



The statistical analysis of highway crash-injury severities: A review and assessment of methodological alternatives

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ARTICLE INFO

Article history:

Received 27 October 2010

Received in revised form 24 March 2011

Accepted 27 March 2011

Keywords:

Injury severity

Crash severity

Discrete outcome

Logit

Probit

ABSTRACT

Reducing the severity of injuries resulting from motor-vehicle crashes has long been a primary emphasis of highway agencies and motor-vehicle manufacturers. While progress can be simply measured by the reduction in injury levels over time, insights into the effectiveness of injury-reduction technologies, policies, and regulations require a more detailed empirical assessment of the complex interactions that vehicle, roadway, and human factors have on resulting crash-injury severities. Over the years, researchers have used a wide range of methodological tools to assess the impact of such factors on disaggregate-level injury-severity data, and recent methodological advances have enabled the development of sophisticated models capable of more precisely determining the influence of these factors. This paper summarizes the evolution of research and current thinking as it relates to the statistical analysis of motor-vehicle injury severities, and provides a discussion of future methodological directions.

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1. Introduction

Due to the incredible economic and emotional burden traffic crashes impose on society, vehicle manufacturers and transportation agencies are under constant pressure to implement improvements in vehicle and roadway design to reduce the frequency of traffic crashes and the degree of injury sustained by those involved in crashes. It is important to note that reducing crash frequency and reducing crash-injury severity may necessitate different strategic approaches. For example, limiting rates of curvature or superelevation are policies aimed at reducing the potential for crashes while crashworthy median barriers and roadway signs are designed to reduce the level of injury sustained in the event such objects are struck during a crash. In addition to roadway features, advances in vehicle design also have the potential to reduce crash frequency (electronic stability control and anti-lock brakes) or severity (safety belts or airbags) while driver-training programs and targeted enforcement may have potential in reducing both frequency and severity.

The development of effective countermeasures requires a thorough understanding of the factors that affect the likelihood of a

crash occurring or, given that a crash has occurred, the characteristics that may mitigate or exacerbate the degree of injury sustained by crash-involved road users. To gain such an understanding, safety researchers have applied a wide variety of methodological techniques over the years. While these various methodological applications have undoubtedly provided new insights, the fundamental characteristics of crash data often result in methodological limitations that are not fully understood or accounted for.

Lord and Mannering (2010) recently provided an assessment of the characteristics of crash-frequency data and the methodological alternatives and limitations for examining such data. The intent of this paper is to provide a similar assessment of data characteristics and methodological alternatives and limitations for examining crash-severity data, highlighting the strengths and weaknesses of each approach and identifying areas of opportunity for future research.¹

¹ A number of recent research studies have sought to jointly model the frequency and injury-severity of crashes (see for example Ma and Kockelman, 2006; Park and Lord, 2007; Ma et al., 2008; El-Basyouny and Sayed, 2009; Agüero and Jovanis, 2010). Although these joint models have definite appeal and future potential, it is important to note that the type of data that they can use is limited. This is because models that simultaneously consider frequency and severity can only use non-crash-specific data (roadway geometry, traffic volumes, etc.). In contrast, traditional injury-severity models, which consider individual crash severities after a crash has occurred, can use detailed post-crash data (number of vehicles involved, age of occupants, weather conditions at the time of the crash, types of vehicles

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2. Characteristics of crash-injury severity data and methodological issues

Injury-severity data are generally represented by discrete categories such as fatal injury or killed, incapacitating injury, non-incapacitating, possible injury, and property damage only.² Beyond the discrete nature of these data, past research has identified a number of data characteristics and methodological issues that are critical considerations in the development and application of an appropriate statistical methodology to study such data. A summary of these characteristics is presented in this section of the paper.

2.1. Underreporting of crashes

Statistical models are generally developed under the assumption that the sample data are randomly selected from a population and that each crash (or crash-involved individual) has an equal probability of being sampled. However, in traditional crash databases, not all crashes are reported because, for example, some states only report those crashes which result in property damage above a specific threshold dollar amount while others require the degree of vehicle damage to be above a certain level (Hauer, 2006). In addition, most crash databases are drawn from police-reported data – and it is well-known that individuals involved in no injury or minor injury crashes are far less likely to have their crashes reported to police (in an effort to avoid the involvement of insurance companies and the possible issuance of traffic citations).³ As evidence of this, Hauer and Hakkert (1989) conducted a meta-analysis of studies comparing police-, hospital-, and insurance-reported injury data and concluded that approximately 20 percent of severe injuries, 50 percent of minor injuries, and up to 60 percent of no-injury crashes are not reported. A subsequent meta-analysis by Elvik and Myssen (1999) found underreporting rates of 30, 75, and 90 percent for serious, slight, and very slight injuries, respectively. A technical report by the National Highway Traffic Safety Administration (2009) estimated that 25 percent of minor injury crashes and half of no-injury crashes are unreported; a sharp contrast to fatal crashes for which the reporting rate is nearly 100 percent (Blincoe et al., 2002).

Given these issues, rather than sampling randomly from the population, crash data are more appropriately described as an

involved, crash type, occupant restraint use, airbag deployment, etc.). A recent study by Anastasopoulos and Mannering (2011) explored the potential loss in model accuracy when using non-crash-specific data as opposed to detailed post-crash data. They found that the loss in forecasting accuracy may be modest, but that there are larger losses in terms of the statistical inferences that can be made with regard to the underlying injury-severity generating process. In other work, Wang et al. (2011) estimated a two-stage frequency-severity model that links the two processes while avoiding the data restrictions imposed by the traditional joint frequency-severity approach.

² This is often referred to as the KABCO scale: fatal injury or killed (K), incapacitating injury (A), non-incapacitating (B), possible injury (C), and property damage only (O). Other types of injury severity data may include detailed information on trauma location and extent of injury which may be a combination of discrete and continuous variables (see, the Abbreviated Injury Scale (AIS) which was originally developed by the American Association for Automotive Medicine, the Organ Injury Scales (OIS) proposed by the American Association for the Surgery of Trauma and the Injury Severity Score (ISS) used by hospitals). The discussion in this paper is limited to the more traditional discrete data forms that are found, for example, in police-reported data.

³ There is also the issue of the accuracy of the data. For example, the distinction between a no-injury crash and a possible-injury crash may vary from one police officer to the next. Hausman et al. (1998) have developed correction procedures for the analysis of misclassified discrete data. This procedure was applied in the crash-injury severity context by Winston et al. (2006). They found that misclassification was not a significant factor in their case, but they used only the two very broad categories of no-injury or injury crashes (thus reducing the likelihood of misclassification that might otherwise be present when more injury outcomes are considered).

outcome-based sample (where the proportion of injury severities in the police-reported sample is not the same as the proportion of injuries among all crashes).⁴ It is well-known that such outcome-based samples result in biased parameter estimates when traditional statistical estimation techniques are used and, in the case of crash injuries, these biases may lead to marginal impacts that are overestimated for key variables such as safety belt use (Yamamoto et al., 2008).⁵

2.2. Ordinal nature of crash and injury severity data

Injury-severity levels are ordinal by nature. For example, the severities become increasingly severe from no injury, to possible injury, to evident injury, to disabling injury, to fatality. Recognizing the ordinal nature of these data is an important consideration in selecting an appropriate methodological approach. In addition, because injury-severity categories are ordered and in sometimes closely related categories (for example no injury and possible injury), there may be shared unobserved effects among adjacent injury categories. This can be problematic for certain types of model estimation methods where failing to account for such correlation can result in biased parameter estimates and incorrect inferences (see Savolainen and Mannering, 2007; Paleti et al., 2010).

2.3. Fixed parameters

The majority of the existing research literature related to injury-severity models includes models comprised of fixed parameters, which restrict the effects of explanatory variables to be the same across individual injury observations. However, unobserved heterogeneity is likely to exist among the population of crash-involved road users (due to differences in risk-taking behavior, physiological factors, etc.), suggesting that these parameters may vary across injury observations. Such effects, if not accounted for, may result in potential bias and erroneous statistical inferences (McFadden and Train, 2000; Train, 2009).

2.4. Omitted variable bias

Often times, the data used to study injury severity are limited due to the amount of information available from crash reports or other factors. Omitting relevant explanatory variables can result in inconsistent parameter estimates if such variables are correlated with other variables that are already included in the model or if the omitted variable is correlated or has different variances among severity levels (Washington et al., 2011). The omission of relevant variables is often a necessary limitation when drawing injury-severity insights to guide safety decisions.

2.5. Small sample size

The volume of crash data that are available is often an important factor in the selection of an appropriate modeling framework. In cases where data are limited, simpler models, such as those which assume fixed parameters, require less data in general to arrive at reasonable estimation results and may be preferred. The size of the sample can guide the methodological-selection process in that small samples may necessitate more simplistic models.⁶

⁴ Outcome-based sampling is commonly referred to as choice-based sampling in the discrete-choice literature (see Ben-Akiva and Lerman, 1985).

⁵ Recent numerical experiments by Ye and Lord (forthcoming) have explored the effects of the underreporting of crashes on various model structures.

⁶ Ye and Lord (2011) provide some guidance on the effects of sample size on parameter estimates for various methodological approaches (ordered probit, multi-

2.6. Endogeneity

Estimation problems can arise when explanatory variables are potentially influenced by injury-severity outcomes. An obvious example would be using an explanatory variable such as a head wound indicator (1 if a head wound was observed and 0 otherwise) to predict injury severity. Such a variable could only be present for injury, disabling injury and fatality, so the no-injury outcome will never exhibit a value of 1 for this variable (here a change in the dependent “left-hand” variable affects an explanatory “right-hand” variable, which is a violation of the assumptions made to derive most statistical models).

However, a more general definition of the problem is one where explanatory variables are correlated with disturbance terms (unobserved heterogeneity), making endogeneity problems far more subtle and difficult to determine. To see this, consider a model that uses the presence of an airbag as an explanatory variable in a model of injury-outcomes. Drivers owning vehicles with airbags may also tend to be more risk averse.⁷ Because the aversion to risk can be difficult to capture with explanatory variables typically available in databases, such effects are generally captured as a part of the disturbance term. If the presence of airbags is included as an explanatory variable in an injury-severity model, its estimated parameter may not only capture the effect of the airbag, but also the fact that risk-averse drivers are more likely to have an airbag in their vehicle. Thus, the effectiveness of the airbag will be overstated because the estimated parameter is capturing these two factors instead of just the airbag’s effectiveness.⁸

2.7. Within-crash correlation

Many crash-severity studies classify the injury level of the crash by the injury level of the most severely injured individual. However, if one seeks to model the injury levels of drivers involved in multivehicle crashes, or of all of the crash-involved individuals, the likelihood of correlation among these injuries must be considered. This is because unobserved elements relating to a specific crash (impact characteristics, etc.) will set up a correlation among crash-injury observations.⁹ Statistical methods that do not account for the correlation among injuries occurring in the same crash are likely to result in biased parameter estimates (Helai et al., 2008).

2.8. Spatial and temporal correlations

Crashes that occur in close proximity in space (for example at the same intersection of roadway segment) or in time (same

day or week) are likely to share unobserved effects (factors that are not taken into account with available measurable explanatory variables). This can significantly complicate the model-estimation structure making estimation of model parameters more difficult. If such correlations are ignored, there will be a loss of efficiency and parameters will be estimated with less precision, thus making statistically defensible inferences more difficult (see Anselin et al., 2005).

3. Methodological approaches

A variety of methodological techniques have been applied to analyze crash-severity data. The statistical methods employed by researchers have primarily relied on the nature of the dependent variable and various methodological issues associated with the data, as discussed previously. The dependent variables of existing crash severity models are typically either a binary response outcome (e.g., injury or non-injury) or a multiple response outcome (e.g., fatality, disabling injury, evident injury, possible injury, or no injury). Dependent variables with multiple response outcomes have been treated as both ordinal (accounting for the ordinal nature of injury data) and nominal (i.e., unordered).

Some researchers have investigated the severity of crashes by considering the injury-severity level of the driver, while others have considered the injury-severity of the most severely injured person, or the injury-severity level of crash-involved individuals (which results in multiple observations per crash). If the injury-severity level of crash-involved individuals is to be considered, one needs to account for within crash correlation among observations which can complicate the model structure significantly.¹⁰

Over the years, a wide variety of statistical techniques have been used to study crash-injury severities.¹¹ A summary listing of these techniques is presented in Table 1 and a discussion of alternate methods is presented below.¹²

3.1. Binary outcome models

Studies that have looked at binary injury-severity outcomes such as injury vs. non-injury crashes or fatal vs. non-fatal crashes have used common discrete-outcome models such as the binary logit and binary probit models. However, because of the characteristics of crash-injury data and the variety of methodological issues associated with these characteristics, a number of variations of the simple binary logit and probit have been employed as discussed below.

nominal logit, and mixed logit models) using a Monte-Carlo approach with both observed and simulated crash data.

⁷ This refers to risk homeostasis (or risk compensation) which states that drivers tend to maintain the same level of risk by adjusting their behavior when the perception of risk changes (see Wilde, 2001; Winston et al., 2006). This theory has been criticized by some (O'Neill and Williams, 1998; Levy and Miller, 2000) and empirically supported by others (Winston et al., 2006).

⁸ Addressing this type of endogeneity problem involves far more complex model structures. Winston et al. (2006) used such a complex structure to study the effectiveness of airbags and anti-lock brakes while accounting for self-selectivity (the fact that purchasers of safety technologies may not be a random sample of the driving population). They found that drivers that first purchased vehicles with airbags and antilock brakes tended to be the safest drivers, noting that if the self-selective tendency of safe drivers to own safety features was not accounted for the analysis of data, the effectiveness of the safety features could be seriously over estimated.

⁹ In multi-vehicle crashes, the speeds of the involved vehicles (which are not likely to be known) are one possible source of the correlation among crash-injury observations. Handling this correlation can be done by considering joint injury severities, or by developing more complex model structures to handle the correlation while considering individual injury severities (for example, the copula-based approach adopted by Eluru et al., 2010).

¹⁰ It should be noted that some U.S. states report the injury severity for only injured passengers. As such, data on uninjured occupants may be underrepresented and lead to biased parameter estimates and misleading inferences. This issue is mitigated when considering only driver injuries or the most severe injury sustained in a particular crash or crash-involved vehicle.

¹¹ Within the area of crash-severity research, discrete outcome models have become the prevailing analytical technique. Such models allow for an examination of crash-severity outcomes while treating injury level as either an ordinal or a nominal variable. Under either modeling framework, tractable models can be estimated using standard maximum likelihood estimation techniques. Recent advances in computational efficiency, as well as in the development and application of simulation methods that can be used to approximate integration, have allowed for the development of more flexible models that allow for new insights into those factors affecting severity outcomes. There are currently a variety of both open source and commercially available software that can be used to employ the statistical analysis techniques described in this paper.

¹² There is a strong methodological connection between crash injury-severity analysis and traditional discrete-choice modeling, with many of the approaches applied in injury-severity modeling having their roots in discrete-choice analysis. Sources such as Ben-Akiva and Lerman (1985), Hensher et al. (2005), and Train (2009) provide a good description of many of these core discrete-choice methods.

Table 1

Summary of previous research analyzing crash-injury severities.

Methodological approach	Previous research
Artificial neural networks	Delen et al. (2006), Chimba and Sando (2009)
Bayesian hierarchical binomial logit	Helai et al. (2008)
Bayesian ordered probit	Xie et al. (2009)
Binary logit and binary probit	Shibata and Fukuda (1994), Farmer et al. (1997), Khattak et al. (1998), Krull et al. (2000), Zhang et al. (2000), Al-Ghamdi (2002), Bedard et al. (2002), Toy and Hammitt (2003), Ballasteros et al. (2004), Chang and Yeh (2006), Sze and Wong (2007), Lee and Abdel-Aty (2008), Pai (2008), Rifaat and Tay (2009), Haleem and Abdel-Aty (2010), Peek-Asa et al. (2010), Kononen et al. (2011), Moudon et al. (2011)
Bivariate binary probit	Lee and Abdel-Aty (2008)
Bivariate ordered probit	Yamamoto and Shankar (2004), de Lapparent (2008)
Classification and regression tree	Chang and Wang (2006)
Generalized ordered logit	Quddus et al. (forthcoming)
Heterogeneous outcome model	Quddus et al. (forthcoming)
Heteroskedastic ordered logit/probit	O'Donnell and Connor (1996), Wang and Kockelman (2005), Lemp et al. (2011)
Log-linear model	Chen and Jovanis (2000)
Markov switching multinomial logit	Malyshkina and Mannering (2009)
Mixed generalized ordered logit	Eluru et al. (2008)
Mixed joint binary logit-ordered logit	Eluru and Bhat (2007)
Multinomial logit	Shankar and Mannering (1996), Carson and Mannering (2001), Abdel-Aty and Abdelwahab (2004), Ulfarsson and Mannering (2004), Khorashadi et al. (2005), Islam and Mannering (2006), Kim et al. (2007), Malyshkina and Mannering (2008), Savolainen and Ghosh (2008), Schneider et al. (2009), Malyshkina and Mannering (2010), Rifaat et al. (2011), Schneider and Savolainen (forthcoming), Ye and Lord (forthcoming, 2011)
Multivariate probit	Winston et al. (2006)
Nested logit	Shankar et al. (1996), Chang and Mannering (1998), Chang and Mannering (1999), Lee and Mannering (2002), Abdel-Aty and Abdelwahab (2004), Holdridge et al. (2005), Savolainen and Mannering (2007), Haleem and Abdel-Aty (2010), Hu and Donnell (2010)
Ordered logit and ordered probit	Khattak et al. (1998), Klop and Khattak (1999), Renski et al. (1999), Khattak (2001), Khattak et al. (2002), Kockelman and Kweon (2002), Quddus et al. (2002), Abdel-Aty (2003), Austin and Faigin (2003), Kweon and Kockelman (2003), Zajac and Ivan (2003), Khattak and Rocha (2003), Donnell and Mason (2004), Khattak and Targa (2004), Abdel-Aty and Keller (2005), Lee and Abdel-Aty (2005), Shimamura et al. (2005), Garder (2006), Lu et al. (2006), Oh (2006), Siddiqui et al. (2006), Pai and Saleh (2007), Das et al. (2008), Gray et al. (2008), Wang and Abdel-Aty (2008), Yamamoto et al. (2008), Chimba and Sando (2009), Wang et al. (2009), Pai (2009), Haleem and Abdel-Aty (2010), Jung et al. (2010), Quddus et al. (forthcoming), Ye and Lord (forthcoming, 2011), Zhu and Srinivasan (2011)

Table 1 (Continued)

Methodological approach	Previous research
Partial proportional odds model	Wang and Abdel-Aty (2008), Wang et al. (2009), Quddus et al. (forthcoming)
Random parameters (mixed) logit	Milton et al. (2008), Kim et al. (2008), Kim et al. (2010), Malyshkina and Mannering (2010), Altwaijri et al. (2011), Anastasopoulos and Mannering (2011), Moore et al. (2011), Ye and Lord (forthcoming, 2011), Morgan and Mannering (forthcoming)
Random parameters (mixed) ordered logit	Srinivasan (2002)
Random parameters ordered probit	Zoi et al. (2010), Paleti et al. (2010)
Sequential binary logit	Sacconanno et al. (1996), Dissanayake and Lu (2002a,b)
Sequential binary probit	Yamamoto et al. (2008)
Sequential logit	Jung et al. (2010)
Simultaneous binary logit	Ouyang et al. (2002)

3.1.1. Bayesian hierarchical binary logit/simultaneous binary logit

When considering the injury-severity level of more than one crash-involved individual in the same crash (necessitating the need to account for within-crash correlations), probit or logit models will likely produce biased parameter estimates. To account for such within-crash correlations, Ouyang et al. (2002) used a simultaneous binary logit model to study injuries in multi-vehicle car-truck collisions in Washington State. They compared their results with simple binary logit models of individual injuries (assuming no correlation among injuries occurring in the same crash) and demonstrated the importance of accounting for injury correlations in crashes. Similarly, Helai et al. (2008) used Bayesian hierarchical binomial logit models to study multi-vehicle crashes with random effects to account for the common unobserved factors shared by drivers involved in the same crash.

3.1.2. Bivariate/multivariate binary probit

As discussed earlier in this paper, one potential concern in crash injury-severity modeling is the possibility that explanatory variables may be endogenous with respect to injury severity. In such cases, including the endogenous variable directly as a predictor can result in biased parameter estimates. As an example of this problem and how it might be addressed, Winston et al. (2006) considered the possibility that drivers' decisions to own a vehicle with airbags and/or anti-lock brakes would be interrelated with their probability of being crash-involved and the severity of their crash. They developed a multivariate model that simultaneously modeled a series of four binary outcomes: the choice of having airbags, the choice of having anti-lock brakes, the likelihood of being involved in a crash (with the dependent variables from the first two models, airbags and antilock brakes, included as explanatory variables), and the likelihood of such a crash resulting in an injury (with the dependent variables from the first two models, airbags and antilock brakes, included as explanatory variables). In other work, Lee and Abdel-Aty (2008) developed bivariate probit models based upon the hypothesis that drivers with passengers may drive differently than those without passengers. Bivariate probit models account for correlation among unobserved effects in the dependent variables – in this case among passenger presence and injury severity. The dependent variable in the first model (passenger presence) is then used as an explanatory variable in the second model (injury severity).

3.2. Ordered discrete outcome models

Accounting for the ordinal nature of injury data (for example, ranging from no-injury, to possible injury, to evident injury, to disabling injury, to fatal injury) is an important consideration in crash injury-severity modeling. To account for the ordinal nature of the data, traditional ordered probability models have been widely applied. The most common approach to the derivation of such models is to start by specifying a latent variable, z that is used as a basis for modeling the ordinal ranking of data. This unobserved variable is most often specified as a linear function for each crash observation, such that $z = \beta\mathbf{X} + \varepsilon$, where \mathbf{X} is a vector of variables determining the discrete ordering for each crash observation, β is a vector of estimable parameters, and ε is a disturbance term (Washington et al., 2011). With this, observed ordinal-injury data, y , for each observed crash is defined as

$$\begin{aligned} y &= 1 && \text{if } z \leq \mu_0 \\ y &= 2 && \text{if } \mu_0 < z \leq \mu_1 \\ y &= 3 && \text{if } \mu_1 < z \leq \mu_2 \\ y &= \dots && \\ y &= i && \text{if } z \geq \mu_{i-1}, \end{aligned} \quad (1)$$

where the μ are estimable threshold parameters that define y , which corresponds to integer ordering and i is the highest integer ordered response. The μ are parameters that are estimated jointly with the model parameters β and, without loss of generality, μ_0 can be set to 0. The estimation problem then becomes one of determining the probability of i specific ordered responses for each crash injury, n . If the error term, ε , is assumed to be normally distributed across observations with a mean of zero and variance of one, an ordered probit model results. Setting the lower threshold, μ_0 , equal to zero results in the outcome probabilities $P(y=i) = \Phi(\mu_i - \beta\mathbf{X}) - \Phi(\mu_{i-1} - \beta\mathbf{X})$, where μ_i and μ_{i-1} represent the upper and lower thresholds for injury severity i . Likewise, if the errors are instead assumed to be logistically distributed across observations, an ordered logit model results (Washington et al., 2011).

However, at least two potential problems arise with a traditional ordered probability approach. First, ordered probability models are particularly susceptible to underreporting of crash-injury data, resulting in biased or inconsistent parameter estimates (see Ye and Lord, forthcoming). If the underreporting rates in the population are known, a weighted maximum likelihood function can be used to analyze outcome-based samples (see Manski and Lerman, 1977) but the true rate of underreporting is generally unknown, making corrective measures difficult.

The second drawback relates to the restriction that traditional ordered probability models place on the way variables influence outcome probabilities. To illustrate this restriction, consider a model of three injury outcomes (no injury, injury, and fatality). Suppose that one of the key factors in determining the level of injury is whether or not an airbag was deployed. As illustrated in Fig. 1, a negative parameter of the airbag indicator (1 if it was deployed and zero otherwise) implies that $-\beta\mathbf{X}$ becomes smaller and thus shifts values that determine outcome probabilities to the left as shown in Fig. 1. In this case, the model constrains the effect of airbags to decrease the probability of a fatality (less area under the y =fatality portion of the density function as $-\beta\mathbf{X}$ shifts to the right as it becomes less negative with the negative parameter of the airbag) and also increase the probability of no injury (more area under the y =no injury portion of the density function). But in reality, it is possible that the deployment of an airbag decreases the probability of a fatality but also decreases the probability of no injury (because the

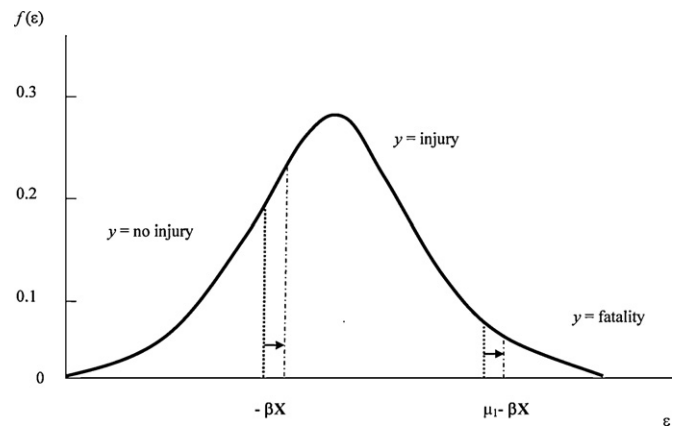


Fig. 1. Illustration of an ordered probability models with an increase in $\beta\mathbf{X}$ (with $\mu_0 = 0$).

airbag itself may cause some minor injuries).¹³ However, as shown in Fig. 1, traditional ordered probability models cannot account for this possibility because the shift in thresholds are constrained to move in the same direction. This is a major restriction of the ordered probability approach.¹⁴

3.2.1. A copula-based multivariate approach

Again, when considering the injury-severity level of more than one crash-involved individual in the same crash (necessitating the need to account for within-crash correlations), in the case of ordered discrete outcome models one needs to take into account both the ordinal nature of the dependent variable and the possible presence of correlated unobserved factors that may simultaneously influence and impact the injury-severity levels of the multiple crash-involved individuals in the same crash. To address this, Eluru et al. (2010) employed a copula-based approach to simultaneously estimate models of injury severities and the number of crash-involved individuals.¹⁵ Their results provide strong evidence of the presence of correlated unobserved factors that affect the levels of injury severity among crash-involved individuals.

3.2.2. Bivariate ordered probit

A bivariate ordered probit model is a hierarchical system of two equations that can be used to model a simultaneous relationship, addressing possible issues of endogeneity such as that of safety-belt use and injury severity. As an illustration of this concern, a study by de Lapparent (2008) estimated bivariate probit models to analyze the decision to fasten a safety belt in a car and the degree of injury sustained as a result of a crash. The approach used in this study accounts for correlation in the error terms of the safety-belt choice and injury severity models, which allows for a determination of whether there is a compensatory effect due to belt use. Ignoring such potential impacts may lead to the effectiveness of safety-belt use being seriously overstated. This is noteworthy because safety-

¹³ For example, Savolainen and Ghosh (2008) show that airbag deployment increases the likelihood of possible and non-incapacitating injuries in comparison to the no injury, incapacitating injury, and fatal outcomes.

¹⁴ Srinivasan (2002) addressed this issue by allowing for variable, random, and correlated injury severity thresholds, though it is noted that such a modeling structure makes interpreting the effects of variables on specific injury levels difficult. More recently, Eluru et al. (2008) have developed a generalized ordered probability model that relaxes the variable restrictions present in standard ordered probability models.

¹⁵ A copula is a multivariate distribution function that generates stochastic dependence among discrete outcomes (in this case injury severities). In addition to Eluru et al. (2010) and Bhat and Eluru (2009) provide a detailed explanation of the approach.

belt use and other covariates may be correlated to unobserved factors that also influence injury severity, giving rise to endogeneity problems as previously discussed.

3.2.3. Heterogeneous choice model

A critical assumption of many models (including ordered probability models) is that the error variances are homoskedastic (have equal variance). However, crash injury-severity data may sometimes produce error terms that are heteroskedastic and, if so, parameter estimates will be biased and inconsistent. An alternative specification for such situations is the heterogeneous discrete outcome model (also known as heteroskedastic ordered logit or location-scale models, see Quddus et al., forthcoming):

$$P(y > i) = \frac{\text{EXP}((\beta \mathbf{X}_n / \sigma_n) - \mu_i)}{[1 + \text{EXP}((\beta \mathbf{X}_n / \sigma_n) - \mu_i)]}, \quad (2)$$

where σ_n is the error variance, which can be parameterized as a function of explanatory variables, some of which may also be included in \mathbf{X}_n . While heterogeneous discrete outcome models can be used to specify models which exhibit heteroskedasticity, there is little evidence of how well these models accomplish this task (Williams, 2009).

O'Donnell and Connor (1996) parameterized the variance in their error terms as a function of occupant age, vehicle speed, vehicle year, and time of crash. Similarly, Wang and Kockelman (2005) parameterized the error variance as a function of speed limit, vehicle type, and vehicle curb weight. Keele and Park (2006) found that when a relevant variable was excluded from the variance equation, parameter estimates were actually more biased than when the unequal variances were ignored altogether. As such, parameterization of the error variance function must be carefully done.

3.2.4. Generalized ordered logit

Another assumption of ordered discrete outcome models, such as the ordered logit and ordered probit models, is that the parameter estimates are constant across severity levels. This condition is referred to as the proportional odds or parallel regression assumption (Quddus et al., forthcoming). The Brant test (Brant, 1990) can be used to test this assumption and, if violated, a viable solution is the generalized ordered logit model. The generalized ordered logit relaxes the proportional odds assumption and generalizes the ordered logit model such that the probability of injury category i resulting can be specified as

$$P(y > i) = \frac{\text{EXP}(\beta_i \mathbf{X}_n - \mu_i)}{[1 + \text{EXP}(\beta_i \mathbf{X}_n - \mu_i)]} \quad (3)$$

where \mathbf{X}_n is a vector of explanatory variables, β_i is a vector of estimable parameters, and μ_i and μ_{i-1} are the upper and lower thresholds for injury severity i . The difference between this formulation and the standard ordered logit is that β_i is free to vary across severity levels.

As the proportional odds assumption inherent in traditional logit analyses may not necessary hold over all severity levels, the generalized logit can be modified such that some, but not all, of the β_i are allowed to vary across outcomes. The resulting model:

$$P(y > i) = \frac{\text{EXP}(\beta_{1i} \mathbf{X}_1 + \beta_2 \mathbf{X}_2 - \mu_i)}{1 + \text{EXP}(\beta_{1i} \mathbf{X}_1 + \beta_2 \mathbf{X}_2 - \mu_i)}, \quad (4)$$

where β_1 is free to vary across different severity levels and β_2 is assumed constant across each severity level i . Partial proportional odds models have been used in recent studies by Wang and Abdel-Aty (2008) and Quddus et al. (forthcoming).

3.2.5. Bayesian ordered probit/mixed generalized ordered logit

Extensions to the traditional ordered probability models include specifications such as Bayesian or mixed/random parameter logit

and probit models. The mixed model, as specified by Srinivasan (2002), allows for random coefficients that are capable of capturing observation-specific differences in the effects of covariates on injury severity. Such a specification can account for unobserved effects, such as personal health or risk taking behaviors on the part of the crash-involved road users. Srinivasan showed the ordered mixed logit to provided superior fit to the traditional ordered logit model.

Similarly, Bayesian ordered probit models allow for the injury severity data to be supplemented by prior knowledge regarding model parameters. Under this context, the model parameters are assumed to follow specific distributions that are reflective of prior knowledge about factors affecting injury severity. Combining these a priori expectations with available data, the likelihood function is used to update the prior distributions and obtain posterior distributions for the model parameters. Xie et al. (2009) used Markov chain Monte Carlo algorithms and found the Bayesian models to produce more reasonable parameter estimates and better predictive capabilities for smaller sample sizes.

Eluru et al. (2008) developed a mixed generalized ordered logit model to assess pedestrian and bicyclist injury severity level in traffic crashes. This generalized version of the standard ordered logit model allows for parameters to vary across individuals and for unobserved heterogeneity effects of variables on the threshold values.

3.3. Unordered multinomial discrete outcome models

Models that do not account for the ordinal nature of injury data have also been popular in the analysis of crash injury-severity data. While not accounting for the ordering of injury-severity outcomes, such models are not afflicted with some of the restrictions imposed by traditional ordered probit and logit models.

3.3.1. Multinomial logit models

Multinomial logit models are traditional discrete outcome models that consider three or more outcomes and do not explicitly consider the ordering that may be present in these outcomes. The general framework used to model the degree of injury severity sustained by a crash-involved individual begins by defining a linear function S that determines the injury outcome i for observation n as

$$S_{in} = \beta_i \mathbf{X}_{in} + \varepsilon_{in}, \quad (5)$$

where β_i is a vector of estimable parameters, \mathbf{X}_{in} is a vector of observable characteristics (driver, vehicle, roadway, and environmental factors) that impact the injury severity sustained by observation n , and ε_{in} is a disturbance term that accounts for unobserved effects. If the disturbance terms are assumed to be independently and identically distributed as generalized extreme value distributed, the multinomial logit model results (see McFadden, 1981)¹⁶:

$$P_n(i) = \frac{\text{EXP}[\beta_i \mathbf{X}_{in}]}{\sum_i \text{EXP}[\beta_i \mathbf{X}_{in}]} \quad (6)$$

In addition to not accounting for the ordering of injury-severity outcomes, the multinomial logit model is particularly susceptible to correlation of unobserved effects from one injury-severity level to the next. Such correlation causes a violation of the model's independence of irrelevant alternatives (IIA) property (see Washington

¹⁶ The multinomial probit model can be similarly derived under the assumption that the errors are normally distributed. However, the absence of a closed form solution makes multinomial probit estimation more cumbersome and, as a result, the model has not been used to the authors' knowledge to study crash-injury severities.

et al., 2011).¹⁷ On the plus side, traditional multinomial models do not impose the sometimes unrealistic parameter restrictions that traditional ordered probability models do. Further, if the IIA property holds, it can be shown that in the presences of underreporting of crashes all parameters will still be correctly estimated except for the constant term (see Washington et al., 2011).

3.3.2. Sequential logit and probit models

The sequential logit/probit is a generalized version of the ordered logit/probit model, which relaxes the restrictions imposed by standard ordered probability models (Eluru et al., 2008). The generalized standard ordered logit model allows the treatment of the severity thresholds across the ordered response levels by separate parameter coefficients for explanatory variables and heterogeneity in the effects of injury severity determinants.

Yamamoto et al. (2008) showed that sequential models could provide superior performance to traditional ordered-response probit models, which assume the same factors correlate across all levels of severity. Even when underreporting is a concern, the parameter estimates remain unbiased except for the constant terms, similar to multinomial logit models (Yamamoto et al., 2008). One limitation of sequential models is that they assume the error terms associated with each level of injury severity are independent.

3.3.3. Markov switching multinomial logit

Malyshkina and Mannering (2009) developed a two-state Markov switching multinomial logit model to study crash-injury severity under the assumption that there exist two unobserved states of roadway safety. These two states account for the possibility of unobserved heterogeneity effects, which may be caused by unobservable risk factors that influence crash severity. Because the risk factors can interact and change, roadway entities can switch between the two states over time.

3.3.4. Nested logit model

When the independence of irrelevant alternatives (IIA) assumption is violated (see discussion above), the nested logit model is an appropriate generalization of the multinomial logit model. To estimate a nested logit model, injury severity levels are partitioned into nests that are comprised of severity outcomes that share some unobserved elements specific to only those outcomes. The nested logit overcomes the IIA limitation of multinomial logit models and potentially improves upon the sequential logit model by allowing for correlations among error terms across different severity levels.

Following Eq. (4) and assuming the disturbances are generalized extreme value distributed, the nested logit can be written as (see McFadden, 1981):

$$P_n(i) = \frac{\text{EXP}[\beta_i \mathbf{X}_{in} + \phi_i LS_{in}]}{\sum_{\forall I} \text{EXP}[\beta_I \mathbf{X}_{In} + \phi_I LS_{In}]} \quad (7)$$

$$P_n(j|i) = \frac{\text{EXP}[\beta_{ji} \mathbf{X}_n]}{\sum_{\forall J} \beta_{Ji} \mathbf{X}_n} \quad (8)$$

$$LS_{in} = \text{LN} \left[\sum_{\forall J} \text{EXP}(\beta_{Ji} \mathbf{X}_{jn}) \right], \quad (9)$$

where $P_n(i)$ is the unconditional probability of crash n resulting in injury outcome i , \mathbf{X} are vectors of measurable characteristics

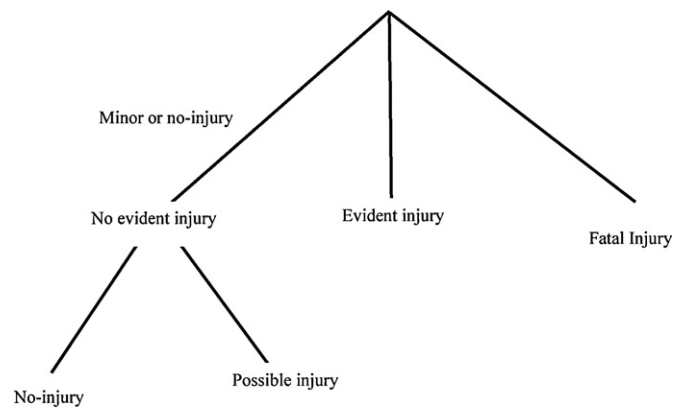


Fig. 2. Example of a nested model structure.

that determine the probability of injury severities, β are vectors of estimable parameters, $P_n(j|i)$ is the probability of crash n having injury outcome i conditional on the outcome being in outcome category i (for example, in the nested structure shown in Fig. 2 the severity category i would be no evident injury and $P_n(j|i)$ would be the binary logit model of injury outcomes no injury and possible injury), J is the conditional set of outcomes (conditioned on i), I is the unconditional set of outcome categories (the upper three branches of Fig. 2), LS_{in} is the inclusive value (logsum), and ϕ_i is an estimable parameter.¹⁸

One note regarding the appropriateness of nested logit models is that, in order to be consistent with McFadden's generalized extreme value derivation of the model, the parameter estimate for the inclusive value must fall between zero and one. If this parameter is equal to one, there is no correlation between the severity levels in the nest and the model reduces to the simple multinomial logit model. If the inclusive-value parameter is equal to zero, perfect correlation is implied among the severity levels in the nest, indicating that the process by which crashes result in particular severity levels is deterministic.

3.3.5. Mixed logit models

Mixed logit models (also commonly referred to as random parameters logit models) are a relatively recent development for the analysis of discrete data (McFadden and Train, 2000). The mixed logit addresses the limitations of the multinomial logit by allowing for heterogeneous effects and correlation in unobserved factors (Train, 2009). Unlike the multinomial probit, its error terms are not limited to normal distributions. A mixed logit model is derived with the addition of a second error term to the severity function (see Eq. (5) for comparison):

$$S_{in} = \beta_i \mathbf{X}_{in} + [\eta_{in} + \varepsilon_{in}] \quad (10)$$

where η_{in} is a random error term with zero mean, ε_{in} is the error term that is independent and identically distributed, and does not depend on underlying parameters or data, and all other variables are as previously defined.

¹⁷ There are well known tests for determining if the multinomial logit model's independence of irrelevant alternatives assumption is being violated (see Small and Hsiao, 1985). Empirical studies have shown that the violation of this property is very much data dependent, with some times the property holding (Shankar et al., 1996) and other times not (Savolainen and Mannering, 2007).

¹⁸ Many of the initial studies involving nested logit models involved sequential estimation in which the lower nest(s) was estimated first (Eq. (8)), the logsum is calculated for all observations within the sample (Eq. (9)) and these logsums were then used as independent variables in the upper level model (Eq. (7)). The drawback of this sequential estimation approach is that the variance-covariance matrices are too small, thus inflating the t statistics by 10–15 percent. This problem can be resolved by estimating the entire equation system simultaneously using full information maximum likelihood estimation. See Savolainen and Mannering (2007) for an application of this approach.

The mixed logit is a generalization of the multinomial structure which allows the parameter vector β_i to vary across each crash-involved road user. The injury outcome-specific constants and each element of β_i may be either fixed or randomly distributed over all parameters with fixed means, allowing for heterogeneity in effects. A mixing distribution is introduced to the model formulation, resulting in injury severity probabilities as follows (Train, 2009):

$$P_{in}(i) = \int_{\mathbf{x}} \frac{\text{EXP}[\beta_i \mathbf{x}_{in}]}{\sum_j \text{EXP}[\beta_j \mathbf{x}_{in}]} f(\beta|\varphi) d\beta \quad (11)$$

where $q(\beta|\varphi)$ is a density function of β and φ is a vector of parameters which describe the density function, with all other terms as previously defined (Milton et al., 2008). The injury severity outcome probability is then simply a mixture of logits (Train, 2009). The distribution is flexible in that β can also be fixed and when all parameters are fixed, the model reduces the standard multinomial logit formulation. In instances where β is allowed to vary, the model is open form and the probability of an observation having a particular outcome can be calculated through integration.

3.4. Other methods

Aside from the commonly used methods described above, a number of other methods have been applied to study crash-injury severities. For example, a few researchers have used artificial neural networks in the injury severity research literature (Abdelwahab and Abdel-Aty, 2001; Delen et al., 2006; Chimba and Sando, 2009). In broad terms, neural networks search for data patterns and allow for potentially non-linear relationships between the injury severity levels and covariates. They provide an analytical technique which utilizes information from previous observations to predict new observations. Such techniques can provide a robust function for prediction and classification problems. While Abdelwahab and Abdel-Aty (2001) found that a multi-layer neural network was able to more accurately classify severity outcomes than the ordered logit model, neural networks may be better served for prediction of injury outcomes rather than a determination of specific parameter impacts.

Various data mining techniques have also been used. For example, Chang and Wang (2006) used a classification and regression tree approach, a data mining technique that does not require a parametric assumption of the underlying relationship between the degree of injury severity sustained and its associated explanatory variables. While a classification and regression tree provides an efficient data mining technique, it does not provide for the interpretive capabilities of discrete outcome models.

4. Directions for future research

The preceding discussion shows that a wide variety of methodological approaches have been used to study crash-injury severities and that there has been a steady methodological evolution that has sought to overcome known deficiencies in analysis and provide new insights. For example, issues relating to the trade-offs between ordered and unordered models have begun to be addressed and recent applications of random parameter and multi-state models have opened up new ways to account for unobserved heterogeneity. The discussion also suggests that the appropriate methodological approach can often depend heavily on the available dataset, including the number of observations, quantity and quality of explanatory variables, and other data-specific characteristics.

Although crash-injury severity research has made impressive progress over the years, there are at least four areas that still present formidable methodological challenges. First, the whole issue of the

spatial and temporal correlation of crash-injury severity data has not been fully addressed. Injury severities in the same crash, at the same intersection, on the same roadway segment, or those occurring during the time of the same weather event, are all likely to share important unobserved effects. While there are many known methods to account for correlations among unobserved effects (from fields such as spatial econometrics), the complexity of the model structure when discrete data are involved is a major barrier to model development. Still, the precision of estimates and resulting inferences could be substantially improved when fully accounting for spatial and temporal correlations.

Second, although the endogeneity of explanatory variables has been dealt with in some previous research efforts (see for example Winston et al., 2006; Lee and Abdel-Aty, 2008; Paleti et al., 2010), the crash-injury severity field has only begun to scratch the surface on this important issue. To provide another example of this problem (in addition to the ones provided earlier in the paper), consider the effectiveness of helmet use in determining motorcycle rider injury severity. If an indicator variable were to be included in the severity model (1 if a helmet was used and 0 if not), one would expect this variable to significantly reduce the probability of a fatality (resulting from a head injury). However, while some variables such as age and gender may be proxies for the risk behavior of the rider, many risk-behavior elements are not likely to be known (past crash and near-crash experiences, genetic predisposition to risk, etc.). Because these unobserved factors are likely to be correlated with the probability of wearing a helmet, the effectiveness of helmet use in reducing severities is likely to be incorrect unless this correlation is explicitly accounted for in the model. For example, if risk-averse riders (which are not fully identified with available data) are more likely to wear helmets, the effectiveness of helmet use in preventing motorcycle fatalities is likely to be significantly overstated because the helmet indicator will be capturing both the actual effectiveness of the helmet and the fact that such riders are more likely to wear helmets. In this case, if the physics of helmets were such that they were completely ineffective in reducing fatalities, the model would still predict that helmets were effective due to the correlation between helmet use and unobserved risk factors. Ignoring such endogeneity can lead to incorrect inferences and the potential implementation of ineffective safety countermeasures.

Third, while a few recent research efforts have begun to look at the potential impact of the under-reporting of crashes (Yamamoto et al., 2008; Ye and Lord, forthcoming), much additional work is needed. This is an extremely important matter because estimating models with under-reported data can easily lead to erroneous inferences, particularly because less severe crashes are far more likely to be underreported than more severe crashes, which results in a data sample which is non-random in its dependent variable thereby violating the fundamentals of traditional statistical and econometric model derivations.

Fourth, the likely availability of new types of crash injury-severity data in the not too distant future will open up many new methodological challenges. While the discussions in this paper are based on the use of traditional police-reported injury data (broad discrete injury categories), which have been the basis of almost all crash-injury research to date, one can imagine more detailed data with specific injury types (head, arm, torso, etc.), lengths of hospital stays, days to recovery, and so on. In the United States, the Crash Outcomes Data Evaluation System (CODES), which links crash data with health outcome data, is an example of such a database. Such data will likely include both discrete and continuous dependent variables that will require the development and application of new methodological approaches. Still, many of the fundamental issues with the traditional data methods described in this paper will also apply to models estimated with these richer databases.

In summary, although substantial progress has been made in crash injury-severity modeling over the years, major methodological and data challenges have yet to be fully resolved. Because injuries in highway crashes represent a major impact on societies throughout the world, addressing these residual challenges must be a priority in future crash-injury research.

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