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# Environmental policy on the back of an envelope: A Cobb-Douglas model is not just a teaching tool<sup>☆</sup>

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## ABSTRACT

To clarify and interpret the workings of a large computable general equilibrium (CGE) model of environmental policy, we build an aggregated Cobb-Douglas (CD) model that can be solved easily and analytically. Its closed-form expressions show exactly how key parameters determine the sign and size of effects from a large new carbon tax on emissions, revenue, prices, output, and welfare. Data and parameters from the detailed, dynamic CGE model of Goulder and Hafstead (2018) are used in the CD model to calculate results that can be compared with theirs. Results from the CD model track aggregates from the large CGE model quite closely, even though the CD model omits much detail such as the number of sectors, intermediate inputs, and international trade. A CGE model is quite useful to generate detailed numerical results and to reflect on particular aspects of environmental policy, but the simpler CD model provides a transparent view of exactly how the policy affects key outcomes.

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A Computable General Equilibrium (CGE) model uses detailed economic data to perform large numerical simulations of producer and consumer behavior in order to calculate equilibrium prices and outputs in many markets simultaneously. Such models have been used widely to study effects of environmental taxes or regulations on emissions, outputs, prices, revenue, and economic welfare.<sup>1</sup> The most recent and fully described CGE model for climate policy is in a book by Goulder and Hafstead (GH) (2018). It analyzes the myriad effects of a carbon tax, cap-and-trade system, and alternative policies. It includes careful modeling of the carbon-intensity of each different production sector, but also the intertemporal dynamics of all U.S. taxes on labor supply, investment, and the incomes of different household groups. The model is

quite useful, but many readers who do not build their own CGE models may find the sheer size and complexity of such models impenetrable.

To understand such models, and to show exactly what features of the model drive key results, this paper builds a new but simplified Cobb-Douglas (CD) version of a CGE model that can be solved on “the back of an envelope” – using not a computer, but just paper, pencil, and a calculator. We use industry data from Goulder and Hafstead (2018) to calculate factor shares and tax rates. Then we perform the same kinds of simulations using the CD model to show how to solve for new post-reform general equilibrium prices and quantities. The CD model has only three sectors and four inputs, and it has no input-output matrix nor dynamics. Our results, therefore, must differ to some degree from those of Goulder and Hafstead, but we show that the simple model can generate aggregate results that are surprisingly close to those of the complicated CGE model. We also show derivations that are very transparent.

We believe this paper makes three contributions. First, it will be extremely useful for teaching general equilibrium or for teaching environmental economics – particularly pollution tax incidence and effects on economic efficiency. For the imposition of a pollution tax, this paper shows step-by-step derivations of all outcomes such as each quantity of output and its price, each input and its price, pollution, abatement

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<sup>1</sup> Jorgenson and Wilcoxon, 1990; Hazilla and Kopp, 1990; Boyd et al., 1995; Bovenberg and Goulder, 1996; Goulder et al., 1999; Parry and Williams, 1999; Parry et al., 1999.

costs, and utility-based changes in household welfare (equivalent variations). Others have used Cobb-Douglas examples for teaching, but none of those is solved analytically, published, and available.<sup>2</sup>

Second, and more importantly, our paper provides an interpretation and commentary on [Goulder and Hafstead \(2018\)](#), the newest and most complete description of a large CGE model. This detailed model is fully capable of analyzing many facets of the U.S. economy, the tax system, and various proposed climate policies. But readers who do not build CGE models themselves may find it hard to penetrate its complexities. Our simpler exposition here is useful to understand the nature of this large, complicated CGE model. Moreover, aggregate results are quite similar. While the complex model is necessary to get detailed numerical results on particular industries, and to solve for adjustment costs during transitions, we show that the detail is not necessary to understand the model nor to get broadly similar results.

In particular, we use their data and their calibration of parameters, adjusted to fit our more aggregated and simpler model. We then use the CD model to study the same questions as in Goulder and Hafstead: what are the effects of a pollution tax when its revenue is rebated to households in a lump-sum fashion or when its revenue is recycled to cut the labor tax? We compare results from our aggregated CD model to results from their dynamic disaggregated CGE model. For impacts of a carbon tax with either lump-sum rebates or with revenue used to cut labor taxes, our simpler model yields key results quite similar to theirs for aggregate carbon dioxide emissions, carbon tax revenue, and welfare costs of the carbon tax.<sup>3</sup>

A third contribution is that we show how to use the simple CD model to analyze three other questions not studied by [Goulder and Hafstead \(2018\)](#). We calculate and show the entire marginal abatement cost curve for each type of carbon tax and revenue use. We calculate and show the second-best optimal rate of tax for each such policy. And we further explore the effect of a carbon tax in only one sector on emissions from the other sector ("leakage"). These results further demonstrate the usefulness of the simple model.

The first section of our paper describes the basic Cobb-Douglas model, and the second section shows how to solve it on the back of an envelope for expressions that show the impact of parameters on outcomes such as emissions, revenue, outputs, prices, and economic welfare. An appendix shows how we aggregate data and parameters from Goulder and Hafstead into fewer sectors for our simple CD model. The third section uses the analytical solutions to calculate those outcomes and to interpret results. The fourth section compares those CD model results to the detailed results of [Goulder and Hafstead \(2018\)](#). The fifth section reports other results from our CD model. The sixth section discusses key findings and concludes.

## 1. Model description

Here, we develop a simplified Cobb-Douglas version of a CGE model. We assume a closed economy with a large number  $n$  of identical households, with perfect competition, perfect mobility, and no uncertainty. Each household is endowed with a fixed amount of capital ( $\bar{K}$ ) and a

fixed amount of time available for labor ( $\bar{L}$ ). Each gets utility from consumption of a composite good  $X$ , plus direct consumption by the household of electricity  $E_H$ , fossil fuels  $F_H$ , and leisure or home production  $L_H$ . In our model, household utility over consumption goods takes the Cobb-Douglas form<sup>4</sup>:

$$U = U(X, E_H, F_H, L_H) = X^a E_H^b F_H^c L_H^{1-a-b-c} \quad (1.1)$$

Consumers maximize that utility subject to their budget, but their welfare also negatively depends on total emissions of carbon dioxide ( $nC$ ) from the use of fossil fuels. We assume a constant marginal effect on welfare from emissions,  $\theta$ , so full welfare is  $U - \theta nC$ . With many identical households, each ignores the impact of their own emissions on total emissions  $nC$ .

In this simplified Cobb-Douglas model, goods are produced according to<sup>5</sup>:

$$X = X(K_X, L_X, E_X, F_X) = A K_X^\alpha L_X^\beta E_X^\gamma F_X^{1-\alpha-\beta-\gamma} \quad (1.2)$$

$$E = E_X + E_H = E(K_E, L_E, F_E) = B K_E^\delta L_E^\epsilon F_E^{1-\delta-\epsilon} \quad (1.3)$$

$$F = F_E + F_X + F_H = G L_F \quad (1.4)$$

$$C = \rho F \quad (1.5)$$

Production functions are all assumed to be Cobb-Douglas, but total-factor productivity (TFP) parameters differ (denoted  $A$ ,  $B$ , and  $G$ ). Production of  $X$  requires capital  $K_X$ , labor  $L_X$ , electricity  $E_X$ , and direct use of fossil fuels  $F_X$ . Production of electricity itself requires capital  $K_E$ , labor  $L_E$ , and fossil fuels  $F_E$ . Fossil fuels are produced using labor,  $L_F$ , only.<sup>6</sup> In our model, emissions are proportional to fossil fuel use, where a unit of fossil fuels emits  $\rho$  tons of carbon dioxide,  $C$ .

In this static model, households have no saving decision; the capital stock is fixed.<sup>7</sup> Households allocate their fixed time endowment between labor supply ( $L$ ) and leisure ( $L_H$ ). The government levies a tax  $\tau_L$  on market labor income and provides a lump-sum transfer of all tax revenue back to households as a per-household lump-sum rebate ( $R$ ). This tax rate is defined as a fraction of the net wage or price of labor ( $P_L$ ), so the gross wage is  $P_L(1 + \tau_L)$ . We consider a policy reform that raises the pollution tax and uses all added revenue to cut the distorting labor tax (holding constant the initial level of government transfers).

This reform can include a tax  $\tau_{CE}$  per metric ton of carbon dioxide ( $\text{CO}_2$ ) emissions in the production of electricity and a tax  $\tau_{CO}$  per metric ton of  $\text{CO}_2$  emissions from all "other" uses of fossil fuels. Given a fixed ratio of emissions to fossil fuel use,  $\rho$ , each such rate is equivalent to a tax at rate  $\rho\tau_{CE}$  per unit of fossil fuel used in electricity and  $\rho\tau_{CO}$  per unit of fossil fuel used in other sectors ( $F_H$  and  $F_X$ ). For simplicity, we ignore taxes on output and on capital, in order to focus only on the possible carbon tax in the presence of a pre-existing labor tax. The

<sup>2</sup> A Cobb-Douglas example appears in the lecture notes of James Markusen (2018), but to obtain those lecture notes would require knowledge of their existence and sending an email to ask for them. Thomas Rutherford (1999) and Shoven and Whalley (1984, 1992) also provide simplified CGE models for pedagogical purposes, but solutions for these models require a computer. Kimbell and Harrison (1986) provide analytical solutions for more complicated models with constant elasticity of substitution production. In contrast, the goal here is to provide a complete and comprehensive Cobb-Douglas example in published form that can be solved analytically.

<sup>3</sup> As discussed below, results are similar despite multiple differences between the two models. Unlike the CD model, the GH model incorporates adjustment costs, with disaggregated electricity generators differentiated by carbon intensity. Some of the many differences between the models likely affect results in opposite directions.

<sup>4</sup> In contrast, the GH model uses constant elasticity of substitution (CES) utility between consumption and leisure, but a Cobb Douglas form to combine different goods into a composite consumption good. We comment later on which differences between the models are most relevant to driving differences in results.

<sup>5</sup> Constant returns to scale (CRTS) removes any importance of the scale of production, so we define all inputs and outputs as amounts per household. The GH model includes  $X$  as a potential input to  $X$ ,  $E$ , and  $F$ .

<sup>6</sup> Production of fuels are capital intensive in GH, but adding capital to fuel production here would complicate our derivations and solutions. A different simplification could combine labor and capital into one composite input, but then we would have to use the labor tax rate from GH on the composite input, or else adjust the tax rate. Our simplified model is not "accurate", but still can obtain results close to those from the GH model.

<sup>7</sup> In contrast, GH have a dynamic model with quasi-fixed capital, giving rise to adjustment costs that imply windfall gains or losses with important distributional implications. GH also obtain short-run impacts that differ from long-run impacts. In these respects, we cannot compare our results to theirs, because (1) our static CD model has no such dynamics, and (2) we have only one type of household and thus no distributional results.

government budget constraint says that the sum of pollution tax revenues and labor tax revenues equals the lump-sum tax rebate<sup>8</sup>:

$$R = \rho\tau_{CE}F_E + \rho\tau_{CO}(F_H + F_X) + \tau_L P_L L \quad (1.6)$$

We can then show how this simple model produces the standard “double dividend” results when we calculate the effects of a carbon tax with revenue used to reduce the labor tax. We will also discuss two different kinds of carbon tax: a uniform tax on all carbon or a tax on carbon only in the electricity sector.

Households choose their consumption of goods and leisure to maximize utility subject to their budget constraint, which requires that their “full” income ( $I$ ) not be exceeded by the sum of expenditures on the composite good, electricity, fossil fuel, and leisure. Their full income includes receipts from fixed factor endowments and the lump-sum tax rebate:

$$I = P_K \bar{K} + P_L \bar{L} + R = P_X X + P_E E_H + (P_F + \rho\tau_{CO})F_H + P_L L_H$$

where  $P_X$  and  $P_E$  are prices faced by consumers. Also,  $P_F$  denotes the net-of-tax price of fossil fuels, so  $P_F + \rho\tau_{CO}$  is the price faced by consumers (gross of tax). The rental price for capital is denoted as  $P_K$  and is untaxed. Only relative prices matter in our general equilibrium model, but instead of fixing one price we anchor the overall price level by assuming that the nominal amount of full income is fixed ( $I = \bar{I}$ ). Then the Cobb-Douglas utility function implies that expenditure on each consumption good is also fixed (as a fraction of that fixed income):

$$P_X X = aI \quad (1.7)$$

$$P_E E_H = bI \quad (1.8)$$

$$(P_F + \rho\tau_{CO})F_H = cI \quad (1.9)$$

$$P_L L_H = (1-a-b-c)I \quad (1.10)$$

Competitive firms are price takers in input and output markets, and they choose input quantities to maximize profits subject to their production functions. The Cobb-Douglas forms yield very simple factor demands. For example, firms in  $X$  always use  $\alpha$  of net sales revenue,  $P_X X$ , to buy units of capital  $K_X$ , each of which cost  $P_K$ . Similarly, for other inputs:

$$P_K K_X = \alpha P_X X \quad (1.11)$$

$$P_L (1 + \tau_L) L_X = \beta P_X X \quad (1.12)$$

$$P_E E_X = \gamma P_X X \quad (1.13)$$

$$(P_F + \rho\tau_{CO})F_X = (1-\alpha-\beta-\gamma)P_X X \quad (1.14)$$

$$P_K K_E = \delta P_E E \quad (1.15)$$

$$P_L (1 + \tau_L) L_E = \epsilon P_E E \quad (1.16)$$

$$(P_F + \rho\tau_{CE})F_E = (1-\delta-\epsilon)P_E E \quad (1.17)$$

$$P_L (1 + \tau_L) L_F = P_F F \quad (1.18)$$

In our Cobb-Douglas economy, the three conditions for general equilibrium are income balance, zero profit, and market clearance. Income balance conditions are the household and government budget constraints above. The zero profit conditions say that the value of output produced by each industry must equal the sum of the values of inputs employed in production [satisfied by Eqs. (1.11)–(1.18)]. Market

clearing implies that the total quantity of each commodity (e.g.,  $F$  or  $E$ ) supplied by firms must equal the total quantity demanded [in Eqs. (1.3) and (1.4) above]. In addition, the quantity of each primary factor used in all sectors must sum to the household's endowment of that factor:

$$\bar{K} = K_X + K_E \quad (1.19)$$

$$\bar{L} = L + L_H = L_X + L_E + L_F + L_H \quad (1.20)$$

All equations above hold in the original equilibrium (indicated by superscript “o”) and in the new equilibrium (with primes). For a change in uniform carbon tax rate from  $\tau_C^o$  to  $\tau_C'$ , we can then compare outcomes such as new emissions  $C'$  to old emissions  $C^o$ .

## 2. Deriving new equilibrium outcomes

This section is designed to demonstrate the simplicity and ease of successive substitution to solve for the new general equilibrium prices and quantities “on the back of an envelope”. We study four of the GH model's scenarios: (1) a uniform carbon tax with lump-sum rebate, (2) a uniform carbon tax with a labor tax cut, (3) a carbon tax on the electricity sector only with lump-sum rebate, and (4) a carbon tax on the electricity sector only with a labor tax cut. In this section, we show the first ten equations that pertain to all of those scenarios, which are enough to demonstrate the method. Then Appendix A shows remaining additional steps, including details that differ for each of the four scenarios.

First, of course, Cobb-Douglas forms yield very simple factor demands. Eqs. (1.7) and (1.11) yield Eq. (2.1), while Eqs. (1.13) and (1.15) yield Eq. (2.2):

$$K'_X = \frac{\alpha P'_X X'}{P'_K} = \frac{\alpha a I}{P'_K} \quad (2.1)$$

$$K'_E = \frac{\delta P'_E E'}{P'_K} = \frac{\delta(b + \gamma a) I}{P'_K} \quad (2.2)$$

Substitute Eqs. (2.1) and (2.2) into the market-clearing condition (1.19) and manipulate to derive an expression for the new factor price of capital:

$$P'_K = \frac{\alpha a + \delta(b + \gamma a) I}{\bar{K}} \quad (2.3)$$

Then substitute Eq. (2.3) back into Eqs. (2.1) and (2.2) to solve for new uses of capital  $K'_X$  and  $K'_E$ .

Since full income includes receipts from the lump-sum tax rebate and fixed factor endowments (labor and capital), that  $P'_K$  in Eq. (2.3) can be used to find the net-of-tax wage as:

$$P'_L = \frac{I - P'_K \bar{K} - R'}{\bar{L}} \quad (2.4)$$

Then use labor demands in Eqs. (1.12), (1.16), and (1.18), plus the market-clearing Eq. (1.20) to obtain labor used in each sector. With  $F' = GL'_F$ , these equations yield:

$$L'_X = \frac{\beta a I}{P'_L (1 + \tau'_L)} \quad (2.5)$$

$$L'_E = \frac{\epsilon(b + \gamma a) I}{P'_L (1 + \tau'_L)} \quad (2.6)$$

$$L'_H = \frac{(1-a-b-c)I}{P'_L} \quad (2.7)$$

<sup>8</sup> In contrast, the government sector in the GH model provides fixed real transfers and services, where services are produced using labor, capital, fuel, electricity, and other inputs.

$$F' = GL'_F = G(\bar{L} - L'_H - L'_X - L'_E) \quad (2.8)$$

The production of fossil fuels Eq. (1.4) and zero profit condition (1.18) yield the first equation for the price of fuel below, and the expression for  $P'_L$  in Eq. (2.4) yields the second:

$$P'_F = \frac{P'_L(1 + \tau'_L)}{G} = \frac{(I - P'_K \bar{K} - R')(1 + \tau'_L)}{\bar{L}G} \quad (2.9)$$

In the case with lump-sum rebate of carbon revenue, we assume no change in labor tax rate ( $\tau'_L = \tau'_L$ ). Given the solution for  $P'_K$  in Eq. (2.3), the only unknown on the right side of Eq. (2.9) is  $R'$ . Our approach is to express all other unknown outcomes as functions of the equilibrium rebate  $R'$ . Then we solve for  $R'$  and substitute back to get all other outcomes. For example, in the uniform carbon tax case ( $\tau'_C = \tau'_{CE} = \tau'_{CO}$ ), we use the government budget Eq. (1.6) to get:

$$R' = \rho \tau'_C (F'_X + F'_E + F'_H) + \tau'_L P'_L (L'_X + L'_E + L'_F),$$

and manipulate to derive the new equilibrium total quantity of fossil fuels:

$$F' = \frac{R' - \tau'_L P'_L (\bar{L} - L'_H)}{\rho \tau'_C} = \frac{R' - \tau'_L (I - P'_K \bar{K} - R') + \tau'_L (1 - a - b - c)I}{\rho \tau'_C} \quad (2.10)$$

We equate Eqs. (2.8) and (2.10) and insert expressions for  $P'_K$  from Eq. (2.3) and  $P'_L$  from Eq. (2.4) to solve for  $R'$ .

In the case with labor tax recycling, an increase in carbon tax revenue is offset by a cut in labor tax rate, with no change in rebate ( $R' = R^0$ ). Our approach in this case is to express all unknown variables as functions of the new labor tax rate  $\tau'_L$ . Then we solve for  $\tau'_L$  and substitute back that expression for  $\tau'_L$  to obtain other new equilibrium outcomes. For example, for the uniform carbon tax case, we equate Eqs. (2.8) and (2.10) to solve for  $\tau'_L$ . Remaining steps are all listed explicitly in Appendix A. Here, we proceed to solve for welfare.

Demand functions for consumption goods and leisure are obtained from expenditure share Eqs. (1.7)–(1.10). Here, we substitute these demands into the utility function Eq. (1.1) to obtain the indirect utility function:

$$V(\text{prices}, I) = \left(\frac{aI}{P_X}\right)^a \left(\frac{bI}{P_E}\right)^b \left(\frac{cI}{P_F(1 + \rho \tau_{CO})}\right)^c \left(\frac{(1 - a - b - c)I}{P_L}\right)^{1 - a - b - c} \\ = \left(\frac{P_X}{a}\right)^a \left(\frac{P_E}{b}\right)^b \left(\frac{P_F(1 + \rho \tau_{CO})}{c}\right)^c \left(\frac{P_L}{1 - a - b - c}\right)^{1 - a - b - c}$$

Then we define  $\bar{P} \equiv \left(\frac{P_X}{a}\right)^a \left(\frac{P_E}{b}\right)^b \left(\frac{P_F(1 + \rho \tau_{CO})}{c}\right)^c \left(\frac{P_L}{1 - a - b - c}\right)^{1 - a - b - c}$  as the “ideal” price index, a function of all prices faced by households.<sup>9</sup> The result is a simple expression for indirect utility,  $U = I/\bar{P}$ . We then invert that function to obtain the expenditure function  $\text{Exp}(U, \bar{P})$ :

$$I = \text{Exp}(U, \bar{P}) = U \times \bar{P}$$

In the case with fixed full income, Mas-Colell et al. (1995, p.82) show that the equivalent variation can be measured as:

$$EV = \text{Exp}(\bar{P}^0, U') - \text{Exp}(\bar{P}', U') = \text{Exp}(\bar{P}^0, U') - \text{Exp}(\bar{P}^0, U^0)$$

<sup>9</sup> Following prior literature, the bars over earlier variables indicate they are fixed ( $\bar{K}$ ,  $\bar{L}$ , and  $\bar{I}$ ). Following other literature, however, the bar over  $\bar{P}$  is used to represent the “ideal” price index. Since all prices can change from the old equilibrium to new equilibrium, the value of this price index can change (from  $\bar{P}^0$  to  $\bar{P}'$ ).

In the Cobb-Douglas case,

$$EV = \bar{P}^0 \times U' - \bar{P}^0 \times U^0 = \bar{P}^0 \times (U' - U^0)$$

This expression has a ready interpretation, since households essentially purchase “utils” at the price  $\bar{P}$  per util. Then the equivalent variation is the change in non-environmental utility ( $U' - U^0$ ) valued at old prices ( $\bar{P}^0$ ), while the compensating variation (CV) is the same change in  $U$  valued at new prices ( $\bar{P}'$ ).

We calibrate this model to the U.S. economy in a way that allows us to study multiple questions listed above. In each case, we compare results of our simple aggregated CD model to results of the large, dynamic, disaggregated CGE model in Goulder and Hafstead (2018). To see how results depend on model structure per se, we compare results of the two models using the same data and parameters. We use the data and calibration from GH, adjusted to fit our more aggregated and simpler model. We also supplement their data with GDP by industry from the US Bureau of Economic Analysis and primary energy use by source from the US Energy Information Administration. Table 1 summarizes data and central parameters for our CD model, including expenditure shares ( $a$ ,  $b$ , and  $c$ ) and factor shares ( $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , and  $\varepsilon$ ). It also shows the central value of other parameters that we vary in sensitivity analysis, such as the initial fraction of available household time spent at work ( $h=0.667$ ) and the ratio of labor to capital in production of electricity ( $k=0.208$ ). All details of this calibration appear in Appendix B. We undertake the same kinds of simulations as in GH, but using the CD model, and we show how to solve for new post-reform general equilibrium prices and quantities.

### 3. Derive and interpret numerical results

We study the effects of a large change in the carbon tax on prices, inputs, outputs, and welfare.<sup>10</sup> In GH's central case, the carbon tax is raised from zero to \$6.67 per metric ton in 2017, rises to \$20 in 2019, and then increases by 4% every year until it is capped at \$60 in 2048, 2049, and 2050. This section illustrates how to obtain simple numerical results, so we first calculate effects of introducing an arbitrary round-number tax of \$15 per metric ton on all sectors' CO<sub>2</sub> emissions while using additional revenue to cut the labor tax rate. Table 2 summarizes key outcomes. The first two columns list variables and their definitions. The third column shows benchmark equilibrium values with no carbon tax (from our calibration, which matches GH data). Column 4 presents the new equilibrium outcomes in the CD model after implementing the carbon tax with labor tax recycling, which we solve using all equations above. Column 5 of Table 2 shows percentage changes for key variables.

#### 3.1. Impacts on outputs, inputs, and prices

The carbon tax with revenue recycling implies no change in tax revenue, so a higher emissions tax rate must be accompanied by a reduced labor tax rate. With fixed nominal income  $I$ , the net-of-tax wage and capital price are unchanged (in the bottom two rows of Table 2), but the cut in labor tax reduces the gross-of-tax wage cost (by 1.05%, not shown). Because of our simplifying assumption that the production of fossil fuels uses only labor in production, a tax on CO<sub>2</sub> is equivalently a tax on fossil fuels or on labor used in production of fossil fuels. Table 2 shows that both  $F$  and  $L_F$  fall by the same percentage (9.69%).

The first three rows of Table 2 show the impact of the carbon tax on outputs, where fossil fuels is the output most affected. Though the table shows that the net-of-tax price of  $F$  received by producers falls by −1.05%, the gross-of-tax cost of fossil fuels increases 11% (from \$1/unit to \$1.11/unit, not shown in Table 2). The production of electricity is much more fossil-fuel intensive than the production of  $X$  (the

<sup>10</sup> The model framework can be used to study other environmental policies, but we focus on carbon pricing.



**Table 1**

Data and parameters for the Cobb-Douglas model.

Panel A: Data for inputs and outputs (in \$B, also quantities since initial prices are 1.0)					
Inputs	Production Sectors			Household	Row sum
	X	E	F	Sector	
K	$K_X^0=7350.86$	$K_E^0=226.841$	–	–	$\bar{K}=7577.70$
L	$L_X^0=3827.38$	$L_E^0=47.187$	$L_F^0=425.706$	$L_H^0=2215.29$	$\bar{L}=6515.56$
E	$E_X^0=377.600$	–	–	$E_H^0=169.100$	$E^0=546.700$
F	$F_X^0=352.475$	$F_E^0=244.245$	–	$F_H^0=85.4360$	$F^0=682.157$
X	–	–	–	$X^0=14,214.0$	–
$\tau_L$	$\tau_L^0 L_X^0=2305.65$	$\tau_L^0 L_E^0=28.4262$	$\tau_L^0 L_F^0=256.450$	–	$R^0=2590.53$
Column Sum	$X^0=14,214.0$	$E^0=546.700$	$F^0=682.157$	$I=16,683.8$	–
Panel B: Share parameters (the corresponding entry in Panel A, divided by the column sum)					
Inputs	Production sectors			Household	
	X	E	F	Sector	
K	$\alpha=0.517$	$\delta=0.415$	–	–	–
L	$\beta=0.431$	$\epsilon=0.138$	1.00	–	$(1-a-b-c)=0.133$
E	$\gamma=0.027$	–	–	–	$b=0.010$
F	$(1-\alpha-\beta-\gamma)=0.025$	$(1-\delta-\epsilon)=0.447$	–	–	$c=0.005$
X	–	–	–	–	$a=0.852$
Column Sum	1.00	1.00	1.00	–	1.00
TFP	$A=2.990$	$B=2.897$	$G=1.602$	–	–
Panel C: Other central case parameters, as described in Appendix B					
Parameter	Definition			Value	
$\theta$	Marginal effect on welfare from emissions			26.657	
$\rho$	Emission conversion rate (ton CO <sub>2</sub> per dollar of fossil fuels)			0.00786	
$\tau_L$	Labor tax as a fraction of net wages			0.602	
$h$	Fraction of household time spent at work			0.667	
$k$	Ratio of labor to capital in electricity			0.208	

The numbers in Panel A are all provided with at least six digits (regardless of how many digits are left or right of the decimal point). Also, in Panel A, the value of national income is not the sum of the last column, because some of  $F$  and  $E$  are inputs to  $X$ . Instead,  $I = 16,683.8 = \bar{L}P_L + \bar{K}P_K + R$ . Panel B shows parameters specific to production and utility. Panel C shows other parameters described in the text (and calibrated in Appendix B).

**Table 2**

Summary of key values and their changes.

(for a carbon tax with revenue used to cut the labor tax rate).

(1)	(2)	(3)	(4)	(5)
Variable	Definition	Value at $\tau_c=0$	Value at $\tau_c=15$	Change (%)
Panel A	Quantities			
X	Output of X	14,213.964	14,223.170	0.065
E	Output of E	546.700	523.124	−4.312
F	Output of F	682.157	616.041	−9.692
$K_X$	Capital in Production of X	7350.859	7350.859	0.000
$K_E$	Capital in Production of E	226.841	226.841	0.000
K	Fixed Total Capital Supply	7577.700	7577.700	0.000
$L_X$	Labor in Production of X	3827.375	3868.133	1.065
$L_E$	Labor in Production of E	47.187	47.690	1.065
$L_F$	Labor in Production of F	425.706	384.446	−9.692
L	Market Labor Supply	4300.269	4300.269	0.000
$L_H$	Labor at Home	2215.290	2215.290	0.000
$\bar{L}$	Fixed Total Labor Endowment	6515.559	6515.559	0.000
$E_X$	Electricity in Production of X	377.600	361.317	−4.312
$E_H$	Electricity Consumption at Home	169.100	161.808	−4.312
$F_X$	Fossil Fuel in Production of X	352.475	318.313	−9.692
$F_E$	Fossil Fuel in Production of E	244.245	220.573	−9.692
$F_H$	Fossil Fuel Consumption at Home	85.436	77.155	−9.692
R	Tax Revenue	2590.531	2590.531	0.000
Panel B	Prices			
$\tau_L$	Labor Tax Rate	0.602	0.586	−2.803
$P_X$	Consumer Price of X (no tax on X)	1.000	0.999	−0.065
$P_E$	Consumer Price of E (no tax on E)	1.000	1.045	4.507
$P_F$	Net-of-Tax Price of F	1.000	0.989	−1.054
$P_K$	Factor Price of Capital (no tax on K)	1.000	1.000	0.000
$P_L$	Net-of-Tax Wage	1.000	1.000	0.000

composite good).<sup>11</sup> Thus, due to the rise in the fuel input price, the consumer price of  $E$  rises by 4.5%, and output of  $E$  falls by 4.3%. In contrast, the consumer price of  $X$  falls by only 0.065% (and output of  $X$  rises by 0.065%).

The next nine rows of Table 2 show impacts of this reform on uses of primary factors in each sector. No change is reported in the use of capital, since the factor price of capital is unchanged (and our static model considers a fixed supply of factor endowments). The 9.69% reduction in labor used by sector  $F$  allows for labor use in larger sectors  $X$  and  $E$  to rise by 1.07%. Total labor supply does not change, so labor use at home ( $L_H$ ) is also unaffected.<sup>12</sup>

Cobb-Douglas utility and production functions ensure that the values of fossil fuels and electricity used in production and at home are fixed proportions of fixed nominal income. So changes in fossil fuels and electricity are proportional to changes in their prices. Also, since the carbon tax is levied on all sectors, they all face the same gross-of-tax price of fossil fuels ( $P_F + \rho\tau_C$ ). Thus, the percentage change in fossil fuels used in each sector is the same as the percentage change in total fossil fuels  $F$ . Similarly, the percentage change in electricity deployed in each sector is the same as the percentage change in total electricity  $E$ .

### 3.2. Impacts on welfare

In our numerical results, the EV is -\$6.692 billion from this tax-swap experiment – the introduction of a \$15 tax per metric ton of CO<sub>2</sub> with revenue used to cut the labor tax rate from 60.2% to 58.6% of the net wage. Since the reform reduces welfare, this EV is negative, so we reverse its sign to represent the positive cost of the policy. The EV shows that the higher CO<sub>2</sub> tax has a negative effect on welfare that exceeds the positive effect of reducing the wage tax. Thus, in our numerical model, the revenue-neutral substitution of the pollution tax for an existing distortionary labor tax has a positive gross cost.<sup>13</sup>

How big is this welfare loss? We divide total welfare loss from the policy by total CO<sub>2</sub> abated to obtain the average abatement cost (AAC). In our example, the tax-swap experiment reduces 0.520 billion metric tons of carbon dioxide emissions compared to the benchmark, so the AAC is \$13.40 per ton ( $=0.962/0.520$ ). This AAC means that on average, and before considering the environment-related benefit, the tax-swap reform costs \$13.40 to reduce each metric ton of CO<sub>2</sub> emissions. Below, we show marginal abatement cost (MAC) curves.

By including the amount of carbon dioxide emissions in the welfare function ( $W = U - \theta nC$ ), we are also able to calculate environmental benefits and thus net benefits of the reform. The environmental benefit of the tax reform is total emission reduction (0.52 billion tons) times the social cost of carbon (SCC). GH use the U.S. government's estimate of  $SCC = \$43$  per metric ton of CO<sub>2</sub>.<sup>14</sup> Thus, after accounting for the environment-related benefit ( $0.520\text{billion} \times 43 = \$22.36$ ), the reform offers a net welfare gain of \$15.40.

<sup>11</sup> Panel B of Table 1 shows that the cost share for fossil fuels in the production of the composite good (0.023) is much less than its cost share in the production of electricity (0.407).

<sup>12</sup> Cobb-Douglas utility means that households spend a fixed fraction of fixed nominal income on each good (e.g.,  $P_L L_H = (1 - a - b - c)I$ ). The unchanged net-of-tax wage  $P_L$  then implies unchanged  $L_H$ . Yet this result is consistent with numerical findings in GH, given their choice of elasticities (see GH's Table 5.7, p.106).

<sup>13</sup> Thus, these numerical results do not support the "strong double dividend" claim, defined by Goulder (1995) as the assertion that the environmental tax not only improves the environment but also increases non-environmental welfare through revenue recycling. GH study welfare costs of a carbon tax under various recycling methods such as cuts in payroll tax, individual income tax, or corporate tax. They find evidence of a strong double dividend only with low abatement amounts and with corporate income tax cuts.

<sup>14</sup> Page 318 of GH cites this study, which is available at [https://www.epa.gov/sites/production/files/2016-12/documents/sc\\_co2\\_tsd\\_august\\_2016.pdf](https://www.epa.gov/sites/production/files/2016-12/documents/sc_co2_tsd_august_2016.pdf).

### 4. Comparisons to results from the large CGE model

As shown in previous sections, our simplified Cobb-Douglas GE model can be solved by readers using a paper and pencil on the back of an envelope. That is an advantage relative to large CGE models whose sheer size and complexity can make them impenetrable for many readers who do not build their own CGE models. Thus, the simple Cobb-Douglas GE model with its great accessibility allows readers to explore and understand exactly and explicitly what features of the model drive key results. And especially, it assists new researchers who seek to build their first own CGE models to decide what features to include in their models.

In this section, we show that the simple model generates key aggregate results that are similar to those of a complicated CGE model, but where derivations are more transparent. In particular, we compare results from our simple aggregated Cobb-Douglas GE model to results from the dynamic disaggregated CGE model of Goulder and Hafstead (2018). The book by GH includes not only careful modeling of the carbon intensity of 35 different production sectors, but also of intertemporal dynamics and of all U.S. taxes on labor, on capital, and on the incomes of different household income groups. For our purpose and length restriction in this paper, we do not undertake every simulation in GH. Instead, we analyze effects of a carbon tax with lump-sum rebates and with revenue used to reduce employee payroll taxes, which enables direct comparison of our results to their results.<sup>15</sup>

On the one hand, our model maintains several key features of the complex CGE model. First, it satisfies a set of standard competitive general equilibrium requirements such as utility maximization, zero economic profits, Walras Law, and market clearing conditions. Second, though we aggregate the 35 industries from GH into only a few sectors, we retain the key fossil-fuel sector and the electricity sector (grouping all other sectors into one sector  $X$ ). Third, we utilize data from GH in order to match closely their initial labor tax rate, their factor shares by sector, and their total expenditure share for each carbon-intensive product.

On the other hand, we note several key differences between our model and their model. First, our Cobb-Douglas model implies unitary elasticities of substitution in both consumption and production, while GH's elasticities of substitutions are constant but can be different from one.<sup>16</sup> Second, the Cobb-Douglas model implies that the uncompensated leisure demand elasticity is  $-1$ , which differs from the GH model. Also, the uncompensated labor supply elasticity depends on a parameter  $h$ , defined as the fraction of total time spent at work. If we use the GH assumption that workers have 60 h per week available and spend 40 h at work, then  $h=0.667$  and our uncompensated labor supply elasticity is  $1 \times 0.333/0.667 = 0.50$ . In contrast, the GH uncompensated elasticity of labor supply is 0.05. Third, for simplicity, our model assumes away all other taxes on capital and goods, leaving only a preexisting tax on labor.<sup>17</sup> GH include treatments of various *ad-valorem* sales and exercise taxes, plus tax deductions and tax credits.<sup>18</sup> Fourth, as mentioned above, our model is a static GE model, while GH is a dynamic model with growth. Fifth, the GH model can consider distributional impacts across industries or household groups, but our model cannot. GH also include import and export activities in their model, while our modeled economy is closed. Therefore, we certainly expect our results to differ to some degree from those of GH.

Goulder and Hafstead in their book study different carbon tax time profiles from 2016 to 2050. In their central case, the carbon tax is raised

<sup>15</sup> GH consider four revenue-recycling methods of a carbon tax revenue, such as: lump-sum rebates, cuts in employee payroll taxes, cuts in individual income taxes, and cuts in corporate income taxes.

<sup>16</sup> Most elasticities of substitution range from 0.5 to 1.10, see Table 4.3 on Page 64 of GH.

<sup>17</sup> Our Cobb-Douglas model can easily accommodate other taxes on goods and capital, but we want to keep our model as simple as possible while still sufficient for the pedagogical purpose as well as for our main point that a simple Cobb-Douglas GE model can yield similar results to a complicated CGE model.

<sup>18</sup> See Tables 4.1 and 4.4 in GH for a list of tax features they consider in their model.

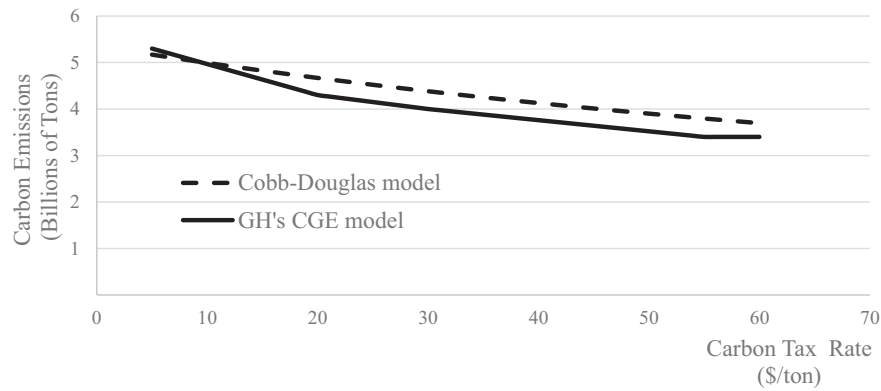


Fig. 1. Impacts on carbon dioxide emissions. (Uniform carbon tax with lump-sum rebates).

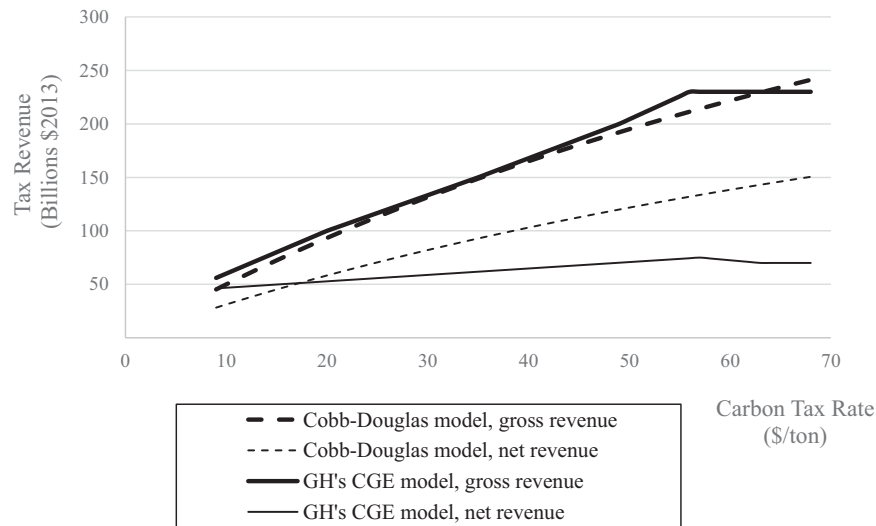


Fig. 2. Revenue from a uniform carbon tax (with lump-sum rebates).

from zero to \$6.67 in 2017, followed by \$13.33 in 2018, and \$20 in 2019. Then it increases 4% every year after 2020 until it is capped at \$60 in 2048, 2049, and 2050. Our model is static, but we vary our tax rates from \$5 to \$60 for alternative calculations where each tax rate leads to a new static equilibrium. Then we calculate outputs, inputs, prices, emissions, and welfare at each tax rate. We plot those equilibrium outcomes against the carbon tax rate, and then we compare our plots with analogous plots from GH.

First, we compare the resulting impacts on economy-wide carbon emissions from our CD model and their CGE model. Since GH only shows the time profile for CO<sub>2</sub> in the case of a carbon tax with lump-sum rebates, we present a similar plot in Fig. 1, which shows tax rates in ascending order from left to right on the horizontal axis and corresponding levels of emissions on the vertical axis. The dashed line shows results from our CD model, and the solid line represents GH's results.<sup>19</sup> The figure shows that a more stringent carbon tax policy results in a lower level of emissions. The solid line for the GH results and the dashed line for the CD results are very close to each other. Emissions fall by more than one-third from 5.3 billion metric tons in the initial equilibrium with no carbon tax to <3.5 billion metric tons by the year

2050, both in the GH model and in our Cobb-Douglas model when the carbon tax reaches \$60 per metric ton (the 2050 tax rate in GH).

Second, also only for the case with lump-sum rebate, GH's Fig. 5.4 on page 92 shows the time profile of carbon tax revenues (for years with rising tax rates). Similarly, we plot gross and net carbon tax revenues from our CD model against carbon tax rates in Fig. 2. Gross carbon tax revenue is the product of the carbon tax rate and total CO<sub>2</sub> emissions. We also calculate the GH definition of net carbon tax revenue, which equals the gross carbon tax revenue minus the "tax-base effect". In GH, the tax-base effect refers to impacts of the carbon tax policy on the base of other taxes and thus on revenues generated by those taxes. In our CD model, the labor tax is the only other tax, so the tax-base effect is measured here by changes in labor tax revenue due to the imposition of the carbon tax. The solid lines show results from the GH model, and dashed lines show results from the CD model. Heavier lines present gross revenues, while lighter lines are net revenues. For example, the carbon tax rate in 2019 is \$20 per metric ton, with gross revenue of about \$B100 in the GH model (about the same as for a rate of \$20 in the CD model). Gross revenue reaches about \$B230 by 2050 in GH with their tax rate of \$60/ton, or at the tax rate of \$60 in the CD model.

Fig. 2 shows that gross tax revenues from our CD model and the GH model follow each other closely as carbon tax rates rise. Accounting for the tax base effect in both models, impacts on net carbon tax revenue are smaller and flatter than gross revenue. Net carbon tax revenue is higher and steeper in our CD model than in the GH model, because our model

<sup>19</sup> In a similar figure, GH show emissions on the vertical axis and year on the horizontal axis, for both their reference case with no carbon tax and their central case with a carbon tax and lump-sum rebates (their Fig. 5.5 on p. 90). In Fig. 1 here, we replace their horizontal time axis with each year's corresponding tax rate.

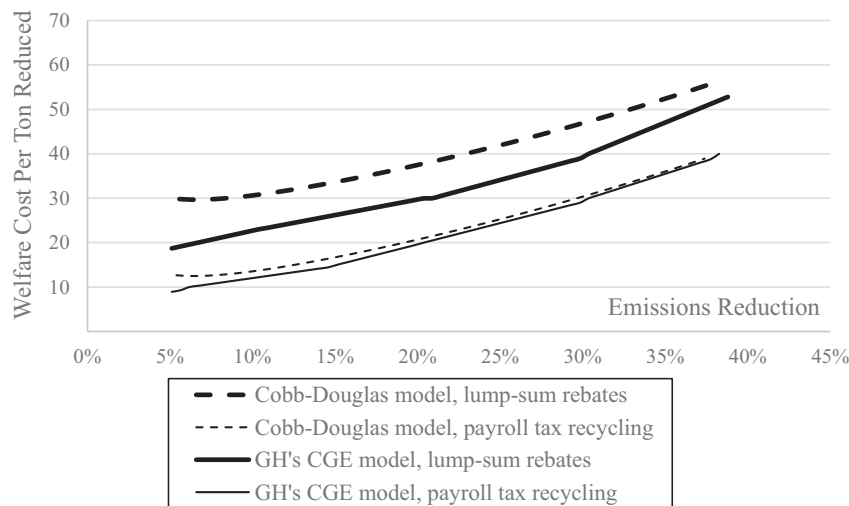


Fig. 3. Welfare costs of a uniform carbon tax under alternative recycling methods.

considers only the preexisting labor tax, while GH has other preexisting taxes. Thus, our tax-base effect is less than their tax-base effect.<sup>20</sup>

Third, GH provides thorough welfare analysis of carbon tax policy under different recycling methods. They calculate various measures of the policy costs such as percent GDP reduction, welfare costs per ton of CO<sub>2</sub> reduced, or welfare costs per dollar of gross revenue. In particular, their Fig. 5.6 graphs welfare cost per ton of emissions reduced against percent emission reductions, so our Fig. 3 plots their results as well as our calculated welfare cost per ton of emissions reduced (average abatement cost, ignoring environmental benefits). Again, solid lines show GH results, while dashed lines are CD results. Heavier lines show the case of lump-sum rebates, and lighter lines show the labor tax recycling scenario.

Our welfare costs per ton of abatement follow the GH results closely. It is especially close for the labor tax-recycling scenario, because the labor tax cut raises the real net wage by almost exactly the same amount that the carbon tax reduces it. And, for this reason, both our model and the GH model find no or little change in total labor supply. Thus, the difference in labor supply elasticities between our CD model and the GH model does not affect that welfare cost calculation. But, in the lump-sum rebate scenario, the real net wage falls, and so total labor supply falls – in a way that does depend on the elasticity of labor supply. More elastic labor supply exacerbates the labor distortion via the tax interaction effect. Our CD model has a more elasticity labor supply than in the GH model, as discussed above, so in this lump-sum case our welfare cost is larger than that of the GH model. Also, Fig. 3 shows that carbon pricing with lump-sum rebates is a costlier policy than a carbon tax with labor tax revenue recycling. Finally, welfare costs increase with emissions reductions in both models.

We can also compare the CD and GH model results on some other outcomes such as prices, outputs, and labor use. Tables 5.5–5.7 in the GH book show detailed impacts of the uniform carbon tax on these outcomes for each of their disaggregated industries. We cannot match details in GH, but Table 3 shows our CD model's calculated policy impacts of both the uniform carbon tax and power-sector-only carbon tax, also with either lump-sum rebates or labor tax recycling (at our CD model's aggregated industry levels).

GH tables report changes in these outcomes from reference case values for years 2020 and 2035 (their carbon tax rate in 2020 is \$20.8/ton and in 2035 is \$37.5). Table 3 shows our calculated policy impacts at carbon tax rates of \$20 and \$40. In general, our Cobb-Douglas results agree with GH's results that producer prices rise, while all outputs and

labor demand falls. From either Table 3 here or Tables 5.5–5.7 in GH, fossil-fuel output is the most affected, and it falls the most in percentage terms. Labor demand in the production of *F* also falls the most in both models. The percent reduction in total labor supply is a bit larger in the CD model, partially due to our assumption that labor is the only input to the production of fossil fuels. Therefore the reduction in *L<sub>F</sub>* has more weight in determining the overall reduction of the equilibrium quantity of labor in our model than in theirs. Another influence is that our labor supply is more elastic than theirs. Prices received by fossil fuel producers fall in both models, while prices paid for fossil fuels rise – so other producer prices rise.

Table 3

Impacts on output, producer prices, and labor demand.  
(Percentage changes from initial equilibrium values).

Variable	Definition	Lump-sum rebate		Labor tax cut	
		(1)	(2)	(3)	(4)
		Value at $\tau_c=20$	Value at $\tau_c=40$	Value at $\tau_c=20$	Value at $\tau_c=40$
Panel A: Overall economy carbon tax					
$X$	Output of $X$	−0.142	−0.302	0.079	0.090
$E$	Output of $E$	−5.871	−10.818	−6.137	−10.457
$F$	Output of $F$	−12.908	−22.987	−15.271	−22.492
$P_X$	Consumer Price of $X$ (no tax)	0.142	0.303	−0.079	−0.090
$P_E$	Consumer Price of $E$ (no tax)	6.237	12.130	5.489	11.678
$P_F$	Net-of-Tax Price of $F$	−0.894	−1.581	−1.387	−2.412
$P_L$	Net-of-tax wage	−0.894	−1.581	0.000	0.000
$L_X$	Labor in Production of $X$	0.902	1.607	1.353	2.471
$L_E$	Labor in Production of $E$	0.902	1.607	1.353	2.471
$L_F$	Labor in Production of $F$	−12.908	−22.987	−15.271	−22.492
$L$	Market Labor Supply	−0.465	−0.828	0.000	0.000
Panel B: Power-sector-only carbon tax					
$X$	Output of $X$	−0.024	−0.031	−0.116	0.046
$E$	Output of $E$	−6.156	−6.850	−6.075	−6.784
$F$	Output of $F$	−4.572	−4.704	−4.421	−4.563
$P_X$	Consumer Price of $X$ (no tax)	0.024	0.031	0.116	−0.046
$P_E$	Consumer Price of $E$ (no tax)	6.560	7.354	6.467	7.278
$P_F$	Net-of-Tax Price of $F$	−0.319	−0.315	−0.483	−0.480
$P_L$	Net-of-Tax Wage	−0.319	−0.315	0.000	0.000
$L_X$	Labor in Production of $X$	0.320	0.316	0.486	0.482
$L_E$	Labor in Production of $E$	0.320	0.316	0.486	0.482
$L_F$	Labor in Production of $F$	−4.572	−4.704	−4.421	−4.563
$L$	Market Labor Supply	−0.165	−0.164	0.000	0.000

<sup>20</sup> The GH model also includes inflation-indexed transfers and government spending, so their tax base effect also accounts for increases in spending necessary to hold fixed the real the production of government services.



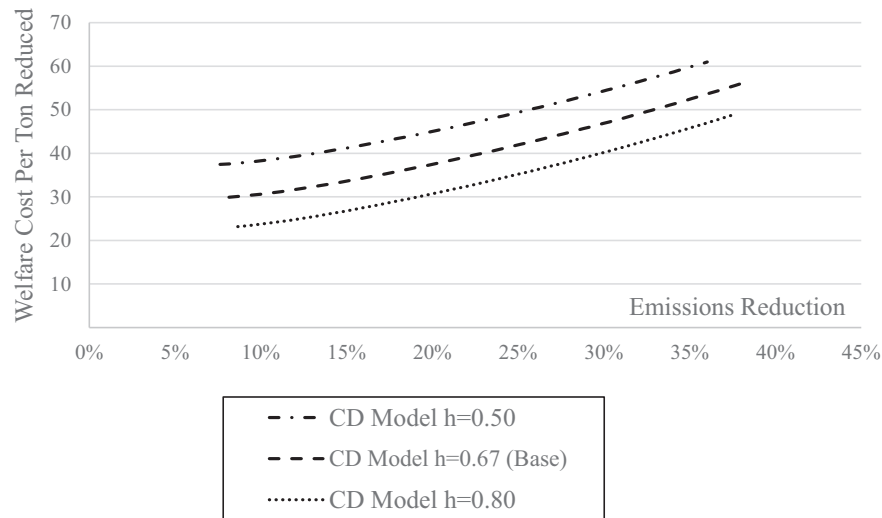


Fig. 4. Welfare cost of a uniform carbon tax with lump-sum rebates, effects of varying  $h$  (the fraction of time spent at work).

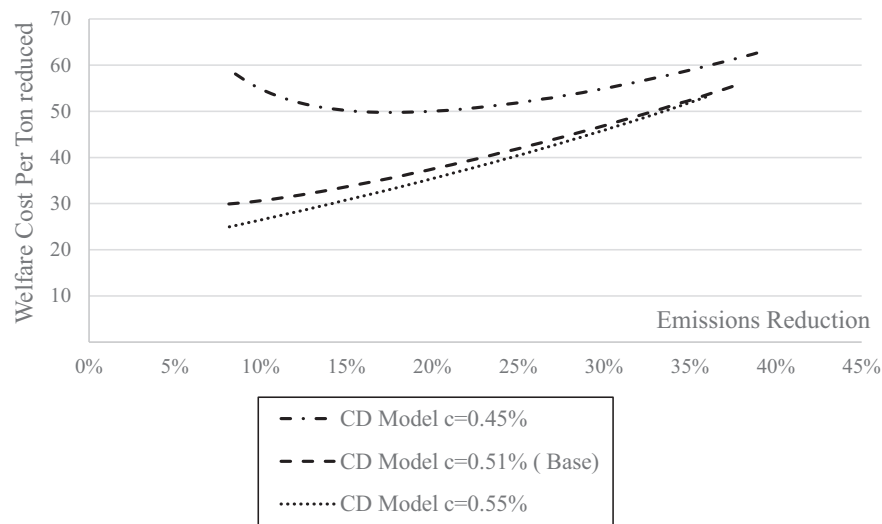


Fig. 5. Welfare cost of a uniform carbon tax with lump-sum rebates, effects of varying  $c$  (the expenditure share on fossil fuels).

So, what are the key elements that drive the similarity of outcomes from our CD model and the GH model? Our equations above show that all outcomes depend on initial values of factor shares, expenditure shares, and elasticities. But how much? We now turn to consider which of these parameter choices substantially affects outcomes in the CD model, in order to see where our use of the GH data and parameters drives the similarity of results. We do not have space to vary all of the model's parameters, but we consider three that are most likely to matter for our results. In twelve sets of simulations, we calculate how these three parameters affect four outcomes: total carbon emissions, net carbon tax revenues, welfare costs with lump-sum rebates, and welfare costs with labor tax recycling. First, we vary the ratio of labor to capital in production of electricity ( $k = L_E/K_E$ ), and we find it has virtually no effect whatever on any of these four outcomes (and so we omit figures showing that lack of sensitivity). The reason is that the relative wage/rental ratio changes very little in the CD model, so the labor/capital ratio has little effect on prices, incentives, or outputs.<sup>21</sup>

Second, we vary the fraction of time spent at work ( $h$ ), and finding it also has virtually no effect on carbon emissions, net carbon revenues, or welfare costs with labor tax recycling. It has some effect on welfare costs with lump-sum rebates, however, as shown in Fig. 4. In this figure, three lines represent three alternative values of  $h$  in the CD model. We do not match the GH leisure demand elasticity, because of our Cobb-Douglas assumption, so the same  $h$  in both models leads to different labor supply elasticities. Because a change in  $h$  alters the labor supply elasticity, Fig. 4 shows how that elasticity affects the CD model's welfare results for the lump sum rebate. The elasticity does not matter with labor tax cuts to recycle the revenue, because the real wage does not change. Since neither  $k$  nor  $h$  affects emissions or revenues, a possible conclusion here is that elasticities matter more than share parameters.

Third, we vary the share of household expenditure on fossil fuels ( $c$ ), by arbitrarily changing their spending on fossil fuels. Also, shares must add to one, so a change in this share ( $c$ ) requires changing the other shares ( $a$  and  $b$ ). We find that changes in expenditure shares have virtually no effect on total carbon emissions or net carbon tax revenues. For welfare costs, however, Fig. 5 shows that the three lines diverge, meaning that welfare costs are sensitive to the expenditure share on fossil fuels  $c$  (as well as other shares  $a$  and  $b$ ). Here, our use of the GH data has helped to drive the similarity of results.

<sup>21</sup> As stated above, fixed nominal income  $I$  means that the net-of-tax wage and capital price are unchanged (in Table 2), but the cut in labor tax reduces the gross-of-tax wage cost (by 1.05%).

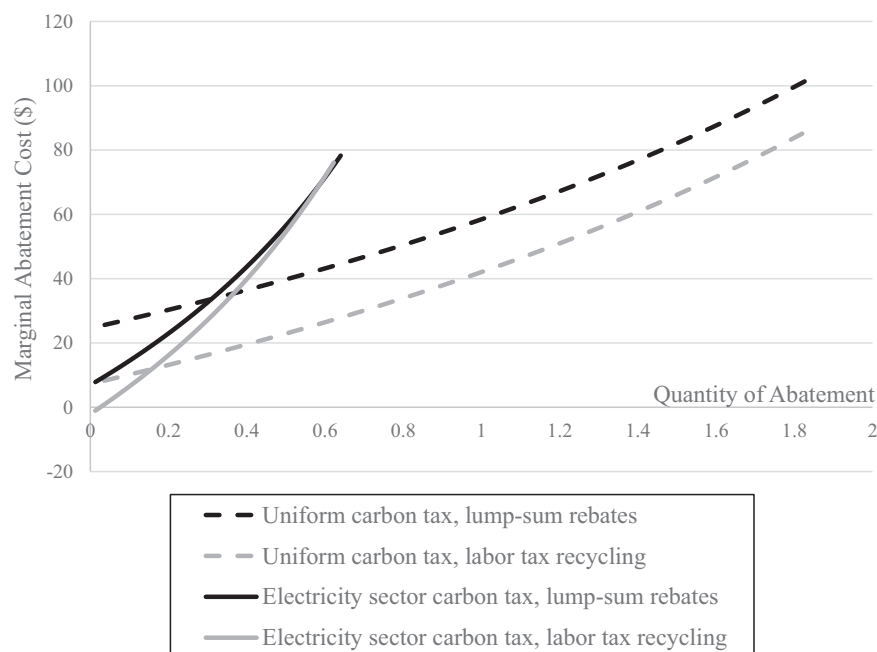


Fig. 6. Marginal abatement cost curve (Cobb-Douglas model only).

Next, we discuss why other key differences between the two models may not lead to very different results. One restriction of our CD model is unitary elasticities of substitution in both production and utility, so we are unable to match our elasticities to those in GH. As chosen by GH, however, elasticities of substitution in production are not very far from one. Moreover, our aggregation tends to reduce the differences in substitution elasticities among the 35 industries in the GH model. For example, if any two industries in GH have different elasticities above one and below one, then their aggregation leads to a weighted-average elasticity closer to one. Thus, our CD model would *not* match results of a CGE model that employs substitution elasticities all substantially less than one (or all greater than one).

Another key difference is dynamic modeling. GH assume a growth rate of 1.5% in the steady state, but adjustment costs affect factor allocations and growth during any transition. We cannot investigate effects of this difference using our one-period CD model with perfect mobility, but two points are relevant. First, while we retain a fossil fuel sector and electricity sector, our aggregation of other industries into only one sector effectively removes any costs of adjustment between those other industries. Second, we avoid transition issues by comparing our CD model results to their medium-run results.<sup>22</sup>

## 5. Other results

The CD model is easily used to calculate other effects of a carbon tax that are not studied or fully reported in the book by Goulder and Hafstead (2018).

### 5.1. Marginal abatement costs

Another highly useful measure of the cost of a CO<sub>2</sub>-emission policy is its marginal abatement cost (MAC), defined as the cost at each level of emission-reduction of eliminating one more metric ton of CO<sub>2</sub> emissions. Then the total cost of achieving a particular level of abatement is the integral under the MAC curve to that point. Policymakers can

compare these costs across different policies to find the most cost-effective means of reducing emissions.

Therefore, we consider many small increases in the CO<sub>2</sub> tax, in increments of \$1/ton. For example, consider the change from \$15 to \$16 per ton. As shown in previous sections, we obtain a set of outcomes at each CO<sub>2</sub> tax rate. The extra cost of abatement is the new welfare cost at \$16/ton minus the prior welfare cost at \$15/ton, equal to \$0.579 billion. The extra CO<sub>2</sub> abated is 0.036 billion tons. Then the marginal cost of abatement (MAC) is the change in welfare cost per unit change in CO<sub>2</sub> abatement ( $0.579/0.031$ ), equal to \$18.46 per ton. The interpretation is that it costs an extra \$18.46 for a marginal metric ton of CO<sub>2</sub> abatement.

Fig. 6 graphs MAC curves for different carbon tax policies. It shows a carbon tax on all sectors with lump-sum rebates (black dashed line) and with labor tax recycling (grey dashed line), a carbon tax in the power sector only with lump-sum rebates (black solid line) and with labor tax recycling (grey solid line). These MAC curves all slope upward, indicating that costs increase with the quantity of emissions abated. For a given level of abatement, and for either type of carbon tax, the MAC with lump-sum rebates is higher than the MAC with labor tax recycling – because reducing the labor tax distortion reduces welfare cost.

Somewhat surprising in Fig. 6 is that the MAC of the power-sector-only carbon tax is initially less than the MAC of the uniform carbon tax. Similar results are also found in GH for cap-and-trade policies (Fig. 5.13 on page 138 of GH). In a first best world with no pre-existing tax, a uniform carbon tax can never be more costly than a specific-sector tax, because a uniform tax allows use of the lowest cost marginal abatement across sectors (as explained in GH on page 139). With pre-existing taxes, however, the MAC includes firm-level abatement cost plus an interaction effect between the carbon tax and other taxes. Our numerical simulation results in Table 3 show that the power-sector-only carbon tax reduces the real net wage by less than does the uniform carbon tax of the same tax rate. Thus, the labor tax distortion is exacerbated more by the uniform carbon tax than by the other policy.

At a higher level of abatement, the tax interaction advantage of the power-sector-only carbon tax is weaker relative to its disadvantage of higher firm-level abatement cost. Thus in Fig. 6, the MAC curves for a power-sector carbon tax start below those of a uniform tax, but rise steeply for more abatement, then cross and become higher than for a uniform tax.

<sup>22</sup> With no short-run rigidity in the CD model, our results might approximate medium-run GH results. Their short-run results differ because of adjustment costs, and their long-run results differ because of growth.

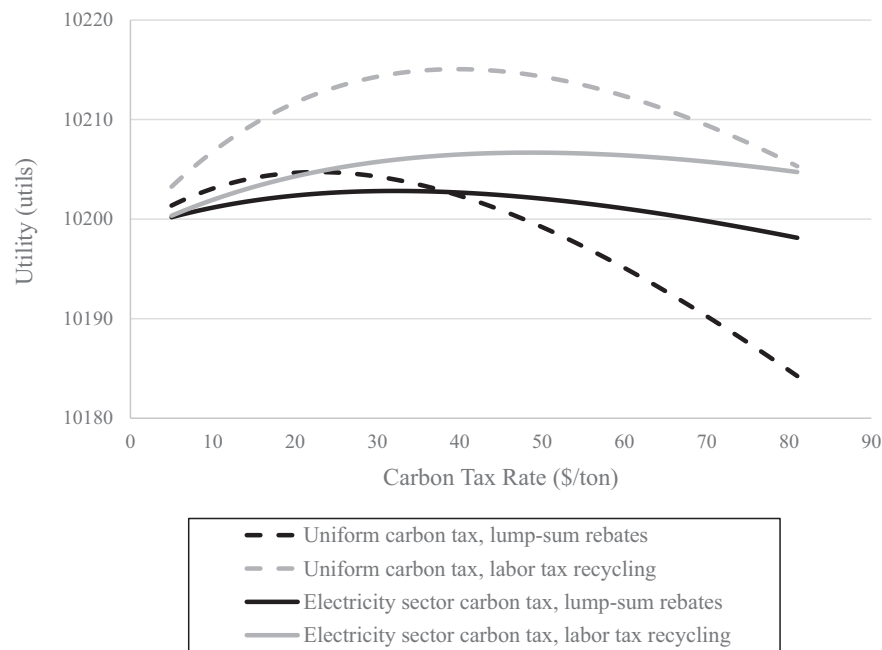


Fig. 7. Second-best optimal carbon tax rates.

### 5.2. Second-best optimal tax rates

In previous sections, we calculate the effects of the pollution-labor tax swap on welfare, pollution, the wage, and labor supply. This section answers a related but different question: given the pollution externality and labor supply distortion, what is the second best optimal (SBO) pollution tax, and how does it compare to the first best optimal (FBO) tax?

In general, the optimal pollution tax induces the level of pollution where the marginal social cost of pollution reduction equals the marginal social benefit of pollution reduction (the marginal social damage from pollution, such as the social cost of carbon, SCC). In the first-best case without distortionary taxes, the optimal pollution tax rate equals the marginal social damage of pollution (the Pigouvian tax level, in Baumol and Oates, 1988, chapter 4). In the second-best case with distortionary taxes, the optimal pollution tax can lie either below or above the marginal social damage, depending on the use of additional pollution tax revenue, the marginal cost of public fund, the magnitude of the revenue-recycling effect, the magnitude of the tax-interaction effect, and the normalization of other tax rates.<sup>23</sup> Our simple numerical Cobb-Douglas model provides an example of those prior findings.

Using the simple Cobb-Douglas model, Fig. 7 plots household utility against the CO<sub>2</sub> tax rate for different carbon tax policies. We obtain hump-shaped curves, where utility rises with the pollution tax rate up to some utility-maximizing tax rate and then falls with further increases in the tax rate. It falls because the adverse distorting effect of increasing the emission tax rate outweighs the benefit of its effect on emissions reduction. The black dashed curve represents a uniform carbon tax scenario with lump-sum rebates. The grey dashed curve represents the case of a uniform carbon tax with labor tax recycling. The solid curves show the case of a carbon tax on the power sector only.

For a uniform tax, the SBO tax rates are below the FBO Pigouvian tax rate of \$43/ton of CO<sub>2</sub> emissions (the SCC). For example, the calculated utility-maximizing SBO uniform tax rate with lump-sum tax rebates is \$22/metric ton of CO<sub>2</sub> emissions – and with labor tax recycling it is \$37/ton. In contrast, the SBO tax rates of a carbon tax on power sector only with lump-sum rebates is \$33, and with labor tax recycling is \$49.

To explain these results, note that the amount of abatement is not shown in Fig. 7e. The SBO uniform carbon tax rates are lower than the power-sector carbon tax rates, because each dollar per ton can achieve more abatement. Also, for each dollar per ton, the uniform carbon tax has a bigger tax interaction effect than the power-sector-only carbon tax. Our numerical results in Table 3 show that each dollar per ton of uniform tax reduces the real net wage much more than does the carbon tax just on the power sector. But, with labor tax recycling, the revenue recycling effect offsets the tax interaction effect, and thus the SBO tax rates in the case with labor tax recycling are always higher than with lump-sum rebates.

### 5.3. Leakage

A concern with any pollution restriction on one sector is that it might cause “leakage”, increases in other sectors’ emissions that offset part of the reduction in pollution-restricted sectors. Goulder and Hafstead do not report leakage results when they discuss the carbon tax (on p.136). They do discuss leakage results for a cap-and-trade program in the power sector only (on p.140). They find leakage of 1.2% to 1.7%, but that page does not report the permit price. In our numerical example, we consider a carbon tax on the power sector only. With lump-sum rebates, we calculate a leakage rate of 4.3%, meaning that additional emissions in other sectors offset 4.3% of emission reductions in the power sector. With labor tax recycling, the leakage rate is 6.6%, which is half again larger than leakage with lump-sum rebates.<sup>24</sup> Accuracy of calculated leakage in the CD model is probably limited because of its high level of aggregation and the absence of a foreign sector. The point of the CD model is not accuracy, however, but understanding the nature of results from large CGE models.

<sup>23</sup> See Bovenberg and de Mooij (1994), Bovenberg and van der Ploeg (1994), Goulder (1995), Bovenberg and Goulder (1996), and Fullerton (1997).

<sup>24</sup> We are unable to decompose total leakage into various channels, as in Baylis et al. (2014). Their paper finds that a pollution limitation on one sector can lead to “negative leakage”, emissions reductions in the other sector under certain conditions. They show that negative leakage is more likely if the elasticity of substitution in the production of electricity is greater than the elasticity of substitution in utility. In our CD model, all elasticities of substitution are unity, which explains why we are unlikely to see a net negative leakage in our CD model.

## 6. Discussion and conclusions

For any proposed policy, political leaders are most often concerned about actual effects on jobs, outputs of each industry, changes in trade deficits, and overall GDP. They have little inclination to discuss conceptual issues in environmental policy analysis, or even to sort out the cause-and-effect relationships. They therefore need the best possible numerical results from the most recent and most detailed CGE model. As of 2018, one of the best and most recent CGE models for climate policy is in the book by Goulder and Hafstead. It is a monumental achievement. But such models can only be produced by the best-trained academics who do understand the conceptual issues and all the cause-and-effect relationships.

For political leaders, our paper does not produce the best possible numbers for the effects of a carbon tax on outputs, GDP, or even emissions. For academics, however, our paper does provide interesting results. It shows very directly, and in simple terms, exactly what are the key ingredients of a CGE model that *can* generate detailed numerical results. In particular, with just three sectors and without the input-output matrix of the GH model, we obtain very similar results on aggregated outcomes such as economy-wide emissions, the welfare cost of carbon taxes, and total labor supply. As long as we identify the fossil fuel sector and the electricity sector, our results are not much affected by the sheer number of disaggregated industries (35 in GH) or the many interactions between them. The main intuitions about climate policy also need not depend on having a dynamic model such as in GH, with capital investments, taxes on capital, or even changes in growth. Our model has none of those, nor international trade.

A key force is the assumed elasticities in production and consumption. More elastic behavior in production or in consumption means that a carbon tax has a larger impact on carbon emissions, output, and welfare. Our Cobb-Douglas model makes solutions for outcomes very simple, but the elasticities all equal to 1.0 are not that different from the elasticities in GH (between 0.5 and 1.0). Also relevant, of course, is carbon-intensity of production, factor shares, and the shares of total expenditure on each carbon-intensive product. We borrow these data directly from GH. A third key feature is whether the carbon tax applies to all carbon in the economy or just to carbon use in one sector. A fourth influence on the effect of carbon policy is the pre-existence of distorting taxes and the use of carbon revenue. When our CD model is built to replicate the large GH model in all these key elements, we show that results are very close to those of the large CGE model.

Our numerical CD model also provides a clean illustration of many concepts and analytical findings in the prior literature on optimal carbon policy and on the impacts of a carbon tax. Our simple CD model has a labor tax as the only pre-existing distortionary tax, for example, so the CD model allows us to study and compare magnitudes of the tax interaction effect between different carbon tax policies and the pre-existing labor tax in a very transparent fashion. As in the analytical chapter and the later simulation chapters of the GH book, we show that the tax interaction effect raises the cost of the carbon tax policy, and its magnitude increases with the carbon tax rate. Revenue recycling effects work in the opposite direction and reduce the cost of the carbon tax policy. Magnitudes of these tax interaction effects and revenue recycling effects help determine second-best optimal carbon tax rates.

In conclusion, our simple Cobb-Douglas model shows readers how to solve a CGE model using a paper and pencil on the “back of an envelope.” Thus, it is a useful tool for teaching general equilibrium to graduate students or for those who have no experience building or solving a CGE model. By comparing numerical results from a simple static CD model to those from a dynamic CGE model, we show how a well-constructed simple CD model can yield several results that match with those from a large-scale CGE model. An advantage of a Cobb-Douglas model is that results are easier to understand using exact analytical solutions for each price, output, welfare loss, and the utility-maximizing pollution tax rate. Therefore, our paper is not just a teaching tool for a

graduate economics class, but it can also be used by new researchers as an entrée to building their own CGE model and by other seasoned researchers to understand CGE results.

## Appendix A. Remaining steps to solve for new equilibrium outcomes

### Uniform carbon tax with lump-sum rebate

The initial equilibrium has no carbon tax ( $\tau_{CE}^0 = \tau_{CO}^0 = \tau_C^0 = 0$ ), but imposition of a uniform carbon tax implies a known  $\tau_{CE}' = \tau_{CO}' = \tau_C' > 0$ . We assume no change in labor tax ( $\tau_L' = \tau_L^0$ ). We use Eqs. (2.8) and (2.10) to get an equation with only one unknown variable,  $R'$ :

$$G\bar{L}\left(1 - \frac{(1 + \tau_L')(1 - a - b - c) + \beta a + \epsilon(b + \gamma a)I}{(I - P_K'\bar{K} - R')(1 + \tau_L')}\right) = \frac{R' - \tau_L'(I - P_K'\bar{K} - R') + \tau_L'(1 - a - b - c)I}{\rho\tau_C'} \quad (A.1)$$

To solve for  $R'$ , first shorten that expression using  $d \equiv (I - P_K'\bar{K} - R')(1 + \tau_L')$ , and obtain:

$$-d^2 + d[I - P_K'\bar{K} + \tau_L'(1 - a - b - c)I - \rho\tau_C'G\bar{L}] + \rho\tau_C'G\bar{L}((1 + \tau_L')(1 - a - b - c) + \beta a + \epsilon(b + \gamma a)I) = 0$$

We solve this quadratic equation to get  $d$  and then  $R'$ .

Then substitute  $R'$  into Eq. (2.4) to get  $P_L'$ , into Eq. (2.9) to get  $P_F'$ , and into Eqs. (2.5), (2.6), and (2.8) to get labor used for each sector. Use Eqs. (1.14), (1.17), and (1.18) to get fossil fuels and thus carbon emissions in each sector. We use the production function for  $E$  to get output  $E$  and price  $P_E$ , and then find  $E_X$ ,  $E_H$ ,  $X$ , and  $P_X$ .

### Uniform carbon tax with labor-tax recycling

In the case with labor tax recycling and no change in rebate ( $R' = R^0$ ), we use Eqs. (2.8) and (2.10) to get an equation with  $\tau_L'$  as the only unknown variable:

$$G\left(\bar{L} - L_H' - \frac{\beta a I}{P_L'(1 + \tau_L')} - \frac{\epsilon(b + \gamma a)I}{P_L'(1 + \tau_L')}\right) = \frac{R' - \tau_L'P_L'(\bar{L} - L_H')}{\rho\tau_C'} \quad (A.2)$$

Through algebraic derivations, we solve this quadratic equation for  $\tau_L'$  using Eq. (A.2) to get:

$$(\bar{L} - L_H')(\tau_L' + 1)^2 + \left(\bar{L} - L_H' - \frac{R'}{G\rho\tau_C'}\right)G\rho\tau_C' - (\bar{L} - L_H') \times (1 + \tau_L') - (\beta a + \epsilon(b + \gamma a)I)G\rho\tau_C' = 0$$

Then substitute  $\tau_L'$  into Eq. (2.9) to get  $P_F'$ , and into Eqs. (2.5), (2.6), and (2.8) to get labor used in each sector. Use Eqs. (1.14), (1.17), and (1.18) to get fossil fuels and carbon emissions in each sector. We use the production function for  $E$  to get output  $E$  and price  $P_E$ , and then find  $E_X$ ,  $E_H$ ,  $X$ , and  $P_X$ .

### Power-sector-only carbon tax with lump-sum rebate

The power-sector-only carbon tax implies a known  $\tau_{CE}' > 0$ ,  $\tau_{CO} = 0$ , and unknown  $R'$ . From Eqs. (1.9), (1.14), and (2.9), we derive fossil fuels used in sectors  $X$  and  $H$ :

$$F_X' = \frac{G(1 - \alpha - \beta - \gamma)aI}{P_L'(1 + \tau_L')} = \frac{G(1 - \alpha - \beta - \gamma)a\bar{L}}{(I - P_K'\bar{K} - R')(1 + \tau_L')} \quad (A.3)$$



$$F'_H = \frac{GcI}{P'_L(1+\tau'_L)} = \frac{Gc\bar{L}}{(I-P'_K\bar{K}-R')(1+\tau'_L)} \quad (\text{A.4})$$

Sum Eqs. (A.3), (A.4), and (1.10) to get total fossil fuels, and then set that equal to the new total fossil fuels in Eq. (2.8) to get:

$$\begin{aligned} G\bar{L} \left( 1 - \frac{(1+\tau'_L)(1-a-b-c) + \beta a + \epsilon(b+\gamma a)}{(I-P'_K\bar{K}-R')(1+\tau'_L)} \right) \\ = \frac{R-\tau'_L(I-P'_K\bar{K}-R') + \tau'_L(1-a-b-c)I}{\rho\tau'_{CE}} \\ + \frac{Gc\bar{L}}{(I-P'_K\bar{K}-R')(1+\tau'_L)} + \frac{G(1-\alpha-\beta-\gamma)a\bar{L}}{(I-P'_K\bar{K}-R')(1+\tau'_L)} \end{aligned} \quad (\text{A.5})$$

Manipulate the above Eq. (A.5):

$$-d^2 + (I-P'_K\bar{K} + \tau'_L(1-a-b-c)I - \rho\tau'_{CE}G\bar{L})d + \rho\tau'_{CE}G\bar{L}(\tau'_L(1-a-b-c) + (\epsilon-1)(b+\gamma a) - \alpha a + 1)I = 0$$

where  $d \equiv (I-P'_K\bar{K}-R')(1+\tau'_L)$ . Solve for  $R'$ , and then substitute  $R'$  successively into other equations, as in the case above for the uniform carbon tax with lump-sum rebate.

#### Power-sector-only carbon tax with labor-tax recycling

This case has a known  $\tau'_{CE} > 0$ ,  $\tau_{CO} = 0$ , and  $R' = R^0$ , but  $\tau'_L$  is unknown. From Eqs. (1.9), (1.14), and (2.9), we derive fossil fuels used in sectors  $X$  and  $H$  as Eqs. (A.3) and (A.4). Then we use the government budget constraint Eq. (1.6) and manipulate to derive the new equilibrium quantity of total fossil fuels used in the power sector:

$$\begin{aligned} R' = \rho\tau'_{CE}F'_E + \tau'_LP'_L(L'_X + L'_E + L'_C) \\ F'_E = \frac{R' - \tau'_LP'_L(\bar{L} - L'_H)}{\rho\tau'_{CE}} \end{aligned} \quad (\text{A.6})$$

Sum Eqs. (A.3), (A.4), and (A.6) to get total fossil fuels, and then equate that total fossil fuels with Eq. (2.8) to get:

$$\begin{aligned} \frac{G(1-\alpha-\beta-\gamma)aI}{P'_L(1+\tau'_L)} + \frac{GcI}{P'_L(1+\tau'_L)} + \frac{R' - \tau'_LP'_L(\bar{L} - L'_H)}{\rho\tau'_{CE}} \\ = G \left( \bar{L} - L'_H - \frac{\beta aI}{P'_L(1+\tau'_L)} - \frac{\epsilon(b+\gamma a)I}{P'_L(1+\tau'_L)} \right) \end{aligned} \quad (\text{A.7})$$

Manipulate the above Eq. (A.7) to get:

$$\begin{aligned} 0 = (1+\tau'_L)^2 \frac{(\bar{L} - L'_H)}{G\rho\tau'_{CE}} + \left( \bar{L} - L'_H - \frac{R' + (\bar{L} - L'_H)}{G\rho\tau'_{CE}} \right) (1+\tau'_L) \\ - (\epsilon(b+\gamma a) + (1-\alpha-\gamma)a + c)I \end{aligned}$$

and then solve for  $\tau'_L$ . To get solutions for all new outcomes, substitute that  $\tau'_L$  successively into other equations, as in the case of a uniform carbon tax with labor-tax recycling.

#### Appendix B. Calibration

We define our electricity sector  $E$  to include four GH industries: electric transmission and distribution, coal-fired electricity generation, other-fossil electricity generation, and non-fossil electricity generation. We aggregate seven industries of GH into our fossil fuel production sector  $F$ , including coal mining, oil and gas extraction, mining support

activities, natural gas distribution, petroleum refining, and pipeline transportation. All other private sector industries are aggregated into  $X$ , our composite good sector.

Panel A of Table 1 shows data for inputs and outputs in the production and household sectors (both dollar values and quantities). We employ the “unit convention,” which defines a unit of each good or primary factor as the amount such that its net-of-tax cost is one dollar in the initial equilibrium:  $P_X^0 = P_E^0 = P_F^0 = P_K^0 = P_L^0 = 1$ . No tax is imposed on capital  $K$  or on goods  $X$ ,  $E$ , and  $F$ . Thus, we infer the initial quantity of each such good from its production value, and the quantity of capital  $K$  from the sum of its values in production of all outputs. Because of the unit convention ( $P_L^0 = 1$ ), labor supply equals total labor compensation divided by  $(1+\tau_L)$ . Then, using data in Panel A of Table 1, we calculate and show share parameters in production and household sectors in Panel B of the same table.

We consider the choice between labor and leisure, using the GH assumption that workers spend  $h=66.7\%$  of their time at work (e.g., 40 h week of 60 h maximum).<sup>25</sup>

We aggregate their various labor income taxes into a single rate. Their calculated labor income tax rate is 26.70%, employer payroll tax rate is 6.23%, and employee payroll tax rate is 6.95%. Thus, our single labor tax rate is 60.24% of the net wage.<sup>26</sup> Total labor compensation is \$6890.8 billion, so  $L=6890.8$  billion. Then we calculate the dollar value of non-market time (leisure at home) as  $P_L^0 L_H^0 = (34/66) \times 6890.80 / (1.6024) = \$52215.29$ , so the quantity of non-market labor is  $L_H^0 = 2215.29$ . Fixed nominal “full” income  $I$  is the sum of this value of leisure plus gross domestic product in the private sector from the U.S. Bureau of Economic Analysis (BEA).<sup>27</sup> We also take household spending on electricity and fossil fuels directly from GH, and then our spending on the composite good  $X$  is the income left after paying for electricity, fossil fuels, and non-market time. By the unit convention, these dollar value expenditures are also quantities (see Household Sector column of Panel A of Table 1).

We also calculate quantities and dollar values of electricity and fossil fuel used in each sector. First, based on our aggregation and the GH data, we calculate the total dollar value of the electricity sector, and  $E^0 = \$546.70$ . Electricity is used either at home or in sector  $X$  in our model, so the dollar value of electricity input in the production of  $X$  is just the dollar value of total electricity,  $E$ , minus the dollar value of electricity used at home,  $E_H$ . These numbers all appear in Panel A of Table 1. Next, we need dollar values of fossil fuels used in sectors  $E$  and  $X$ , which are not readily available from GH, so we use household expenditures on fossil fuels and reported sectoral shares of fossil-fuel use from the U.S. Energy Information Administration (EIA).<sup>28</sup> By our calculation for 2013, the residential sector uses about 12.52% of total fossil fuels, the electricity sector uses 35.80%, and all other sectors use 51.67%. Thus,  $F_E$  is about three times the fossil fuels used at home,  $F_H$  (and  $F_X$  is about four times  $F_H$ ). Total fossil fuel used in the economy is the sum of fossil fuel use in all sectors.

Next, we find labor and capital used in each sector. Capital input data are not reported in GH, and our production functions do not include all GH inputs (capital, labor, energy, and materials). Therefore, we supplement GH data with Bureau of Economic Analysis (BEA) data. The production of electricity in our model uses only three inputs: capital  $K_E$ , labor  $L_E$ , and fossil fuels  $F_E$ . Thus, this sector's total payment to labor and capital must equal the value of electricity minus the cost of fossil fuels calculated above. We use the BEA data to calculate the ratio of all

<sup>25</sup> The relevance and impact of this assumption is demonstrated in Ballard (1999).

<sup>26</sup> Suppose  $w$  is the wage actually paid, so the gross-of-all-tax wage paid for each unit of labor is  $w \times (1 + 0.0623)$ . The net-of-all-tax wage is  $P_L = w \times (1 - 0.0695 - 0.267)$ . Thus, our single labor tax rate as a fraction of the net wage is:  $\tau'_L = (1 + 0.0623) / (1 - 0.0695 - 0.267) - 1 = 0.6024$ . This labor tax rate is 60.24% of the net wage, equivalent to a tax rate of 0.6024 / (1.6024) = 37.59% of the gross wage.

<sup>27</sup> We choose the BEA's 2013 data to be consistent with input-output data from GH.

<sup>28</sup> <https://www.eia.gov/energyexplained/>.

labor costs to capital costs, about one-third, and we use it for our aggregated electricity sector.<sup>29</sup> Then we use this ratio and the labor tax rate (0.6024) to calculate the labor to capital ratio  $k \equiv L_E/K_E = (1/3)/(1 + 0.6024) = 0.208$ . This ratio is used to identify separate the factor quantities in Table 1, where  $L_E^0 = 47.19$ ,  $K_E^0 = 226.84$  (and their ratio is 0.208).

As labor is the only input to production of  $F$ , the payment to labor in  $F$  is the value of  $F$  calculated above. The payment to labor used in  $X$  is total labor compensation minus labor compensation paid in  $E$  and  $F$ . Total payment to capital in  $X$  is the dollar value of  $X$  minus the above-calculated costs of electricity, fossil fuel, and labor inputs. Using our unit convention and the initial labor tax rate, we get quantities of labor and capital used in each sector, as shown in the first two columns of panel A in Table 1.

Next, we use output value for each sector to calculate the household expenditure share for each consumption good, and we use input values in each sector to calculate production cost shares. In the Cobb-Douglas model, those shares are also parameters in utility and production functions. Also, we calculate the TFP parameters in Cobb-Douglas production functions using benchmark equilibrium quantities of outputs and inputs. Given  $X = AK_X^\alpha L_X^{\beta_E} E_X^{1-\alpha-\beta}$ , for example, we solve backwards for the parameter  $A = X/(K_X^\alpha L_X^{\beta_E} E_X^{1-\alpha-\beta})$ .

To analyze welfare impacts of the pollution tax, we follow GH by using \$43 per metric ton of CO<sub>2</sub> as the social cost of carbon (SCC). We take this SCC (in dollars per ton) times the marginal utility of income (in utils per dollar) to get the marginal utility cost of emissions (in utils per ton). This marginal disutility is assumed to be a constant parameter  $\theta$  in household welfare ( $W = U - \theta nC$ ). In the Cobb-Douglas case, with homothetic utility, the marginal utility of income equals the ratio of total utility from consumption of goods and leisure to the full income ( $U/I$ ). This calculation yields  $\theta = 26.66$  (utils per ton).

Finally, to calculate the emission conversion parameter  $\rho$ , divide total CO<sub>2</sub> emissions (5.360 billion metric tons as in GH) by total fossil fuels  $F$  (682.157 billion). Thus,  $\rho = 0.00786$ .

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<sup>29</sup> The BEA data on composition of output by industry only allow us to calculate the labor-capital compensation ratio for “Utilities,” which include our aggregated electricity industry plus natural gas distribution and water utilities. However, output from the electricity industry is the major part of all utilities.