CS 395 Homework 10

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Grade: ____________________

PROBLEMS

1. 7.1-1

Consider the array $A = \{13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11\}$. The PARTITION(A) algorithm operations are as follows:

a).

p,j											r
13	19	9	5	12	8	7	4	21	2	6	11

b).

p,i	j										r
13	19	9	5	12	8	7	4	21	2	6	11

c).

p,i		j									r
13	19	9	5	12	8	7	4	21	2	6	11

d).

i	p		j								r
9	19	13	5	12	8	7	4	21	2	6	11

e).

	i	p	j								r
9	5	13	19	12	8	7	4	21	2	6	11

f).

p	i	p			j						r
9	5	13	19	12	8	7	4	21	2	6	11

g).

p		i				j					r
9	5	8	19	12	13	7	4	21	2	6	11

h).

p			i				j				r
9	5	8	7	12	13	19	4	21	2	6	11

i).

p				i			j				r
9	5	8	7	4	13	19	12	21	2	6	11

j).

p				i				j			r
9	5	8	7	4	13	19	12	21	2	6	11

k).

p					i				j		r
9	5	8	7	4	2	19	12	21	13	6	11

l).

p						i				j	r
9	5	8	7	4	2	6	12	21	13	19	11

m).

p						i				j	r
9	5	8	7	4	2	6	11	21	13	19	12

2. 7.2-2

In Quicksort, if every element is the same, then the call to PARTITION(A) in Quicksort always returns r - 1. This leads to the worst case partitions, and Quicksort runtime = $T(n) = \Theta(n^2)$ (worst case runtime).

3. 7.3-1

The randomization won't improve the worst case, it just makes the chances of hitting the worst case small. So we don't consider it.

4. 7.4-1

Considering the equation $T(n) = max_{0 \leq q \leq n-1}(T(q) + T(n-q-1)) + \Theta(n)$, we will show that the running time $T(n) = \Omega(n^2)$.

Inductive Hypothesis

Given $T(n) = \Omega(n^2)$, we assume:

$$T(n) \geq cn^2 \tag{1}$$

Where c is a constant.

Further:

$$T(n) \geq max_{0 \leq q \leq n-1}(q^2 + c(n-q-1)^2) + \Theta(n) \tag{2}$$
$$= (c)max_{0 \leq q \leq n-1}(q^2 + (n-q-1)^2) + \Theta(n) \tag{3}$$
$$\tag{4}$$

The minimum for $q^2 + (n-q-1)^2$ can be found in the range $0 \leq q \leq n-1$. We can then say:

$$max_{0 \leq q \leq n-1}(q^2 + (n-q-1)^2) \leq (n-1)^2 = n^2 - 2n + 1 \tag{5}$$
$$\tag{6}$$

Furthermore:

$$T(n) \geq cn^2 - c(2n-1)\Theta(n) \tag{7}$$
$$\geq cn^2 \tag{8}$$

Therefore, runtime $T(n) = \Omega(n^2)$.