Homework 9
Assigned: Tuesday, April 24, 2012
Due: **Monday, April 30, 2012**

Heapsort
Assigned problems:

1. (problem 6.1-1) What are the minimum and maximum numbers of elements in a heap of height h?

2. (problem 6.1-6) Is the array with values $\langle 23, 17, 14, 6, 13, 10, 1, 5, 7, 12 \rangle$ a max-heap?

3. (problem 6.2-1) Using Figure 6.2 as a model, illustrate the operation of MAX-HEAPIFY$(A, 3)$ on the array $A = \langle 27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0 \rangle$.

4. (problem 6.3-1) Using Figure 6.3 as a model, illustrate the operation of BUILD-MAX-HEAP on the array $A = \langle 5, 3, 17, 10, 84, 19, 6, 22, 9 \rangle$.

5. (problem 6.4-1) Using Figure 6.4 as a model, illustrate the operation of HEAP-SORT on the array $A = \langle 5, 13, 2, 25, 7, 17, 20, 8, 4 \rangle$.

6. (problem 6.5-1) Illustrate the operation of HEAP-EXTRACT-MAX on the heap $A = \langle 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 \rangle$.

7. (problem 6.5-2) Illustrate the operation of MAX-HEAP-INSERT(A,10) on the heap $A = \langle 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 \rangle$.

8. (problem 6.5-6) Each exchange operation in line 5 of HEAP-INCREASE-KEY typically requires three assignments. Show how to use the idea of inner loop of INSERTION-SORT to reduce the three assignments down to just one assignment.

Suggested problems:

1. (problem 6.1-2) Show that an n-element heap has height $\lfloor \lg n \rfloor$.

2. (problem 6.2-6) Show that the worst-case running time of MAX-HEAPIFY on a heap of size n is $\Omega(\lg n)$. (Hint: For a heap with n nodes, give node values that cause MAX-HEAPIFY to be called recursively at every node on a simple path from the root down to a leaf.)

3. (problem 6.4-2) Argue the correctness of HEAPSORT using the following loop invariant:

> At the start of each iteration of the **for** loop of lines 2–5, the subarray $A[1..i]$ is a max-heap containing the i smallest elements of $A[1..n]$, and the subarray $A[i + 1..n]$ contains the $n - i$ largest elements of $A[1..n]$, sorted.

4. (problem 6.4-3) What is the running time of HEAPSORT on an array of length n that is already sorted in increasing order? What about decreasing order?

5. (problem 6.5-3) Write pseudocode for the procedures HEAP-MINIMUM, HEAP-EXTRACT-MIN, HEAP-DECREASE-KEY, and MIN-HEAP-INSERT that implement a min-priority queue with a min-heap.