CSE512 Fall 2018 Machine Learning - Homework 7

Your Name: Caitao Zhan

Solar ID: 111634527

NetID email address: caitao.zhan@stonybrook.edu

Names of people whom you discussed the homework with: Ting Jin

1 Manual calculation of one round of EM for a GMM

1.1 M step

1. The likelihood of data, and also the log likelihood. We optimize the log likelihood.

$$\begin{aligned} P(D;\pi,\mu,\sigma) &= \prod_{i=1}^{m} p(x^i;\pi,\mu,\sigma) \\ &= \prod_{i=1}^{m}\sum_{z^i=1}^{k} p(x^i|z^i;\mu,\sigma)\pi_{z^i} \\ \log P(D;\pi,\mu,\sigma) &= \sum_{i=1}^{m}\log\sum_{z^i=1}^{k} p(x^i|z^i;\mu,\sigma)\pi_{z^i} \end{aligned}$$

   where π_{z^i} is the probability of z^i is the true cluster label, μ is the mean, σ is the standard deviation.

2. $\pi_1 = \frac{1}{3}(1+0.3+0) = 0.433$
   $\pi_2 = \frac{1}{3}(0+0.7+1) = 0.567$

3. $\mu_1 = (1\times 1 + 0.3\times 10 + 0\times 20)/(1+0.3+0) = 3.077$
   $\mu_2 = (0\times 1 + 0.7\times 10 + 1\times 20)/(0+0.7+1) = 15.882$

4. $\sigma_1 = \sqrt{[1\times(1-\mu_1)^2 + 0.3\times(10-\mu_1)^2 + 0\times(20-\mu_1)^2]/(1+0.3+0)} = 3.792$
   $\sigma_2 = \sqrt{[0\times(1-\mu_2)^2 + 0.7\times(10-\mu_2)^2 + 1\times(20-\mu_2)^2]/(0+0.7+1)} = 4.922$

1.2 E step

1. There are two clusters, so $c \in \{1,2\}$
   Gaussian distribution: $g(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
   $p(x^i|z^i=c;\mu_c,\sigma_c) = g(x^i|\mu_c,\sigma_c)$

2. $\begin{pmatrix} 0.988 & 0.012 \\ 0.277 & 0.723 \\ 0 & 1 \end{pmatrix}$

2 GAN

3 RNN