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The applications of Cobb-Douglas Production Function in remanufacturing industry

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Abstract. The study was conducted in a heavy equipment remanufacturing industry. The problem faced by the industry is in order to make the right decisions about the number of products that must be produced and the labor and parts needed to carry out the remanufacturing process. All decisions regarding this matter consider the cost of the new and reprocessed parts and the workforce used, since it depends on the condition of the product returned. In this regard, it will be useful to provide recommendations for developing the production process framework for remanufacturing. The main purpose of this paper is to measure the appropriate production quantity with an existing remanufacturing alternative, during the 3 months observations. This study based on the Cobb-Douglas production function with labor and parts as the input. The remanufacturing process itself consists of three stages; disassembly, reconditioning and reassembly. There were two cases that occurred in reassembly process during 3 months of observation. The input for reassembly process uses new parts and reconditioned parts. In Case I all parts from disassembly process are forwarded to reconditioning process without any constraints. In Case II the parts are binding by the constraints of the return parts. As a result, the existing production quantity from reassembly process is below the maximum output, except for month 2. This indicates that production framework needs to pay attention to the proportion of input parameters. It means that the maximum quantity of the new products could get from a given combination of labor and parts at the reassembly process.
Keywords: remanufacturing, Cobb-Douglas Production Function, returns to scale.

1. INTRODUCTION
Inventory management issues have been widely discussed, especially in terms of deterministic models. The development of inventory management tends towards a global scale. Just-in-time philosophy, vendor managed inventory and consignment are some of the development concepts of the inventory model [1]. In recent years, inventory management is also developed for raw materials such as components or spare parts that come from a used return product and then to be reused to produce a new product after going through a process called remanufacturing [2]. In remanufacturing, inventory management is more complex due to the unknown condition of the used return products [3]. The inventory management framework for components obtained from used products is different from the conditions for new components purchased from suppliers [4]. This study was carried out in the heavy equipment remanufacturing industry. The purpose of this study is to design the production planning of remanufacturing process using Cobb-Douglas production function for a certain engine component.

Cobb-Douglas Production Function is widely used in economics and productivity for representing the relationship between input and output. The production function states the physical relationship between production input and output. The analysis of production function is often used in research, hence with the limited resources can provide the maximum results. In his thesis Hassani [5] has developed the use of Cobb-Douglas production function for a cost function model that shows the relationship between labor and equipment to the costs required if the construction project schedule is shortened from the desired time. This paper discusses the combination from a given labor and material/parts to achieve the maximum output of production in a heavy equipment remanufacturing industry.

2. METHODS

The Cobb-Douglas production function is a function or equation that involves two or more variables, dependent variable / Y (which is explained) and independent variable / X (which explains). This approach is used as a production function to show the effect of the input used on the desired output. In this case the data is processed into a natural logarithm (Ln) and converted to the regression equation as $Y = a + bX$ or $Y = Ln.Q$ and $X = Ln.I$. Therefore, the the regression equation can be written as;

$$Ln.Q = a + b\left(Ln.I\right) \; ; \; Ln.Q = a + Ln.I^{b} \; ; \; Ln.Q - Ln.I^{b} = a \; \rightarrow \; Q = e^{a}I^{b}$$

The general form of the equation is as follows [2]:

$$Q = \delta.I^{\theta} \tag{1}$$

where, Q = output; I = the type of input used; δ = efficiency index of input use in generating output; θ = production elasticity of the input used. In remanufacturing the production system will pass through disassembly, cleaning, reprocessing, testing and reassembly processes. With Cobb-Douglas production function the decisions will focus on the influence of cost of labor and material. For this purpose, the equation (1) in production system can be written as follows;

$$Q = \delta L^{a} M^{b} \tag{2}$$

where, Q = total output from manufacturing production, L = labor input, M = material input, a = production elasticity of labor, b = production elasticity of material and δ as the efficiency index or total factor productivity. The value of a and b are constant determined by the state of technology in the remanufacturing processes. There are 3 alternatives; 1) if a+b = 1, the production function has constant returns to scale (RTS), 2) If a+b<1, the RTS are decreasing and 3) if a+b>1, the RTS are increasing. For example in case of L and M are increased by 20% respectively, 1) if Q increases exactly by 20%, it becomes constant RTS, 2) if Q increases less than 20%, RTS will decreasing, and 3) if Q increases more than 20%, RTS will increasing. Returns to scale (RTS) examines the changes in total production related to a proportional changes in all inputs.

2.1. Model Formulation

Production process flow in the remanufacturing industry begins with the arrival of used products from consumers. Inspection of used products is conducted by the collection unit. Remanufacturing begins with disassembly and reconditioning process of the return products. The result of the dissasembly process is to group the parts into 3 categories; 1) can not be used and must be replaced with a new one, 2) can still be used through the machining process, and 3) can be used without going through the machining process. In addition, judgment is carried out in the disassembly process to determine the level of damage. The low, medium and high damage level categories are labeled with A, B and C respectively. The next step is to do the washing process of all parts that can be reused. Parts that

cannot be reused are disposed of. In order to meet the requirements at the production floor, the company should make an order to the supplier. After the machining process, the reconditioned parts will be re-inspected, and at the last process new and reconditioned parts will be reassembled into new products.

2.2. Notation and Terminology
The notation and terminology used refer to [2] ;
Decision variables:

L_m	: labor input at reassembly process
L_r	: labor input at disassembly and reconditioning process
M_m	: new parts input at the reassembly process
R	: return products into disassembly process
λ	: Lagrange multiplier for the budget constraint
μ	: Lagrange multiplier for the return products constraints
M_r	: used parts after reconditioning as input at the reassembly process
M	: total parts input at reassembly process $(M = M_m + M_r)$
Q	: output from reassembly process

Parameters

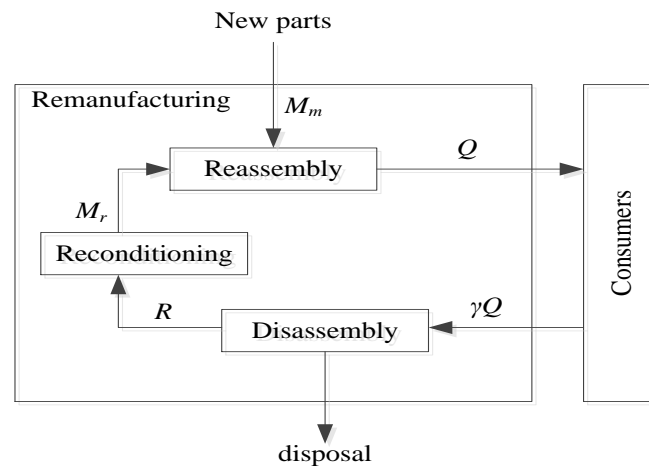|	
γ	: return ratio for the return products, $\gamma < 1$, γQ the maximum number of return products to be disassembled
P_l	: price of input labor
P_m	: price of input material
P_r	: price of used product
C	: budget limit
α, a, b	: positive values of Q
β, c, d	: positive values of M_r

Figure 1 Remanufacturing System

The equations for the output from reassembly process and the ouput from the reconditioning process are represented by the Cobb-Douglas function. It is assumed that returns to scale (RTS) are decreasing, i.e. *a+b<*1 and *c+d<*1.

$$Q = \alpha L_m^a M^b \qquad (3)$$

$$M_r = \beta L_r^c R^d \tag{4}$$

The total parts input at reassembly process are consisted of used parts after reconditioning and new purchased parts. It substitutes equation 4) into 3) for equation $Q = \alpha L_m^a (M_m + M_r)^{b.}$. It obtains as follows;

$$Q = \alpha L_m^a \left(M_m + \beta L_r^c R^d\right)^b \tag{5}$$

The total cost (*TC*) consists of labour cost plus material cost plus returned product cost and it cannot exceed the budget limit, *C*. Furthermore, the returned parts, R, is limited by the maximum number of used products to be diassembled, γQ.

$$TC = P_l\left(L_r + L_m\right) + P_m M_m + P_r.R \le C \tag{6}$$

$$R \le \gamma Q \tag{7}$$

In this case, the problem is to maximize *Q*, the output from reassembly process, subject to the budget limit *C*. Lagrangian function is used in order to maximize the problem, where λ and μ are the Lagrange multipliers for equation (6) and (7), respectively. The Lagrangian function is the sum of equation (5), (6) and (7) as follows [2];

$$L = \alpha L_m^a \left(M_m + \beta L_r^c R^d\right)^b + \lambda\left(C - P_l\left(L_r + L_m\right) - P_m M_m - P_r R\right) + \mu\left(\gamma Q - R\right) \tag{8}$$

2.3. Reconditioning process

The return parts from reconditioning and disassembly process should be maximized, subject to the budget limit for these processes, namely C_R. The Lagrangian function is the sum of equation (4), (6) for reconditioning process only and equation (7);

$$L_R = \beta L_r^c R^d + \lambda_R\left(C_R - P_l L_r - P_r R\right) + \mu_R\left(Q - \gamma R\right) \tag{9}$$

The limited budget for the reconditioning process will bind the labor at reconditioning and disassembly, reconditioning parts and Lagrange multipliers for λ_R, L_R, and $M_R > 0$. Since there are inequality constraints, the non-linear programming problem needs to apply Kuhn Tucker conditions to find the optimal value [6]. The first order Kuhn Tucker conditions are:

$\dfrac{\partial L_R}{\partial L_r}$; $\dfrac{\partial L_R}{\partial R}$; $\dfrac{\partial L_R}{\partial \lambda_R}$ and $\dfrac{\partial L_R}{\partial \mu_R}$.

The solution for the decision variables are as follows;

$$L_r^* = C_R c P_l^{-1}\left(c + d y_R\right)^{-1} \tag{10}$$

$$R^* = C_R d P_r^{-1} y_R\left(c + d y_R\right)^{-1} \tag{11}$$

$$\text{where,} \quad y_R = P_r\left(P_r + \frac{\mu_R}{\lambda_R}\right)^{-1} \tag{12}$$

The output of the decision variable is;

$$M_r^* = C_R^{c+d} \beta c^c d^d P_l^{-c} P_r^{-d} y_R^d \left(c + d y_R\right)^{-(c+d)} \tag{13}$$

2.4. Reassembly process

The consequences of a limited budget will also have an impact on the process of reassembly. As the final process of remanufacturing, for M_r, L_r, and $R > 0$. The first order Kuhn Tucker conditions are [2]; $\frac{\partial L}{\partial L_r} = 0$; $\frac{\partial L}{\partial L_m} = 0$; $\frac{\partial L}{\partial M_m} \leq 0$; $\frac{\partial L}{\partial R} = 0$; $\frac{\partial L}{\partial \lambda} = 0$ and $\frac{\partial L}{\partial \mu} \mu = 0$

The inequality is in the condition of $\frac{\partial L}{\partial M_m} \leq 0$. In Case I, all parts from disassembly process are forwarded to reconditioning ($R^* < \gamma Q^*$ with $\mu = 0$ and $y = 1$). The input for reassembly process consists of reconditioned parts ($M_r > 0$) and new parts ($M_m > 0$). The formulation for this condition becomes, as follows [2];

$$L_m^* = \frac{aP_l^{-1}C}{a+b}\left(1 + hy^{d/(1-c-d)}\left(1-c-dy\right)\right) \tag{14}$$

$$M_m^* = \frac{bP_m^{-1}C}{a+b}\left(1 - hy^{d/(1-c-d)}\left(\frac{a+bc}{b}+dy\right)\right) \tag{15}$$

$$L_r^* = cP_l^{-1}Chy^{d/(1-c-d)} \tag{16}$$

$$R^* = dP_r^{-1}Chy^{1-c/(1-c-d)} \tag{17}$$

$$Q^* = \frac{\alpha a^a b^b P_l^{-1} P_m^{-b} C^{a+b}}{(a+b)^{a+b}}\left(1 + hy^{d/(1-c-d)}\left(1-c-d\right)\right)^{a+b} \tag{18}$$

$$\lambda^* = \alpha a^a b^b P_l^{-a} P_m^{-b}\left(\frac{a+b}{C(1+h(1-c-d))}\right)^{1-a-b} \tag{19}$$

where, $$h = \beta^{1/(1-c-d)} c^{c/(1-c-d)} d^{d/(1-c-d)} P_l^{-c/(1-c-d)} P_m^{1/(1-c-d)} P_r^{-d/(1-c-d)} C^{-1} \tag{20}$$

According to [2], if the value of h is less than $b/(a+bc+bd)$, then the solution is optimal. For Case II the value of $y = y_{II}$ and it can be calculated using the following formula;

$$h = \frac{by_{II}^{-d/(1-c-d)}}{a+bc+bdy_{II}} \tag{21}$$

3. Result & Discussion
In this paper, the Cobb Douglas production function is implemented in industries that remanufacture heavy equipment. This study focuses on observing one type of product that is part of excavator equipment, namely core engine type XX. For example; the core engine requires 210 items that must be replaced. These items consist of 6325 parts. Inspections are carried out by workers to group between parts that require reconditioning process and parts that are directly used in the disassembly process. There are 2974 parts need to be reconditioned and 3351 parts do not require a reconditioned process. Return ratio for the core engine's parts is, $\gamma = 2974/6325 = 0.47$.

Technical efficiency at reassembly process (α) and disassembly process (β) are 4 and 2, respectively. These values indicate that the use of technology is more dominant in the reassembly process than the disassembly process. Parameters $a, b, c,$ and d indicate the elasticity of the rate of

change in output as a consequence of the input used. The parameter values a, b, c, and d are 0.1, 0.6, 0.1 and 0.3, respectively. The elasticity value of a = 0.1 means that the reassembly process requires only a few workers because the use of technology is more dominant. The value of b = 0.6, means the use of new parts is greater than the reconditioned parts. Obviously, the number of reconditioned parts used as input in the reassembly process is less than the new parts, it is shown by c = 0.1. The value of R (input return) is denoted by d = 0.3, since the input parts used in the reassembly process consists of new parts, reconditioned parts and returned parts without machining. The total labor cost for the core engine is USD 3523.24 (P_l). The price of returned core engine is USD 6427 (P_r). The total price for all new components (P_m) is USD 27767.93.

3.1. Case I

In Case I, the input for reassembly process uses new parts ($M_m > 0$) and reconditioned parts ($M_r > 0$). All parts from disassembly process are forwarded to reconditioning without any constraints ($\mu = 0$ and $y = 1$). The budget limit for producing a new core engine (C) is USD 90,000. Referring to equation (20), it obtains h = 1.0718. The results show the number of workers in the disassembly process (L_m) is 3 workers and reassembly (L_r) is 6 workers. The number of return parts to the reconditioning process (R) is 5. The number of new materials (M_m) is equal to 2. The total reconditioning material (M_r) is 4, and the optimal amount of production (Q) is 12 units. In summary, the result are $L_r^* = 3$, $R^* = 5$, $L_m^* = 6$, $M_m^* = 2$, $M_r^* = 4$, and $Q^* = 12$ units.

3.2. Case II

Similar to Case I, the new parts ($M_m > 0$) and the reconditioned parts ($M_r > 0$) are the input to the reassembly process. In Case II, the Lagrange multiplier value for the constraints of return parts is greater than 0 ($\mu > 0$) and the value of y is less than 1 (y < 1). The budget limit for producing the new core engine for Case II is USD 100,000. Using equation (20) with C = 100,000, the value of h is 0.9646. Case II optimal if the value of $h < b/(a + bc + bd)$ = 0.6/(0.1+ (0.6 x 0.1) + (0.6 x 0.3)) = 0.6/0.34 = 1.765. It shows h (= 0.9646) less than 1.765. Using equation (21) for Case II with h equals to 0.9646, then

$$0.9646 = \frac{0.6 \, y_{II}^{\,0.3/(1-0.1-0.3)}}{0.1 + (0.6 \text{ x } 0.1) + (0.6 \text{ x } 0.3 \text{ x } y_{II})}$$, it obtains y_{II} equals to 0.5942, where y = y_{II}.

Table 1 shows the resources allocation for Case I and II during 3 month observations.

Table 1. Resources allocation for 3 month observations

Case	Decision Variables					
	R^*	L_r^*	L_m^*	M_m^*	M_r^*	Q^*
I	5	3	6	2	4	12
II	2	3	7	3	3	13

The existing production quantity that produced by reassembly process in month 1, 2 and 3 were 5, 12 and 2 units, respectively. It is shown that only in month 2 has the same production quantity with Cobb-Douglas production function approaches. The existing production quantity in month 1 and 3 are below the maximum output of the reassembly process.

4. Conclusion

The main purpose of this paper is to measure the appropriate production quantity with an existing remanufacturing alternative, during the 3 months observations. The existing production quantity for the last 3 months in month 1, 2 and 3 are 5 units, 12 units and 2 units, respectively. The final result of the Cobb-Douglas production function approach shows that at Case I optimal production has been achieved in month 2, while in month 1 and 3 the production are not optimal. Similar with Case 1, the optimal conditions in Case II is achieved at period 2. For further research the application of the Cobb Douglas production function can be tried again for different parameter values. It is shown that production framework needs to pay attention to the proportion of input parameters so that the influence of production inputs can have a positive influence on production results. The recommendations are to induce the industry to search a way to get as much output as possible from the existing combination of inputs.

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