Particle-Based Fluid Simulation for Interactive Application

Presentation Date: Sep 15th

Chrissie C. Cui

Outline

- Main Contribution
- Motivation
- Particle Based Fluids Modeling
- Visualization
- Implementation
- Conclusion and Results

Main Contribution

- Simulate fluids with free surface
- Fluids modeling Based on Smoothed Particles Hydrodynamics (SPH)
- Model the surface tension forces
- Design new smoothing kernels for interactivity purposes
- Efficient Surface Tracking and Rendering

Motivation

- Real-time Fluid Simulation
  - Online Computation
  - Less Accuracy
  - High Interactivity
- Possible Applications:
  - Industrial Design Validation
  - **Medical Simulator(Blood Flow)**
  - Computer games
  - Virtual Reality

Smoothed Particle Hydrodynamics(SPH)

- An interpolation method for particle systems
- General Format

$$A_S(\mathbf{r}) = \sum_j \boxed{m_j} \frac{\boxed{A_j}}{\boxed{\rho_j}} W(\mathbf{r} - \boxed{\mathbf{r}_j}, h),$$

The mass of particle j

The field value of particle j

The density of particle j

The position of particle j

- W(**r**, **h**) – the smoothing kernel with core radius **h**
  - Even function, i.e. W(**r**, **h**) = W(**-r**, **h**)
  - Normalization, i.e. $\int W(\mathbf{r})d\mathbf{r} = 1.$
- SPH based Gradient

$$\nabla A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla W(\mathbf{r} - \mathbf{r}_j, h)$$

- SPH based Laplacian

$$\nabla^2 A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla^2 W(\mathbf{r} - \mathbf{r}_j, h).$$

Modeling Fluids with Particles I

- Eulerian Fomulation
  - Conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

  - Naiver Stokes Equation (Conservation of Momentum)

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{v}$$

- Simplification based on SPH
  - Mass Conservation Equation unnecessary
  - Convective Term: $\mathbf{v} \cdot \nabla \mathbf{v}$ unnecessary
  - $\partial \mathbf{v} / \partial t$ can be replaced by $D\mathbf{v}/Dt$
- The acceleration for particle I

$$\mathbf{a}_i = \frac{d\mathbf{v}_i}{dt} = \frac{\mathbf{f}_i}{\rho_i}$$

Modeling Fluids with Particles II

- SPH based Pressure Term

$$\mathbf{f}_i^{\text{pressure}} = -\nabla p(\mathbf{r}_i) = -\sum_j m_j \frac{p_j}{\rho_j} \nabla W(\mathbf{r}_i - \mathbf{r}_j, h).$$

- Symmetrization

$$\mathbf{f}_i^{\text{pressure}} = -\sum_j m_j \frac{p_i + p_j}{2\rho_j} \nabla W(\mathbf{r}_i - \mathbf{r}_j, h)$$

- Evaluate the pressure at particle locations
  - Step I: Compute the density of the particle location

$$\rho_S(\mathbf{r}) = \sum_j m_j \frac{\rho_j}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h) = \sum_j m_j W(\mathbf{r} - \mathbf{r}_j, h)$$

  - Step 2: Compute pressure via the ideal gas state equation

$$p = \boxed{k}(\rho - \boxed{\rho_0})$$

Gas Constant ***Rest Density***

Modeling Fluids with Particles III

- SPH based viscosity term $\mu\nabla^2\mathbf{v}$:

$$\mathbf{f}_i^{\text{viscosity}} = \mu\nabla^2\mathbf{v}(\mathbf{r}_a) = \mu\sum_j m_j \frac{\mathbf{v}_j}{\rho_j}\nabla^2 W(\mathbf{r}_i - \mathbf{r}_j, h)$$

- Symmetrization

$$\mathbf{f}_i^{\text{viscosity}} = \mu\sum_j m_j \frac{\mathbf{v}_j - \mathbf{v}_i}{\rho_j}\nabla^2 W(\mathbf{r}_i - \mathbf{r}_j, h).$$

- Interpretation:

Look at the neighbors of particle *i from i's own moving frame of reference.* Then particle *i is accelerated in the direction of the relative* speed of its environment.

Force Modeling

- Surface Tension:
  - Physics Property:
    - Generated by unbalanced inter-molecule forces at the free surface
    - Act along surface normal towards the fluid
    - Tend to minimize the curvature
  - SPH representation
    - Color field: $c_S(\mathbf{r}) = \sum_j m_j \frac{1}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h).$
    - Force Density acting near the surface

$$\mathbf{f}^{\text{surface}} = \sigma \kappa \mathbf{n} = -\sigma \nabla^2 c_S \frac{\mathbf{n}}{|\mathbf{n}|}$$

σ ***Tension Coefficient***

- External Forces
  - Gravity, Collision Forces, Force caused by user interaction
  - No SPH

Smoothing Kernels I

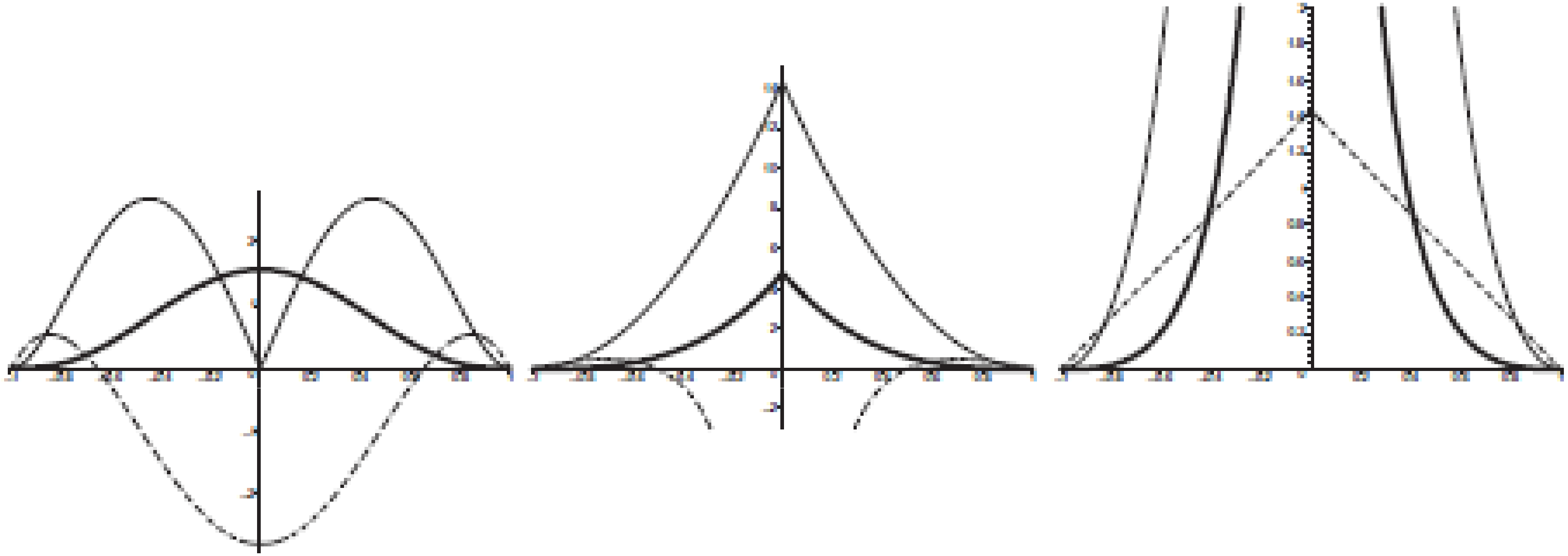
- Desired Properties
  - Stability, accuracy and computational speed
  - Second order interpolation error bound
  - Zero with vanishing derivatives at the boundary
- Designed Kernels: poly6, spiky & viscosity

$$W_{\mathrm{poly6}}(\mathbf{r},h) = \frac{315}{64\pi h^9}\begin{cases}(h^2-r^2)^3 & 0\le r\le h\\ 0 & \text{otherwise}\end{cases}$$

$$W_{\mathrm{spiky}}(\mathbf{r},h) = \frac{15}{\pi h^6}\begin{cases}(h-r)^3 & 0\le r\le h\\ 0 & \text{otherwise,}\end{cases}$$

$$W_{\mathrm{viscosity}}(\mathbf{r},h) = \frac{15}{2\pi h^3}\begin{cases}-\frac{r^3}{2h^3}+\frac{r^2}{h^2}+\frac{h}{2r}-1 & 0\le r\le h\\ 0 & \text{otherwise.}\end{cases}$$

Smoothing Kernels II



The three smoothing kernels W_{poly6}, W_{spiky} *and* $W_{viscosity}$ *(from left to right) used in the fluid simulations. The thick lines show the kernels, the thin lines their gradients in the direction towards the center and the dashed lines the Laplacian. Note that the diagrams are differently scaled. The curves show 3-d kernels along one axis through the center for smoothing length* $h = 1$.

Surface Tracking and Visualization

- Surface particle Tracking: Identify a particle as a surface particle if $|\mathbf{n}(\mathbf{r}_i)| > l$. $\mathbf{n} = \nabla c_S$ and c_S is the color field
- Visualization
  - Point Splatting
  - Marching Cubes

Implementation

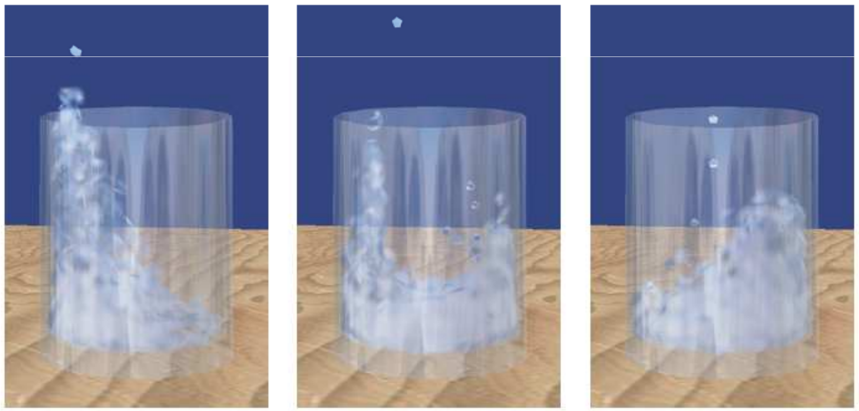
- Reduce Computational Complexity
  - Set unit grid cells size at h, h is the finite support of SPH kernel
  - $O(n^2)$ to $O(mn)$, m is the average number of particles per grid
- Store copies of the particle objects in the grid cells instead of reference to the particles
- Good data structure for fast neighbor searches
- Possible further speedup: Better clustering using Hilbert space filling curves

Conclusion & Results

- Present a particle-based method for interactive fluid simulation and rendering.
- The physical model is based on Smoothed Particle Hydrodynamics
- Use special purpose kernels to increase stability and speed.
- Present technique to track and render the free surface of fluids.

(a) (b) (c)

This figure shows a swirl in a glass induced by a rotational force field. Image (a) shows the particles, (b) the surface using point splatting and (c) the iso-surface triangulated via marching cubes.



The user interacts with the fluid causing it to splash.

Pouring water into a glass at 5 frames per second.

Thanks for your attention.☺
Questions???