Journal of Economic Perspectives—Volume 26, Number 2—Spring 2012—Pages 223–236

Retrospectives
The Introduction of the Cobb–Douglas Regression

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This feature addresses the history of economic terms and ideas. The hope is to deepen the workaday dialogue of economists, while perhaps also casting new light on ongoing questions. If you have suggestions for future topics or authors, please write to Joseph Persky of the University of Illinois at Chicago at ⟨jpersky@uic.edu⟩.

Introduction

At the 1927 meetings of the American Economic Association, Paul Douglas presented a paper entitled "A Theory of Production," which he had coauthored with Charles Cobb. The paper proposed the now familiar Cobb–Douglas function as a mathematical representation of the relationship between capital, labor, and output. The paper's innovation, however, was not the function itself, which had originally been proposed by Knut Wicksell, but the use of the function as the basis of a statistical procedure for estimating the relationship between inputs and output.[1] The paper's least squares regression of the log of the output-to-capital ratio in manufacturing on the log of the labor-to-capital ratio—the first Cobb–Douglas

[1] Wicksell proposed the function as a hypothetical representation of the relationship between inputs and output in 1896, and continued to use it in subsequent works to help illustrate theoretical propositions (Samuelson 1979). It was largely through Douglas's empirical studies, however, that the function became familiar to economists, and thus it came to be known as the Cobb–Douglas function. While the present essay deals with the early history of the function as a basis for statistical estimation of production relationships, the function also plays a role in the interesting and related history of mathematical modeling in economic theory.

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http://dx.doi.org/10.1257/jep.26.2.223 doi=10.1257/jep.26.2.223

regression—was a realization of Douglas's innovative vision that a stable relationship between empirical measures of inputs and outputs could be discovered through statistical analysis, and that this stable relationship could cast light on important questions of economic theory and policy.

This essay provides an account of the introduction of the Cobb–Douglas regression: its roots in Douglas's own work and in trends in economics in the 1920s, its initial application to time series data in the 1927 paper and Douglas's 1934 book *The Theory of Wages*, and the early reactions of economists to this new empirical tool.

Paul Douglas and the Origin of the Cobb–Douglas Regression

Paul H. Douglas received his Ph.D. in economics from Columbia University in 1920. He had begun his graduate education at Columbia in 1913 and had taken his first college teaching post in 1915. In 1920, he accepted a position at the University of Chicago, where he would remain on the faculty until 1948. Douglas was a prolific researcher and began in the late teens to publish a steady stream of articles and books, usually on topics related to labor legislation and working-class living standards. In 1921, he entered an ongoing debate on the trend in real wages in the U.S. since 1890 (Douglas and Lamberson 1921), and in 1924 started work on *Real Wages in the United States, 1890–1926*, a statistical exploration of recent trends in wages, prices, employment, and unemployment rates (Douglas 1930). As Douglas assembled this evidence, he was also developing a theoretical framework through which to interpret it. In 1926 he submitted a "treatise on the theory of wages" to a competition sponsored by the clothing manufacturer Hart, Schaffner, and Marx, and was awarded the $5,000 first prize. The prize-winning manuscript, which included "a more or less original explanation of general wages drawn in terms of relative elasticities of supply" and "the theory of occupational and geographical differences in wage rates," was too long to be published, and Douglas agreed to distill it into a book. Seven years passed before this book appeared under the title *The Theory of Wages* (Douglas 1934, p. xi). It was substantially altered from its 1926 form, and at its core was the Cobb–Douglas regression.

Douglas (1971, pp. 46–47) recounted the "origin story" of the Cobb–Douglas regression in several places, including this version in his autobiography:

> One spring day in 1927, while lecturing at Amherst, I charted on a logarithmic scale three variables I had laboriously compiled for American manufacturing for the years 1899 to 1922: an index of total fixed capital corrected for the change in the cost of capital goods (C), an index of the total number of wage earners employed in manufacturing (L), and an index of physical production (P). I noticed that the index of production lay between those for capital and labor and that it was from one third to one quarter of the relative distance between the lower index of labor and the higher index of capital. After consulting with my friend Charles W. Cobb, the mathematician, we

> decided to try to find on the basis of these observations the relative contributions which each of the two factors of production, labor and capital, had upon production itself. We chose the Euler formula of a simple homogeneous function of the first degree, which that remarkable Englishman Philip Wicksteed had developed some years before ($P = bL^k C^{1-k}$). We found the values of k and $1 - k$ by the method of least squares to be .75 and .25, and that b was merely 1.01.[2]

Thus, Cobb and Douglas estimated k in their hypothesized relationship $P = bL^k C^{1-k}$ by using the indexes Douglas had constructed to fit the linear regression $\text{Log}(P/C) = b + k\ \text{Log}(L/C)$.[3] They then plugged annual values of C and L, along with the estimated values of b and k, into their nonlinear "Euler formula" and calculated a series of predicted or "theoretical" values for P, denoted P'. Douglas was encouraged by the high correlation between actual P and predicted P', as well as the fact that the estimate for the share of manufacturing value added represented by wages and salaries over the period 1909–1918 from the National Bureau of Economic Research was almost identical to the estimate of k. The first public presentation of this research was the above-mentioned paper, "A Theory of Production," which appeared a few months later in the *American Economic Review* (Cobb and Douglas 1928). When Douglas published *The Theory of Wages* six years later, it included as a central feature a description of the statistical data, methods, and results from "A Theory of Production," accompanied by accounts of results of estimating the Cobb–Douglas regression with time series data from Massachusetts and New South Wales.

In the meantime, Cobb (1930) had published a paper on his own using the Massachusetts data and an estimation method designed to ameliorate problems created by measurement error in the data.[4] Cobb obtained bizarre results with this method, and was unable to make any sense of them. He never again published work involving the Cobb–Douglas regression. Through the years, however, Douglas would consistently give Cobb half of the credit for establishing the research program.

The Cobb–Douglas Regression and Theories of Production and Distribution

In the 1920s, a "theory of production" was understood to be an explanation of the determinants of the level of output. The amount produced, it was generally agreed, depended upon the level of technological knowledge and the quantities

[2] Similar accounts can be found in Douglas (1948, p. 6) and Douglas (1976, p. 904). Samuelson (1979, p. 926) discusses Douglas's apparent confusion between the work of Wicksteed and Wicksell.

[3] This specification embodied the restriction that the sum of the coefficients of capital and labor equaled one. Douglas would relax this restriction in his later work with the function.

[4] Cobb's estimation method was "diagonal mean regression," a method proposed by Ragnar Frisch for estimating linear relationships between variables when all variables are measured with error. See Cobb (1939) for details on the technique.

of the factors of production employed. A theory of production offered an account of the forces making for changes in these determinants of output, and/or details about the quantitative relationships between inputs and output—for example, hypotheses about the circumstances under which the "law of diminishing return" was applicable. A theory of distribution, on the other hand, explained the determination of the division of output between various members of society. Classical economists had theorized about distribution in terms of the shares of the product received by three social classes—land owners, laborers, and capitalists—who controlled the three productive factors of land, labor, and capital, with the principles explaining the share going to land owners being distinct from those governing the shares received by laborers or capitalists. Marx's version of distribution theory ran in terms of two classes—workers and capitalists—describing the ways in which the social and economic institutions of capitalism allowed capitalists to expropriate much of the output attributable to the efforts of the working class. During the late 1800s, a number of economists introduced theories of distribution based on the now-standard principle that in a market system, the payment received by the owner of any factor of production was determined by the marginal productivity of that factor. Some of these marginal productivity theorists presented their ideas in terms of the classical trio of factors, while others rejected the relevance of those categories, preferring to emphasize the commonality, from the point of view of distribution theory, of any tool, substance, or service that could contribute to the production of final goods. By the 1920s, the economics profession displayed no consensus regarding the appropriate approach to theorizing about distribution, with approaches based on the marginal productivity principle competing with those rooted in the classical tradition and those emphasizing the ways in which social and economic institutions influenced the bargaining power of various groups. Among the important points of disagreement between advocates of the various versions of distribution theory were the extent to which distribution was a function of malleable human institutions versus relatively unchangeable aspects of human nature and the physical world, and the extent to which policies intended to alter distributive shares would influence the overall level of output.

As is clear from the title of the 1927 paper, the Cobb–Douglas regression was first presented as a contribution to production theory. Douglas opened the paper with a list of questions that could be addressed if an empirical relationship between capital, labor, and output could be discovered, including whether the increase in output apparent in the data was "purely fortuitous, whether it was primarily caused by technique, and the degree if any, to which it responded to changes in the quantity of labor and capital," and whether his proposed statistical procedure might provide "an historical approach to the theories of decreasing imputed productivity (diminishing increment to the total product)" that would open the way towards "further attempts to secure quantitative approximations to these tendencies, if indeed there should turn out to be historical validity to them" (Cobb and Douglas 1928, p. 139). The paper made no explicit reference to theories of distribution. There was a mention of the question of whether the "processes of distribution are modeled at all closely upon those of the production

of value" but no explicit discussion of the link between the two provided by marginal productivity theory.

By contrast, in *The Theory of Wages* the general discussion of the empirical estimation of production relationships was embedded in a detailed explication of the marginal productivity theory and a defense of that theory as a framework for inductive study of production and distribution. The estimated elasticities of curves of marginal productivity, which as of 1934 Douglas seemed to regard as the most important quantities revealed by his innovative statistical analysis, could, in light of the marginal productivity theory, also be regarded as elasticities of aggregate demand curves for capital and labor. In the 1934 volume, there was no question as to the theoretical framework motivating Douglas's numerous comparisons between estimates of the value of labor's marginal product, derived from his regression, and measures of real wages or labor's share of the value of output.

The Cobb–Douglas Regression and Empirical Economics in the 1920s

One can see in Douglas's innovation of 1927 a blending of several characteristics of the empirical economics literature of the 1920s. First, it reflected the period's emphasis on the importance of creating reliable statistical measures of economic activity. Because government programs for collecting economic statistics were still in their infancy, one of the more important skills for empirically oriented economists was the ability to construct, from the fragmentary statistical evidence available on a phenomenon, a credible quantitative account of that phenomenon. Thus, the researcher had to locate the relevant data sources, to extrapolate from time periods or sectors for which data were relatively complete to time periods or sectors in which they were scant, and to defend or assess the likely accuracy of the results using logic, implicit theorizing, and various consistency checks across data from different sources. Researchers also needed to persuade readers not only that the steps taken to produce the estimates were the most reasonable ones under the circumstances, but that the resulting statistical picture, with all its shortcomings, was accurate enough to be useful. This type of work was a crucial prerequisite to the estimation of the Cobb–Douglas regression in 1927, with ten of the 24 pages of "A Theory of Production" devoted to explaining and defending Douglas's methods of constructing time series for fixed capital and labor. Judged by the standards of the time, Douglas's construction of these series by itself would have been considered an important contribution to empirical economics.

Second, Douglas's use of least squares regression, as well as correlation coefficients and indexes, placed him at the cutting edge of statistical practice in economics as of 1927. Prior to World War I, with the exception of the work of a few pioneers such as Wesley Mitchell, Warren Persons, and Irving Fisher, most empirical economic research simply presented raw numbers or percentage shares, and included no derived statistical measures such as means, standard deviations, or index numbers. This began to change during the 1920s, and by the end of that decade, the well-trained empirical economist understood basic statistical theory

and applied it in constructing index numbers, tabulating frequency distributions, and calculating summary statistics.[5] Douglas's own development as an economic statistician paralleled these changes in what represented good statistical practice for economists, as he moved from reporting numbers and percentages in tables and text (Douglas 1918, 1919), to calculating means and measures of average deviation to illustrate relevant points (Douglas 1930), to the use of correlation coefficients and linear regression in "A Theory of Production."

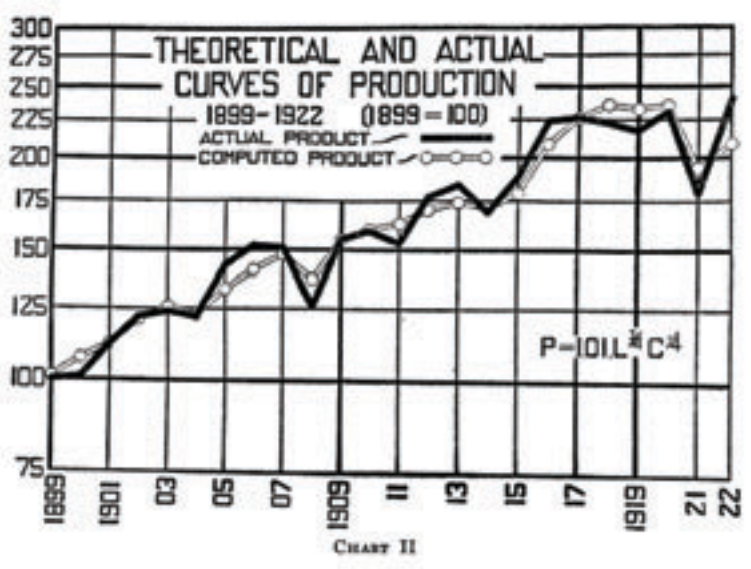
Finally, in attempting to statistically estimate the key functional relationships underlying marginal productivity theory, Douglas was expanding the boundaries of a newly emerging research program in empirical economics. During the 1920s and early 1930s, a growing literature sought to apply regression techniques to estimate the real world counterparts of theoretical supply and demand curves. As Morgan (1990) has shown, this work played an important role in shaping the approach to combining statistical methods and economic theory that would become the standard econometric practice in the later decades of the twentieth century, but in the 1920s it was still quite esoteric. Douglas (1934, p. xii), however, saw it as the wave of the future, and he explicitly linked his own research to it:

> It has long seemed to me that the inductive, statistical, and quasi-mathematical method must be used if we are ever to make economics a truly fruitful and progressive science. The neoclassical school has constructed a valuable theoretical scaffolding according to which the value of commodities and the rates of return to land, labor, and capital are fixed at the intersection of the various supply and the demand curves. This is a beginning but only a beginning. For in order to make the analysis precise, to forecast, and to detect interactions in economic society it is plainly necessary to determine the slopes of the demand and supply curves. . . . An excellent beginning has been made in this direction by such scholars as Henry L. Moore, Schultz, Ezekiel, Bean, Working, and Marschak . . .
>
> There is a need for a similar approach to the problems of distribution. We need to know whether the assumed curves of diminishing incremental productivity are merely imaginative myths or whether they are real, and if the latter, what their slopes are. We need to know more about the supply functions of the factors of production and whether the actual processes of distribution furnish any degree of corroboration to the inductive tendencies discovered. This book is an attempt to do just that.

Thus, the Cobb–Douglas regression represented a bold attempt to join up-to-date statistical methods with the still controversial marginal productivity theoretical framework.

[5] In Biddle (1999), I document this transition, while Ayres (1927) provides a contemporary account.

Figure 1
One of Five "Charts" from Cobb and Douglas's "A Theory of Production"



Source: Chart II from "A Theory of Production" (Cobb and Douglas 1928, p. 153).

In both "A Theory of Production" and *The Theory of Wages*, Douglas needed to convince readers that his regression procedure provided reliable measures of real and interesting economic quantities, a task made more difficult by the novelty of his approach. He resorted to an array of arguments to do so. His primary argument was based on the fit of the regressions. The time series Cobb–Douglas regressions tended to generate a close correspondence between the actual values of the output variable (P) and values predicted by the regression in the manner described above (P'). This correspondence, Douglas argued, strongly suggested that the regressions captured a true relationship between inputs and output. He demonstrated this goodness of fit using correlation coefficients, tables, and graphs, such as Figure 1, reproduced from "A Theory of Production" (1928), showing the relationship between actual and "theoretical" (predicted) output. He raised the possibility that the good fits were due to spurious correlation between trending series, noting that "it has some times been charged that . . . equally good results would be secured by comparing the relative movement of hogs in Wisconsin, cattle in Wisconsin, with the physical product in manufacturing" but responded to this concern by arguing that there was an a priori theoretical connection between capital, labor, and output that did not exist for pigs, cattle, and output and by showing that deviations of P and P' from their three-year moving averages were also highly correlated (Cobb and Douglas 1928, p. 160).

In an interesting twist on the fit argument, Douglas pointed out that observations (years) showing large differences between actual P and estimated P', because they were abnormal in some way, actually strengthened the case that the Cobb–Douglas

regression reflected the normal relationship between capital, labor, and product. Years in which P was below P' were recession years, and those in which P was above P' were years of prosperity. Since the capital index measured existing capital rather than capital utilized, and the labor index was men rather than man-hours, one would expect this pattern: in a recession, when plants were idled and overtime eliminated, the capital and labor indexes overstated the amount of the inputs actually employed, and so the equation produced a predicted output that was too high. Likewise, prosperities were periods of full capital utilization and long, intense hours for workers, leading the indexes to understate true input use (Douglas 1934, pp. 160–61).

Douglas also made much of evidence that various measures of labor's compensation tended to correspond to measures of marginal productivity derived from his estimates. This included the striking similarity between the .75 estimate for k produced by the Cobb–Douglas regression and the estimate from the National Bureau of Economic Research of labor's share of the total value added in manufacturing, but Douglas added to this by showing positive correlations between moving averages of real wages in various industries and moving averages of the measures of labor's marginal product in those industries implied by his function. There was some ambiguity, however, as to how readers were to regard this evidence. At times, Douglas seemed to be arguing that the regression provided a means of testing the marginal productivity theory, and that a close concurrence between wages and estimated marginal products represented a verification of this theory. More often, however, Douglas pointed to the consistency of his results with the predictions of marginal productivity theory as an additional proof of the plausibility of his procedure, a line of argument that essentially assumed the truth of the marginal productivity theory.[6]

Critiques and Responses

Douglas made bold claims in "A Theory of Production" and *A Theory of Wages*: Using generally available data and accessible statistical techniques, he had shown that the actual relationship between capital, labor, and output in manufacturing could be closely approximated by a simple function, one which embodied and allowed quantification of the hypothesis of diminishing marginal productivity. He had demonstrated a relationship between the characteristics of this "law of production" and the distribution of income between labor and capital, a relationship posited by a well-known but still-contested theory of distribution. Unsurprisingly, the Cobb–Douglas regression attracted considerable attention from Douglas's fellow economists.

[6] This ambiguity about whether the Cobb–Douglas procedure should be regarded as a means of testing various predictions of marginal productivity theory, as a way of measuring the parameters of a theory maintained to be true, or both, was not clarified in Douglas's subsequent discussions of the procedure, and was at times a point of contention in the literature on production function estimation in subsequent decades. See, for example, Mendershausen (1941), and the exchange between Shaikh (1974) and Solow (1974).

Sumner Slichter, assigned to discuss Cobb and Douglas's paper at the 1927 AEA meetings, was a decidedly unfriendly critic of the work (Slichter 1928). The bulk of his remarks were devoted to detailing problems with the index of fixed capital constructed by Douglas, but his complaints went beyond issues of data quality as he believed the entire project to be wrong-headed. Despite the fact that the marginal productivity theory was not explicitly mentioned in the paper, Slichter thought he could see a hidden agenda, and he did not approve. "Professors Cobb and Douglas conclude that it has been statistically demonstrated that the relationship between the agents of production on one hand and the volume of output on the other meets the requirements of the marginal productivity hypothesis." Slichter disputed this specific claim and argued more generally that marginal productivity theory had little to offer as a framework for thinking about distribution. He closed his comments with a final indictment of the research (p. 170):

> There is probably no more important single cause for our meagre knowledge of the distributive process than the fact that the subject has been so largely studied within the narrow limits imposed by the assumptions of static economics. . . . Quantitative economics, by helping to provide the raw materials for a realistic theory, can be of great use in liberating the study of distribution from the tyranny of economic statics. But it can be of little assistance if statisticians and mathematical economists are too completely preoccupied with verifying the propositions of static doctrine.

In *The Theory of Wages,* Douglas (1934) responded to critics like Slichter who complained of the lack of realism in the marginal productivity theory. He included a chapter on the assumptions of the theory, both explicit and implicit, with long discussions of the extent to which each was valid for the United States. After arranging the implicit assumptions on a scale ranging from "largely valid but not wholly so" to "partially true but on the whole not true," Douglas (pp. 94–95) commented:

> Many, who have seen the degree of variance between real life and the assumptions of the productivity school, have in their impatience declared that because of this defective basis, the conclusions which have been drawn from the productivity theory are not worthy of credence and hence should be disregarded. But such an attitude as this ignores the fact that the assumptions do represent real tendencies which in the aggregate are more powerful than those of a conflicting nature.

Such critics, Douglas argued, seemed ignorant of the fact that the method of deduction and abstraction used to build the marginal productivity theory was also the method that had achieved great success in the natural sciences.

Another type of unfriendly critic was exemplified by Douglas's colleague Frank Knight, who argued that the key concepts of economic theory were essentially static and abstract, while historical data was dynamic, reflecting the action of forces that were assumed away in static theory. Thus, statistical methods could never quantify

theoretical concepts.[7] Douglas (1934, pp. 106, 107) dismissed such arguments rather undiplomatically in *The Theory of Wages*:

> [T]he high priests of "pure" theory are never tired of pointing out that they are dealing only with static conditions—as of one moment in time for one community. When statistical series dealing with time sequences or even relative distributions in space are brought forward, the armchair theorists brush these aside on the ground that they may include shiftings of the curves or different curves. These series are then dismissed as being merely historical or empirical.
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> Now it is of course true that one of the aims of statistical economics . . . should be to approximate as far as possible the static concepts and to give concrete meaning and definite values to them. But if this cannot be completely carried out . . . [s]hould we abandon all efforts at the inductive determination of economic theory and remain in the ivory tower of "pure" theory[?] If this is what is done, we may as well abandon all hope of further developing the science of economics and content ourselves with merely the elaboration of hypothetical assumptions which will be of little aid in solving problems since we will not know the values. Or shall we try to make economics a progressive science?

There were also friendly critics of the Cobb–Douglas paper who, while finding fault with various details of what Douglas had done, expressed considerable enthusiasm for the "method of attack" represented by the research and offered constructive suggestions for pushing the research program forward. One such friendly critic was Douglas's colleague and prominent empirical economist Henry Schultz. Schultz accepted the point that Cobb and Douglas's statistical procedure, which employed time series data but made no adjustment for secular changes, could not result in a verification of a static theory like the marginal productivity theory.[8] He argued, however, that that the Cobb–Douglas method of estimating production relations should be "modified, not abandoned." Referring to his own approach to estimating the supply and demand curves implied by neoclassical theory, Schultz (1929) described the strategy of adjusting the data to remove long-term trends, then correlating deviations from those trends in order to isolate relationships that were closer in principle to the concepts of static theory.

Another prominent friendly critic was J. M. Clark, who within a few months of the appearance of "A Theory of Production" published an article devoted solely to discussing issues raised by the Cobb–Douglas paper (Clark 1928). His criticisms were

[7] Letter from F. Knight to Douglas, 10/12/1932; see also Douglas (1976, p. 905). Knight, whose opinion carried considerable weight with a number of younger Chicago faculty members at the time, was generally hostile to empirical work in economics, and was highly critical of Henry Schultz's work as well (Reder 1982).

[8] As Morgan (1990, sec 5.2) explains, possible approaches to testing a static theory with time series data were much discussed in the literature on estimating supply and demand relationships, to which Schultz was a major contributor.

numerous, but most were constructive, aimed at improving the Cobb–Douglas analysis of marginal productivity rather than discrediting it. Like Slichter, he questioned the accuracy of Douglas's capital and labor series, but took it for granted that "they will be improved and refined as the authors continue their researches." A central feature of the article was a suggestion for an augmented version of the Cobb–Douglas regression. Clark believed that the Cobb–Douglas equation offered a good account of the "normal" or long–run relationship between labor, capital, and output, but did a poor job of representing the effect of cyclical fluctuations in labor and capital utilization. He proposed altering the function by including a factor representing cyclical swings, and suggested $P = L^k C^{1-k}(L/L_n)^m$, where L_n represented the "normal" level of employment, or that which the capital stock was designed to accommodate. This formula allows the marginal increases in labor input to have a magnified impact on output if they are allowing idled capital to be put back into use. Moreover, Clark fit this function to Douglas's data, although not by the method of least squares. Clark was also troubled by the fact that the Cobb–Douglas regression left no room for improvements in technology to affect productivity, but believed that this problem could be solved, if not with data based on historical aggregates, then with comparative studies of "simultaneous" data from different industries. The Cobb–Douglas study was "a bold and significant piece of pioneer work in a hitherto neglected field," which Clark clearly hoped would be followed up by others.

In *The Theory of Wages*, Douglas responded positively to many of Clark's comments and suggestions, even reporting some of his own experiments with Clark's modified formula. He also devoted several pages to the question of how the substantial technological progress over his sample period affected the meaning of his estimates, a concern raised by other commentators in addition to Clark. He did not go so far as to develop a modification of his regression procedure that would accommodate technological change, nor would he ever, as he would soon change the focus of his research program to the estimation of the regression with cross-section datasets, for which the issue of accounting for technological progress was no longer important.[9]

Modern economists may sense an air of the familiar in the initial reactions to Douglas's new empirical procedure. While few economists today would agree with Sumner Slichter's rejection of the marginal productivity theory as a framework for the inductive study of distribution, it is not unusual to see an empirical research program being criticized on the grounds that it is guided—or the critic might say constrained—by a too-literal reliance on the assumptions of an oversimplified theoretical framework and will thus miss important aspects of the phenomenon under study and/or produce meaningless results. It is also not uncommon to see the argument that the quality of existing data is not adequate to the needs of a

[9] Moving to cross sections of industry-level data introduced the new concern that different industries might have different production functions, a problem that Douglas recognized but never addressed empirically. A solution to the problem of estimating an aggregate time series production function while also accounting for and even measuring technological improvement was introduced in Solow's (1957) seminal article on growth accounting.

proposed new empirical technique. Such arguments may be offered as a reason to reject the new technique, as Slichter seemed to be doing in his discussion of "A Theory of Production," but some who raise issues of data quality take the attitude of J. M. Clark that the eventual development of better data is to be expected, making it worthwhile to experiment with and develop a better understanding of the technique in the meantime.

Finally, the differing reactions of Frank Knight and Henry Schultz to the Cobb–Douglas regression reflect a difference of opinion that persisted, and arguably still persists, among economists. Both men were strong believers in the usefulness for economic research of neoclassical equilibrium models and the abstract deductive method that produced them, but they disagreed on whether it was worthwhile or even feasible to attempt to use data from a messy, dynamic world to test these models or to extract empirical measures of their key components. Schultz believed that the bridging of the theory–data gap through the further development of statistical techniques and data was both possible and one of the most important tasks of modern economics, and he applauded Douglas's work; Knight believed that economic theoretical concepts like the market demand curve for a good were essentially ideal types that could never manifest themselves in data generated by a dynamic real world economy, and saw work like that of Douglas and Schultz as a waste of time at best, and perniciously misleading at worst.

After the appearance of *The Theory of Wages*, a significant set of friendly criticisms came from mathematical economists who embraced marginal productivity theory and who were trying to make sense of the relationship between Douglas's regression equation and the equations of their theoretical systems. These included Wassily Leontief (1934), Jacob Marschak (1936), and David Durand (1937). It was in these articles, as well as a very critical article by Mendershausen (1938) that the phrase "production function" was first consistently applied to the relationship that Douglas was attempting to estimate, although Douglas quickly adopted it himself. At the time, the phrase was rare in the economics literature, used almost exclusively by those concerned with mathematical formalization of neoclassical theory and/or the statistical estimation of the components of the resulting models. As mentioned earlier, Douglas saw his work as a part of the program to estimate the relationships of neoclassical theory, so the young econometricians who referred to Douglas's regression as a production function were affirming Douglas's conception.

However, while these neoclassically oriented econometricians were embracing Douglas's program as complementary to their own, they were also redefining the objectives of the program and developing criteria for evaluating Douglas's methods and results that Douglas himself would not have accepted. This outcome arose partly because they thought about marginal productivity theory within the context of Walrasian general equilibrium theory, in which the production function was a characteristic of a firm. As a result, they were crucially concerned with whether the estimated coefficients revealed anything useful about the true parameters of firm-level production functions, or perhaps were in some sense averages of those parameters. But this question, which became increasingly important to

economists engaged in production function estimation as time passed, was never crucial for Douglas. As Douglas (1971, p. 29) noted in his autobiography, he was taught theory by J. B. Clark, and received "a thorough drilling in (the marginal productivity) principle, which served me well . . . when I started my own inductive work in the theory." But Clark's formal analysis ran in terms of aggregates: the basic wage rate and the interest rate depended on the marginal products of "social" capital and "social" labor (Stigler 1941, p. 307). A student of Clark would have had no trouble thinking of an aggregate production function as a primal entity to be estimated, and its parameters as significant theoretical quantities in themselves.

Aftermath

Douglas's work with the Cobb–Douglas production function continued for another 14 years after *The Theory of Wages*, as he and various coauthors estimated Cobb–Douglas regressions using both aggregate time series data and cross-section industry-level data, while developing a series of arguments defending the procedure as a way of obtaining important knowledge about the economy from empirical data. Douglas's research career ended in 1948 with his election to the U.S. Senate, but in the ensuing decades the procedure of regressing the log of a measure of output on the logs of measures of various inputs became a standard and accepted empirical procedure in a number of areas of microeconomics and macroeconomics, while spawning several other, more complex approaches to estimating empirical production functions. There remain open questions about the scientific value of this procedure in each of the contexts in which it is applied, some of which are variations of the friendly and unfriendly questions raised by Douglas's initial critics. However, measured by the extent to which it has been embraced, applied, and elaborated upon by subsequent economists, Douglas's innovative 1927 idea that one could use statistical analysis to uncover meaningful empirical relationships between inputs and outputs, as well as his specific implementation of that idea using the Cobb–Douglas functional form and least squares regression, was an overwhelming success.

■ *The author would like to thank Ross Emmett, David Autor, Timothy Taylor, Charles Johnson, and John List for helpful comments.*

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