University of BRISTOL

COMS 30115

Rendering: Raytracing

Carl Henrik Ek - carlhenrik.ek@bristol.ac.uk

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http://www.carlhenrik.com

Introduction

Info

- Coursework
  - Find a partner

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- Reddit
  - its reddit, cat gifs are OK etc.
  - if you found an awesome graphics resource etc.

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  - its annonomous, tell me if I'm an idiot and the lecture/coursework being stupid

Today

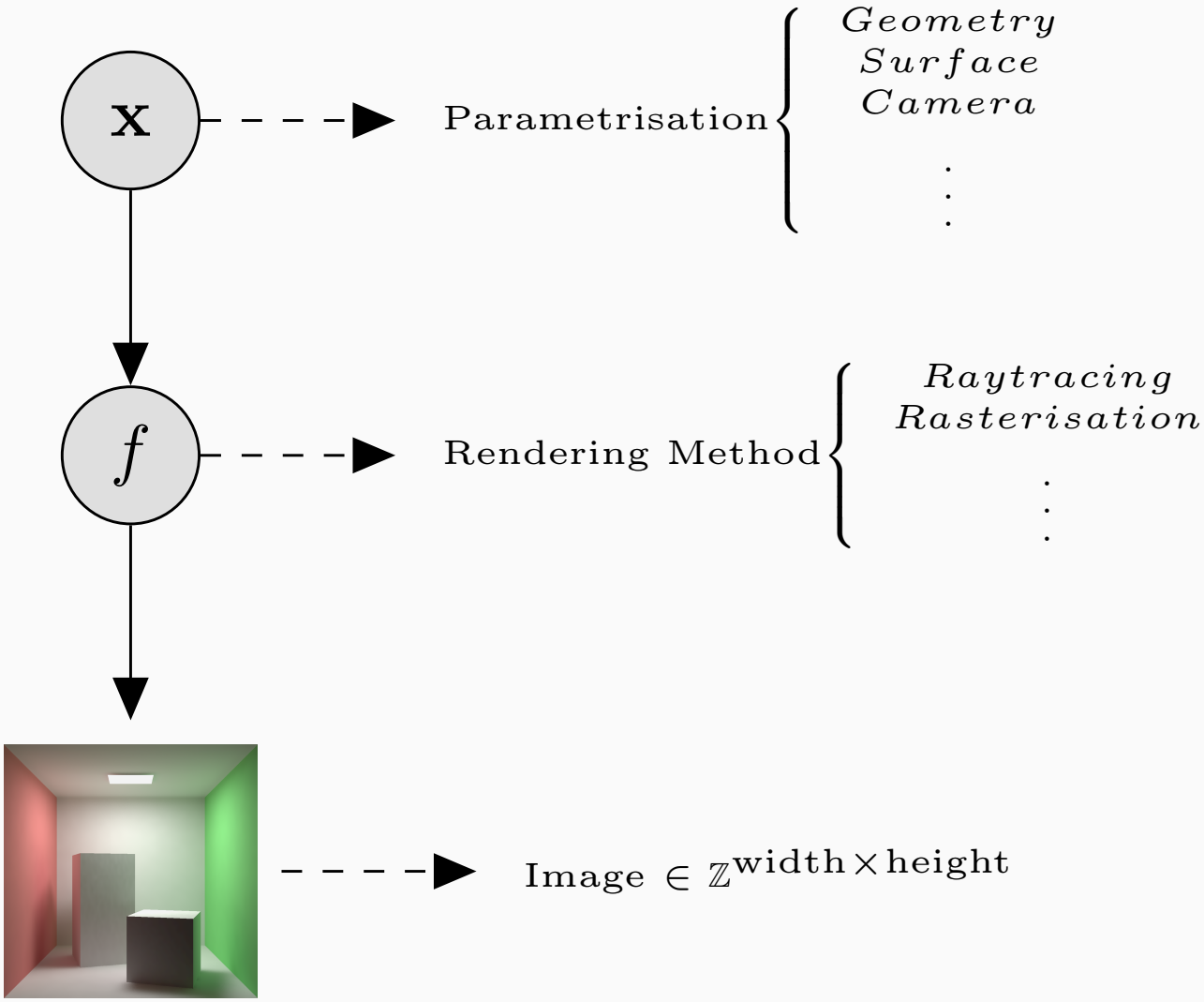
- Generative Model and Assumptions
- First rendering method
  - Raytracing/Raycasting
  - Cameras
  - Intersection calculations
- After today you should be able to start with Lab 1 (sorry)

Scratchapixel

- Introduction to Ray Tracing: a Simple Method for Creating 3D Images
- An Overview of the Ray-Tracing Rendering Technique

Generative Model

Generative Model



Images

- Rectangular grid of elements
- Parametrization
  - Location (x, y)
  - Colour p

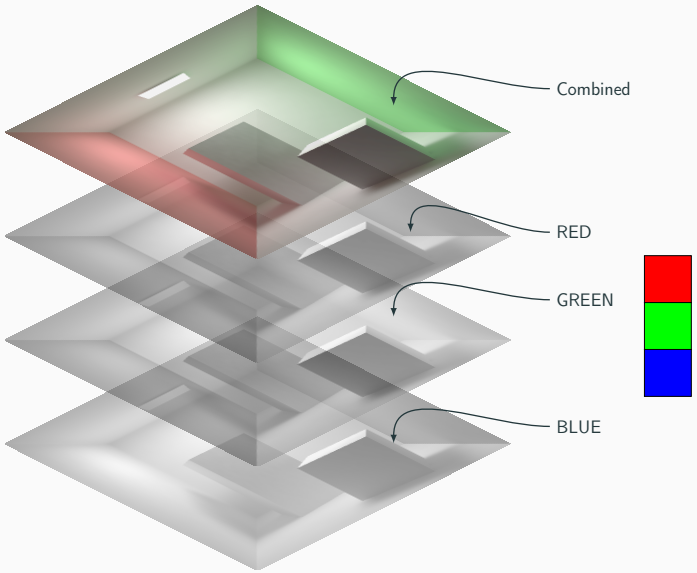
Images

Encoding

- Indexed Images
  - GIF, PNG have this format
  - Palette (look-up)
  - Efficient if few colours in image
  - Older hardware implemented this
- (True) colour Images
  - Directly stores Colour values
  - Simpler, not limited to palette size
  - Less efficient for low colour images

Pixels

- Colour
  - RGB - `RED`, `GREEN`, `BLUE`
  - HSV - `HUE`, `SATURATION`, `VALUE`
- Nr of bits known as the depth
- What is the effect of the parametrisation?
- Similarity



Pixel Parametrisation

The Dress that broke the Internet[1]

[1] https://en.wikipedia.org/wiki/The_dress

Colour

- Psychological
  - colours encode meaning
  - meaning is highly subjective
- Physiological
  - internal processes of colour
  - optical illusions etc.

Colour

- Psychological
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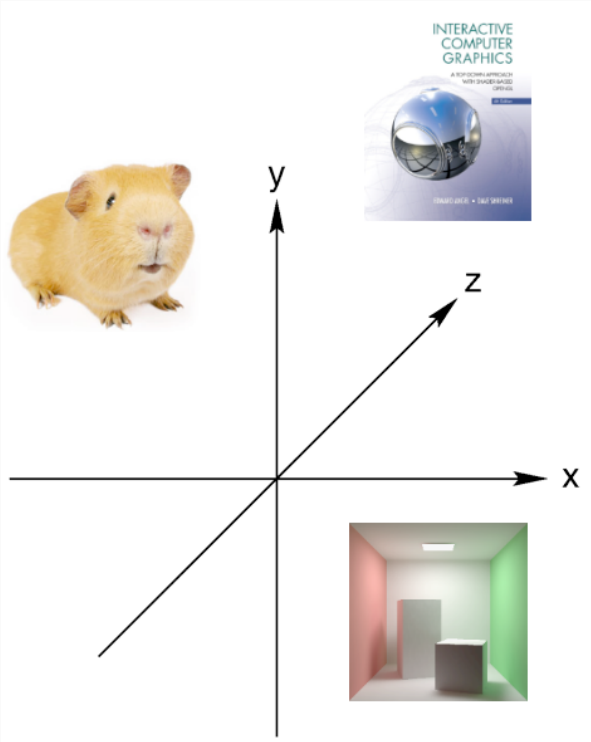
- *an image is interpreted by us, we never see the pixels*

Colour like a coder

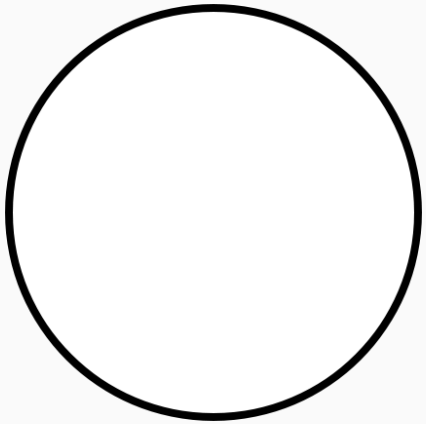
Red is not a colour it is `0x00ff0000`

Image Parametrisation

- RGB pixel $p \in \mathbb{Z}^3$
- Image as a point in a vector space
- $I \in (\mathbb{R}^3)^{width \times height}$
- Typical Image size
  $1920 \times 1080 \times 3 = 6220800$



Manifolds



"In mathematics, a manifold is a topological space that near each point resembles Euclidean space"[2]

[2] http://en.wikipedia.org/wiki/Manifold

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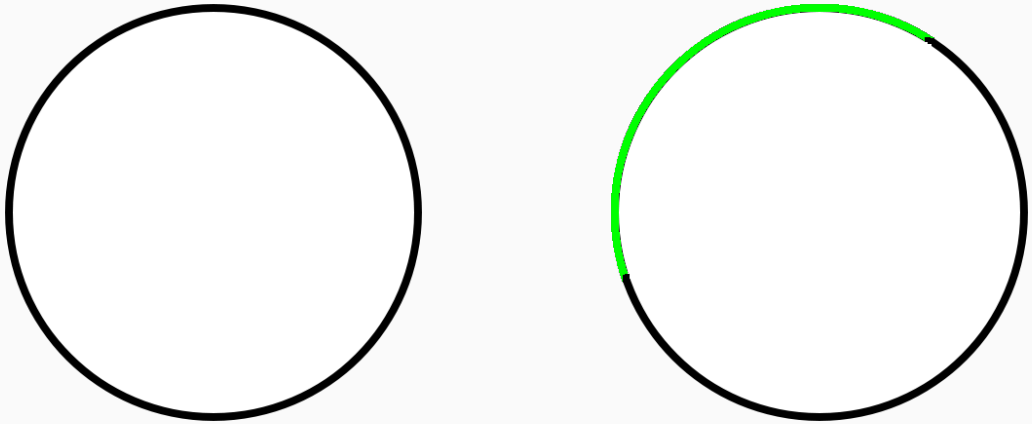

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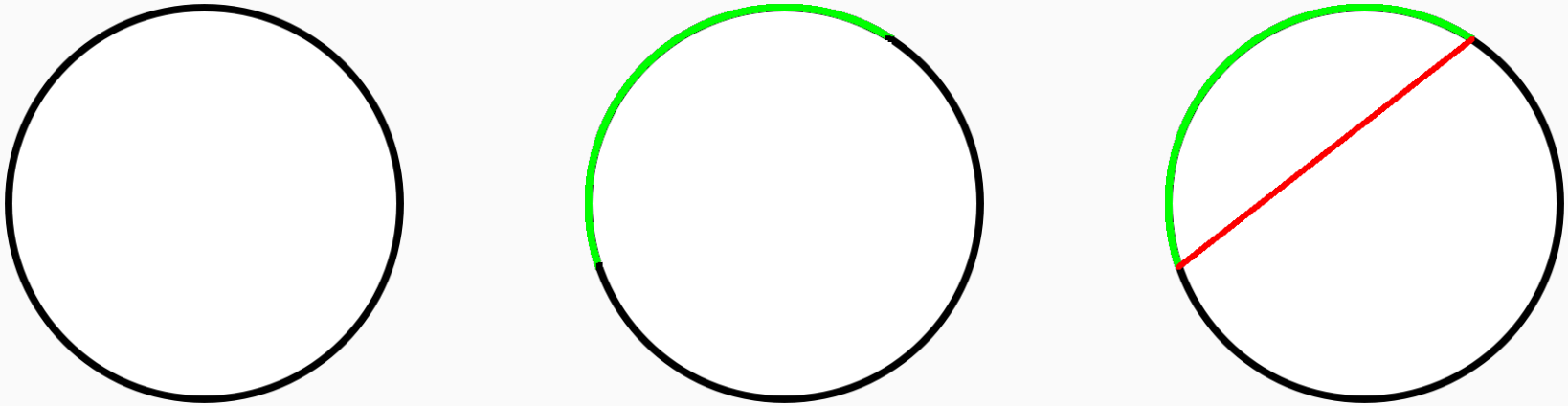

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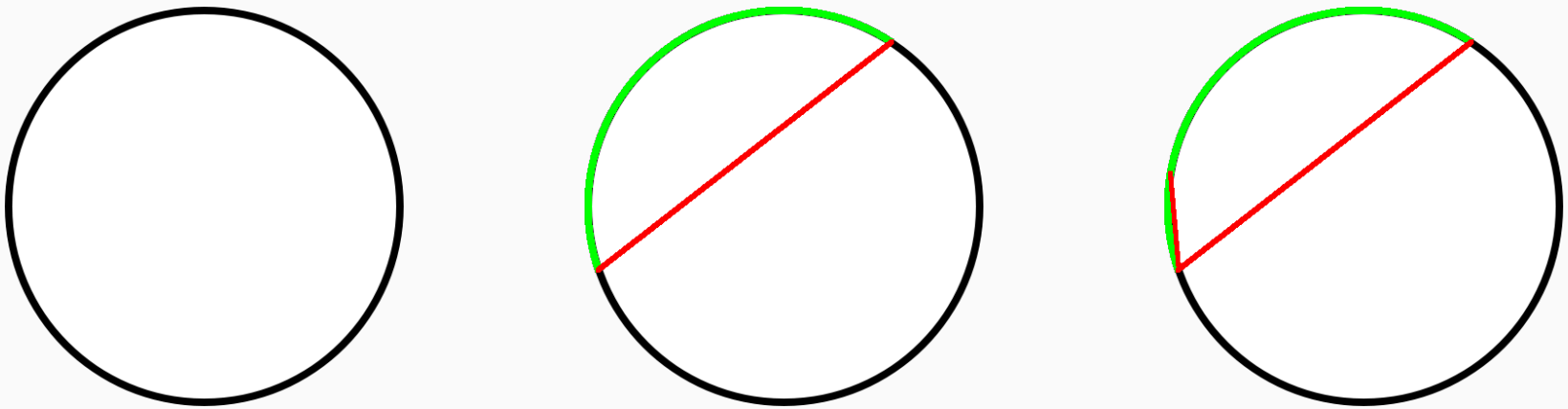

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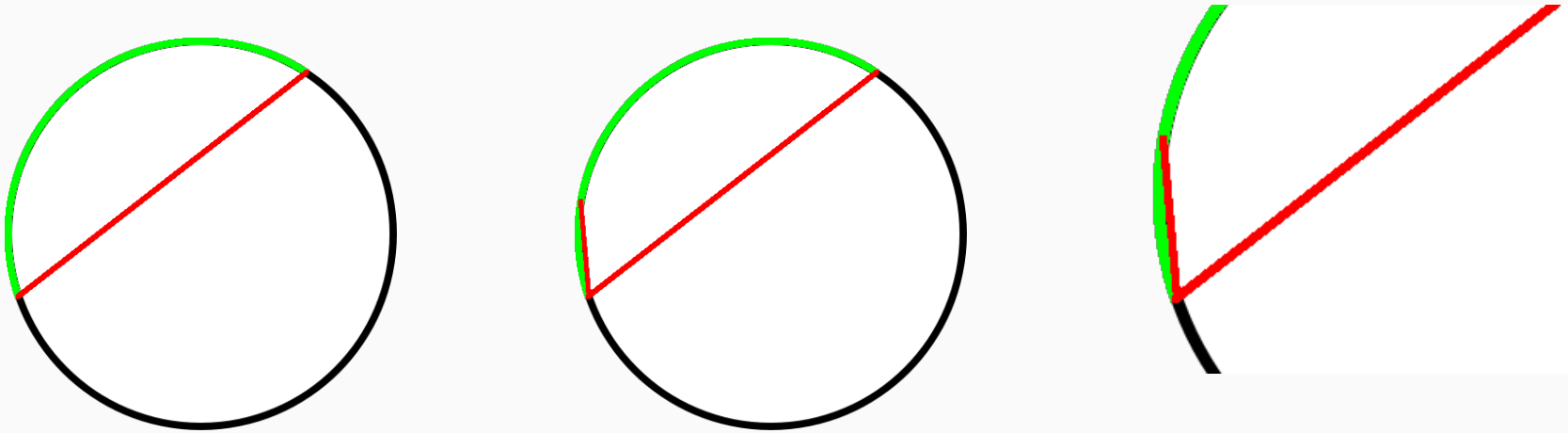

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Parametrisation

- Image space is a manifold in space of matrices with positive values
- Degrees of freedom is coordinates along manifold
    - Each parameter configuration generates valid image
- *how can we find these parametrers?*

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  - Geometry Light bounces of surfaces to eye/camera, i.e. we see surface
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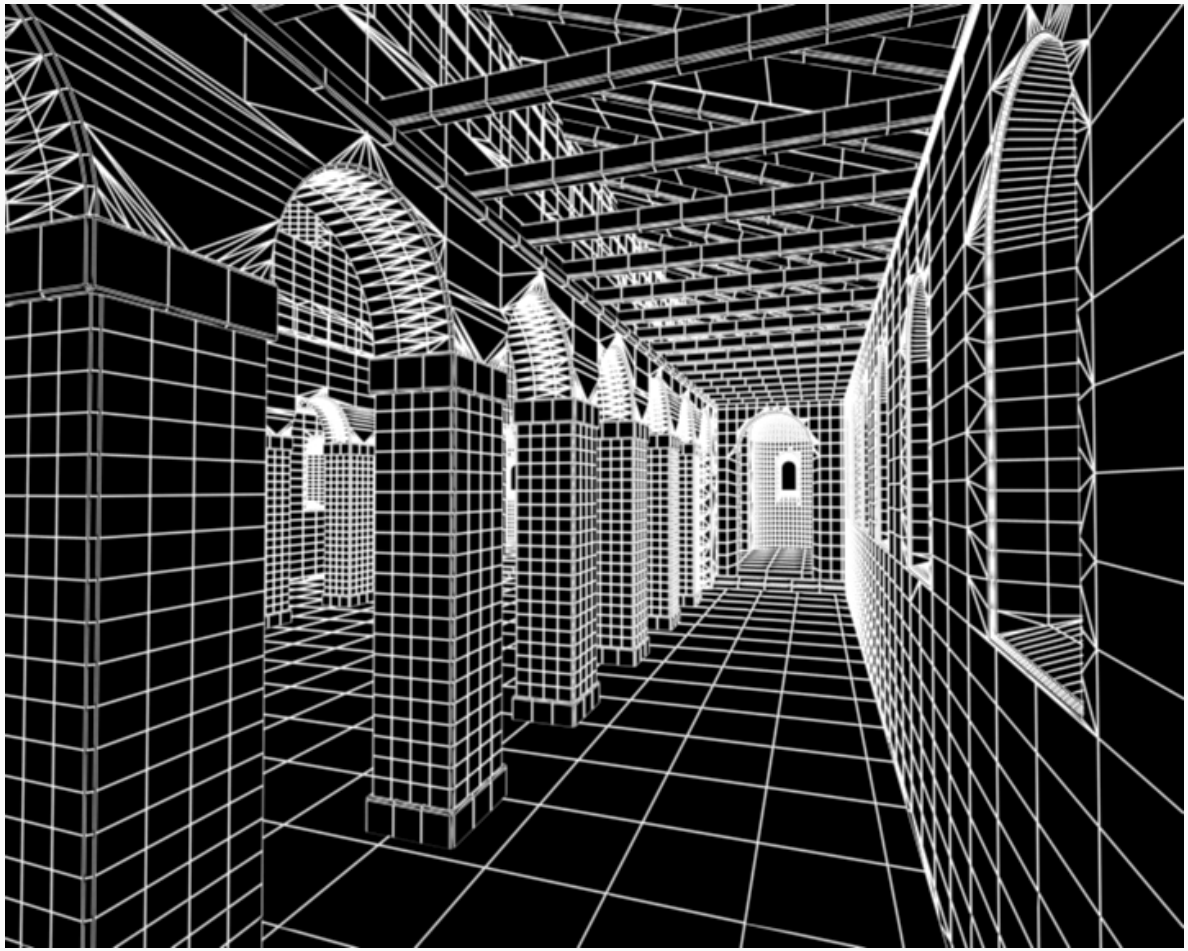
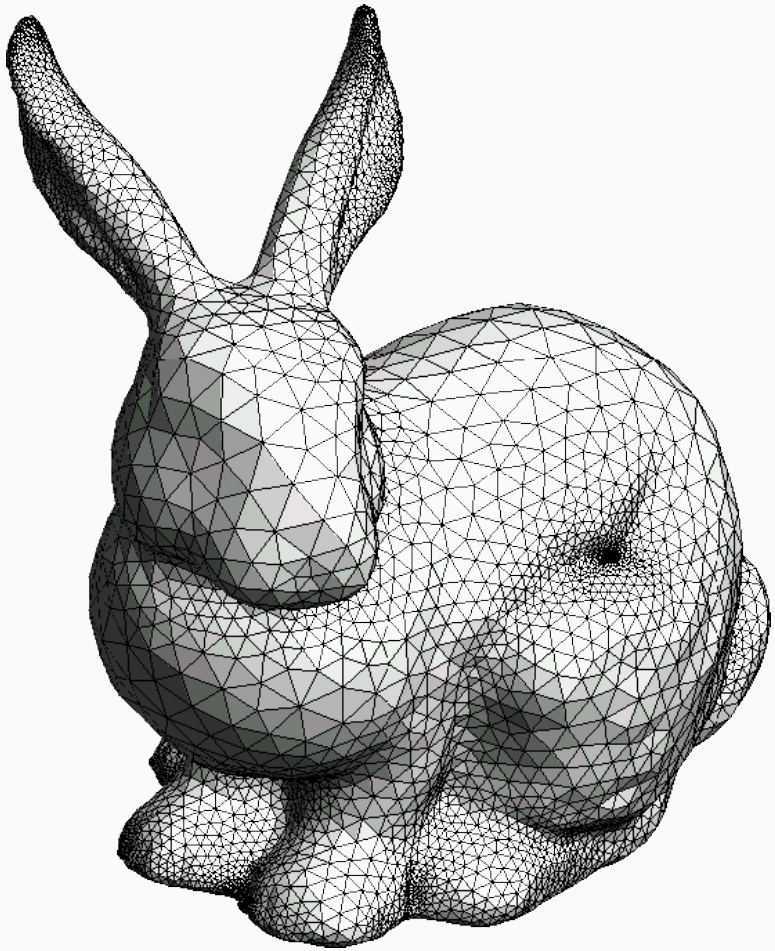
- We will study both the parametrisation and the generative mapping

Parametrisation

Geometry

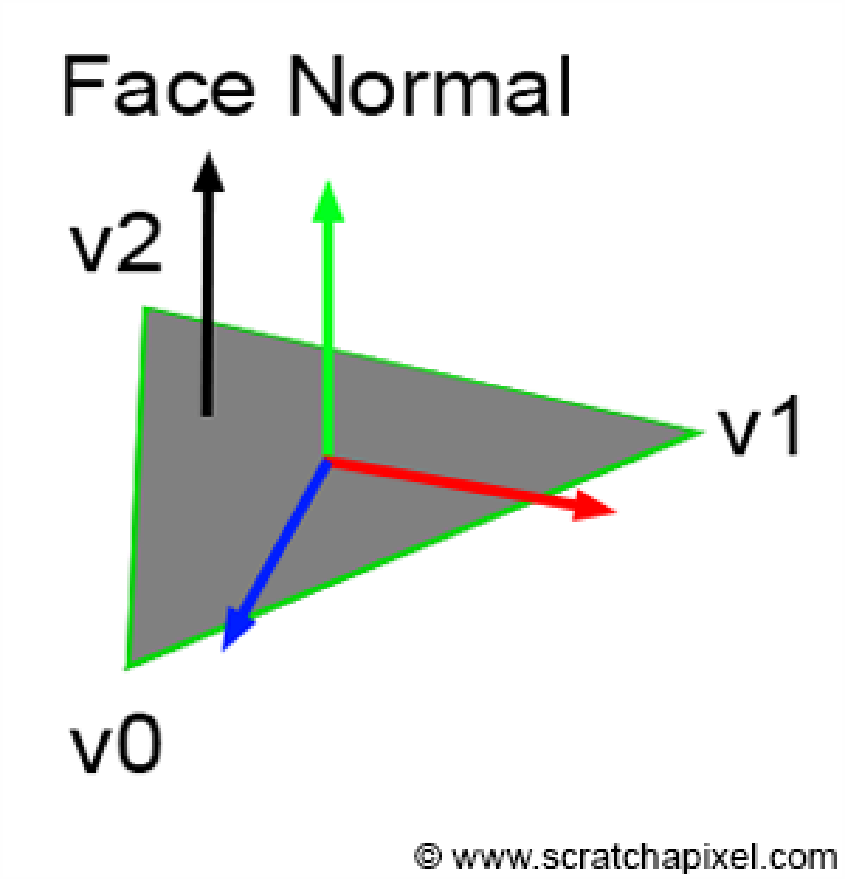


Meshes[3]

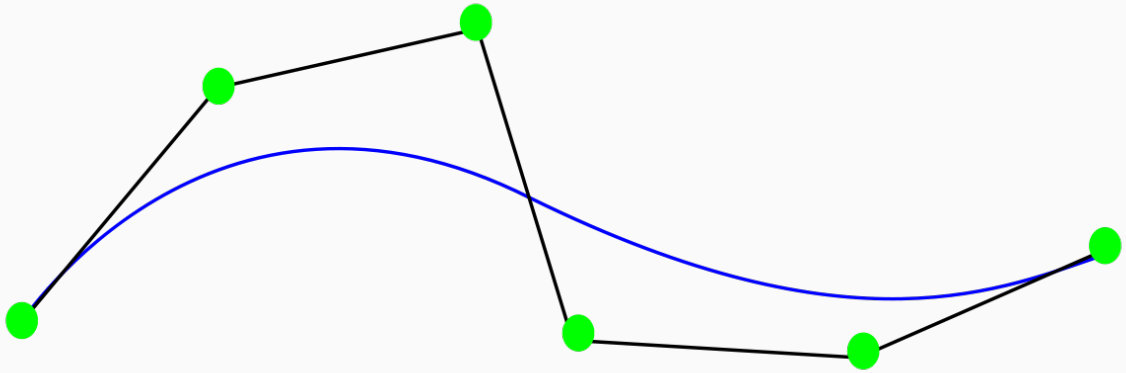


[3] https://en.wikipedia.org/wiki/Stanford_bunny

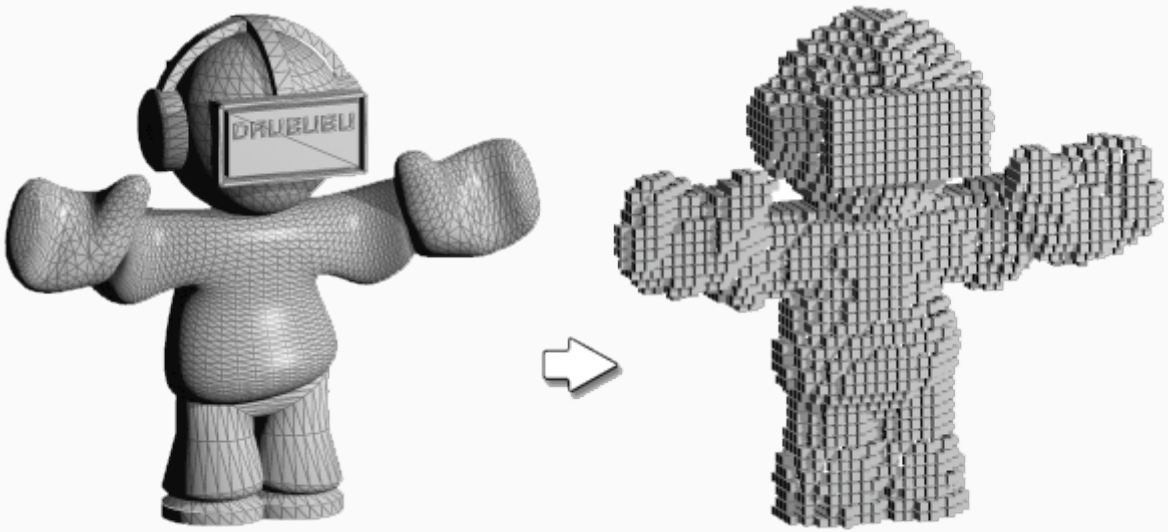
Triangles



Non-planar Geometry



Voxels[4]



[4] https://www.gamersnexus.net/gg/762-voxels-vs-vertexes-in-games

Assumptions

- The world is locally flat
- Parametrise each object as a set of triangles
- Each object is just an empty hull
- Everything has a fixed resolution

Illumination

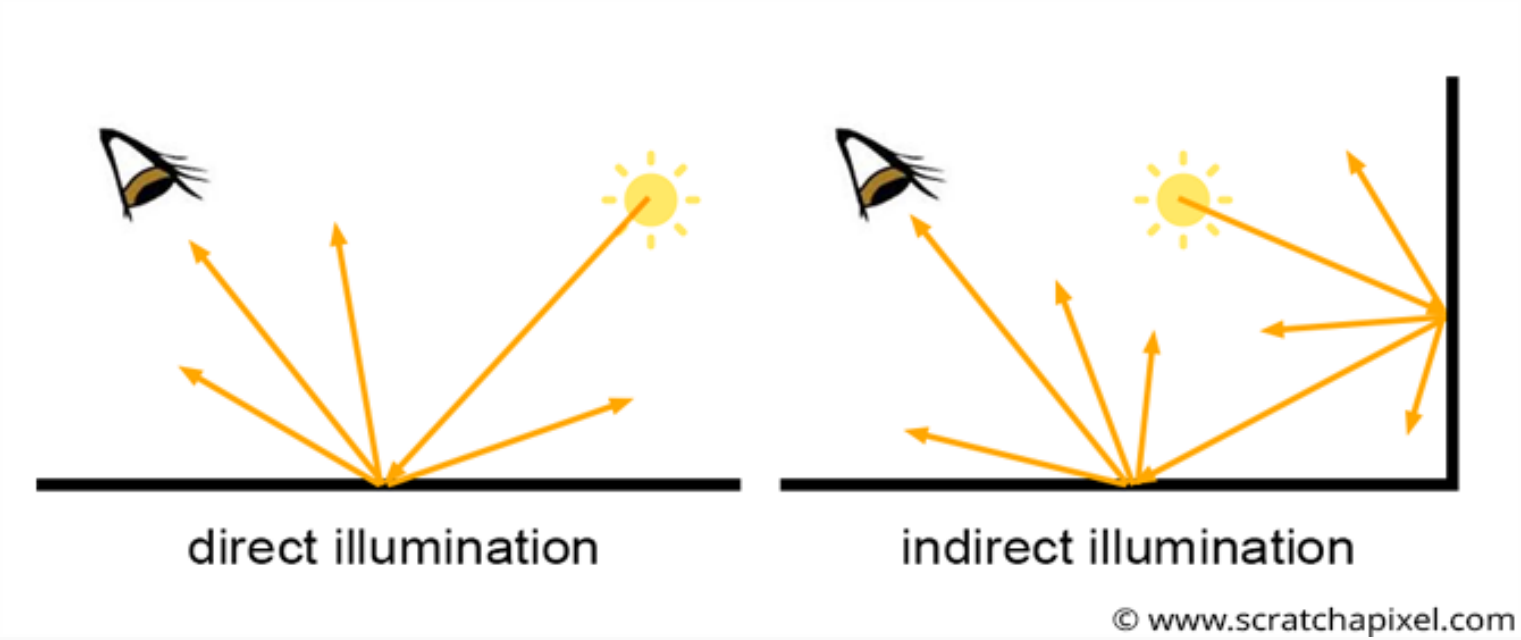
Light

- generated from lightsources (i.e. sun, lamps)
- "bounces" around in the world
- surface visible when hit by light

direct source $\Rightarrow$ surface

indirect surface $\Rightarrow$ surface

Illumination



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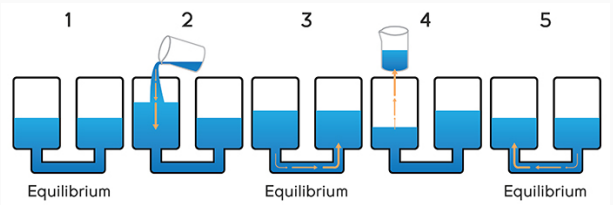
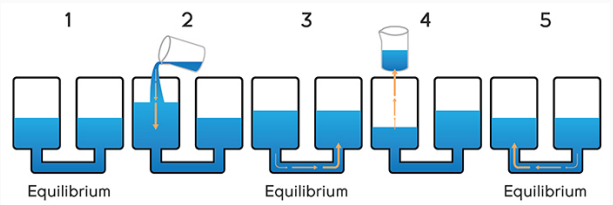


Transport Problem

- Illuminating/Rendering $\Rightarrow$ compute interaction between incident light and surface
- Compute transportation
- Conservation principle
  - nothing is created except for at the source
- light is *very* fast
- Assume that it is at equlibrium



Rendering Equation [5]

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) d\omega_\Psi$$

- Formulates transport problem of light
- Encapsulates "all" rendering techniques
- Intractable

[5] Kajiya, J. T. (1986). The rendering equation. In , ACM Siggraph Computer Graphics (pp. 143–150)

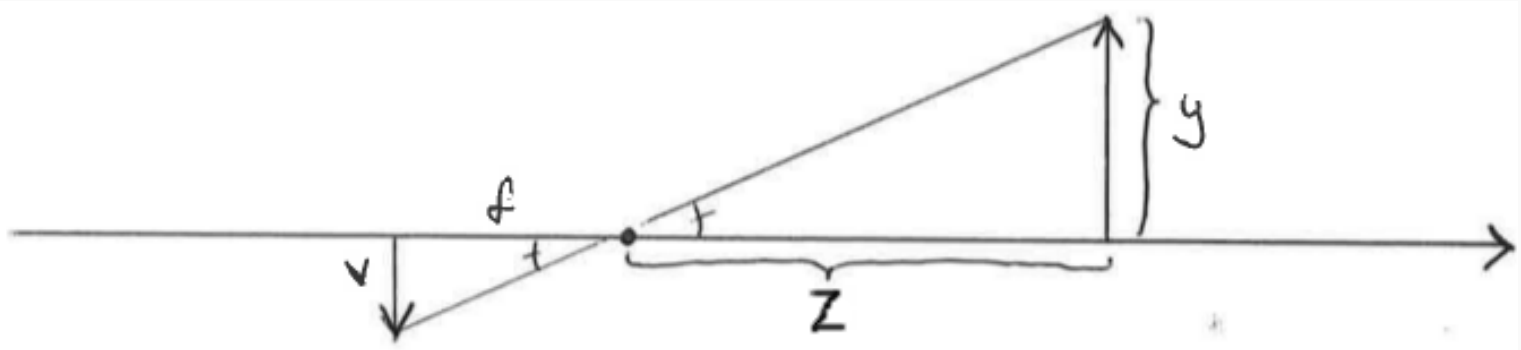
Assumptions

- Only direct lighting
- Indirect lighting can be simulated by ambient light
- All surfaces are perfectly diffuse
    - look the same from each angle
- All lights are points

Camera

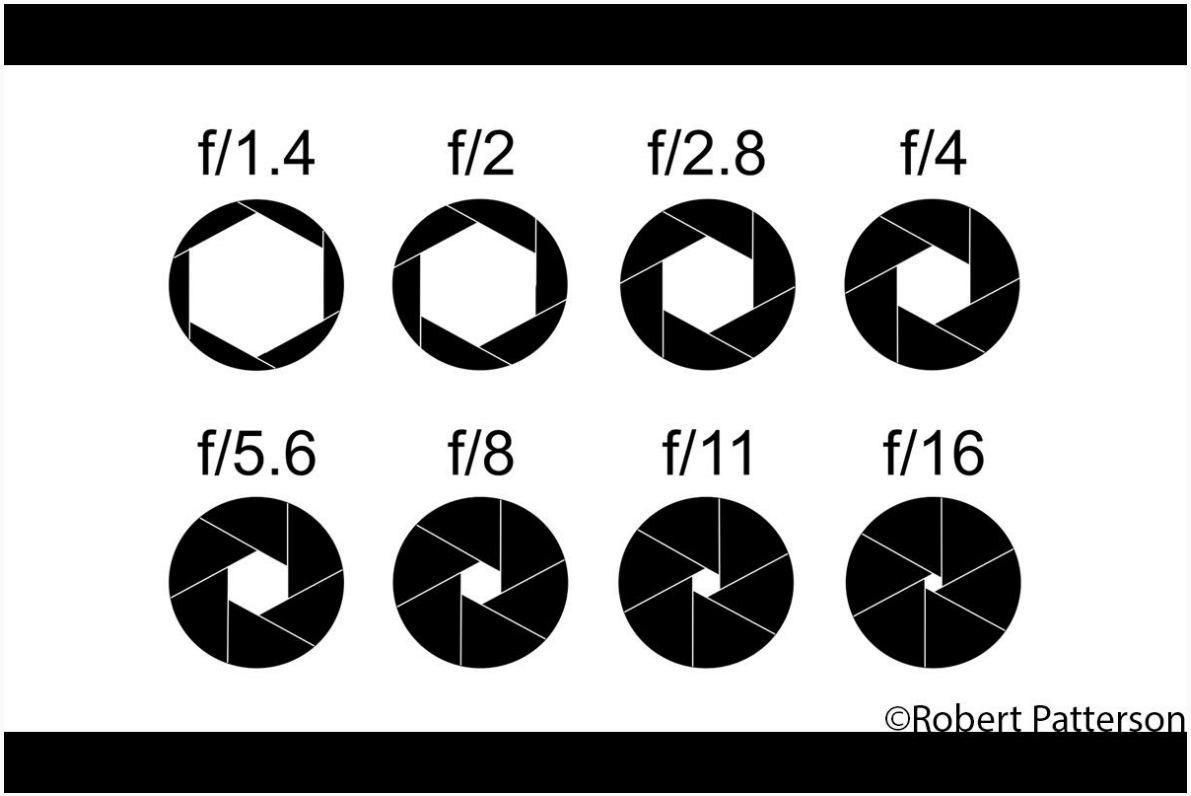
- Cameras create images
- Lens
  - captures rays
- Focal length
- Apature

Pinhole camera



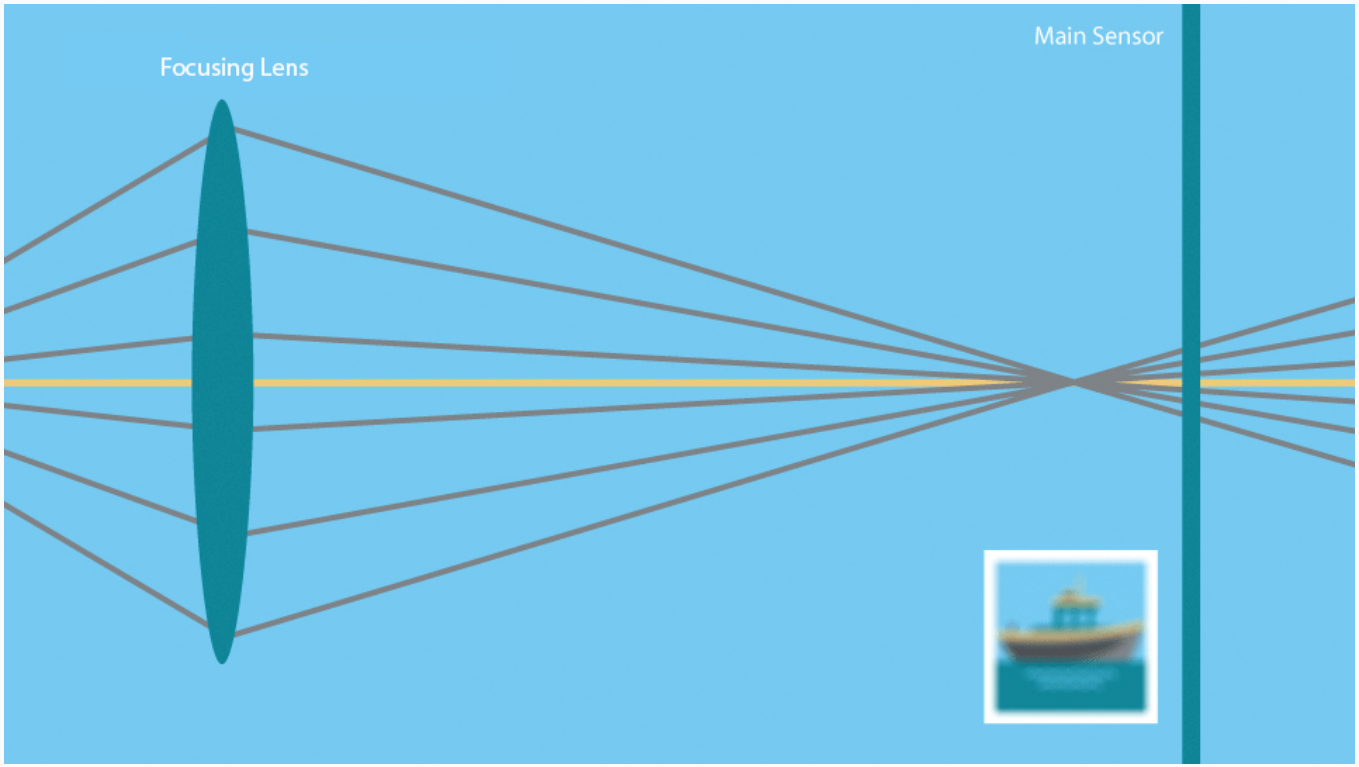
- Simplest form camera, where one ray of light lights up each pixel
- Defined by a *location*, *focal length*, *view-direction* and *up-direction*
- is this a realistic camera?

Apature[6]



[6] https://geneseodigitalphotography.wordpress.com/learn-about-your-camera/reference-sheets/aperture/

Depth-of-Field[7]



[7] https://www.bhphotovideo.com/explora/photography/tips-and-solutions/how-focus-works

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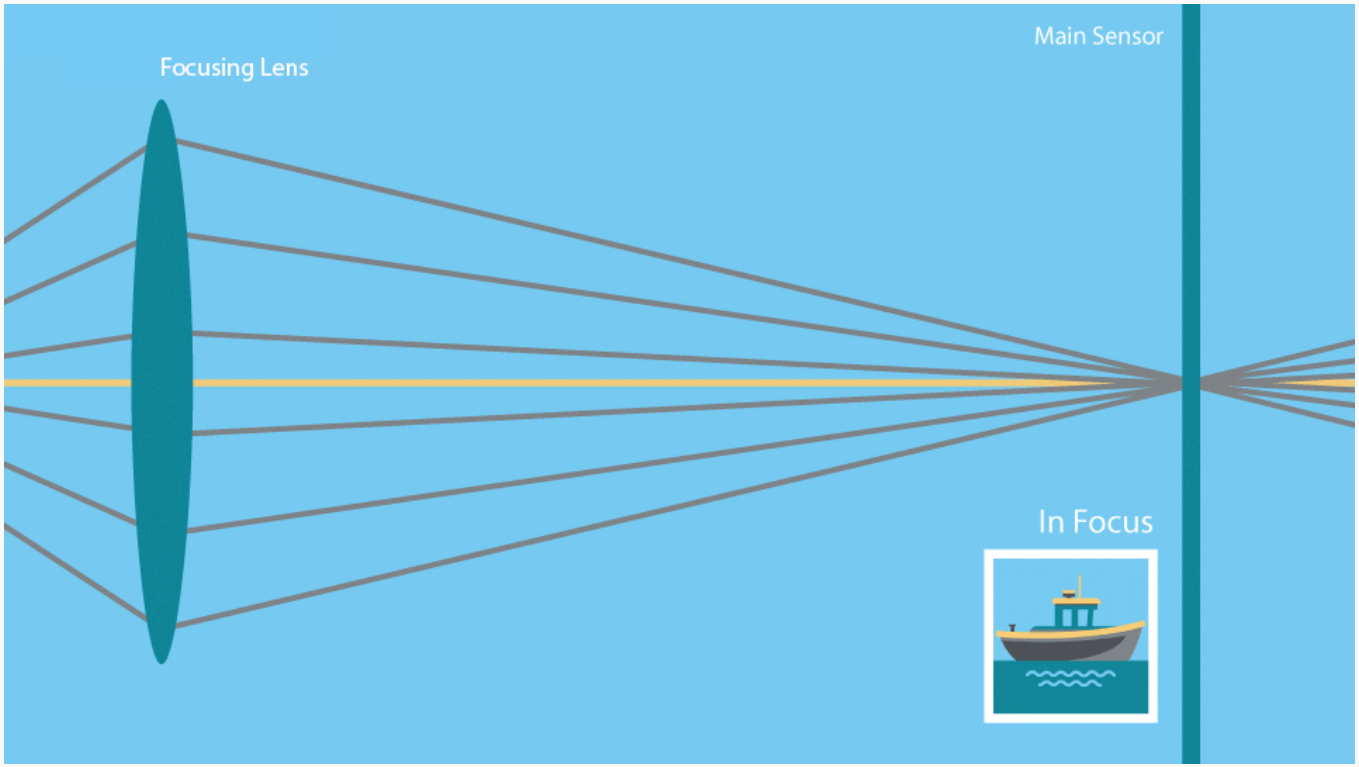

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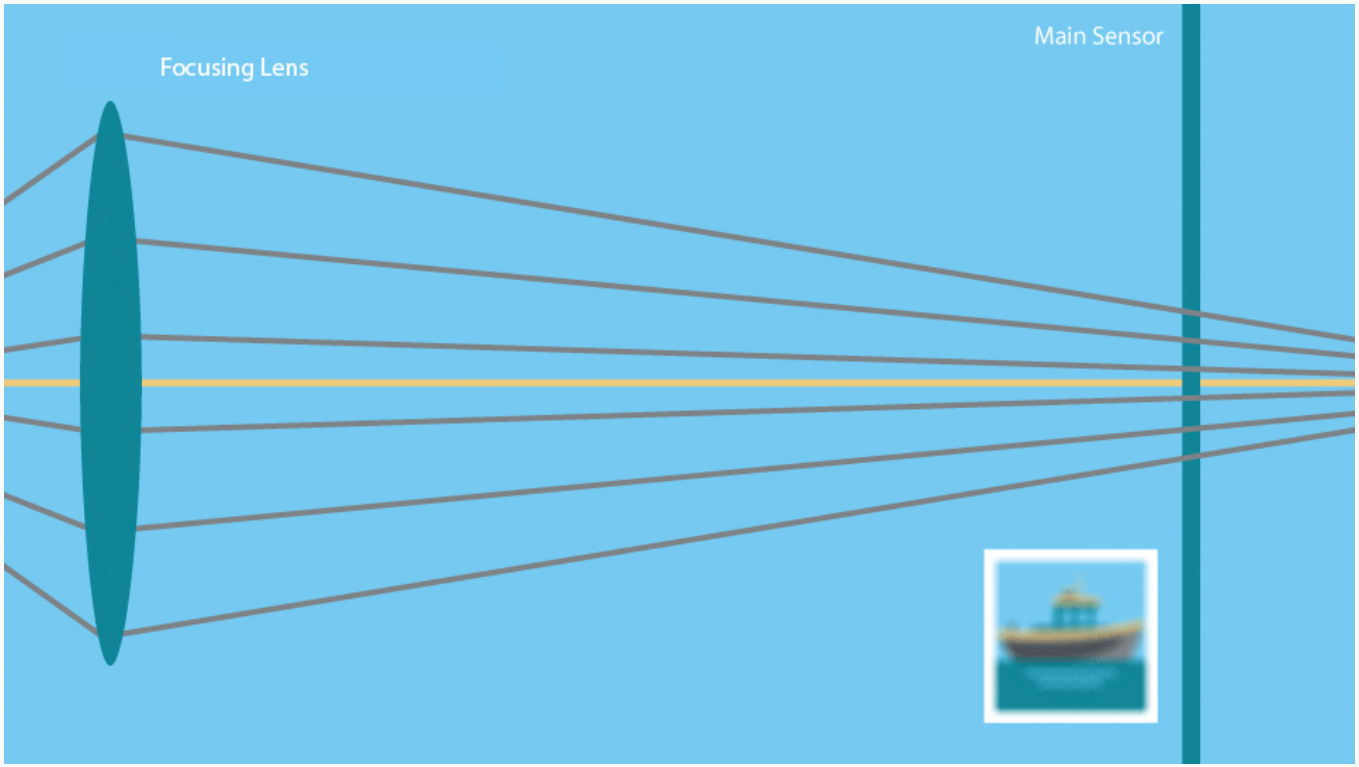

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Lens Flare

Assumptions

- Pinhole camera
  - everything is in focus
  - point aperture
- No lens
  - no lens flare
  - no lens distortion

Raytracing

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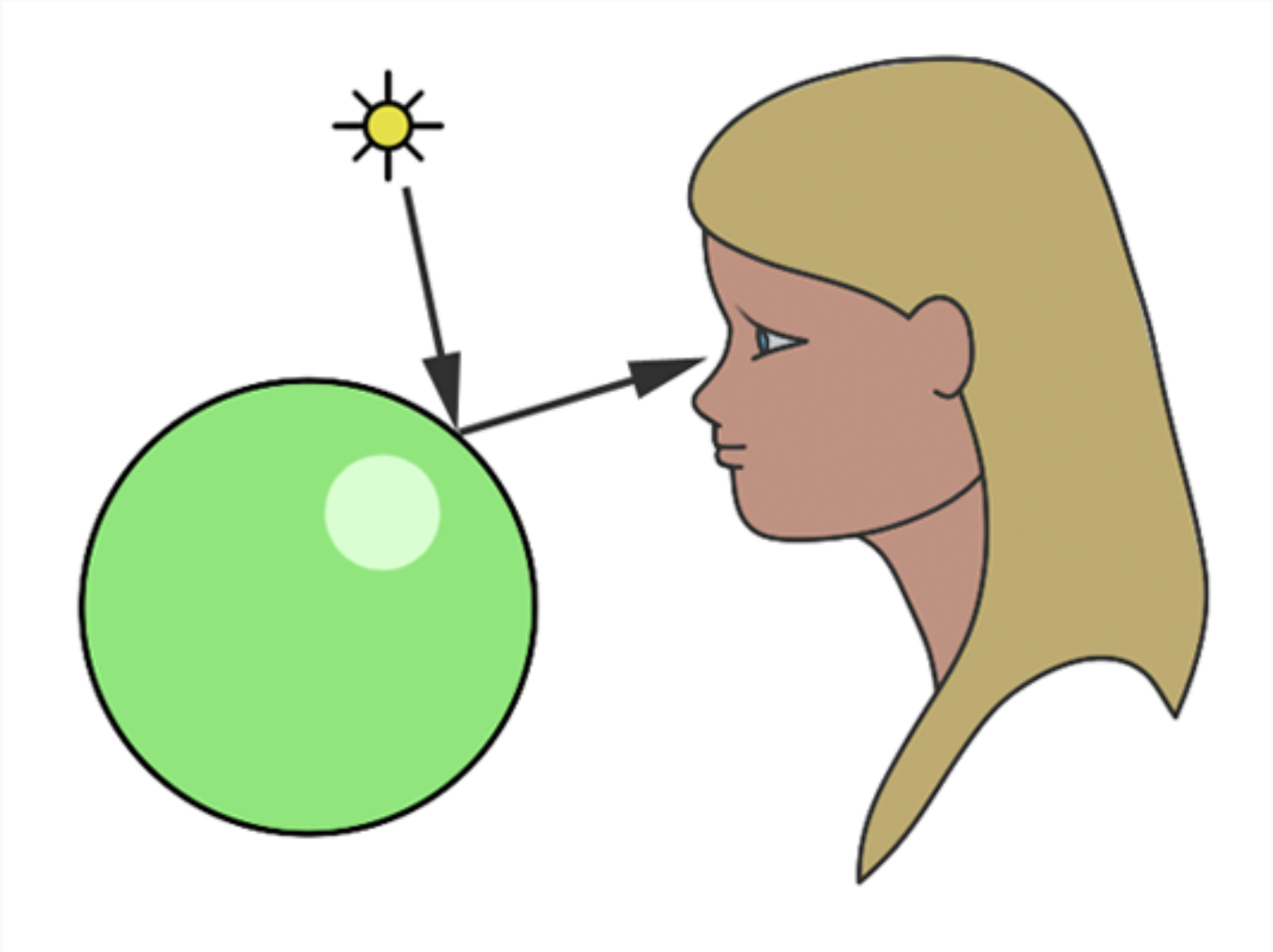

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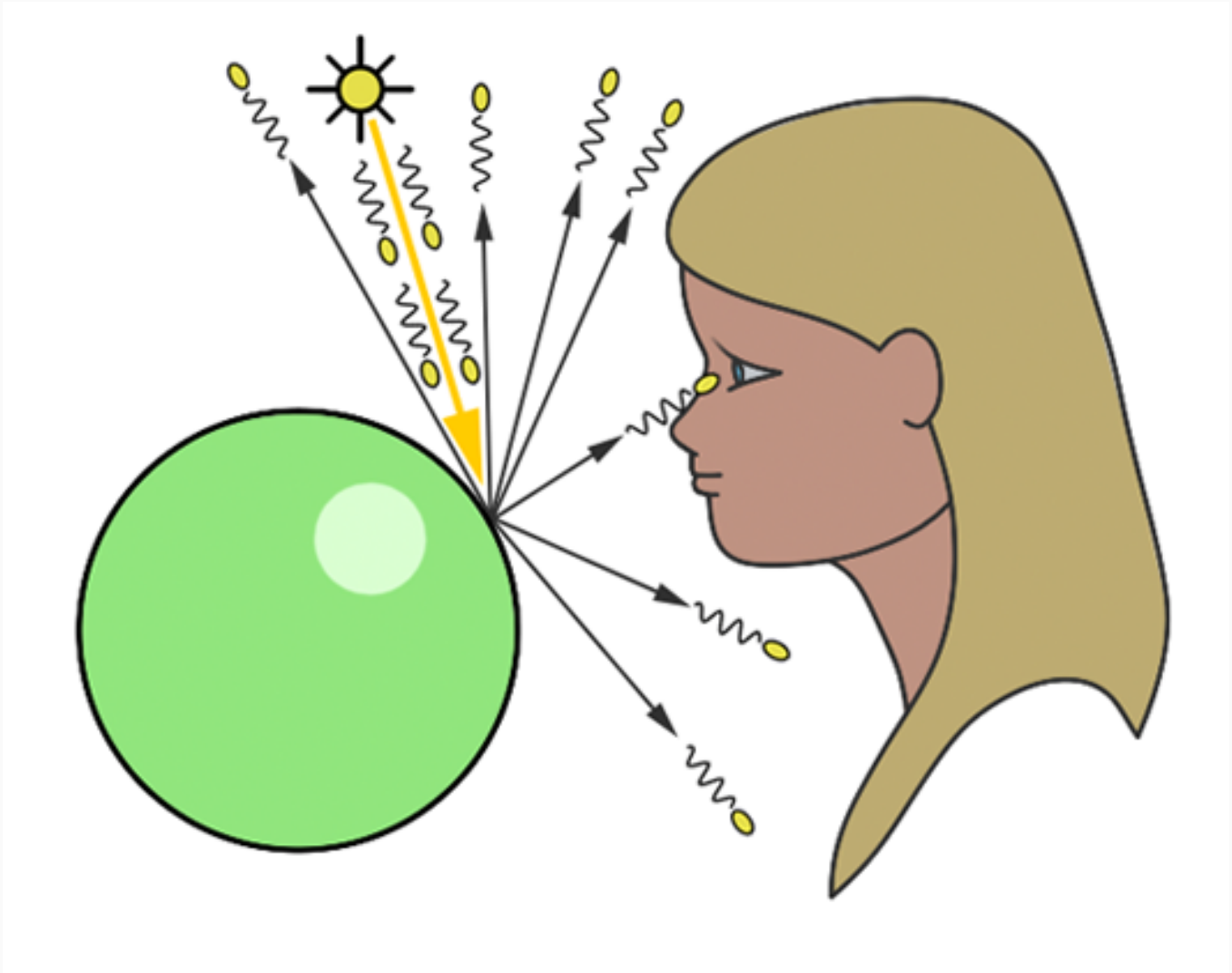

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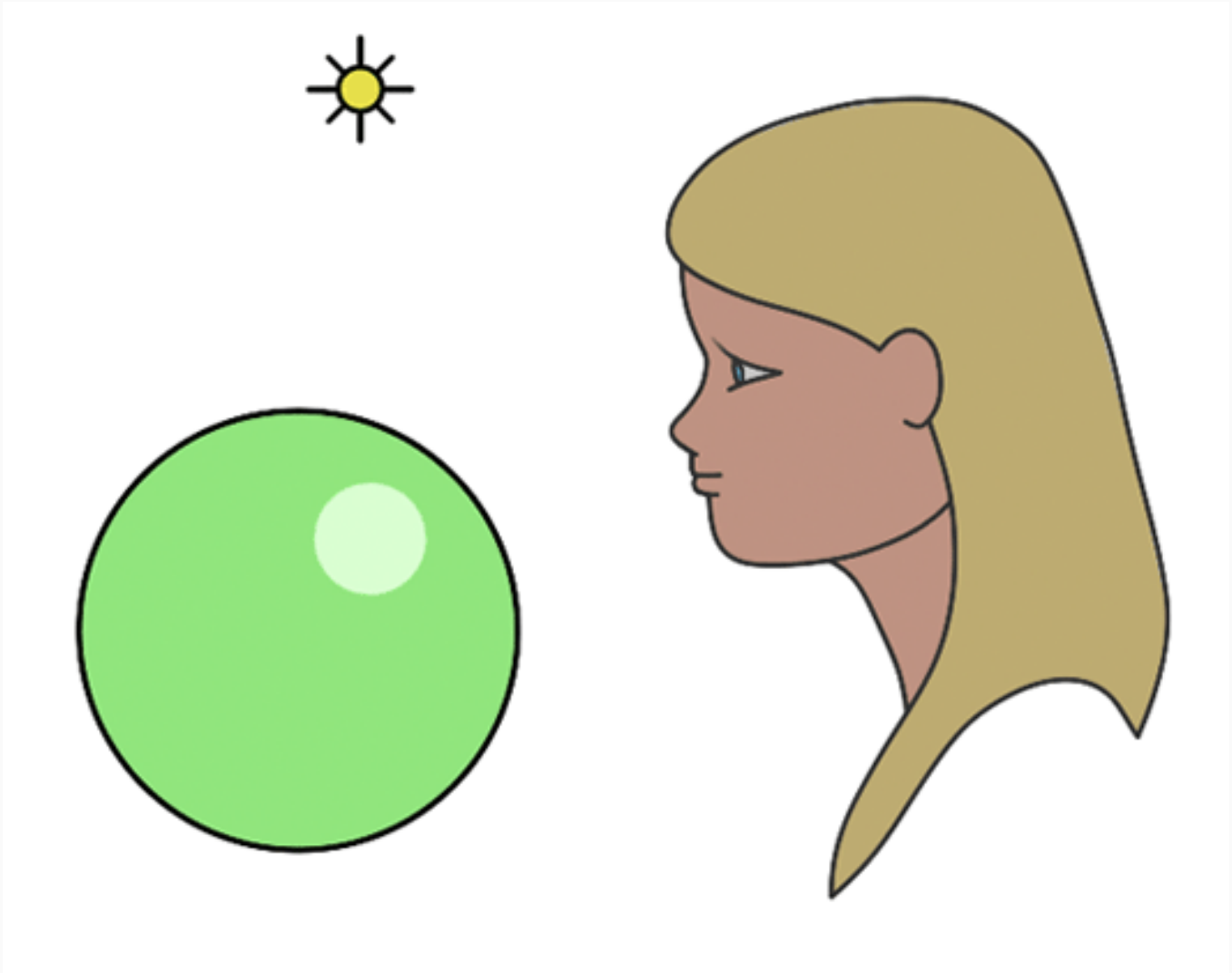

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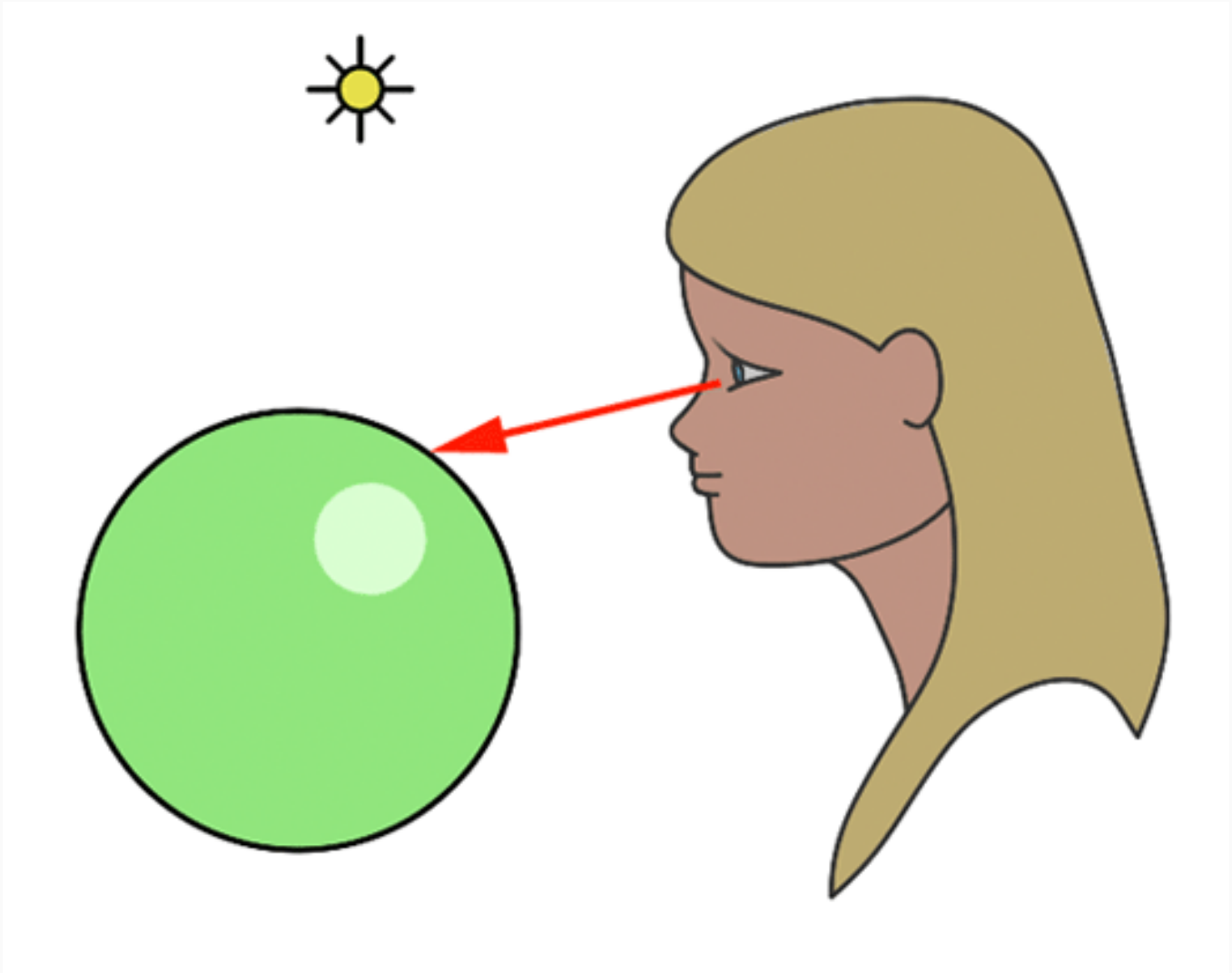

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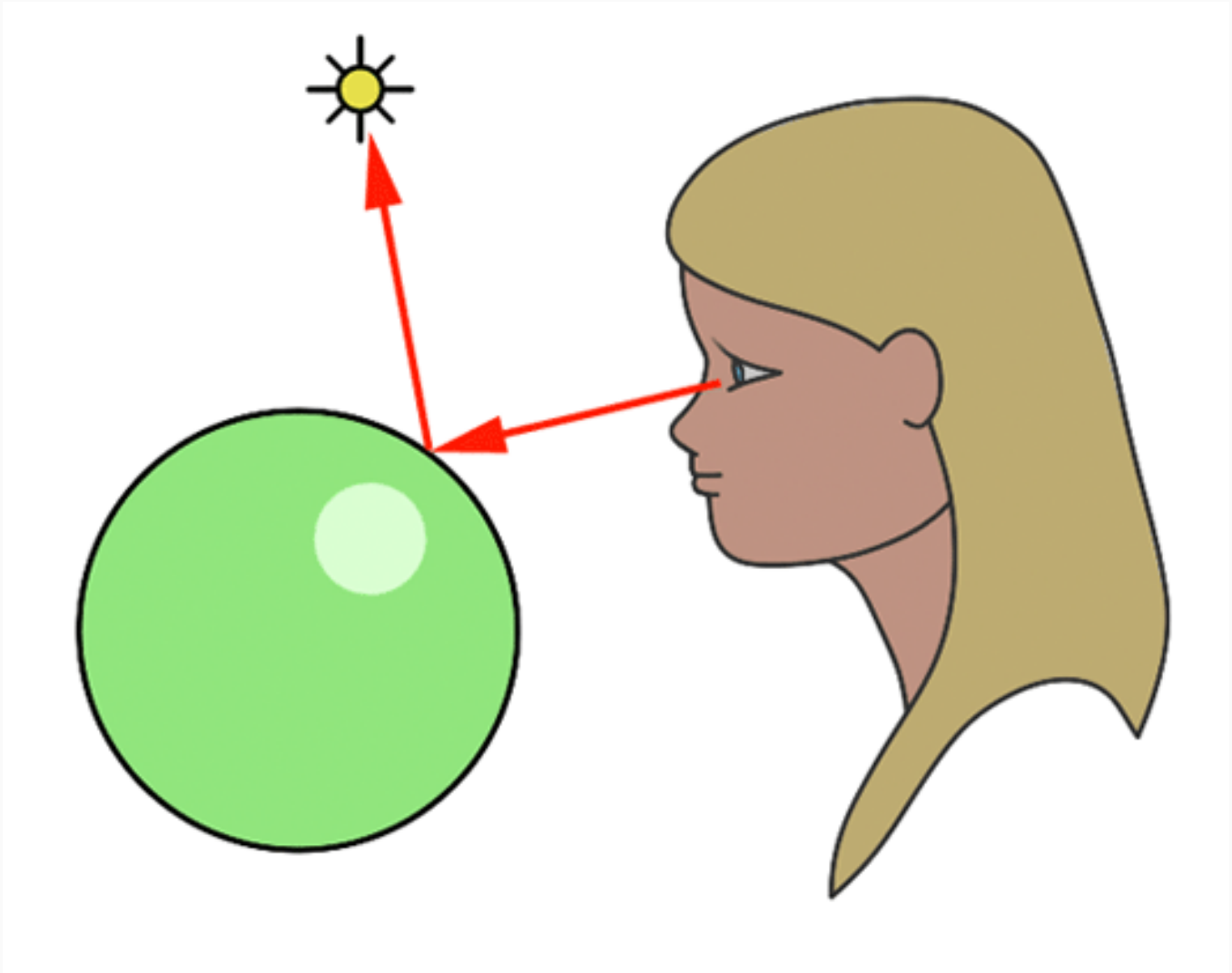

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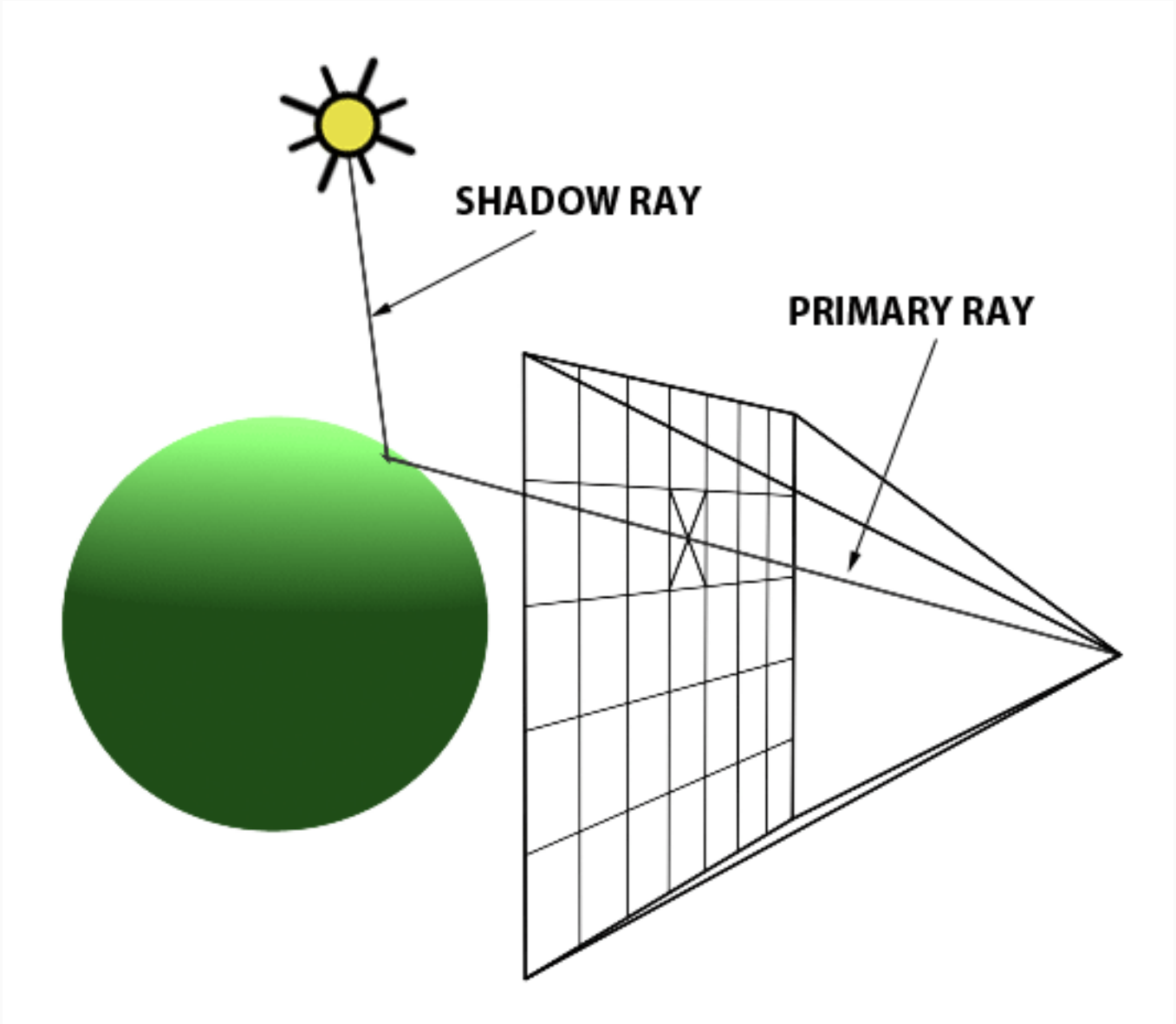

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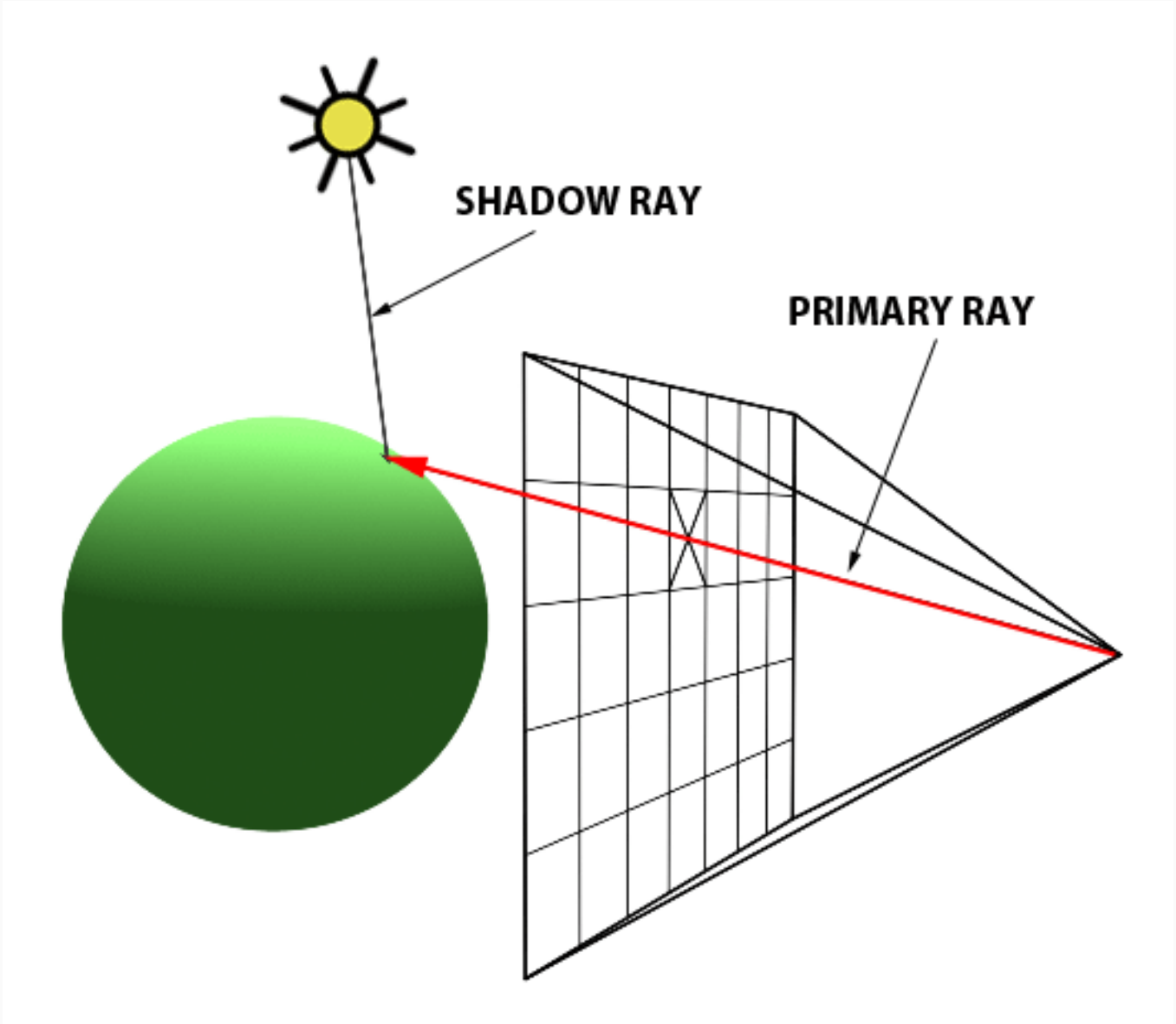

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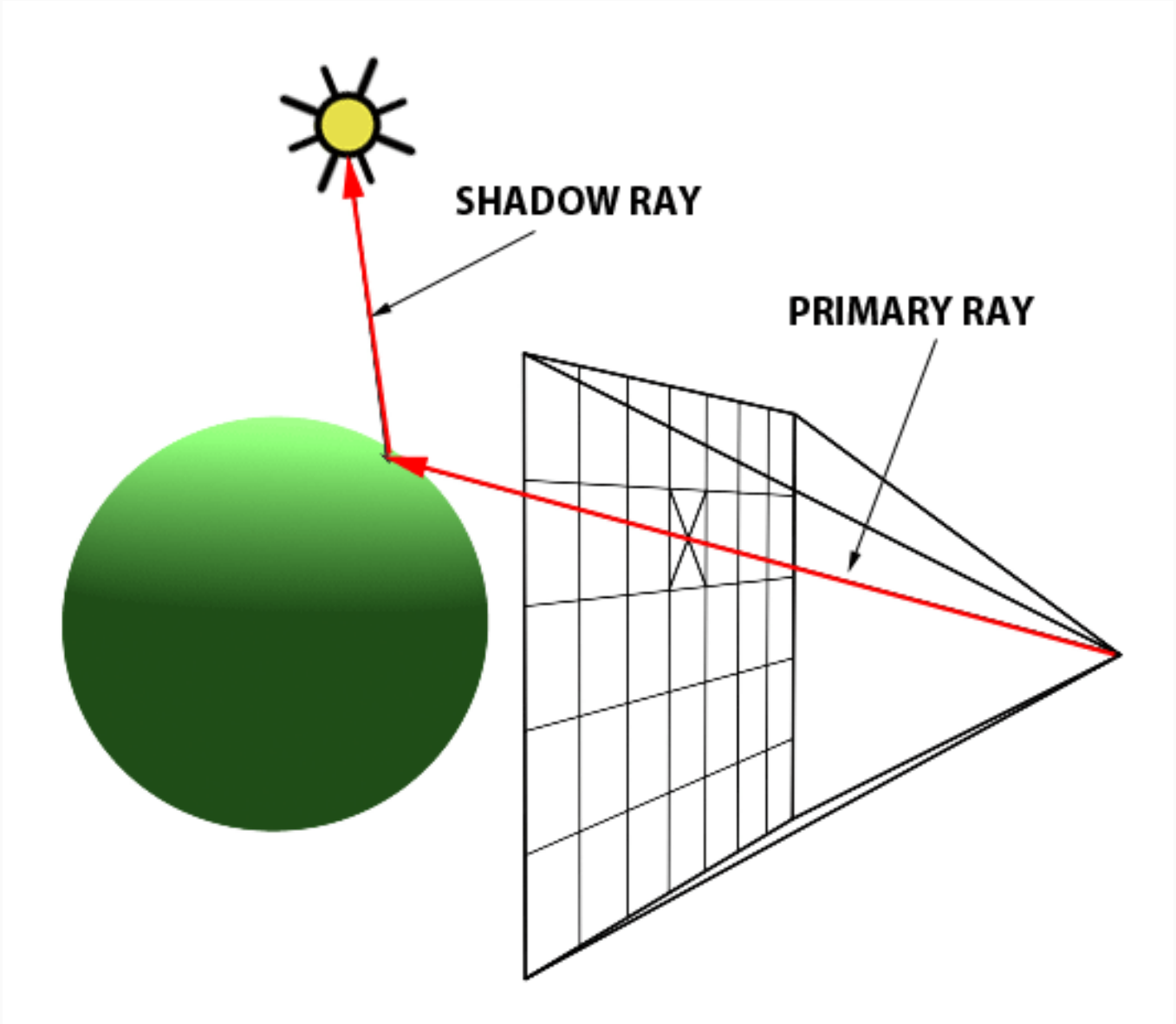

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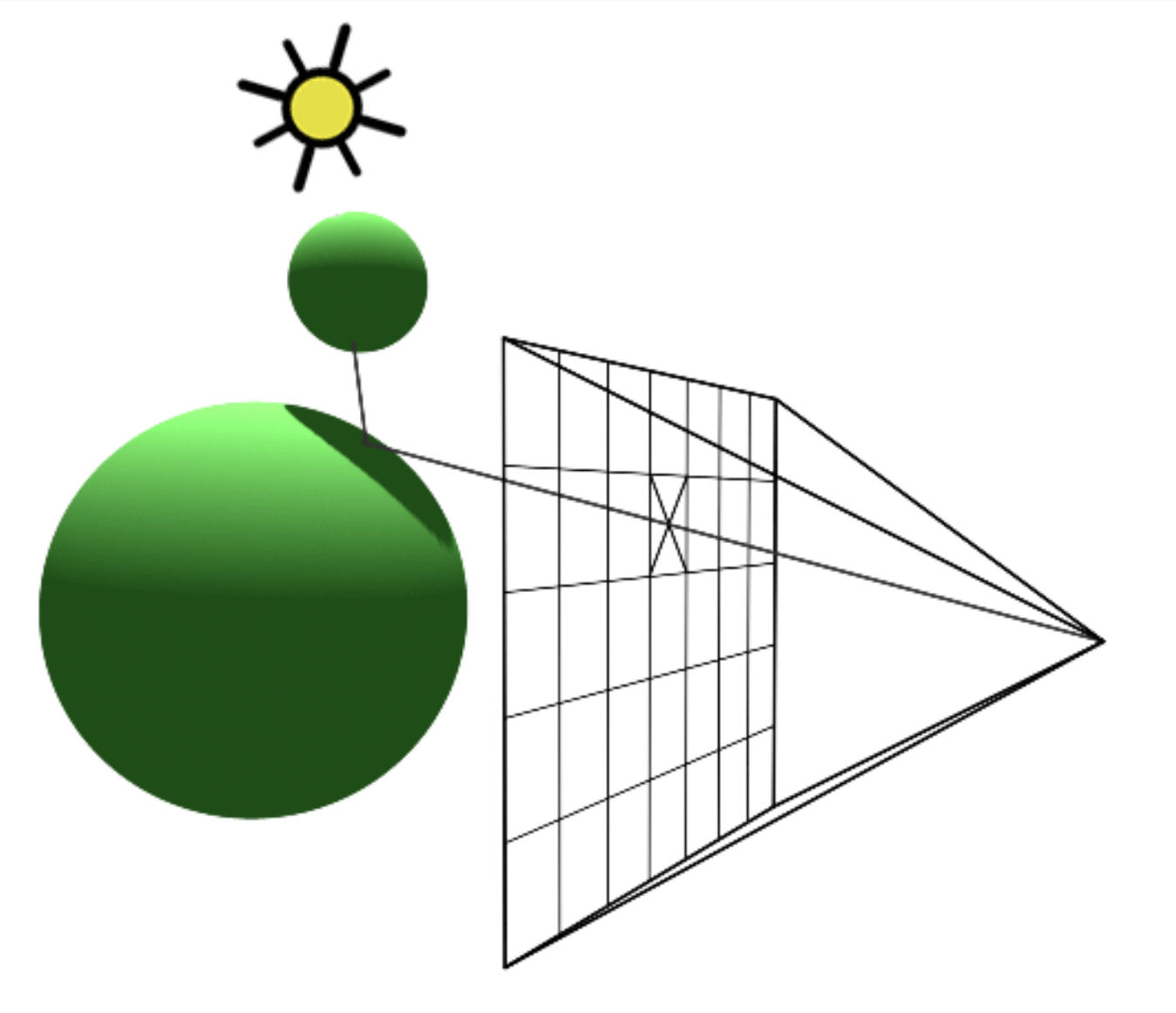

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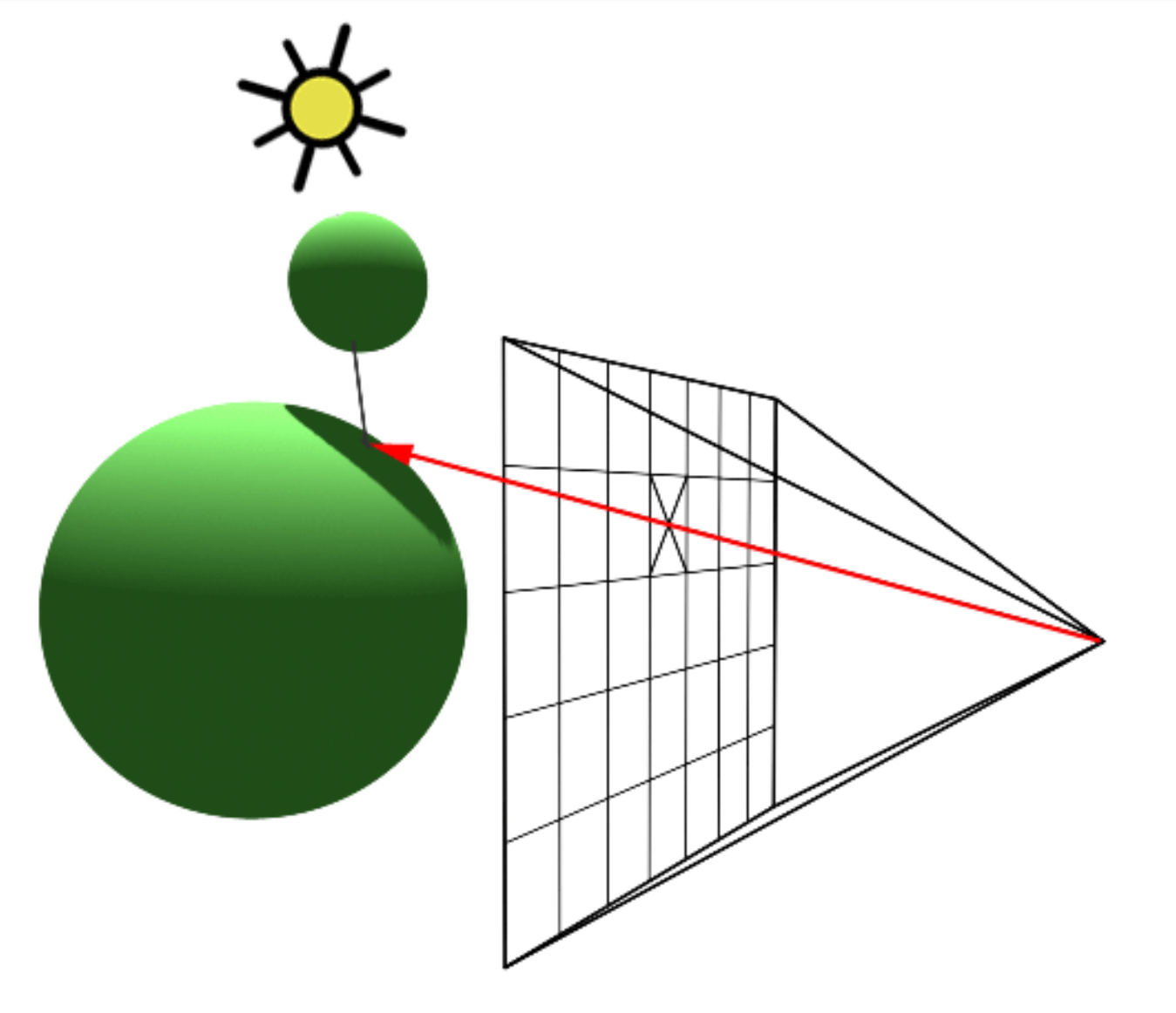

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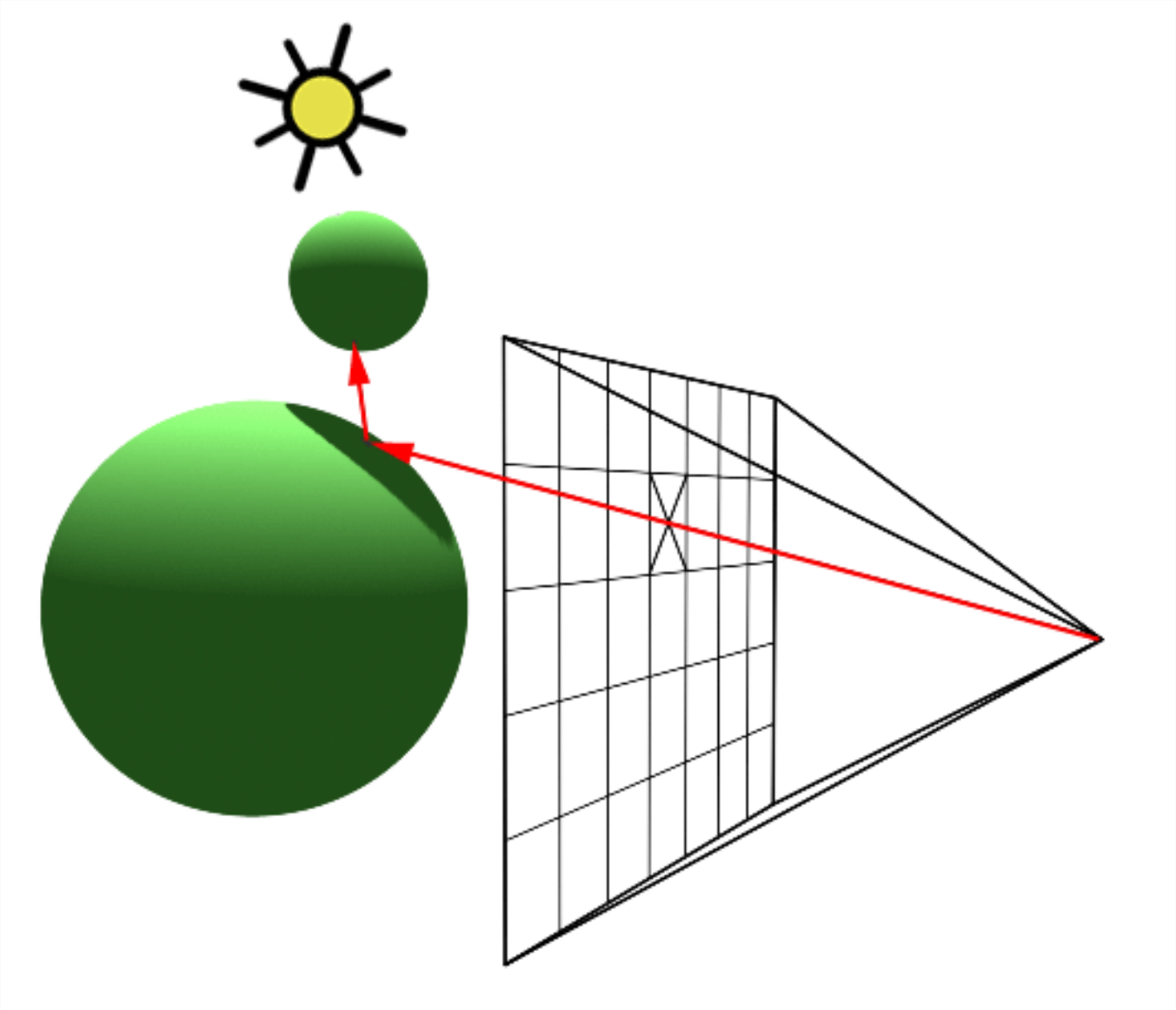
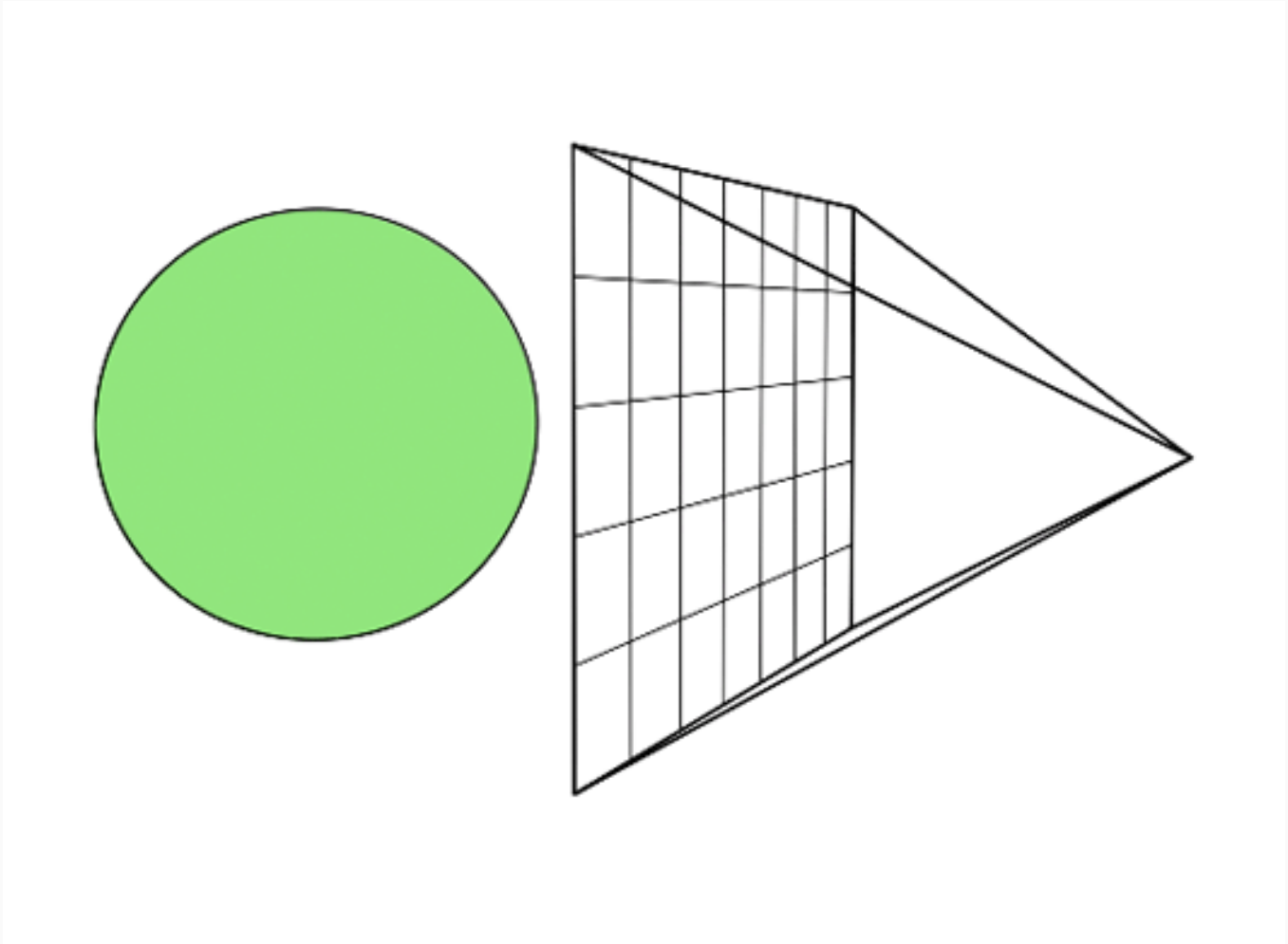


Raycasting Anim



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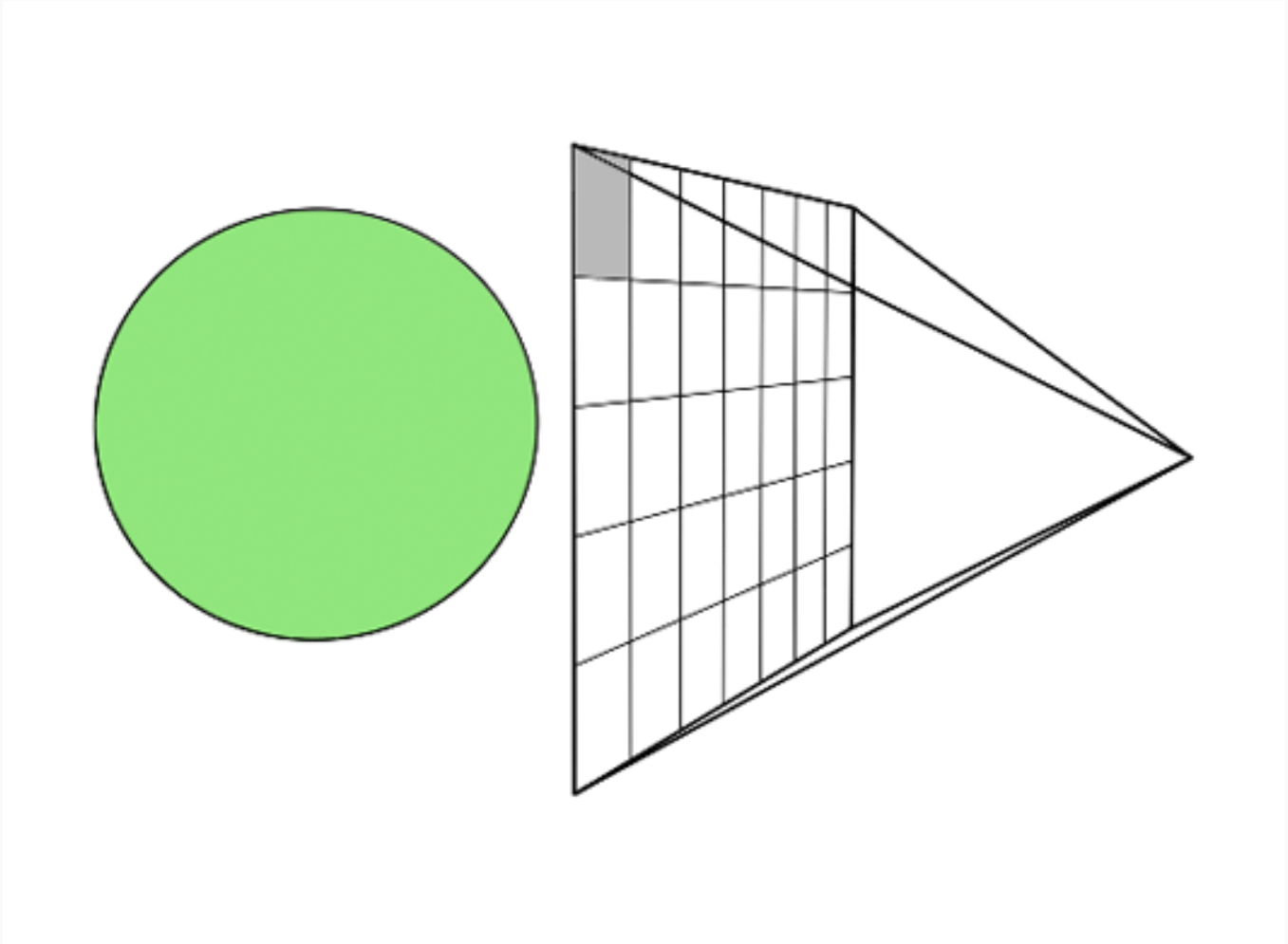

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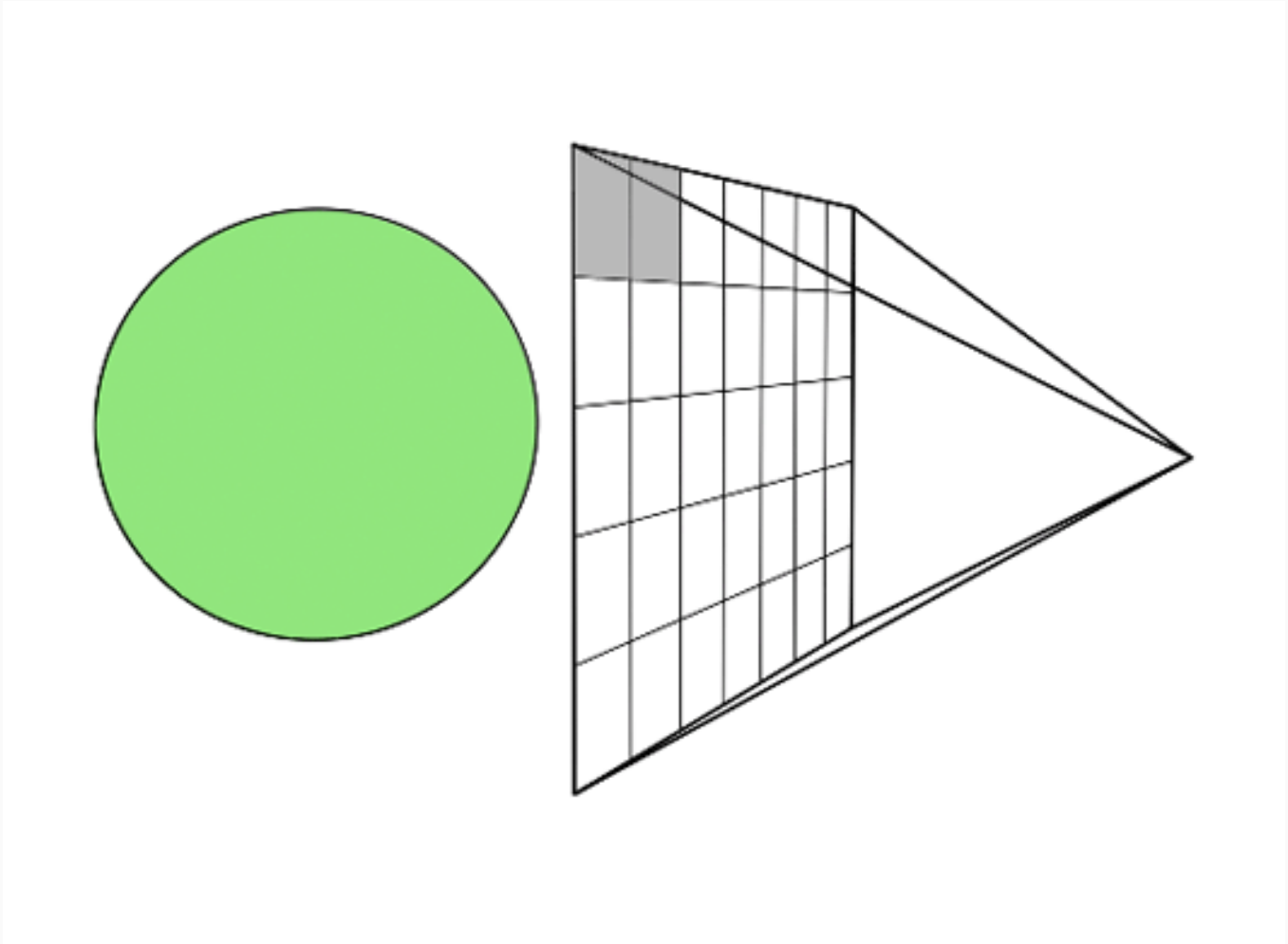

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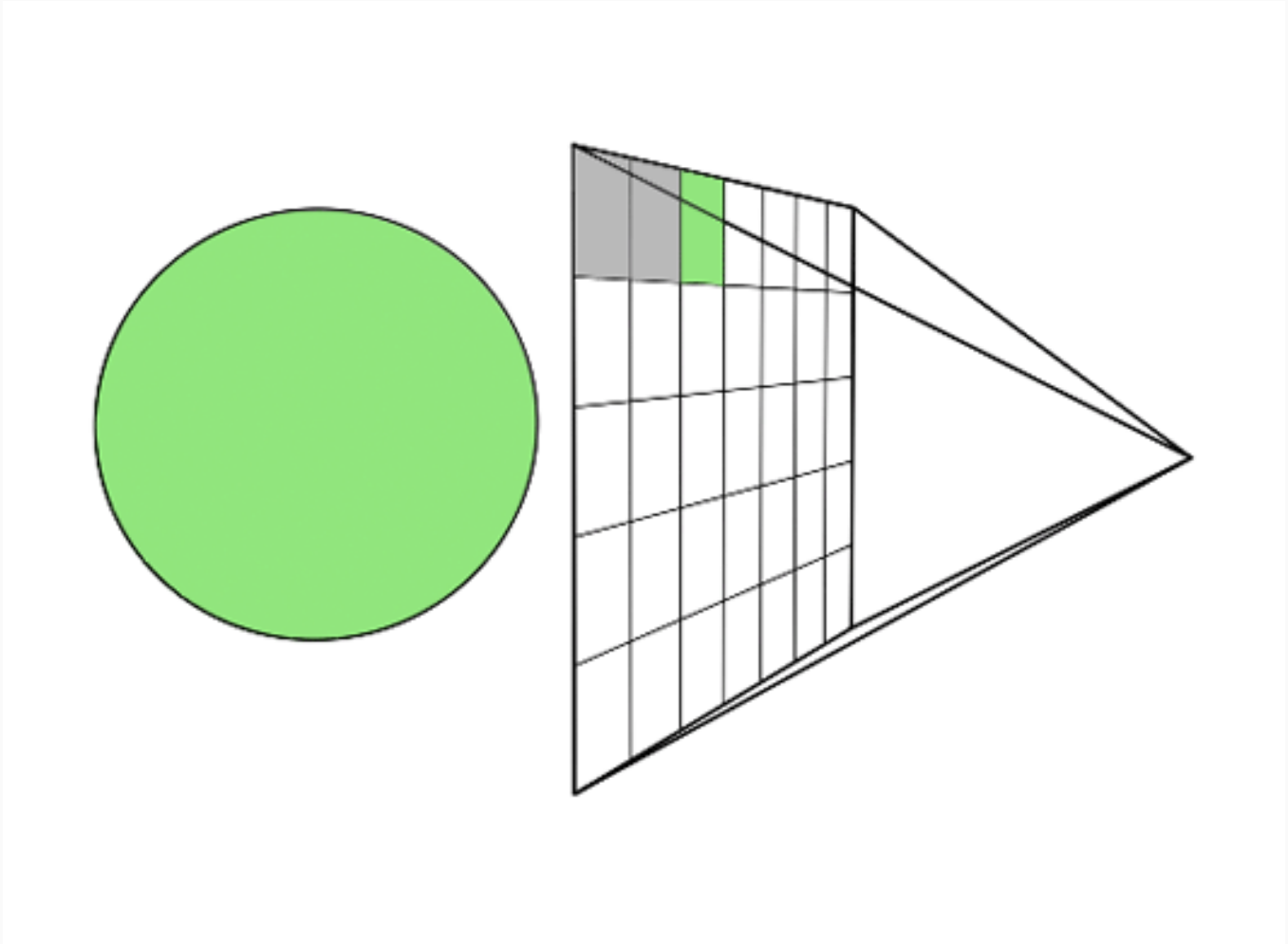

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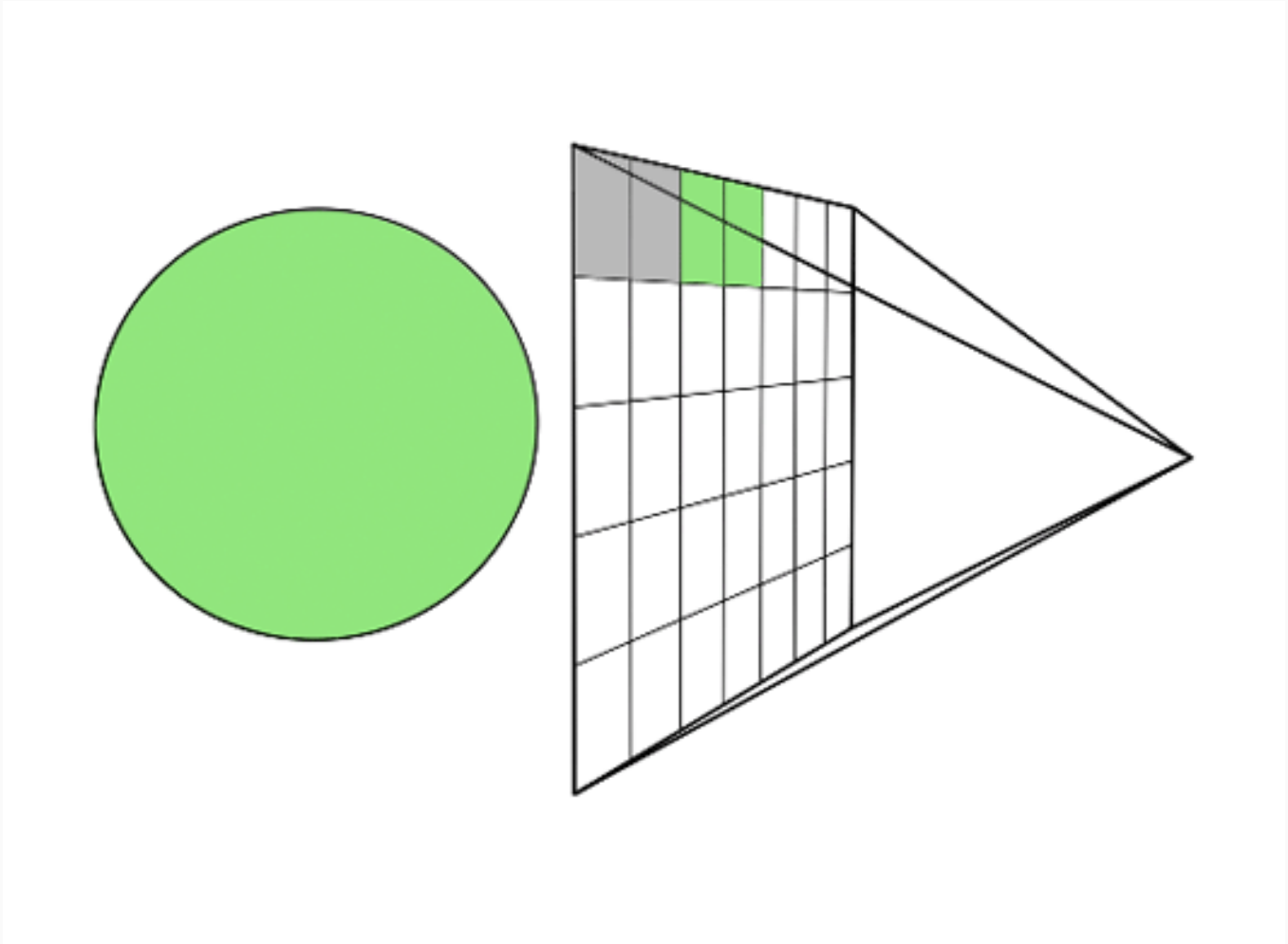

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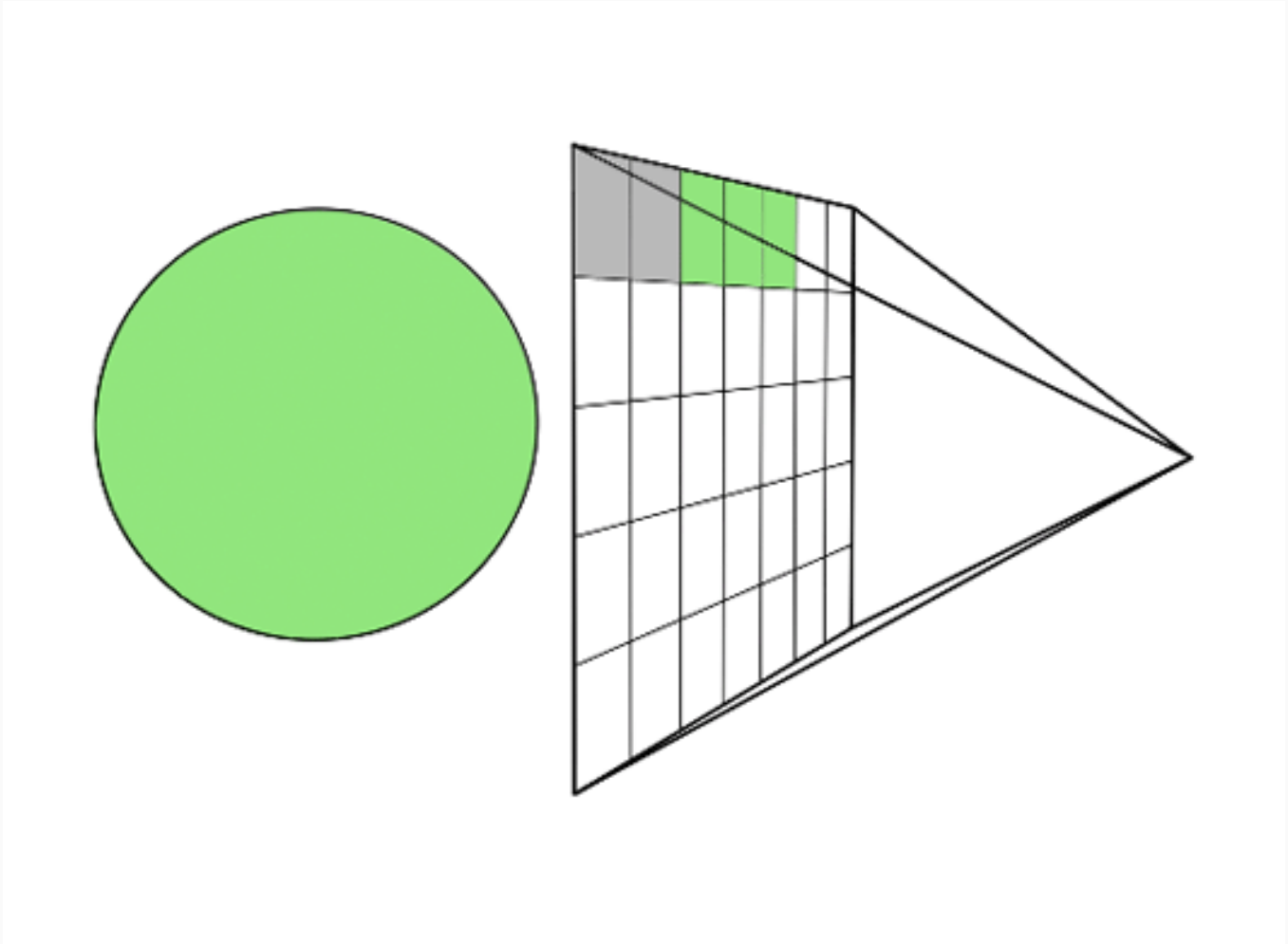

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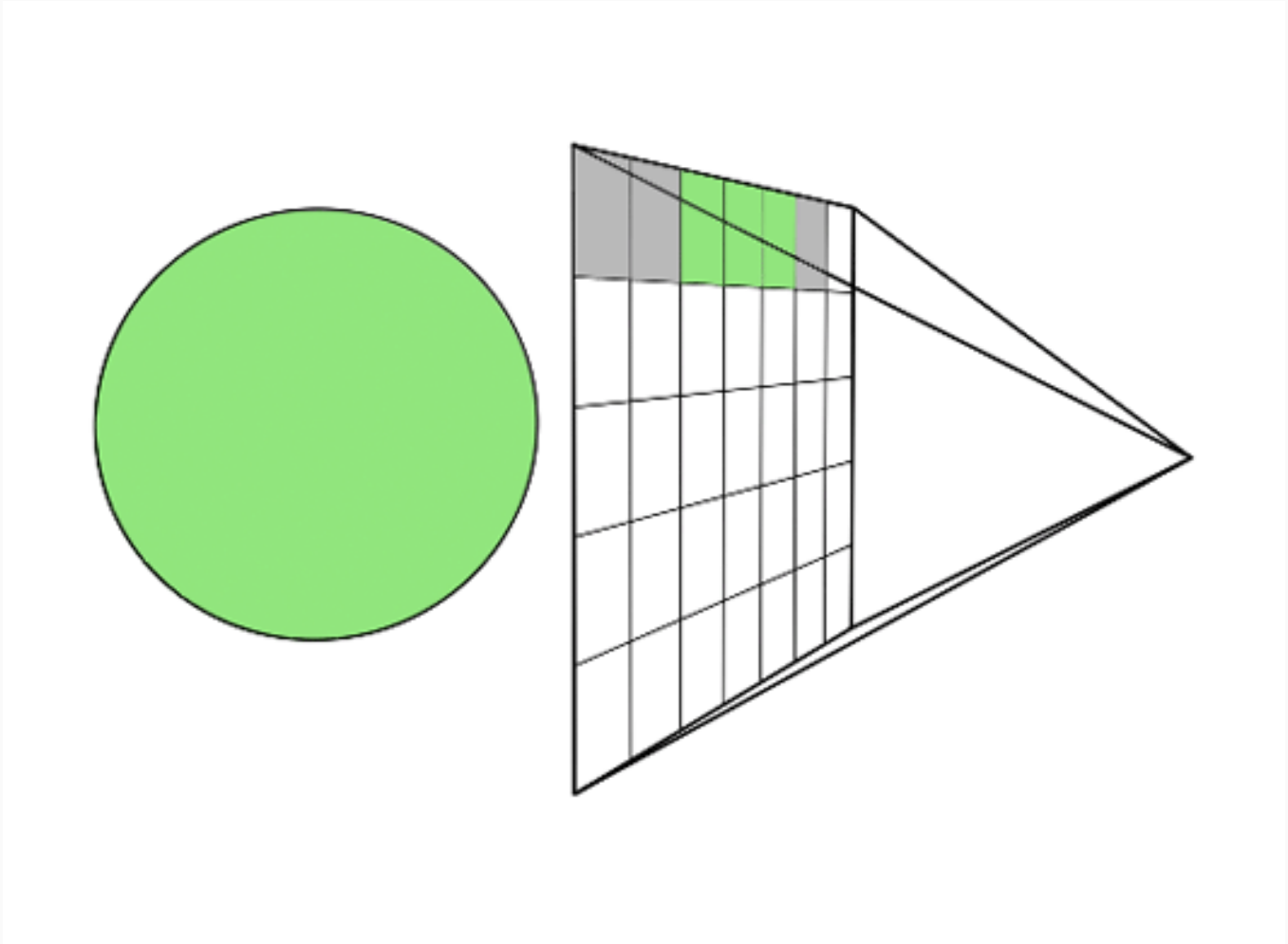
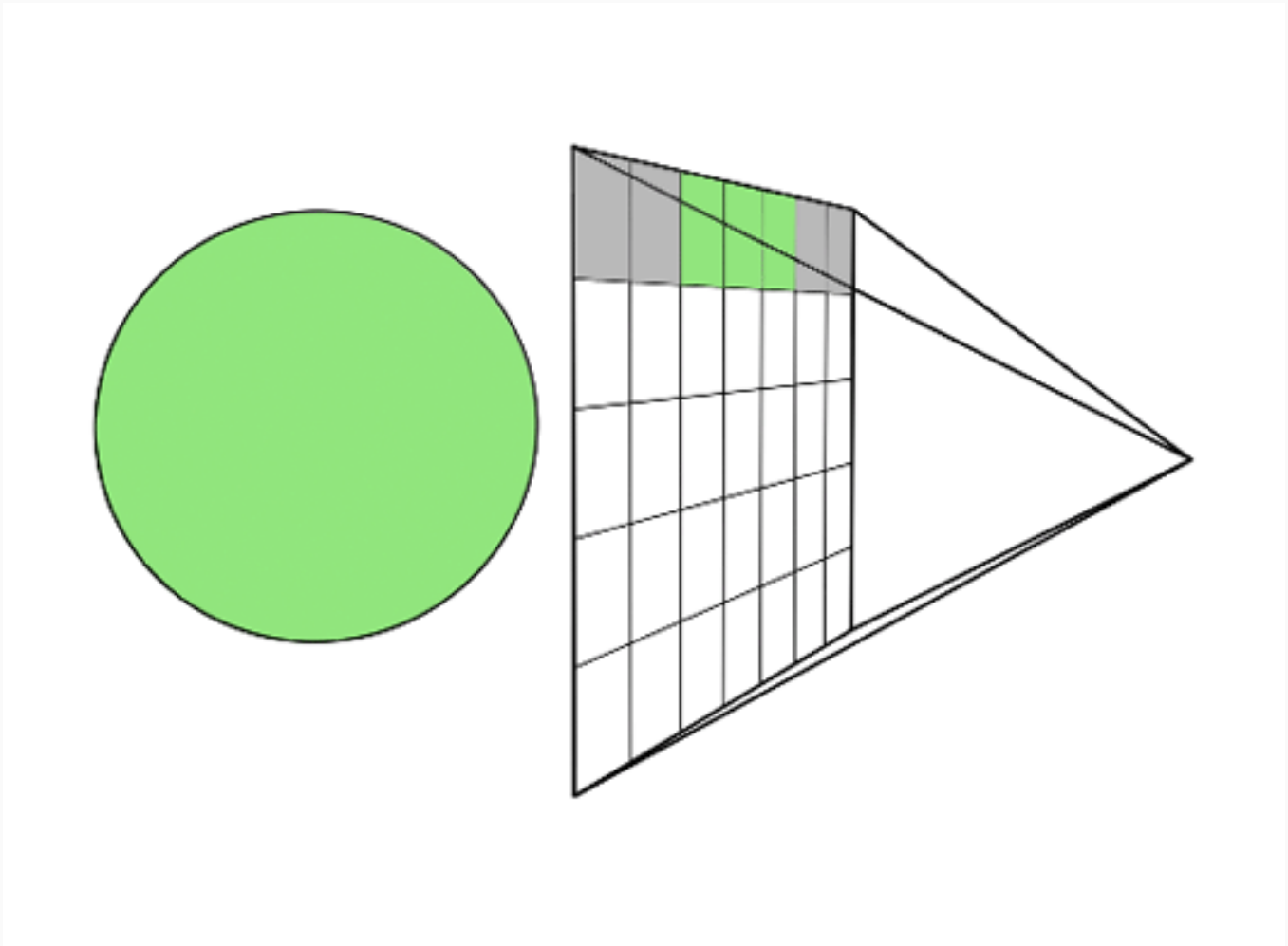


Raycasting Anim



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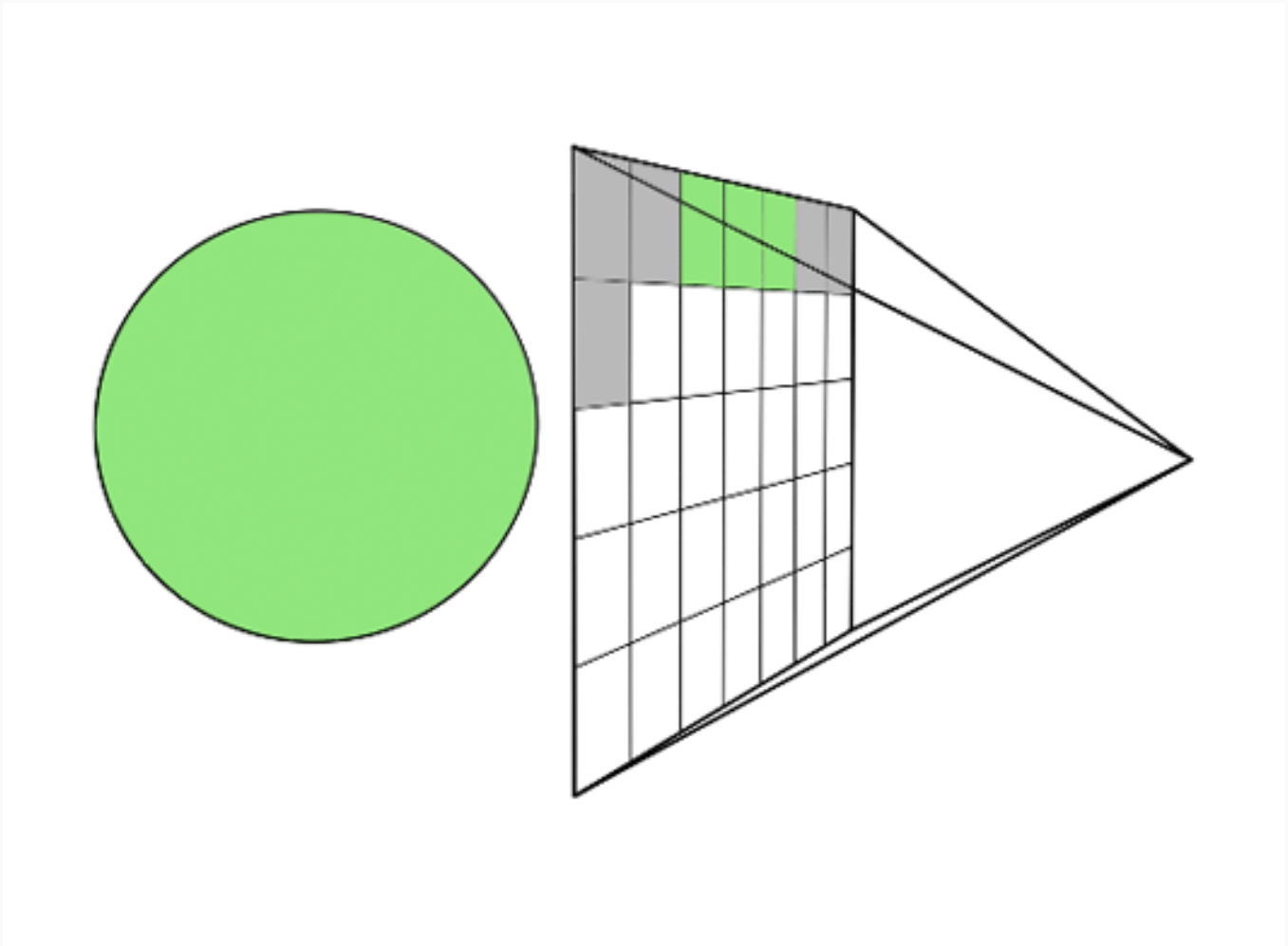

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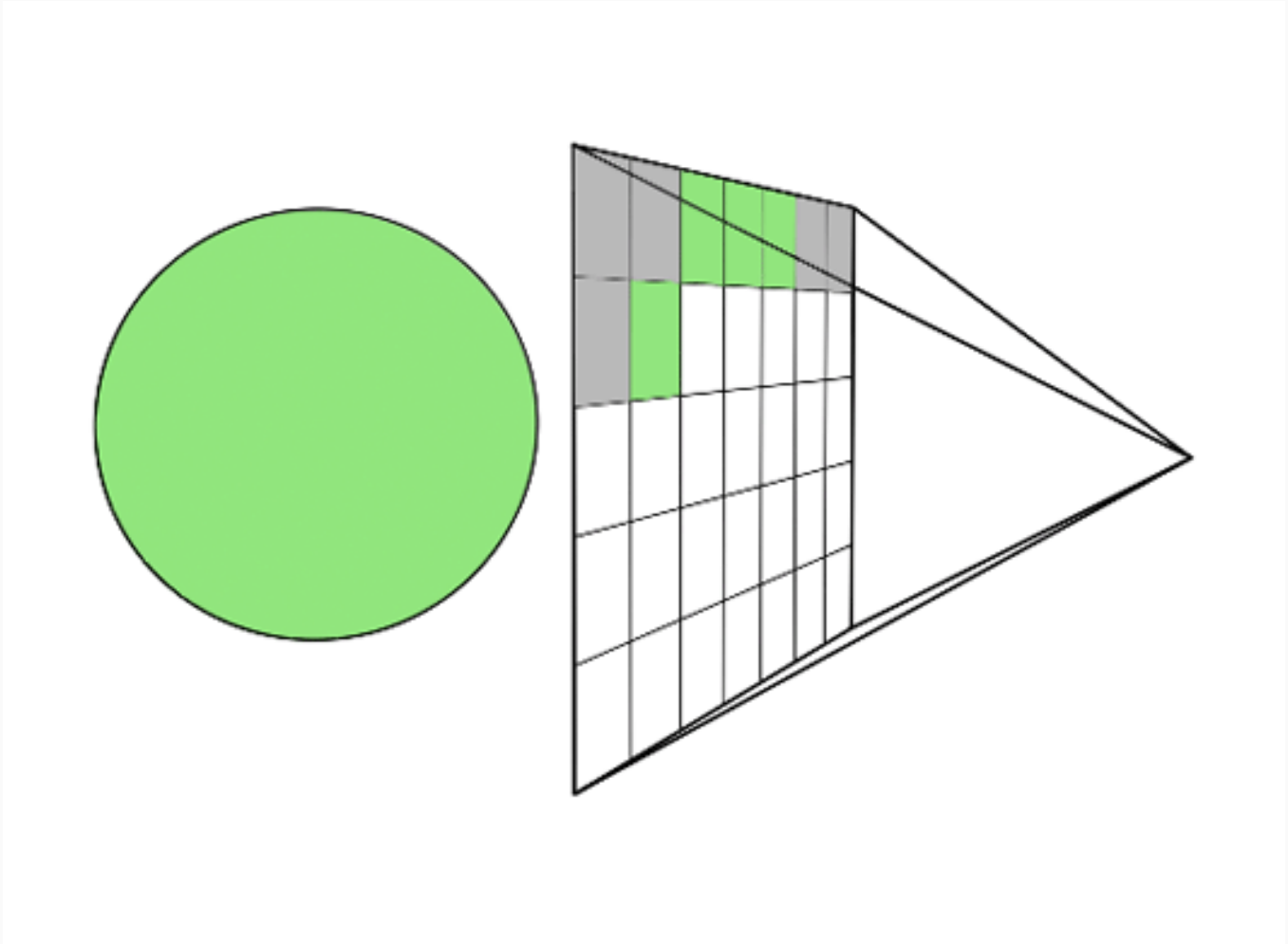

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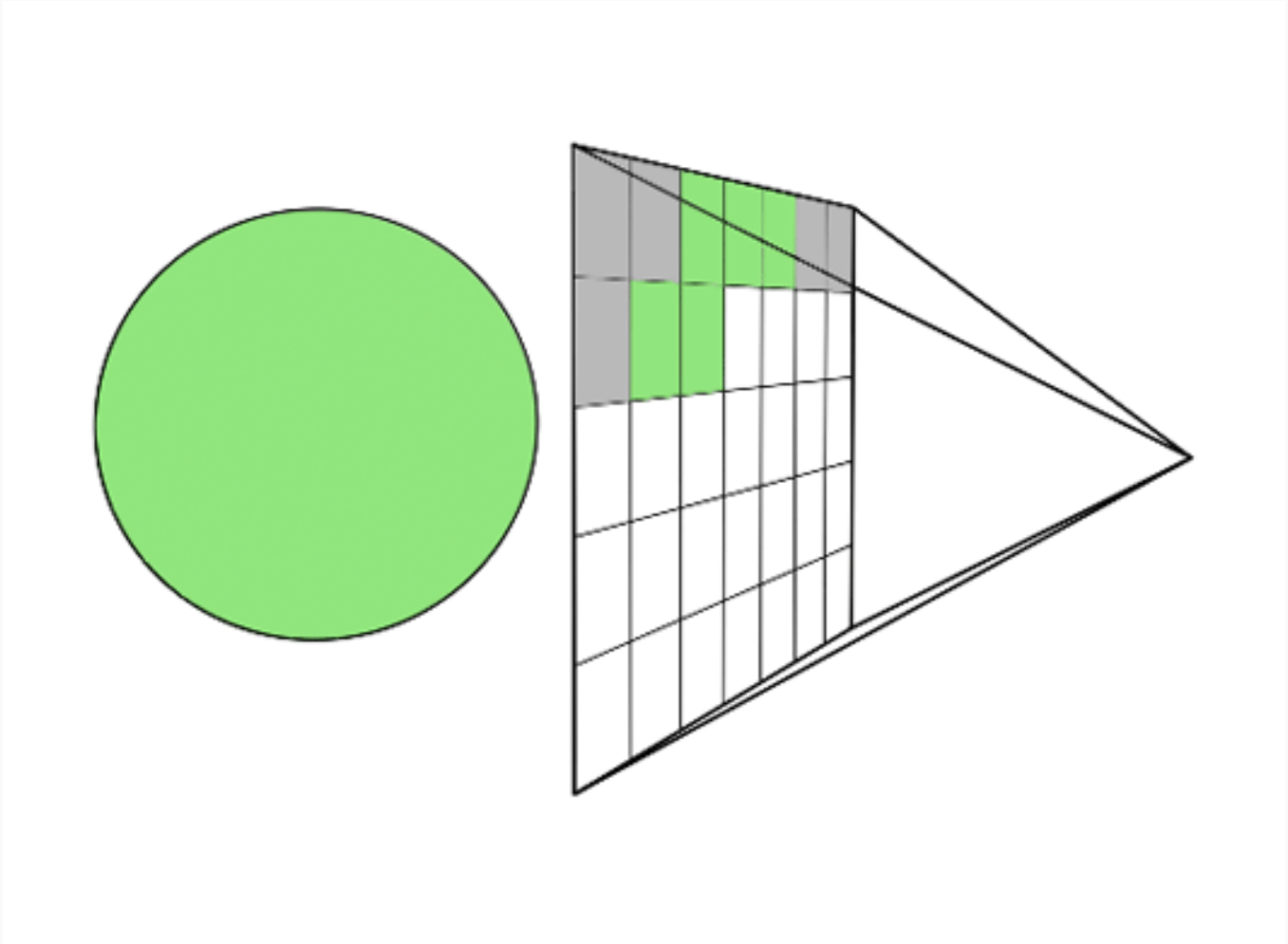

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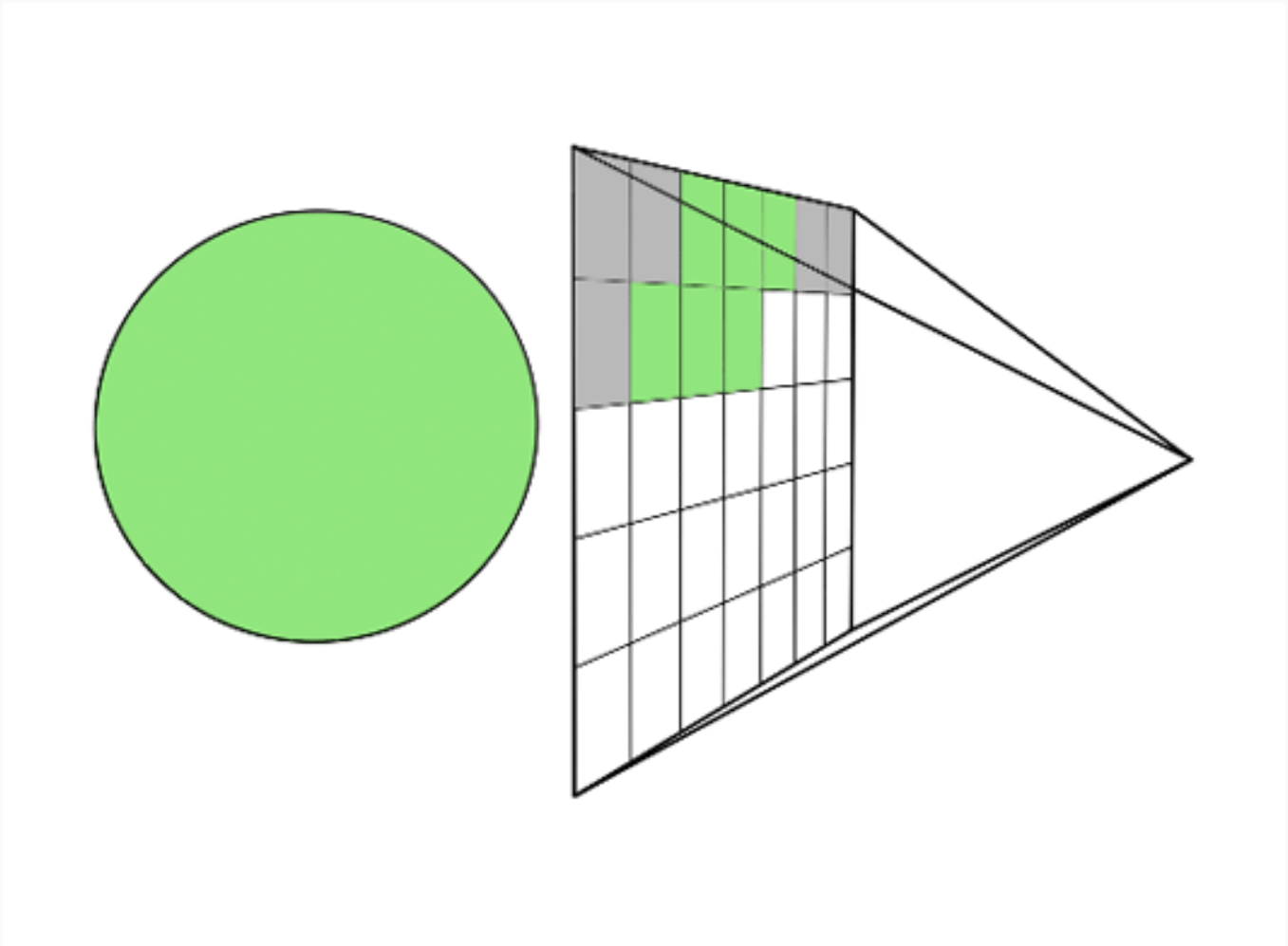

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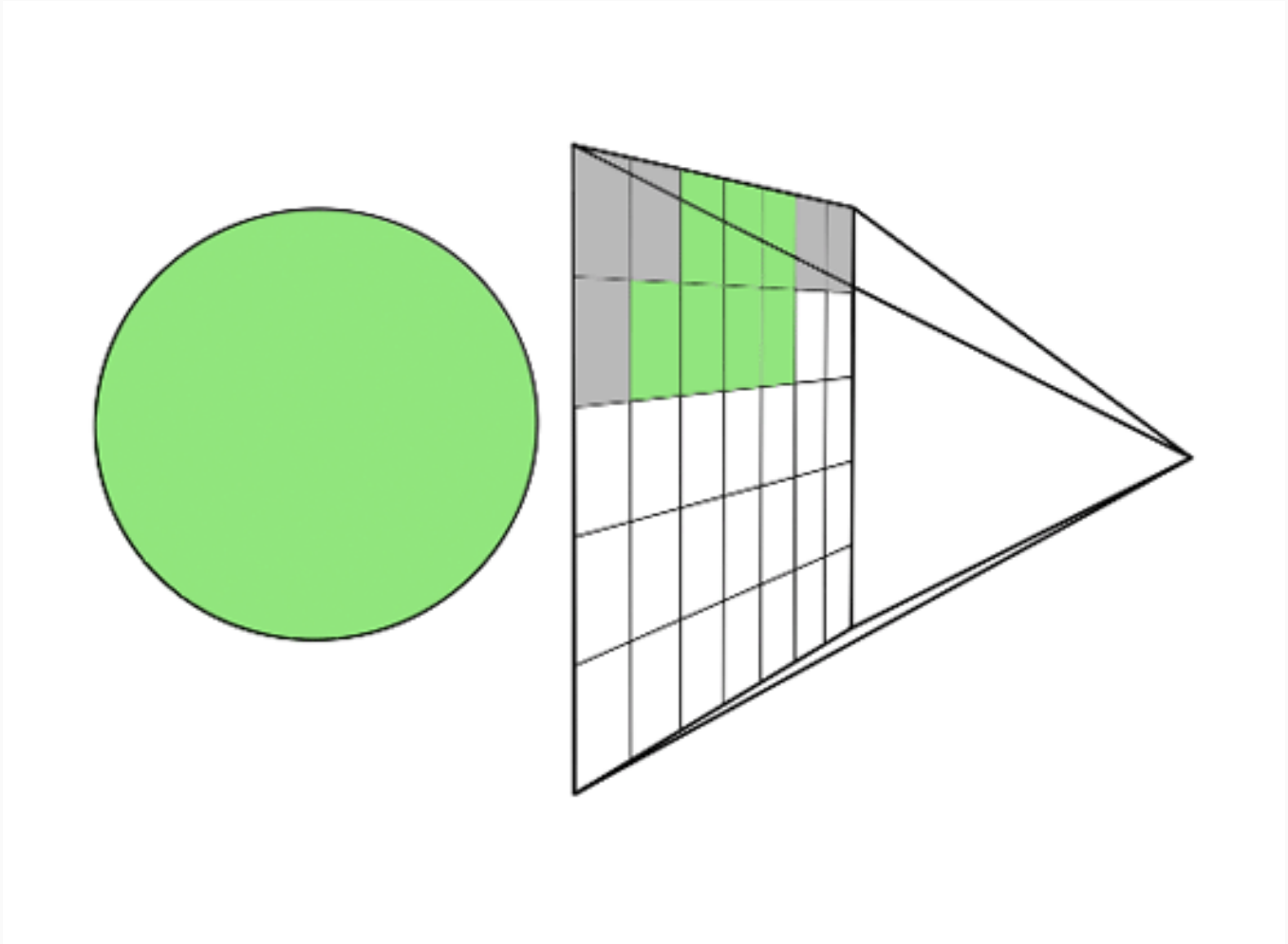

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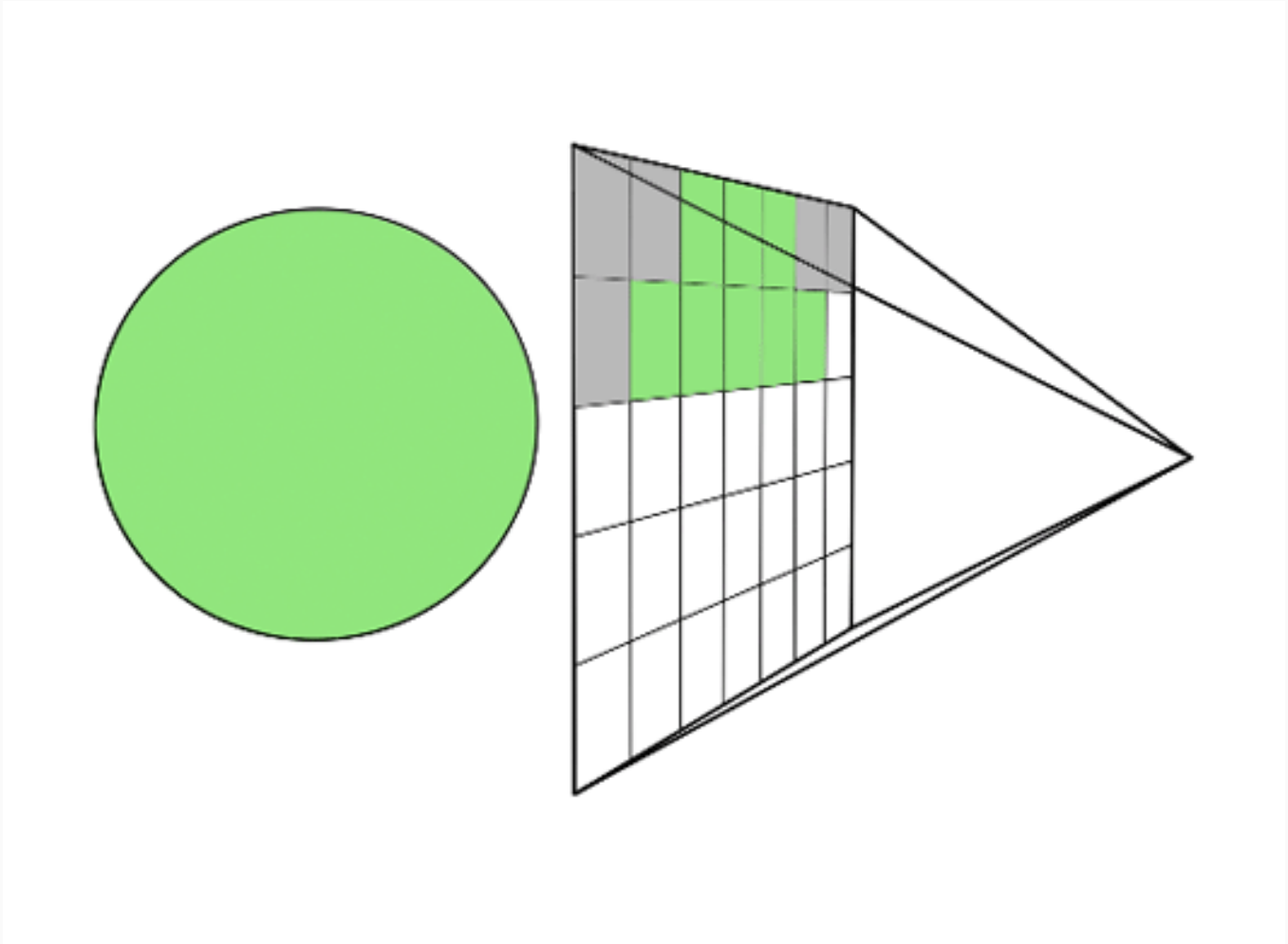

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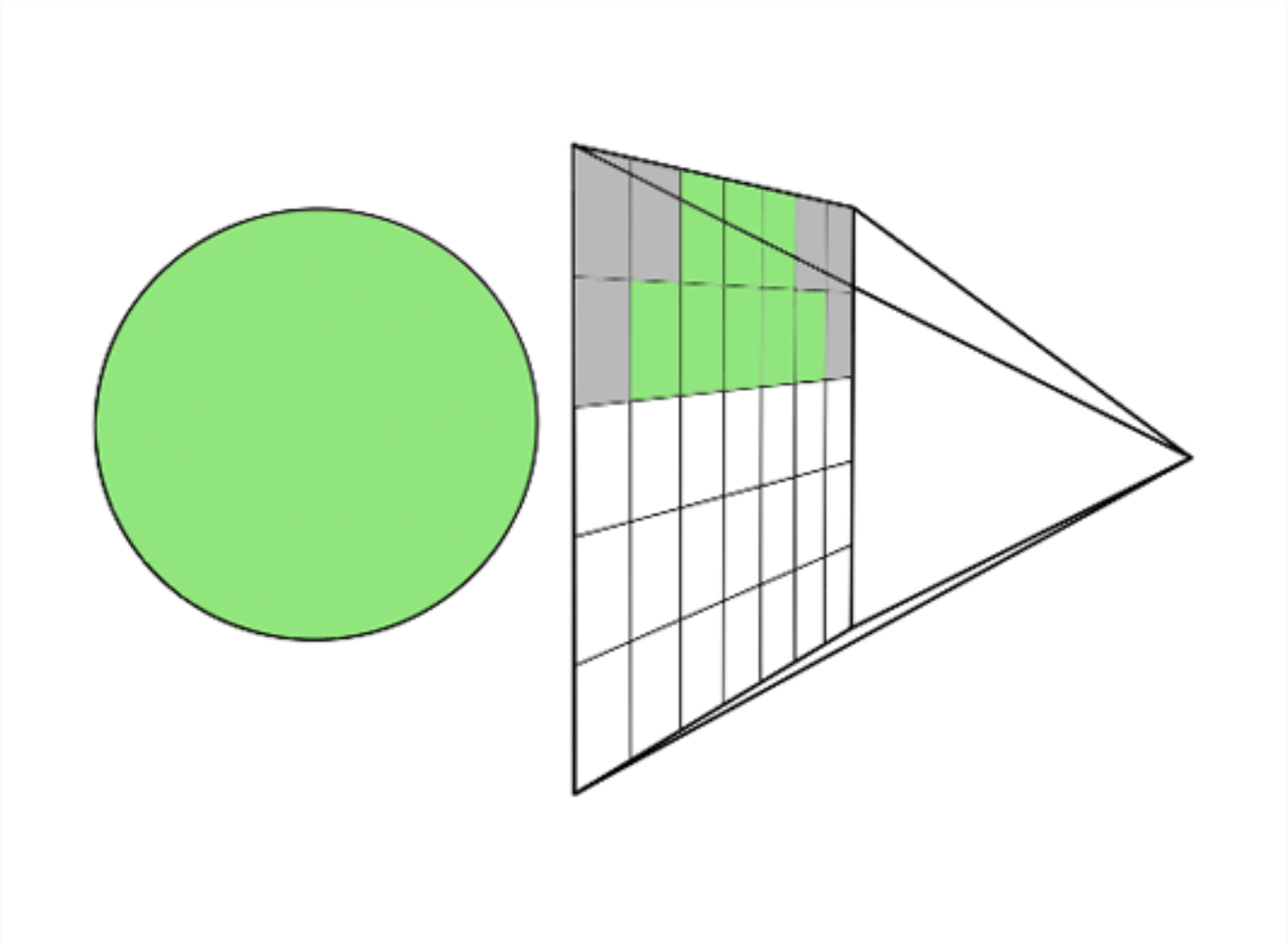

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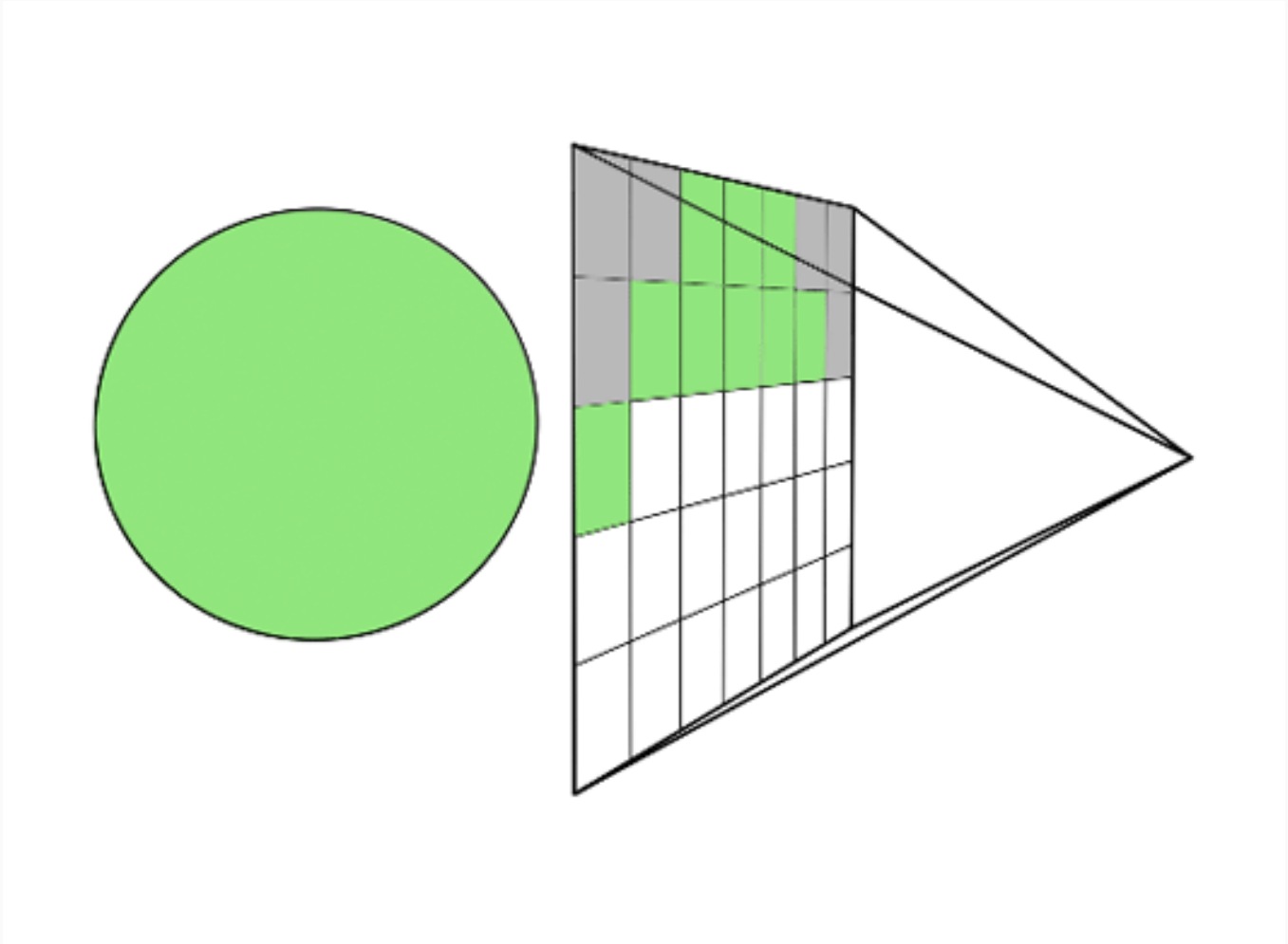

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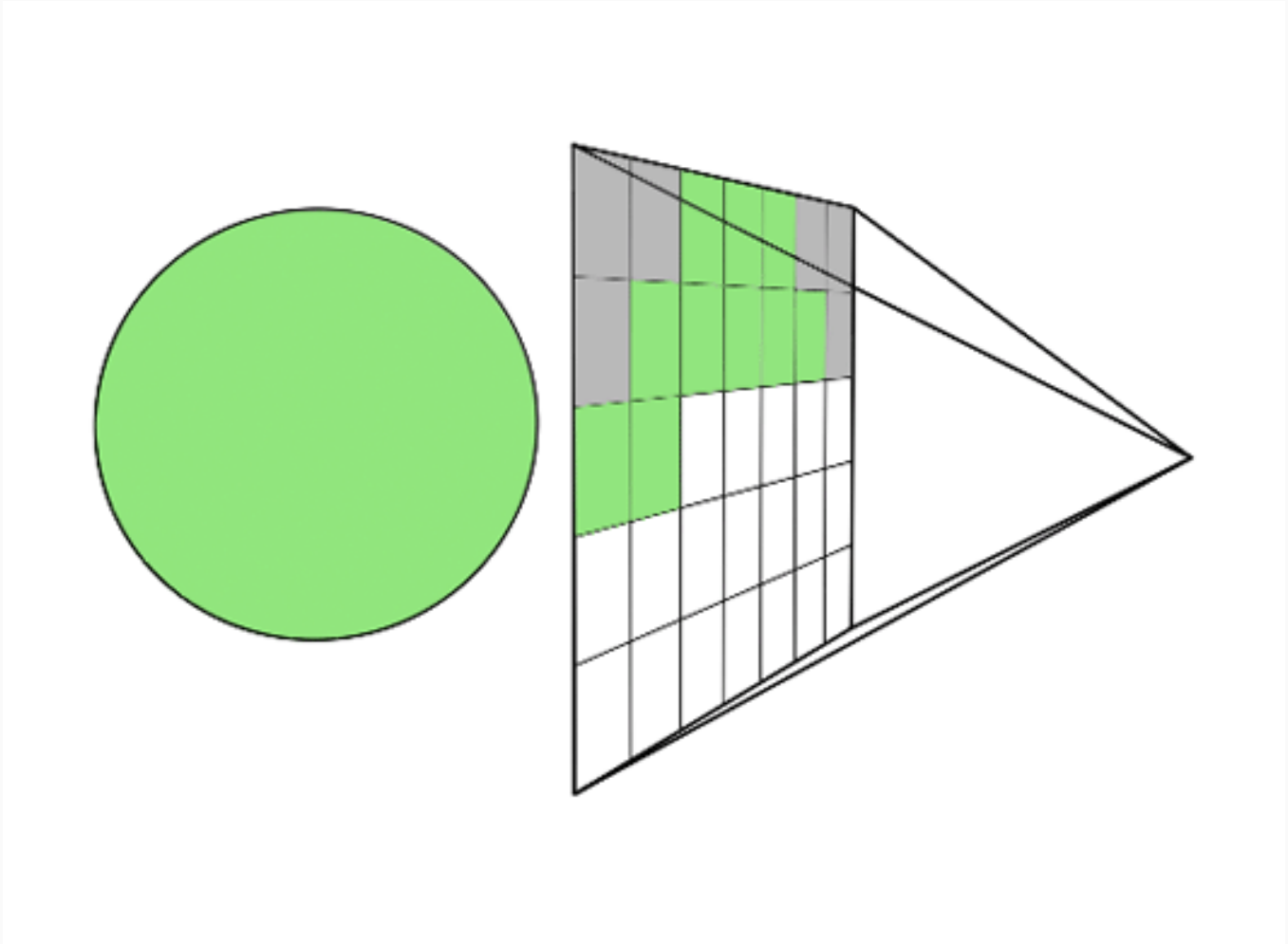

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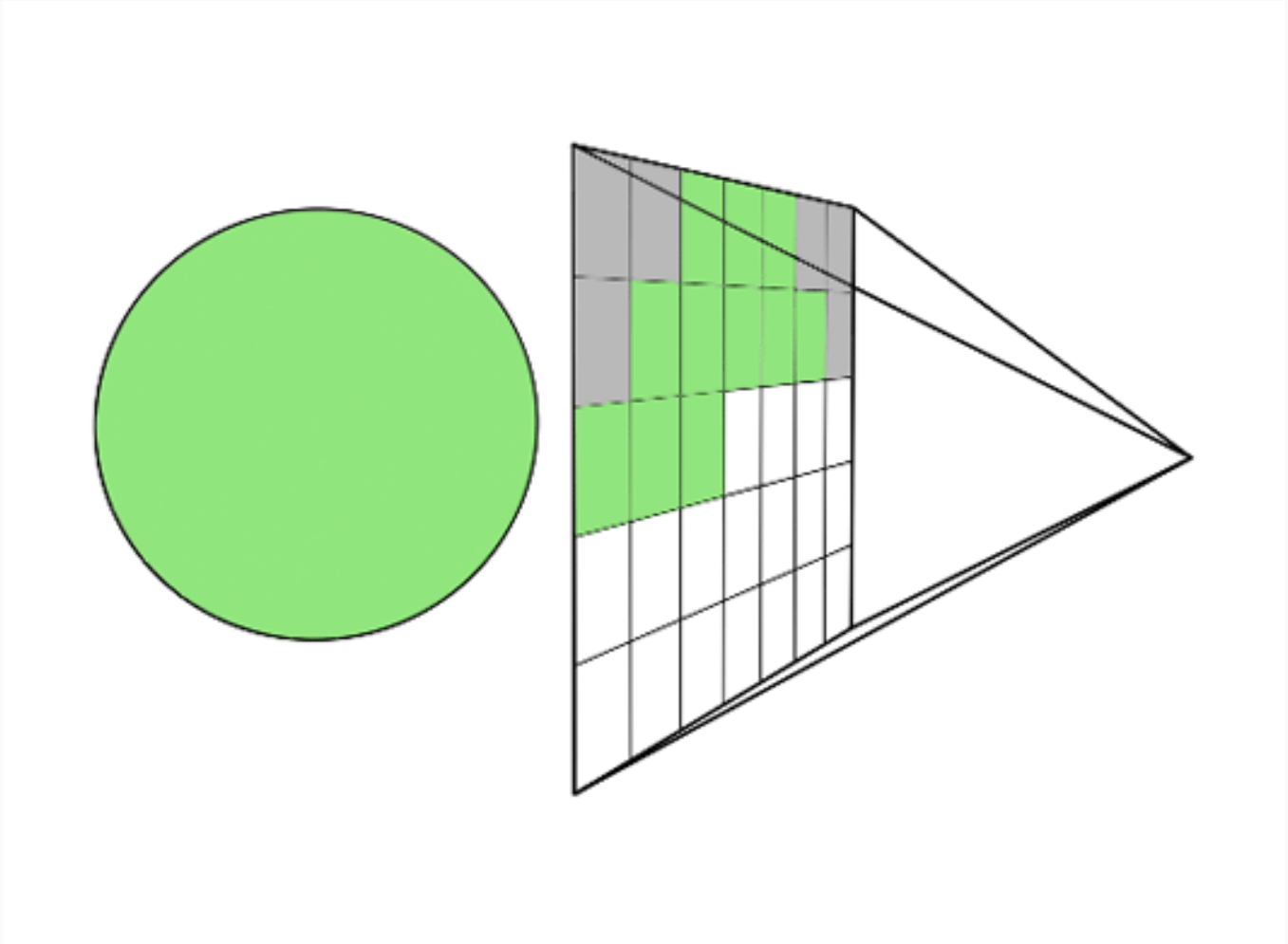

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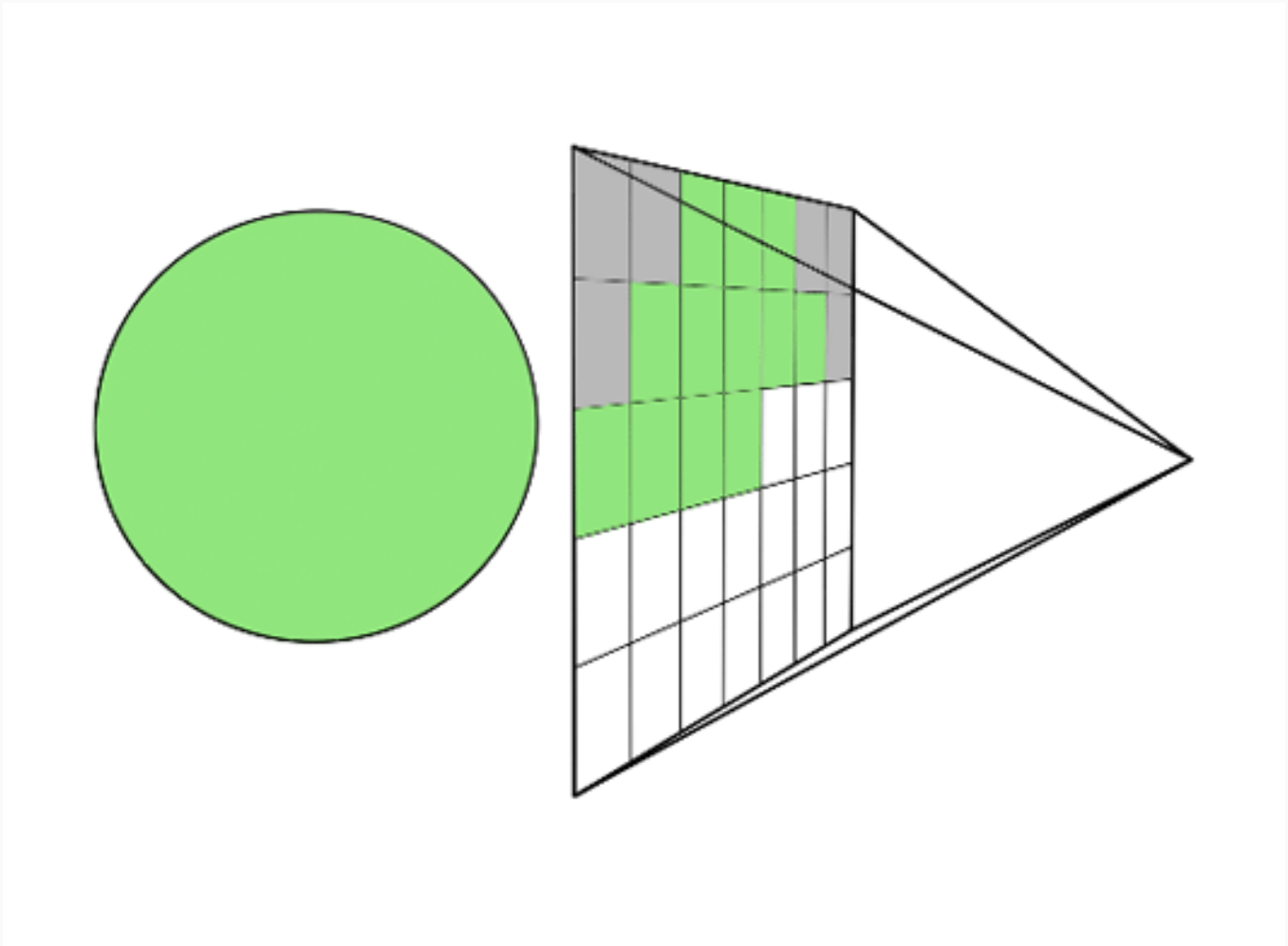

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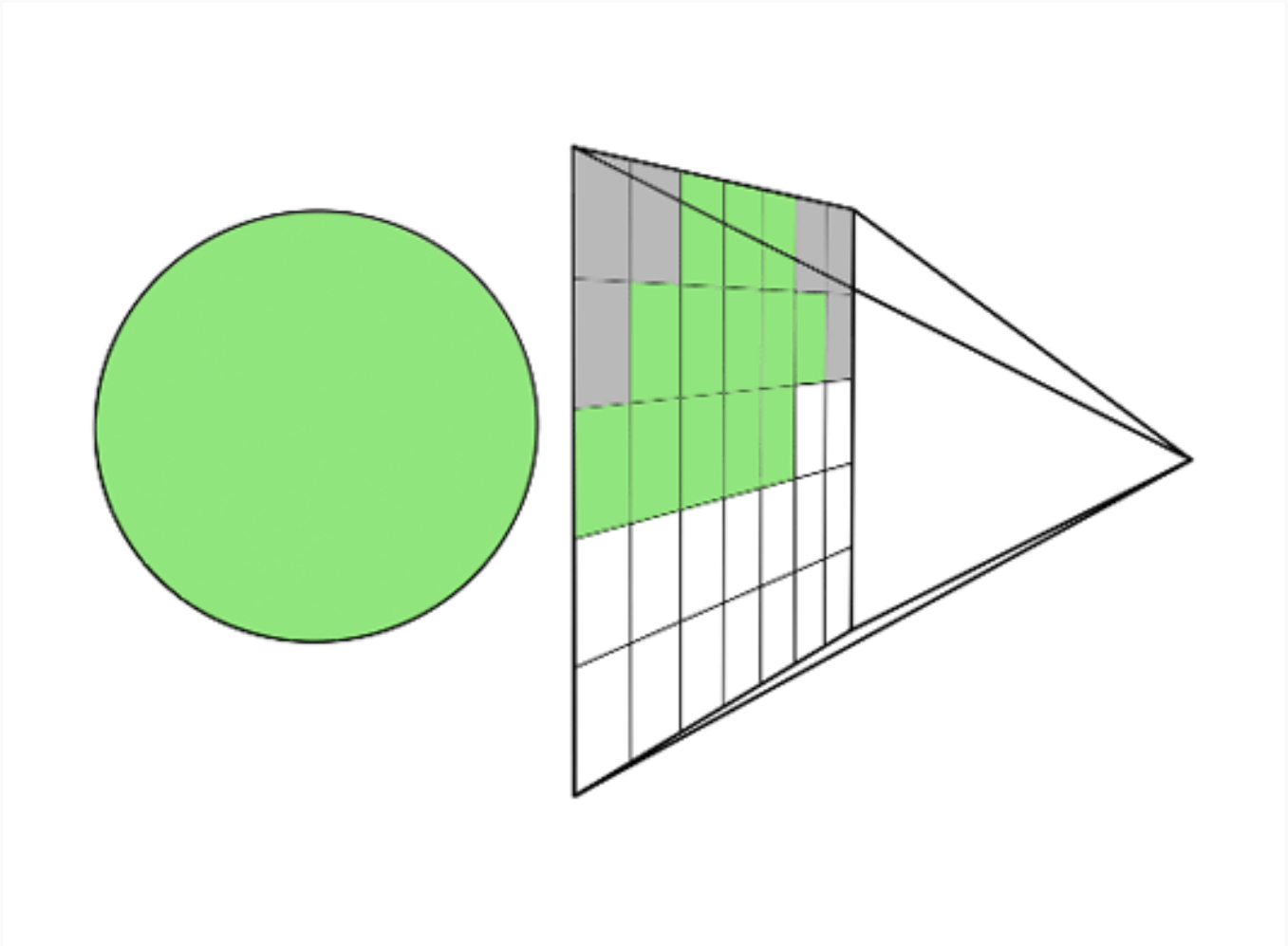

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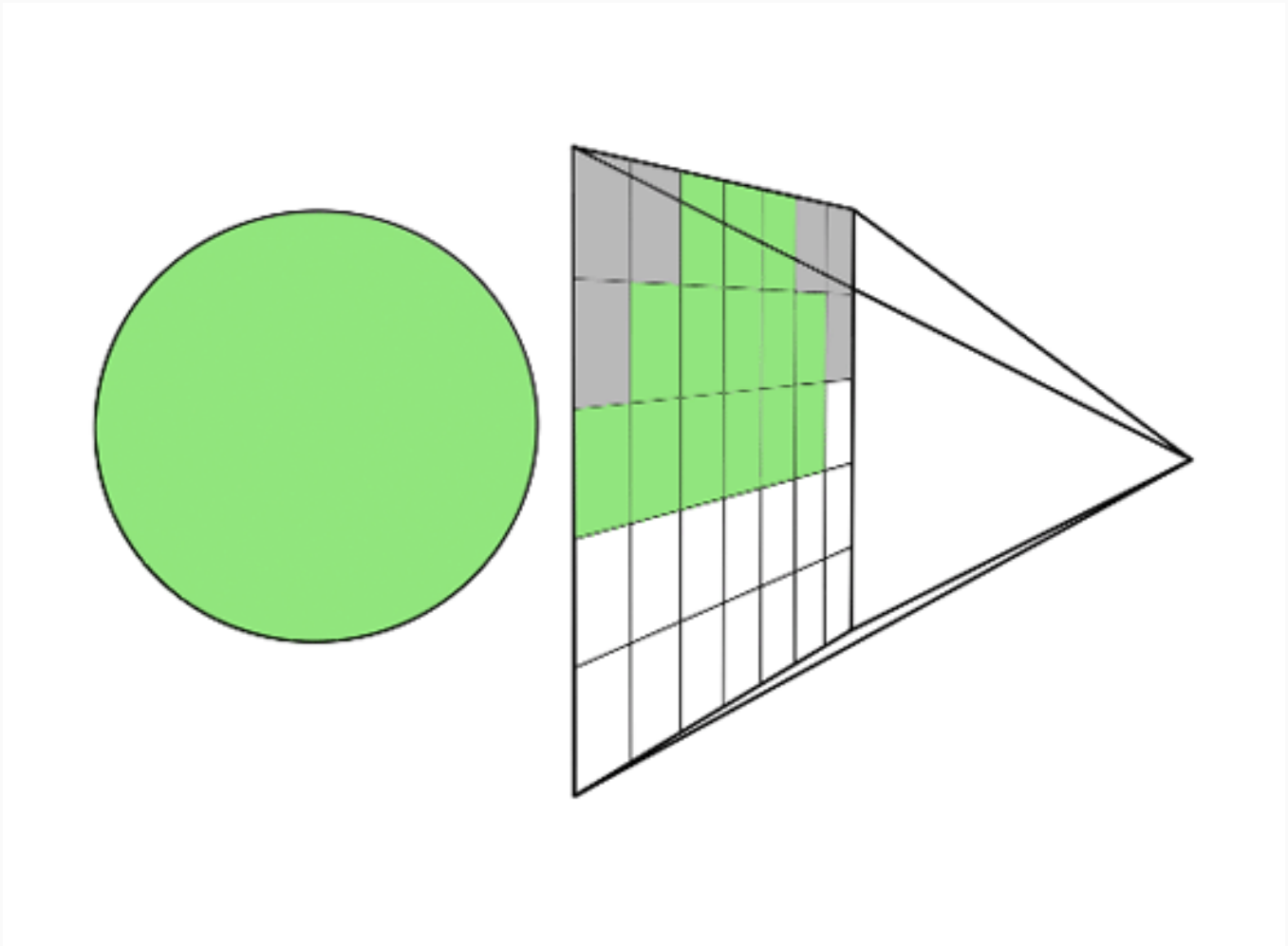
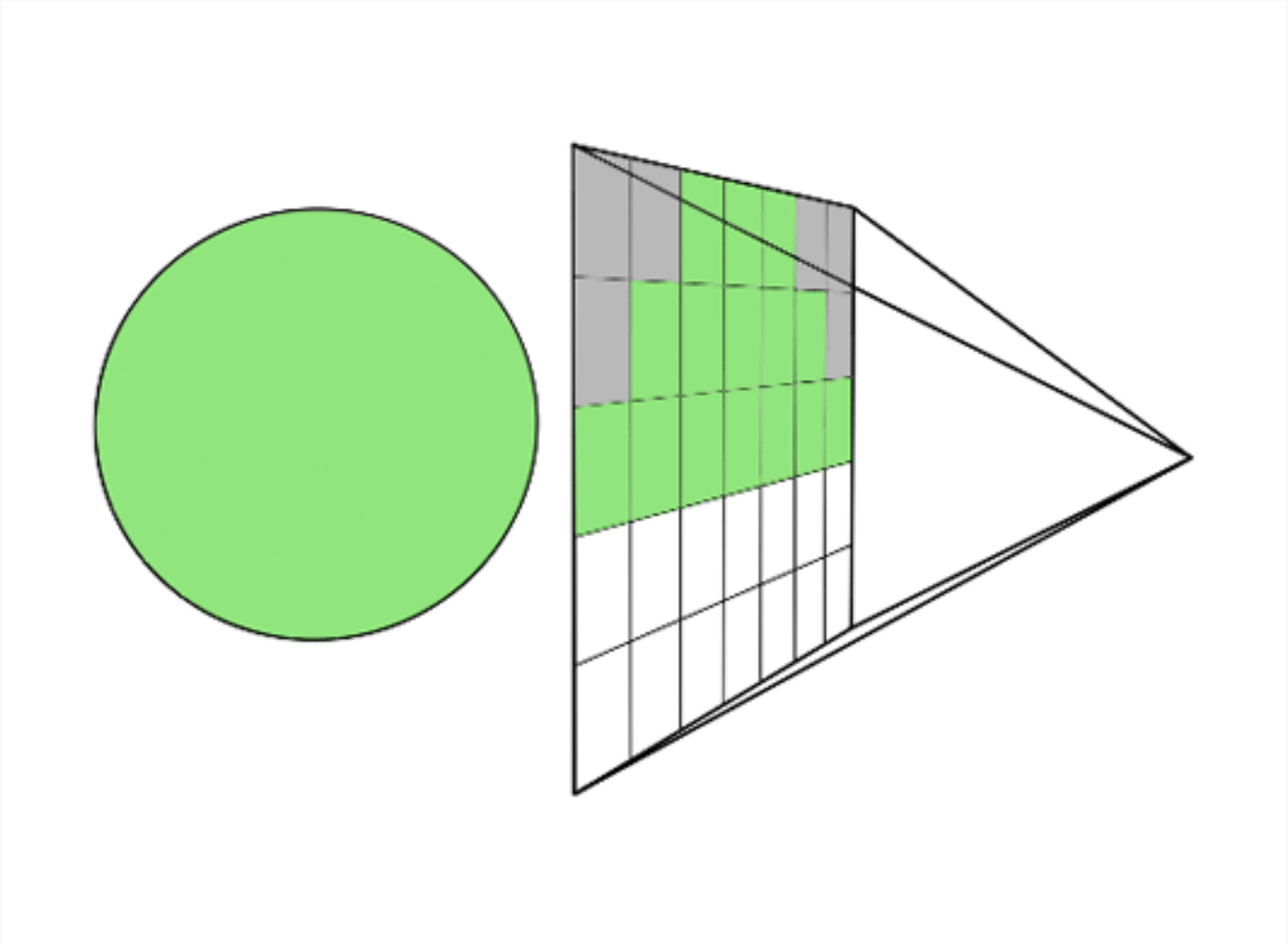


Raycasting Anim



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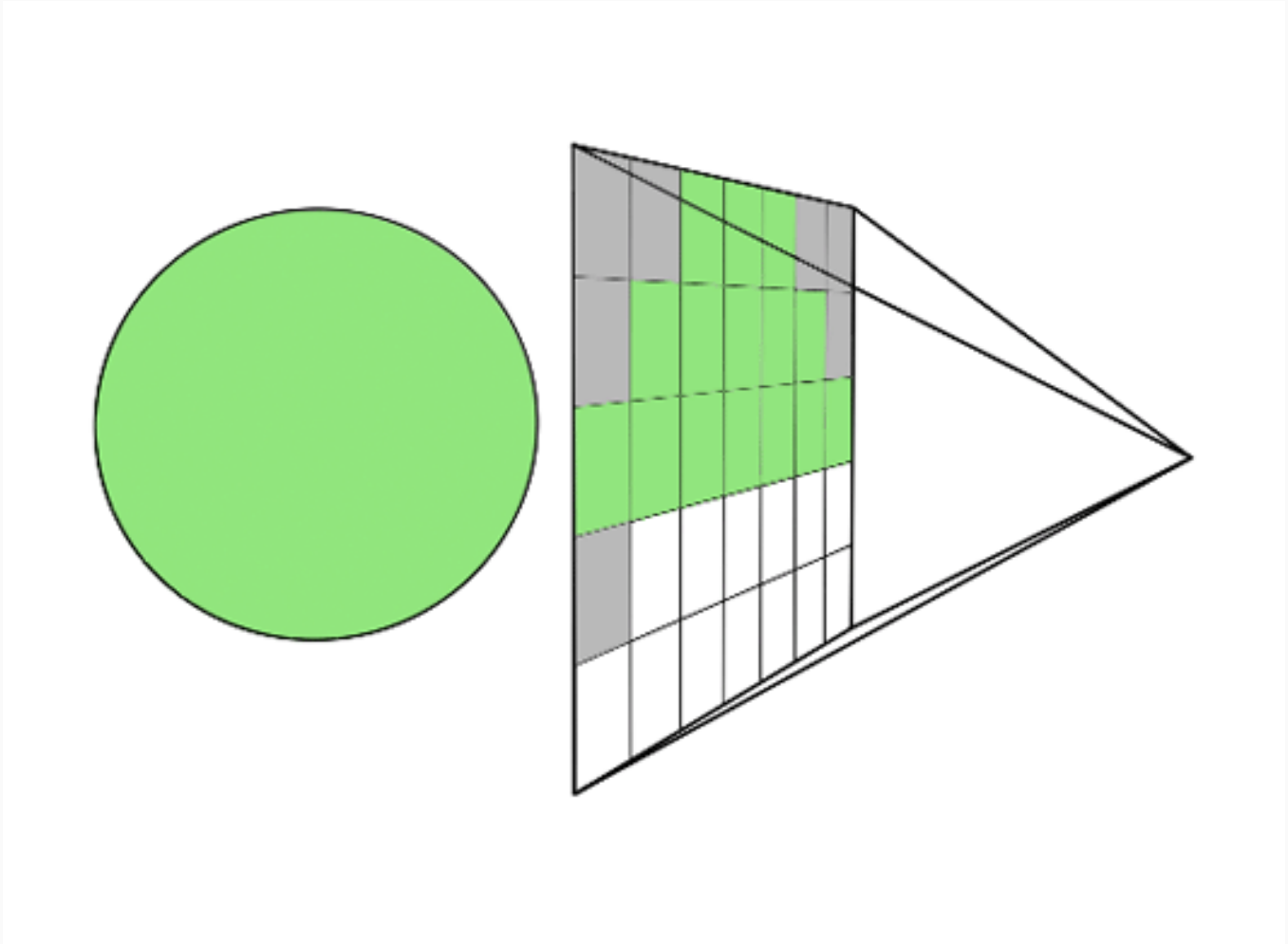

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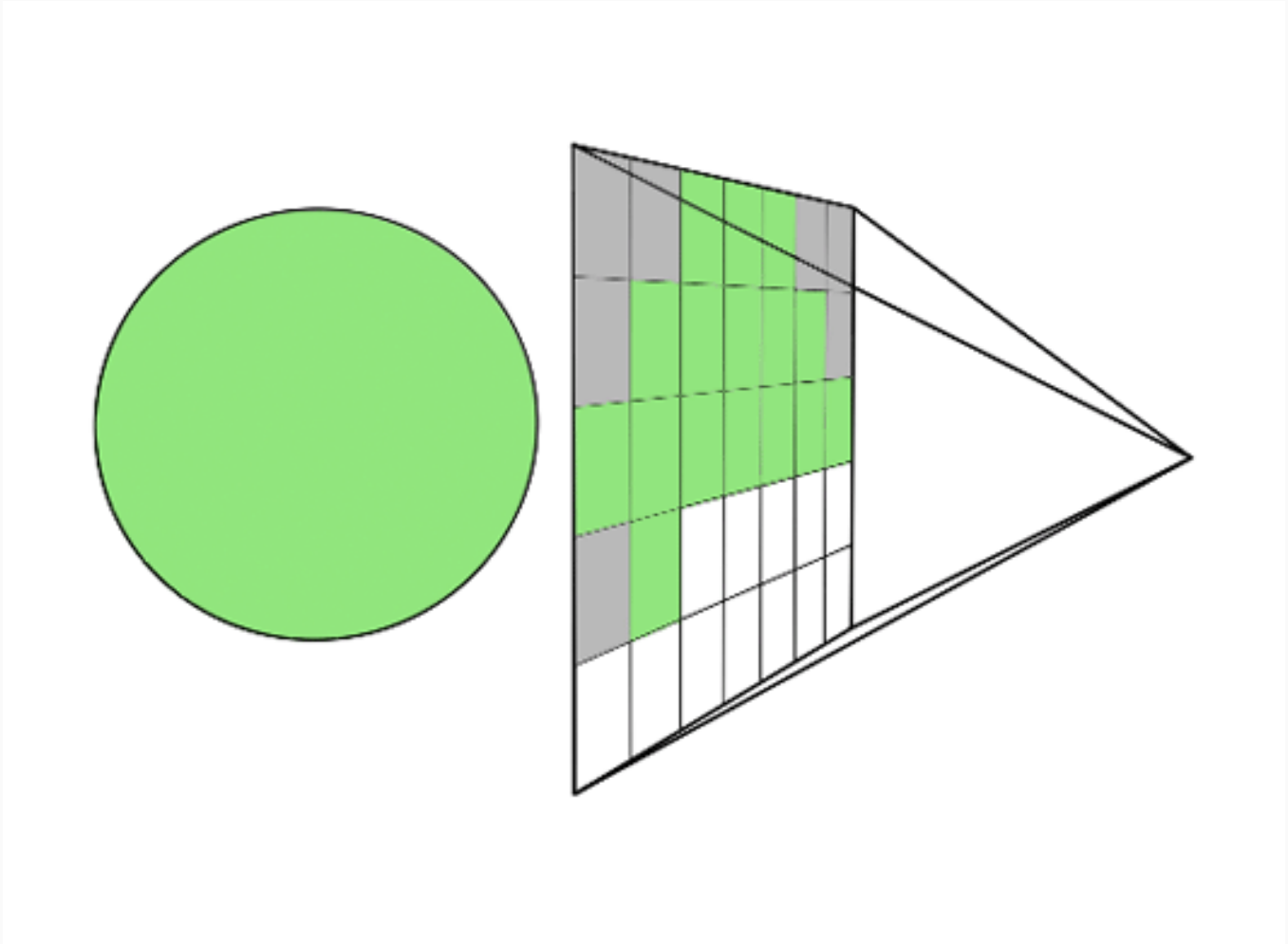

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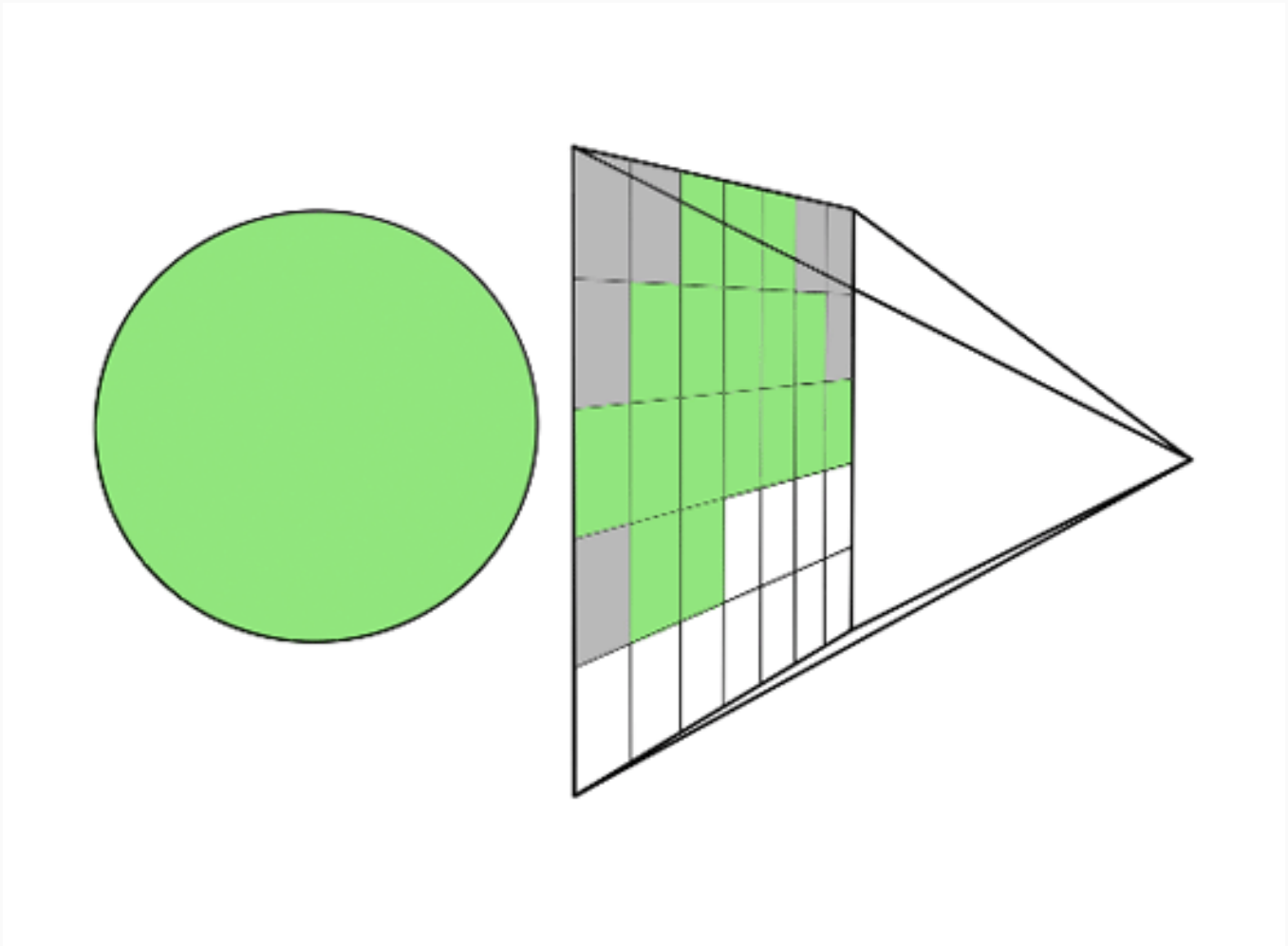

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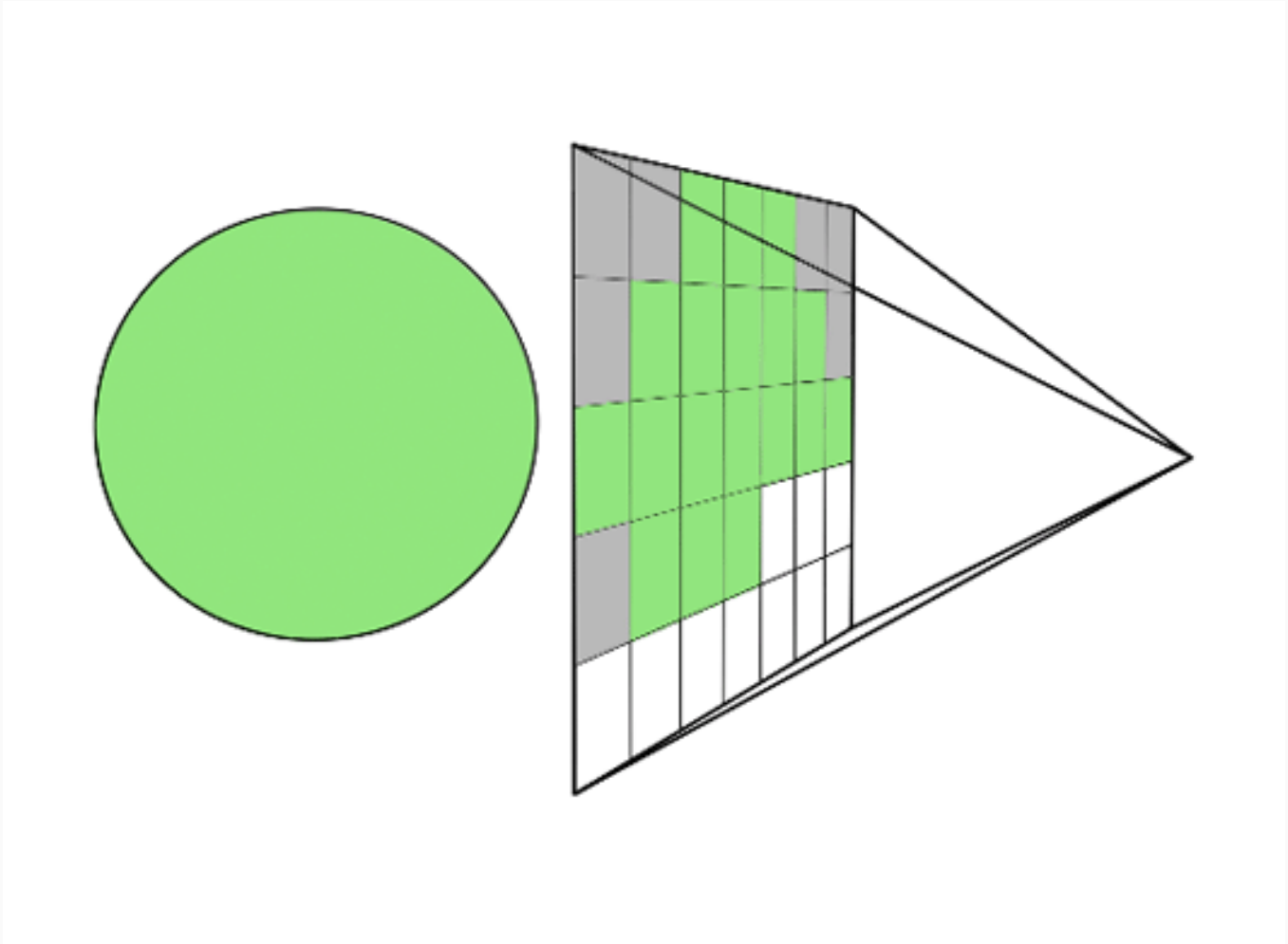

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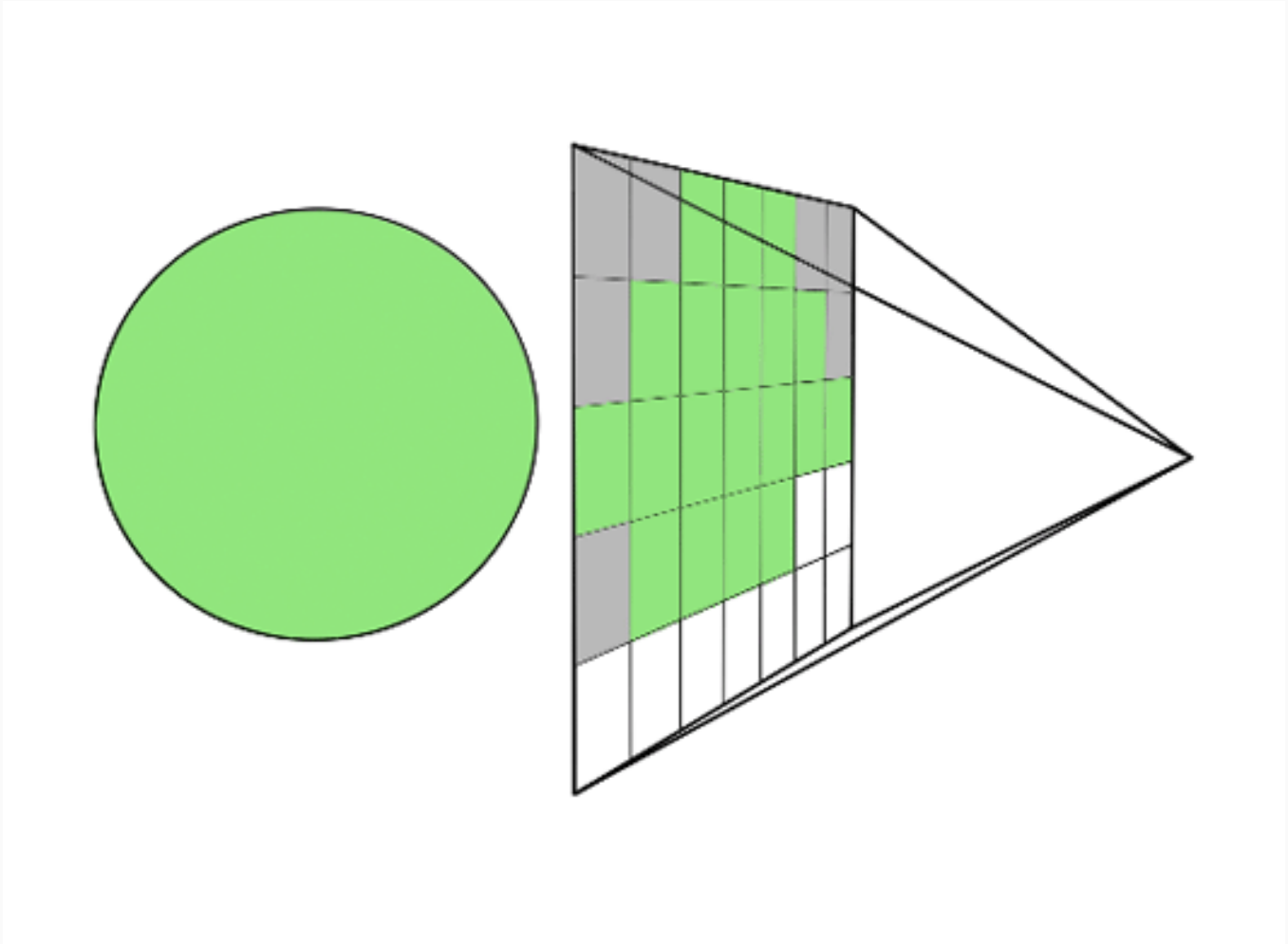

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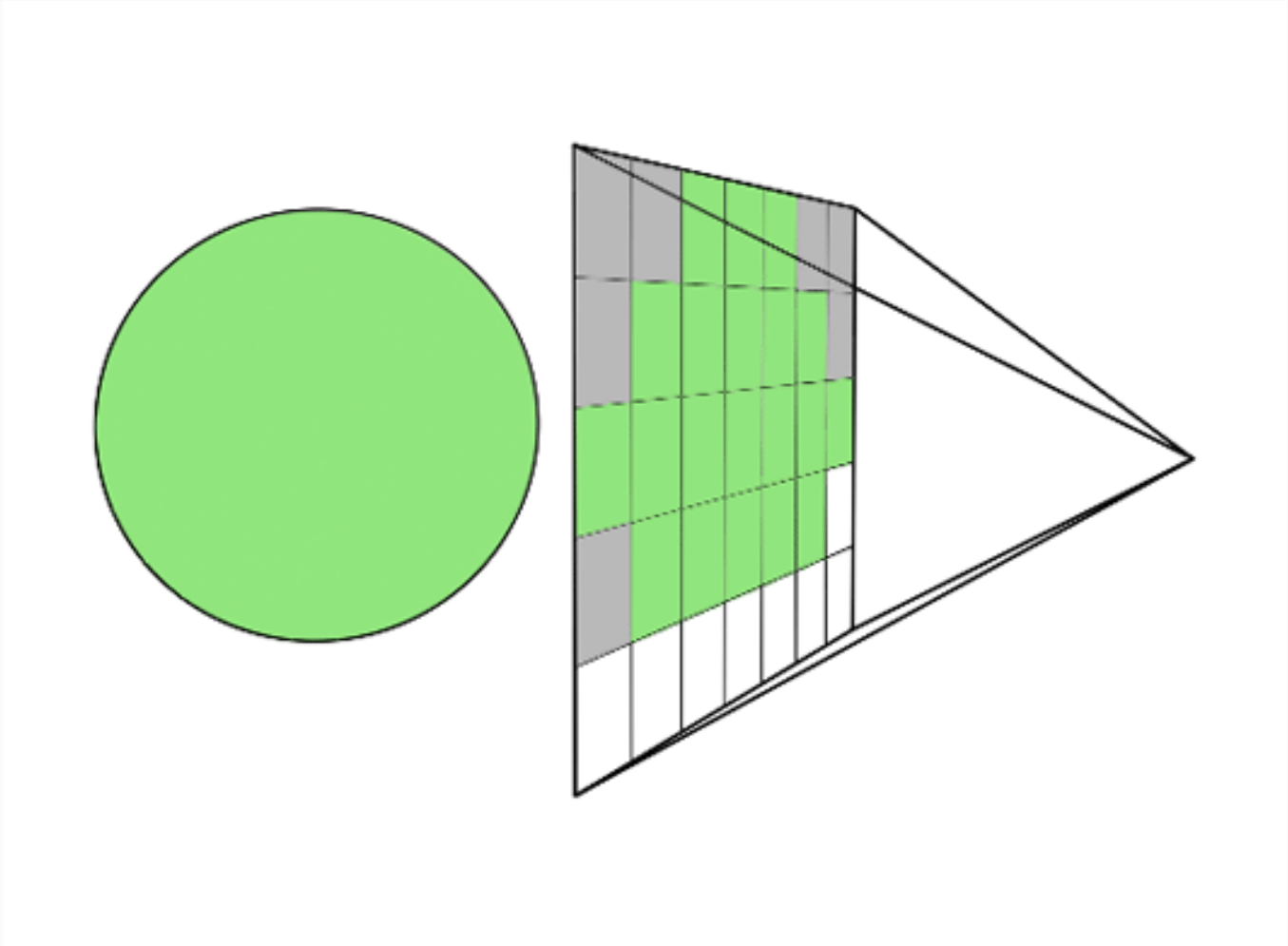

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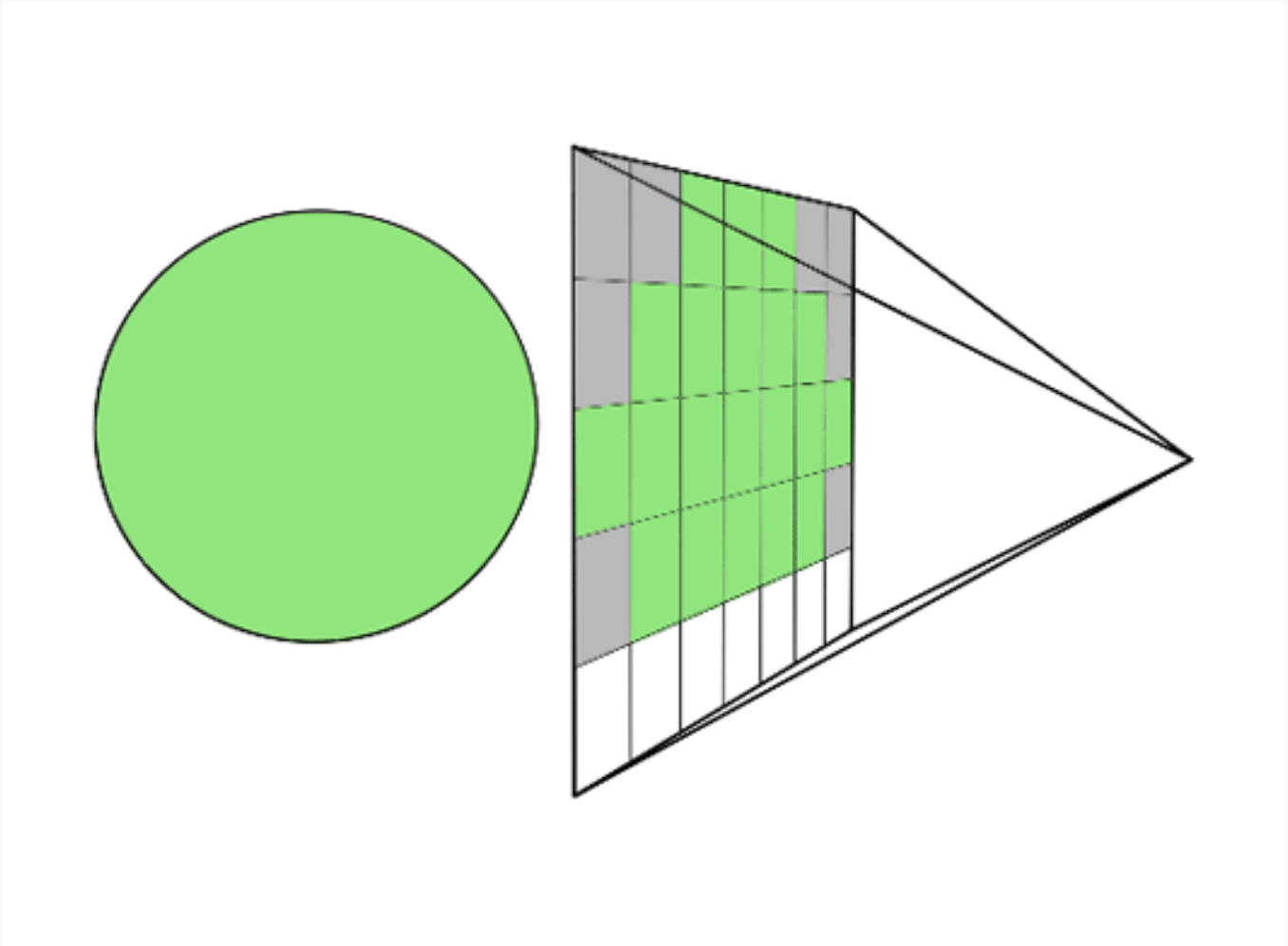

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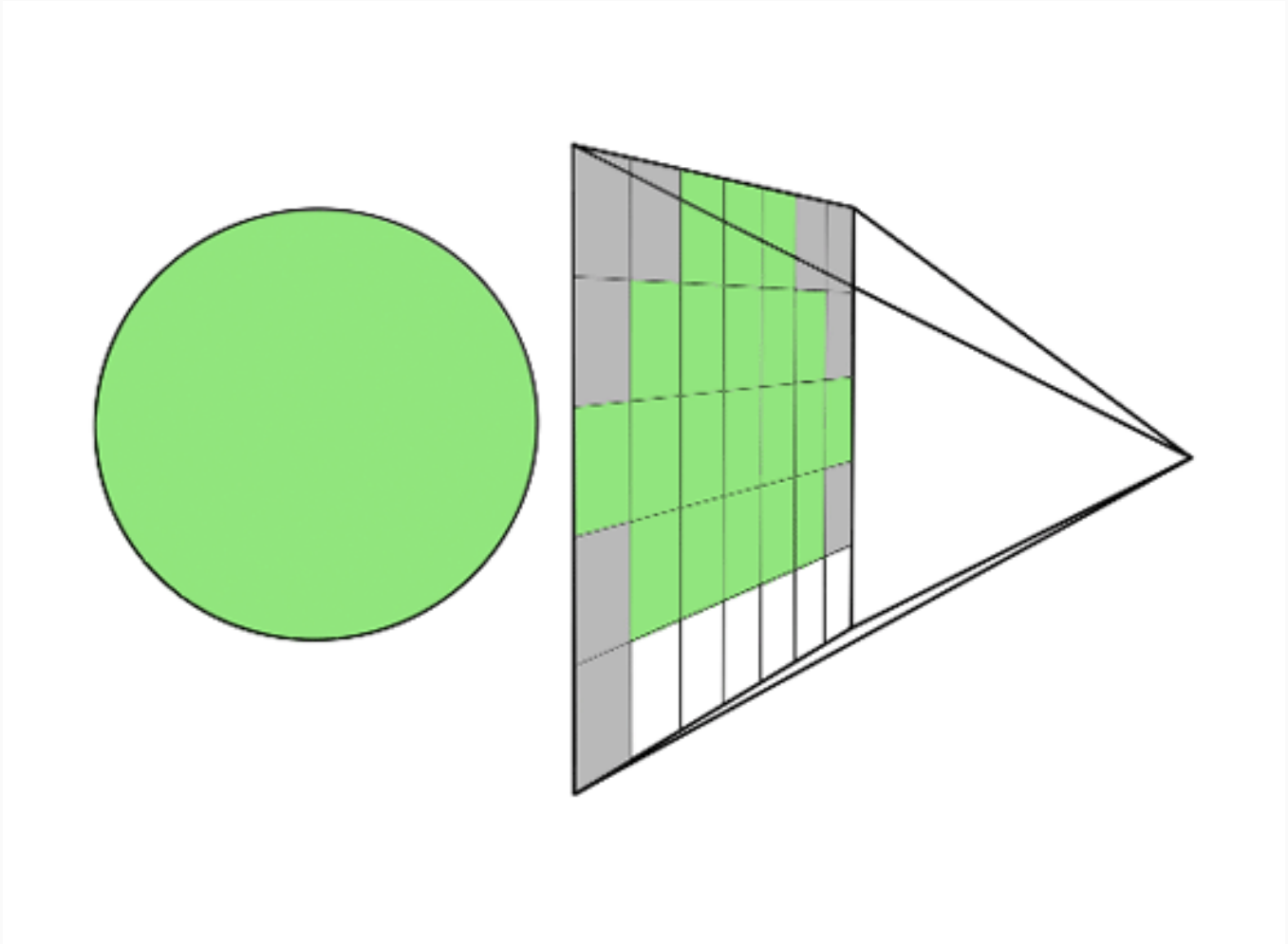

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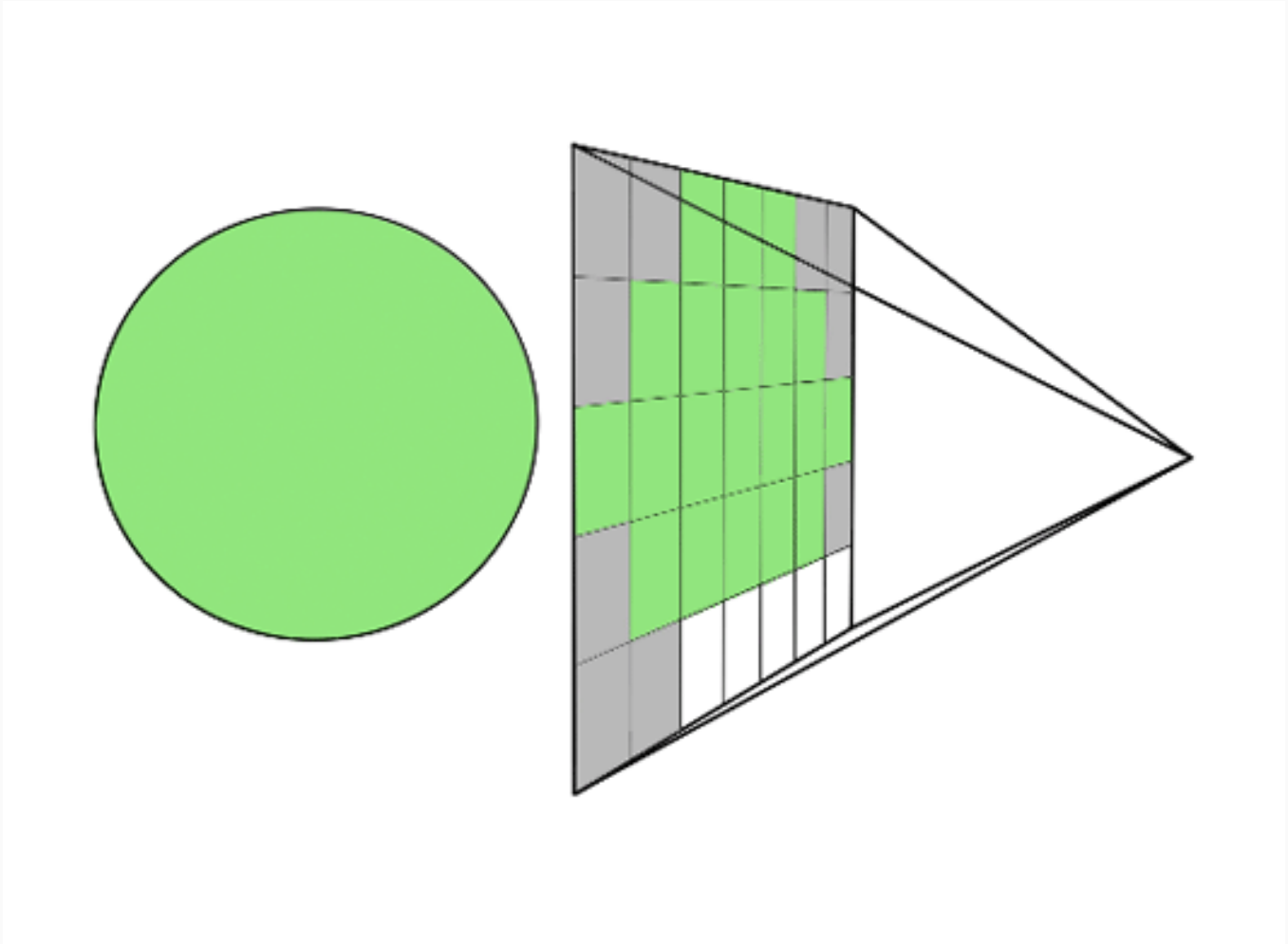

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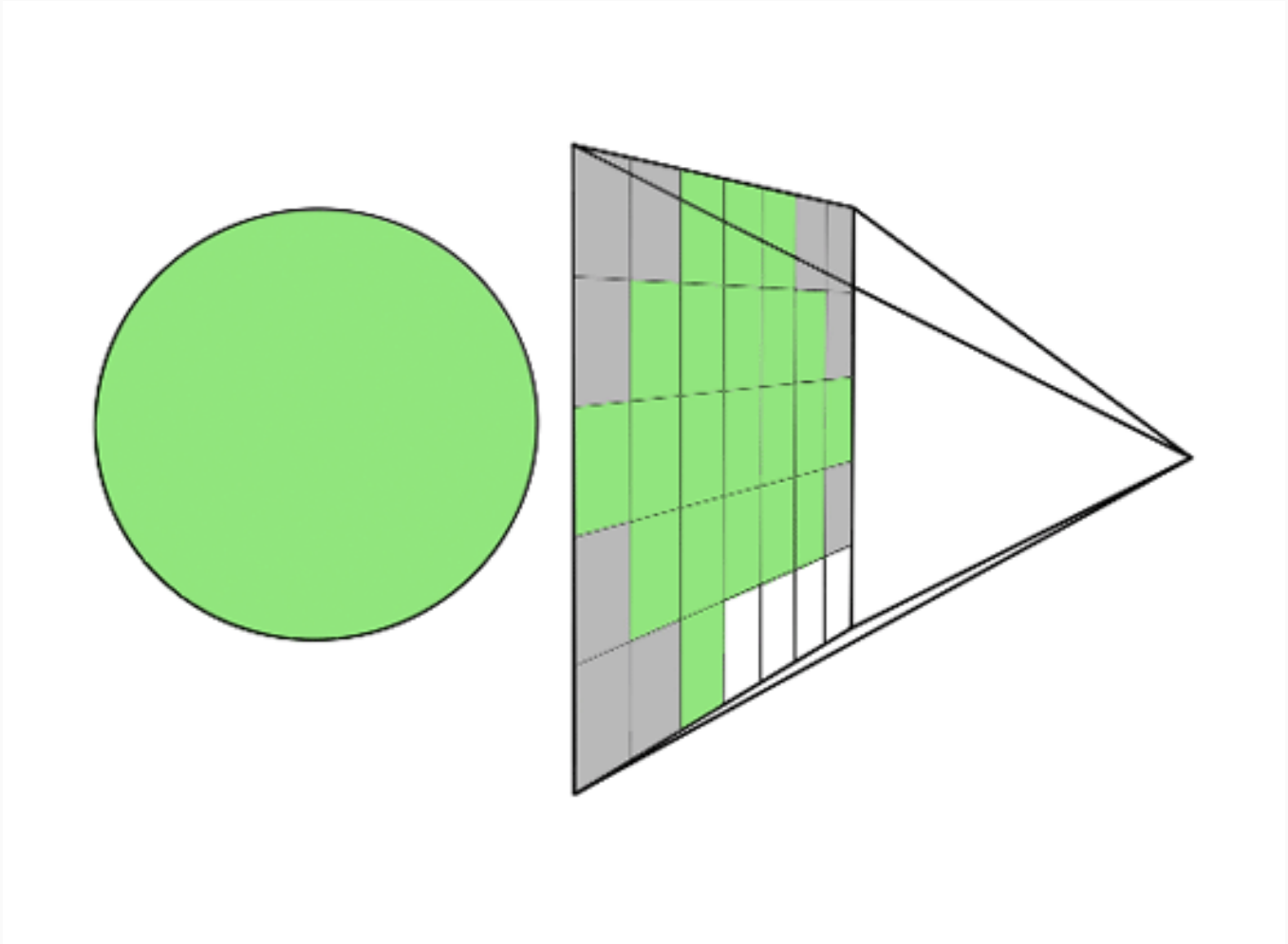

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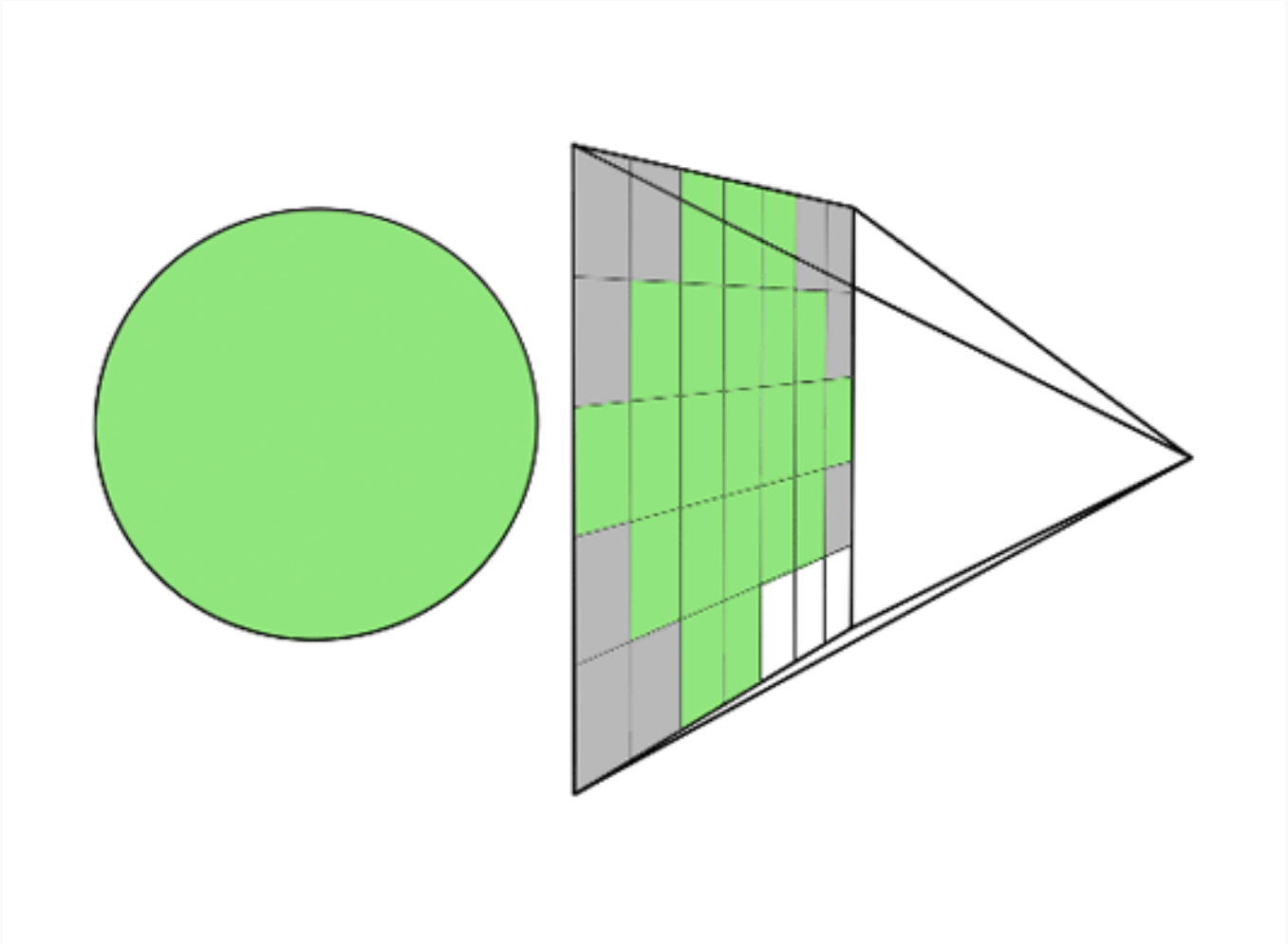

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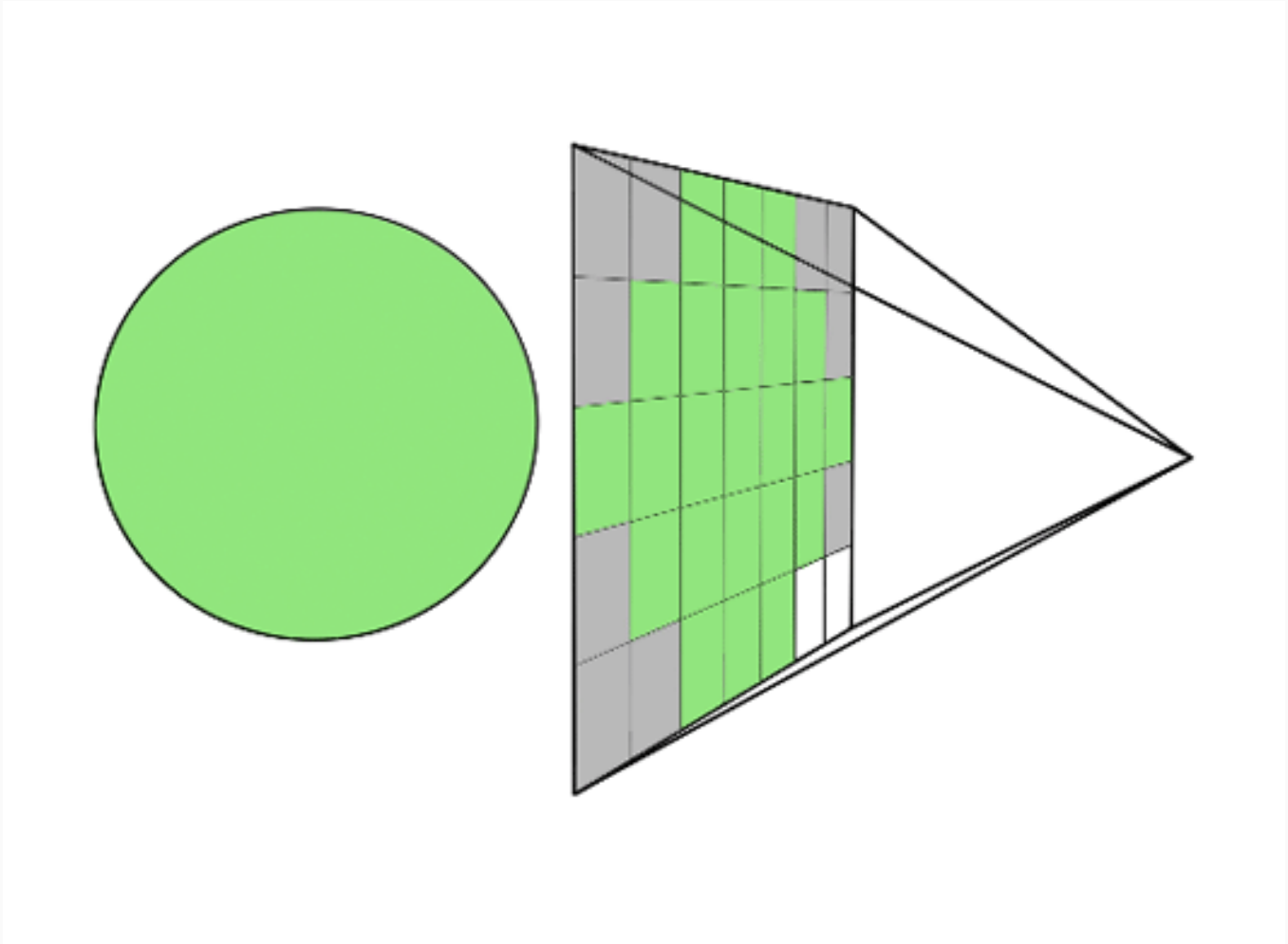

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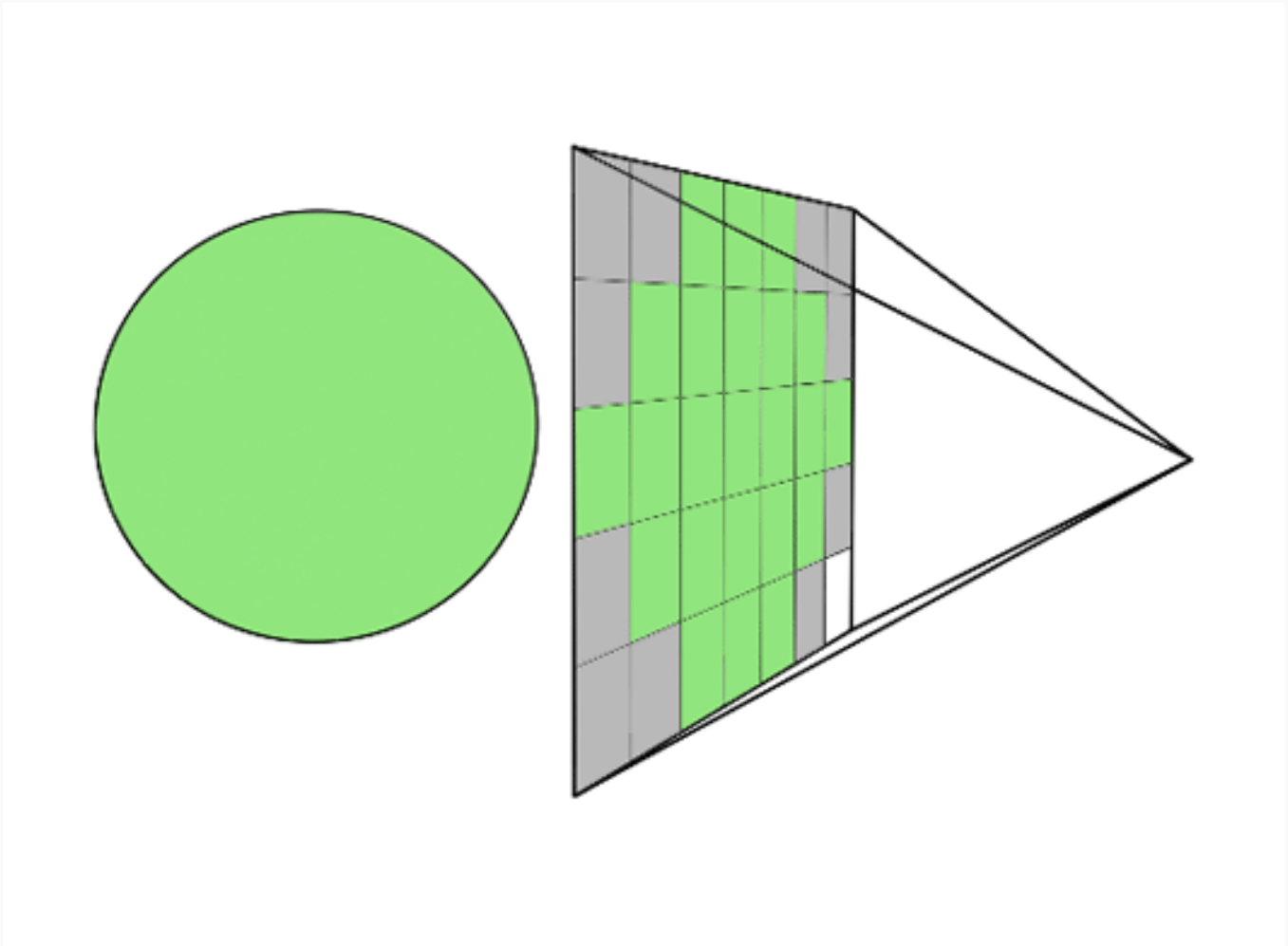

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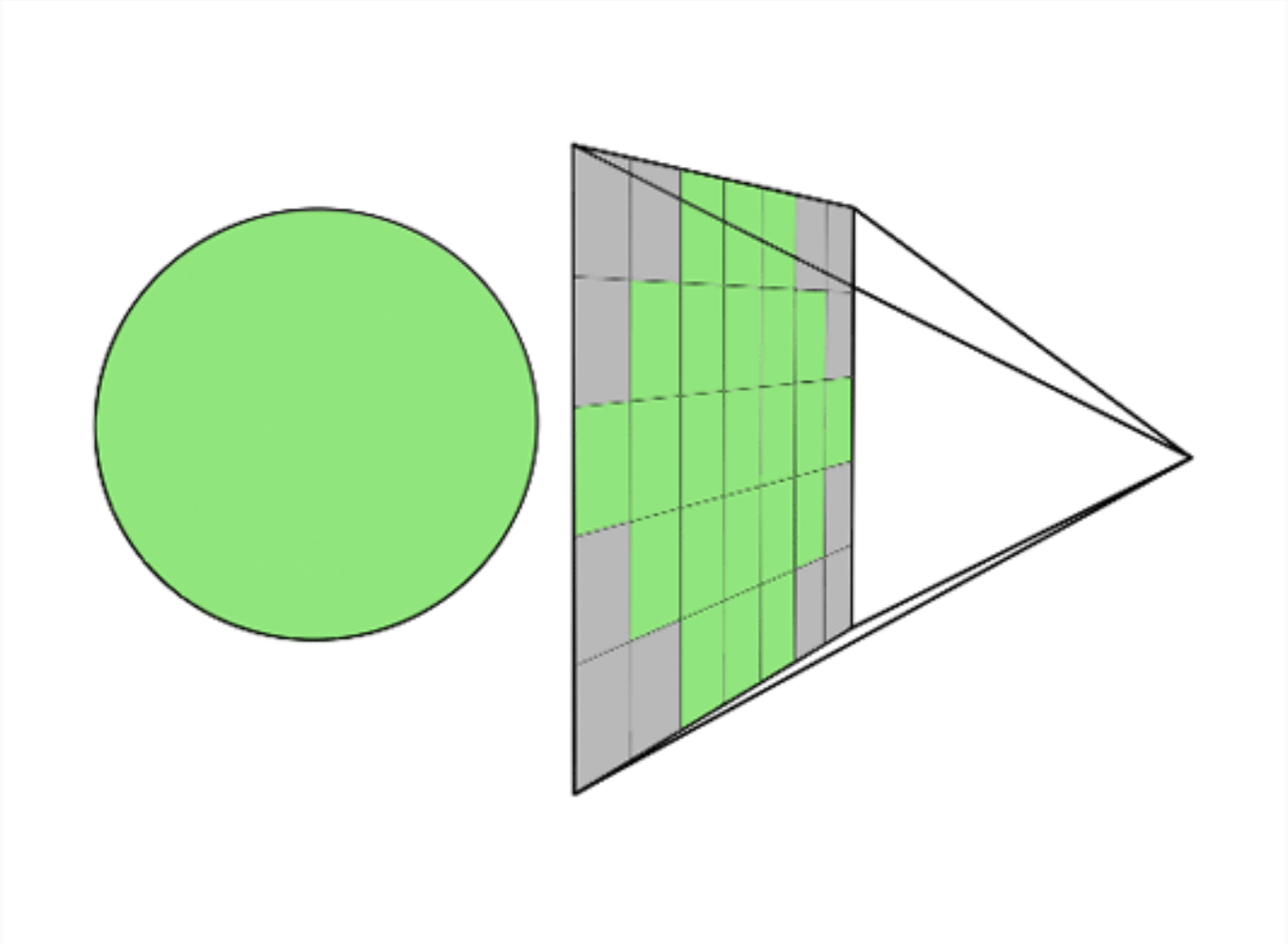


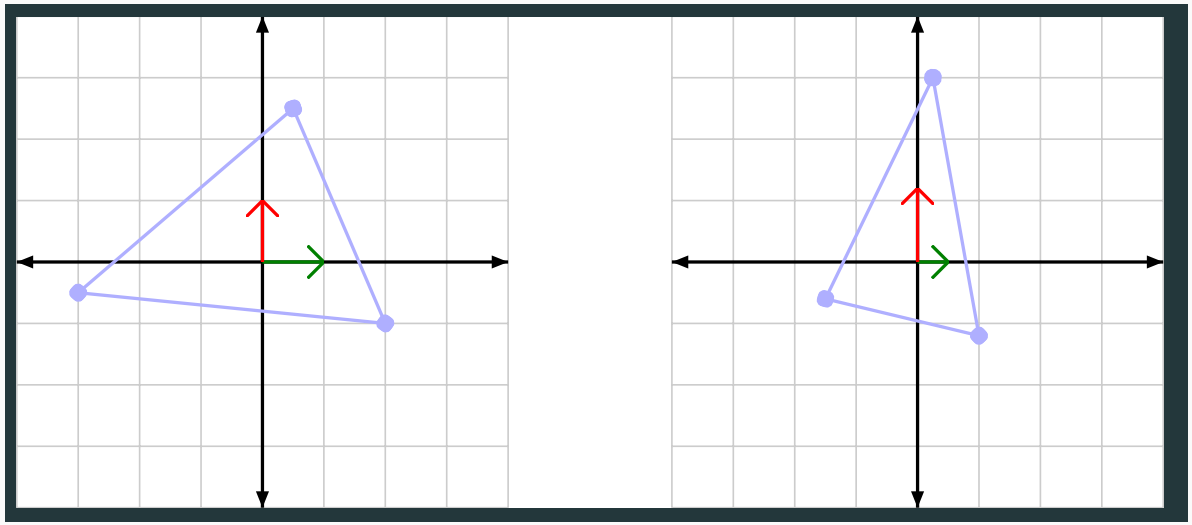
Linear Algebra

Vector Spaces

Point in vector space does not make sense without basis

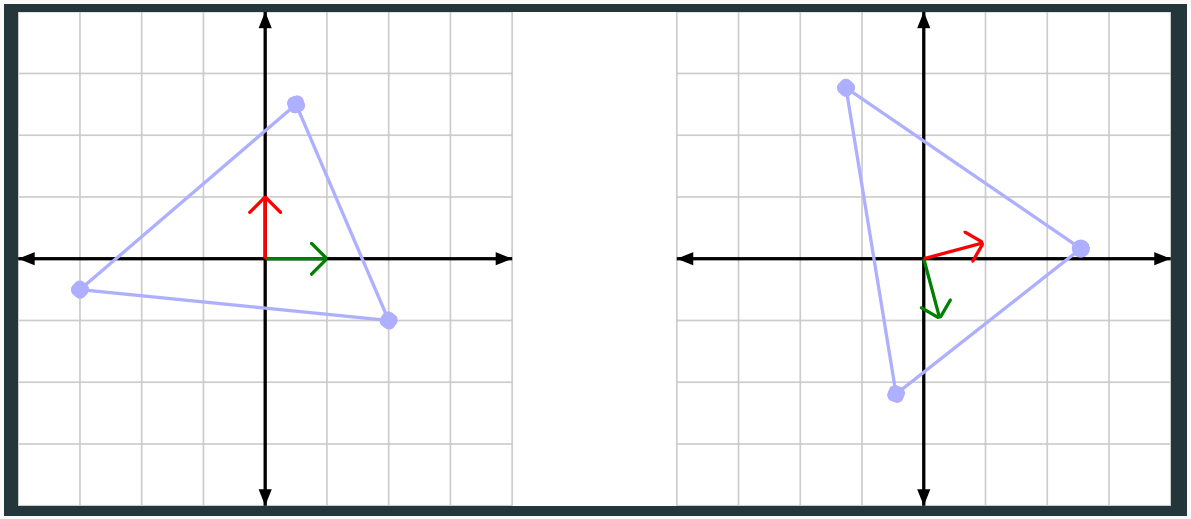
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Scaling



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [v_x \mathbf{e}_1, v_y \mathbf{e}_2, v_z \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} v_x & 0 & 0 \\ 0 & v_y & 0 \\ 0 & 0 & v_z \end{bmatrix} [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Rotation



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \mathbf{R}\left[\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\right] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Rotation

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

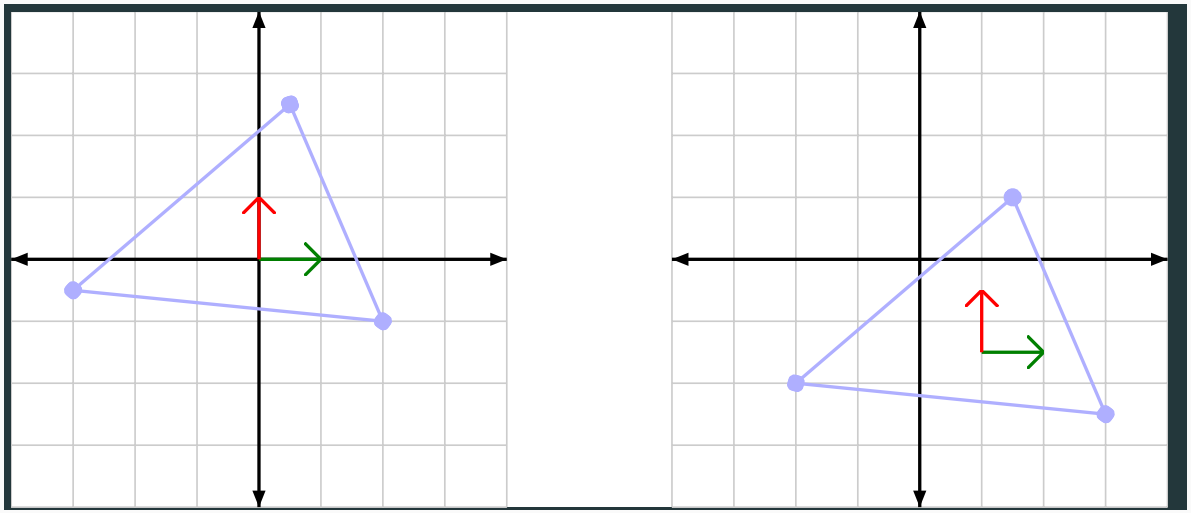
- Matrix multiplication is not commutative
  - $\mathbf{AB} \neq \mathbf{BA}$
  - Need to determine order convention
- Pick and order and stick with it

Rotation

$$R_{xyz} = \begin{bmatrix} \cos\theta_y \cos\theta_z & -\cos\theta_x \sin\theta_z + \sin\theta_x \sin\theta_y \cos\theta_z & \sin\theta_x \sin\theta_z + \cos\theta_x \sin\theta_y \cos\theta_z \\ \cos\theta_y \sin\theta_z & \cos\theta_x \cos\theta_z + \sin\theta_x \sin\theta_y \sin\theta_z & -\sin\theta_x \cos\theta_z + \cos\theta_x \sin\theta_y \sin\theta_z \\ -\sin\theta_y & \sin\theta_x \cos\theta_y & \cos\theta_x \cos\theta_y \end{bmatrix}$$

- Matrix multiplication is not commutative
  - $\mathbf{AB} \neq \mathbf{BA}$
  - Need to determine order convention
- Pick and order and stick with it

Translation



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x + dx \\ y + dy \\ z + dz \end{bmatrix}$$

Homogenous Coordinates[5]

- We do not want to deal with translation differently compared to other transformations
- We can work in *projective* or *homogenous* coordinate representations instead
- We will append the space with one dimension and specify an equivivalence class of all projections

Homogenous Coordinates

- Perspective Projection

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{x}{z} \\ \frac{y}{z} \end{bmatrix}$$

- Specify equivivalence class between all projected points

$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \sim \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$$

- Homogenous coordinates

$$\begin{bmatrix} \frac{2}{4} \\ \frac{3}{4} \\ \frac{4}{4} \end{bmatrix} = \begin{bmatrix} \frac{4}{8} \\ \frac{6}{8} \\ \frac{8}{8} \end{bmatrix} = \begin{bmatrix} \frac{2}{4} \\ \frac{3}{4} \\ 1 \end{bmatrix}$$

Homogenous Coordinates

- Cartesian 3D space

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Allows for representing points in infinity

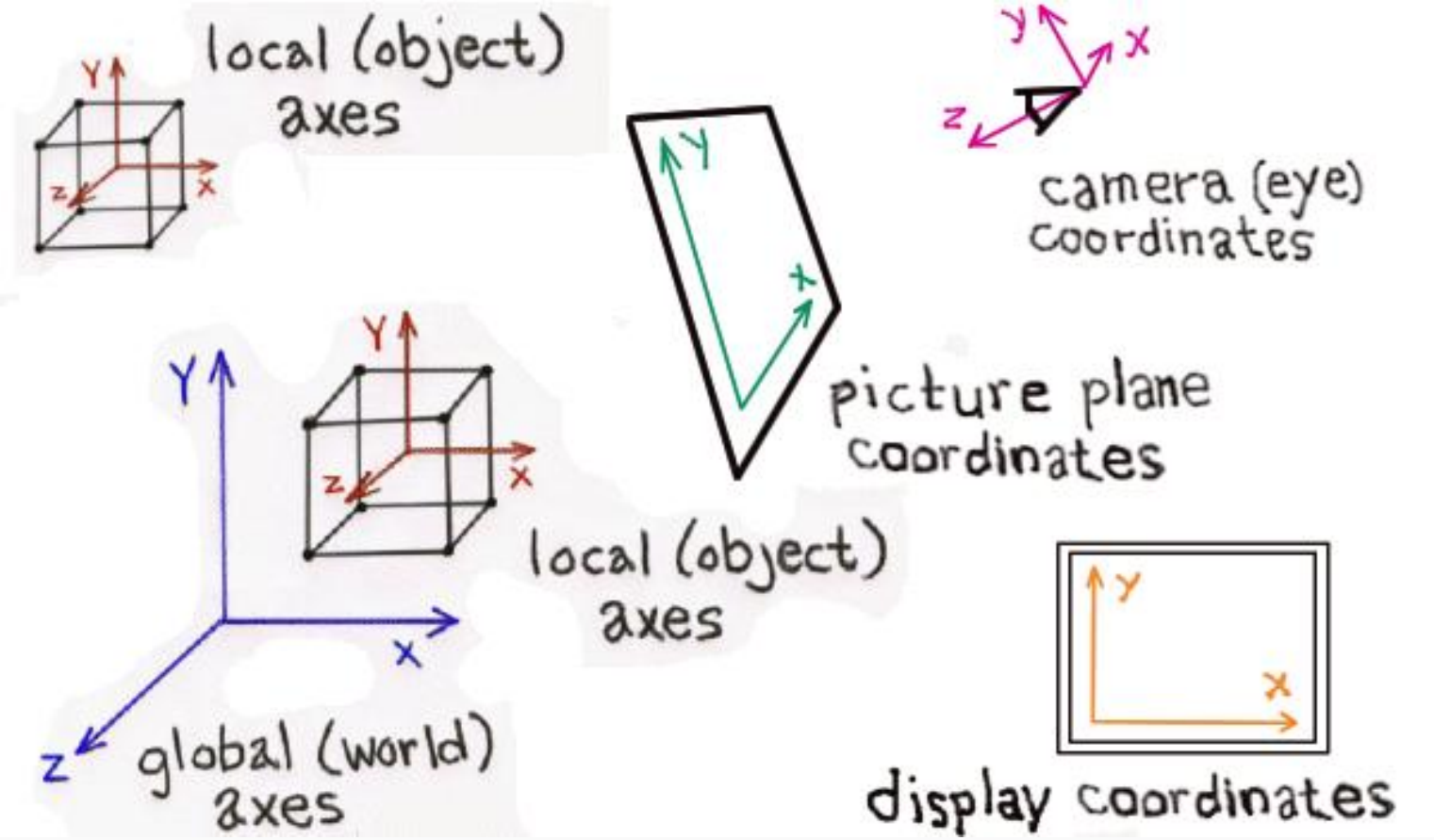
$$\lim_{d \to \infty} \begin{bmatrix} x + d \\ y + d \\ z + d \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

- For example, under projection two parallell lines will meet

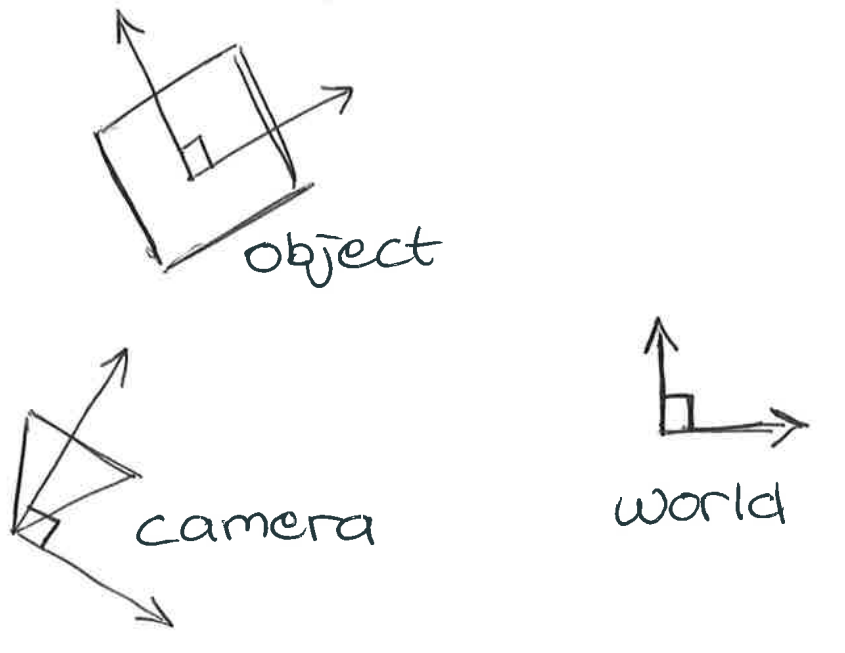
Homogenous Coordinates

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} v_x \cdot R_{11} & R_{12} & R_{13} & dx \\ R_{21} & v_y \cdot R_{22} & R_{23} & dy \\ R_{31} & R_{32} & v_z \cdot R_{33} & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Cordinate Transforms



View Transformations



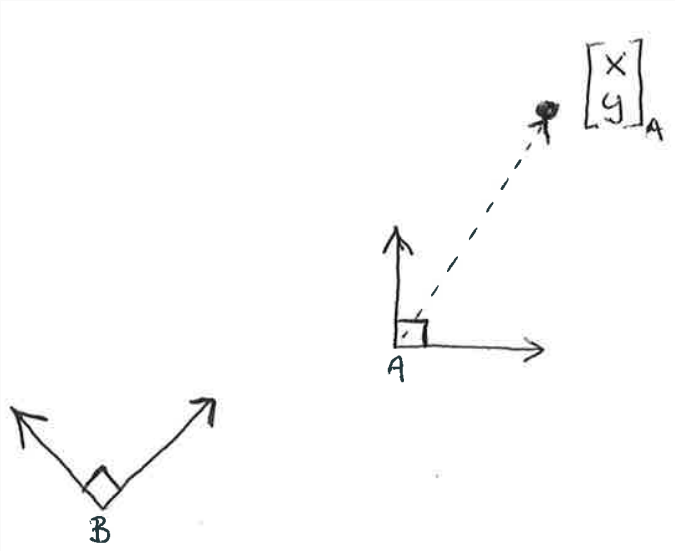
View Transformations

1. Translate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}_A - \begin{bmatrix} x \\ y \end{bmatrix}_{O_B}$$

2. Rotate (negative)

$$\begin{bmatrix} x \\ y \end{bmatrix}_B = \begin{bmatrix} cos(-\theta) & -sin(-\theta) \\ sin(-\theta) & cos(-\theta) \end{bmatrix}_B \begin{bmatrix} x' \\ y' \end{bmatrix}$$



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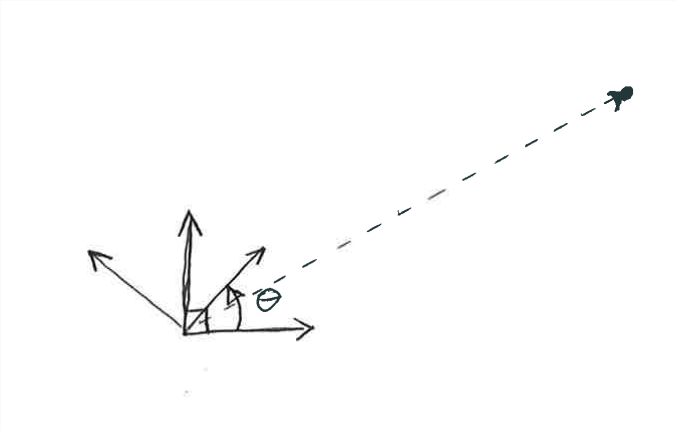

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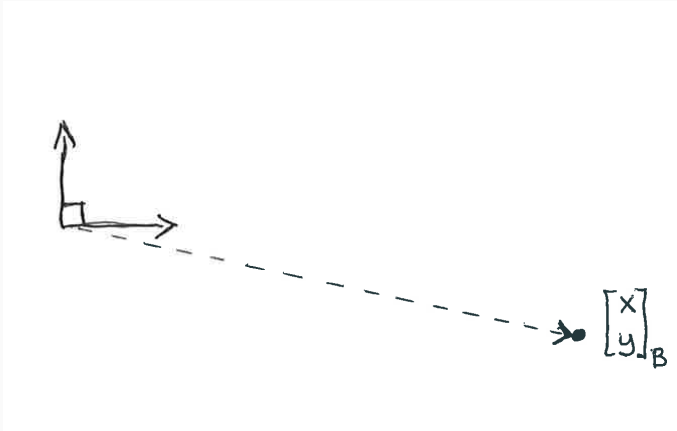

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View Transformations

Transformation

Always the same order for every transformation of coordinate systems

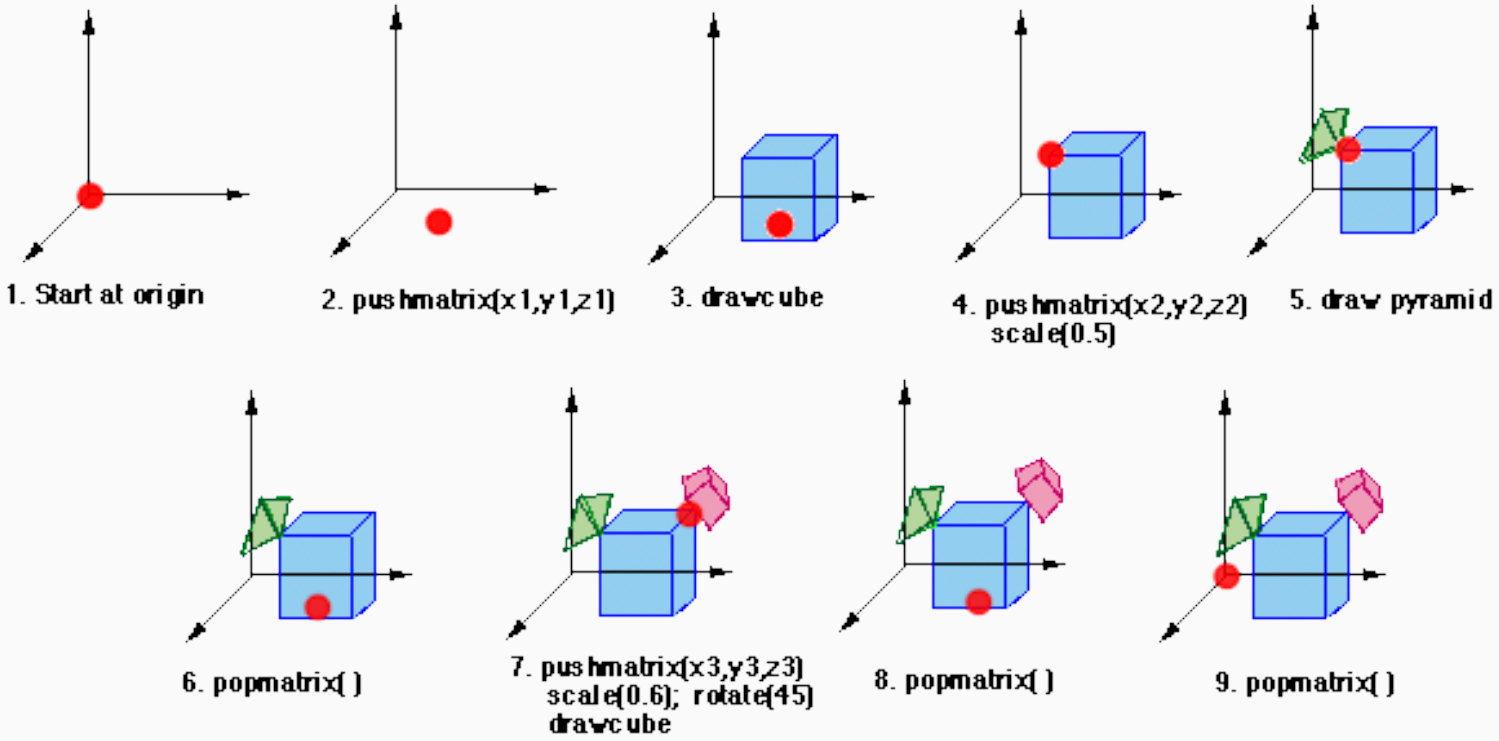
1. Translate
2. Rotate

Transformation Homogenous Coordinates

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} R_{11}(\theta) & R_{12}(\theta) & R_{13}(\theta) & 0 \\ R_{21}(\theta) & R_{22}(\theta) & R_{23}(\theta) & 0 \\ R_{31}(\theta) & R_{32}(\theta) & R_{33}(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Rotate θ degrees around x-axis at $[a, b, c]^{\mathrm{T}}$
- Remember that matrix multiplications are not cummutative

Transformation Stack

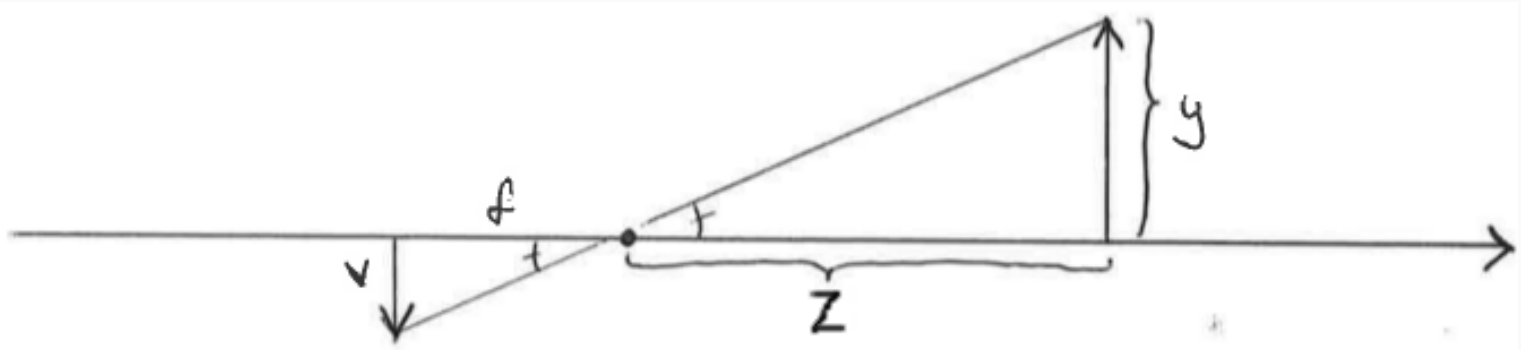


Raytracing

Camera

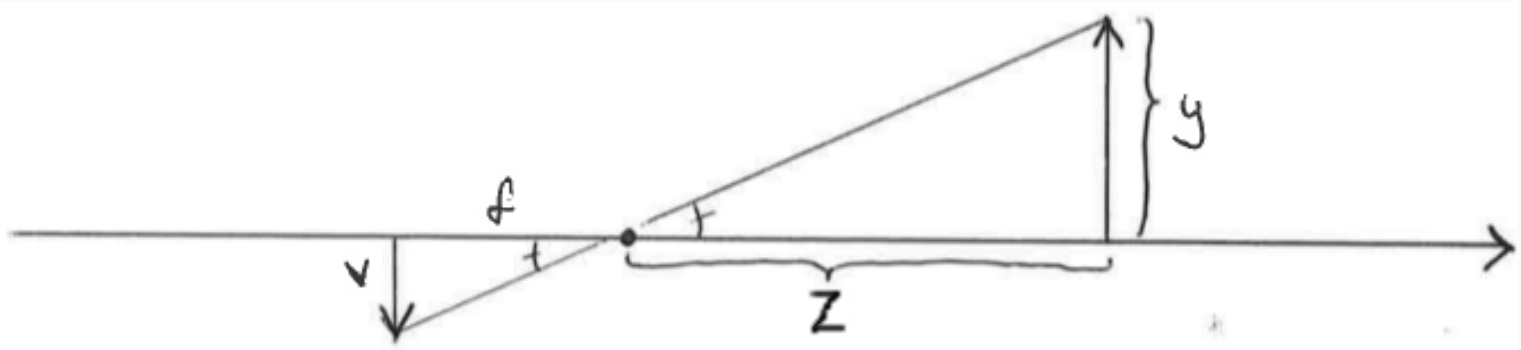
- Cameras create images
- Lens
  - captures rays
- Focal lenth
- Shutter opening

Pinhole camera



- Simplest form camera, where one ray of light lits up each pixel
- Defined by a *location*, *focal length*, *view-direction* and *up-direction*
- is this a realistic camera?

Pinhole camera

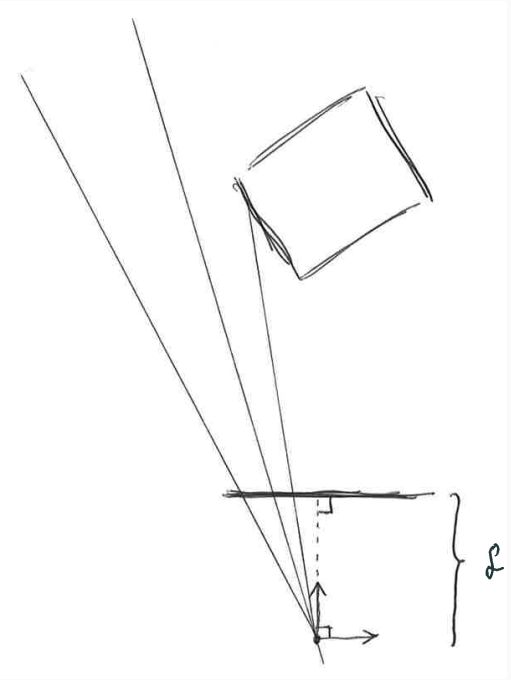


$$\frac{v}{f} = \frac{y}{z}$$

$$\Rightarrow v = \frac{f}{z} y$$

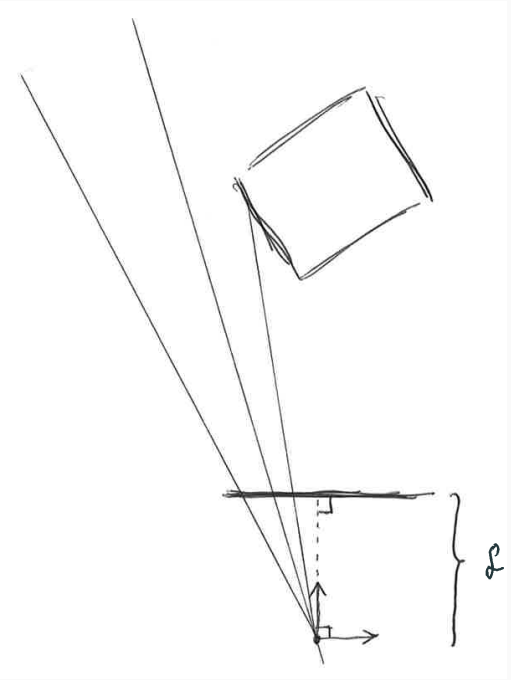
Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$



Raycasting

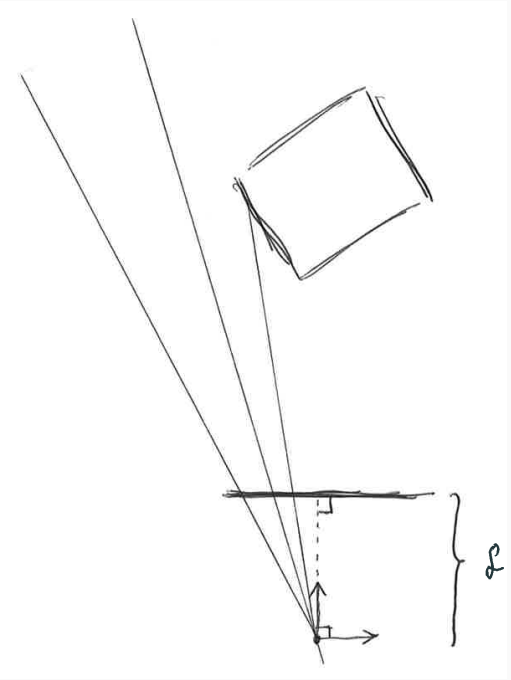
1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$



Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$
3. Image plane

$$\left\{ [u, v, f]^{\mathrm{T}} \,||\, -\frac{W}{2} \leq u < \frac{W}{2}, -\frac{H}{2} < v < \frac{H}{2}, f \right.$$



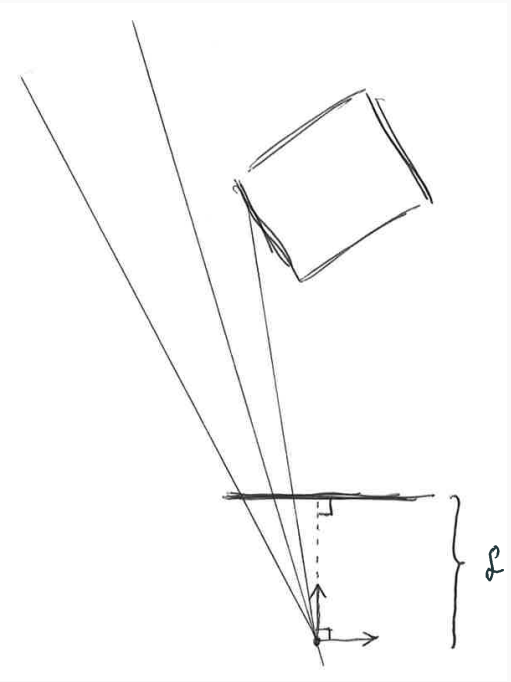
Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$
3. Image plane

$$\left\{ [u, v, f]^{\mathrm{T}} || -\frac{W}{2} \leq u < \frac{W}{2}, -\frac{H}{2} < v < \frac{H}{2}, f \right.$$

4. Compute normalised vector from origin to image-plane

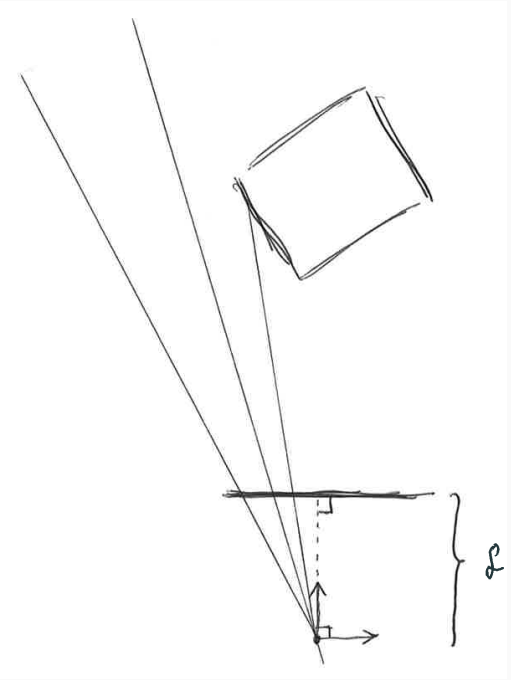
$$\mathbf{d}_{ij} = \frac{1}{\sqrt{[u_i, v_j, f][u_i, v_j, f]^{\mathrm{T}}}} [u_i, v_j, f]^{\mathrm{T}}$$



Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$
3. Image plane

$$\left\{ [u, v, f]^{\mathrm{T}} || -\frac{W}{2} \leq u < \frac{W}{2}, -\frac{H}{2} < v < \frac{H}{2}, f \right.$$



4. Compute normalised vector from origin to image-plane

$$\mathbf{d}_{ij} = \frac{1}{\sqrt{[u_i, v_j, f][u_i, v_j, f]^{\mathrm{T}}}} [u_i, v_j, f]^{\mathrm{T}}$$

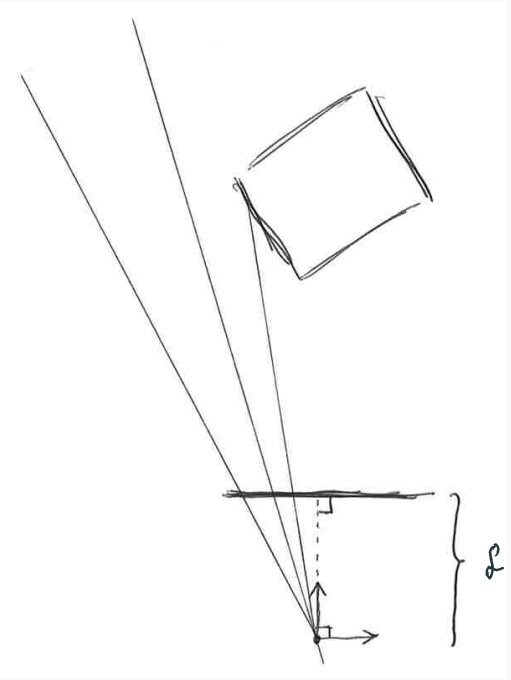
5. Parmatrise Ray

$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$

Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$
3. Image plane

$$\left\{ [u, v, f]^{\mathrm{T}} || -\frac{W}{2} \leq u < \frac{W}{2}, -\frac{H}{2} < v < \frac{H}{2}, f \right.$$



4. Compute normalised vector from origin to image-plane

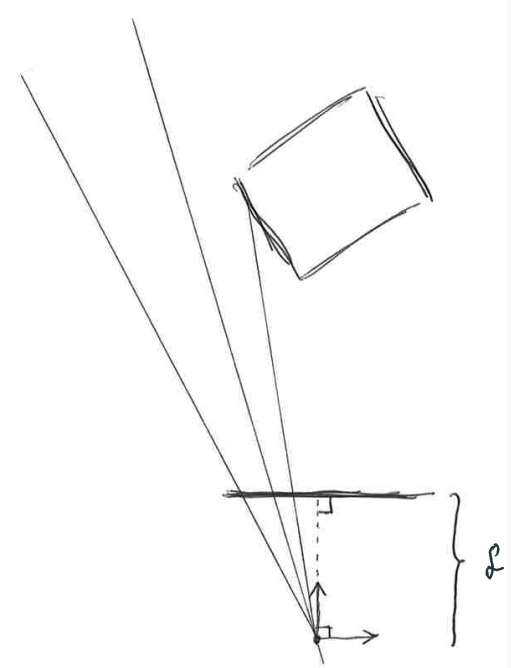
$$\mathbf{d}_{ij} = \frac{1}{\sqrt{[u_i, v_j, f][u_i, v_j, f]^{\mathrm{T}}}} [u_i, v_j, f]^{\mathrm{T}}$$

5. Parmatrise Ray

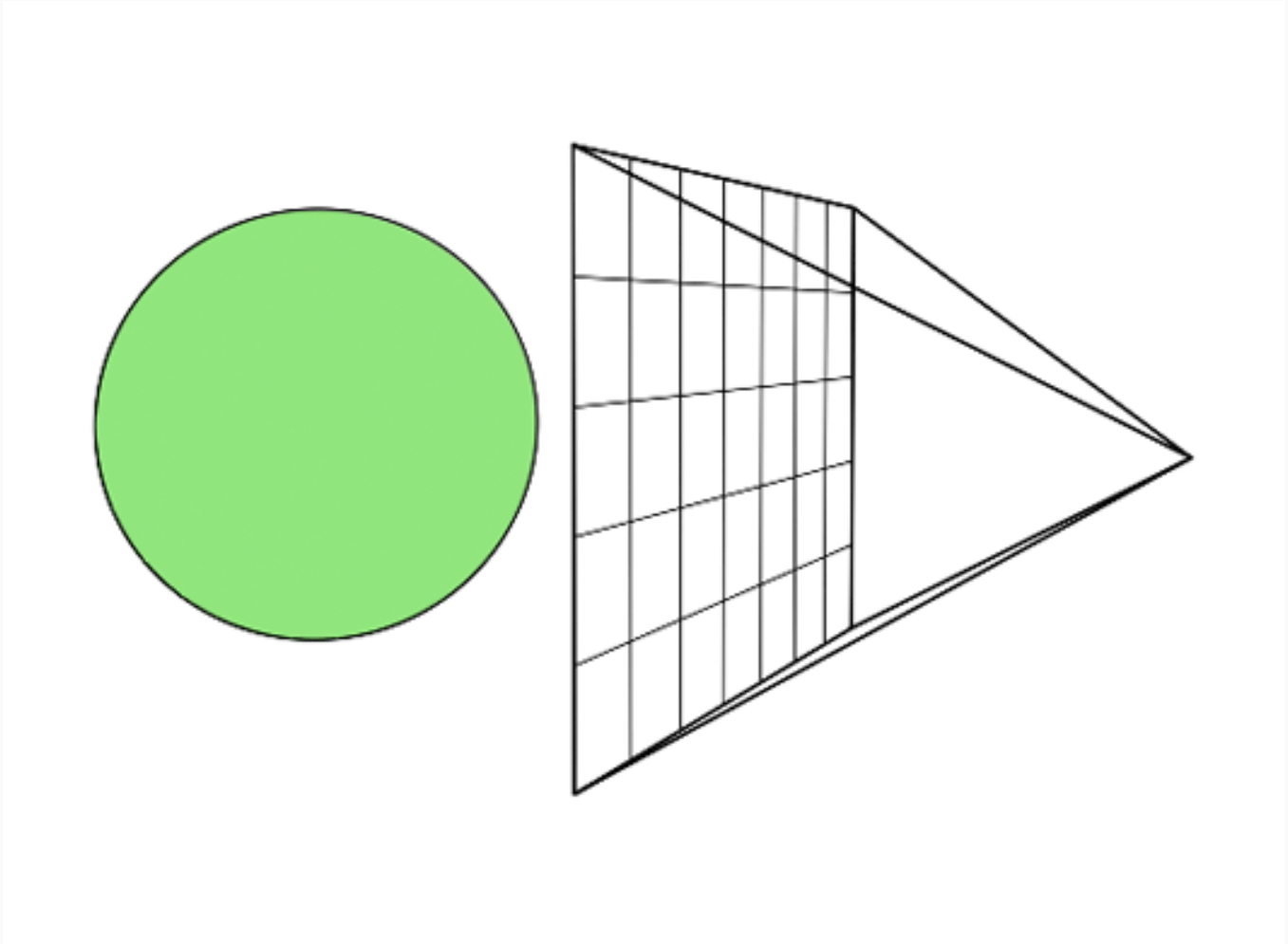
$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$

Raycasting

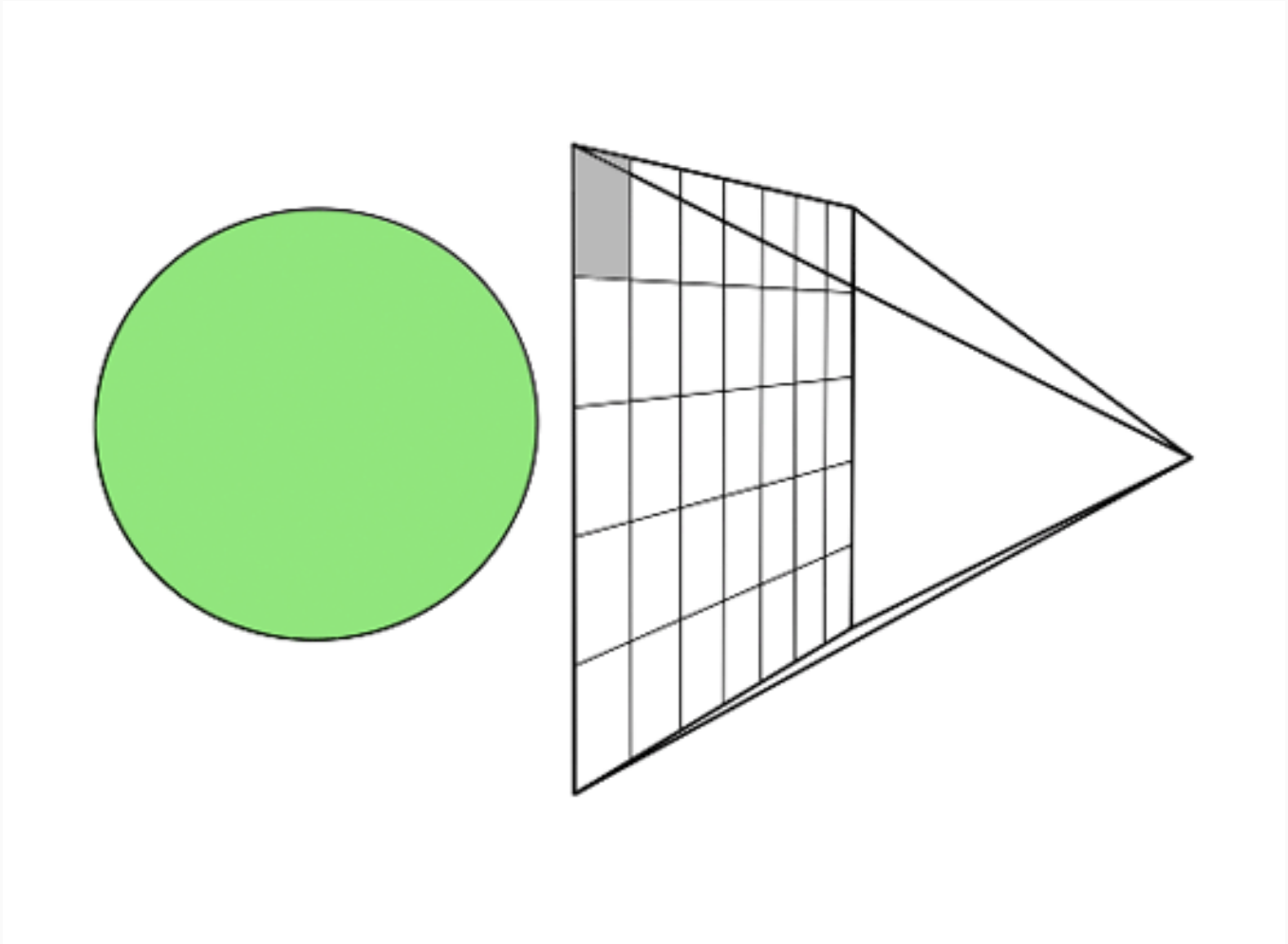
- Trace a ray for each pixel
- Compute intersection with all objects in world
- Plot colour of intersecting surface
- No "lighting" often called Raycaster



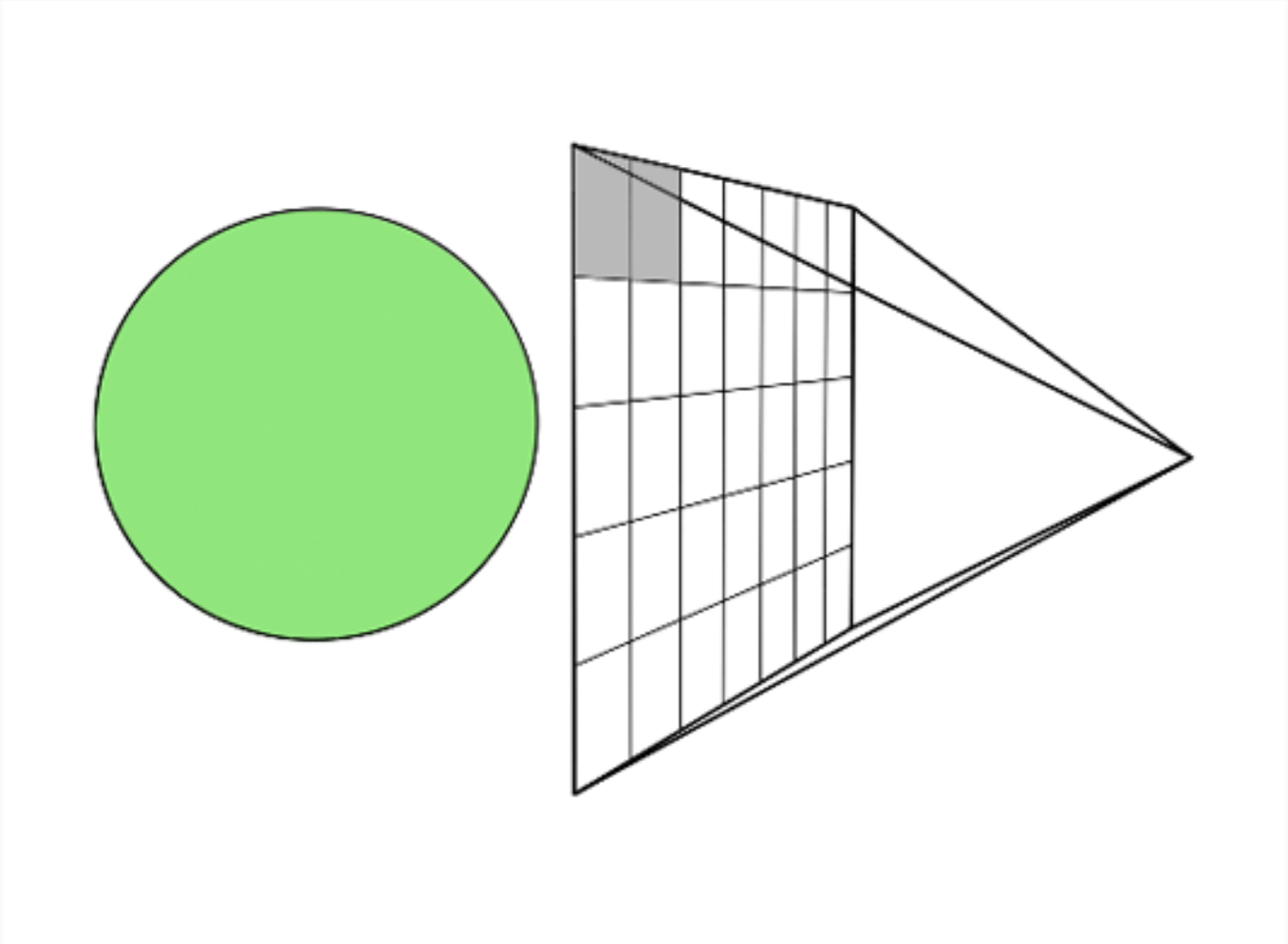
Raycasting Anim



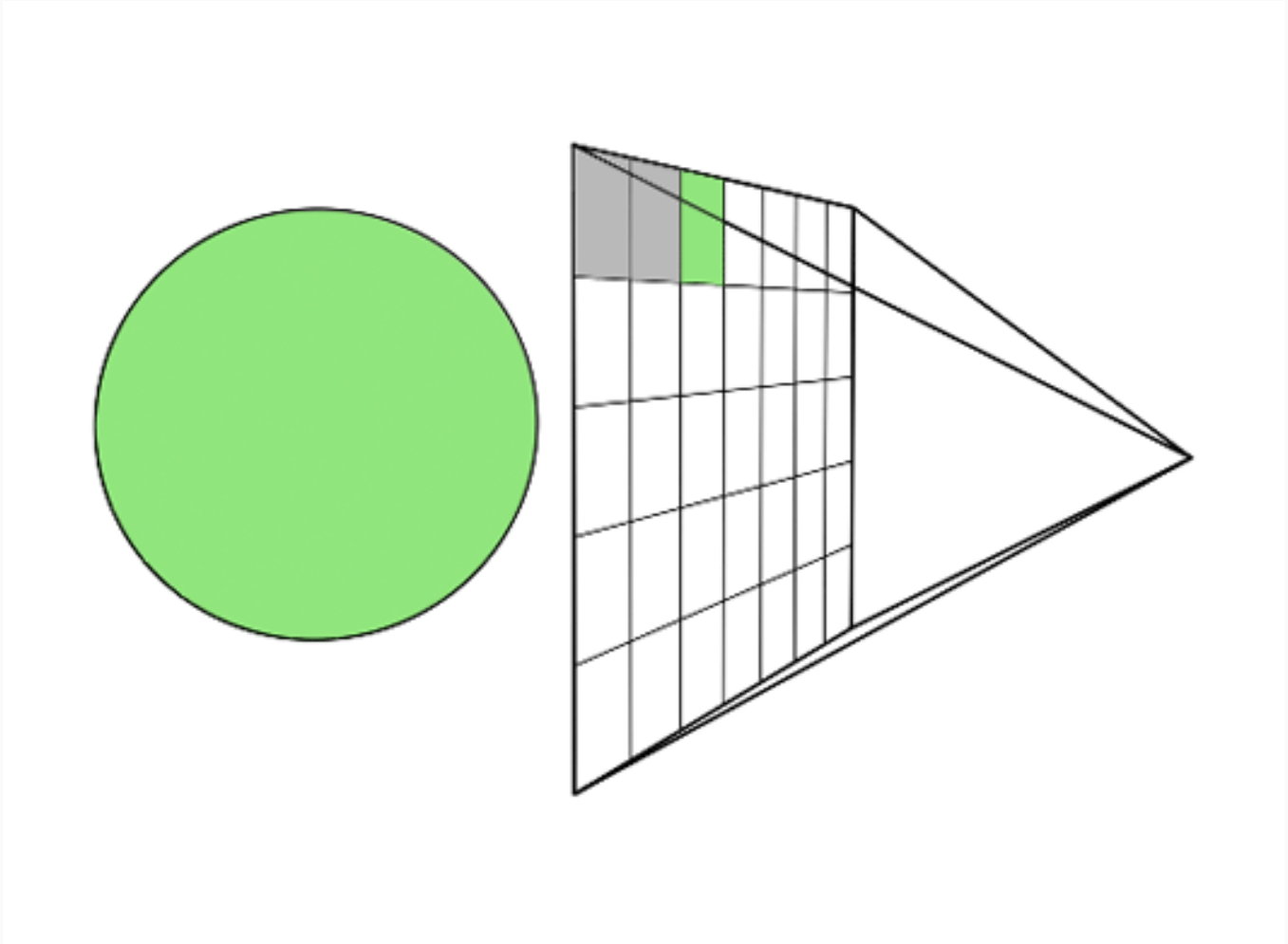
Raycasting Anim



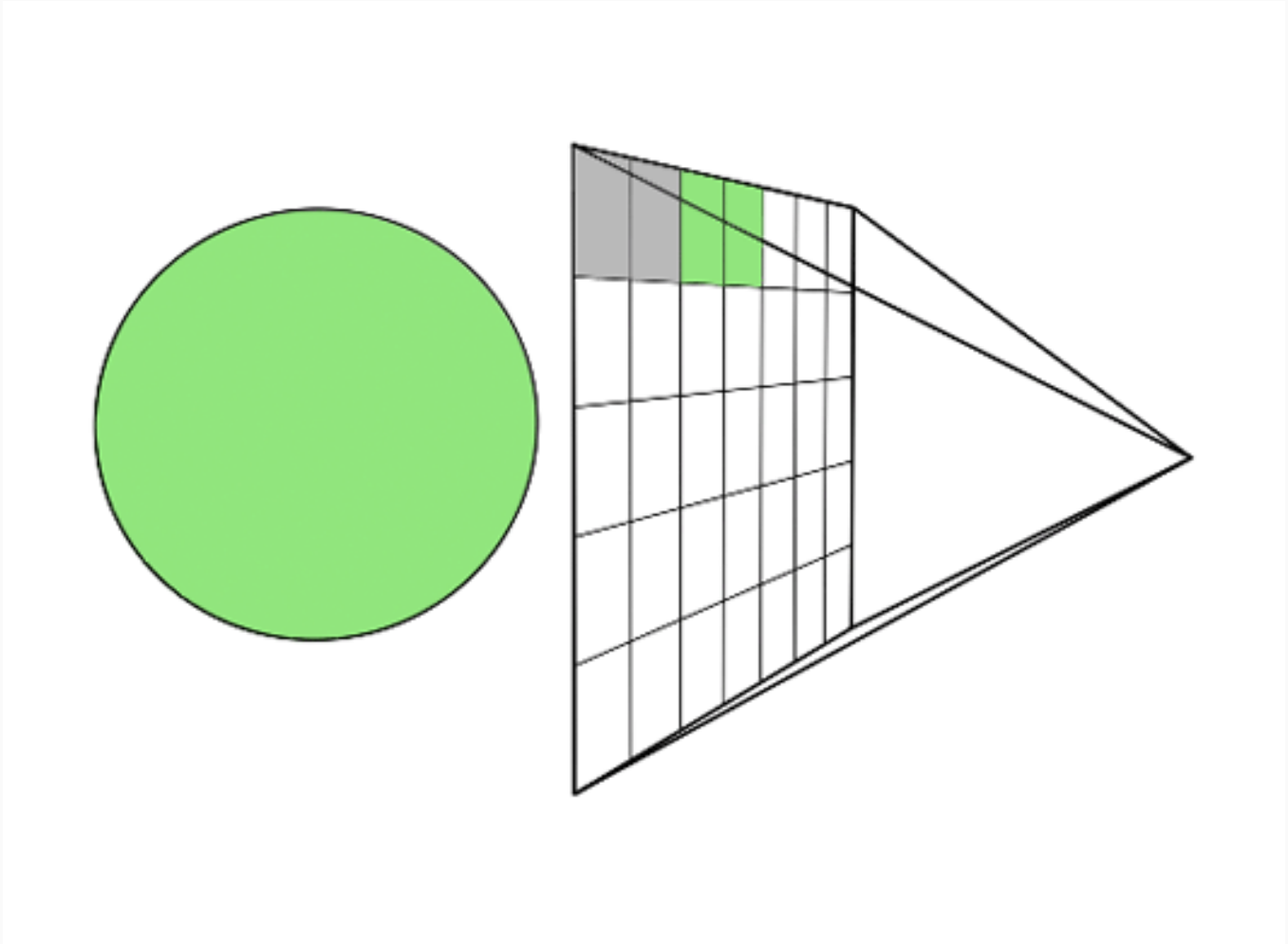
Raycasting Anim



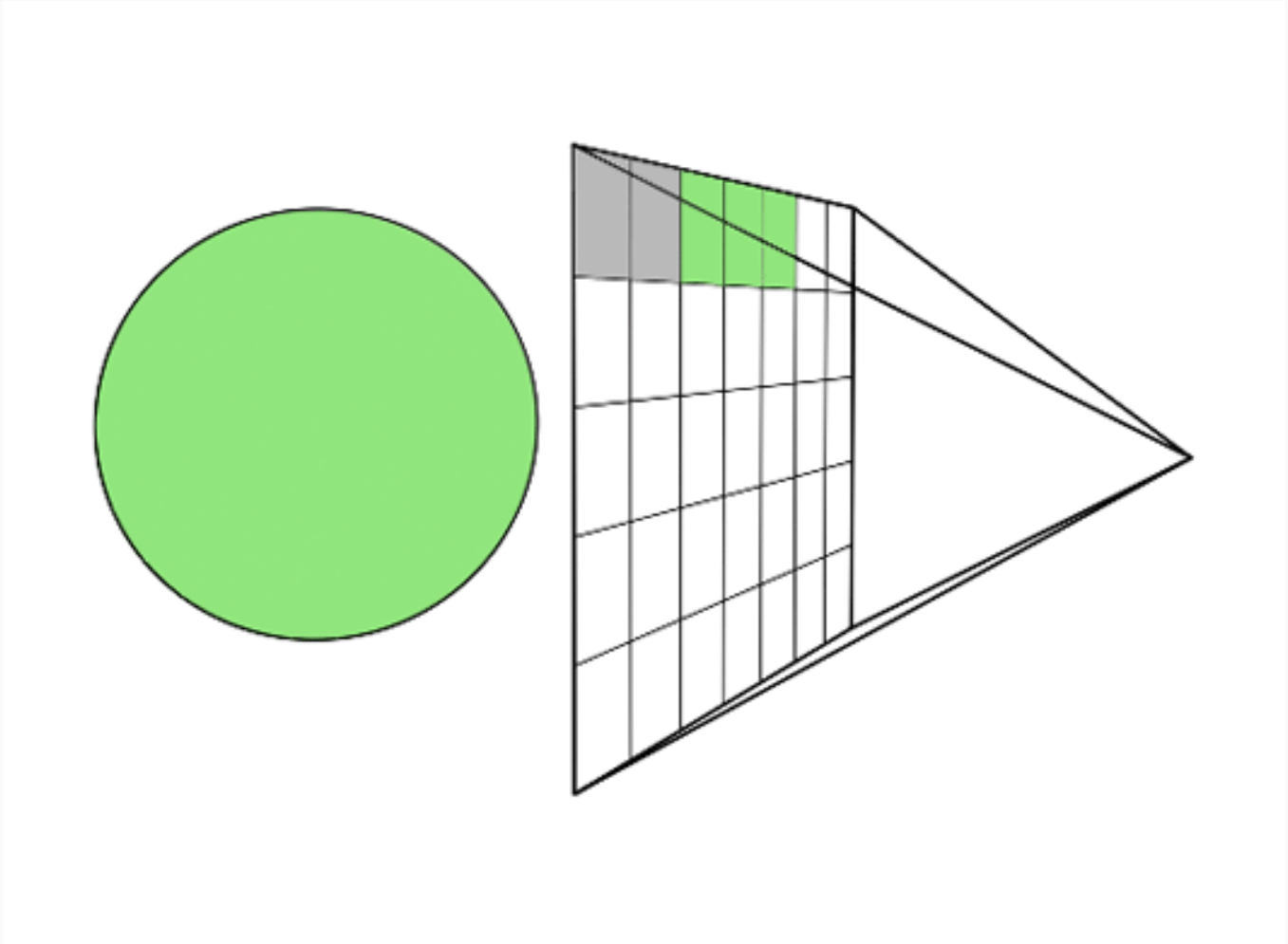
Raycasting Anim



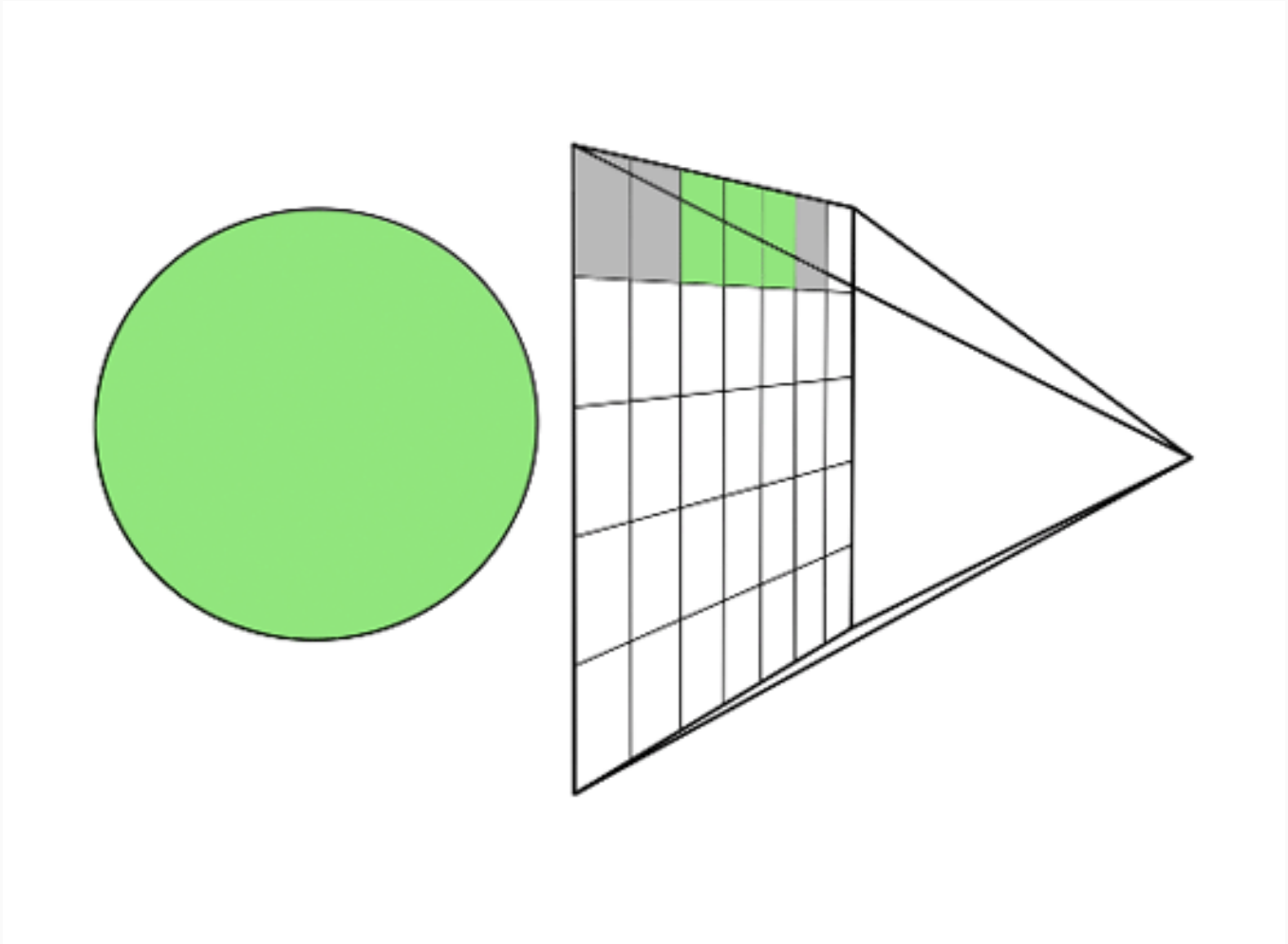
Raycasting Anim



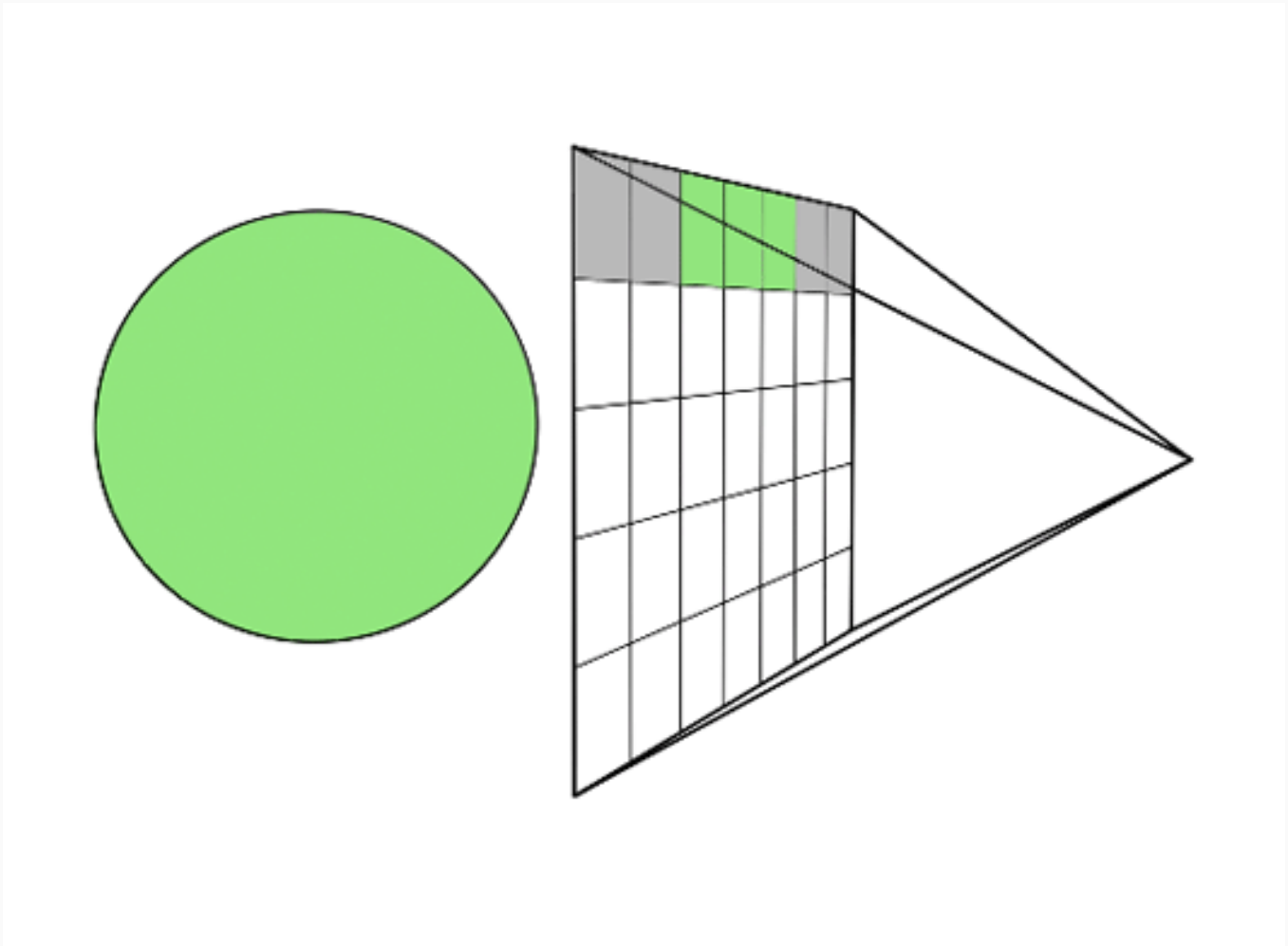
Raycasting Anim



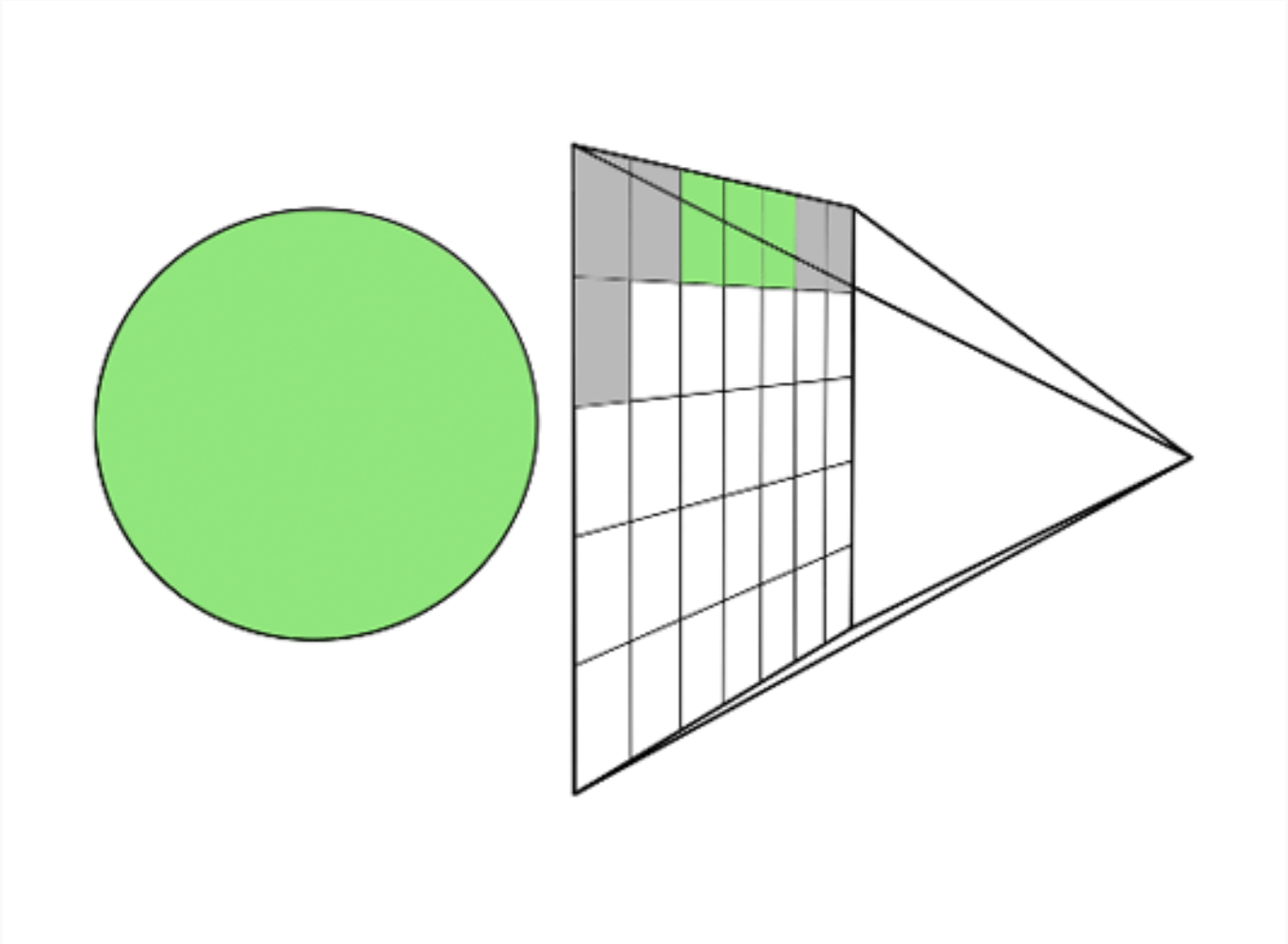
Raycasting Anim



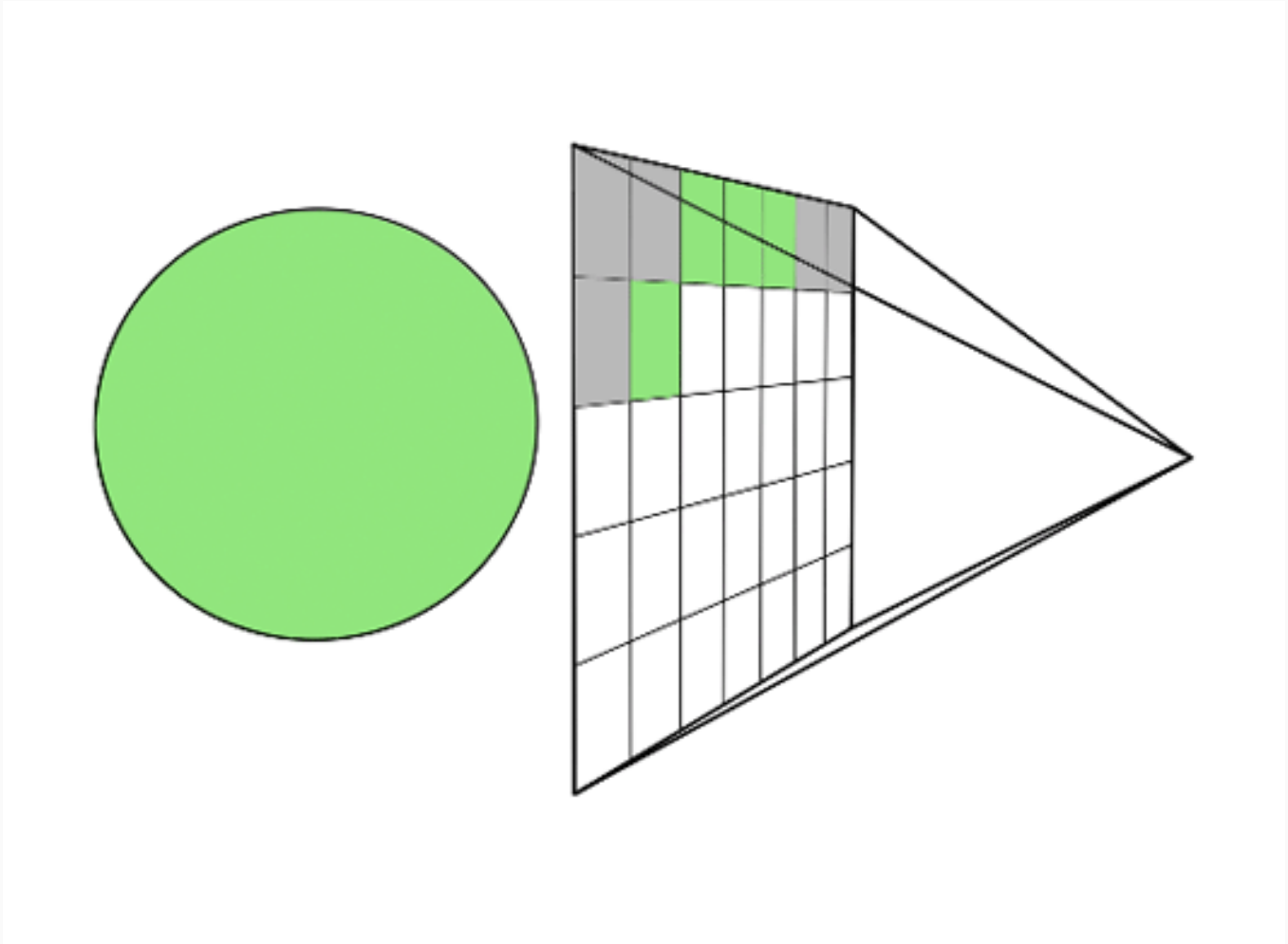
Raycasting Anim



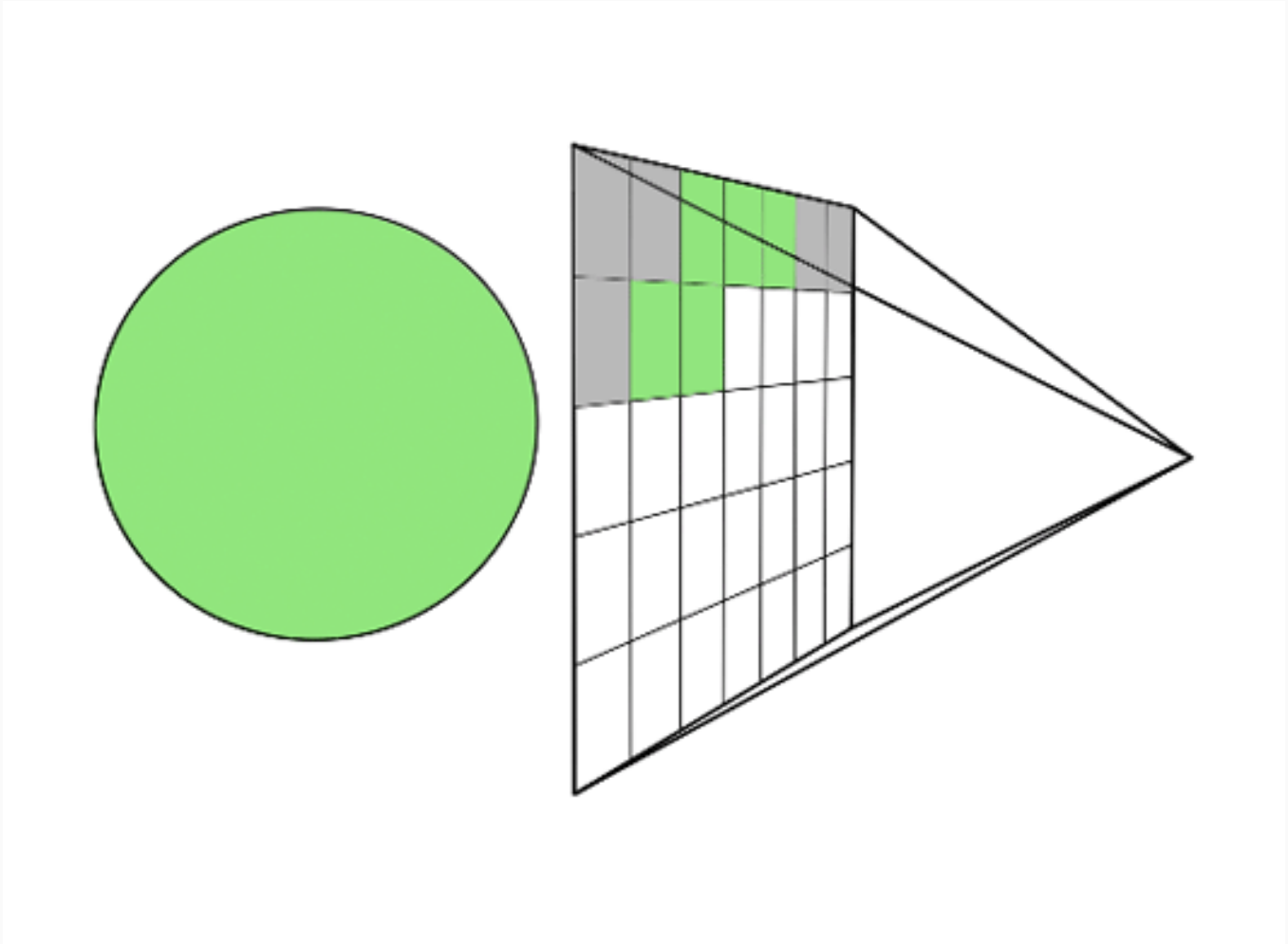
Raycasting Anim



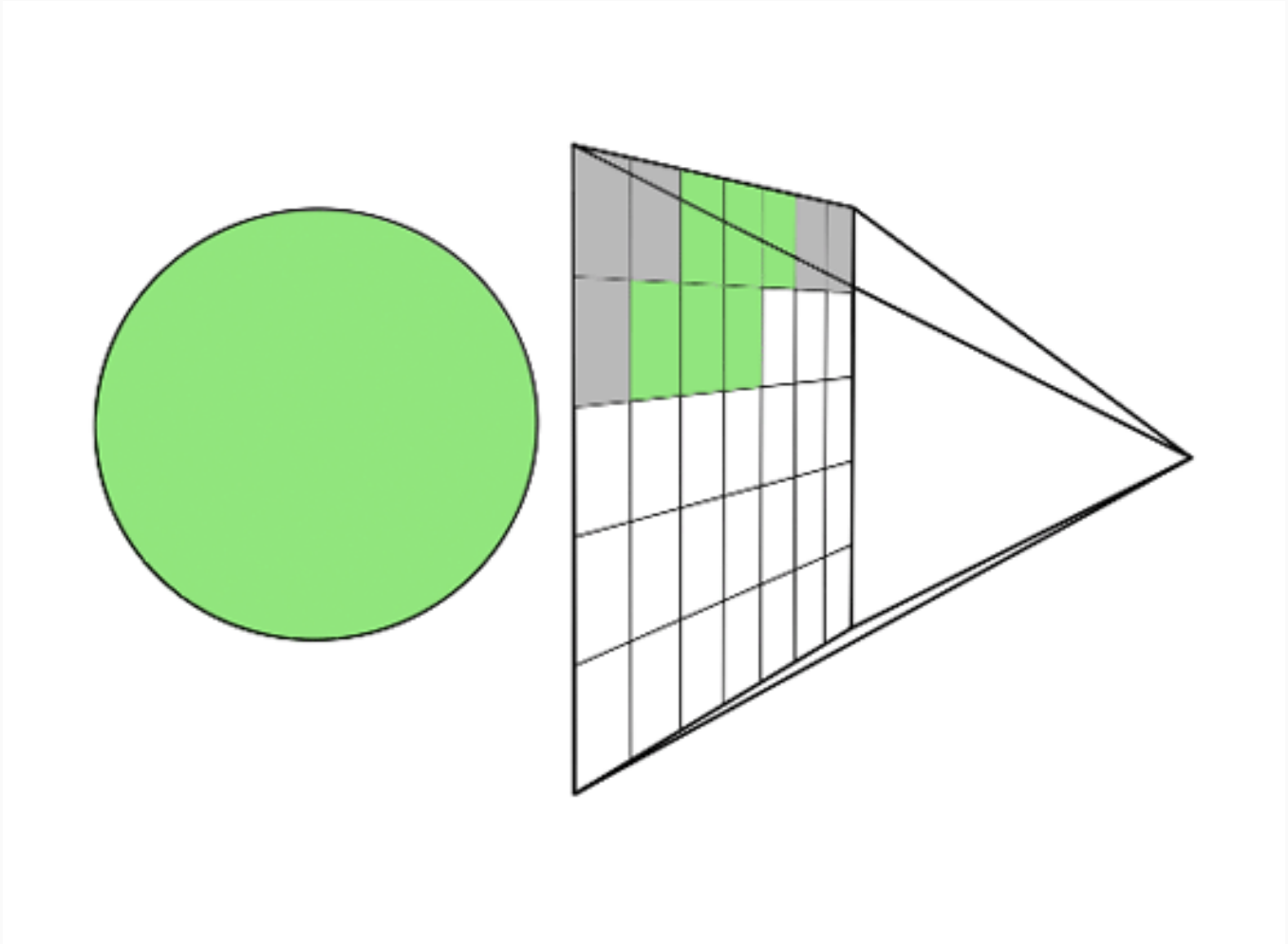
Raycasting Anim



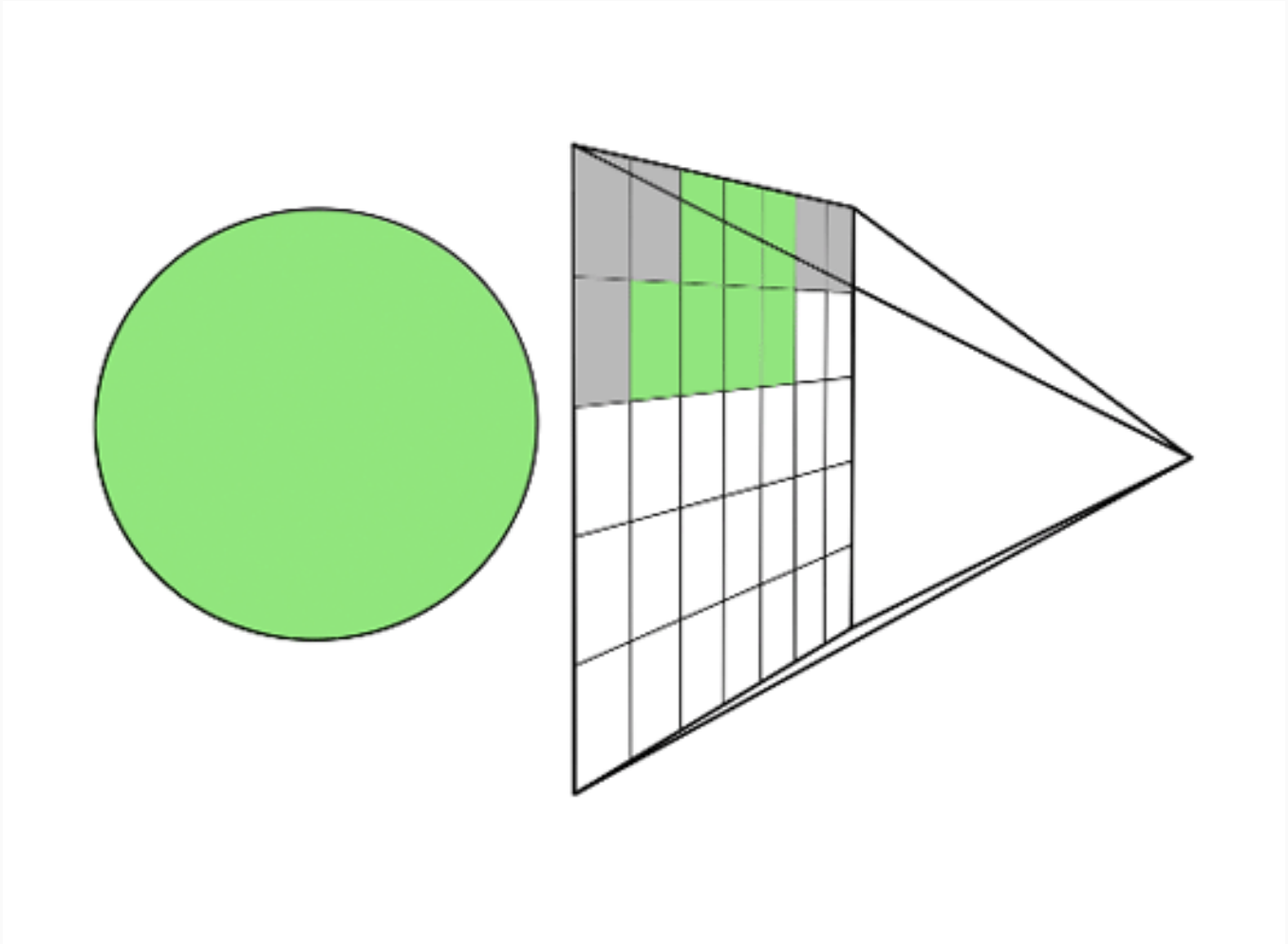
Raycasting Anim



Raycasting Anim



Raycasting Anim



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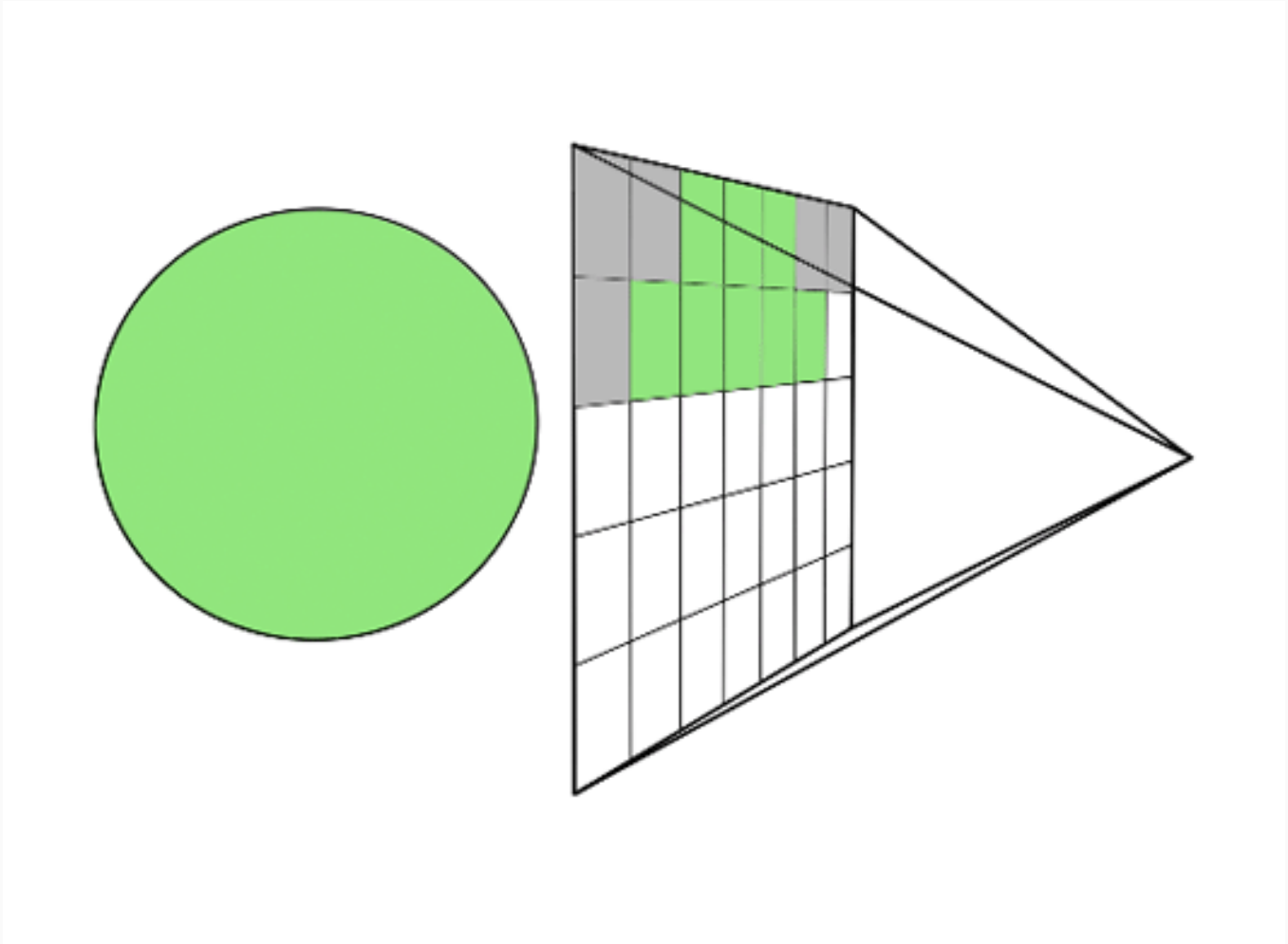
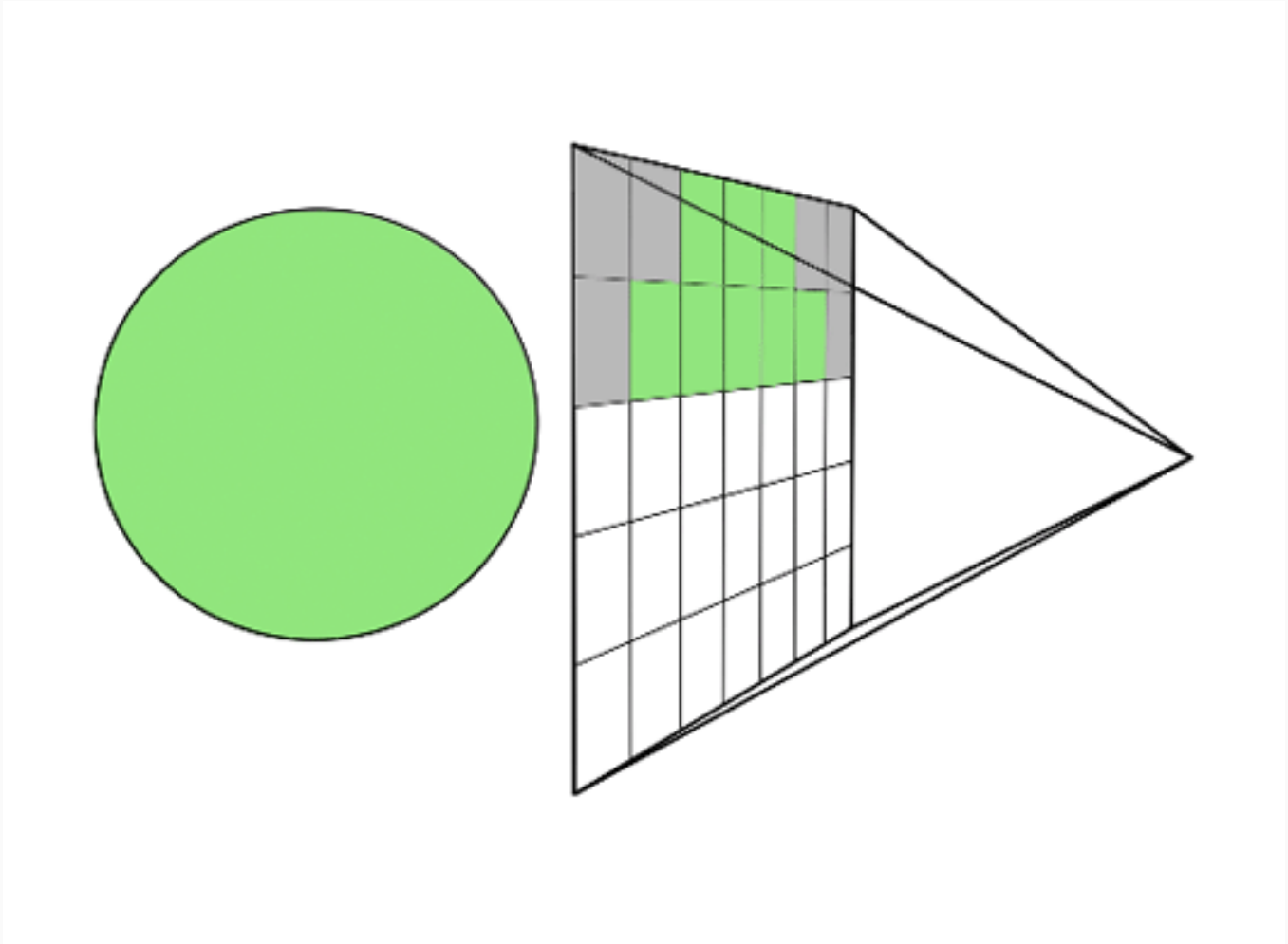


Raycasting Anim



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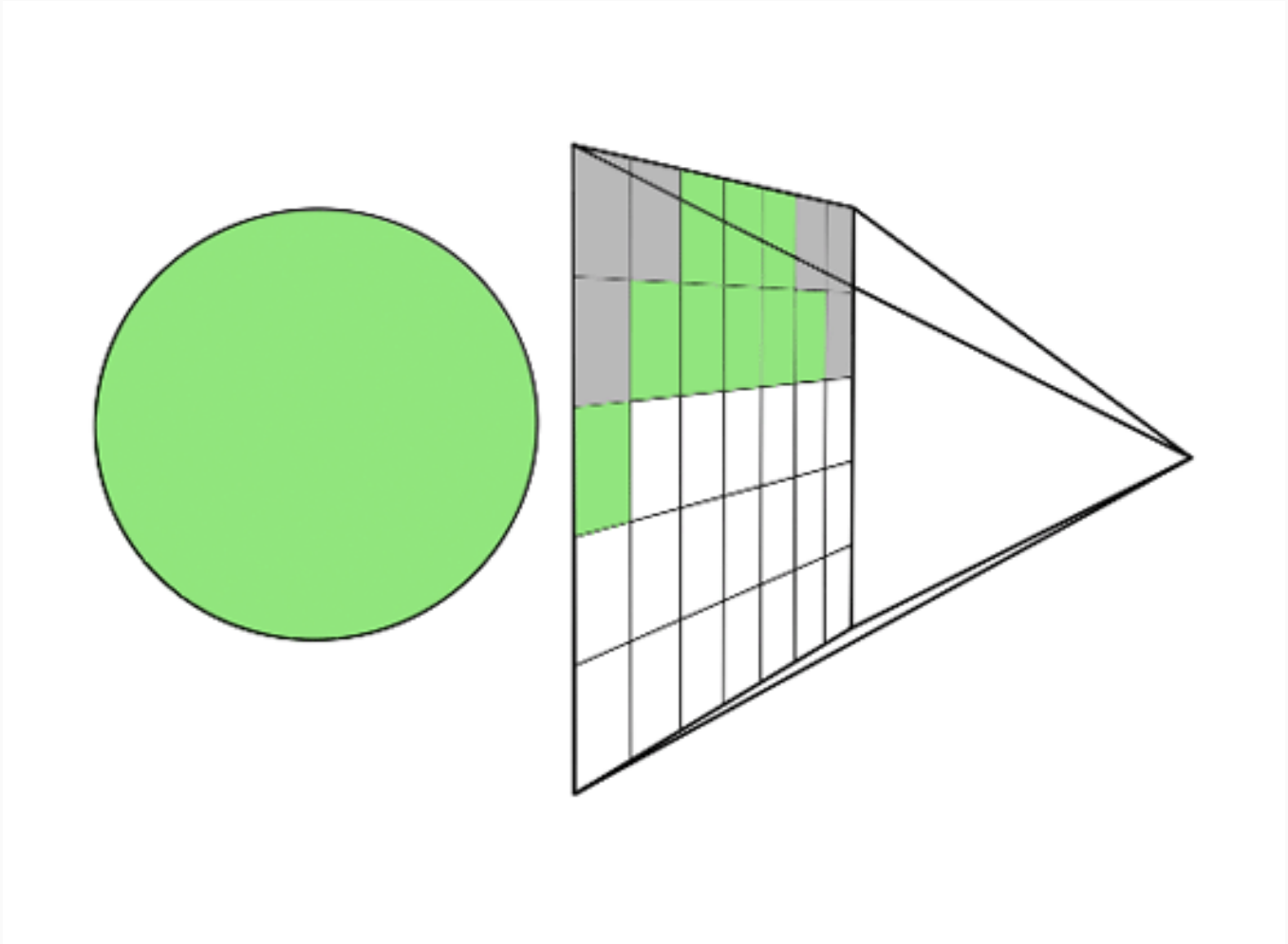
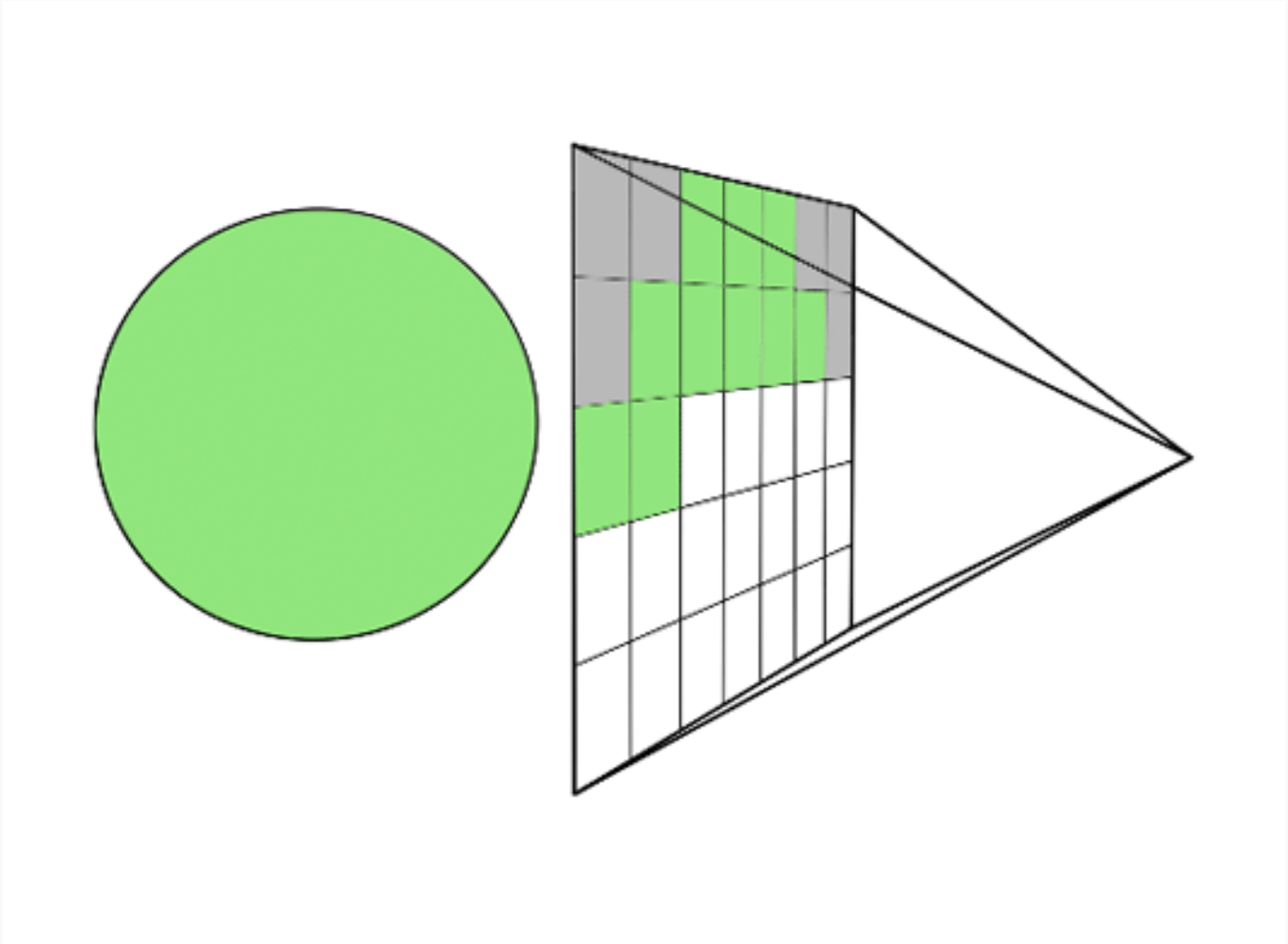
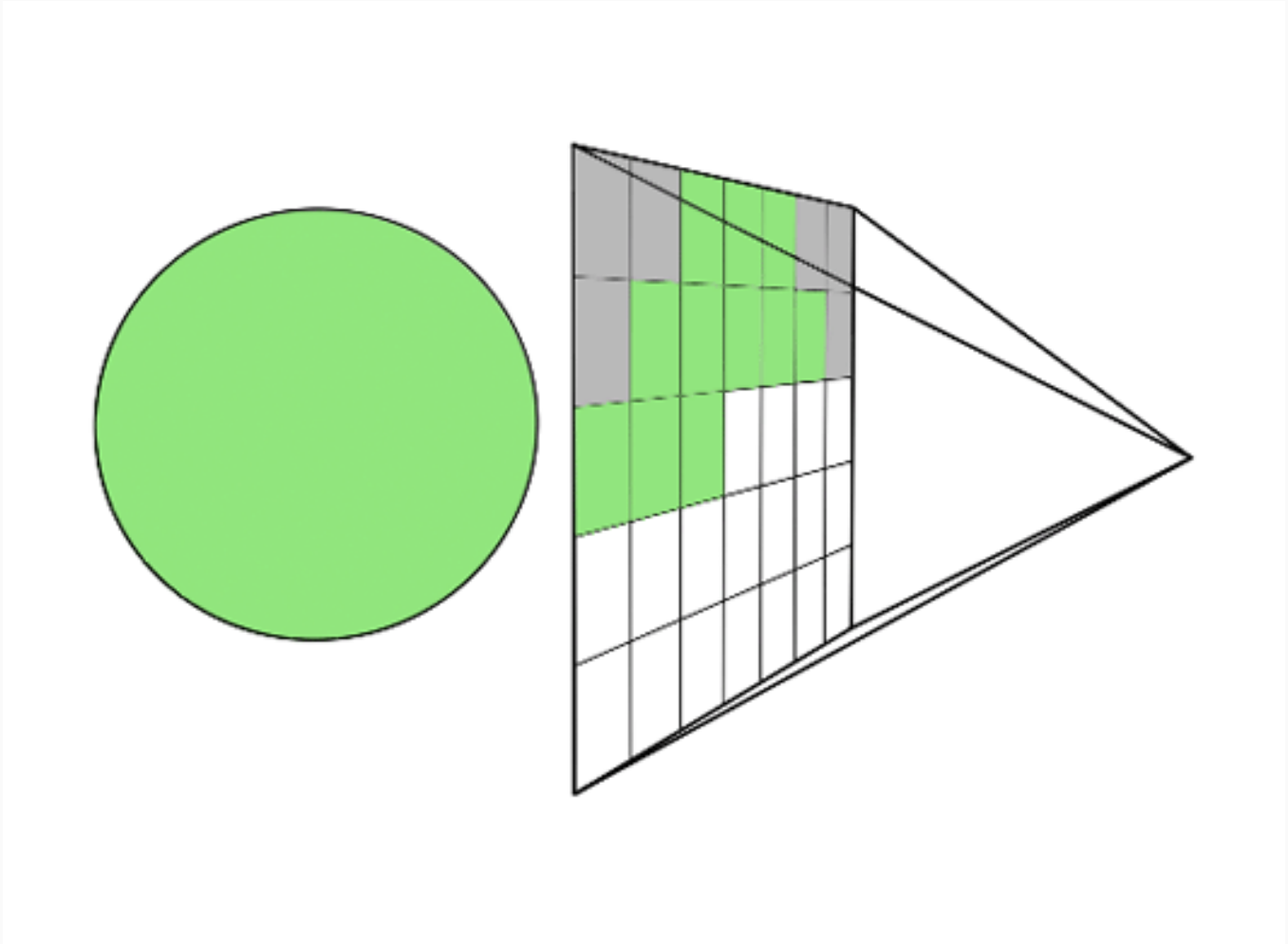


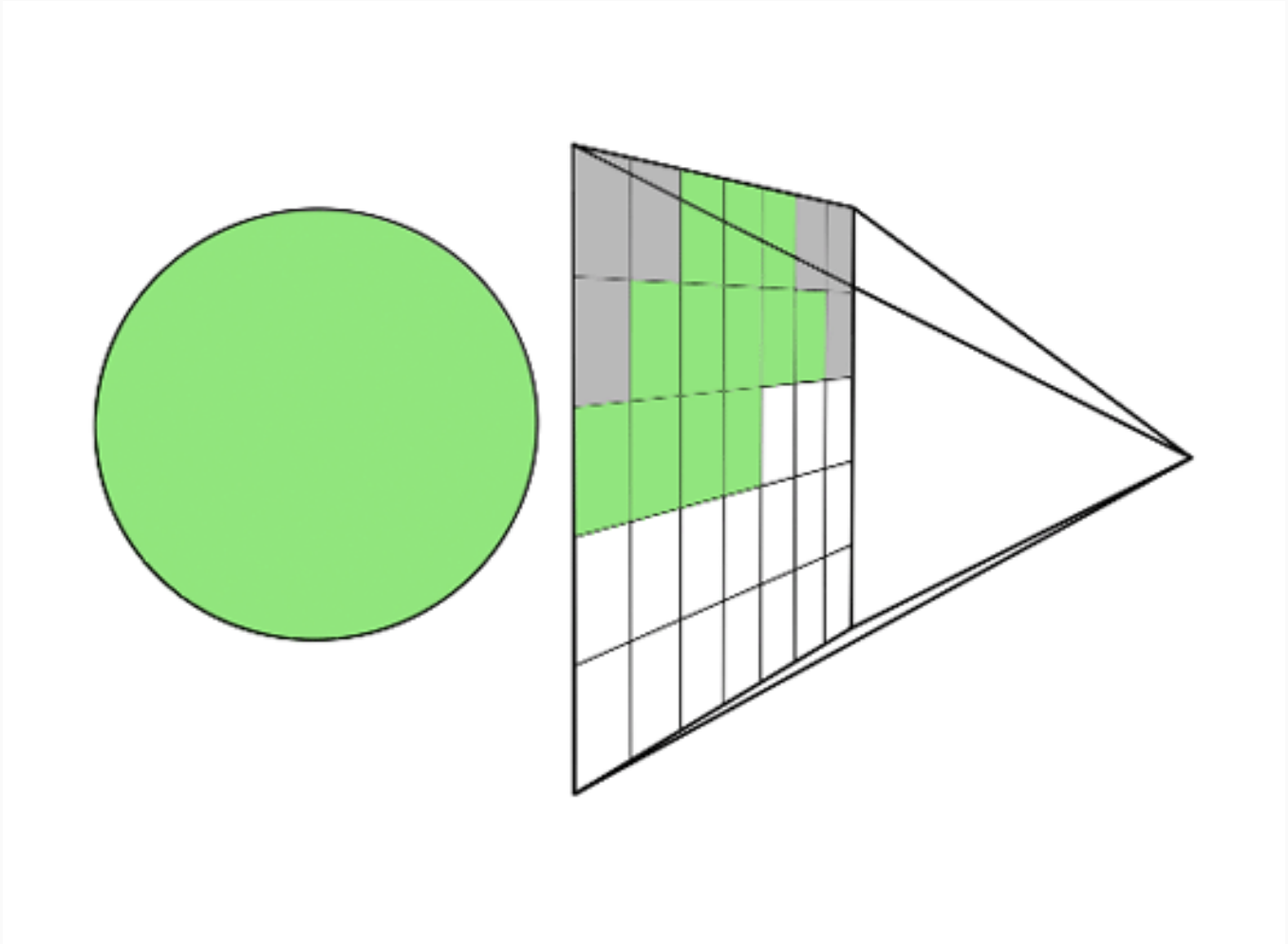
Raycasting Anim



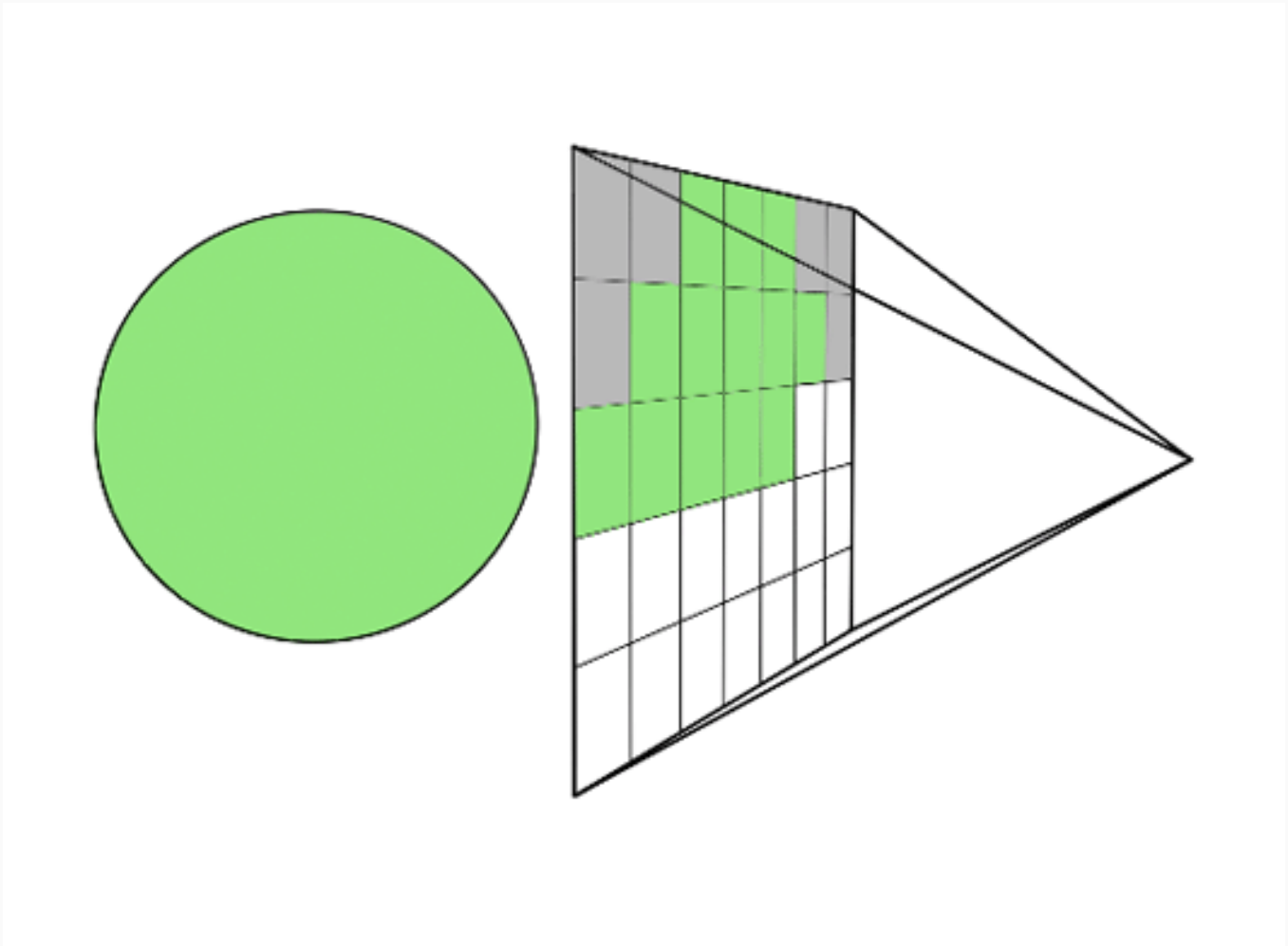
Raycasting Anim



Raycasting Anim



Raycasting Anim



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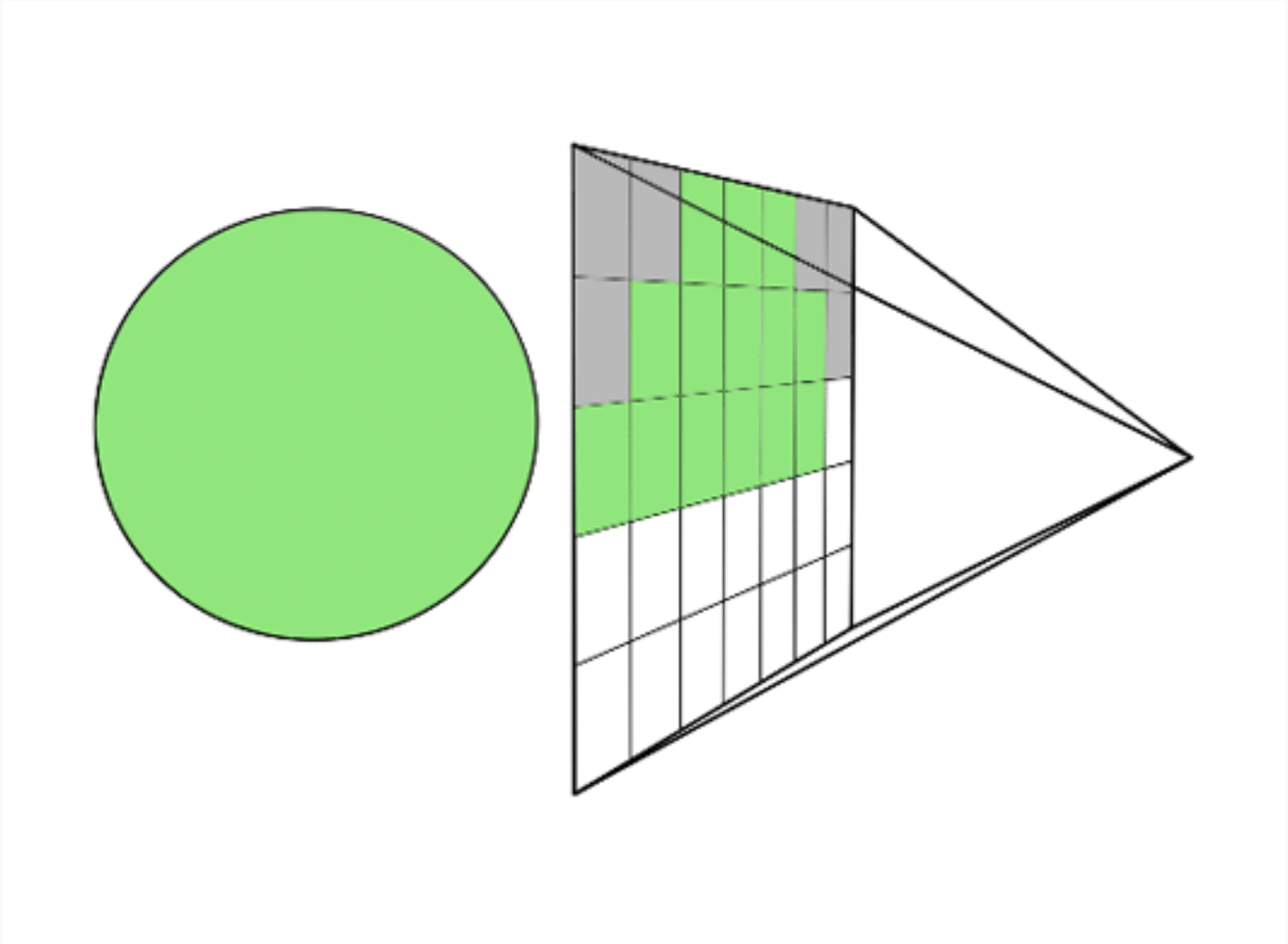
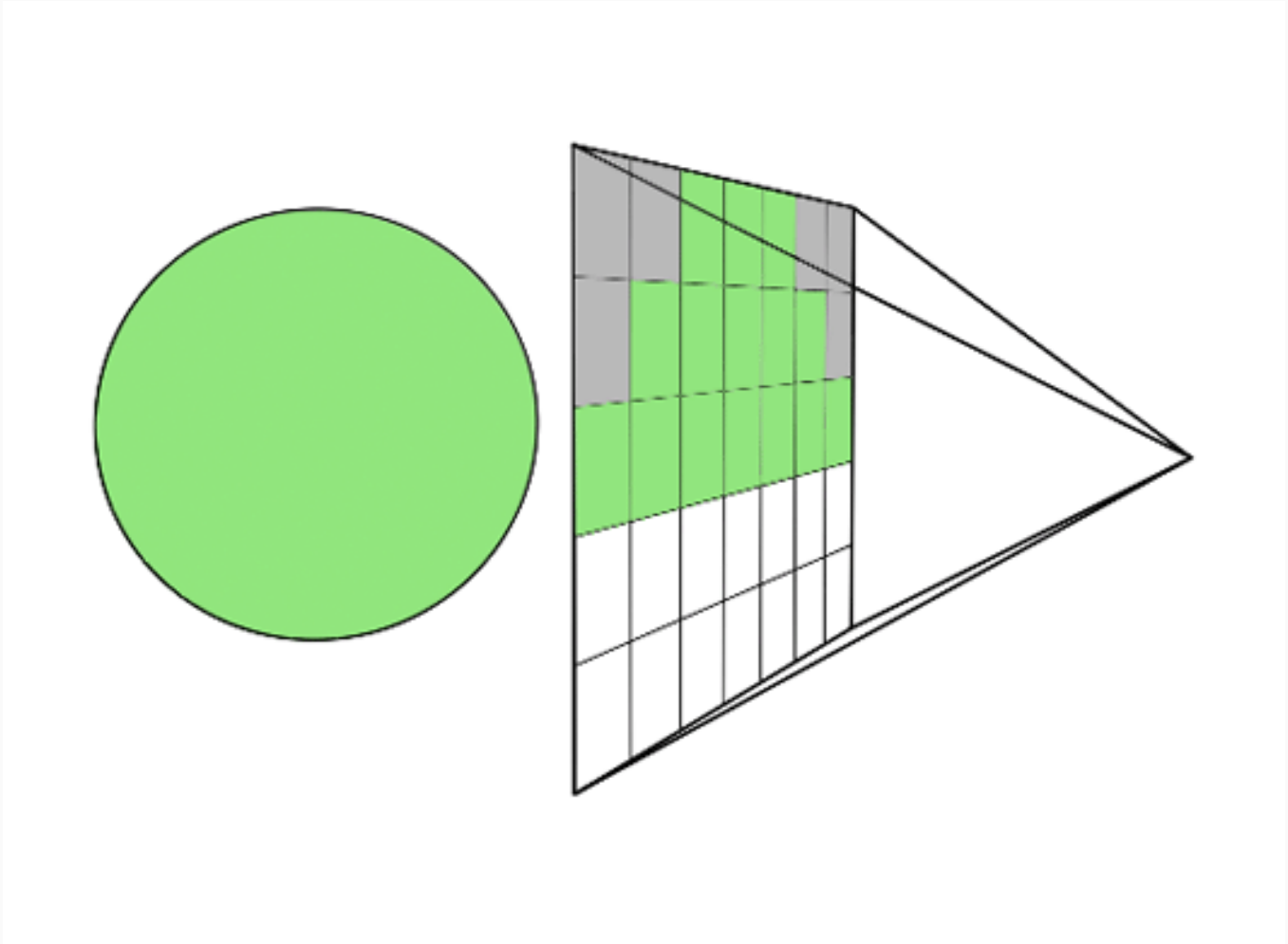


Raycasting Anim



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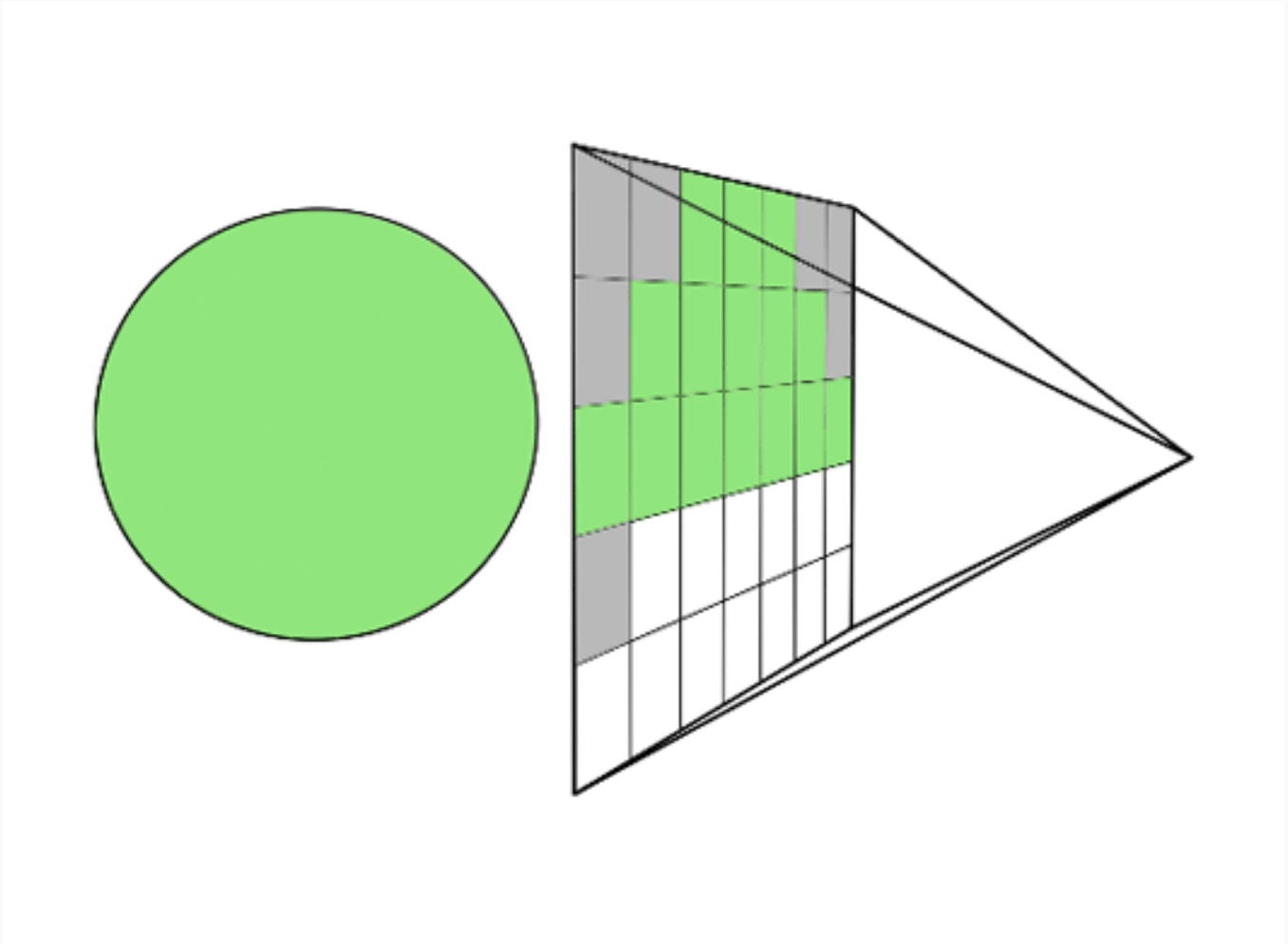
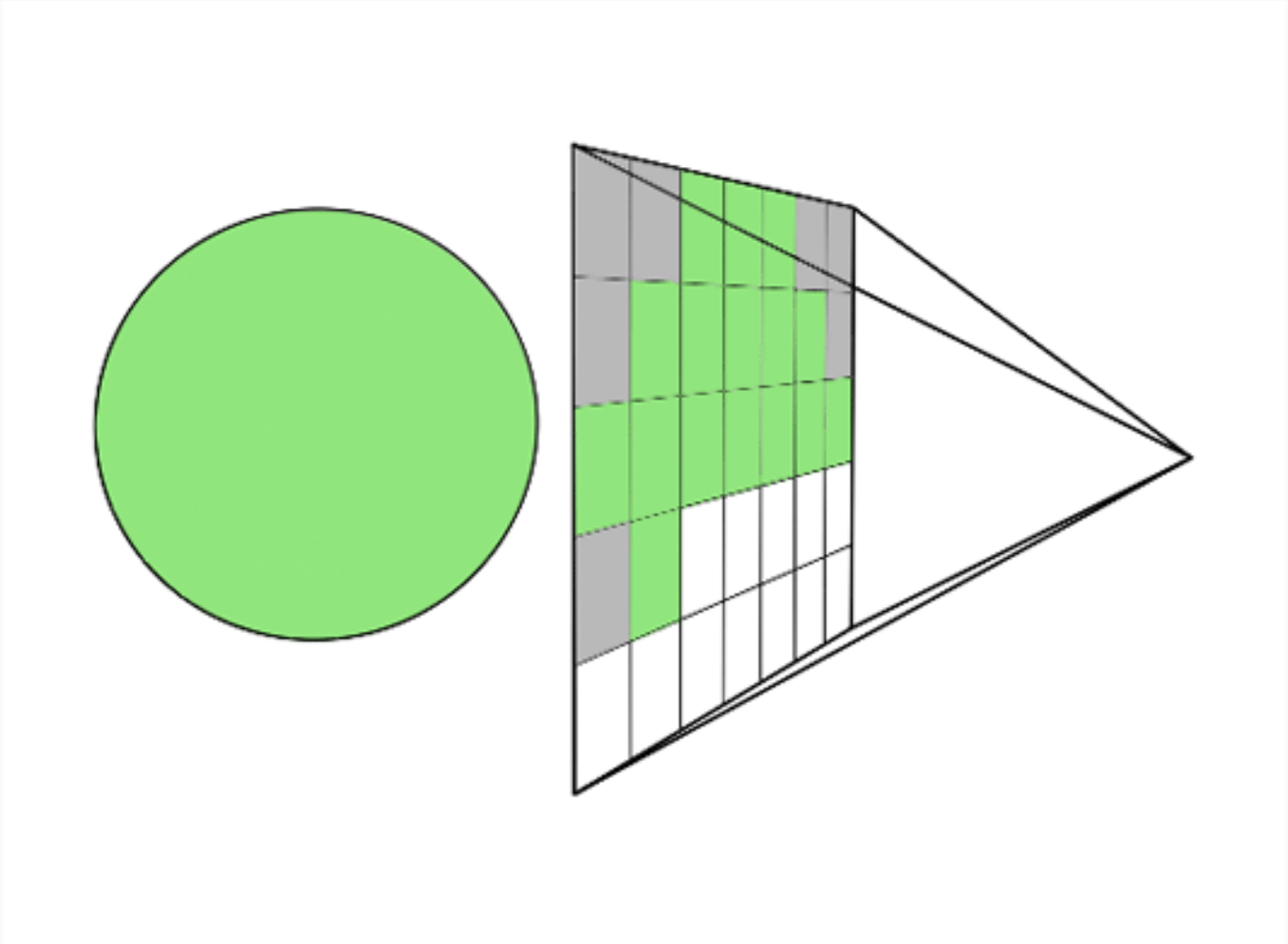


Raycasting Anim



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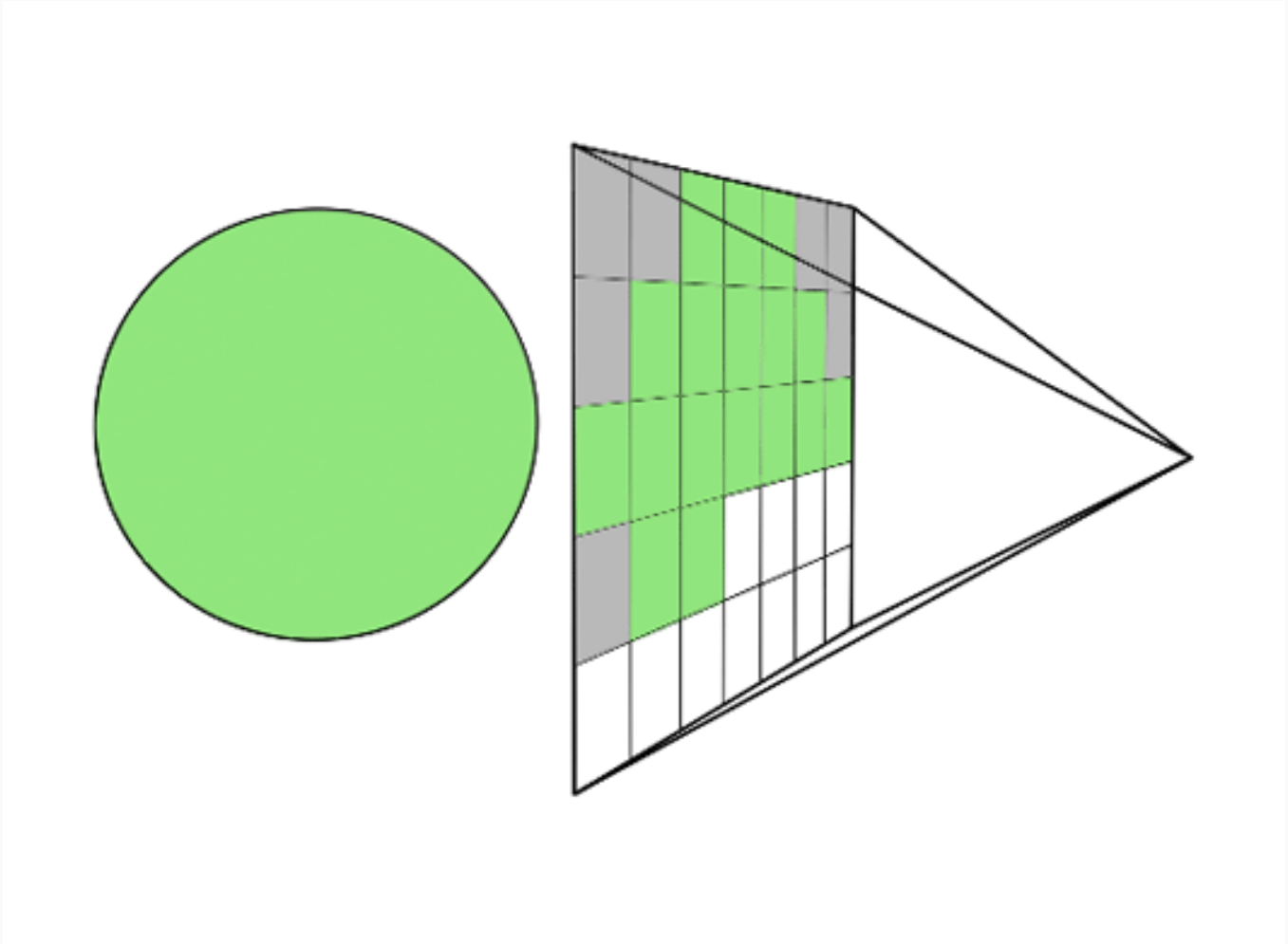

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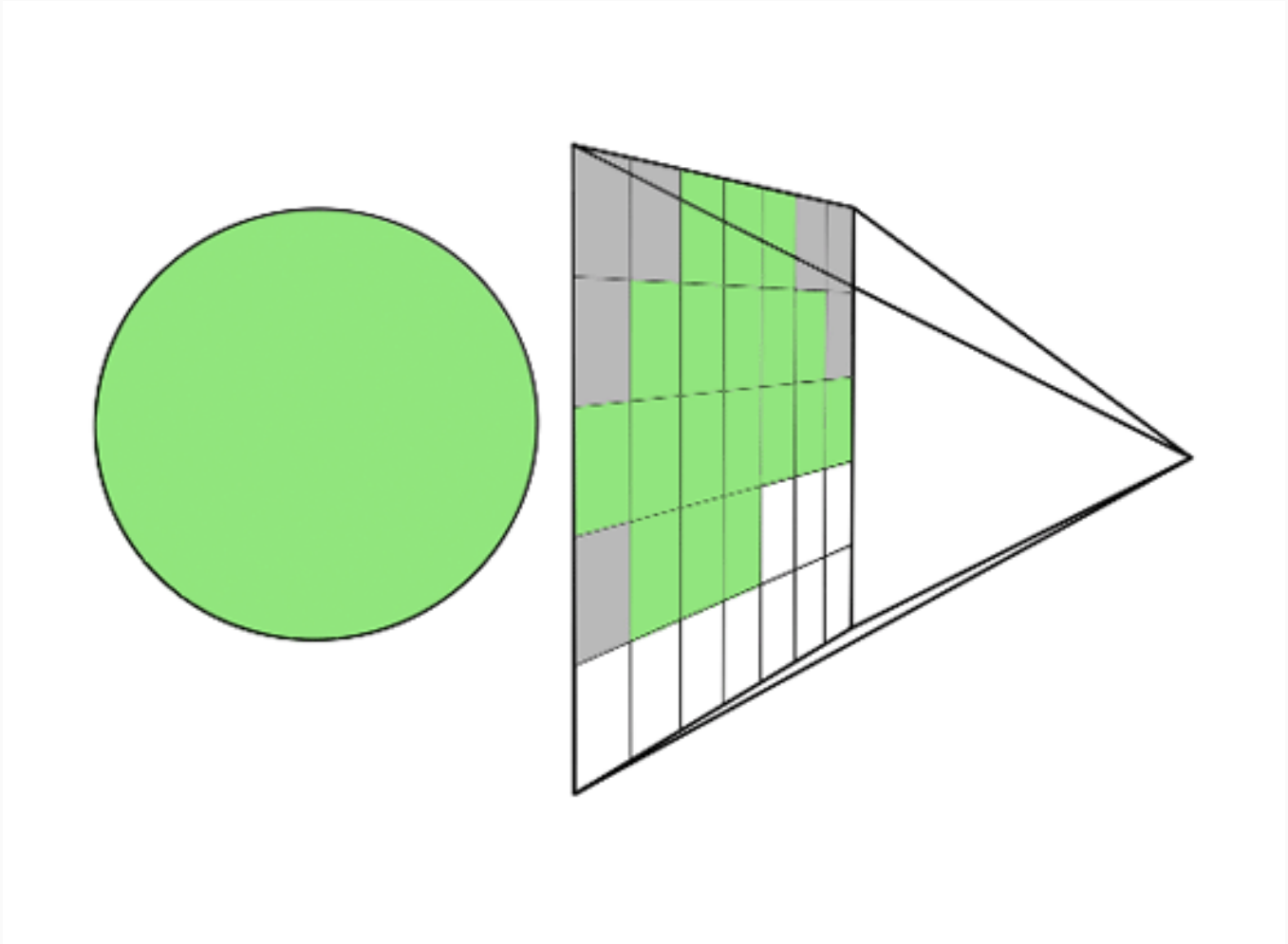

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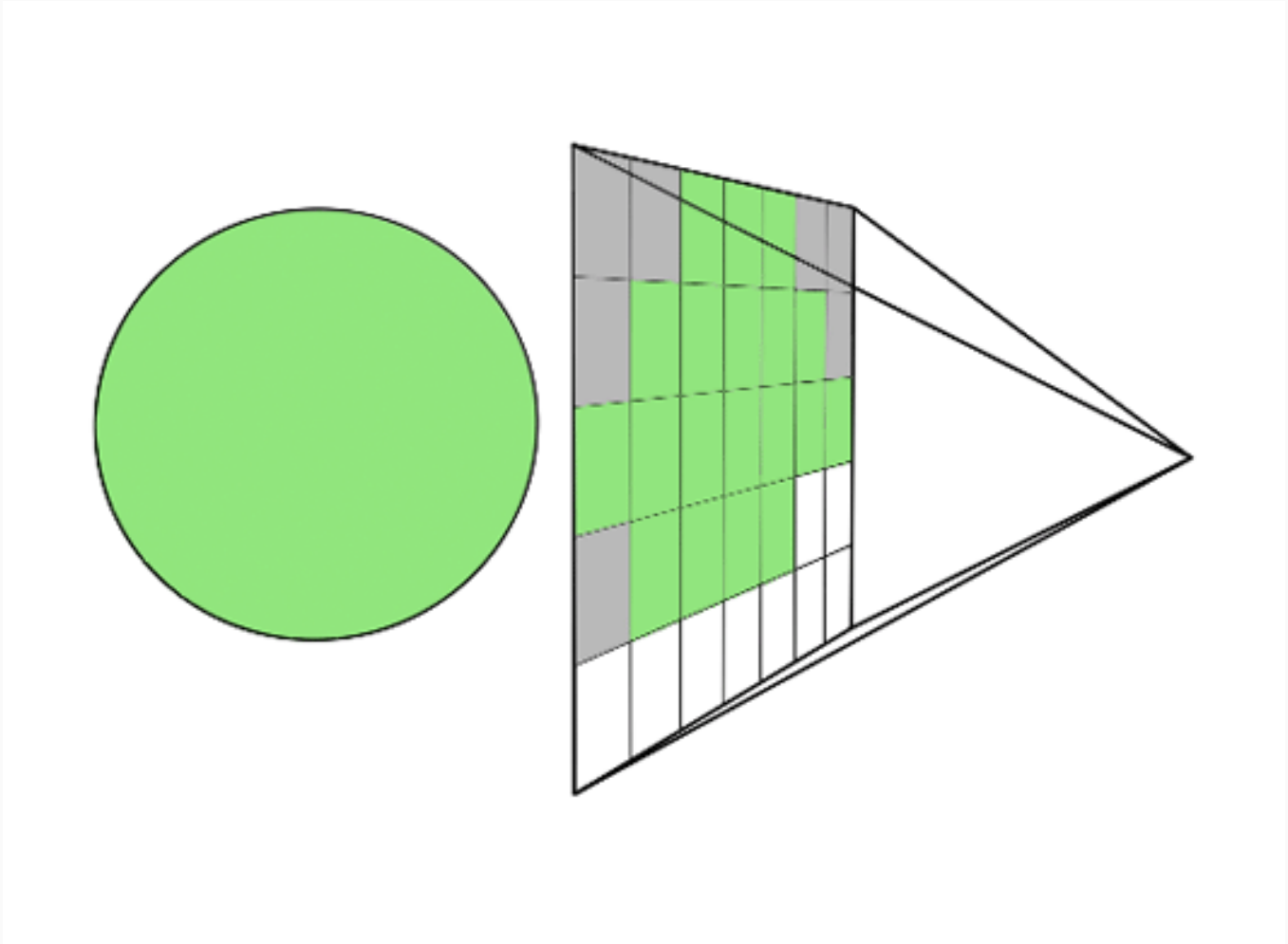

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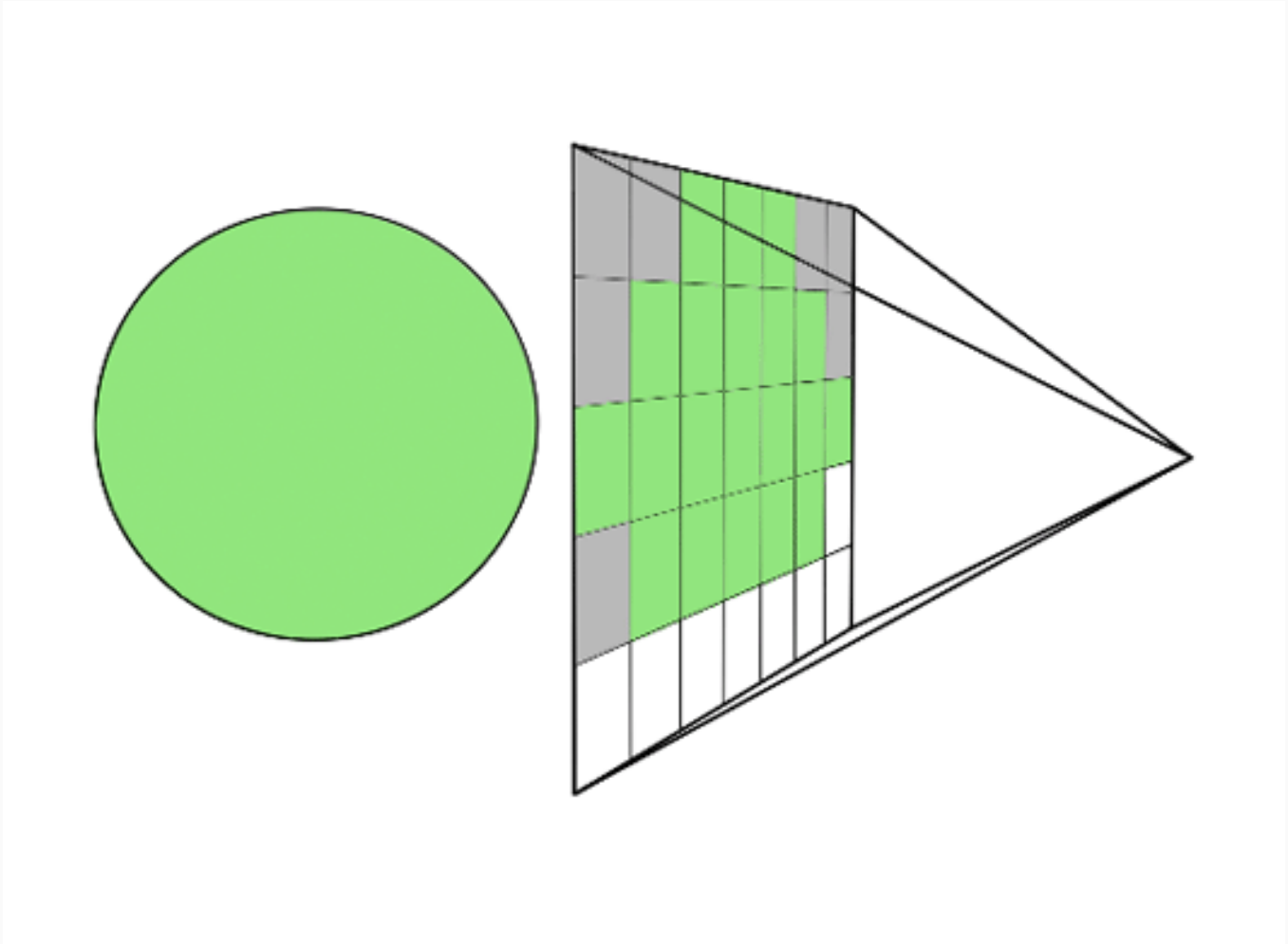
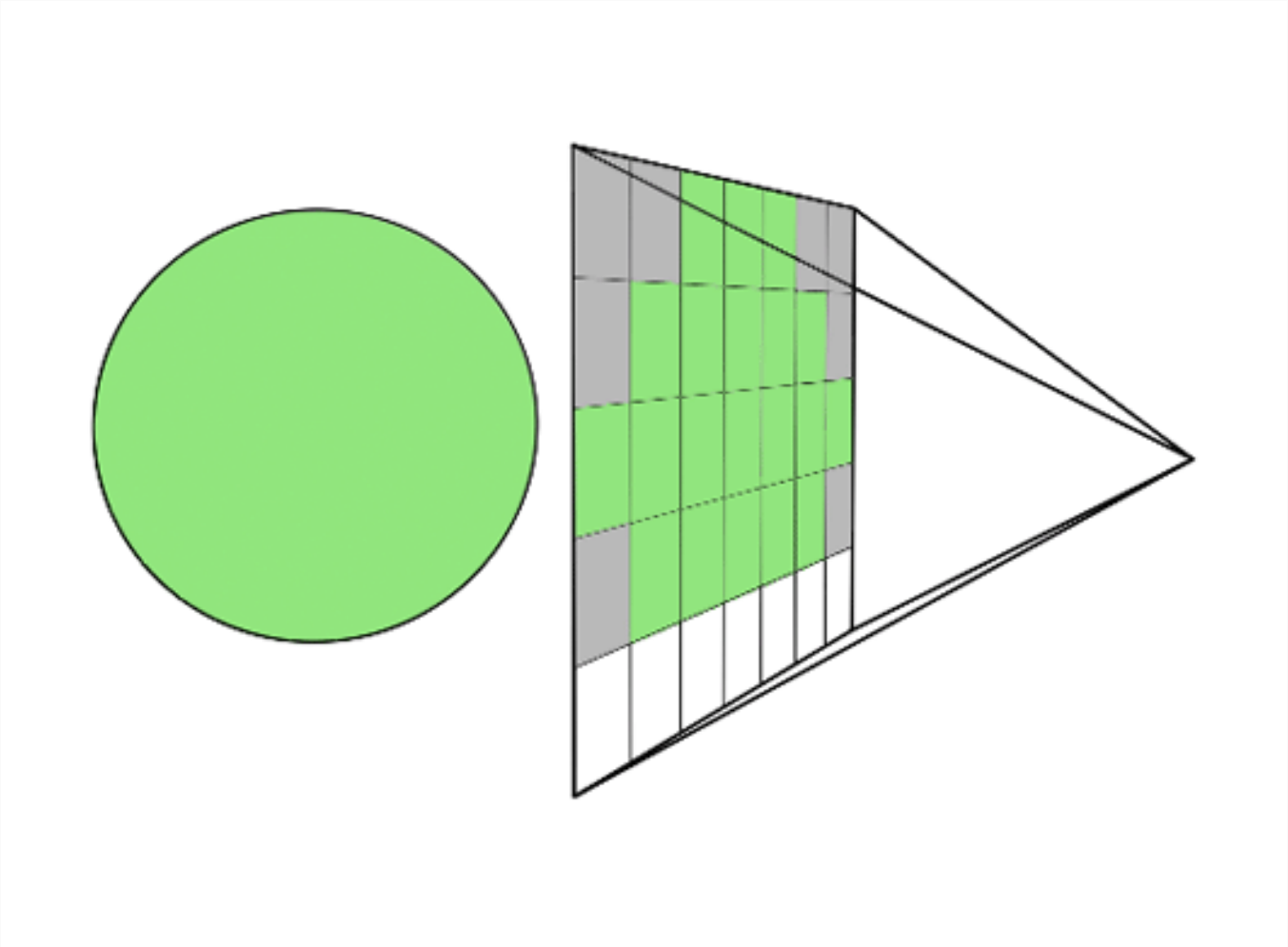
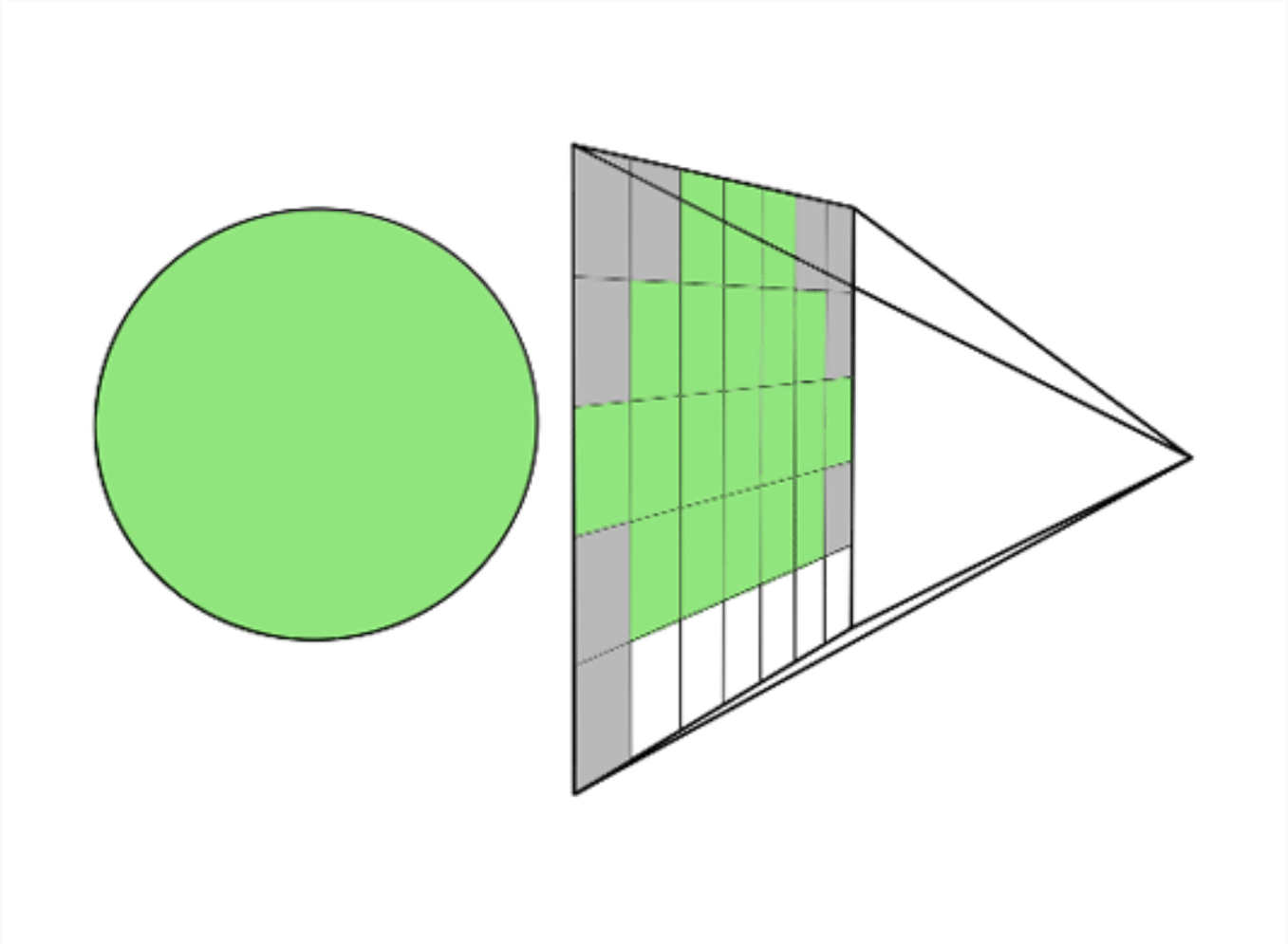


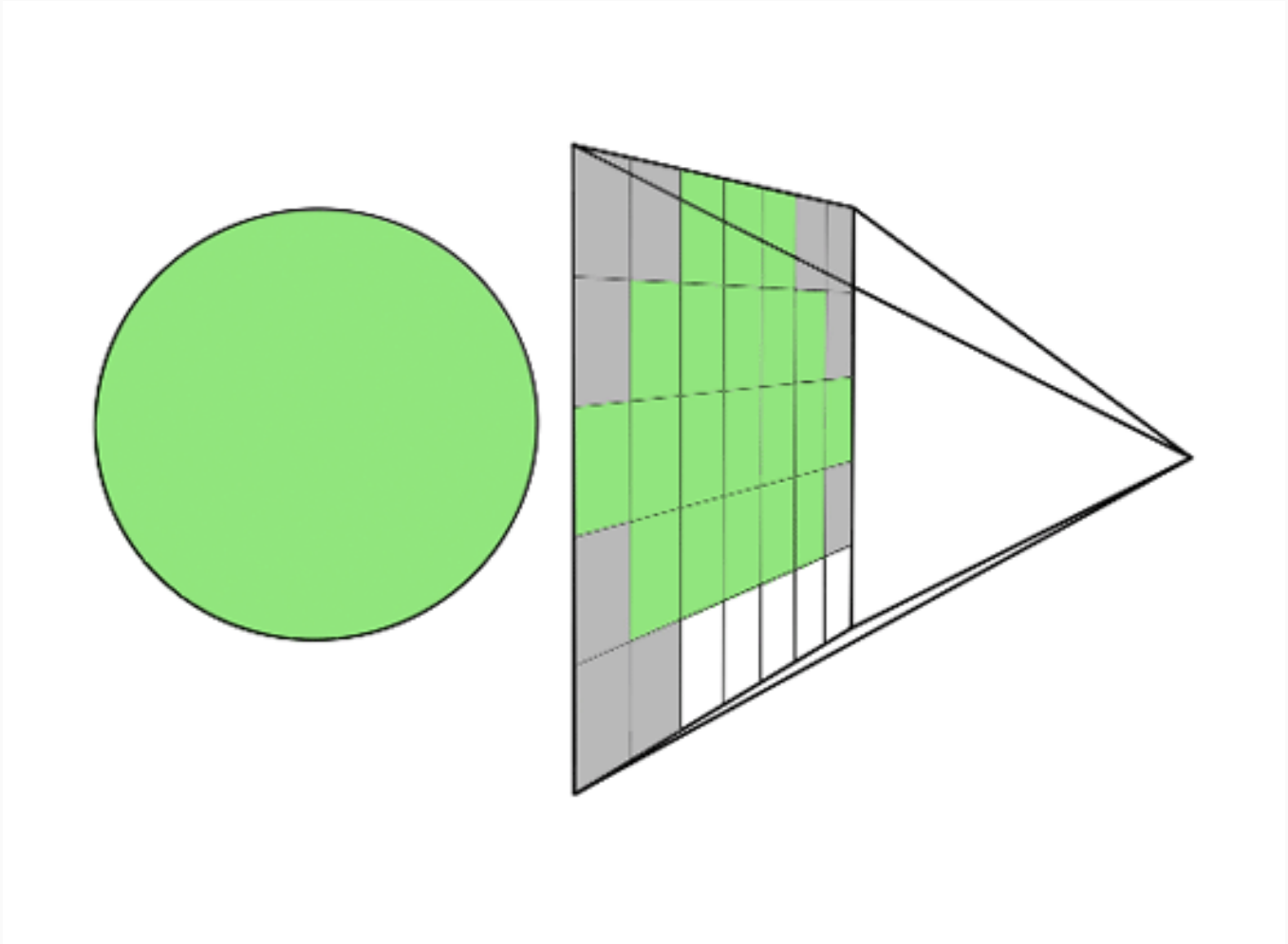
Raycasting Anim



Raycasting Anim



Raycasting Anim



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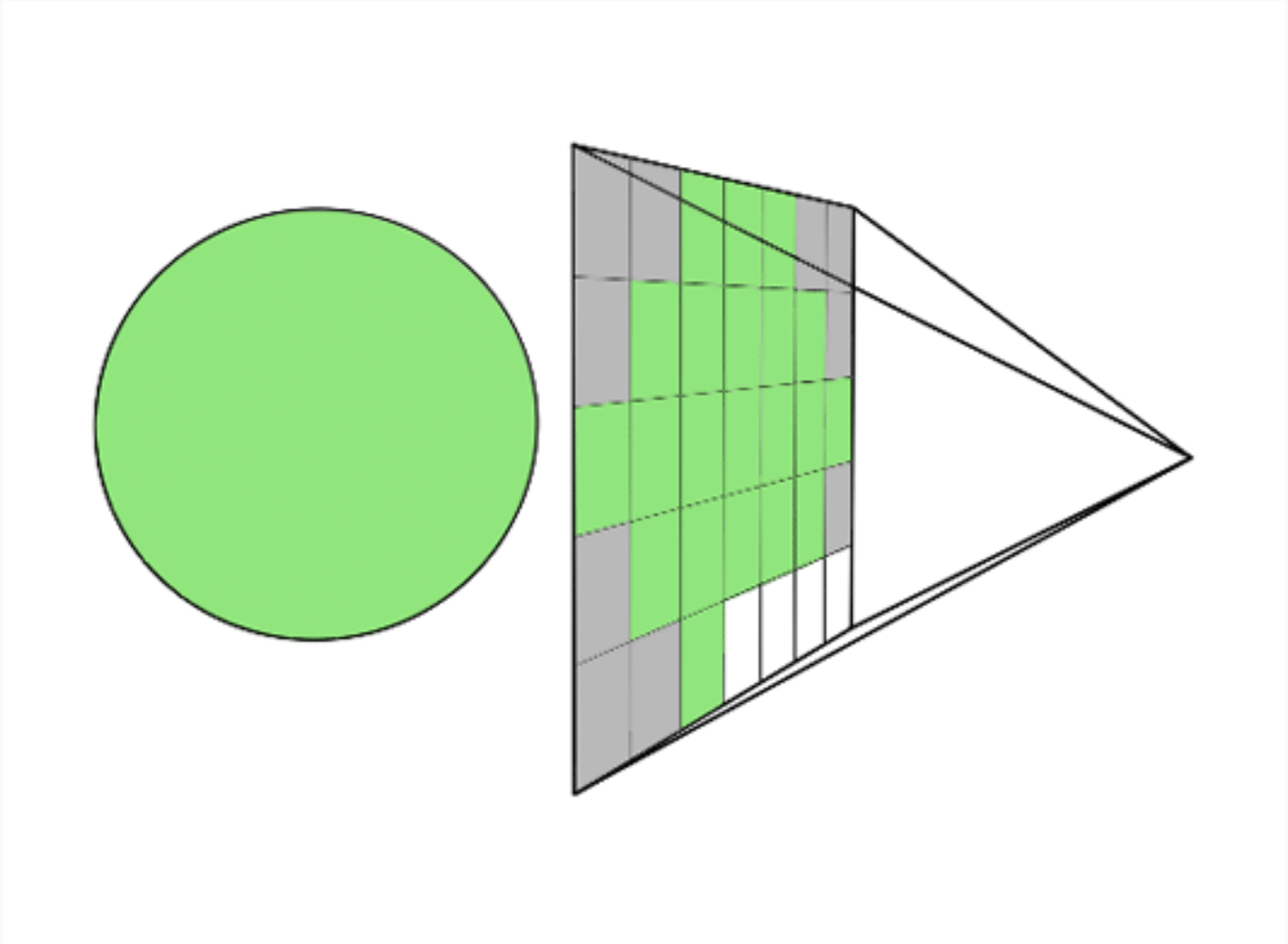
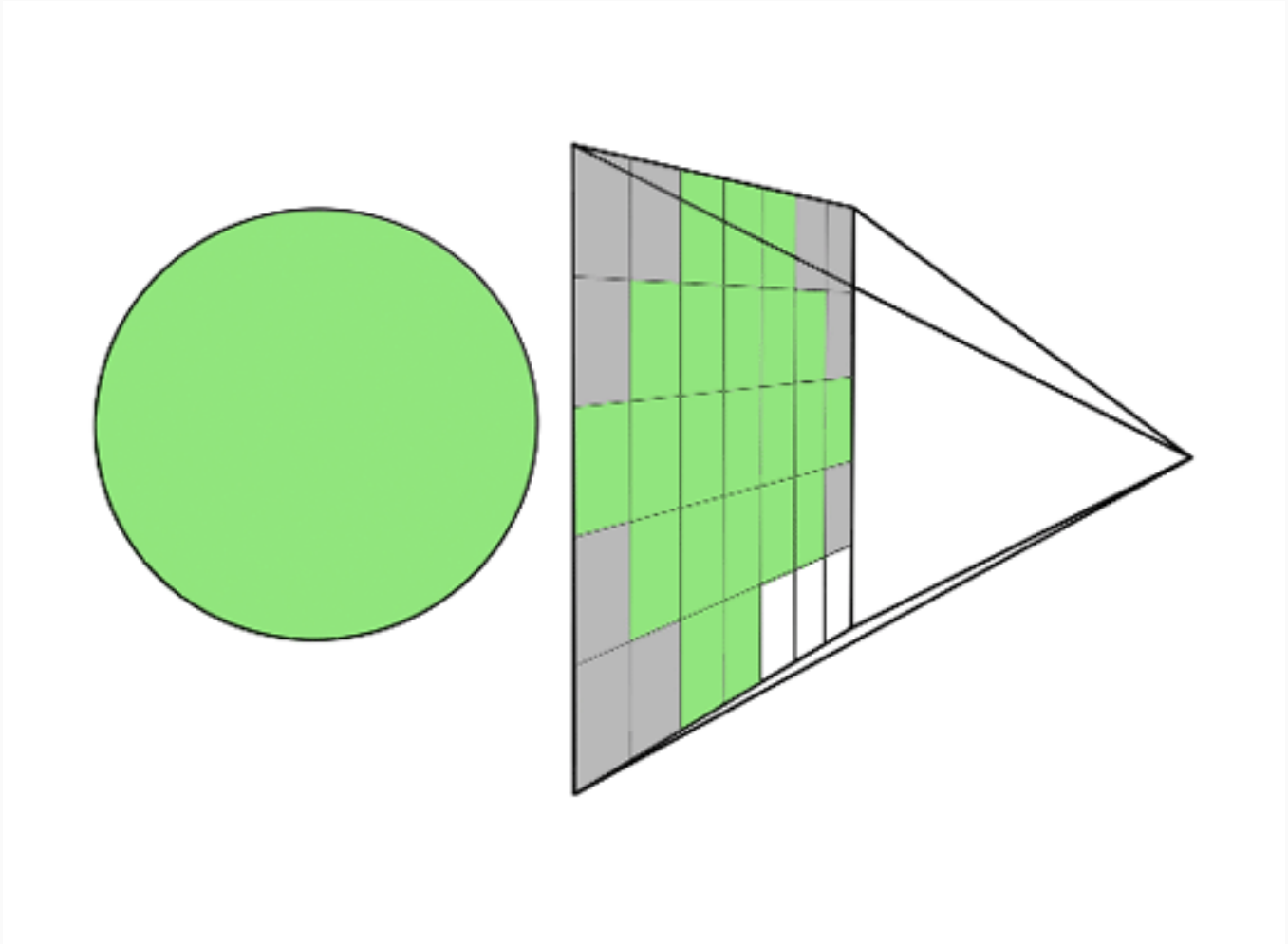
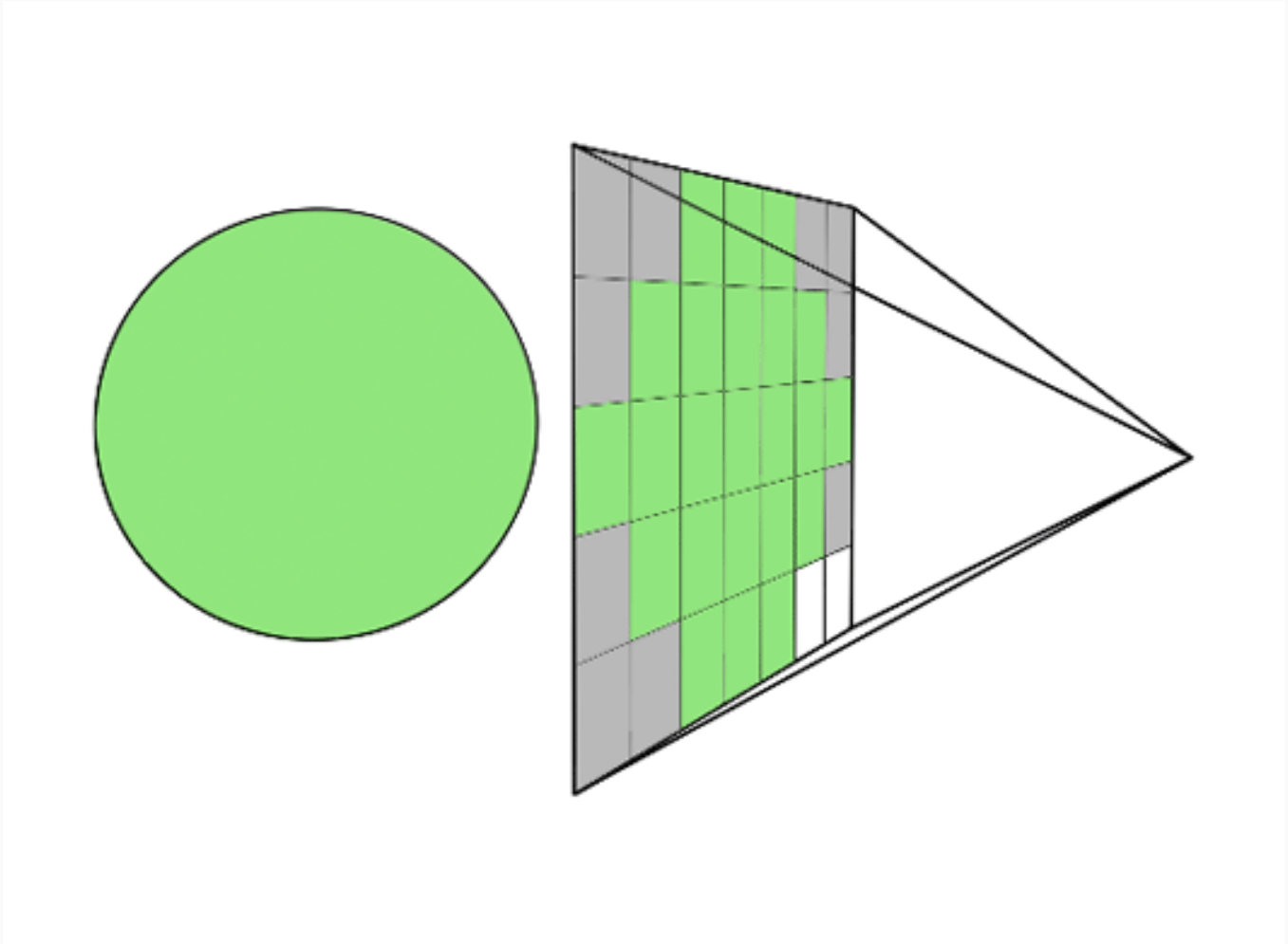


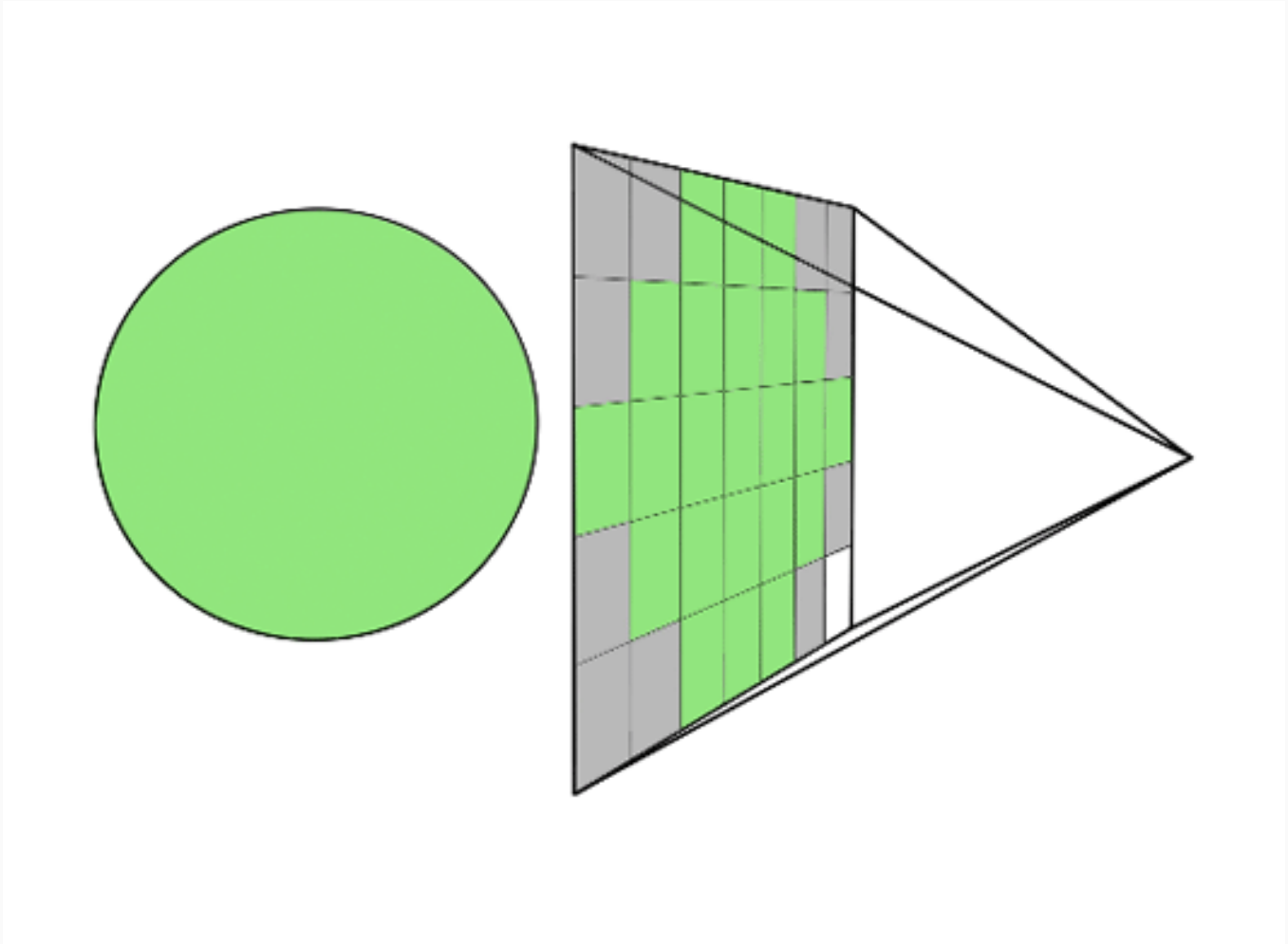
Raycasting Anim



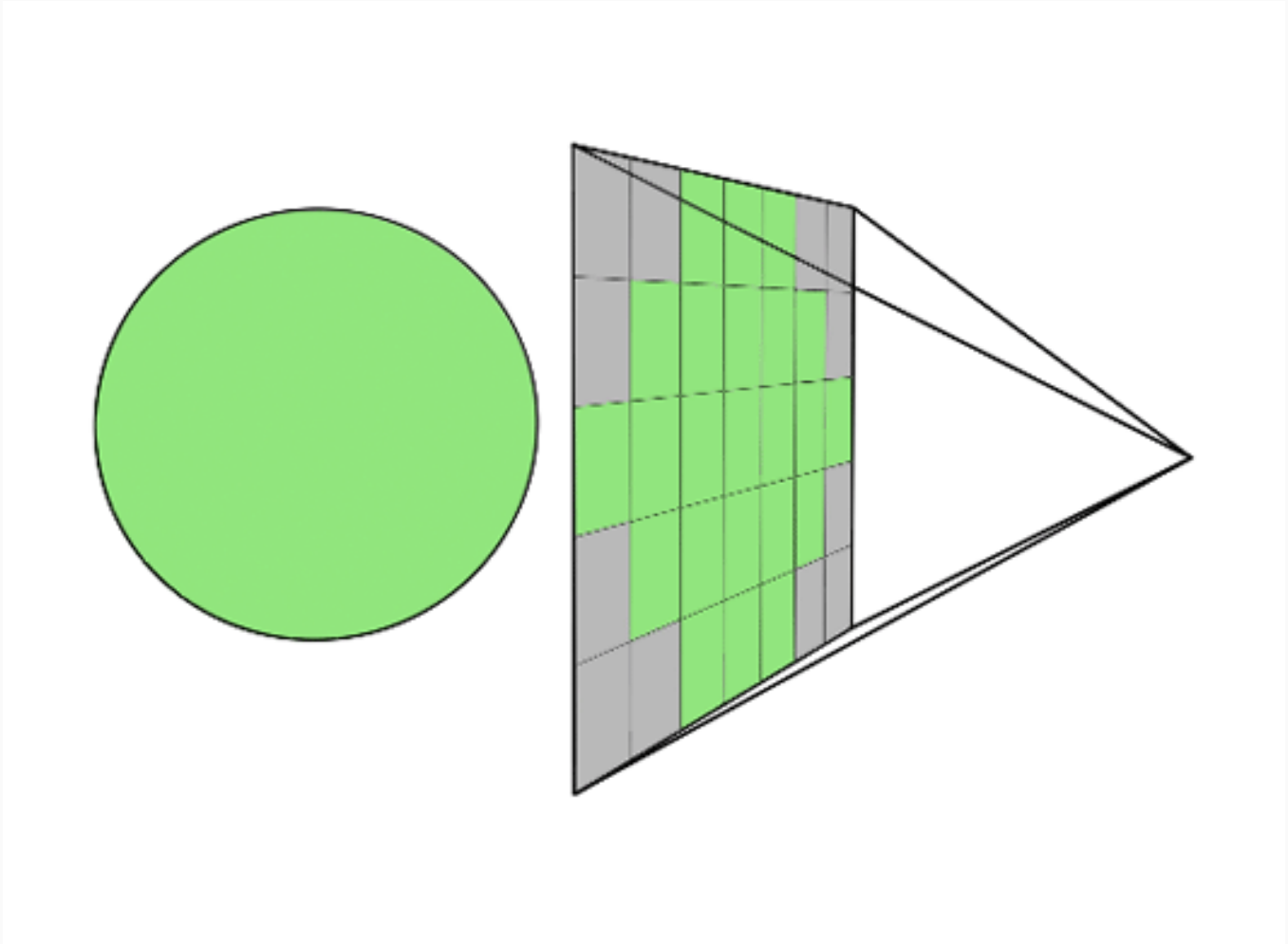
Raycasting Anim



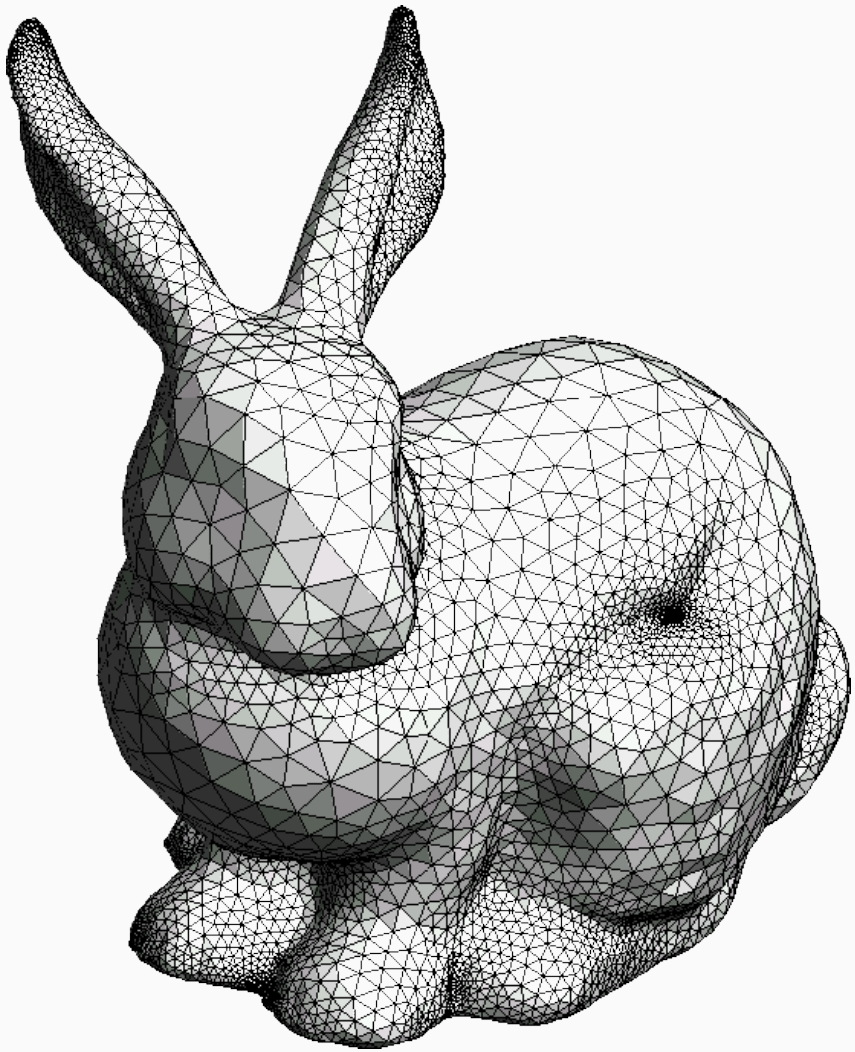
Raycasting Anim



Raycasting Anim



Intersections

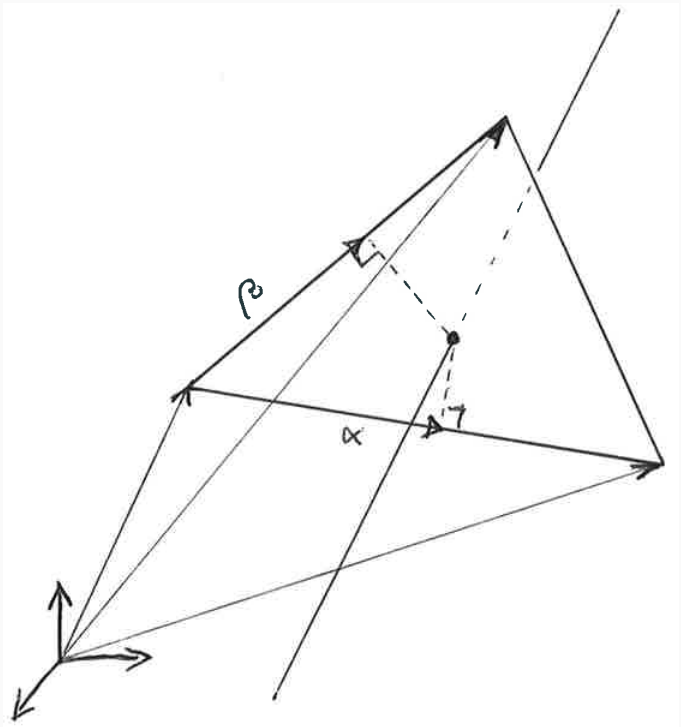


Ray-Triangel Intersection

- Compute two vectors in plane

$$\mathbf{e}_1 = \mathbf{v}_1 - \mathbf{v}_0$$

$$\mathbf{e}_2 = \mathbf{v}_2 - \mathbf{v}_0$$



Ray-Triangel Intersection

- Compute two vectors in plane

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- All points on plane that triangle lies in

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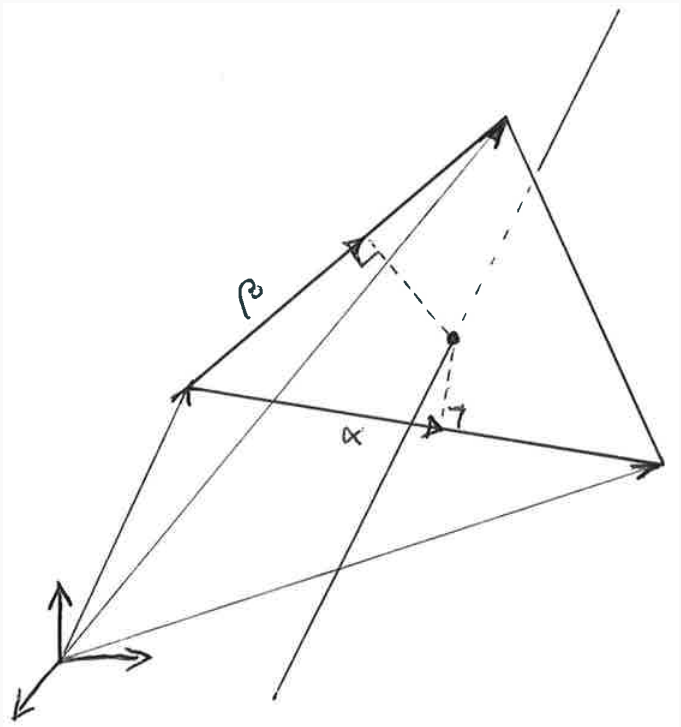

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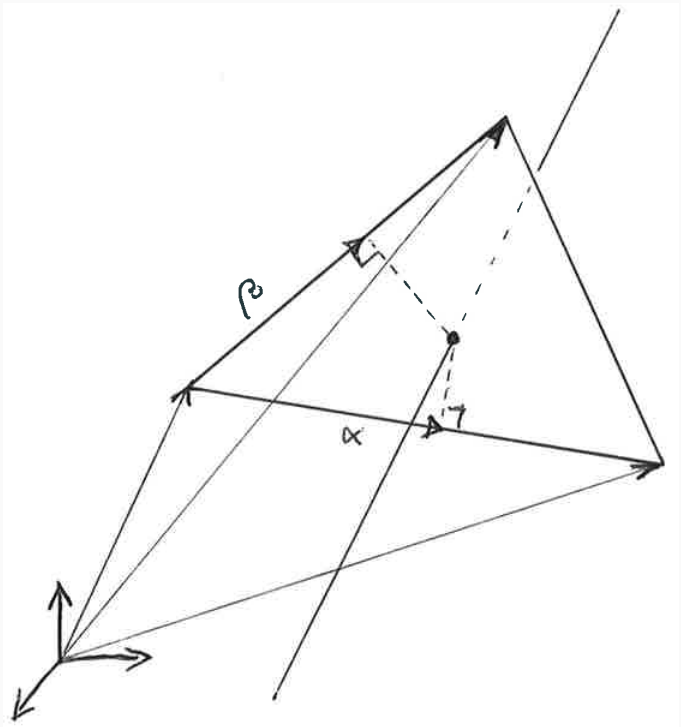

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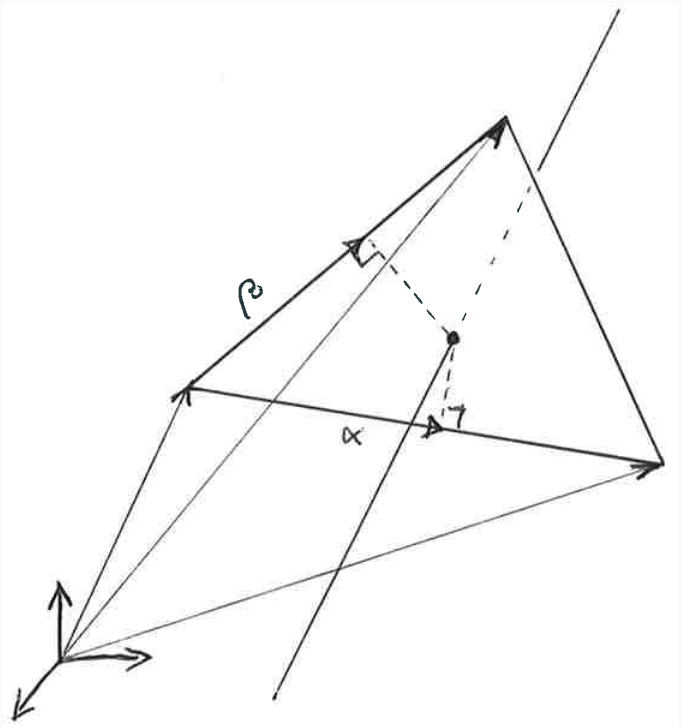
$$\{u, v | u \geq 0, v \geq 0, v+u \leq 1\}$$



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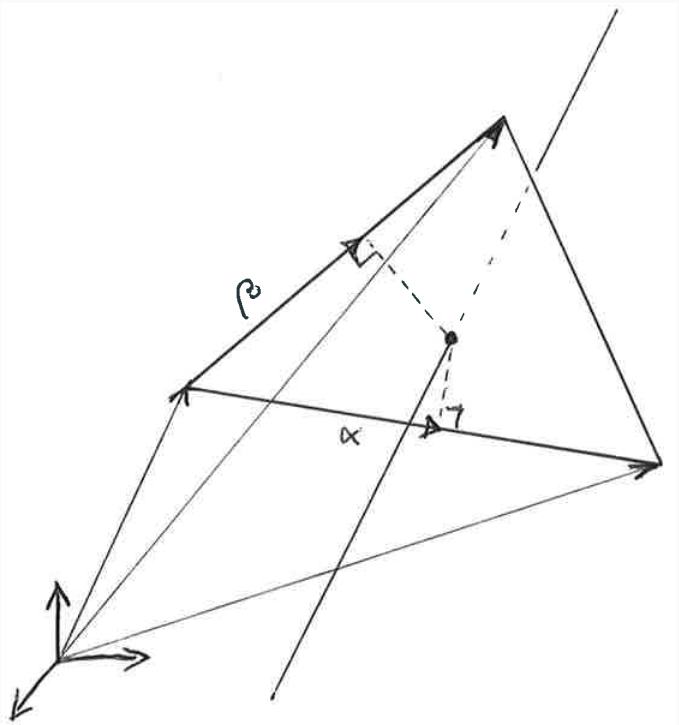

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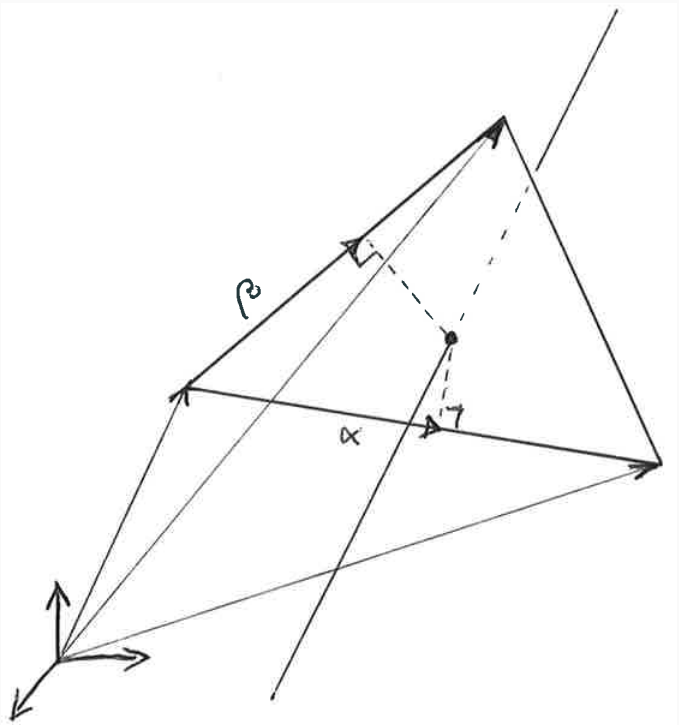

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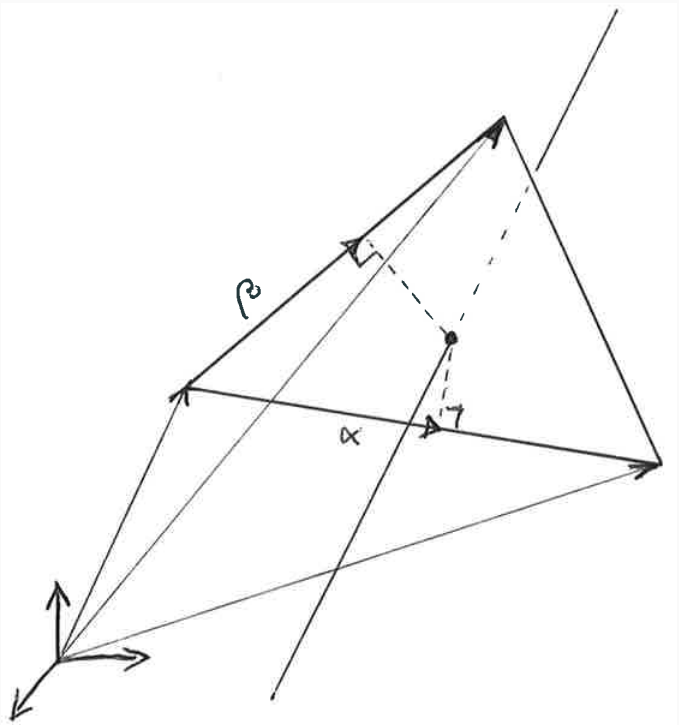

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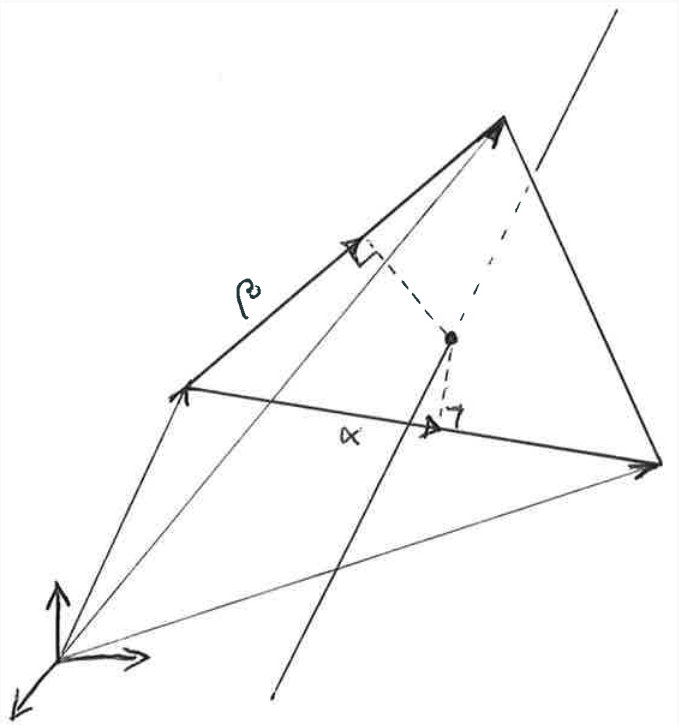

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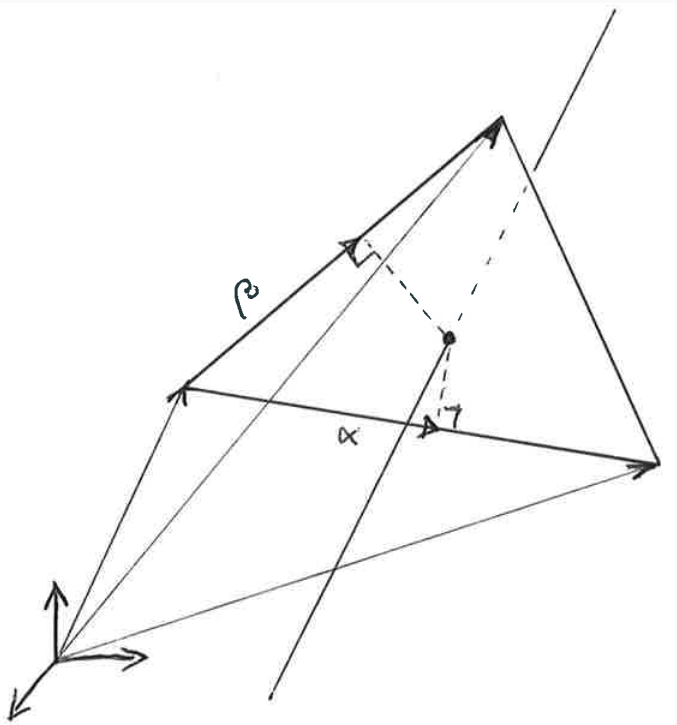
- Check boundary conditions of triangle



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- Ray in plane equation

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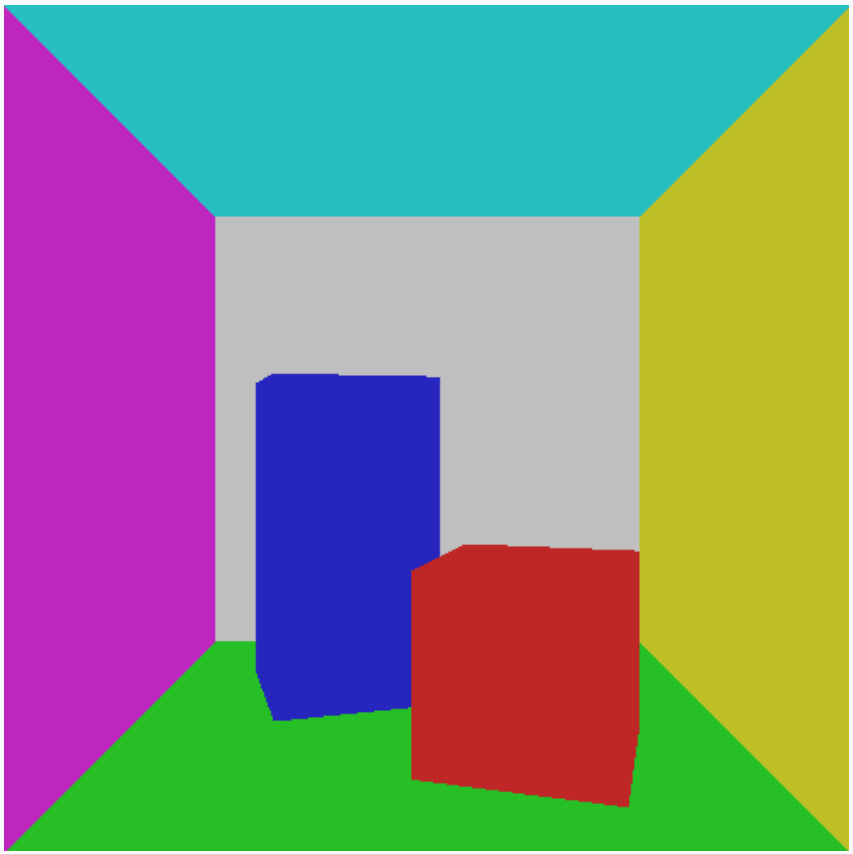
- *silliest solution*

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

Intersections



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- Quadratic expression i t

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Ray Intersections

- A Raytracer spends most its time doing intersection computations (profile your code)
- Rules of Thumb
  - Can we trivially reject something?
  - Can we re-use computations?
- If we need to do several tests do them in order of computational cost
- Realtime

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  - Can we trivially reject something?
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- If we need to do several tests do them in order of computational cost
- Realtime
- *Its really fun optimisations as the code is often very small and the solutions are really cute*

Ray-Sphere Intersection

- Sphere intersection

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- Look at square-root
  - $= 0$ Only one solution, no need to compute square-root
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  - > 0 We have to compute square root (two intersections)
- *Which of the above cases do we have most of the time?*

Raycaster

Code

```
for (int y=-h/2;y<h/2;y++)
  {
    for(int x=-w/2;x<w/2;x++)
      {
        /*compute primary ray*/
        for(int i=0;i<N_primitives;i++)
          {
            /*1. compute intersection
              2. check closest*/
          }
      }
  }
```

Next Time

Next Time

Lab Start with Lab 1

- Implement a camera structure and infrastructure to deal with transformations

Lecture Thursday 2nd of February

- 11-12 WMB 3.31
- Illumination: Light I
- How to decide colour of pixel

END

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