Stepwise construction of simple Agda programs

Steps towards an interactive Agda programming tutor

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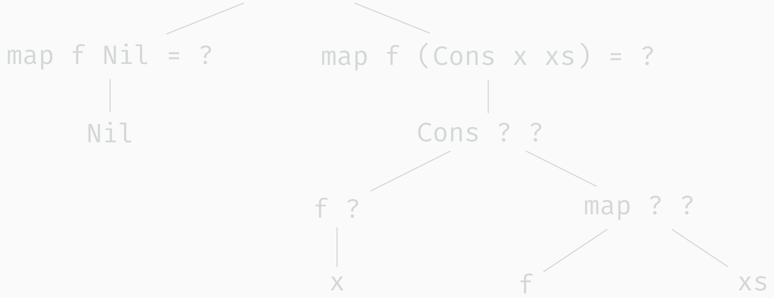
Introduction

Research question

What (sub-)class of semi-decidable programs (or, equivalently, proofs) can be automatically constructed in the programming language Agda through a series of mechanical steps that would normally be performed by a user writing a program, and how can we automatically extract these steps for programs within these classes?

Example

```
map : ∀ {A B n} → (A → B)  → (Vec A n) → (Vec B n)
            map f xs = ?
```



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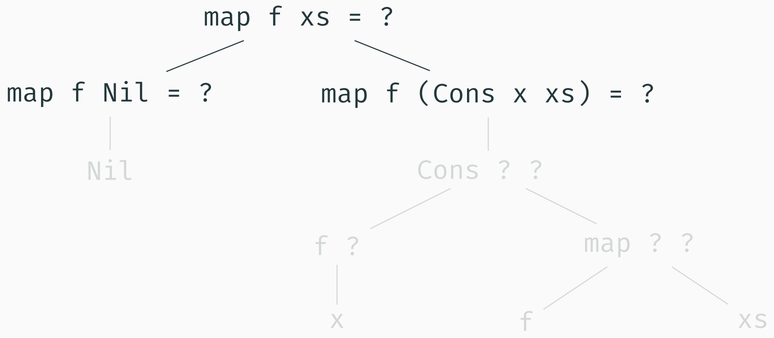

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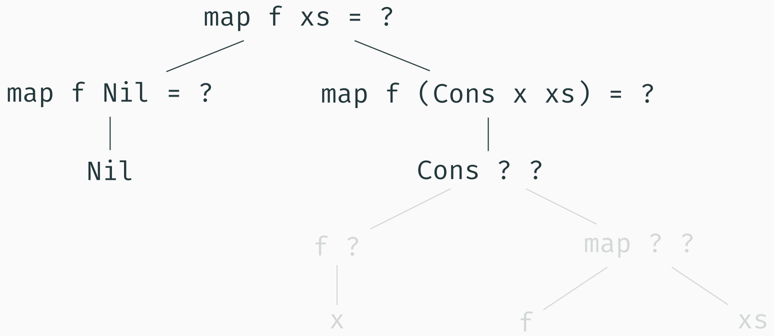

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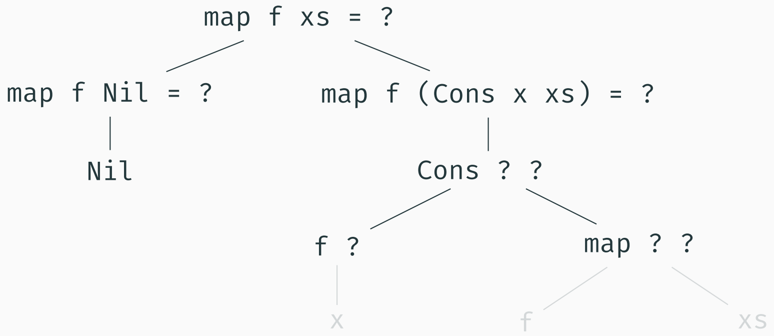

Example

```
map : ∀ {A B n} → (A → B) → (Vec A n) → (Vec B n)
```

$$\begin{tikzcd}
 & \texttt{map f xs = ?} \arrow[dl, no head] \arrow[dr, no head] & & & \\
\texttt{map f Nil = ?} \arrow[d, no head] & & \texttt{map f (Cons x xs) = ?} \arrow[d, no head] & & \\
\texttt{Nil} & & \texttt{Cons ? ?} \arrow[dl, no head] \arrow[dr, no head] & & \\
 & \texttt{f ?} \arrow[d, no head] & & \texttt{map ? ?} \arrow[dl, no head] \arrow[dr, no head] & \\
 & \texttt{x} & \texttt{f} & & \texttt{xs}
\end{tikzcd}$$

Approach

Strategy

Fill Hole

- First try to solve a hole using the proof search algorithm. [1]
- If that fails, invoke the case split strategy.

[1]Based on the paper *Auto in Agda* by Kokke and Swierstra
[2]We bound the depth so we can stop and backtrack

Strategy

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Case-Split

- Generates candidate variables for splitting based on their depth in patterns.
- Candidates are checked one at a time (split may fail).
- When the split succeeds, invokes the fill-hole strategy to recursively solve the newly generated clauses.[2]

[1] Based on the paper *Auto in Agda* by Kokke and Swierstra
[2] We bound the depth so we can stop and backtrack

Proof Search

- the goal type and the types of the definitions in scope are translated into Prolog-like terms
- a term with a given type is found by solving the corresponding Prolog query

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foo : Vec Nat (suc zero)
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```

nil rule: `Vec(`x_A`, zero).`

cons rule: `Vec(`x_A`, suc(`x_n`)) :- `x_A`, Vec(`x_A`, `x_n`).`

goal query: `?- Vec(Nat, suc(zero))`

Demonstration

Future Work

Future Work

- Improve case splitting by finding an effective and computationally cheap heuristic to select the variable.
  - Example: Analyze dependencies between variables and select the one with highest impact.
- Improve proof search to handle some restricted version of higher order unification
- Specify properties that help to prune the proof search state

Conclusion

- Using Agda as a library is hard (and unsupported).
- Expressiveness of dependent type system allows to automatically generate solutions for exercises, no model solutions required.