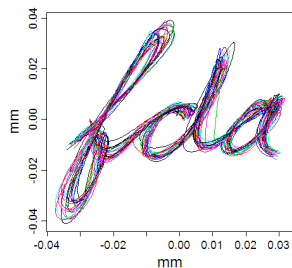


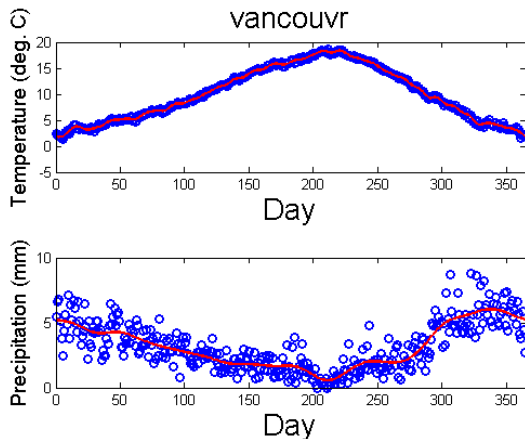
# Back to "What is Functional Data"

Or *What isn't Functional Data?*



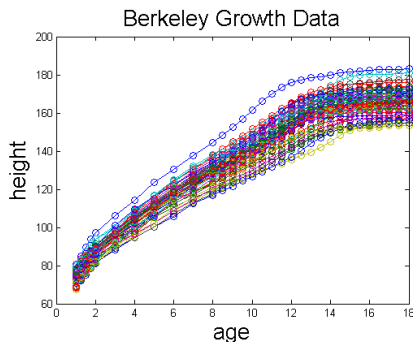
Do my data need to look this good?

Data may be measured more noisily



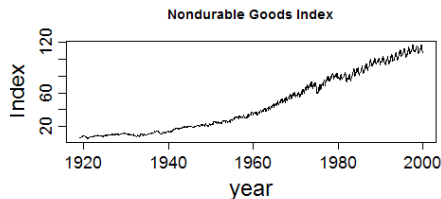
We need to find the *smooth* process under the data.

## Data may be measured more sparsely

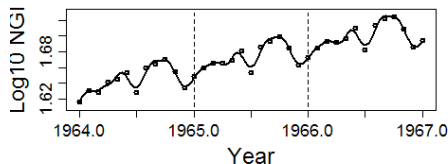


- ▶ Data are low noise but low-resolution
- ▶ Measured at unequal intervals
- ▶ We know that the curves must always increase

# We may not have repeated measurements



- ▶ Single time series
- ▶ But, repeated "shapes" over each year
- ▶ We can use this to investigate variation, development, dynamics



# Necessities for Functional Data

- ▶ must believably derive from a smooth process
- ▶ process should not be easily parameterizable (should not be able to write down a formula)
- ▶ enough data to resolve the essential features of the process (peaks, zero-crossings, speed... will depend on application)
- ▶ some repetition in the process
- ▶ do not need equally-spaced or perfect measurements

# Common Sources

medical monitoring: EEG, ECG, fMRI, blood pressure ...

medical tests: HIV antibodies, flu screens...

biology: animal behavior (whale songs, fly egg-laying...)

environmental monitoring: weather, pollution, solar radiation,  
traffic ...

optotrack experiments: psychology/physiology

economics/marketing: macro-trends, futures markets

web data: e-bay auction prices, google trends

# Essential Questions

Or what can FDA do for me?

- ▶ How do we go from discrete to functional data?
- ▶ How do we describe random variation in functional data?
- ▶ How do we decide if groups of functional data are different?
- ▶ How do we relate functional data to other data? To other functional data?
- ▶ What is special about functional data?
  - ▶ Aligning functions (registration)
  - ▶ Use of rates of change (dynamics)

# Approximate Class Agenda

1. From data to functional data
  - ▶ Basis expansions and smoothing
  - ▶ The `fda` library
  - ▶ Positive and monotone smoothing
2. Exploring Functional Data
  - ▶ Means, variances, covariances
  - ▶ Functional PCA
3. Functional Linear Models