Math/CS 395 - Analysis of Algorithms - Spring 2012

Homework 6

Assigned: Wednesday, March 21, 2012

Due: **Wednesday, March 28, 2012**

Strassen's algorithm

1. (problem 4.2-1) Use Strassen's algorithm to compute the matrix product

$$\begin{pmatrix} 1 & 3 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 6 & 8 \\ 4 & 2 \end{pmatrix}$$

2. (problem 4.2-2) Write pseudocode for Strassen's algorithm.

3. (problem 4.2-5) V. Pan discovered a way of multiplying 68×68 matrices using 132,464 multiplications, a way of multiplying 70×70 matrices using 143,640 multiplications, and a way of multiplying 72×72 matrices using 155,424 multiplications. Which method yields the best asymptotic running time when used in a divide-and-conquer matrix-multiplication algorithm? How does it compare to Strassen's algorithm?

4. (problem 4.2-7) Show how to multiply the complex numbers $a + bi$ and $c + di$ using only three multiplications of real numbers. The algorithm should take a, b, c, and d as input and produce the real component $ac - bd$ and the imaginary component $ad + bc$ separately.

Substitution method

5. (problem 4.3-1) Show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$.

6. (problem 4.3-7) Using the master method in Section 4.5, show that the solution to the recurrence $T(n) = 4T(n/3) + n$ is $T(n) = \Theta(n^{\log_3 4})$. Show that a substitution proof with the assumption $T(n) \leq cn^{\log_3 4})$ fails. Then show how to subtract off a lower-order term to make a substitution proof work.