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Optimization of Cobb-Douglas production functions

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Abstract. The purpose of this research is to know simultaneously from the influence of working capital, land area and labor. This analysis uses descriptive quantitative research methods. The data used is secondary data from the research conducted by Suryati. The data were analyzed using the Cobb-Douglas production function and estimated its parameters using Gauss Newton-Nonlinear Least Square. the optimum production function can be obtained by minimizing the function constraints, therefore the Lagrange method is used to obtain the minimum constraint function. In the results of the research it was found that the input elasticity of land area and labor affected production output. Model of Cobb-Douglas production function in onion production in Sakuru Village, Monta District, Bima Regency is $\hat{Q} = 5,305,000 x_1^{0.5957} x_2^{0.0766}$. The maximum production can be achieved with a land area of 13.19 acres and a workforce of 6 people.

Keywords: Cobb-Douglas, Gauss Newton-Nonlinear Least Square, elasticity.

1. Introduction

Optimization problems are often found in everyday life. In general, optimization is defined as the process of determining the maximum value and depending on its function. optimization is very important to provide optimal solutions with constraints or without constraints. optimization is divided into linear and non-linear. Nonlinear programming is one of the operational research techniques to solve optimization problems using equations and nonlinear inequalities to obtain optimal output by considering sources (inputs) whose supply is limited to a certain value [1].

One common form of nonlinear programming problem is determining $\mathbf{x}' = (x_1, x_2, \dots, x_n)$ to obtain the following results

Purpose Function:

Max / Min : $f(\mathbf{x}')$

With Constraints : $\mathbf{g}_m(\mathbf{x}') \leq \mathbf{b}$ dan $\mathbf{x}' \geq 0$

One optimization method that is often applied is the Lagrange Method. That used to determine the maximum or minimum value of a function with a constraints. constraints will be faced to optimize the objectives of the activities carried out. This method starts with the formation of the Lagrangean function which is defined as:

$$L(\mathbf{x}', \lambda_j) = f(\mathbf{x}') + \sum_{j=1}^m \lambda_j g_m(\mathbf{x}') \quad (1)$$

Where L is a compilation of Lagrange functions whose optimization techniques support the Kuhn Tucker method in calculating nonlinear programs that have constraints. is a framework function in preparing Lagrange functions. Decision variables are optimization goals, while constraints arise in



optimization. The conditions needed for the function $f(x)$ with the constraints $g_m(x) = 0$, with $j = 1, 2, \dots, n$ to have a relative minimum at that point is the first partial derivation of the Lagrange function which is defined as $L = L(x_1, x_2, x_3, \dots, x_n, \lambda_1, \lambda_2, \lambda_3, \dots, \lambda_m)$ for each argument has a value of zero [2].

Lagrange multipliers have an interesting physical meaning [3]. Suppose there is an optimization problem with one obstacle as follows:

Maximum / Minimum : $f(x)$

With constraints : $g(x) = b$

The Lagrange function is

$$L(x, \lambda) = f(x) + \lambda(b - g(x)) \quad (2)$$

Production is an activity that increases the utility value of an object. While the production function is intended as a relationship between input and production output where the function is a function that can be optimized [4]. Analysis of production functions is often used as research because researchers need information about how to manage limited resources such as land, labor, and capital so that maximum production results are obtained. Development of the production function was made around 1928 by mathematician Charles W. Cobb and the Paul H. Douglas economy. The function known as Cobb-Dauglas has a non-linear form. The Cobb-Douglas function has two forms, which can be changed and cannot be transformed into linear forms. Unchangeable functions must be estimated by nonlinear techniques.

The initial model of the Cobb-Douglas production function is as follows:

$$Q = f(x_1, x_2) = \beta_1 x_1^{\beta_2} x_2^{\beta_3} \quad (3)$$

Where:

Q : Output of production (Quantity of Product)

x_1 : Labor input (Labor of Product)

x_2 : Capital of Product Input

$\beta_1, \beta_2, \beta_3$: Constant

Cobb-Douglas production function with multiplicative error term is formulated by:

$$Q_m = \hat{\beta}_1 x_1^{\hat{\beta}_2} x_2^{\hat{\beta}_3} e \quad (4)$$

This form can be transformed in the form of a linear function so that it can be estimated by ordinary linear techniques, its function being:

$$\ln Q_m = \ln \hat{\beta}_1 + \hat{\beta}_2 \ln x_1 + \hat{\beta}_3 \ln x_2 + \ln e \quad (5)$$

The Cobb-Douglas function with an additive error term is formulated by:

$$Q_a = \hat{\beta}_1 x_1^{\hat{\beta}_2} x_2^{\hat{\beta}_3} + e \quad (6)$$

This form cannot be transformed in linear functions. In other words, its function is a nonlinear Cobb-Douglas production function so it must be estimated by nonlinear statistical techniques. According to Sugiarto et al., production elasticity (E) is defined as the percentage change in output divided by the percentage change in input [5]. Production elasticity shows the ratio of relative changes in output produced to changes in the relative amount of input used.

Output elasticity is measured by:

$$\frac{\Delta Q}{\Delta x_i} \times 100\% = E \quad (7)$$

Capital output elasticity can be measured directly through the E coefficient of the Cobb-Douglas production function. Parameters can be estimated for Cobb-Douglas production and cannot use classical parameter estimates such as minimizing the number of squared errors or maximizing the probability function. According to Draper and Smith nonlinear problems can be done with detailed approaches to normal equations and developing repetitive techniques to solve them [6]. One type of is Newton Gauss, where the iterative approach uses the first-order Taylor series. This iteration can be used to minimize the squared error or least square to estimate the β parameter [7].

2. Methodology

Data was obtained from research conducted by Suryati, with the title "The Effect of Working Capital, Land Area and Manpower on Red Onion Farmer Revenues in Sakuru Village, Monta District, Bima Regency". The researcher obtained the data primarily. Data collection techniques are done by giving questionnaires and conducting interviews. The number of samples taken was 141 [8]. Data analysis techniques used in this research used the help of Software R Studio. Data analysis was carried out with the following steps: Specifies the initial parameter value $\beta_0' = (\beta_{10}, \beta_{20}, \beta_{30})$.

1. Calculates Z_0 , $Q - f^0$, $\hat{\beta}_0$ and β_1 at First Iteration In Gauss Newton's and performed on the next iteration until the obtained solution converges.
2. Model production with the Cobb-Douglas production function
3. Perform a partial parameter test
4. Test the assumption of residual normality using the Kolmogorov-Smirnov test
5. Perform the Glejser test
6. Specifies the Lagrange function.

The research, the average yield of shallots produced by the people of Sakuru Village, Monta Subdistrict, Bima Regency was Rp. 11,522,198. The minimum production of the community was Rp. 85,000,000 and a maximum of Rp. 65,000,000. The land used by the community to grow shallots is an average of 11.98 hectares, where the narrowest land is 2 hectares and the largest is 30 hectares. The average number of workers for each farmer is 7 people, farmers with the fewest workers are as many as 4 workers and at most 16 workers [8].

Calculation of descriptive statistics from data can be seen in Table 1.

Table 1. Analysis of Descriptive Statistics

Variabel	Mean	Standard deviation	Minimum	Maximum
Q	26,417,730	11,522,198	8,500,000	65,000,000
x_1	11.98	6.447002	2	30
x_2	7	2.481012	4	16

3. Result and Discussion

3.1. Estimated Parameters of Cobb-Douglas Production Function.

Results the parameter estimates using Gauss Newton-Nonlinear Least Square can be found in Table 2.

Table 2 Parameter Estimation

Number of Iterations = 4		
Parameter	Initial value	Estimated Value
	β_{j0}	β_{j4} or $\hat{\beta}_{NLS}$
β_1	0.1	5.305.000
β_2	0.11	0.5957
β_3	0.11	0.0766

3.2. Model

Estimasi From the table it is known that the number of iterations so as to reach convergent conditions) is 4. In this paper, the initial parameter value is $\beta_0' = (0.1, 0.11, 0.11)$. From the results of $\hat{\beta}_{NLS}$ the production function model formed is:

$$\hat{Q} = 5.305.000 x_1^{0,5957} x_2^{0,0766}$$

β_2 explains that the input elasticity of land area is 0.5957, which means that about 59.57% of the influence of changes in input of land area on the output of shallot production. Then β_3 explains that the elasticity of labor input is 0.0766, which means 7.66% the effect of changes in labor input on the output of red onion production in Sakuru Village, Monta District, Bima Regency.

3.3. Parsial Testing

Test significant of β_j value on the model or $\beta_j \neq 0$ where $j = 1, 2, 3$ hen a partial test is carried out where $j = 1, 2, 3$, while to find the Prob. Value is used by R Studio software. The results of the test can be seen in Table 3.

Table 3 Probability Value of Parsial Test

J	Prob. Value	Decision
1	2.71×10^{-9}	Reject H_0
2	2.33×10^{-16}	Reject H_0
3	0.444	Accept H_0

In the calculation with the value $j = 1, 2$, the decision to reject H_0 is obtained with the conclusion that β_1 and β_2 are different from 0 which means that the constants and inputs of land area have a significant effect on production output. while for β_3 result is different with β_1 and β_2 who receive H_0 so that it can be concluded that labor input no significant effect on production output.

3.4. Assumption Test

Assumption Test include residual normality and homochasticity. Normal testing using the Kolmogorov-Smirnov test. The Kolmogorov-Smirnov value table can be seen in Table 4.

Table 4 Kolmogorov-Smirnov Residual Normal Value.

D	Prob. Value
0.075354	0.04829

Because the Probability value is $0.04829 > 0.01$, it is decided to fail to reject H_0 . It can be concluded that the residue spreads normally. So the assumption of the remaining spread is normally fulfilled.

The next assumption is homoskedasticity. In this paper to test homoskedasticity, a test was used. This test models the residuals using independent variables. The results of the Gelser test can be seen in Table 5.

Table 5. Glejser Testing for Homoscedasticity

Parameter	T	Prob. Value
β_1	0.518	0.605
β_2	0.329	0.742
β_3	-0.542	0.589

Because Prob. value > 0.5, it is decided to fail to reject H_0 . It was concluded that there was no heteroscedasticity in the model or assuming homoskedasticity was fulfilled. From the two tests above, it can be concluded that the assumptions in nonlinear least square are fulfilled.

3.5. Optimization of the Lagrange Method

From the completion above the production function is formed, so that the production function can reach the maximum value, the constraints on the function must be minimized. To minimize the constraint function, the lagrange method is used. The function of the constraints obtained in the study has been explained in Table 1 and the following equation is obtained

$$C = g_{x_1}x_1 + g_{x_2}x_2$$

$$7,631,206 = 512,499x_1 + 150,000x_2$$

So that the lagrange function that is formed as in the equation is as follows

$$L = 5,305,000 x_1^{0.5957} x_2^{0.0766} + \lambda(7,631,206 - 512,499x_1 - 150,000x_2)$$

Next to maximize L, then it is done then for each independent variable

- Derivatives against x_1

$$f_{x_1} = \frac{\partial L}{\partial x_1} = 3,160,188.5x_1^{-0.4043}x_2^{0.0766} - 512,499\lambda_{x_1} = 0$$

$$\lambda_{x_1} = \frac{3,160,188.5x_1^{-0.4043}x_2^{0.0766}}{512,499}$$

- Derivative against x_2

$$f_{x_2} = \frac{\partial L}{\partial x_2} = 406,363x_1^{0.5957}x_2^{-0.9234} - 150,000\lambda_{x_2} = 0$$

$$\lambda_{x_2} = \frac{406,363x_1^{0.5957}x_2^{-0.9234}}{150,000}$$

- Derivative of λ

$$f_{\lambda} = \frac{\partial L}{\partial \lambda} = 7,631,206 - 512,499x_1 - 150,000x_2 = 0$$

The next step is to eliminate Equations

$$\lambda_{x_1} = \lambda_{x_2}$$

$$\frac{3,160,188.5x_1^{-0.4043}x_2^{0.0766}}{512,499} = \frac{406,363x_1^{0.5957}x_2^{-0.9234}}{150,000}$$

$$474,028,275,000x_2 = 208,400,543,364x_1$$

$$x_1 = 2.27x_2$$

Then substitute Equation

$$f_{\lambda} = 7,631,206 - 512,499(2.27x_2) - 150,000x_2 = 0$$

$$x_2 = 5.81 \cong 6$$

So that is obtained

$$x_1 = 2.27x_2 = 2.27(5.81) = 13.19$$

From the solution above, it can be seen that to produce maximum shallot production can be achieved with a capital of Rp. 7,631,206, with an area of 13.19 hectares and the number of workers is 6 people.

To prove that the production produced is maximum, then look for a value of Δ_2 like the following.

$$\Delta_2 = \begin{vmatrix} 0 & 512,499 & 150,000 \\ 512,499 & \left. \frac{\partial f_{x_1}}{\partial x_1} \right|_{13.19;6} & \left. \frac{\partial f_{x_1}}{\partial x_2} \right|_{13.19;6} \\ 150,000 & \left. \frac{\partial f_{x_2}}{\partial x_1} \right|_{13.19;6} & \left. \frac{\partial f_{x_2}}{\partial x_2} \right|_{13.19;6} \end{vmatrix}$$

Because $\Delta_2 > 0$, it can be concluded that the production is maximum.

4. Conclusion

The research it was found that the input elasticity of land area and labor affected production output. Model of Cobb-Douglas production function in onion production in Sakuru Village, Monta District, Bima Regency is

$$\hat{Q} = 5,305,000 x_1^{0.5957} x_2^{0.0766}$$

The maximum production can be achieved with a land area of 13.19 acres and a workforce of 6 people.

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