More Money for Some: The Redistributive Effects of Open Market Operations*

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Abstract

Using a general equilibrium search-theoretic model of money, I study the distributional effects of open market operations. In my model, heterogeneous households trade bilaterally among themselves in a frictional market and save using cash and illiquid short-term nominal government bonds. Wealth effects generate slow adjustments in household portfolios following their trading activity in decentralized markets, giving rise to a persistent and nondegenerate distribution of assets. The model reproduces the distribution of asset levels and portfolios across households observed in the data, which are crucial to quantitatively assess the incidence of monetary policy changes at the individual level. I find that an open market operation targeting a higher nominal interest rate requires increasing the relative supply of bonds, raising the ability of households to self-insure against idiosyncratic shocks. As a result, in the long run, inequality falls, and the inefficiencies in decentralized trading shrink. This leads households that are relatively poor and more liquidity-constrained to benefit the most by increasing their consumption and welfare.

Keywords: open market operations, monetary economics, search theory, heterogeneous agents.
JEL Codes: E21, E32, E52.

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A Appendix

A.1 Solution Method

The numerical method employed here to solve for the stationary equilibrium, to my knowledge, has not been used in the literature of search-theoretic models of money. Although most of the papers in this area use assumptions that guarantee analytical tractability, the few documents that have implemented numerical methods to account for the intertemporal heterogeneity in money holdings have employed solution strategies that are either costly in terms of efficiency, that may sacrifice accuracy, and that are not easily extended to higher dimensions. For example, the numerical method in Molico (2006) and Chiu and Molico (2010, 2011) require to simulate matches between households and then perform a kernel density estimation of the distribution of money with these observations. Given that we must consider every possible meeting, the size of the simulation has to be large enough, so we have several observations for each match. This also implies that the required number of simulated matches increases with the dimension of the portfolio, as the number of possibles matches may increase. In addition to this, having to use kernel estimation may imply to lose accuracy in the estimated distribution.

My method solves iteratively for the agents' decision rules, the terms of trade in decentralized trading, the level of lump-sum transfers consistent with the government balance, and the prices that clear the markets of bonds and money. We start by solving the agents' value functions in (9) and (10). This requires updating $V(m, a; F, \theta)$ and $W(x; G, \theta)$ one after the other. Doing this for W in the decentralized market is standard, as long as we have how to compute the continuation value V. However, when computing the value function in the decentralized market, we need to know the entire distribution $F(m, a) : \mathcal{M} \times \mathcal{A} \to \mathbb{R}$ and the terms of trade for each possible meeting between agents with (m_b, a_b) and (m_s, a_s), where $(m_i, a_i) \in \mathcal{M} \times \mathcal{A}$ for $i = b, s$. Therefore, we must start not only with some guess for W as in any value function iteration approach, but also with guesses for $F(m, a)$, and $\langle Q, D \rangle = \langle q(m_b, a_b, m_s, a_s; F, \theta), d(m_b, a_b, m_s, a_s; F, \theta) \rangle$. To do this, we discretize our state space $\mathcal{M} \times \mathcal{A}$, so that $F(m, a)$ and $\langle Q, D \rangle$, as well as $V(m, a; F, \theta)$ are defined over a finite number of points. Similarly, when evaluating $W(x; G, \theta) : \mathcal{X} \to \mathbb{R}$, we need to discretize $\mathcal{X}$. Given this, we can solve the stationary equilibrium of this economy as follows:

1. Solve for V and W given some $F(m, a)$, some terms of trade $\langle Q, D \rangle$, a vector of prices (ϕ_m, ϕ_a) and some guess for the transfers, τ. As a byproduct of this step,

we should have the decision rules for $m' = g_m(x; G, \theta)$ and $a' = g_a(x; G, \theta)$ in the centralized market.

There are several valid approaches to this. In particular, I use piece-wise cubic splines when evaluating continuation values that depend on W, solve the non-linear labor-leisure condition using the bisection method, and employ the Howard's improvement algorithm to speed up the solution of the CM problem in (10)-(12).

2. Given $F(m, a)$ and the current values in $\langle Q, D \rangle$, we can compute $G(m, a)$. Likewise, using $G(m, a)$ and the decision rules for m' and a', we can compute $F'(m, a)$. Repeatedly update F and G until these distributions converge.

3. Verify if markets for money and bonds are clearing, i.e., check if Equations (5) and (6) hold. If not, adjust the prices (ϕ_m, ϕ_a) and return to Step 1. Repeat this process until clearing both markets. Of course, when returning to Step 1, use the most recently computed value for F.

   When adjusting prices, an alternative is to use the excess demand functions of bonds and money. I do so using a partial updating approach.

4. Check if the government balance in Equation (2) is satisfied. In case it is not, adjust the tax rate in Equation (3) using the most recent value for ϕ_a and go back to Step 1.

5. We finally must update the terms of trade $\langle Q, D \rangle$. The key input here is the value function W, that is required to compute the continuation values in the problem in (7)-(8). As in Step 1, I use piece-wise cubic splines on W. Check if the newly calculated terms of trade are close enough to their previous values. If not, return to Step 1. If so, we are done.

B Persistent, Non-Degenerate Distribution with Only Money

Below I present a version of the model where money is the only asset and, hence, monetary policy do not operate through open market operation but via lump-sum transfers to households. This model economy resembles the one in Lagos and Wright (2005), but with non-quasi-linear preferences. This simplified model illustrates how much heterogeneity this environment can generate as it moves away from having a quasi-linear utility function in the CM and we add more curvature to the labor supply. It is also helpful to explain the numerical solution method in a simpler manner.

As in the full model, each period is divided into decentralized and centralized markets operating sequentially. Matching in the decentralized market occurs as in the full model, with households trading using only money and terms of trade being determined by a "take it or leave it" offer of the buyer to the seller. The households' period utility function is as in (1), with U not restricted to be linear in h. The production technologies in both the decentralized and centralized markets operate as in the full model.

The monetary authority controls the supply of the only available asset to households: fiat money, M. The supply of money grows at the rate μ. At the beginning of every CM, the monetary authority transfers μM to every agent in the economy before the centralized market begins. This implies that the stock of aggregate nominal money evolves according to:

$$M' = (1 + \mu) M \tag{20}$$

where M' is nominal money supply at the middle of the current period, and therefore, at the beginning of next period. Individual nominal money holdings are normalized with respect to the beginning of the period money supply, M. This means that, if a household holds $\hat{m}$ units of nominal money, then it has $m = \hat{m}/M$ units of relative money holdings. Note that money injections μM are equivalent to μ units of relative money.

Using a similar notation as before, the distribution of individuals over m are summarized by the probability measures $F(m)$ and $G(m)$ for the DM and CM, respectively. These distributions evolve according to $G = \Gamma_G(F, \mu)$ and $F' = \Gamma_F(G, \mu, \mu')$. Given this, and the policy implemented by the monetary authority, in equilibrium we have:

$$\int_{\mathcal{M}} m dF(m) = 1 \tag{21}$$

$$\int_{\mathcal{M}} m dG(m) = 1 + \mu \tag{22}$$

Let $V(m; F, \mu)$ and $W(m; G, \mu)$ be the value functions at the beginning of the DM and CM, respectively. As noted before, without loss of generality, we assume that there are not double-coincidence meetings. In single-coincidence meetings, a buyer and a seller with relative money holdings m_b and m_s are matched. Buyers make a "take it or leave it" offer to the seller: the buyer offer to buy q of the differentiated good at the price d. The terms of such an offer are determined according to the following problem:

$$\max_{q,d} \; u(q) + W(m_b + \mu - d; G, \mu) \tag{23}$$

subject to the seller's participation constraint

$$-v(q) + W(m_s + \mu + d; G, \mu) \geq W(m_s + \mu; G, \mu) \tag{24}$$

to the law of motion of the distribution, $G(m) = \Gamma_G(F(m), \mu)$, and to $0 \leq d \leq m_b$, $q \geq 0$. Note that the continuation values of buyers and sellers take into account their money holdings when exiting the current DM and the transfer they will receive at the beginning of the next CM. For each meeting (m_b, m_s), and given an aggregate state $\{F(m), \mu\}$, we have that the terms of trade $q(m_b, m_s; F, \mu)$ and $d(m_b, m_s; F, \mu)$ solve the problem stated above.

In this context, the expected lifetime utility at the beginning of the DM of an agent with money holdings m, i.e., before knowing if they are matched or not, and before knowing what would be their role in an eventual match, is given by the following functional equation:

$$\begin{aligned} V(m; F, \mu) = & \frac{\alpha}{2} \int_{\mathcal{M}} \{ \; u(q(m, m_s; F, \mu)) + W(m + \mu - d(m, m_s; F, \mu); G, \mu)\} \, dF(m_s) \\ & + \frac{\alpha}{2} \int_{\mathcal{M}} \{-v(q(m_b, m; F, \mu)) + W(m + \mu + d(m_b, m; F, \mu); G, \mu)\} \, dF(m_b) \\ & + (1 - \alpha) W(m + \mu; G, \mu) \end{aligned} \tag{25}$$

Let $\tilde{m}$ denote relative money holdings at the beginning of the CM. Money holdings at the beginning of the CM are money holdings at the end of the previous DM, plus lump-sum transfers of money. For an agent with relative money holdings $\tilde{m}$, the lifetime value of their utility at the beginning of the CM is given by:

$$W(\tilde{m}; G, \mu_i) = \max_{c,h,m'} \left\{ U(c, h) + \beta \sum_{j=1}^{N_\mu} \pi^{\mu}_{i,j} V\left(m'; F', \mu_j\right) \right\} \tag{26}$$

subject to the budget constraint

$$c = h + \phi\left(G, \mu_i\right)\left[\tilde{m} - \left(1+\mu\right)m'\right] \tag{27}$$

and to the mapping of $G\left(m\right)$ into $F'\left(m\right)$

$$F'\left(m\right) = \Gamma_F\left(G\left(m\right), \mu_i, \mu'\right) \tag{28}$$

For the utility and cost functions in the decentralized market, as in the full model, I follow Lagos and Wright (2005). However, for the centralized market I drop the quasi-linearity assumption. In particular, the utility function in the CM is concave in both consumption and leisure and given by:

$$U\left(c, h\right) = \log c - \kappa\frac{h^{1+\chi}}{1+\chi} \tag{29}$$

where χ is the inverse of the Frisch elasticity of labor supply and $\kappa > 0$ a scale parameter. Note that (29) nests the quasi-linear utility function in Lagos and Wright (2005) when $\chi = 0$ and $\kappa = 1$.

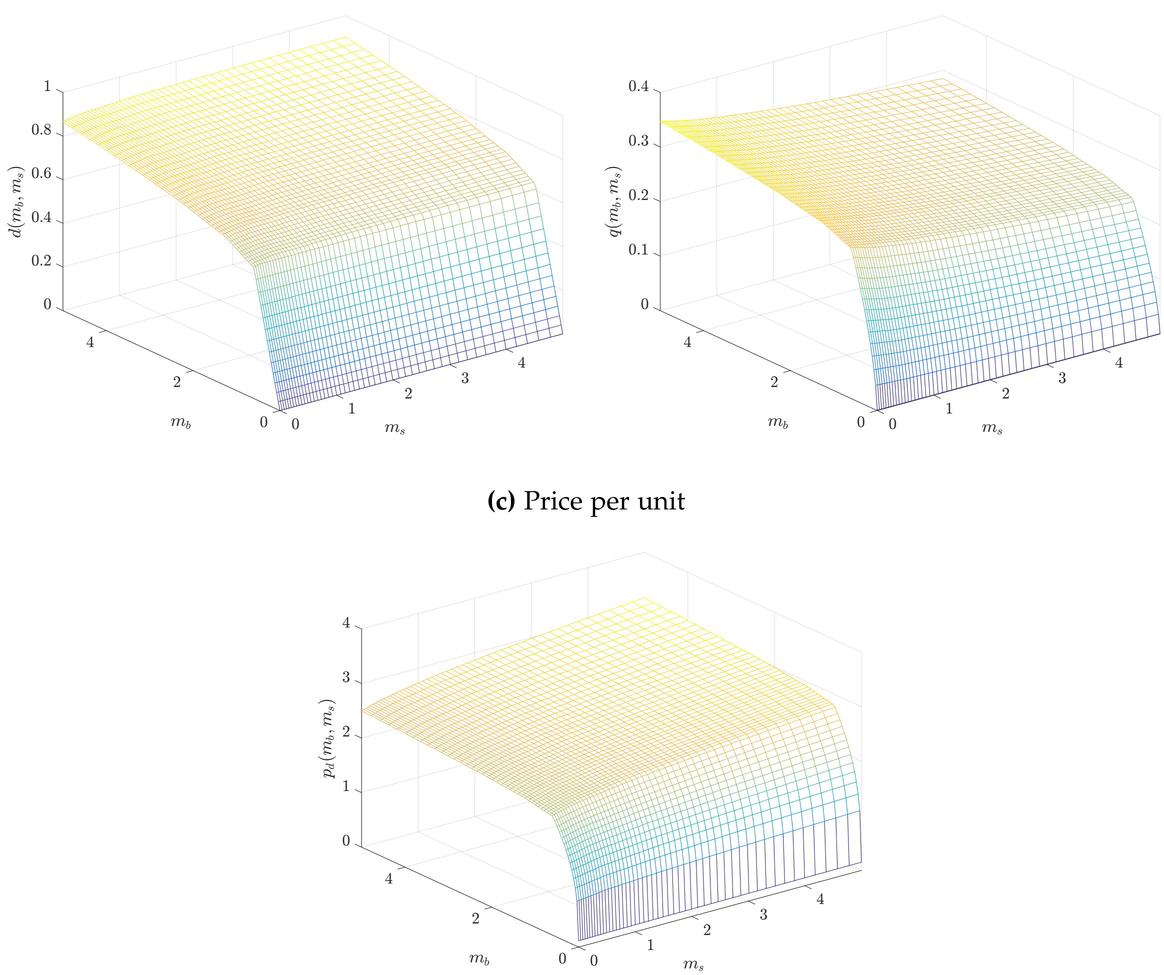
The model is parametrized as closely as possible to Lagos and Wright (2005) for comparison purposes. One model period is equivalent to one year. The discount factor is set to generate a real interest rate of 4 percent, and the money growth rate is consistent with an inflation rate, in the stationary equilibrium, of 2 percent. The probability of being matched in the decentralized market, α, is set to 1 to minimize the role of the search frictions. The inverse of the Frisch elasticity is chosen to be 0.5. The scale parameters B and κ, as the curvature parameter ν pining down the cost of producing in the DM are chosen to calibrate the size of the centralized market to be 10 percent of total output, a velocity of 2 and the average markup of 30 percent reported by Faig and Jerez (2005). Finally, the parameter that governs the curvature of the utility function in the DM, η, is set close to 1, so that the utility function is close to being logarithmic.

B.1 Stationary Equilibrium

In a stationary equilibrium the distribution over relative money holdings remains constant, i.e., $F = \Gamma_F\left(\Gamma_G\left(F, \mu\right), \mu, \mu'\right)$. This, in turn, implies $\phi = \phi'$ and requires the growth rate of prices, π, to be equal to the growth rate of money, μ. In contrast to models like Shi (1997) or Lagos and Wright (2005), the distribution of money holdings is not necessarily

Figure B4: Terms of Trade in the Decentralized Market

(a) Payments **(b)** Quantities



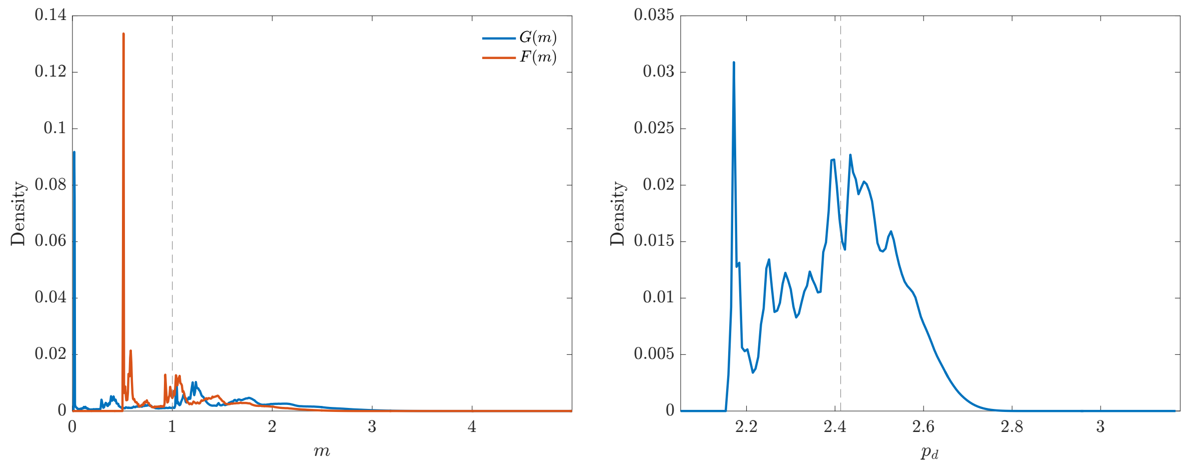
(c) Price per unit

degenerate across periods.

Figure B4 shows the stationary terms of trade $d(m_b, m_s; F, \mu)$ and $q(m_b, m_s; F, \mu)$. As opposed to models where preferences in the CM are linear in labor, here, as in the full model, terms of trade depend not only on the buyer's money holdings but also on the seller's. For a fixed level of m_s, larger values of m_b imply that the buyer can obtain more of the differentiated good but at a higher total cost. On the other hand, keeping fixed m_b, a seller with larger money balances requires a higher payment for lower quantities of the good he produces because of the seller's continuation value (captured by the function W) is now concave in m. Consequently, as Figure B4(c) shows, the price per unit of good exchanged, $p_d(m_b, m_s) = d(m_b, m_s) / q(m_b, m_s)$, mainly responds to the seller's wealth.

Figure B5: Distributions at the Stationary Equilibrium

(a) Distribution of Money Holdings **(b)** Distribution of Prices



The chance of having different types of meetings with heterogeneity in both q and d, plus the fact that preferences in the CM are not quasi-linear, generates a non-degenerate distribution of money holding, as shown in Figure B5(a). In that Figure, we have the distributions $F(m)$ and $G(m)$. The distribution entering the CM, unsurprisingly, is more disperse that the distribution at the beginning of the DM, reflecting the heterogeneity-inducing effect of decentralized trading. We can also observe that, after trading in the DM, some agents end up with very little money holdings (the money injections prevent them from reaching $m = 0$). This is the result of a combination of sequences of meetings with sellers that make the buyer deplete his money holdings. However, even for this "unlucky" type of agent, the centralized market acts as an insurance market that let them replenish, at least partially, their liquidity for the next round of decentralized trading. Note that some agents reaching the lower end of the distribution may experience similar matches in the future as they move away from $m = 0$. This explains the spikes in $F(m)$ and why they get diluted for higher values of money holdings.

Figure B5(b), shows the observed distribution of prices. This distribution comes from taking into account the likelihood of all possible meetings implied by the distribution $F(m)$ and the prices $p_d(m_b, m_s)$ at each one of them. In this context, the model is able to produce a non-degenerate distribution of prices that reflects the differences in assets between different agents in this economy.

B.2 The Role of the Elasticity of Labor Supply

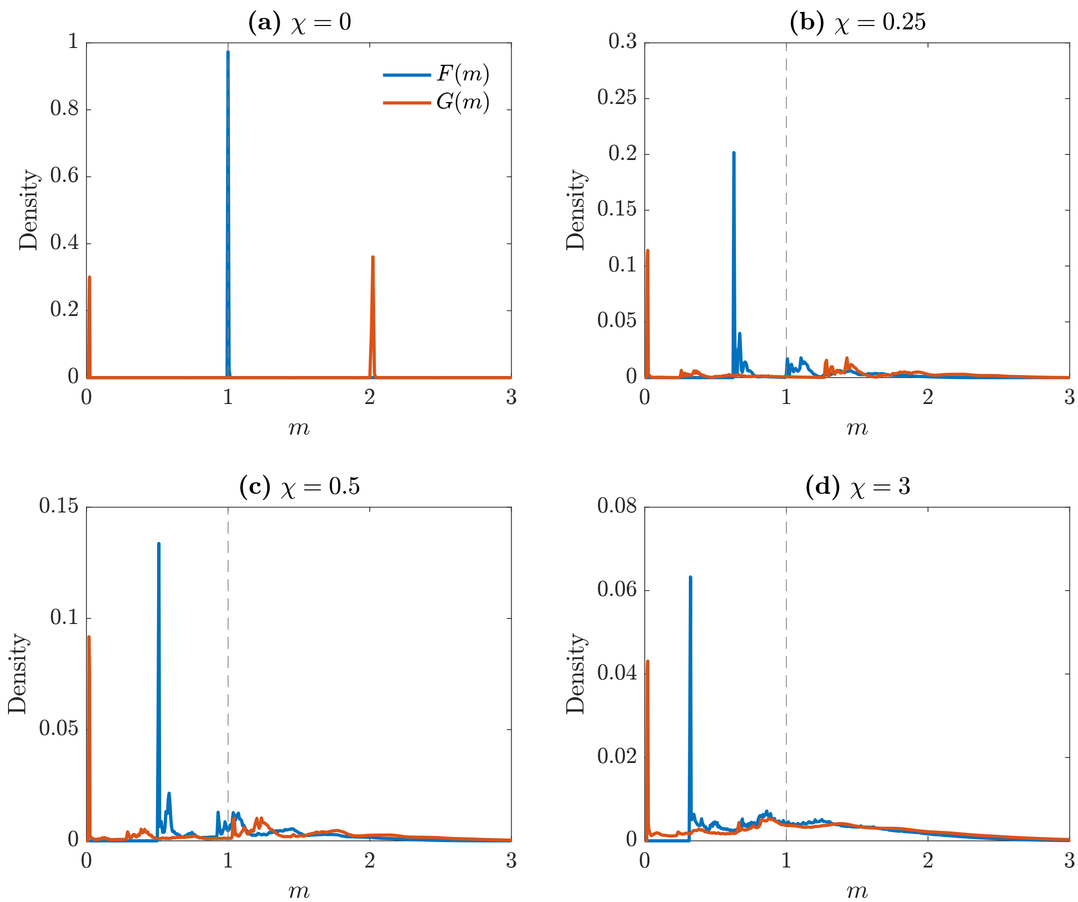
The elasticity of labor supply is crucial in determining the extent to which households can reshuffle their money holdings after trading in the decentralized market. The higher this elasticity, the easier for households to offset their previous idiosyncratic trading histories. In Lagos and Wright (2005), labor supply is infinitely elastic, so agents are able to completely undo in the CM whatever happened in their previous DM meeting. In this model, having a perfectly elastic labor supply is equivalent to $\chi = 0$.

Figure B6 shows the stationary distributions of money at the beginning of the DM and CM for different values of χ, the inverse of the Frisch elasticity of labor. When $\chi = 0$, we obtain a degenerate distribution of money in the DM (blue line in panel (a) of Figure B6): all households are holding the same amount of relative units of money. As these agents trade in decentralized meetings, buyers deplete their money holdings, while sellers end up with about double of what they initially had (red line).[14] However, the fact that, for this case, labor is perfectly elastic, allows agents to reshuffle back their money holdings completely . The decision rules for labor and money holdings in Figure B7(a) and Figure B7(b) show that labor supply changes linearly with the individual state, m, so that households can exactly exit the period with 1 unit of relative money. This is a consequence of the absence of wealth effects under quasi-linear preferences.

When the Frisch elasticity of labor supply ($1/\chi$) becomes finite, the inability of households of offsetting the effects of their previous trading history results in a non-degenerate distribution of money in the DM. In fact, as the elasticity of labor supply decreases, we obtain distributions of money holdings with less concentration.

[14] The masses at two points at which the distribution of money balances at the CM concentrate do not appear to add up to 0.5 in the figure because of a small numerical error. These two points do not necessarily are in the grid $\mathcal{M}$, so that the mass of households gets distributed between the two adjacent points. In any case, adding the masses at those adjacent points gives exactly 0.5 for each one.

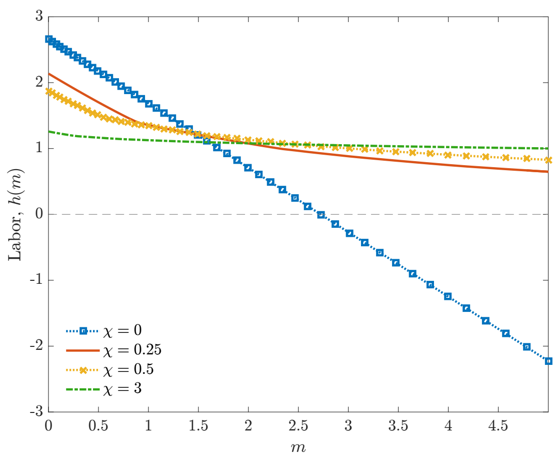
Figure B6: Distributions of Money Balances at the Stationary Equilibrium for Different Levels of the Frisch Elasticity of Labor



Notes: Stationary distributions of money holdings at the beginning of the decentralized market, $F(m)$, and the beginning of the centralized market, $G(m)$, for different values of the Frisch elasticity of labor supply, $1/\chi$.

Figure B7: Decision Rules in the Centralized Market at the Stationary Equilibrium for Different Levels of the Frisch Elasticity of Labor

(a) Decision Rules for Labor



(b) Decision Rules for Money Holdings