$$\frac{v_{y}}{Z_{1}} = \frac{v_{f}}{Z_{eq}} + \frac{v_{f}}{Z_{1}}$$

$$\frac{v_{y}}{Z_{1}} = v_{f} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}}\right)$$

$$v_{y} = v_{f} Z_{1} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}}\right)$$
(5) en (3)

$$\frac{v_x - v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right)}{Z_1} = \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right) - 0}{Z_2} + \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right) - v_f}{Z_2}$$

$$\frac{v_x - v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right)}{Z_1} = \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right)}{Z_2} + \frac{v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right) - v_f}{Z_1}$$

$$\frac{v_x}{Z_1} - v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right) = v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right) - \frac{v_f}{Z_1}$$

Despejando Vx

$$v_x = Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right] - \dots (6)$$

(6) en dos

$$\frac{v_a - v_x}{Z_1} = \frac{v_x - 0}{Z_2} + \frac{v_x - v_y}{Z_1}$$

$$v_y = v_f Z_1 \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right)$$

$$\frac{v_a}{Z_1} - \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} = \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_2} \right]$$

$$+\frac{Z_{1}\left[v_{f}\frac{Z_{1}}{Z_{2}}\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)+v_{f}\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)-\frac{v_{f}}{Z_{1}}+v_{f}\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)\right]}{Z_{1}}-\frac{v_{f}Z_{1}\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)}{Z_{1}}$$

$$\frac{v_a}{Z_1} - \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_1} = \frac{Z_1 \left[v_f \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{v_f}{Z_1} + v_f \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right]}{Z_2} \right]$$

$$+\frac{Z_{1}\left[v_{f}\frac{Z_{1}}{Z_{2}}\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)+v_{f}\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)-\frac{v_{f}}{Z_{1}}+v_{f}\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)\right]}{Z_{1}}-\frac{v_{f}Z_{1}\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)}{Z_{1}}$$

8-may-06 Titulo: Simulación

 $\frac{v_{a}}{Z_{1}} = \frac{Z_{1} \left[v_{f} \frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + v_{f} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{v_{f}}{Z_{1}} + v_{f} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right]}{Z_{2}} + \frac{Z_{1} \left[v_{f} \frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + v_{f} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{v_{f}}{Z_{1}} + v_{f} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right]}{Z_{1}} - \frac{v_{f}}{Z_{1}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \frac{Z_{1} \left[v_{f} \frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + v_{f} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{v_{f}}{Z_{1}} + v_{f} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right]}{Z_{1}} + \frac{Z_{1} \left[v_{f} \frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + v_{f} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{v_{f}}{Z_{1}} + v_{f} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right]}{Z_{1}} \right\}$

factorizando vf

$$\frac{y_{g}}{Z_{1}} = v_{f} \cdot \left\{ \begin{aligned} & Z_{1} \left[\frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right] \\ & Z_{2} \end{aligned} \right. \\ & + \frac{Z_{1} \left[\frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right] \\ & - \frac{Z_{1} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right)}{Z_{1}} + \frac{Z_{1} \left[\frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right] \\ & - \frac{Z_{1} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right)}{Z_{1}} + \frac{Z_{1} \left[\frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right] \\ & - \frac{Z_{1} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right)}{Z_{1}} + \frac{Z_{1} \left[\frac{Z_{1}}{Z_{eq}} + \frac{1}{Z_{1}} \right] + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right] \\ & - \frac{Z_{1} \left[\frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right] \\ & - \frac{Z_{1} \left[\frac{Z_{1}}{Z_{eq}} + \frac{1}{Z_{1}} \right] + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right] \\ & - \frac{Z_{1} \left[\frac{Z_{1}}{Z_{eq}} + \frac{1}{Z_{1}} \right] + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right] \\ & - \frac{Z_{1} \left[\frac{Z_{1}}{Z_{eq}} + \frac{1}{Z_{1}} \right] + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right] \\ & - \frac{1}{Z_{1}} \left[\frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right] + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \\ & - \frac{1}{Z_{1}} \left[\frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} \right] \\ & - \frac{1}{Z_{eq}} \left[\frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} \right] \\ & - \frac{1}{Z_{eq}} \left[\frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} \right] \\ & - \frac{1}{Z_{eq}} \left[\frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} + \frac{1}{Z_{eq}} + \frac{1}{Z_{eq}}$$

$$\frac{v_{g}}{Z_{1}} = V_{g}$$

$$Z_{1} \left[\frac{Z_{1} \left[\frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right]}{Z_{2}} \right]$$

$$Z_{1} \left[\frac{Z_{1} \left[\frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right]}{Z_{1}} - \frac{Z_{1} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right)}{Z_{1}} + \frac{Z_{1} \left[\frac{Z_{1}}{Z_{2}} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) - \frac{1}{Z_{1}} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}} \right) \right]}{Z_{1}} \right]$$

$$\frac{1}{Z_1 \left\{ \frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right\} }{Z_2} + \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right] - \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left[\frac{Z_1}{Z_2} \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \frac{1}{Z_1} + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right] \right\}$$

Simplificando

$$\frac{1}{Z_{1}\left[\frac{Z_{1}}{Z_{2}}\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)+\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)-\frac{1}{Z_{1}}+\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)\right]\left\{\frac{Z_{1}}{Z_{2}}+1-\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)+1\right\}}=\frac{v_{f}}{v_{a}}$$

$$\frac{1}{Z_{1}\left[\frac{Z_{1}}{Z_{2}}\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)+\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)-\frac{1}{Z_{1}}+\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)\right]\left\{\frac{Z_{1}}{Z_{2}}+1-\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)+1\right\}}=\frac{v_{f}}{v_{a}}$$

$$\frac{1}{\left[\frac{Z_{1}}{Z_{2}}\left(\frac{Z_{1}}{Z_{eq}} + \frac{Z_{1}}{Z_{1}}\right) + \left(\frac{Z_{1}}{Z_{eq}} + \frac{Z_{1}}{Z_{1}}\right) - \frac{Z_{1}}{Z_{1}} + \left(\frac{Z_{1}}{Z_{eq}} + \frac{Z_{1}}{Z_{1}}\right)\right]\left\{\frac{Z_{1}}{Z_{2}} + 1 - \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{1}}\right) + 1\right\}} = \frac{v_{f}}{v_{a}}$$

8-may-06 Titulo: Simulación

$$\frac{1}{\left[\left(\frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_1}\right)\left\{\frac{Z_1}{Z_2} + 1 + 1\right\} - \frac{Z_1}{Z_1}\right]\left\{\frac{Z_1}{Z_2} + 1 - \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right) + 1\right\}} = \frac{v_f}{v_a}$$

$$\frac{1}{\left[\left(\frac{Z_{1}}{Z_{eq}}+1\right)\left[\frac{Z_{1}}{Z_{2}}+2\right]-1\right]\left\{\left[\frac{Z_{1}}{Z_{2}}+2\right]-\left(\frac{1}{Z_{eq}}+\frac{1}{Z_{1}}\right)\right\}} = \frac{v_{f}}{v_{a}}$$

$$\frac{1}{\left\{ \left(\frac{Z_1}{Z_{eq}} + 1 \right) \left[\frac{Z_1}{Z_2} + 2 \right] \left[\frac{Z_1}{Z_2} + 2 \right] - \left(\frac{Z_1}{Z_{eq}} + 1 \right) \left[\frac{Z_1}{Z_2} + 2 \right] \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \left(\frac{Z_1}{Z_{eq}} + 1 \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right\}} = \frac{v_f}{v_a}$$

$$\frac{1}{\left\{ \left(\frac{Z_1}{Z_{eq}} + 1 \right) \left[\frac{Z_1}{Z_2} + 2 \right] \left[\frac{Z_1}{Z_2} + 2 \right] - \left(\frac{Z_1}{Z_{eq}} + 1 \right) \left[\frac{Z_1}{Z_2} + 2 \right] \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) - \left(\frac{Z_1}{Z_{eq}} + 1 \right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1} \right) \right\}} = \frac{v_f}{v_a}$$

$$\frac{1}{\left(\frac{Z_1}{Z_{eq}}\frac{Z_1}{Z_2} + 2\frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_2} + 2\right)\left[\frac{Z_1}{Z_2} + 2\right] - \left(\frac{Z_1}{Z_{eq}}\frac{Z_1}{Z_2} + \frac{Z_1}{Z_2} + 2\right)\left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right) - \left(\frac{Z_1}{Z_{eq}} + 1\right) + \left(\frac{1}{Z_{eq}} + \frac{1}{Z_1}\right)} = \frac{v_f}{v_a}$$

$$\frac{Z_{1}}{Z_{eq}} \frac{Z_{1}}{Z_{2}} \frac{Z_{1}}{Z_{2}} + 2 \frac{Z_{1}}{Z_{eq}} \frac{Z_{1}}{Z_{2}} + 2 \frac{Z_{1}}{Z_{eq}} \frac{Z_{1}}{Z_{2}} + 4 \frac{Z_{1}}{Z_{eq}} + \frac{Z_{1}}{Z_{2}} \frac{Z_{1}}{Z_{2}} + 2 \frac{Z_{1}}{Z_{2}} + 2 \frac{Z_{1}}{Z_{2}} + 4 - \left(\frac{Z_{1}}{Z_{eq}} \frac{Z_{1}}{Z_{2}} \frac{1}{Z_{eq}} + \frac{Z_{1}}{Z_{2}} \frac{1}{Z_{1}} + \frac{Z_{1}}{Z_{2}} \frac{1}{Z_{2}} \frac{1}{Z_{2}} + \frac{Z_{1}}{Z_{2}} \frac{1}{Z_{2}} \frac{1}{Z$$

$$\frac{\overline{Z_{1}}}{\overline{Z_{eq}}} \frac{Z_{1}}{Z_{2}} \frac{Z_{1}}{Z_{2}} + 2 \frac{Z_{1}}{\overline{Z_{eq}}} \frac{Z_{1}}{Z_{2}} + 2 \frac{Z_{1}}{\overline{Z_{eq}}} \frac{Z_{1}}{Z_{2}} + 4 \frac{Z_{1}}{\overline{Z_{eq}}} + \frac{Z_{1}}{\overline{Z_{2}}} \frac{Z_{1}}{Z_{2}} + 2 \frac{Z_{1}}{\overline{Z_{2}}} + 2 \frac{Z_{1}}{\overline{Z_{2}}} \frac{Z_{1}}{\overline{Z_{2}$$

$$\frac{1}{\frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} \frac{Z_1}{Z_2} + 4 \frac{Z_1}{Z_{eq}} \frac{Z_1}{Z_2} + 3 \frac{Z_1}{Z_{eq}} + \frac{Z_1}{Z_2} \frac{Z_1}{Z_2} + 4 \frac{Z_1}{Z_2} + 4 \frac{Z_1}{Z_2} \frac{Z_1}{Z_2} \frac{1}{Z_2} \frac{1}{Z_2} \frac{1}{Z_{eq}} - \frac{Z_1}{Z_1} \frac{1}{Z_1} \frac{1}{Z_1} - \frac{Z_1}{Z_2} \frac{1}{Z_1} - \frac{Z_1}{Z_2} \frac{1}{Z_1} - 1} = \frac{v_f}{v_a}$$