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\documentclass[12pt]{article}
\usepackage[utf8]{inputenc}
\usepackage{amsmath}
\usepackage{graphicx}
\title{Math 315 Final}
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\date{December 11th, 2010}
\begin{document}
\maketitle
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\section*{Problem 1}
\subsection*{A.}
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To determine the dimension of the fractal, we refer to the class definition of dimension: \\\

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\begin{equation}
c = s ^ d
\end{equation}
\begin{equation}
d = \frac{\log(c)}{\log(s)}
\end{equation}
where
\begin{description}
d = dimension \\\
s = scaling factor \\\
c = copies to achieve replication.\\\
\end{description}
```

Continuing from here, we can observe that in order for the fractal to replicate itself, it must reproduce itself 3 times in the x and y directions. This give us a scaling factor $s = 3$. We can also see that in the original fractal, there are equivalent sub-parts around the parameter, but non in the middle. So we have 8 sub-parts, instead of $3 \times 3 = 9$. This means $c = 8$. Using equation (2) above:

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\begin{equation}
d = \frac{\log(8)}{\log(3)}
\end{equation}
\begin{equation}
d \approx 1.8928
\end{equation}
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\subsection*{B.}
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\subsection*{C.}

Given the public RSA key parameters: \

$$\begin{aligned} &\text{\begin{equation}} \\ &m = 1633, e = 17 \\ &\text{\end{equation}} \end{aligned}$$

we want to find the encrypted text, C, using the class definition for RSA encryption:

$$\begin{aligned} &\text{\begin{equation}} \\ &C \equiv P^e \pmod{m} \\ &\text{\end{equation}} \end{aligned}$$

We transfer our plaintext letters 'UI' into their integer equivalents, in the order that they are in the alphabet (i.e. A = 00, B = 01, etc): \

$$\begin{aligned} &\text{\begin{equation}} \\ &U = 20, I = 08 \\ &\text{\end{equation}} \\ &\text{\begin{equation}} \\ &'UI' = 2008 \\ &\text{\end{equation}} \end{aligned}$$

Using the definitions for C, m, and e above:

$$\begin{aligned} &\text{\begin{equation}} \\ &C \equiv 2008^{17} \pmod{1633} \\ &\text{\end{equation}} \end{aligned}$$

2008 exponent 17 is a big number, a little hard to process. We use Binary Exponentiation to reduce the number, and to process the modulo:

$$\begin{aligned} &\text{\begin{equation}} \\ &2008^{17} = 2008^{16} + 2008^1 \\ &\text{\end{equation}} \\ &\text{\begin{align}} \\ &2008^1 \equiv 375 \pmod{1633} \\ &2008^2 \equiv 187 \pmod{1633} \\ &2008^4 \equiv 187^2 \equiv 676 \pmod{1633} \\ &2008^8 \equiv 676^2 \equiv 1369 \pmod{1633} \\ &2008^{16} \equiv 1369^2 \equiv 1110 \pmod{1633} \\ &\text{\end{align}} \\ &\text{\begin{align}} \\ &2008^{17} \equiv (1110 * 375) \pmod{1633} \\ &\equiv 1468 \pmod{1633} \\ &\text{\end{align}} \end{aligned}$$

Therefore, C = 1468.

\subsection*{D.}

To find the decryption value, we use the class definition for P decryption:

\begin{equation}

$$P \equiv C^d \pmod{m}$$

\end{equation}

where

\begin{description}

C = encrypted code (i.e. 1468 from Problem 1.B above) \\\

m is part of the RSA public key as explained in Problem 1.B above \\\

d is given by the equation:

\begin{equation}

$$de \equiv 1 \pmod{\varphi(m)}$$

\end{equation}

\end{description}

For d, we need to know that $\varphi(m)$ equals the number of factors of m. We now have to find d to find e, and $\varphi(m)$ to find both. We know from Euler's Totient Function that:

\begin{equation}

$$\varphi(m) = (p-1)(q-1)$$

\end{equation}

where

\begin{description}

p and q are prime factors of m

\end{description}

If we use some computer programming, we can quickly find that the prime factors of m = 1633 are

p = 23, q = 71. We can then tell that $\varphi(m) = 22 \cdot 70 = 1540$. So:

\begin{equation}

$$de \equiv 1 \pmod{1540}$$

\end{equation}

We then apply the Euclidian Algorithm, substituting in the other part of our RSA public key e = 17:

\begin{align}

$$d(17) \equiv 1 \pmod{1540} \\\$$

\\

$$1540 = 17(90) + 10 \\\$$

$$17 = 10(1) + 7 \\\$$

$$10 = 7(1) + 3 \\\$$

$$7 = 3(2) + 1 \\\$$

\\

$$\begin{aligned}
1 &= 7 - 3(2) \\
1 &= 7 - (10 - 7)(2) \\
1 &= 7(3) - 10(2) \\
1 &= (17 - 10)(3) - (10)(2) \\
1 &= 17(3) - 10(3) - (10)(2) \\
1 &= 17(3) - 10(5) \\
1 &= 17(3) - (1540 - 17(90))(5) \\
1 &= 17(3) - 1540(5) + 17(450) \\
1 &= 17(453) - 1540(5) \\
17(453) &= 1540(5) + 1
\end{aligned}$$

We now have an equation of the form $d \equiv 1 \pmod{1540}$:

$$\begin{aligned}
453(17) &\equiv 1 \pmod{1540} \\
d &= 453
\end{aligned}$$

We can now decode with $d = 453$, $m = 1633$, and $C = 1468$ for the original decryption algorithm:

$$\begin{aligned}
P &\equiv C^d \pmod{m} \\
P &\equiv (1468)^{453} \pmod{1633}
\end{aligned}$$

Use the Euclidian Algorithm:

$$\begin{aligned}
1468^{453} &= 1468^{256} + 468^{128} + 1468^{64} + 1468^4 + 1468^1 \\
1468^1 &\equiv 1468 \pmod{1633} \\
1468^2 &\equiv 1097 \pmod{1633} \\
1468^4 &\equiv 1097^2 \equiv 1521 \pmod{1633} \\
1468^8 &\equiv 1521^2 \equiv 1113 \pmod{1633} \\
1468^{16} &\equiv 1113^2 \equiv 955 \pmod{1633} \\
1468^{32} &\equiv 955^2 \equiv 811 \pmod{1633} \\
1468^{64} &\equiv 811^2 \equiv 1255 \pmod{1633} \\
1468^{128} &\equiv 1255^2 \equiv 813 \pmod{1633} \\
1468^{256} &\equiv 813^2 \equiv 1237 \pmod{1633} \\
1468^{453} &\equiv (1237 * 813 * 1255 * 1521 * 1468) \pmod{1633} \\
&\equiv (1386 * 1511 * 1468) \pmod{1633} \\
&\equiv (740 * 1468) \pmod{1633} \\
&\equiv 375 \pmod{1633}
\end{aligned}$$

Notice that from Problem 1.C that $P \equiv 375 \pmod{1633} \equiv 2008 \pmod{1633}$, so we essentially have gotten our original $P = 2008 = \text{'UI'}$ back. In practice, if we pick a m greater than our highest character coder, we will avoid the problem like having $375 \equiv 2008$. This is usually the case, because we use ASCII code with a highest value around 256, so a 4 block code would equal a highest value of something like 02560256. m will be a hundred or so digits long, so the problem is solved.

`\section*{Problem 2}`

`\subsection*{A.}`

The argument that there is no fundamental order or rule to the universe is not currently a strong one. We can easily see th

`\end{document}`