UNIVERSIDAD NACIONAL DE INGENIERÍA

FACULTAD DE CIENCIAS

CALIFICACIÓN

Preg N°	Puntos
1	5.0
2	5.0
3	5.0
4	0.0
5	
6	
Total	

CURSO: Cálculo Vectorial .... COD. CURSO ....

PRACTICA: Calificado N° 4 SECCIÓN: C

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Lima, 11 de Noviembre del 2015 N° Lista ....

NOTA: 15 (En números) — (En letras)

Nombre del Profesor — Firma del Profesor

1.. Sean:

$P: x - 2y + 3z = 8 \longrightarrow \vec{n} = (1,-2,3)$: normal al plano

$l: \frac{x+4}{4} = \frac{z-5}{3}; y = -1 \longrightarrow l: \{P \in \mathbb{R}^3 / P = (-4,-1,5) + t(4,0,3)\}$

Hallando: $\mathcal{X}_0 = \{P \in \mathbb{R}^3 / P = P_0 + s\vec{D}\}$

Donde $P_0 = (0, 2, -1)$

Como $\mathcal{X} \cap \mathcal{X}_0 \neq \emptyset \Rightarrow \exists P \in \mathcal{X} \cap \mathcal{X}_0$

$\longrightarrow P \in \mathcal{X} \wedge P \in \mathcal{X}_0$

$P = (4t-4; -1; 3t+5) \wedge P = P_0 + s\vec{D}$

$\rightarrow P - P_0 = s\vec{D} \rightarrow (4t-4; -1; 3t+5) - (0,2,-1) = s\vec{D}$

$\rightarrow (4t-4; -3; 3t+6) = s\vec{D}$ (*)

Como: $\vec{D} \perp \vec{n}$ $(\mathcal{X}_0 \parallel P) \rightarrow s\vec{D} \perp \vec{n}$

$\rightarrow \langle (4t-4; -3; 3t+6); (+1,-2,3) \rangle = 0$

$\rightarrow 4t - 4 + 6 + 9t + 18 = 0$

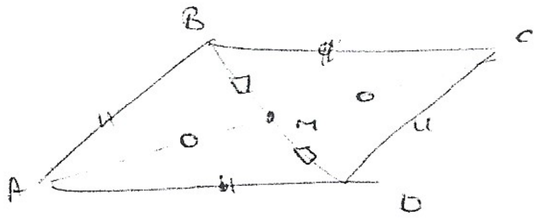
$\rightarrow 13t = -20 \rightarrow t = -20/13$

Reemplazando en (*)

$s\vec{D} = \left(-\frac{132}{13}, -1, \frac{18}{13}\right) \parallel \vec{D} \parallel (-132; -13; 18)$

$\rightarrow \mathcal{X}_0 = \{P \in \mathbb{R}^3 / P = (0,2,-1) + r(-132,-13,18)\}$ ☐

2. Sea el paralelogramo:



$A = (1, 0, 0)$

$B = (0, 1, 0)$

$M = (0, 0, 1)$

Del gráfico:

$C = A + 2\,\overline{AM}$

$C = A + 2M - 2A$

$C = 2M - A$

$C = (0,0,2) - (1,0,0)$

$\boxed{C = (-1,0,2)}$

$D = B + 2\,\overline{BM}$

$D = B + 2M - 2B$

$D = 2M - B$

$D = (0,0,2) - (0,1,0)$

$\boxed{D = (0,-1,2)}$

3a. $L_1 = \{(3,3,4) + t(2,2,3)\}$
$L_2 = \{(1,6,-1) + r(-1,2,0)\}$

Hallar: $\mathcal{L} / \mathcal{L} \perp \mathcal{L}_1 \wedge \mathcal{L} \perp \mathcal{L}_2 \wedge \mathcal{L} \cap \mathcal{L}_1 \neq \phi \quad \mathcal{L} \cap \mathcal{L}_2 \neq \phi$

Afirmo: $\mathcal{L}_1 \cap \mathcal{L}_2 = \phi$

Dem:
Supongamos $\mathcal{L}_1 \cap \mathcal{L}_2 \neq \phi \rightarrow \exists P \in \mathcal{L}_1 \cap \mathcal{L}_2$

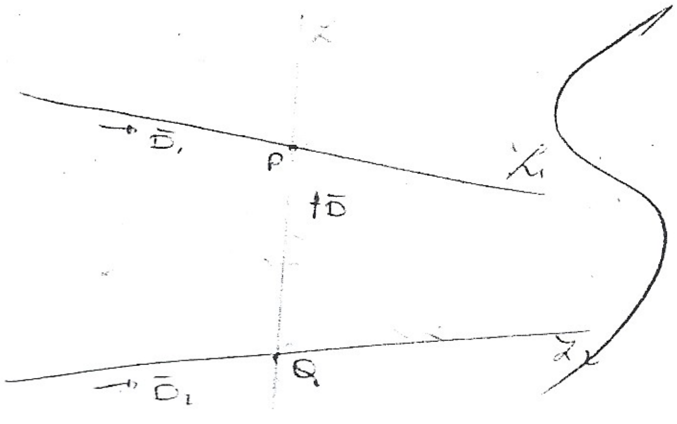
$\rightarrow P = (3+2t, 3+2t; 4+3t) = (1-r; 6+2r; -1)$

Resolviendo:
$\rightarrow$
$4t+3 = -1 \rightarrow t = -5/3$
$1-r = 6+2r \rightarrow r = -5/3 \quad) \quad t = r$

también:
$3+2t = 1-r = 1-t \rightarrow t = -2/3 = -5/3 \quad (\rightarrow\leftarrow)$

$\rightarrow \mathcal{L}_1 \cap \mathcal{L}_2 = \phi$

Gráficamente:



Como: $\bar{D} \perp \bar{D}_1 \wedge \bar{D} \perp \bar{D}_2$

$\rightarrow \bar{D} \parallel \bar{D}_1 \times \bar{D}_2$

luego:
$\bar{D}_1 \times \bar{D}_2 = (2,2,3) \times (-1,2,0)$
$= (-6,-3,6)$

Consideremos:
$\boxed{\bar{D} = (-2,-1,2)} \parallel (-6,-3,6)$

Notemos:
$P = (x,y,z) \in \mathcal{L}_1 \Rightarrow (x,y,z) = (3,3,4) + t(2,2,3)$
$Q = (a,b,c) \in \mathcal{L}_2 \Rightarrow (a,b,c) = (1,6,-1) + r(-1,2,0)$
luego: $\rightarrow \boxed{Q - P = (-2-r-2t; 3+2r-2t; -5-3t)}$
Además:
$PQ \parallel D \rightarrow PQ \perp D_1 \wedge PQ \perp \bar{D}_2$

$\Rightarrow \langle Q-P; (2,2,3)\rangle = 0 \quad \wedge \quad \langle Q-P; (-1,2,0)\rangle = 0$

$\langle(-2-r-2t; 3+2r-2t; -5-3t); (2;2;3)\rangle = 0 \quad \wedge \quad \langle(-2t-r-2; 3+2r-2t; -5-3t); (-1,2,0)\rangle = 0$

$\Rightarrow -4-2r-4t+6+4r-4t-15-9t = 0 \quad \wedge \quad 2t+r+2+6+4r-4t = 0$

$\Rightarrow 2r - 17t = 13 \quad \wedge \quad 5r - 2t = -8$

$\rightarrow t = -1$
$r = -2 \quad \Rightarrow P = (3,3,4) + (-1)(2,2,3) = (1,1,1)$

$\Rightarrow \mathcal{L} = \{(1,1,$ ...

4.

$P_1 \equiv a_1x + b_1y + c_1z = \lambda_1 \quad = P_1 : \langle P, n_1 \rangle = \langle P_{0_1}, n_1 \rangle = \lambda_1$

donde $n_1 = (a_1 ; b_1 ; c_1)$: vector normal a P_1

$P_2 \equiv a_2x + b_2y + c_2z = \lambda_2 \quad > P_1 : \langle P ; n_2 \rangle = \langle P_{0_2} ; n_2 \rangle = \lambda_2$

donde $n_2 = (a_2 ; b_2 ; c_2)$: vector normal a P

Como $\bar{n}_1 \not\parallel \bar{n}_2 \Rightarrow P_1 \cap P_2 \neq \emptyset \quad (*)$

$\Rightarrow \exists P_0 \in P_1 \cap P_2$

Considerando P_0 : punto de paso de $P_1 \wedge P_2$

$$\rightarrow \begin{cases} P_1 = \langle P, n_1 \rangle = \langle P_0 ; n_1 \rangle \\ P_2 = \langle P, n_2 \rangle = \langle P_0 ; n_2 \rangle \end{cases}$$

P.d $= P_1 \cap P_2 = \mathcal{L} : \{ P \in \mathbb{R}^3 \ / \ P = P_0 + t(\bar{n}_1 \times \bar{n}_2) \}$

$(\subset)$ Sea $P \in P_1 \cap P_2$

$\rightarrow \langle P ; n_1 \rangle = \langle P_0 ; n_1 \rangle \Rightarrow \langle P_0P ; n_1 \rangle = 0$

$\rightarrow \langle P ; n_2 \rangle = \langle P_0 ; n_2 \rangle \Rightarrow \langle P_0P ; n_2 \rangle = 0$

$\rightarrow P_0P \perp \bar{n}_1 \wedge P_0P \perp \bar{n}_2$

$\rightarrow P_0P \parallel \bar{n}_1 \times \bar{n}_2$

$\rightarrow P_0P = t(\bar{n}_1 \times \bar{n}_2)$

$\rightarrow P = P_0 + t(\bar{n}_1 \times \bar{n}_2) \quad \rightarrow P \in \mathcal{L}$

$(\supset)$ Sea $P \in \mathcal{L} \Rightarrow P = P_0 + t(\bar{n}_1 \times \bar{n}_2)$

$\rightarrow P_0P = t(n_1 \times \bar{n}_2)$

$\rightarrow P_0P \parallel \bar{n}_1 \times \bar{n}_2$

$\rightarrow P_0P \perp \bar{n}_1 \wedge P_0P \perp \bar{n}_2$

$\rightarrow \langle P_0P , n_1 \rangle = 0 \wedge \langle P_0P ; n_2 \rangle = 0$

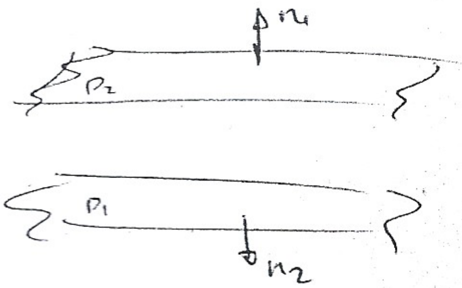
$\rightarrow \langle P_0 , n_1 \rangle = \langle P , n_1 \rangle \wedge \langle P_0 ; n_2 \rangle = \langle P , n_2 \rangle$

$\rightarrow P \in P_1 \wedge P \in P_2$

$\rightarrow P \in P_1 \cap P_2 \quad \square$

Justificación de (*) :

Si $P_1 \cap P_2 = \emptyset \Rightarrow P_1 // P_2$



Notamos que :

$n_1 // n_2$ $(\rightarrow \leftarrow)$