# CS 395 Homework 6

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Due March 27th, 2012

Grade: ____________________

# PROBLEMS

## 1.

First, the matrix product using Strassen's algorithm is as follows:

$$A = \left(\begin{smallmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{smallmatrix}\right) B = \left(\begin{smallmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{smallmatrix}\right) \tag{1}$$

$$A = \left(\begin{smallmatrix} 1 & 3 \\ 7 & 5 \end{smallmatrix}\right) B = \left(\begin{smallmatrix} 6 & 8 \\ 4 & 2 \end{smallmatrix}\right) \tag{2}$$

We then find the $\frac{n}{2} * \frac{n}{2}$ matrices:

$$S_1 = B_{12} - B_{22} = 8 - 2 = 6 \tag{3}$$
$$S_2 = A_{11} + A_{12} = 1 + 3 = 4 \tag{4}$$
$$S_3 = A_{21} + A_{22} = 7 + 5 = 12 \tag{5}$$
$$S_4 = B_{21} - B_{11} = 4 - 6 = -2 \tag{6}$$
$$S_5 = A_{11} + A_{22} = 1 + 5 = 6 \tag{7}$$
$$S_6 = B_{11} + B_{22} = 6 + 2 = 8 \tag{8}$$
$$S_7 = A_{12} - A21 = 3 - 5 = -2 \tag{9}$$
$$S_8 = B_{21} + B_{22} = 4 + 2 = 6 \tag{10}$$
$$S_9 = A_{11} - A_{21} = 1 - 7 = -6 \tag{11}$$
$$S_{10} = B_{11} + B_{12} = 6 + 8 = 14 \tag{12}$$

We then compute the product of A and B matrices as follows:

$$P_1 = A_{11} * S_1 = 1 * 6 = 6 \tag{13}$$
$$P_2 = S_2 * B_{22} = 4 * 2 = 8 \tag{14}$$
$$P_3 = S_3 * B_{11} = 12 * 6 = 72 \tag{15}$$
$$P_4 = A_{22} * S_4 = 5 * (-2) = -10 \tag{16}$$
$$P_5 = S_5 * S_6 = 6 * 8 = 48 \tag{17}$$
$$P_6 = S_7 * S_8 = -2 * 6 = -12 \tag{18}$$
$$P_7 = S_9 * S_{10} = (-16) * 14 \tag{19}$$
$$= -84 \tag{20}$$

The $\frac{n}{2} * \frac{n}{2}$ sub matrices of the product:

$$\begin{aligned}
C_{11} &= P_5 + P_4 - P_2 + P_6 && (21)\\
&= 48 - 10 - 8 - 12 && (22)\\
&= 18 && (23)\\
C_{12} &= P_1 + P_2 && (24)\\
&= 6 + 8 && (25)\\
&= 14 && (26)\\
C_{21} &= P_3 + P_4 && (27)\\
&= 72 - 10 && (28)\\
&= 62 && (29)\\
C_{22} &= P_5 + P_1 - P_3 - P_7 && (30)\\
&= 48 + 6 - 72 + 84 && (31)\\
&= 66 && (32)
\end{aligned}$$

The resulting matrix for $C$ is:

$$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \quad (33)$$

$$C = \begin{pmatrix} 18 & 14 \\ 62 & 66 \end{pmatrix} \quad (34)$$

**2.**

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STRASSENS_ALG(A,B)

n = rows in A

A = matrix with n/2 x n/2 dimensions
B = matrix with n/2 x n/2 dimensions
C = matrix with n/2 x n/2 dimensions

 if == 2
        S1 = B12 - B22
        S2 = A11 + A12
        S3 = A21 + A22
        S4 = B21 - B11
        S5 = A11 + A22
        S6 = B11 + B22
        S7 = A12 - A22
        S8 = B21 + B22
        S9 = A11 + A21
        S10 = B11 + B12

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        P1 = A11 * S1
        P2 = S2 * B22
        P3 = S3 * B11
        P4 = A22 * S4
        P5 = S5 * S6
        P6 = S7 * S8
        P7 = S9 * S10
        C11 = P5 + P4 - P2 + P6
        C12 = P1 + P2
        C21 = P3 + P4
        C22 = P5 + P1 - P3 - P7

return C
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**3.**

Consider the recurrence for each:

$$\begin{aligned} T(n) &= 132464T(n/68) + n^2 \\ &= \Theta(n^{lg_{68}132464}) \\ &\approx \Theta(n^{2.795128}) \end{aligned}$$

$$\begin{aligned} T(n) &= 143640T(n/70) + n^2 \\ &= \Theta(n^{lg_{70}143640}) \\ &\approx \Theta(n^{2.795128}) \end{aligned}$$

$$\begin{aligned} T(n) &= 155424T(n/72) + n^2 \\ &= \Theta(n^{lg_{72}155424}) \\ &= \Theta(n^{2.795147}) \end{aligned}$$

Strassen's algorithm runs in $\Theta(n^{lg7}) \approx \Theta(n^{2.81})$. All of the above outperform Strassen's algorithm.

**4.**

Let $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$.

Now consider the following:

$$p_1 = (a + b) * (c + d) \tag{35}$$

$$p_2 = a * c \tag{36}$$

$$p_3 = b * d \tag{37}$$

$$ac - bd = p_2 - p_3 \tag{38}$$

$$ad + bc = p_1 = p_2 - p_3 \tag{39}$$

Using only $p_1, p_2$, and $p_3$, the product of two complex numbers can be computed.

**5.**

Using the substitution method, we will prove that $T(n) = T(n - 1) + n$ is $O(n^2)$.

**Assumption**

$T(n) = T(n-1) + n$

**Substitution**

$$T(n) = T(n-1) + n \tag{40}$$
$$T(n-1) = T(n-2) + (n-1) \tag{41}$$
$$T(n-2) = T(n-3) + (n-2) \tag{42}$$
$$... \tag{43}$$

Adding up the equations as follows:

$$T(n-1) + T(n-2) + ...T(2) + T(1) + n + (n-1) + n(-2) + ...3 + 2 + 1 \tag{44}$$
$$T(n) = n + (n-1) + (n-2) + ...2 + 1 \tag{45}$$
$$= \frac{n(n+1)}{2} \tag{46}$$
$$= O(n^2) \tag{47}$$

Therefore, we have shown that $T(n) = O(n^2)$.

## 6.

We will show that $T(n) = 4T(\frac{n}{3}) + n$ is $T(n) = \Theta(n^{log_3 4})$. Consider $a = 4, b = 3$. $n^{log_b a} = n^{log_3 4} = \Theta(n^2), f(n) = O(n^{log_3 4 - \epsilon})$, when $\epsilon = 1$. Applying case 1 of the Master Theorem, we conclude that the solution is $T(n) = \Theta(n^2)$.

Using the **Substitution Method**, we assume with the induction hypothesis that $T(n/3) \leq c(\frac{n}{3})^2 - c(\frac{n}{3})$. Consider:

$$T(n) = 4T(\frac{n}{3}) + n \tag{48}$$