MSA400: Technical Project 1

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- Calvin: Introduction, Data, Methods (except Block Maxima daily VaR), Results, Discussion and Conclusions, Coding implementation.
- Kingsley: Block Maxima daily VaR section in methods.

Contents

1 Introduction and aim

The purpose of this project is to evaluate a few different methods for extreme value statistics. The methods are used on stock price data from the Scandinavian airline company SAS. The following methods will be looked at:

- Peaks Over Threshold (PoT).
- Block Maxima (BM).
- Assuming Normality.

2 Data

The data consists of daily closing prices of the Scandinavian airline companany SAS between 2001 and 2021. For the purpose of this project the data has been split into two different sets:

- **sas06**: 2006/01/02 - 2012/12/28, 1756 observations.
- **sas21**: 2016/01/04 - 2021/04/21, 1331 observations.

The dataset **sas06** includes the 2009 financial crisis and the set **sas21** includes the financial crisis due to Covid-19.

Figure 1: Daily closing prices of SAS between 2006-01-02 and 2012-12-28

Figure 2: Daily closing prices of SAS between 2016-01-04 and 2021-04-21

From these two datasets, the daily returns have been calculated as:

$$R_t = \frac{S_{t+1} - S_t}{S_t}$$

Where S_t is the price of the stock at time t. Further , the returns have been transformed so that a positive return indicates a loss and a negative returns indicates a profit.

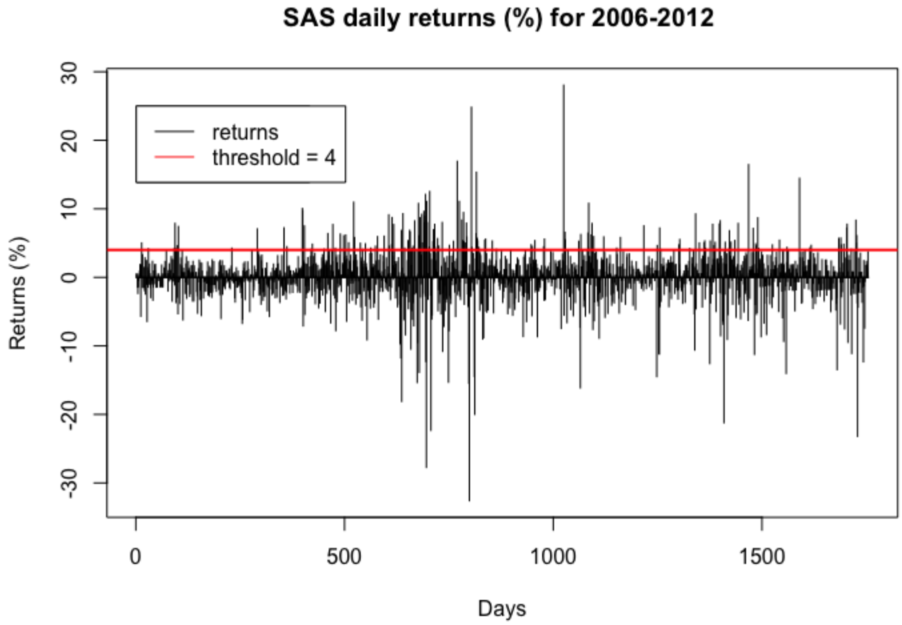

Figure 3: Daily returns of SAS between 2006-01-02 and 2012-12-28

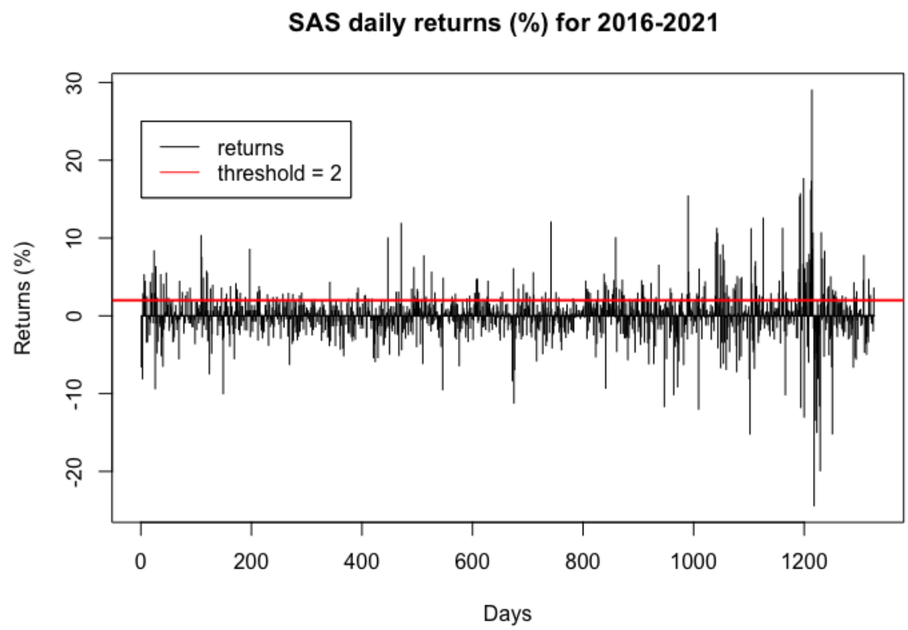

Figure 4: Daily returns of SAS between 2016-01-04 and 2021-04-21

3 Methods

The calculations have been performed in the statistical programming language R.

3.1 Estimating daily VaR

The general outline of estimating daily Value-at-Risk (VaR) is the following:

- Load data of daily returns.
- Fit the data to the desired distribution (GP,GEV and Gaussian) using Maximum Likelihood.
- Obtain the high quantiles of the distribution that are of interest.

3.1.1 PoT

In Peaks over Threshold modelling it is assumed that exceedances over a given threshold are Generalized-Pareto distributed and that the exceedances occur as a Poisson process. The conditional probability of exceedances of a random variable X over a threshold μ can be written as:

$$P(X > x|X > \mu) = (1 + \frac{\gamma}{\sigma}(x - \mu))^{-1/\gamma}. \tag{1}$$

Since the exceedances are assumed to occur as a Poisson process we can estimate the probability $P(X > u)$ by $\frac{N}{n}$. Where N denotes the number of exceedances of a certain threshold and n is the total number of observations. From this we get that

$$P(X > x) = \frac{N}{n}(1 + \frac{\gamma}{\sigma}(x - \mu))^{-1/\gamma} = 1 - F(X) = \bar{F}(x). \tag{2}$$

3.1.2 BM

The block maxima (BM) approach in extreme value theory (EVT), consists of dividing the observation period into non-overlapping periods of equal size and restricts attention to the maximum observation in each period.The new observations thus created follow approximately an extreme value distribution, Gγ for some real γ. Parametric statistical methods for the extreme value distributions are then applied to those observations.

3.1.3 Assuming normality

This method simply assumes that returns are normally distributed. The quantiles can then be easily computed.

3.2 Estimating Expected Shortfall (ES)

The formula used for estimating the p-th quantile Expected Shortfall (ES) based on the PoT method is:

$$ES_p(L) = VAR_p(L) + \frac{\sigma + \gamma(VAR_p(L) - \mu)}{1 - \gamma}, \tag{3}$$

where

$$VAR_p(L) = \frac{\sigma}{\gamma}((\frac{1-p}{p_\mu})^{-\gamma} - 1) + \mu, \text{ for } p > p_\mu \tag{4}$$

3.3 Estimating monthly block maxima VaR

Let M_T be the maximum in the time interval $[0, T]$, then the relationship between the PoT and the BM methods for $x > 0$ is

$$P(M_T < x + \mu) = \exp\left(-(1 + \gamma\frac{x - ((\lambda T)^\gamma - 1)\sigma/\gamma}{\sigma(\lambda T)^\gamma})^{-1/\gamma}\right) \tag{5}$$

which is a GEV-distribution with updated location and scale parameters, where σ and γ are taken from the PoT method. Thus, in the above formula we replace

σ and γ with the ML-estimates $\hat{\sigma}$ and $\hat{\gamma}$ obtained from the GP-distribution and the intensity λ is estimated by $\lambda = N/n$, the number of excesses divided by the total number of observations. The updated location and scale parameters thus become

$$\mu_{bm} = ((\hat{\lambda}T)^{\hat{\gamma}} - 1)\frac{\hat{\sigma}}{\hat{\gamma}}$$

$$\sigma_{bm} = \hat{\sigma}(\hat{\lambda}T)^{\hat{\gamma}}$$

In our application we have chosen $T = 20$ days since there is approximately 20 traiding days in one month.

3.4 Backtesting VaR

When using backtesting we have expanded our two datasets. The data set **sas06** is expanded to include returns from 2001 and the set **sas21** from 2006. The general outline of our implementation is as follows:

1. Train a model on the first 6 years of data.
2. Calculate the daily %p VaR.
3. Check if the return of day 6 years + 1 exceeds the VaR.
4. Repeat steps 1-3. But now training the model on the first 6 years + 1 day and checking if the return on day 6 years + 2 exceed the new VaR.
5. Repeat until the last day.
6. Count the number of times returns exceeds VaR (violations).

The violations are then compared to the expected number of violation which are estimated by $t * (1 - p)$, where t is the number of days from 6 years to the last day.

4 Results

4.1 Daily VaR

	95%	96%	97%	98%	99%	99.9%	99.99%
Empirical	5.31	5.87	6.82	7.85	9.27	18.87	27.48
Gaussian	6.38	6.79	7.29	7.95	8.98	11.89	14.29
GP	5.33	5.87	6.6	7.71	9.8	19.07	33.33
GEV	4.34	4.83	5.52	6.59	8.75	20.24	43.41

Table 1: Daily Value at Risk for **sas06**.

	95%	96%	97%	98%	99%	99.9%	99.99%
Empirical	4.55	5.33	6.21	7.84	11.03	25.27	67.5
Gaussian	7.32	7.79	8.36	9.12	10.32	13.69	16.46
GP	4.63	5.26	6.17	7.67	10.99	33.70	98.64
GEV	3.73	4.22	4.93	6.12	8.81	28.86	93.18

Table 2: Daily Value at Risk for **sas21**.

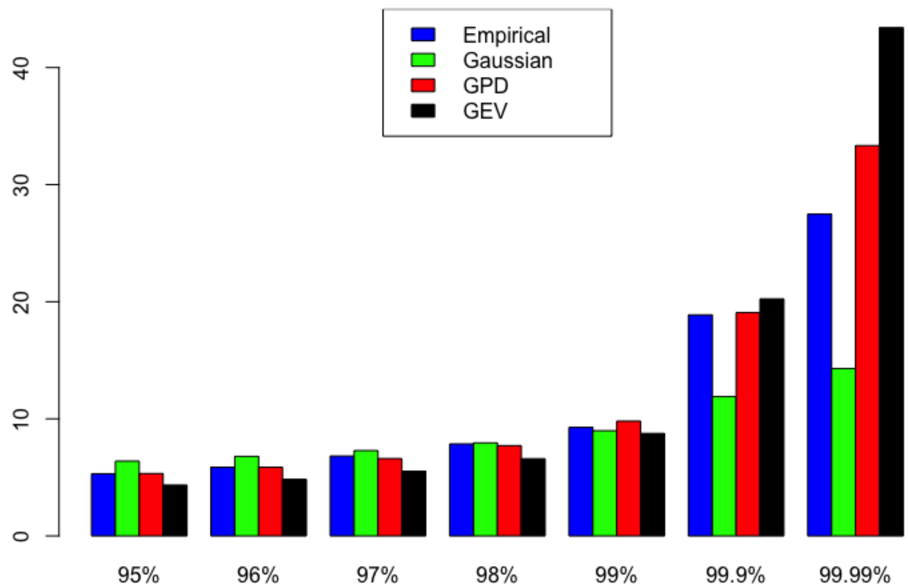

Figure 5: Barplot of daily estimations of VaR for the **sas06** dataset.

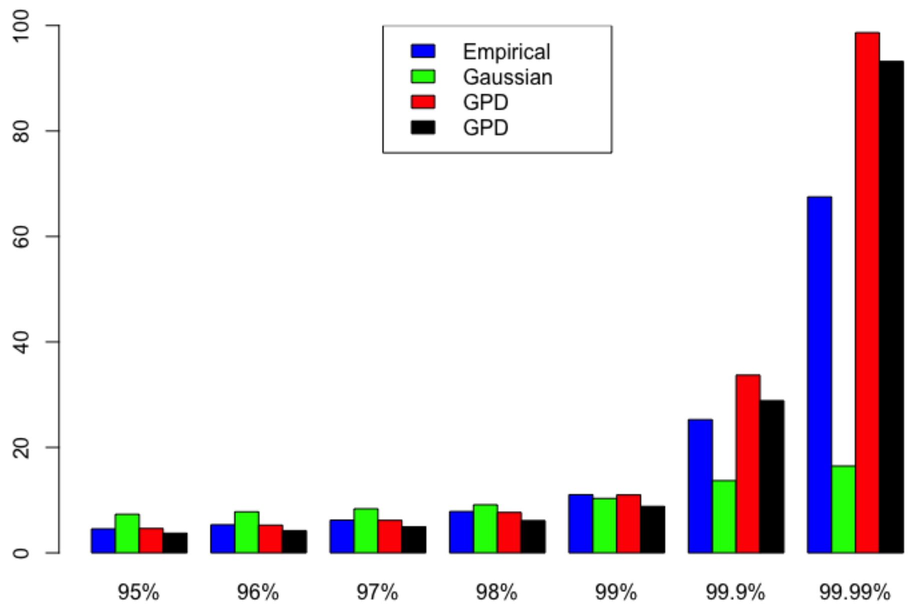

Figure 6: Barplot of daily estimations of VaR for the **sas21** dataset.

4.2 Expected Shortfall

	95%	96%	97%	98%	99%	99.9%	99.99%
sas06	8.26	8.93	9.83	11.19	13.76	25.17	42.71
sas20	9.55	10.71	12.38	15.14	21.24	63.01	182.41

Table 3: Expected shortfall for both datasets.

4.3 Monthly Block Maxima VaR from PoT estimation.

	95%	96%	97%	98%	99%	99.9%	99.99%
sas06	16.36	17.94	20.09	23.36	29.68	59.57	110.34
sas20	35.78	40.81	48.17	60.57	88.73	301.53	998.17

Table 4: Monthly BM VaR from GP estimation.

4.4 Backtesting

	95%	96%	97%	98%	99%	99.9%	99.99%
sas06	122(87.75)	90(70.2)	71(52.65)	51(35.1)	36(17.55)	9(1.755)	5(0.1755)
sas20	72(66.5)	60(53.2)	49(39.9)	40(26.6)	22(13.3)	3(1.33)	1(0.133)

Table 5: Backtesting results, violations of 1-day VaR, GP method. Expected no of violations in parentheses.

	95%	96%	97%	98%	99%	99.9%	99.99%
sas06	178(87.75)	138(70.2)	115(52.65)	74(35.1)	47(17.55)	5(1.755)	0(0.1755)
sas20	91(66.5)	77(53.2)	59(39.9)	43(26.6)	28(13.3)	2(1.33)	1(0.133)

Table 6: Backtesting results, violations of 1-day VaR, GEV method. Expected no of violations in parentheses.

5 Discussion and Conclusions

a) Differences and similarities of the results obtained with different methods.

Looking at the results from the estimations of daily VaR for the different methods, we can see a pretty clear pattern. For the lower quantiles, the methods produce similar results, but the higher we go the larger the differences. This is not surprising as both the PoT and the BM are methods created for for estimating tail risks, and hence can predict risks "outside" the scope of our data. The Gaussian method clearly falls short for high quantiles as it is unable to fit the tails of our data. This becomes apparently evident when we compare the 99.99% empirical quantile and the correspsonding Gaussian for the **sas06** dataset. The empirical VaR is 27.48% (which is the highest daily loss in the dataset), while the Gaussian estimation is 14.29%. This means that the Gaussian method produces a VaR that is lower than the actual maximum observed loss, which is far from ideal when trying to predict future risk.

Furthermore, when comparing the results from the GP and the GEV methods, we can see from table 5 and 6 that they both consistently underestimate the risks compared to the expected number of violations of the daily VaR. However, the GEV method gives results that are closer to the expected number of violations for higher quantiles, while the GP method gives better results for lower quantiles (although both methods are bad for lower quantiles). I am not sure how to explain this, because in theory PoT should give better results than BM when it comes to daily VaR, since it observes all the exceedances over a given threshold rather than using the max over a certain period. However, the thresholds μ and the period T were chosen somewhat randomly, and can possibly have affected the results. The treshold μ should probably be higher.

b) Does VaR from the earlier period predict risk for the 2021 period in a reasonable way?

No. For the lower quantiles, the VaR from both periods seem to be pretty similar. However, as the quantiles increase, so does the difference between the risks of the two periods. For example, using the 99.99% daily VaR from the earlier period to predict the risks of the 2021 period is a very bad idea. The GP 99% daily VaR from the earlier period is 33.33% with a corresponding ES of 42.72%, compared to the 2021 period which is 98.64% and an ES of 182.42%.

c) Differences between VaR in the two periods.

The VaR estimates for the **sas21** dataset are considerably higher than the corresponding estimates for the **sas06** dataset for high quantiles. This seems quite reasonable. Both datasets includes a massive financial crisis, but the cause of them are totally different. Looking at the figures 1 and 2, we see two very different stock movements. The stock price during the 2008 financial crisis moves "steadily downwards" for an extended period of time, while in the stock price during the pandemic, we see an immediate drop, pretty much a straight vertical line, before it stabilizes quickly. It seems quite obvious that the **sas06** dataset reflects an actual financial crisis, with stock prices declining as a response to the economic situation, while the **sas21** dataset reflects a situation where the sudden drop in stock price is not necessarily based on any financial issues but rather on uncertainty and fear of how the pandemic might affect the economy (and the world) in the future. That the VaR estimates for this period are higher is a good thing, since our goal is to model risks of unexpected events.

d) Does ES contribute more useful information than VaR?

Value at Risk is a good metric for assessing the potential risks of an extreme event. However, it does not give any information about the magnitude of the loss given that an extreme event occurs. Expected Shortfall on the other hand, is the expected loss during a certain period, given that the loss exceeds our (in this case) VaR. In other words, it gives us an indication of how much we could potentially lose, given that an extreme event occurs. For example, say that we have calculated a 99% daily VaR of 10%. This means that we believe that there is a 1% risk of a loss exceeding 10% in one day. However, depending on the shape of the distribution of losses, especially the tails, this metric can be misleading. Two different distributions of losses can have the same 99% daily VaR, but if the distributions above our VaR behave differently, the actual loss given that our returns exceeds the VaR can be much different.

e) Is covid-10 a black swan?

The professor and statistician Nassim Taleb termed the coin ”black swan” in 2007. One of his criterias for something to be called a ”black swan” is ”First, it is an outlier, as it lies outside the realm of regular expectations, because nothing in the past can convincingly point to its possibility...”
The occurrence of pandemics and epidemics are a well documented and researched area and have been shown to be the number 1 mass killer of all time. We know that pandemics break out from time to time and that they can have a catastrophical impact. For example, the Spanish Flu in 1917 is estimated to have killed between 20 and 50 million people. Moreover, warnings about future pandemics have been issued for years by health experts, professors and even politicians. Saying that the Covid-19 pandemic is ”something that lies outside the realm of regular expectations, because nothing in the past can convincingly point to its possibility” seems like a missjudgement.