CSE512 Fall 2018 Machine Learning - Homework 6

Your Name: Caitao Zhan

Solar ID: 111634527

NetID email address: caitao.zhan@stonybrook.edu

Names of people whom you discussed the homework with: Ting Jin

1 PCA via Successive Deflation

1. $\mathbf{A}$ and $\mathbf{B}$ are symmetric, $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T = \mathbf{BA} \Rightarrow \mathbf{v}_1\mathbf{v}_1^T\mathbf{X}\mathbf{X}^T = \mathbf{X}\mathbf{X}^T\mathbf{v}_1\mathbf{v}_1^T$
$\mathbf{v}_1\mathbf{v}_1^T\mathbf{X}\mathbf{X}^T\mathbf{v}_1\mathbf{v}_1^T = \mathbf{v}_1\mathbf{v}_1^T n\lambda_1\mathbf{v}_1\mathbf{v}_1^T = n\lambda_1\mathbf{v}_1\mathbf{v}_1^T = \mathbf{X}\mathbf{X}^T\mathbf{v}_1\mathbf{v}_1^T$

$$\begin{aligned}
\tilde{\mathbf{C}} &= \frac{1}{n}\tilde{\mathbf{X}}\tilde{\mathbf{X}}^T \\
&= \frac{1}{n}(\mathbf{I} - \mathbf{v}_1\mathbf{v}_1^T)\mathbf{X}\mathbf{X}^T(\mathbf{I} - \mathbf{v}_1\mathbf{v}_1^T) \\
&= \frac{1}{n}(\mathbf{X}\mathbf{X}^T - \mathbf{X}\mathbf{X}^T\mathbf{v}_1\mathbf{v}_1^T - \mathbf{v}_1\mathbf{v}_1^T\mathbf{X}\mathbf{X}^T + \mathbf{v}_1\mathbf{v}_1^T\mathbf{X}\mathbf{X}^T\mathbf{v}_1\mathbf{v}_1^T) \\
&= \frac{1}{n}(\mathbf{X}\mathbf{X}^T - \mathbf{X}\mathbf{X}^T\mathbf{v}_1\mathbf{v}_1^T - \mathbf{X}\mathbf{X}^T\mathbf{v}_1\mathbf{v}_1^T + \mathbf{X}\mathbf{X}^T\mathbf{v}_1\mathbf{v}_1^T) \\
&= \frac{1}{n}(\mathbf{X}\mathbf{X}^T - n\lambda_1\mathbf{v}_1\mathbf{v}_1^T) \\
&= \frac{1}{n}\mathbf{X}\mathbf{X}^T - \lambda_1\mathbf{v}_1\mathbf{v}_1^T
\end{aligned}$$

2.

$$\begin{aligned}
&\mathbf{C}\mathbf{v}_j = \lambda_j\mathbf{v}_j \\
\Rightarrow &\frac{1}{n}\mathbf{X}\mathbf{X}^T\mathbf{v}_j = \lambda_j\mathbf{v}_j \\
\Rightarrow &\frac{1}{n}\mathbf{X}\mathbf{X}^T\mathbf{v}_j - \lambda_1\mathbf{v}_1\mathbf{v}_1^T\mathbf{v}_j = \lambda_j\mathbf{v}_j \\
\Rightarrow &(\frac{1}{n}\mathbf{X}\mathbf{X}^T - \lambda_1\mathbf{v}_1\mathbf{v}_1^T)\mathbf{v}_j = \lambda_j\mathbf{v}_j \\
\Rightarrow &\tilde{\mathbf{C}}\mathbf{v}_j = \lambda_j\mathbf{v}_j
\end{aligned}$$

Therefore, $\mathbf{v}_j$ is also a principal eigenvector of $\tilde{\mathbf{C}}$ with the same eigenvalue λ_j

3. Let the eigenvalues of $\mathbf{C}$ rank in decreasing order

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{d-1} \geq \lambda_d > 0 \tag{1}$$

Let the eigenvalues of $\tilde{\mathbf{C}}$ rank in decreasing order

$$\lambda_1^{'} \geq \lambda_2^{'} \geq \cdots \geq \lambda_{d-1}^{'} > 0 \tag{2}$$

$\mathbf{u}$ is the principal eigenvector of $\tilde{\mathbf{C}}$, so

$$\tilde{\mathbf{C}}\mathbf{u} = \lambda_1^{'}\mathbf{u} \tag{3}$$

From the above question 1.2, we have the conclusion that for $j \neq 1$, $\mathbf{v}_j$ is also a principal eigenvector of $\tilde{\mathbf{C}}$ with the same eigenvalue λ_j.
Let $j = 2$, then $\mathbf{v}_2$ is a eigenvector with eigenvalue λ_2, so

$$\tilde{\mathbf{C}}\mathbf{v}_2 = \lambda_2\mathbf{v}_2 \tag{4}$$

From formulas (1), (2), (3), (4), we can conclude that $\lambda_1^{'} = \lambda_2$ and $\mathbf{u} = \mathbf{v}_2$

4. I wrote a piece of python code to represent psuedo code.

```
def first_k_vector(C, k, f):
    lambdas, principal_basis = [], []
    for _ in range(k):
        lamb, u = f(C)
        lambdas.append(lamb)
        principal_basis.append(u)
        C = C - lamb * numpy.matmul(u, u.T)
    return lambdas, principal_basis
```

2 Action recognition with CNN