01Simulating risk data

December 19, 2019

1 Simulating risk data

Author: Miao Cai miao.cai@slu.edu

1.1 Data generating process

The data generating process of real-life transportation risk is very complex. As an illustrating example, here we assume that the risk was generated from a Poisson distribution with the following data generating process:

$$
\begin{aligned}
Y_i &\sim \text{Poisson}(d_i \cdot \lambda_i) && (1)\\
\log(\lambda_i) &= \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i && (2)\\
\epsilon_i &\sim \text{Normal}(0, 2^2), && (3)
\end{aligned}
$$

where d_i is the distance traveled in the i-th trip, x_1 is precipitation, and x_2 is road traffic.

We assume the sample size $N = 10,000$ and the parameters and data has following values or distributions:

- $\beta_0 = -3$,
- $\beta_1 = 0.5$,
- $\beta_2 = 0.9$,
- $d \sim \text{Poisson}(1000)$
- $x_1 \sim \text{Bernoulli}(0.15)$,
- $x_2 \sim \text{Beta}(2, 2)$

1.2 Simulating data

1.2.1 Import packages and print package version

[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import beta
import sys

print("Python version: " + sys.version)
print("pandas version: " + pd.__version__)
print("numpy version: " + np.__version__)
```

```
Python version: 3.7.5 (default, Oct 31 2019, 15:18:51) [MSC v.1916 64 bit
(AMD64)]
pandas version: 0.25.3
numpy version: 1.17.4
```

1.2.2 set parameters and generate data

[2]:

```
def simulate_distance(N_size):
    return(np.random.poisson(lam = 1000, size = N_size))
def simulate_precipitation(N_size):
    return(np.random.binomial(n = 1, p = 0.15, size = N_size))
def simulate_traffic(N_size):
    return(np.random.beta(a = 2, b = 2, size = N_size))
```

[3]:

```
np.random.seed(123) # set random seed
N = 10**4
b0, b1, b2 = -10, 0.5, 0.9

d = simulate_distance(N)
x1 = simulate_precipitation(N)
x2 = simulate_traffic(N)
epsilon = np.random.normal(loc = 0, scale = 1, size = N)

lambda_i = np.exp(b0 + b1*x1 + b2*x2 + epsilon)
y = np.random.poisson(d*lambda_i)
```

1.2.3 Plot distributions of simulated data

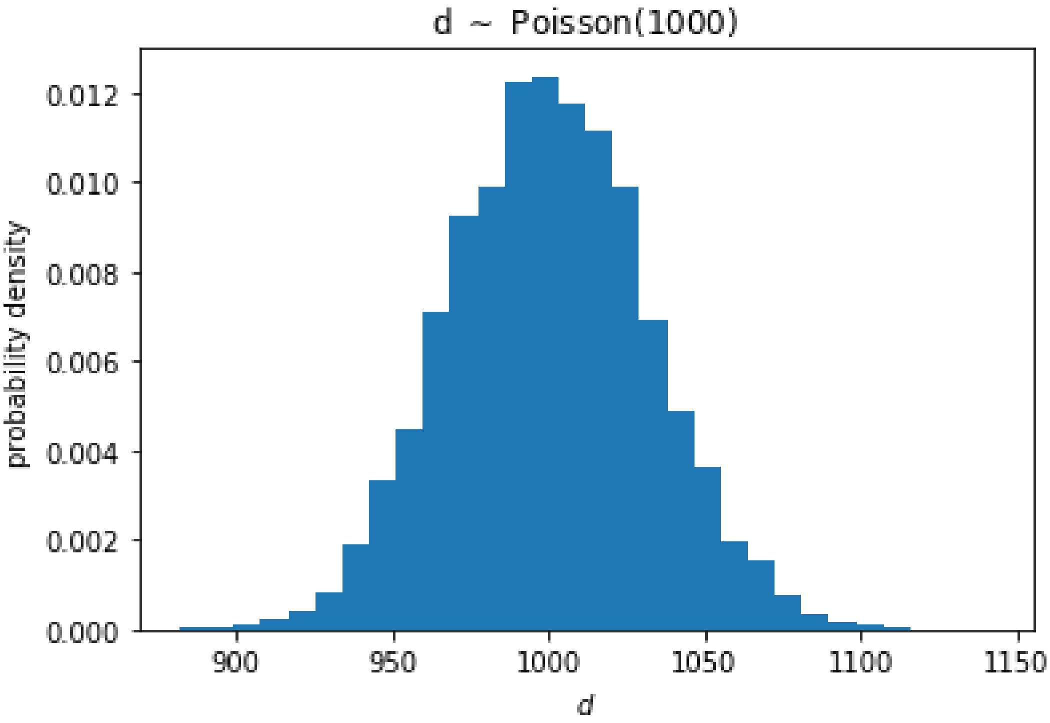
[4]:

```
# The distribution of y
from collections import Counter
print("The maximum potential crash in a trip is " + str(max(y)))
print("The distribution of y is: \n" + str(Counter(y)))
```

```
The maximum potential crash in a trip is 5
The distribution of y is:
Counter({0: 8910, 1: 937, 2: 117, 3: 24, 4: 6, 5: 6})
```

[5]:

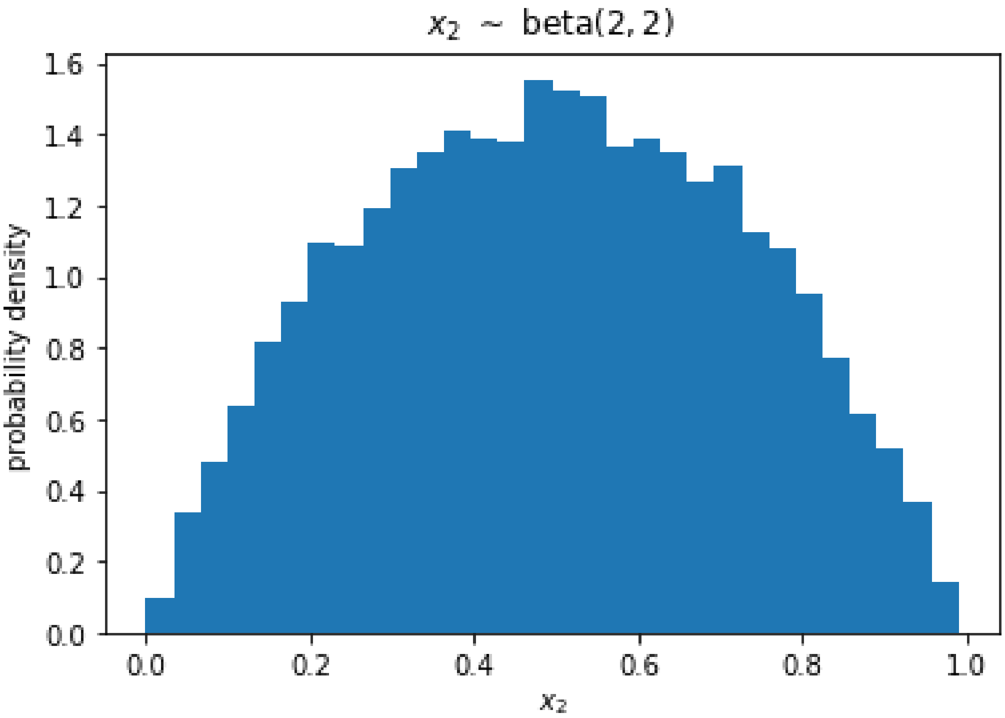
```
# The distribution of d
from scipy.special import factorial
count, bins, ignored = plt.hist(d, 30, density=True)
plt.title("d $\sim$ Poisson$(1000)$")
plt.xlabel('$d$')
plt.ylabel('probability density')
plt.show()
```



```
[6]: # The distribution of x1
     Counter(x1)
```

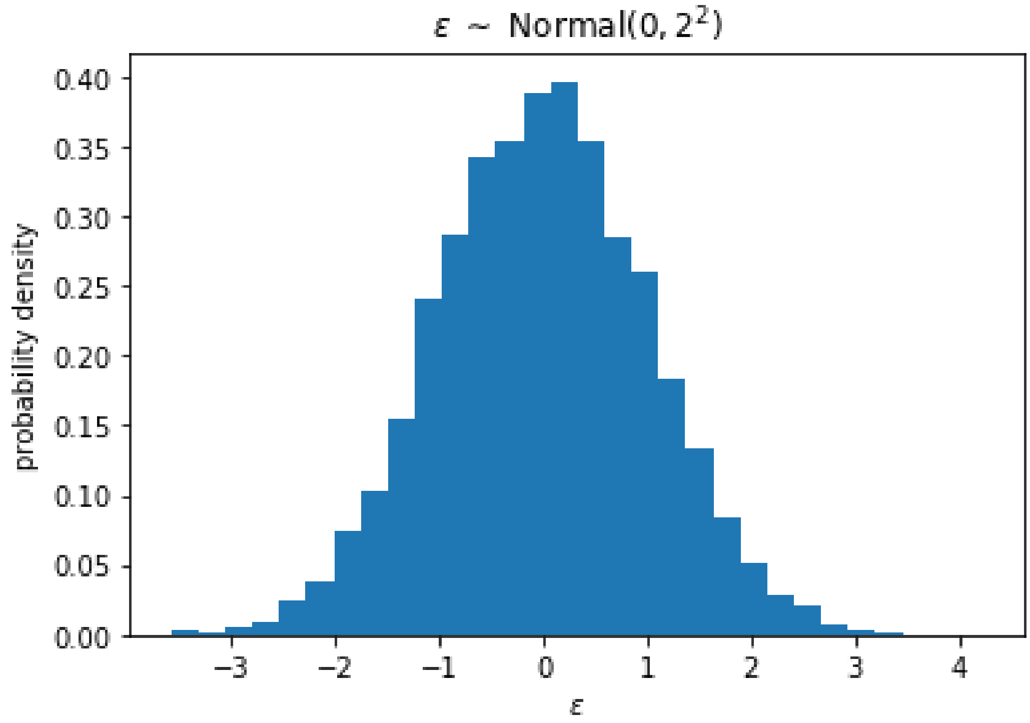
```
[6]: Counter({0: 8508, 1: 1492})
```

```
[7]: # The distribution of x2
     count, bins, ignored = plt.hist(x2, 30, density=True)
     plt.title("$x_2$ $\sim$ beta$(2,2)$")
     plt.xlabel('$x_2$')
     plt.ylabel('probability density')
     plt.show()
```



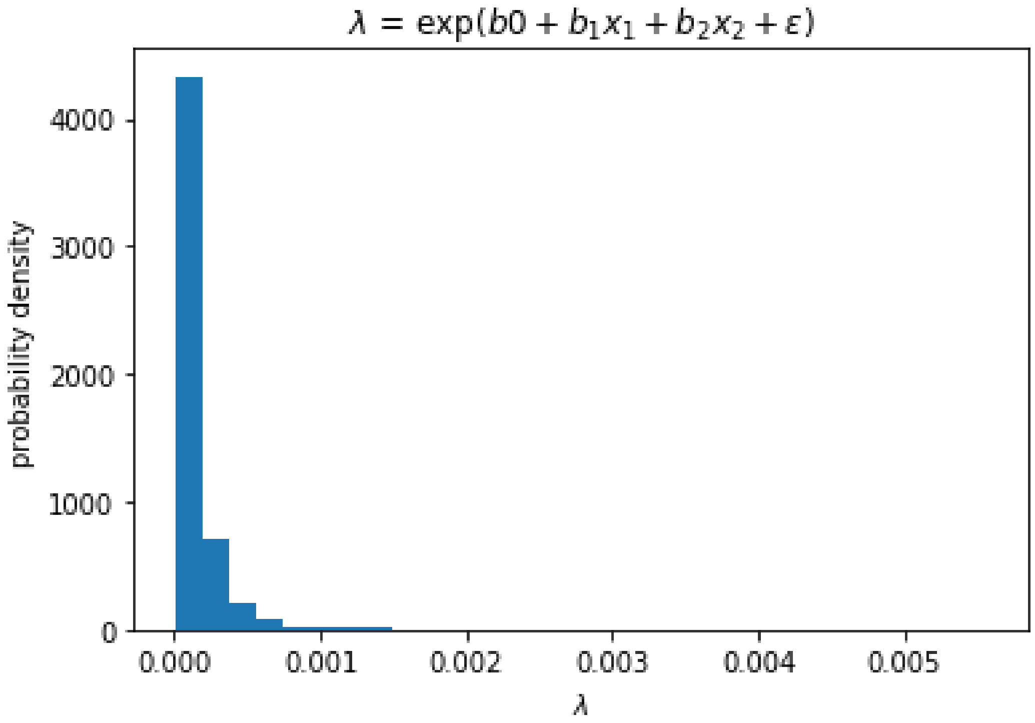
[8]:

```
# The distribution of $\epsilon$
count, bins, ignored = plt.hist(epsilon, 30, density=True)
plt.title("$\epsilon$ $\sim$ Normal$(0,2^2)$")
plt.xlabel('$\epsilon$')
plt.ylabel('probability density')
plt.show()
```



[9]:

```
# The distribution of $\lambda$
count, bins, ignored = plt.hist(lambda_i, 30, density=True)
plt.title("$\lambda$ = exp($b0 + b_1x_1 + b_2x_2 + \epsilon$)")
plt.xlabel('$\lambda$')
plt.ylabel('probability density')
plt.show()
```



1.3 Create and save dataframe

```
[10]: df = pd.DataFrame({
          'y': y,
          'Distance': d,
          'Precipitation': x1,
          'Traffic': x2
      })
      df.head(10)
```

```
[10]:    y  Distance  Precipitation   Traffic
      0  0      1018              0  0.299886
      1  0       973              0  0.565617
      2  0      1021              0  0.414564
      3  0       998              0  0.559767
      4  0       985              0  0.777217
      5  0       994              0  0.716722
      6  0       991              0  0.782984
      7  1       967              0  0.587694
      8  0      1003              0  0.595906
      9  0      1012              0  0.742293
```

```
[11]: df.to_csv("data/simulated_data.csv", sep=',', encoding='utf-8')
```

1.4 Simulate precipitation and traffic variables for links data

```
[12]: links = pd.read_csv('data/links.csv')
      links.head(20)
```

```
[12]:       Node A   Node Z  Distance
      0    node 1   node 7       800
      1    node 1  node 11       800
      2    node 1  node 12      2400
      3    node 2   node 6      1200
      4    node 2  node 10       900
      5    node 3   node 6      1100
      6    node 3   node 8       800
      7    node 3  node 12       600
      8    node 4   node 8       700
      9    node 4  node 10       700
      10   node 4  node 14      2800
      11   node 5   node 6      2000
      12   node 5   node 7       500
      13   node 5  node 11       300
      14   node 6  node 13      2000
      15   node 7  node 10       500
      16   node 9  node 12      1000
      17   node 9  node 13       600
      18   node 9  node 14      1100
      19  node 10  node 11       500
```

```
[13]: np.random.seed(0)
      links['Precipitation'] = simulate_precipitation(links.shape[0])
      links['Traffic'] = simulate_traffic(links.shape[0])
      links.head(20)
```

```
[13]:       Node A   Node Z  Distance  Precipitation   Traffic
      0    node 1   node 7       800              0  0.254345
      1    node 1  node 11       800              0  0.243435
      2    node 1  node 12      2400              0  0.254188
      3    node 2   node 6      1200              0  0.424037
      4    node 2  node 10       900              0  0.573477
      5    node 3   node 6      1100              0  0.834188
      6    node 3   node 8       800              0  0.190679
      7    node 3  node 12       600              1  0.689211
      8    node 4   node 8       700              1  0.507221
      9    node 4  node 10       700              0  0.129693
      10   node 4  node 14      2800              0  0.192684
      11   node 5   node 6      2000              0  0.334005
      12   node 5   node 7       500              0  0.568231
      13   node 5  node 11       300              1  0.499119
```

```
14   node 6  node 13      2000              0  0.441698
15   node 7  node 10       500              0  0.167268
16   node 9  node 12      1000              0  0.776060
17   node 9  node 13       600              0  0.514337
18   node 9  node 14      1100              0  0.122787
19  node 10  node 11       500              1  0.741051
```

[14]:

```python
links.to_csv("data/links_traffic_precipitation.csv", sep=',', encoding='utf-8')
```