The Redistributive Effects of Changes in Public Liquidity*

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Abstract

Using a general equilibrium search-theoretic model of money, I study the distributional effects of permanent changes in the provision of public liquidity. In my model, heterogeneous agents trade bilaterally among themselves in a frictional market and save using cash and illiquid short-term nominal government bonds. Wealth effects generate slow adjustments in agents' portfolios following their trading activity in decentralized markets, giving rise to a persistent and non-degenerate distribution of assets. The model reproduces the distribution of asset levels and portfolios across households observed in the data, which is crucial to quantitatively assess the incidence of monetary policy changes at the individual level. I find that changes in public liquidity targeting a higher nominal interest rate requires increasing the relative supply of bonds, raising the ability of agents to self-insure against idiosyncratic shocks. As a result, in the long run, inequality falls, and the inefficiencies in decentralized trading shrink. This leads agents that are relatively poor and more liquidity-constrained to benefit the most by increasing their consumption and welfare.

Keywords: public liquidity, monetary economics, search theory, heterogeneous agents.
JEL Codes: E21, E32, E52.

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Appendix

D Persistent, Nondegenerate Distribution with Only Money

Below I present a version of the model where money is the only asset and, hence, monetary policy does not operate through OMOs but via lump-sum transfers to agents. This model economy resembles the one in Lagos and Wright (2005), but with non-quasi-linear preferences. This simplified model illustrates how much heterogeneity this environment can generate as it moves away from having a quasi-linear utility function in the CM and we add more curvature to the labor supply. It is also helpful to explain the numerical solution method in a simpler manner.

As in the full model, each period is divided into decentralized and centralized markets operating sequentially. Matching in the DM occurs as in the full model, with agents trading using only money and terms of trade being determined by a "take it or leave it" offer of the buyer to the seller. The agents' period utility function is as in the full model, with U not restricted to be linear in h. The production technologies in both the decentralized and centralized markets operate as in the full model.

The monetary authority controls the supply of the only available asset to agents: fiat money, M. The supply of money grows at the rate μ. At the beginning of every CM, the monetary authority transfers μM to every agent in the economy before the CM begins. This implies that the stock of aggregate nominal money evolves according to:

$$M' = (1+\mu) M, \tag{D.1}$$

where M' is the nominal money supply at the middle of the current period, and therefore, at the beginning of next period. Individual nominal money holdings are normalized with respect to the beginning of the period money supply, M. This means that if an agent holds $\hat{m}$ units of nominal money, then they have $m = \hat{m}/M$ units of relative money holdings. Note that money injections μM are equivalent to μ units of relative money.

Following a similar notation as before, the distribution of agents over m are summarized by the probability measures $F(m)$ and $G(m)$ for the DM and CM, respectively. These distributions evolve according to $G = \Gamma_G(F, \mu)$ and $F' = \Gamma_F(G, \mu)$. Given this, and the policy implemented by the monetary authority, in equilibrium we have:

$$\int_{\mathcal{M}} mdF(m) = 1 \tag{D.2}$$

$$\int_{\mathcal{M}} mdG(m) = 1 + \mu. \tag{D.3}$$

Let $V(m; F, \mu)$ and $W(m; G, \mu)$ be the value functions at the beginning of the DM and CM, respectively. As noted before, without loss of generality, we assume that there are not double-coincidence meetings. In single-coincidence meetings, a buyer and a seller with relative money holdings m_b and m_s are matched. The buyer makes a "take it or leave it" offer to the seller: the buyer offers to buy q of the differentiated good at the price d. The terms of such an offer are determined according to the following problem:

$$\max_{q,d} \; u(q) + W(m_b + \mu - d; G, \mu) \tag{D.4}$$

subject to the seller's participation constraint

$$-v(q) + W(m_s + \mu + d; G, \mu) \geq W(m_s + \mu; G, \mu) \tag{D.5}$$

to the law of motion of the distribution, $G(m) = \Gamma_G(F(m), \mu)$, and to $0 \leq d \leq m_b$, $q \geq 0$. Note that the continuation values of buyers and sellers take into account their money holdings when exiting the current DM and the transfer they will receive at the beginning of the next CM. For each meeting (m_b, m_s), and given an aggregate state $\{F(m), \mu\}$, we have that the terms of trade $q(m_b, m_s; F, \mu)$ and $d(m_b, m_s; F, \mu)$ solve the problem stated above.

In this context, the expected lifetime utility at the beginning of the DM of an agent with money holdings m, i.e., before knowing if they are matched or not, and before knowing their role in an eventual match, is given by the following functional equation:

$$\begin{aligned} V(m; F, \mu) = & \frac{\alpha}{2} \int_{\mathcal{M}} \{ \;\; u(q(m, m_s; F, \mu)) + W(m + \mu - d(m, m_s; F, \mu); G, \mu)\} \, dF(m_s) \\ & + \frac{\alpha}{2} \int_{\mathcal{M}} \{-v(q(m_b, m; F, \mu)) + W(m + \mu + d(m_b, m; F, \mu); G, \mu)\} \, dF(m_b) \\ & + (1 - \alpha) W(m + \mu; G, \mu). \end{aligned} \tag{D.6}$$

Let $\tilde{m}$ denote relative money holdings at the beginning of the CM. Money holdings at the beginning of the CM are those at the end of the previous DM, plus lump-sum transfers of money. For an agent with relative money holdings $\tilde{m}$, the lifetime value of their utility at the beginning of the CM is given by:

$$W(\tilde{m}; G, \mu) = \max_{c,h,m'} \left\{ U(c,h) + \beta V\left(m'; F', \mu\right) \right\} \tag{D.7}$$

subject to the budget constraint

$$c = h + \phi(G, \mu)\left[\tilde{m} - (1+\mu)\, m'\right] \tag{D.8}$$

and to the mapping of $G(m)$ into $F'(m)$

$$F'(m) = \Gamma_F\left(G(m), \mu\right). \tag{D.9}$$

For the utility and cost functions in the DM, as in the full model, I follow Lagos and Wright (2005). However, for the CM I drop the quasi-linearity assumption. In particular, the utility function in the CM is concave in both consumption and leisure and given by:

$$U(c,h) = \log c - \kappa \frac{h^{1+\chi}}{1+\chi}, \tag{D.10}$$

where χ is the inverse of the Frisch elasticity of labor supply and $\kappa > 0$ a scale parameter. Note that (D.10) nests the quasi-linear utility function in Lagos and Wright (2005) when $\chi = 0$ and $\kappa = 1$.

The model is parametrized as closely as possible to Lagos and Wright (2005) for comparison purposes. One model period is equivalent to one year. The discount factor is set to generate a real interest rate of 4 percent, and the money growth rate is consistent with an inflation rate, in the stationary equilibrium, of 2 percent. The probability of being matched in the DM, α, is set to 1 to minimize the role of the search frictions. The inverse of the Frisch elasticity is chosen to be 0.5. I select B and κ (scale parameters), as well as ν (the curvature parameter that pins down the cost of producing in the DM) to reproduce the following three targets: (1) the size of the CM to be 10 percent of total output, (2) a velocity of money of 2, and (3) an average markup in DM meetings of 30 percent (Faig and Jerez, 2005). Finally, the parameter that governs the curvature of the utility function in the DM, η, is set close to 1, so that the utility function is close to being logarithmic.

D.1 Stationary Equilibrium

In a stationary equilibrium, the distribution over relative money holdings remains constant, i.e., $F = \Gamma_F(\Gamma_G(F, \mu), \mu)$. This, in turn, implies $\phi = \phi'$ and requires the growth rate of prices, π, to be equal to the growth rate of money, μ. In contrast to models like Shi (1997) or Lagos and Wright (2005), the distribution of money holdings is not necessarily degenerate across periods.

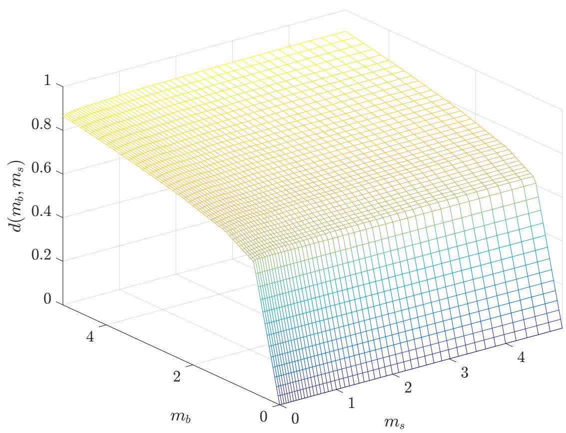
Figure D1 shows the stationary terms of trade $d(m_b, m_s; F, \mu)$ and $q(m_b, m_s; F, \mu)$. As opposed to models where preferences in the CM are linear in labor, here, as in the full model, terms of trade depend not only on the buyer's money holdings but also on the seller's. For a fixed level of m_s, larger values of m_b imply that the buyer can obtain more of the differentiated good but at a higher total cost. On the other hand, keeping fixed m_b, a seller with larger money balances requires a higher payment for lower quantities of the good he produces. This is because of the seller's continuation value (captured by the function W), which is now concave in m. Consequently, as Figure D1(c) shows, the price per unit of good exchanged, $p_d(m_b, m_s) = d(m_b, m_s) / q(m_b, m_s)$, mainly responds to the seller's wealth.

The chance of having different types of meetings with heterogeneity in both q and d, plus the fact that preferences in the CM are not quasi-linear, generates a nondegenerate distribution of money holding, as shown in Figure D2(a). In that figure, we have the distributions $F(m)$ and $G(m)$. The distribution entering the CM, unsurprisingly, is more dispersed than the distribution at the beginning of the DM, reflecting the heterogeneity-inducing effect of decentralized trading. We can also observe that, after trading in the DM, some agents end up with very little money holdings (the money injections prevent them from reaching $m = 0$). This is the result of a combination of sequences of meetings with sellers that make the buyer deplete her money holdings. However, even for this "unlucky" type of agent, the CM acts as an insurance market that lets her replenish, at least partially, her liquidity for the next round of decentralized trading. Note that some agents reaching the lower end of the distribution may experience similar matches in the future as they move away from $m = 0$. This explains the spikes in $F(m)$ and why they get diluted for higher values of money holdings.

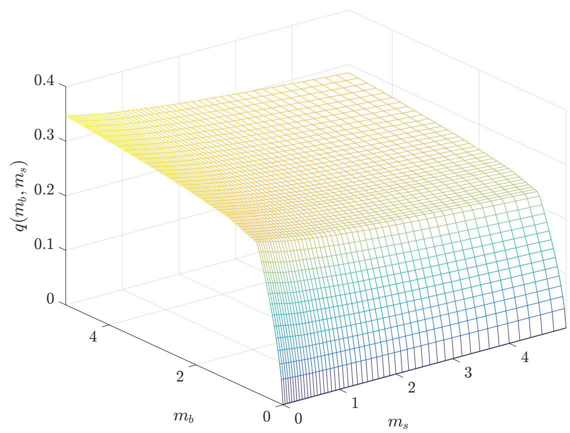
Figure D2(b), shows the observed distribution of prices. This distribution comes from taking into account the likelihood of all possible meetings implied by the distribution $F(m)$ and the prices $p_d(m_b, m_s)$ at each one of them. In this context, the model is able

Figure D1: Terms of Trade in the Decentralized Market

(a) Payments



(b) Quantities



(c) Price per unit

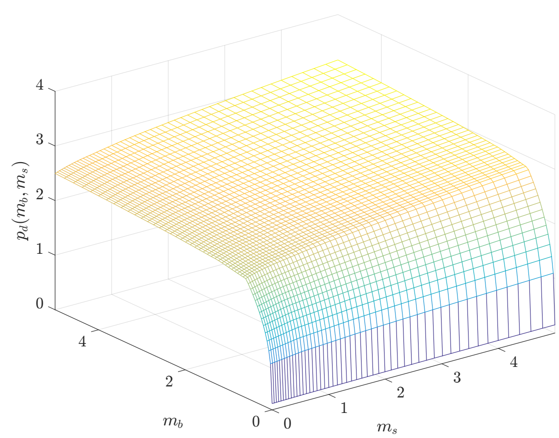
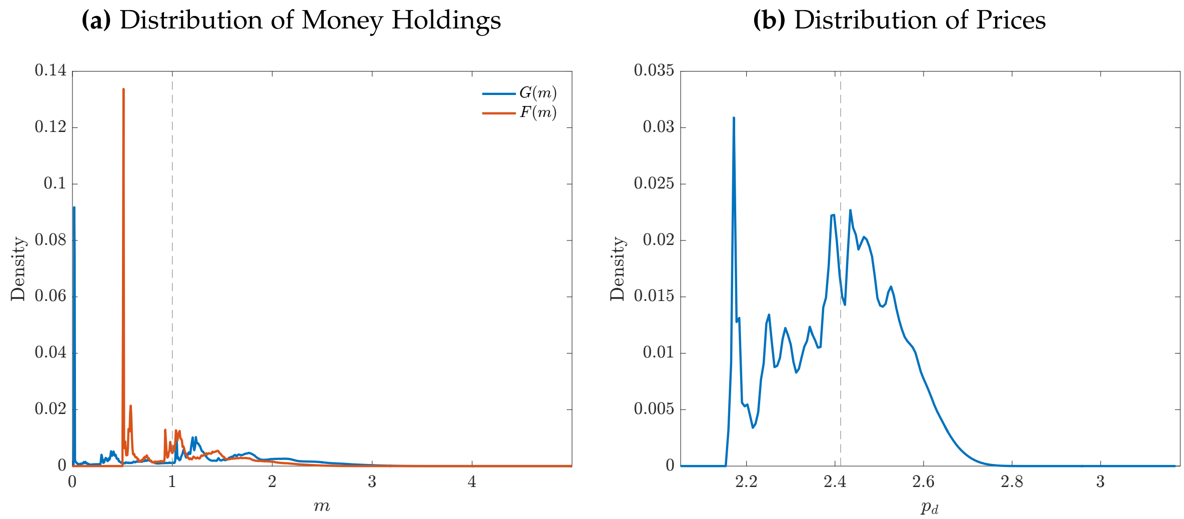


Figure D2: Distributions at the Stationary Equilibrium



to produce a nondegenerate distribution of prices that reflects the differences in assets between different agents in this economy.

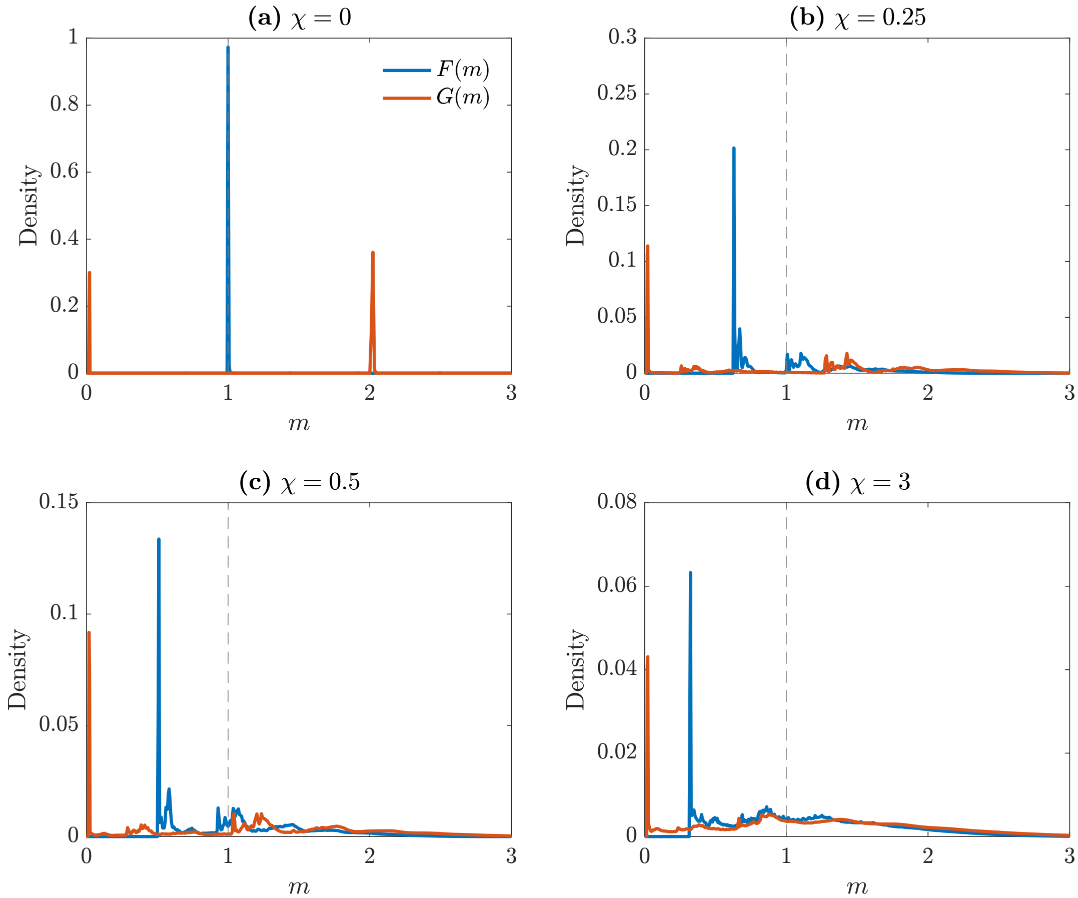
D.2 The Role of the Elasticity of Labor Supply

The elasticity of labor supply is crucial in determining the extent to which agents can reshuffle their money holdings after trading in the DM. The higher this elasticity, the easier for agents to offset their previous idiosyncratic trading histories. In Lagos and Wright (2005), labor supply is infinitely elastic, so agents are able to completely undo in the CM whatever happened in their previous DM meeting. In this model, having a perfectly elastic labor supply is equivalent to $\chi = 0$.

Figure D3 shows the stationary distributions of money at the beginning of the DM and CM for different values of χ, the inverse of the Frisch elasticity of labor. When $\chi = 0$, we obtain a degenerate distribution of money in the DM (blue line in panel (a) of Figure D3): all agents are holding the same amount of relative units of money. As these agents trade in decentralized meetings, buyers deplete their money holdings, while sellers end up with about double what they initially had (red line).[19] However, the fact that, for this case, labor is perfectly elastic allows agents to reshuffle back their money holdings

[19]When $\chi = 0$, the CM distribution is split equally between two points, each one with mass 0.5. In Figure D3, this does not appear to happen because of a small numerical error. These two points are not necessarily in the grid $\mathcal{M}$, so that the mass of agents gets distributed between their adjacent points. In any case, adding the masses at those adjacent points gives exactly 0.5 for each one.

Figure D3: Distributions of Money Balances at the Stationary Equilibrium for Different Levels of the Frisch Elasticity of Labor



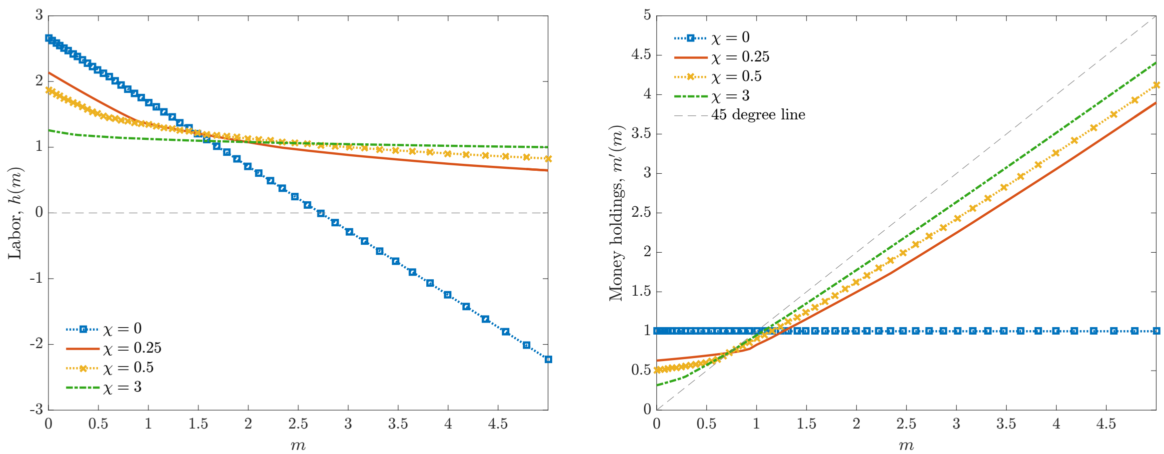
Notes: Stationary distributions of money holdings at the beginning of the decentralized market, $F(m)$, and the beginning of the centralized market, $G(m)$, for different values of the Frisch elasticity of labor supply, $1/\chi$.

completely. The decision rules for labor and money holdings in Figure D4(a) and Figure D4(b) show that labor supply changes linearly with the individual state, m, so that agents can exactly exit the period with 1 unit of relative money. This is a consequence of the absence of wealth effects under quasi-linear preferences.

When the Frisch elasticity of labor supply ($1/\chi$) becomes finite, the inability of agents to offset the effects of their previous trading history results in a nondegenerate distribution of money in the DM. In fact, as the elasticity of labor supply decreases, we obtain distributions of money holdings with less concentration.

Figure D4: Decision Rules in the Centralized Market at the Stationary Equilibrium for Different Levels of the Frisch Elasticity of Labor

(a) Decision Rules for Labor **(b)** Decision Rules for Money Holdings



D.3 Solution Method

My method solves iteratively for the agents' decision rules, the terms of trade in decentralized trading, and the price of money that clears its market. We start by solving the agents' value functions in (D.6) and (D.7). This requires updating $V(m; F, \mu)$ and $W(x; G, \mu)$ one after the other. Doing this for W in the decentralized market (DM) is standard, as long as we have how to compute the continuation value V. However, when computing the value function in the DM, we need to know the entire distribution $F(m) : \mathcal{M} \to \mathbb{R}$ and the terms of trade for each possible meeting between agents with m_b and m_s. Therefore, we must start not only with some guess for W, as in any value function iteration approach, but also with guesses for $F(m)$ and $\langle Q, D\rangle = \langle q(m_b, m_s; F, \mu), d(m_b, m_s; F, \mu)\rangle$. To do this, we discretize our state space $\mathcal{M}$, so that $F(m)$ and $\langle Q, D\rangle$, as well as $V(m; F, \mu)$, are defined over a finite number of points. Similarly, when evaluating $W(x; G, \mu) : \mathcal{X} \to \mathbb{R}$, we need to discretize $\mathcal{X}$. Given this, we can solve the stationary equilibrium of this economy as follows:

1. Solve for V and W given some $F(m)$, some terms of trade $\langle Q, D\rangle$, and a price ϕ_m. As a byproduct of this step, we should have the decision rule for $m' = g_m(x; G, \mu)$ in the centralized market (CM).

   There are several valid approaches to this. In particular, I use piece-wise cubic splines when evaluating continuation values that depend on W, solve the nonlinear labor-leisure condition using the bisection method, and employ Howard's improvement algorithm to speed up the solution of the CM problem in (D.7)-(D.9).

2. Given $F(m)$ and the current values in $\langle Q, D \rangle$, compute $G(m)$. Likewise, using $G(m)$ and the decision rule for m', compute $F'(m)$. Repeatedly update F and G until these distributions converge.

3. Verify if the market for money is clearing, i.e., check if Equations (D.2) and (D.3) hold. If not, adjust the price of money ϕ_m and return to Step 1. Repeat this process until guaranteeing market clearing. Of course, when returning to Step 1, use the most recently computed value for F.
   When adjusting prices, an alternative is to use the excess demand function for money. I do so using a partial updating approach.

4. Finally, update the terms of trade $\langle Q, D \rangle$. The key input here is the value function W that is required to compute the continuation values in the problem in (D.4)-(D.5). As in Step 1, I use piece-wise cubic splines on W. Check if the newly calculated terms of trade are close enough to their previous values. If not, return to Step 1. If so, we are done.

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