## ${\rm CSE}512$ Fall 2018 Machine Learning - Homework 1

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## 1 Question 1

1. The expectation:  $E(X) = \frac{N^2-1}{6N}$ 

The problem is discrete. There are  $N^2$  cases with equal probability, since X1 and X2 are independent and both are uniformly distributed.

When  $X2 \le X1, X = 0$ , so only need to consider cases all cases where X2 > X1.  $P(X = 1) = \frac{N-1}{N^2}$ 

$$P(X=2) = \frac{N-2}{N^2}$$

$$P(X=i) = \frac{N-i}{N^2}$$

$$E(X) = \sum_{i=1}^{N} i P(i) = \sum_{i=1}^{N} i \cdot \frac{N-i}{N^2} = \frac{N^2-1}{6N}$$

2. The variance:  $Var(X) = \frac{2N^4 - N^2 - 1}{36N^2}$ 

$$Var(X) = E(X^2) - E(X)^2$$

We already derived E(X), so we only need to derive  $E(X^2)$ . We just need to slightly alter the process of computing E(X)

$$P(X = 1^2) = \frac{N-1}{N^2}$$

$$P(X=2^2) = \frac{N-2}{N^2}$$

$$P(X=i^2) = \frac{N-i}{N^2}$$

$$E(X^2) = \sum_{i=1}^{N} i^2 P(i) = \sum_{i=1}^{N} i^2 \cdot \frac{N-i}{N^2} = \frac{N^2-1}{12}$$

$$Var(X) = E(X^2) - E(X)^2 = \frac{2N^4 - N^2 - 1}{36N^2}$$

3. The covariance:  $Cov(X, X1) = \frac{-N^2+1}{24}$ 

 $Cov(X, X1) = E[(X - E(X) \cdot (X1 - E(X1))] = E(X \cdot X1) - E(X)E(X1)$ We already derived E(X), and  $E(X1) = \frac{N+1}{2}$ . So we need to derive  $E(X \cdot X1)$ 

$$\begin{split} E(X \cdot X1) &= \frac{1}{N^2} \sum_{X1=1}^{N} \sum_{X2=1}^{N} [max(X2 - X1, 0) \cdot X1] \\ &= \frac{1}{N^2} \sum_{X1=1}^{N} \sum_{X2=X1+1}^{N} [(X2 - X1) \cdot X1] + \frac{1}{N^2} \sum_{X1=1}^{N} \sum_{X2=1}^{X2} \cdot 0 \\ &= \frac{1}{N^2} \sum_{X1=1}^{N} [\frac{(X1 + 1 + N)(N - X1)}{2} - X1(N - X1)]X1 \\ &= \frac{1}{N^2} \sum_{X1=1}^{N} [X1^3 - (2N + 1)X1^2 + (N^2 + N)X1] \\ &= \frac{N^3 + 2N^2 - N - 2}{24N} \end{split}$$

$$Cov(X, X1) = Cov(X, X1) - E(X)E(X1) = \frac{N^3 + 2N^2 - N - 2}{24N} - \frac{N^2 - 1}{6N} \cdot \frac{N + 1}{2} = \frac{-N^2 + 1}{24N}$$

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## 2 Question 2

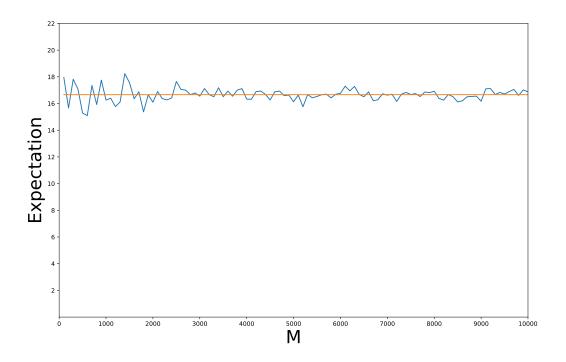


Figure 1: Expectation as a function of M. Theoretic E(X) = 16.665

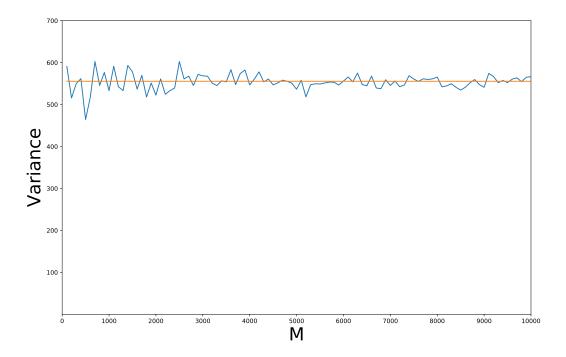


Figure 2: Variance as a function of M. Theoretic Var(X) = 555.527775

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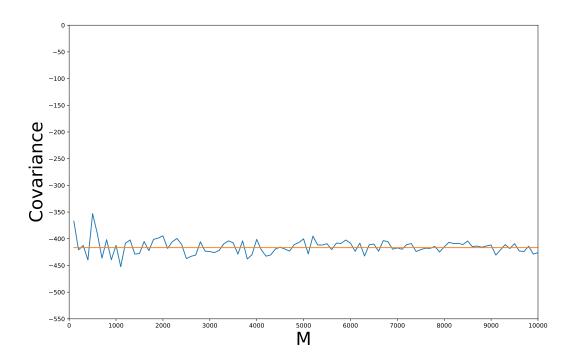


Figure 3: Variance as a function of M. Theoretic Cov(X) = -416.625

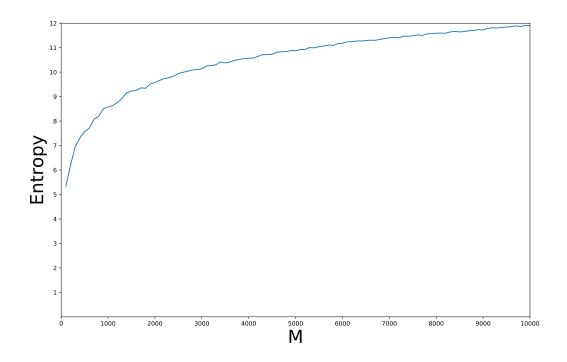


Figure 4: Entropy as a function of M.

Note: The python plotting script is in the jupyter notebook.