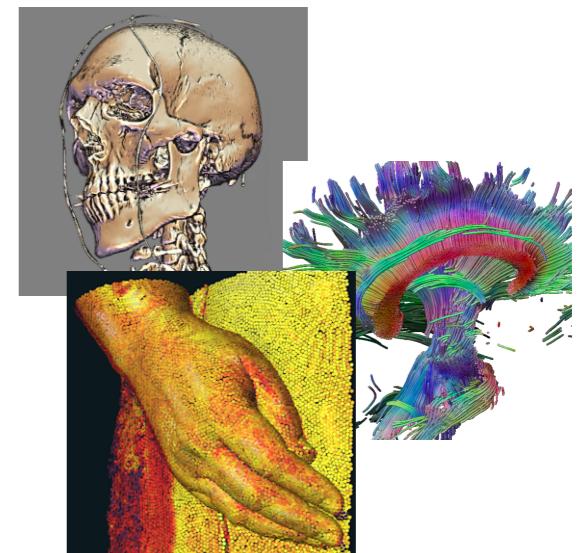


# Addressing Domain-Specific Computational Needs with a New Programming Language

Charisee Chiw



# Diderot: A DSL for image analysis

Diderot is a domain specific language for image analysis and scientific visualization

## [CHIW PLDI]

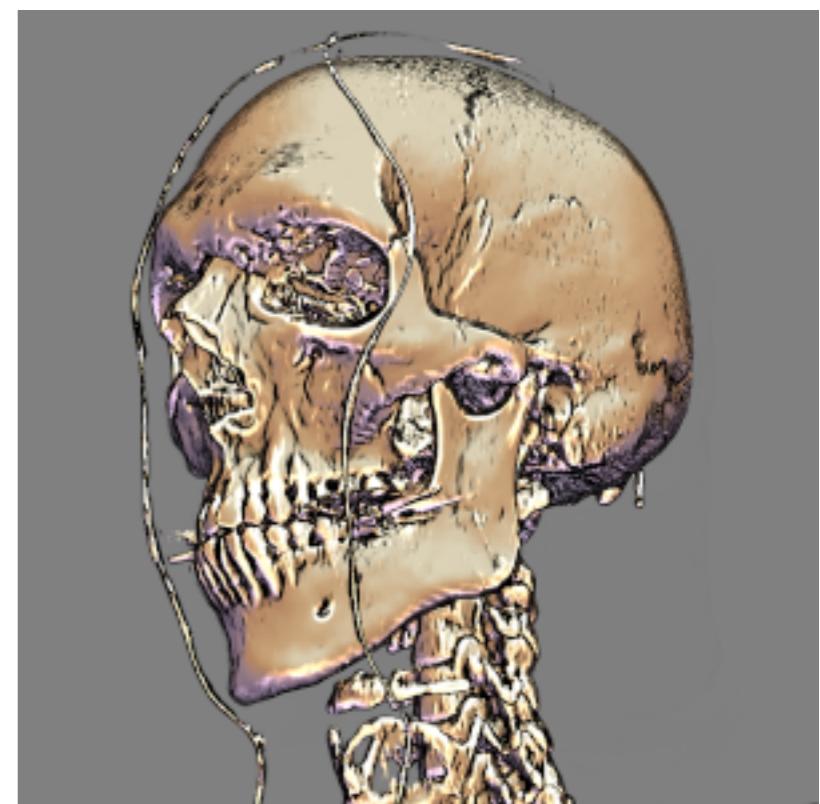
### Diderot: A Parallel DSL for Image Analysis and Visualization

Charisee Chiw   Gordon Kindlmann   John Reppy   Lamont Samuels   Nick Seltzer \*\*  
University of Chicago  
[\(chiw,gk,jr,amorts,nositzerj\)@cs.uchicago.edu](mailto:(chiw,gk,jr,amorts,nositzerj)@cs.uchicago.edu)

#### Abstract

Research scientists and medical professionals use imaging technology, such as computed tomography (CT) and magnetic resonance imaging (MRI) to measure a wide variety of biological and physical objects. The increasing sophistication of imaging technology creates demand for equally sophisticated computational techniques to analyze and visualize the image data. Analysis and visualization codes are often crafted for a specific experiment or set of images.

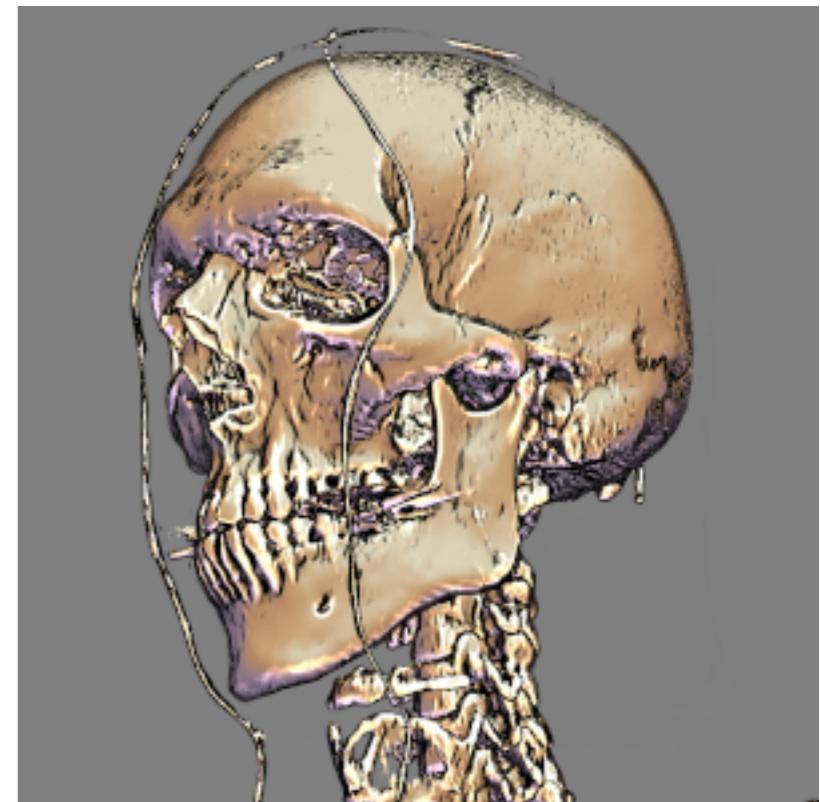
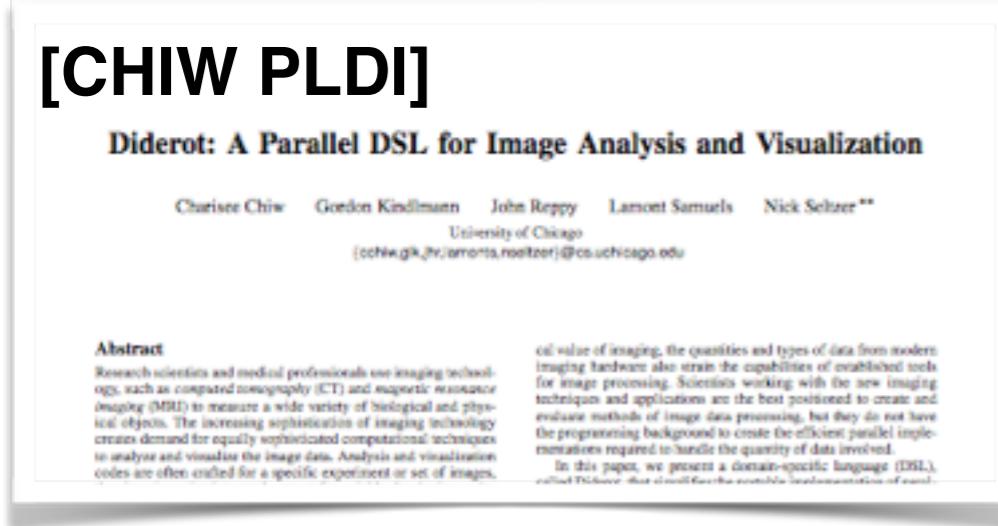
In this paper, we present a domain-specific language (DSL), called *Diderot*, that allows for the parallel implementation of anal-



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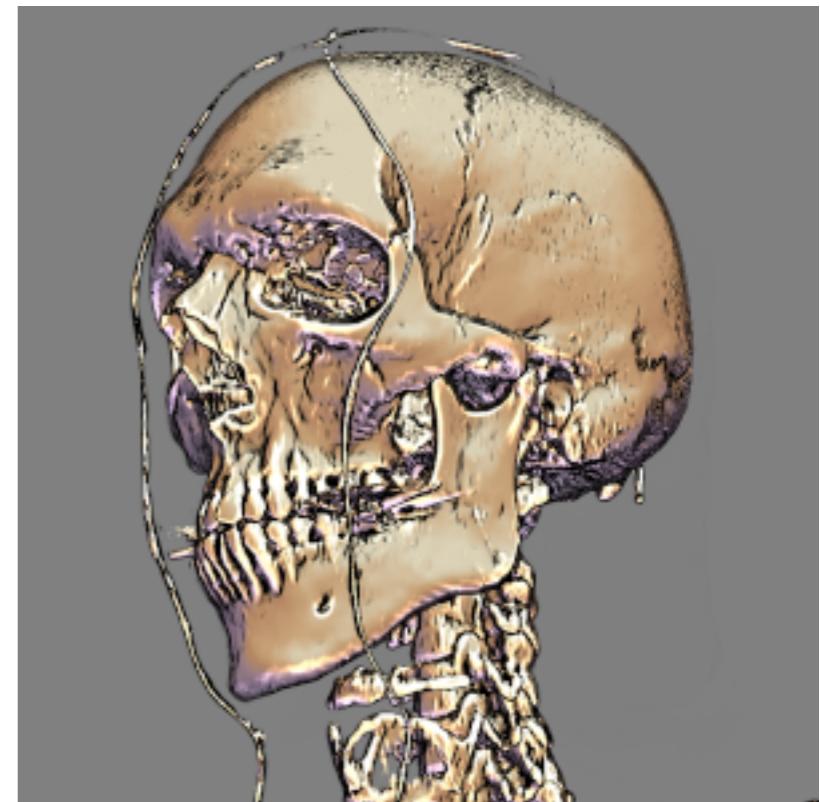
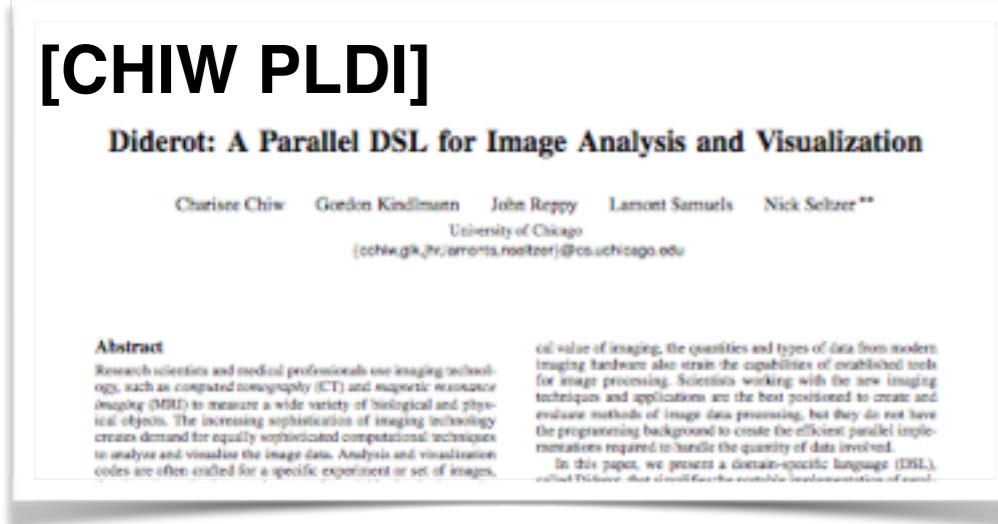
- Provide a notation can **express** statements in the domain space



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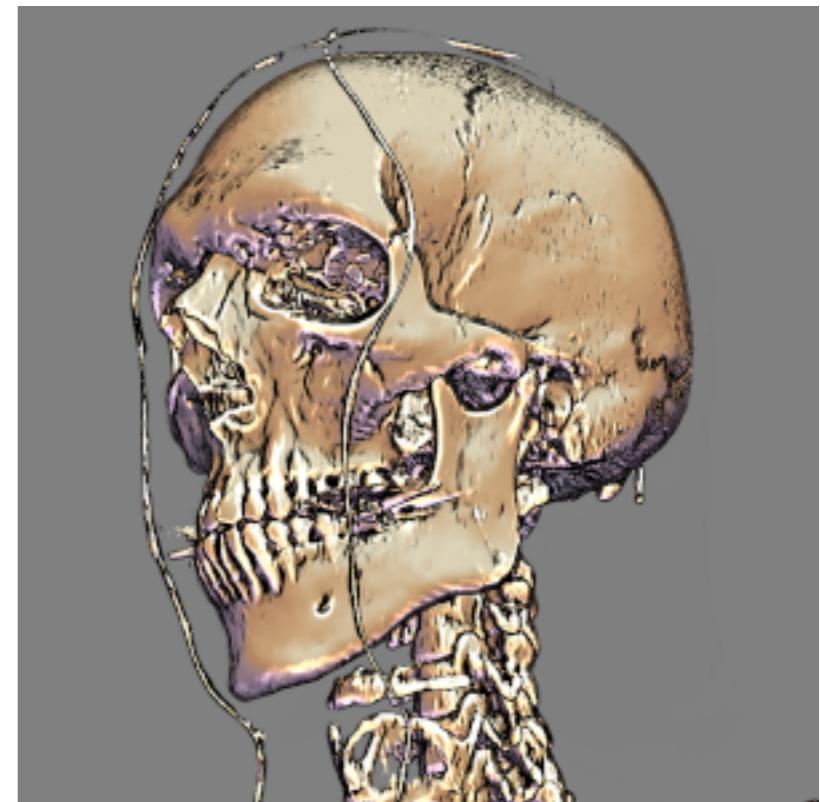
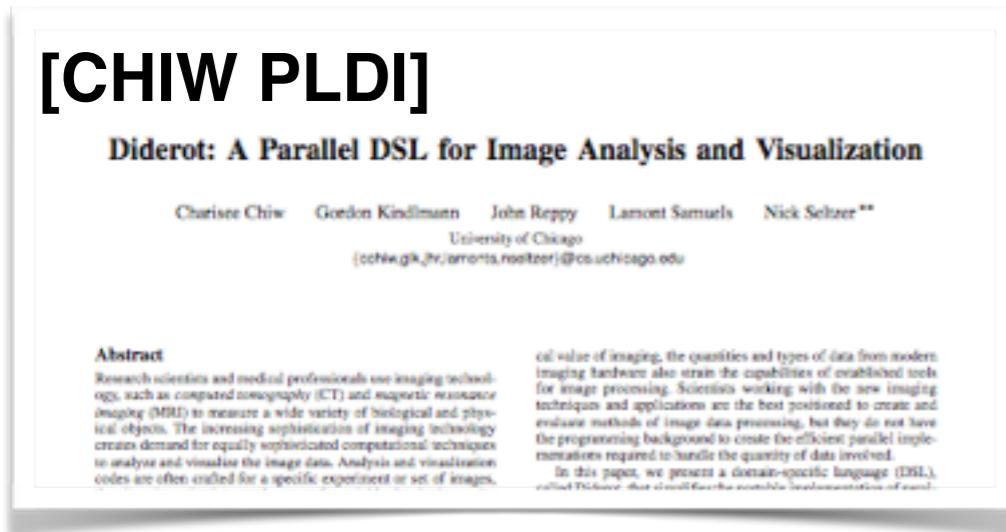
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- **Efficient** execution of expensive computations



# Diderot: A DSL for image analysis

Diderot is a domain specific language for image analysis and scientific visualization

- Provide a notation can **express** statements in the domain space
- **Efficient** execution of expensive computations
- Enable cutting-edge research



# Roadmap

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## 1. Implementing expressivity

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2. Automated Testing of compiler implementation

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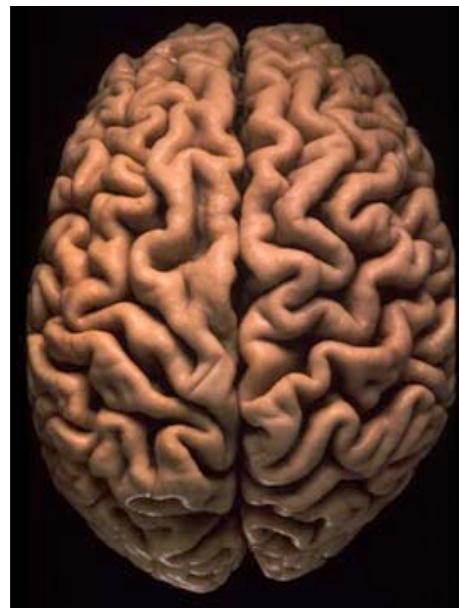
0. Image analysis and scientific visualization
1. Implementing expressivity
2. Automated Testing of compiler implementation
3. Extension to a new domain

# Why image analysis is important

- Scientists need software tools to extract structure from many kinds of image data.
- Creating new visualization programs is part of the experimental process.
- Visualization helps explore data.

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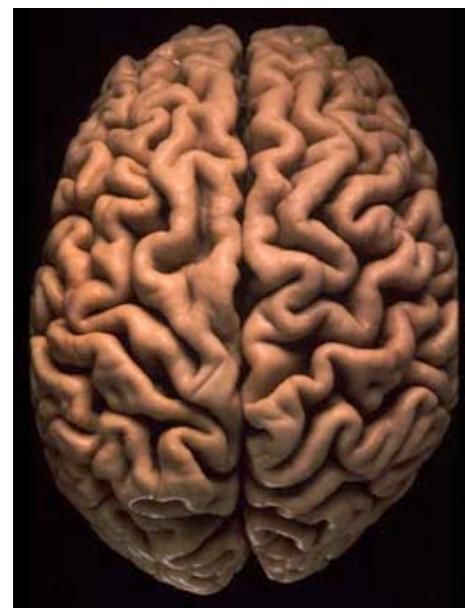
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**Physical object**

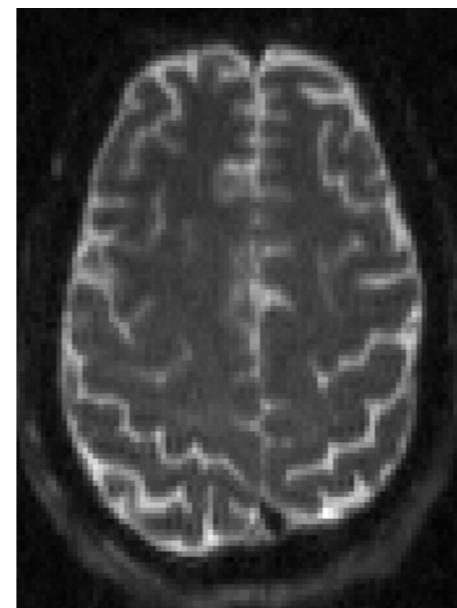
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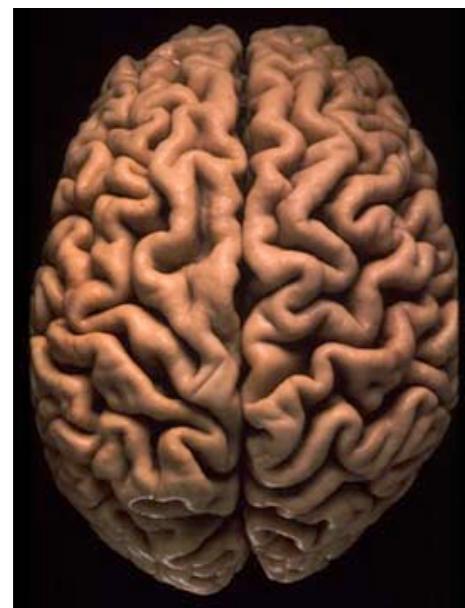
*Imaging*



**Image data**

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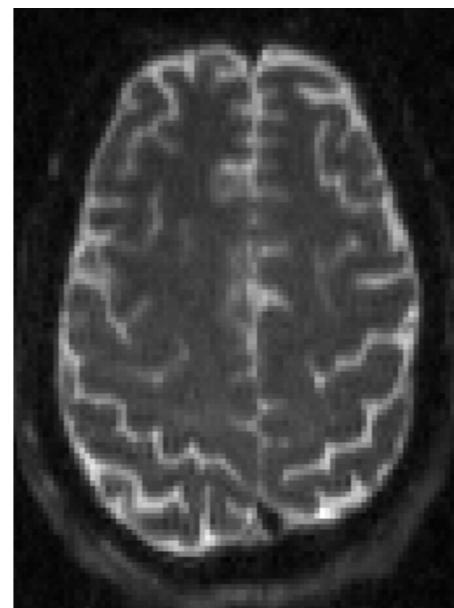
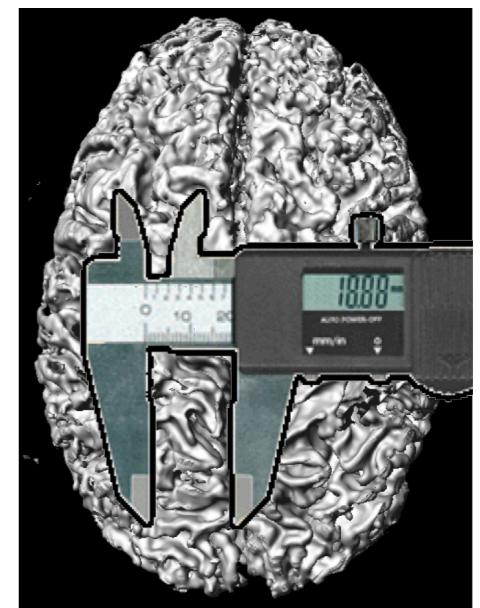


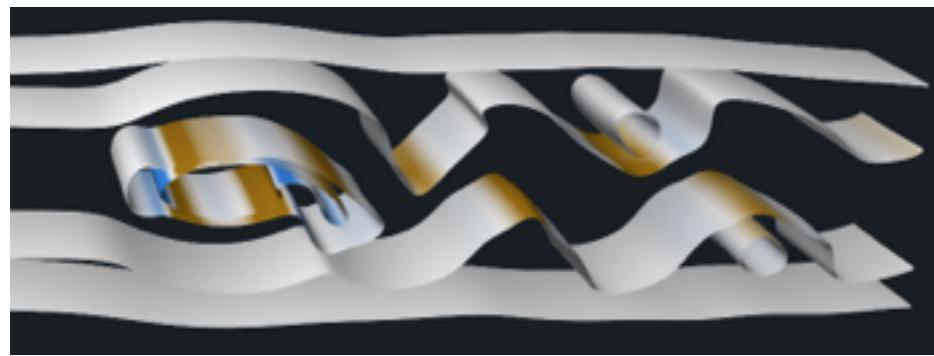
Image data

*Visualization*  
*Analysis*



Computational  
representation

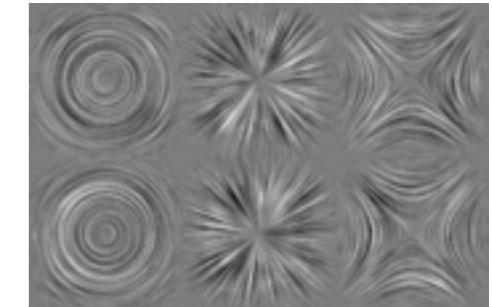
# Visualization Applications



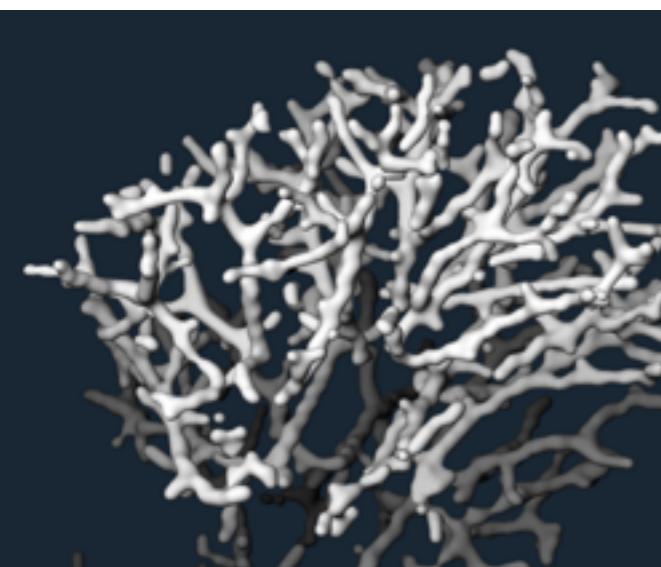
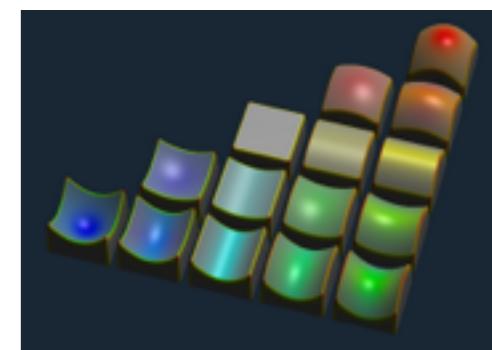
Fluid flow



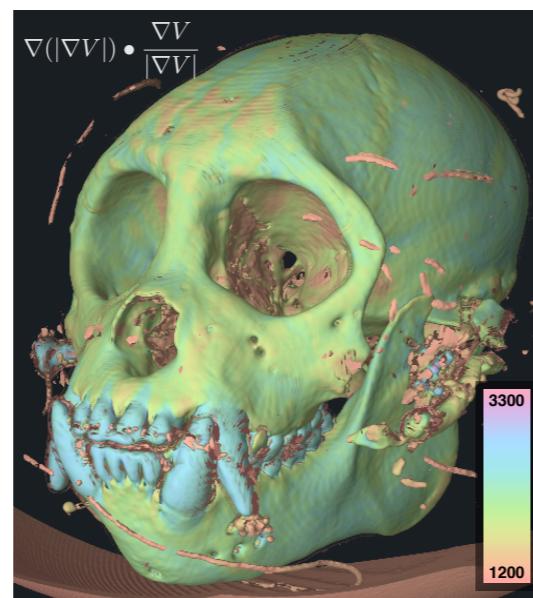
Diderot ^ The guy



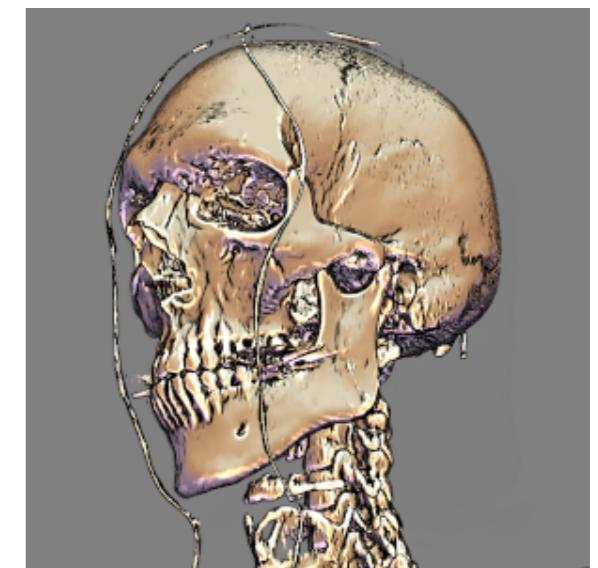
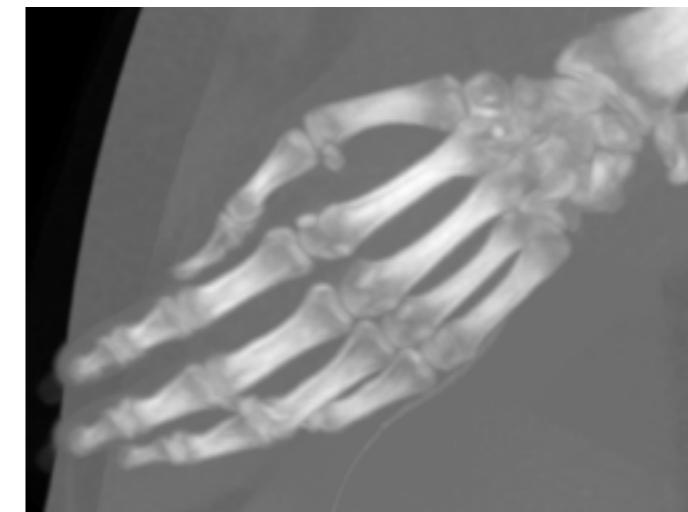
Synthetic data



Lung data



X-Ray data and CT Scans  
6



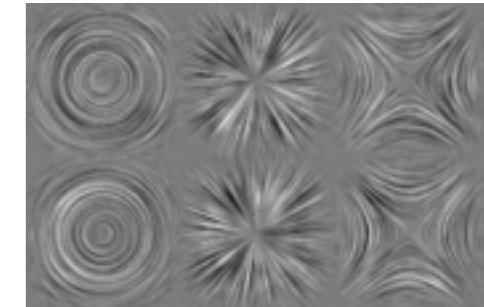
# Visualization Applications

Visually explore data and compute features

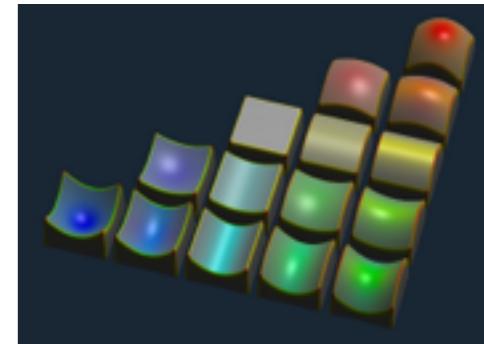
Flow Magnitude



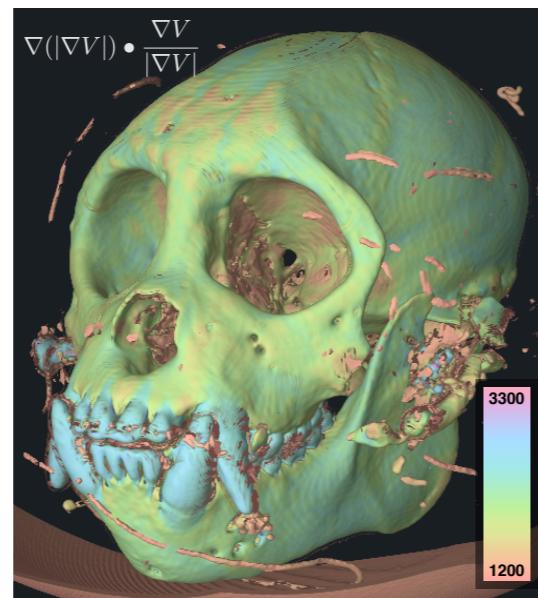
LIC



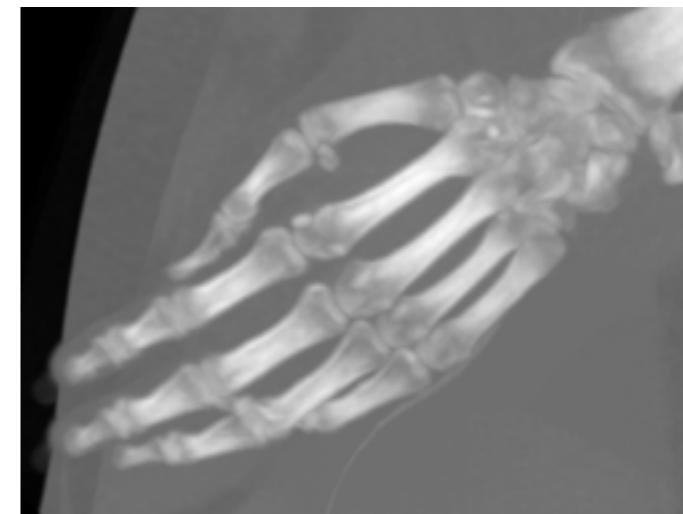
Surface  
Curvature



Canny Edges

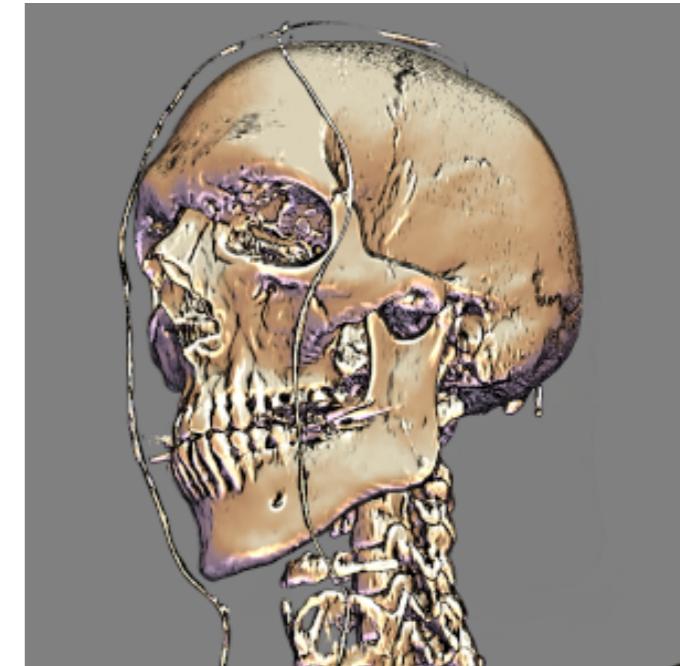


Isocontour Surface  
MIP



Ridge Detection

Volume rendering



# Types and definitions

**Tensor:** reals, vectors, matrices,...

9

**Tensor operators:** addition, subtraction, multiplication

9+3

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$F(3) = 9$

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$\nabla F = 2x$

**Applying the derivative operator**

$\nabla(F+G) \Rightarrow \nabla F + \nabla G$

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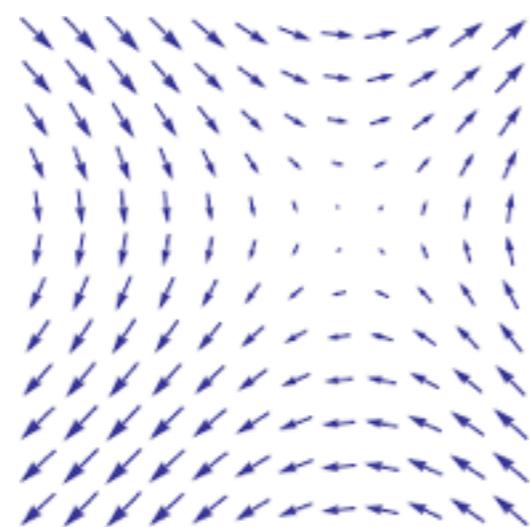
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vector field is a type of field



vector is a type of tensor

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**Tensor operators:** addition, subtraction, multiplication

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**Field:** function from d-dimensional space to tensors

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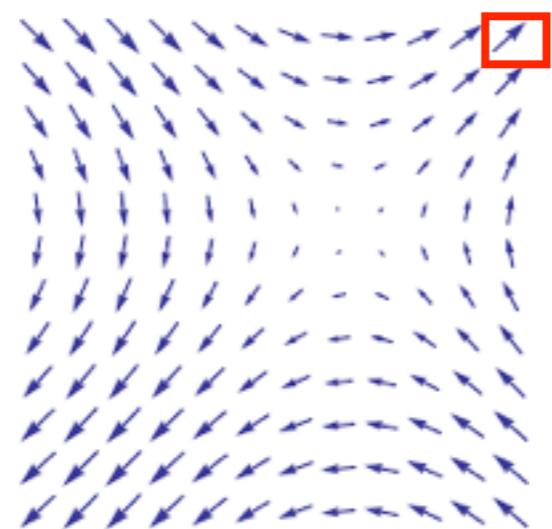
$F(3) = 9$

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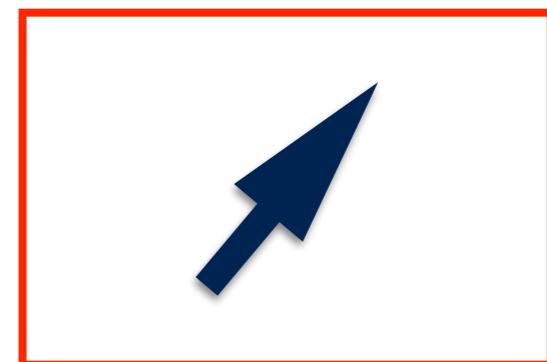
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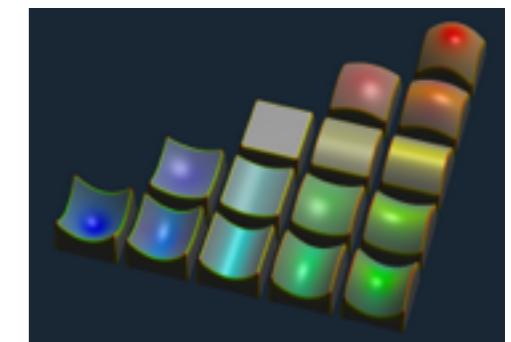
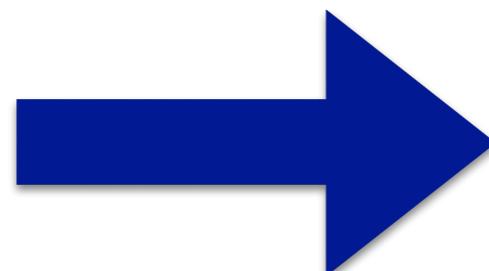
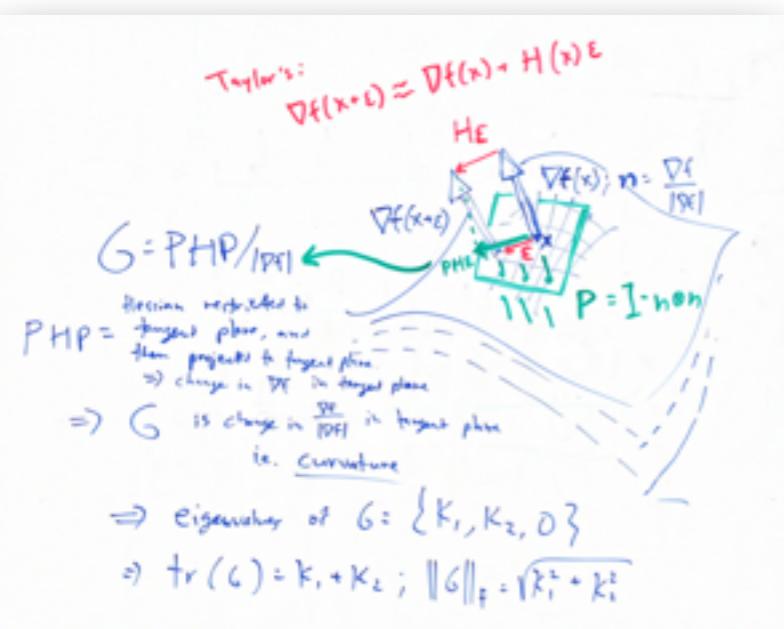
vector field is a type of field



vector is a type of tensor

# Challenges

- What do I want to implement?



Visualization concept

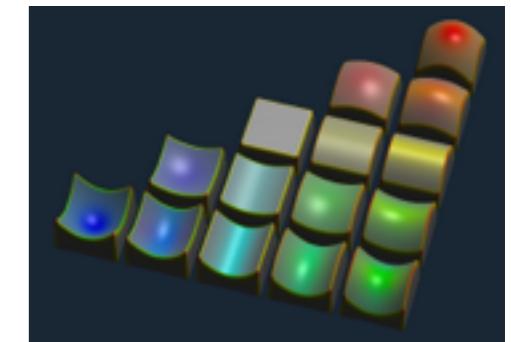
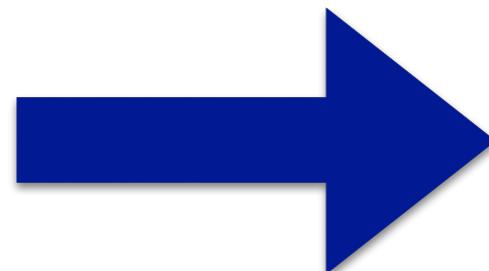
Implementation

Image

# Challenges

- What do I want to implement?

```
grad = - $\nabla F$ ;  
norm = normalize(grad);  
H =  $\nabla \otimes \nabla F$ ;  
P = identity[3] - norm $\otimes$ norm;  
G = -(P $\bullet$ H $\bullet$ P)/|grad|;  
disc =  $\sqrt{2.0 * |G|^2 - \text{trace}(G)^2}$ ;  
k1 =  $\frac{\text{trace}(G) + disc}{2}$ ;  
k2 =  $\frac{\text{trace}(G) - disc}{2}$ ;
```



Visualization concept

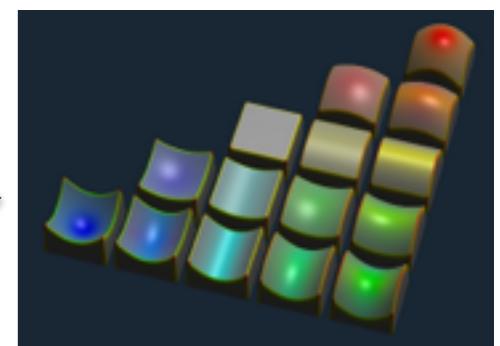
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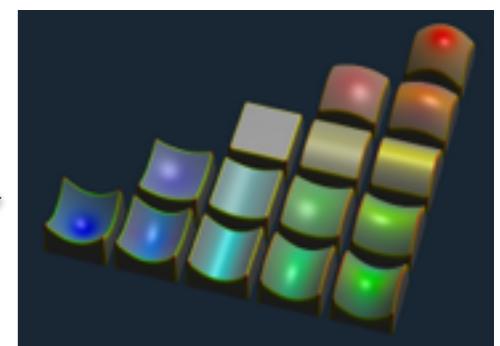
Implementation

Image

# Challenges

- What do I want to implement?
- How do I implement this?
- How do I implement it efficiently?

```
grad = - $\nabla F$ ;  
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H =  $\nabla \otimes \nabla F$ ;  
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```



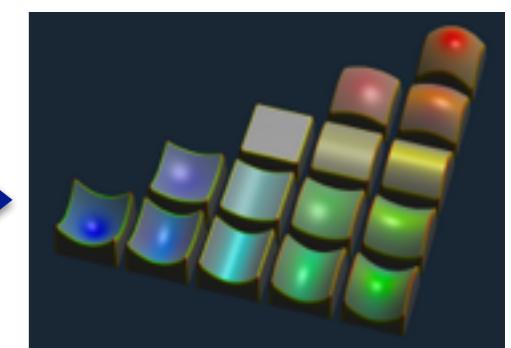
Visualization concept

Implementation

Image

# Programmability: From whiteboard to code

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Visualization concept

Diderot code

Image

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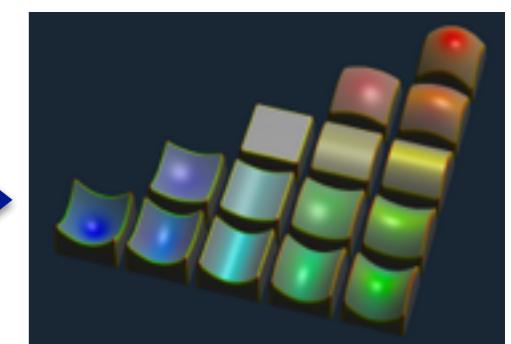
grad =  $-\nabla F$ ;  
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```
vec3 grad = - $\nabla F$  (pos);
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```



Surface  
Curvature



Visualization concept

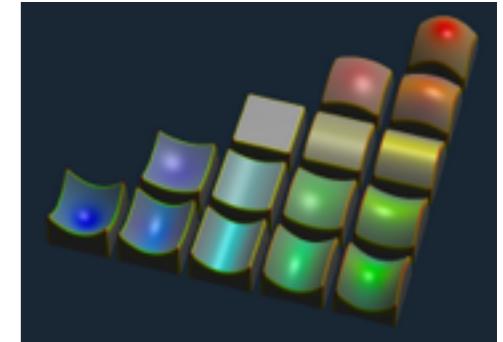
Diderot code

Image

# Diderot program

```
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field#0(2)[3] RGB = tent ⋅ image("2d-bow.nrrd");
...
strand RayCast (int ui, int vi) {
    ...
    update
    ...
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**Derivative:** rate of change

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# Motivation

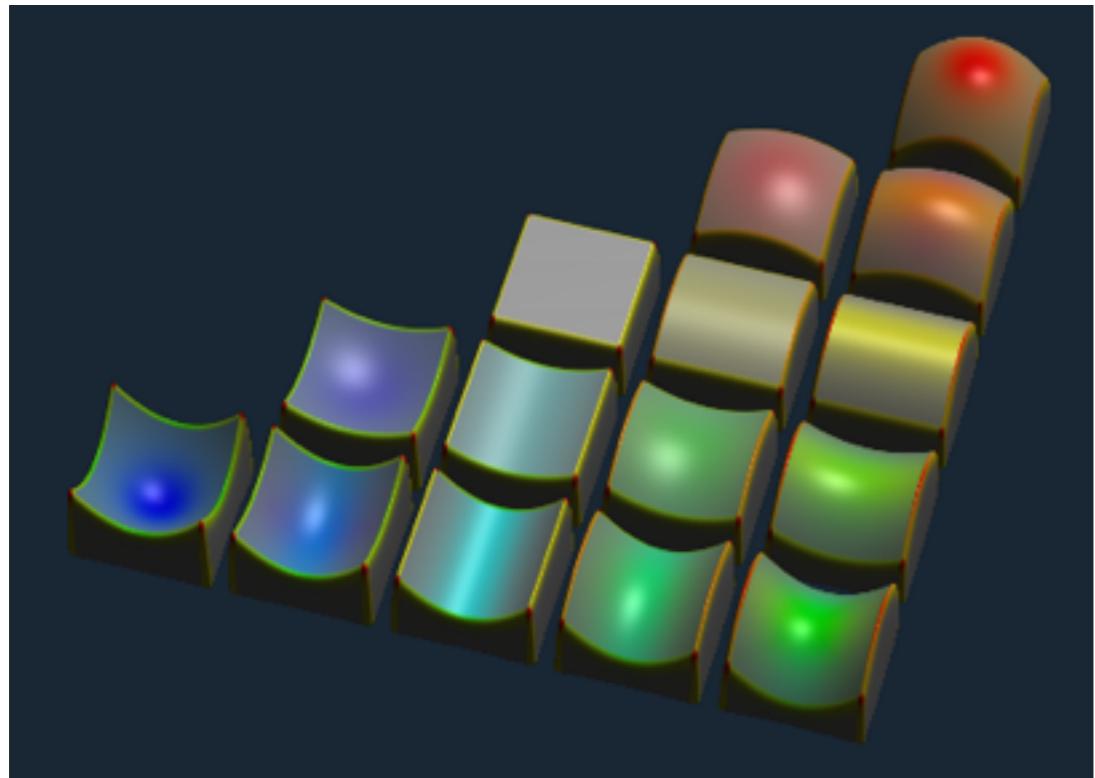
- Language restrictions make it difficult to implement new ideas

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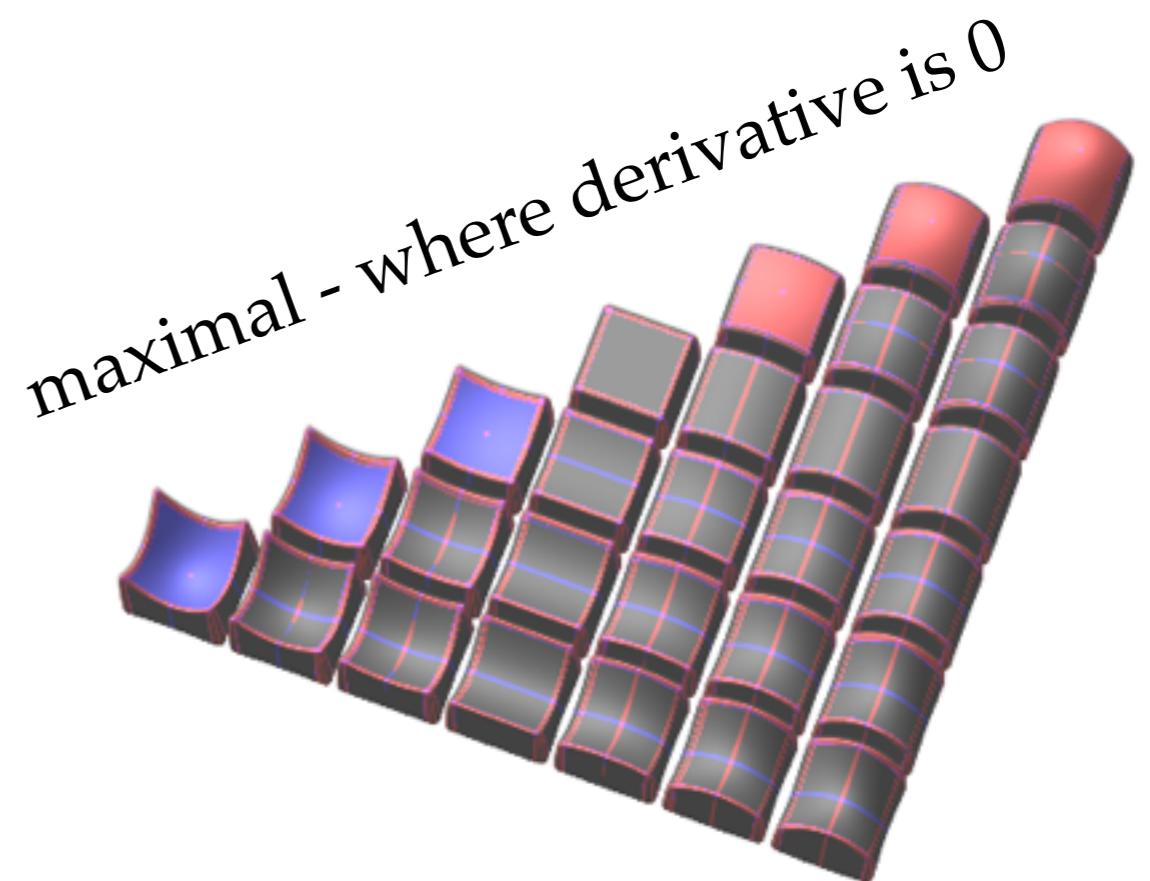
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- Curvature and Crest Lines are related visualization properties

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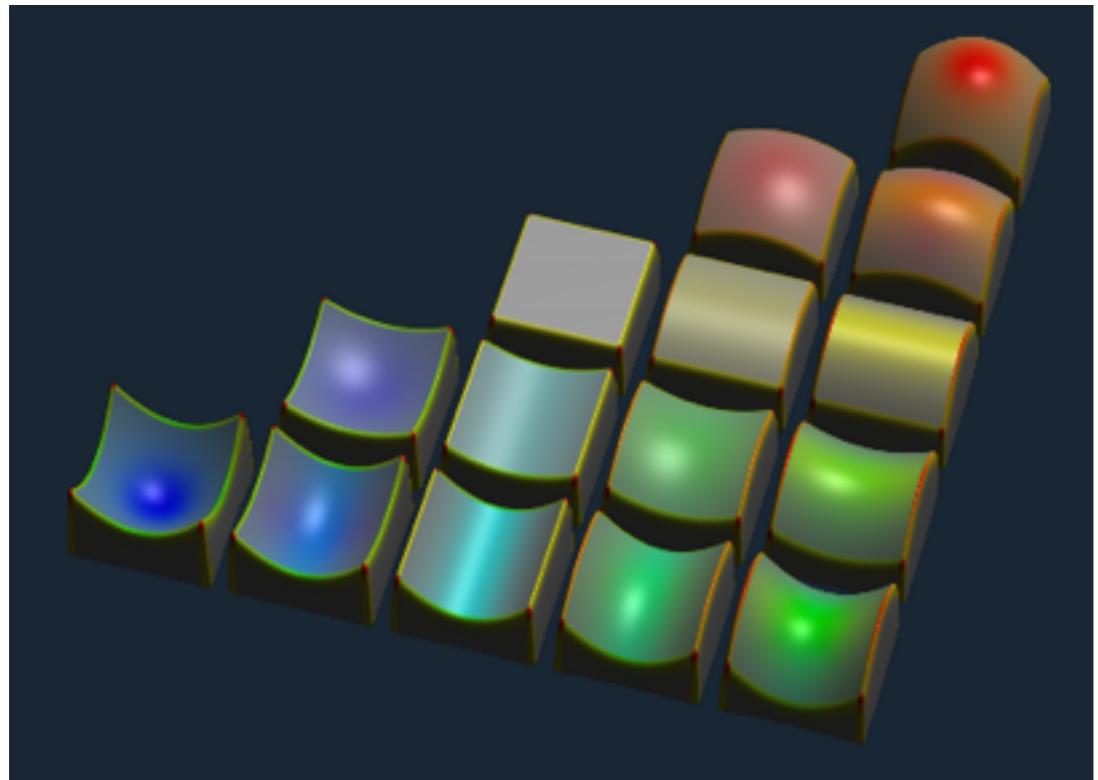
**Curvature**



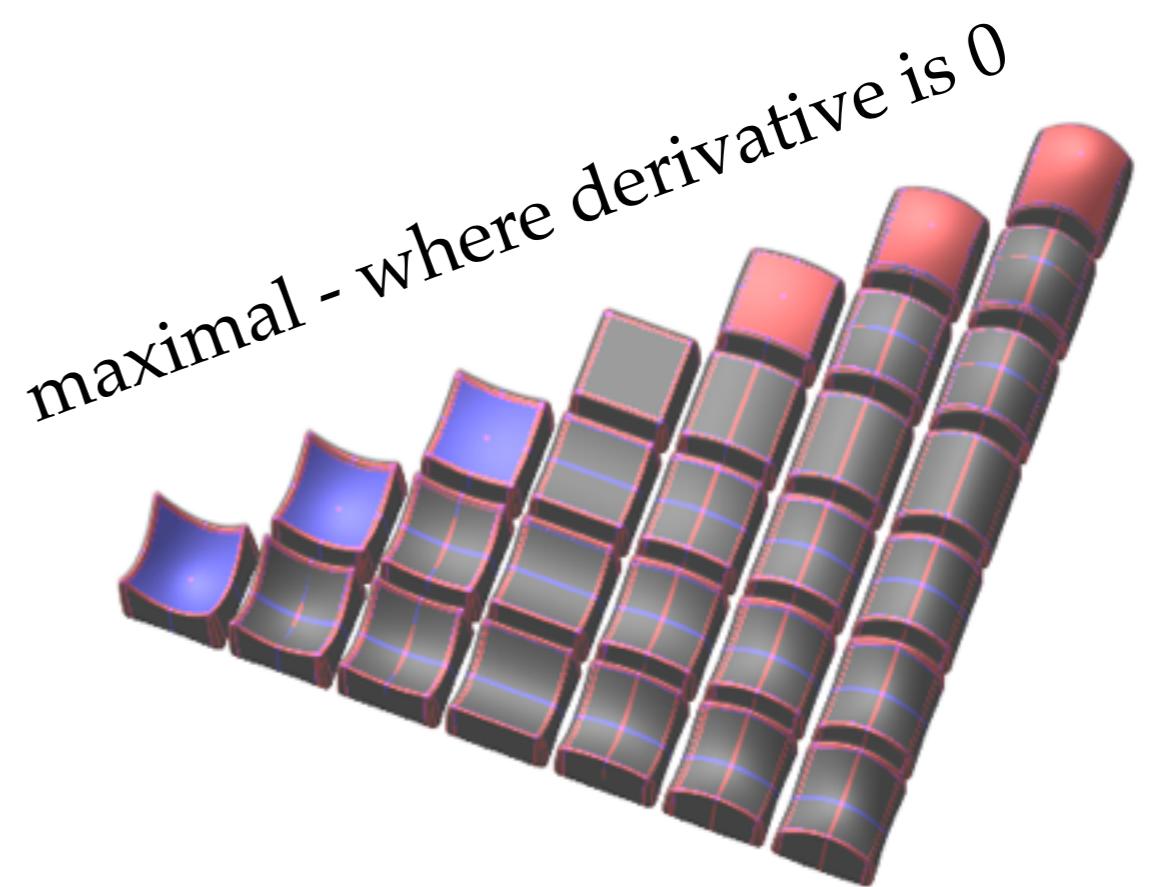
**Crest Lines** are places where the surface curvature is maximal along the **curvature** direction.

# Motivation

- Language restrictions make it difficult to implement new ideas
- Curvature and Crest Lines are related visualization properties
- Should be able to build code



**Curvature**



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    tensor[3,3] P = identity[3] - norm $\otimes$ norm;
    tensor[3,3] G = -(P $\bullet$ H $\bullet$ P)/|grad|;
    real disc = sqrt(2.0 * |G|^2 - trace(G)^2);
    real k1 = (trace(G) + disc)/2.0;
    real k2 = (trace(G) - disc)/2.0;
    vec3 matRGB = ...RGB(...k1, ...k2));
    real out = ... $\nabla \otimes \nabla$ k1...
}
...
}
```

Directly add line  
15

# Motivation

```
field#4(3) [] F = bspln5 ⋘ image("quad-patches.nrrd");
field#0(2)[3] RGB = tent ⋘ image("2d-bow.nrrd");
...
strand RayCast (int ui, int vi) {
    ...
    update
    ...
    vec3 grad = - $\nabla$ F (pos);
    vec3 norm = normalize(grad);
    tensor[3,3] H =  $\nabla \otimes \nabla$  F (pos);
    tensor[3,3] P = identity[3] - norm $\otimes$ norm;
    tensor[3,3] G = -(P $\bullet$ H $\bullet$ P)/|grad|;
    real disc = sqrt(2.0 * |G|^2 - trace(G)^2);
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    vec3 matRGB = ...RGB(...k1, ...k2));
    real out = ... $\nabla \otimes \nabla$ k1...
}
...
}
```

Not permitted  
15

# Motivation

first-order	higher-order
$q = \det V(x)$	
$\det_{\text{tr}} = q[0,0,:] + q[1,1,:] + q[2,2,:]$	
$E = V(x) - \text{trace}^* \text{identity}[3]/3$	
$\text{val} = 3 \cdot \sqrt{6} \cdot \det(E / \sqrt{E:E})$	
$K = (\text{identity}[3] \otimes \det_{\text{tr}})/3$	
$\det E = \det V(x) - K$	$E = V - \text{trace}(V) \cdot \text{identity}[3]$
$N1 = \det E \cdot \sqrt{E:E}$	$F = 3 \cdot \sqrt{6} \cdot \det(E /  E )$
$\text{inner} = 2 \cdot E : \det E$	$\alpha = \text{alpha}(F(x),  \nabla F(x) )$
$B = (\text{inner} /  E:E )/2$	
$N2 = E \bullet B$	
$dQ = (N1 - N2) / E:E$	
$Q = E / \sqrt{E:E}$	
$\text{part1} = dQ[0,0,:] \cdot (Q[1,0] \cdot Q[2,2] - Q[1,2] \cdot Q[2,0])$	
$+ Q[0,0] \cdot (Q[2,2] \cdot dQ[1,1,:] + Q[1,0] \cdot dQ[2,2])$	
$- dQ[1,2] \cdot Q[2,0] + Q[1,2] \cdot dQ[2,0])$	
$\text{part2} = dQ[0,1,:] \cdot (Q[1,0] \cdot Q[2,1] - Q[1,2] \cdot Q[2,0])$	
$+ Q[0,1] \cdot (Q[2,1] \cdot dQ[1,0] + Q[1,0] \cdot dQ[2,1])$	
$- dQ[1,2] \cdot Q[2,0] + Q[1,2] \cdot dQ[2,0])$	
$\text{part3} = dQ[0,2,:] \cdot (Q[0,0] \cdot Q[2,1] - Q[1,1] \cdot Q[2,0])$	
$Q[0,2] \cdot (Q[2,1] \cdot Q[1,0] + Q[1,0] \cdot dQ[2,1])$	
$- dQ[1,1,:] \cdot Q[2,0] + Q[1,1] \cdot dQ[2,0])$	
$\det M = \text{part1} - \text{part2} + \text{part3}$	
$\text{grad} = 3 \cdot \sqrt{6} \cdot \det M$	
$\alpha = \text{alpha}(\text{val}, \text{lgrad})$	

# Motivation

The compiler, not the programmer, should do the work.

first-order	higher-order
$q = \det V(x)$	
$\det tr = q[0,0,:] + q[1,1,:] + q[2,2,:]$	
$E = V(x) - \text{trace}^* \text{identity}(3)/3$	
$\text{val} = 3\sqrt{6} \det(E/\sqrt{E:E})$	
$K = (\text{identity}(3) \otimes \det tr)/3$	
$\det E = \det V(x) - K$	$E = V - \text{trace}(V) \cdot \text{identity}(3)$
$N1 = \det E \cdot \sqrt{E:E}$	$F = 3\sqrt{6} \det(E/ E )$
$\text{inner} = 2 \cdot E : \det E$	$\alpha = \text{alpha}(F(x),  \nabla F(x) )$
$B = (\text{inner} /  E:E )/2$	
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$\text{part1} = dQ[0,0] \cdot (Q[1,0] \cdot Q[2,2] - Q[1,2] \cdot Q[2,0])$	
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$- dQ[1,2] \cdot Q[2,0] + Q[1,2] \cdot dQ[2,0,:])$	
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# Implementing Expressivity

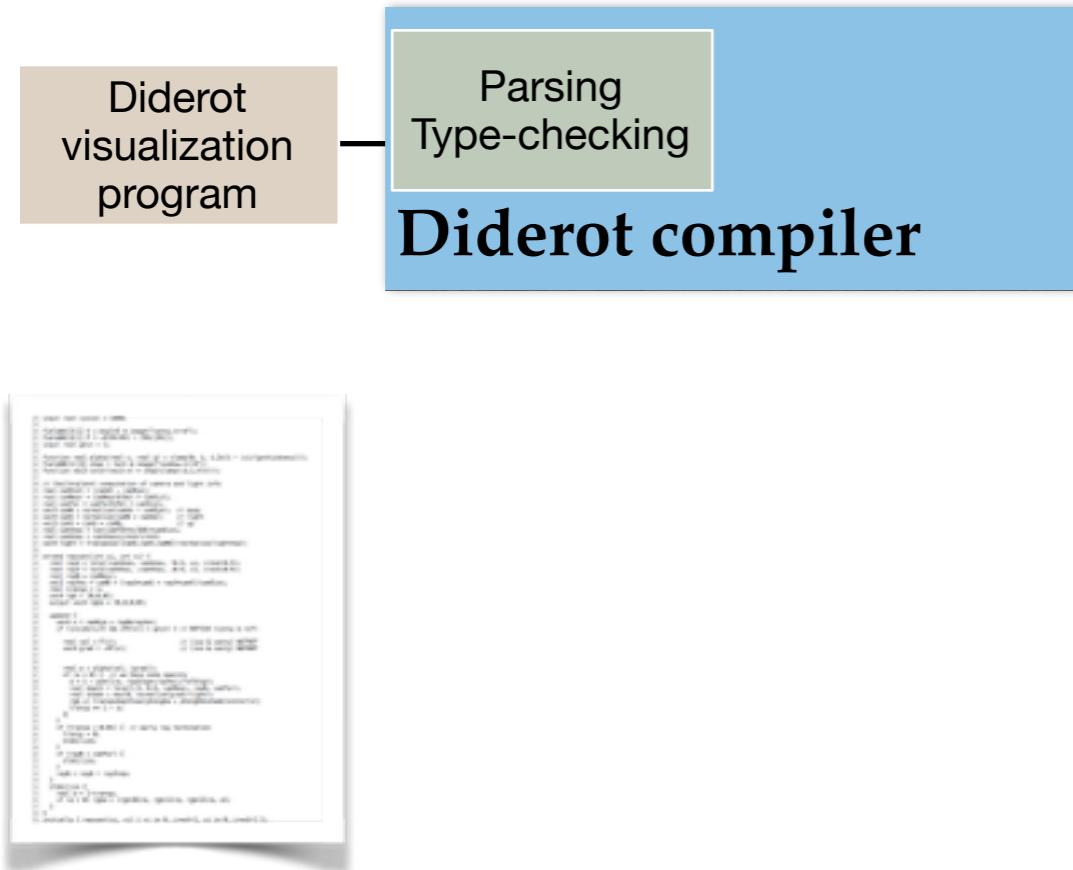
- Motivated our need for higher-level notation
- Overview of Diderot compiler
- Then focus on two stages

# Implementing Expressivity

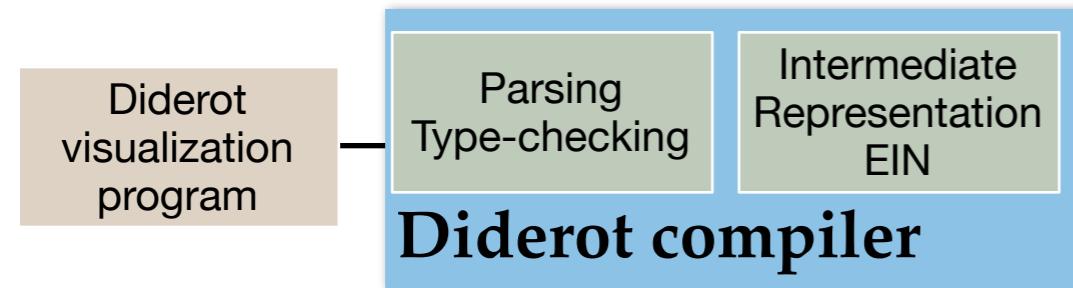
Diderot  
visualization  
program



# Implementing Expressivity

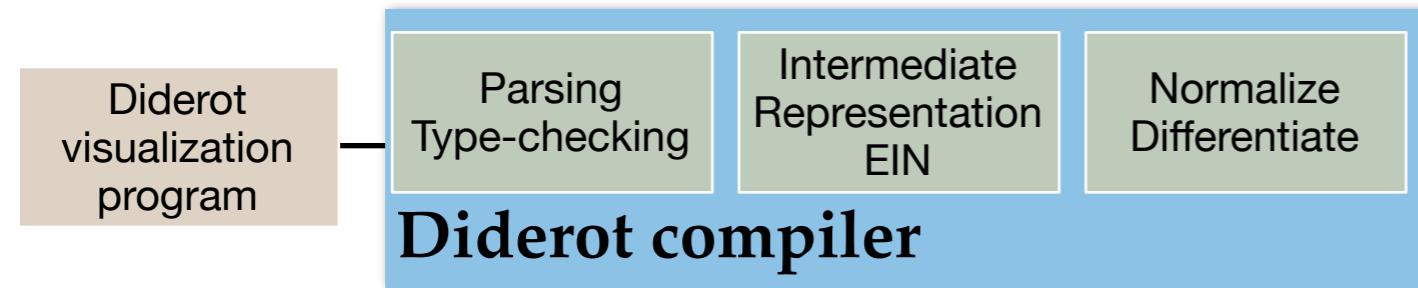


# Implementing Expressivity



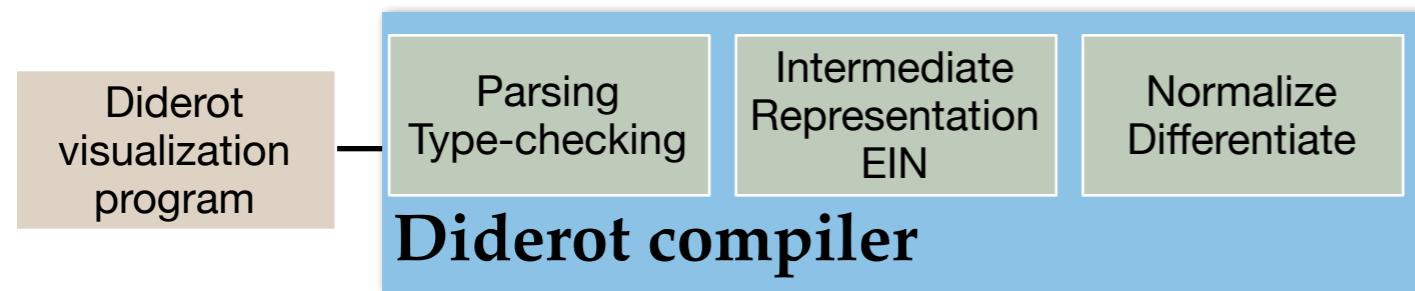
Phase that represents tensor math

# Implementing Expressivity



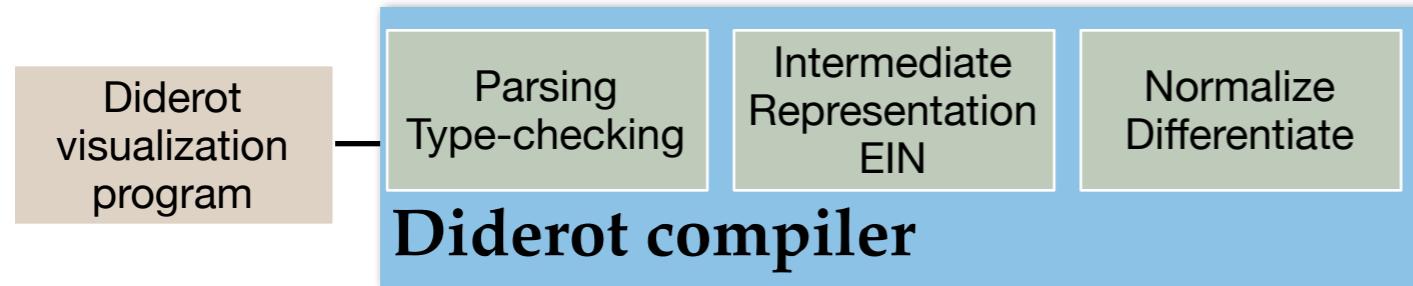
A screenshot of a terminal window displaying the output of a Diderot compilation process. The text is too small to read in detail but shows command-line arguments and standard output from a compiler run.

# Implementing Expressivity



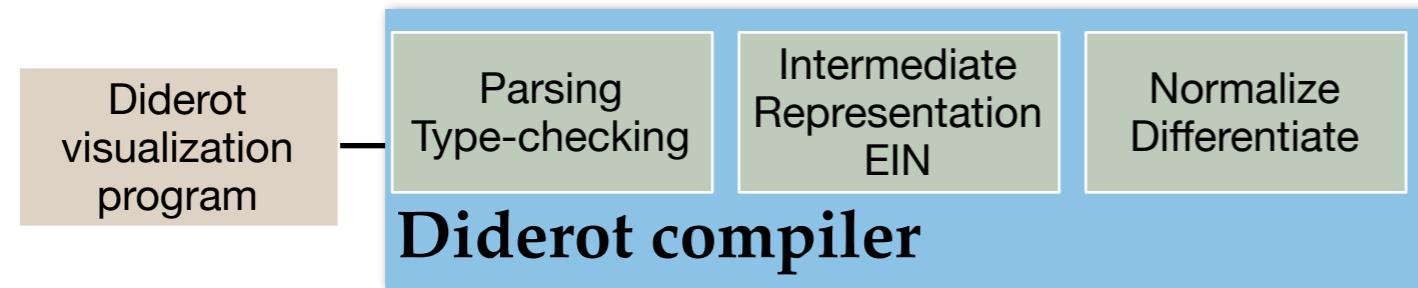
$$\nabla |\nabla F|(x)$$

# Implementing Expressivity



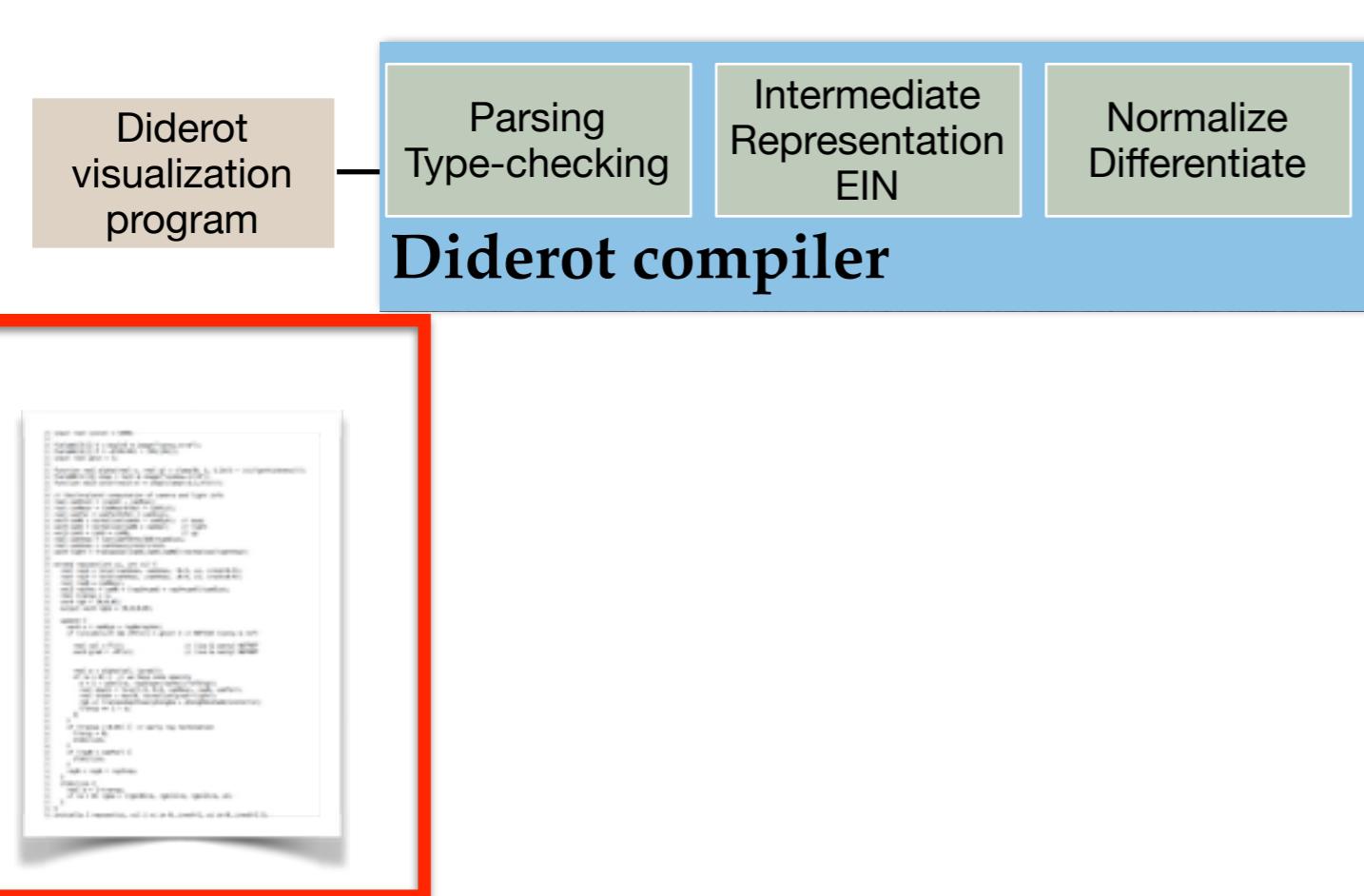
$$\nabla|\nabla F|(x) \longrightarrow \frac{1}{2} \frac{(\nabla \otimes \nabla F(p) \bullet \nabla F(p)) + (\nabla F(p) \bullet \nabla \otimes \nabla F(p))}{\sqrt{\nabla F(p) \bullet \nabla F(p)}}$$

# Implementing Expressivity



$$\nabla|\nabla F|(x) \longrightarrow \frac{1}{2} \frac{(\nabla \otimes \nabla F(p) \bullet \nabla F(p)) + (\nabla F(p) \bullet \nabla \otimes \nabla F(p))}{\sqrt{\nabla F(p) \bullet \nabla F(p)}}$$

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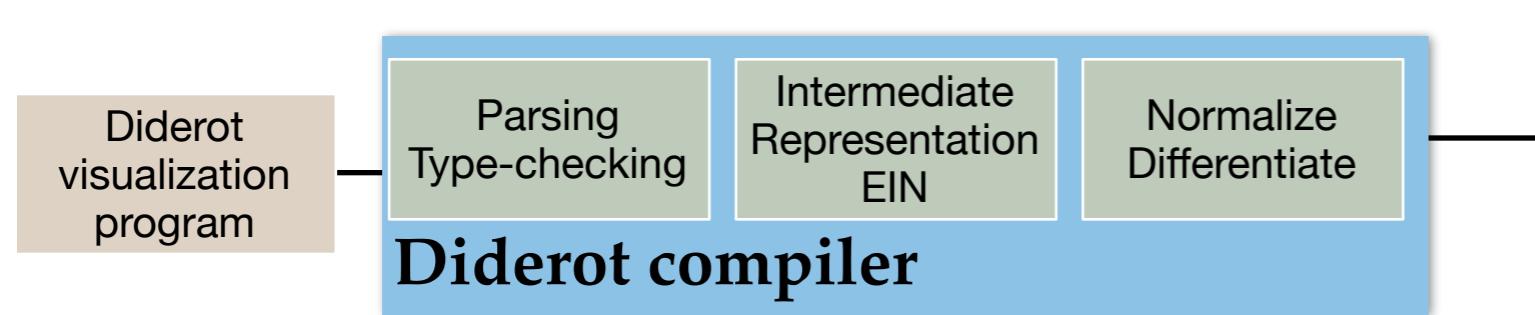


Compilation Issue



Programs could not compile or took too long to compile

# Implementing Expressivity



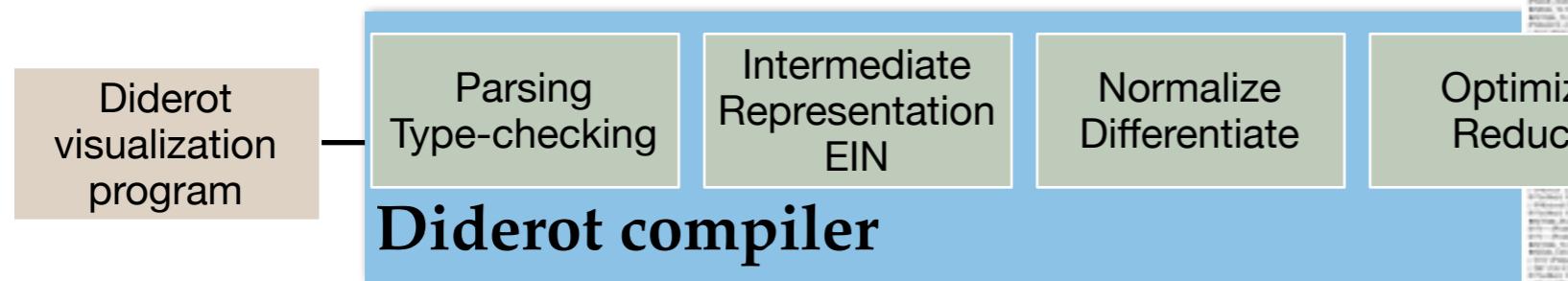
## Compilation Issue

Program compilation of the Diderot visualization program failed due to a bug in the differentiation module. In fact, this bug caused the compiler to run indefinitely, which is a serious problem for a production system. This bug was discovered during a stress test of a distributed system, which involved running many parallel tasks. One task was a simulation of a turbulent flow, while others were performing various calculations related to the simulation. The bug was triggered when one of the parallel tasks tried to differentiate a tensor field. The differentiation module was supposed to handle this operation correctly, but it instead got stuck in an infinite loop, causing the entire system to grind to a halt. This kind of bug can have serious consequences, especially if it occurs in a critical system like a medical device or a nuclear reactor. It's important to catch such bugs early on in the development process, before they cause major problems in the field.

Programs could not compile or took too long to compile

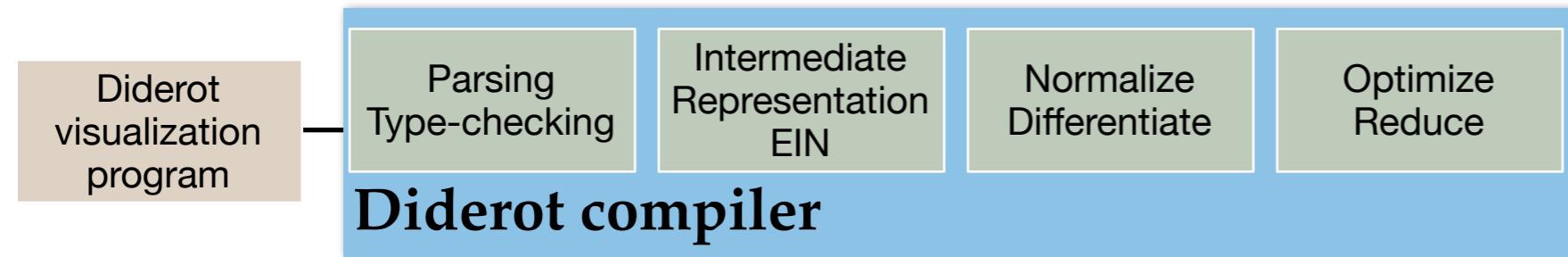
# Implementing Expressivity

## Compilation Issue



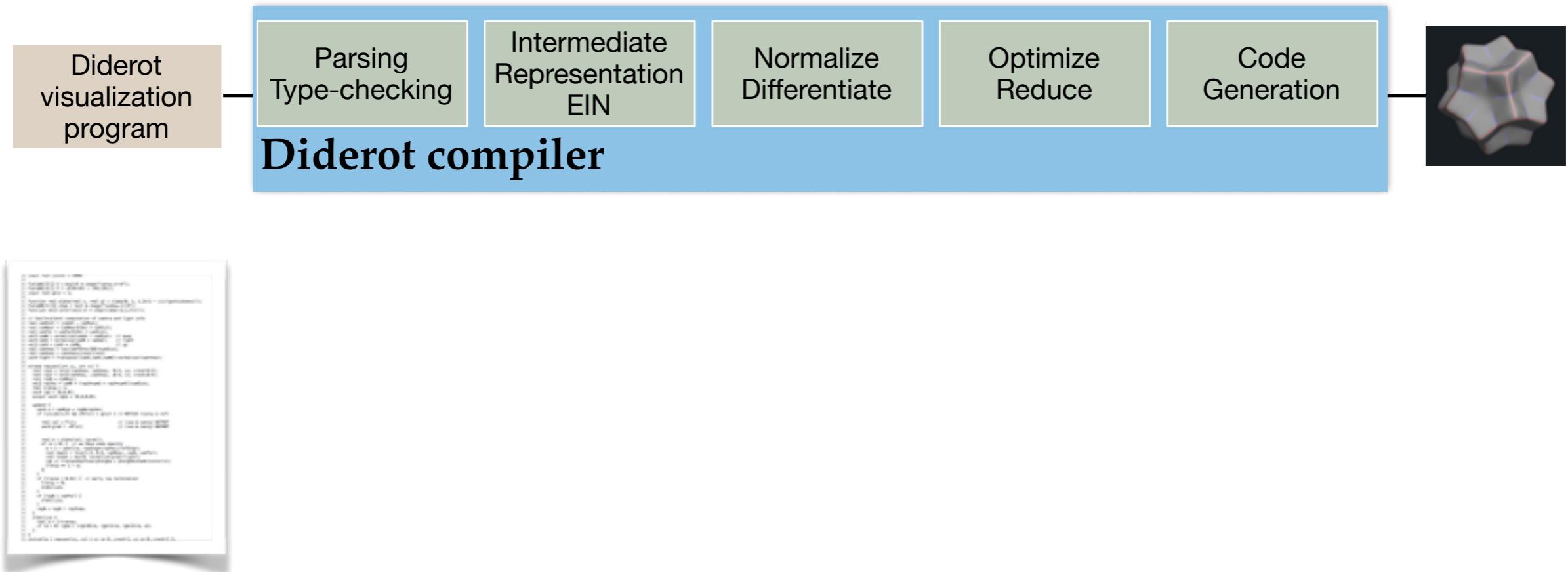
Programs could not compile or took too long to compile

# Implementing Expressivity

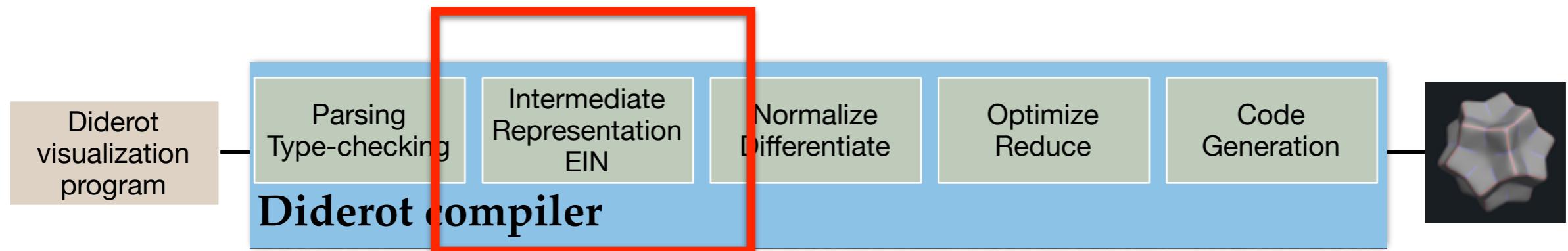


Phase that enables effective compilation of tensor math

# Implementing Expressivity



# EIN Intermediate Representation



Phase that represents tensor math

# EIN Intermediate Representation

- EIN represents tensors and fields

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$$M_{ij} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

EIN Term      Matrix M

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- EIN represents operators on tensor and fields

$$M_{ji} = \begin{bmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{bmatrix}$$

Switch row and column

$$M_{11} = M_{11} .$$

Single entry

$$\sum M_{ii} = M_{11} + M_{22}$$

Add diagonal components

$$M_{i1} = \begin{bmatrix} M_{11} \\ M_{21} \end{bmatrix}$$

Select a column

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$$M_{\boxed{11}} = M_{11}$$

Single entry

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$$M_{ji} = \begin{bmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{bmatrix}$$

Switch row and column

$M_{11} = M_{11}$

An EIN term can concisely represent tensor math

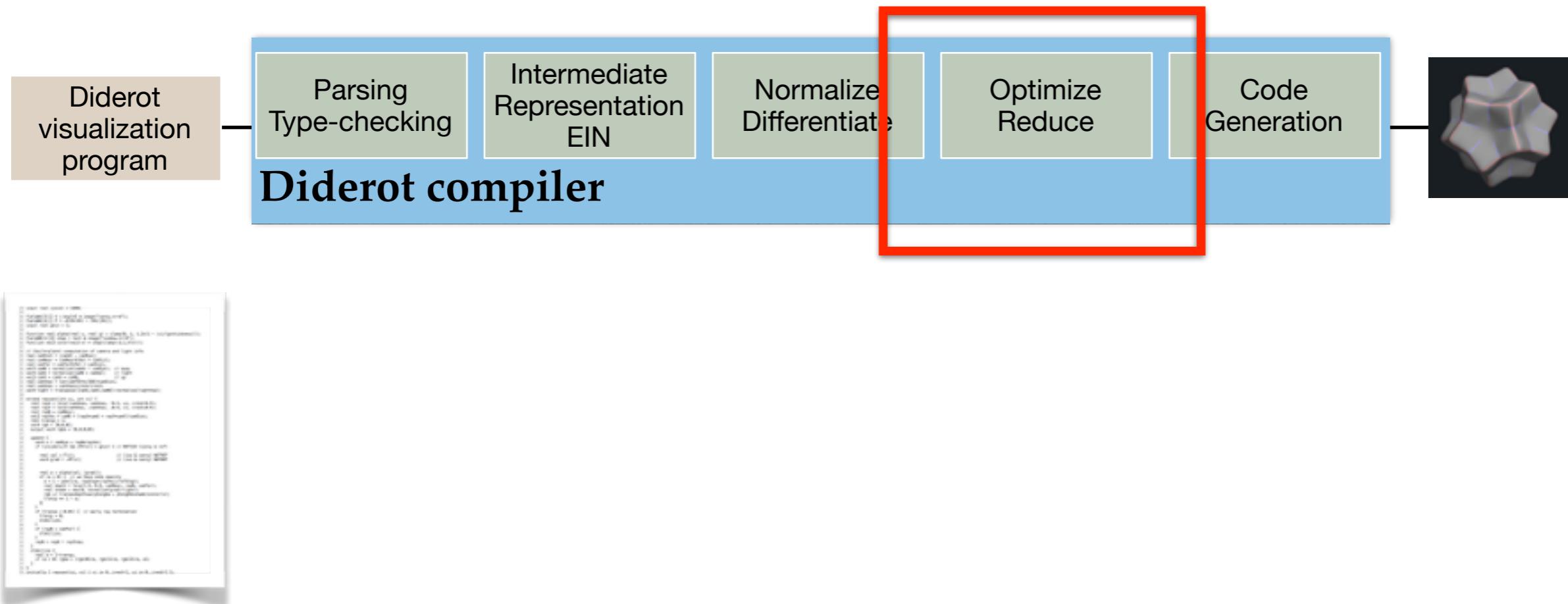
$$\sum M_{ii} = M_{11} + M_{22}$$

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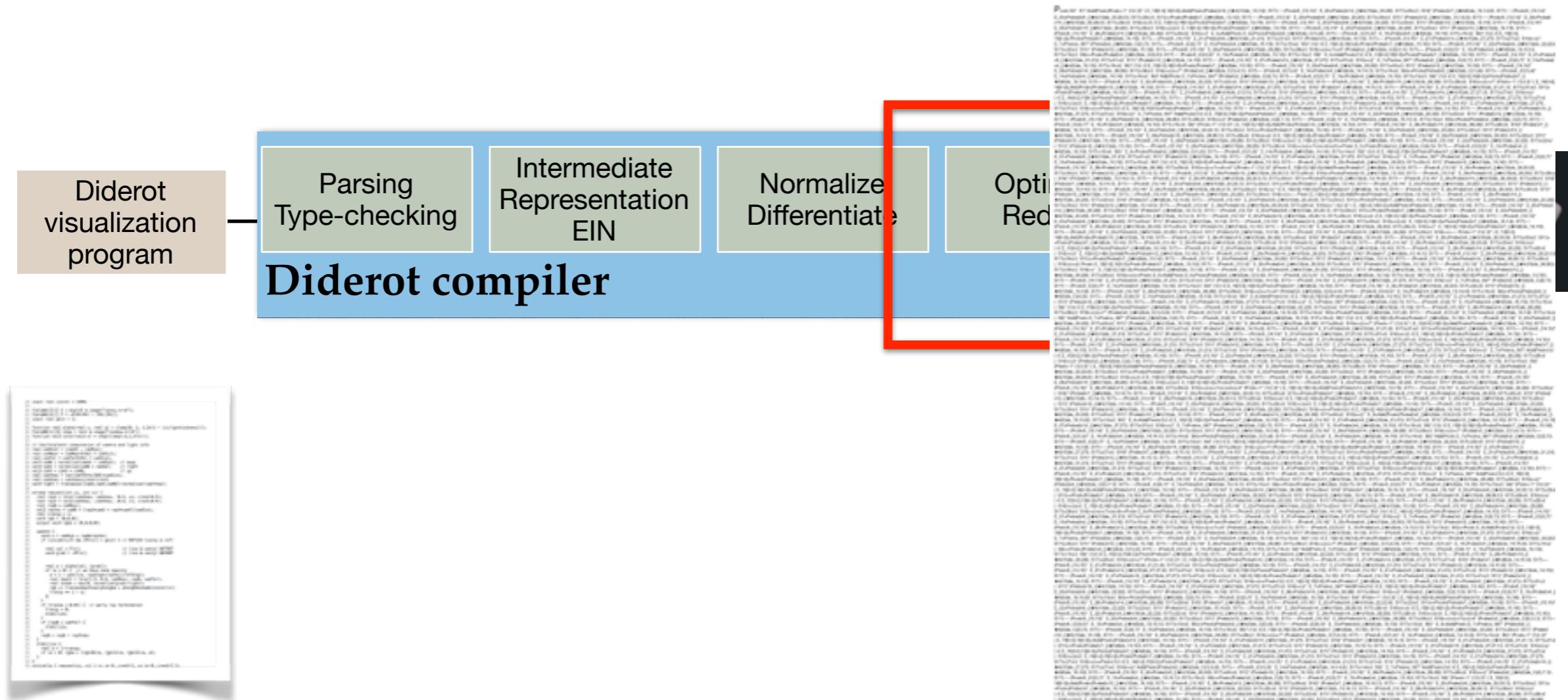
Select a column

# Implementing Expressivity



Phase that enables effective compilation of tensor math

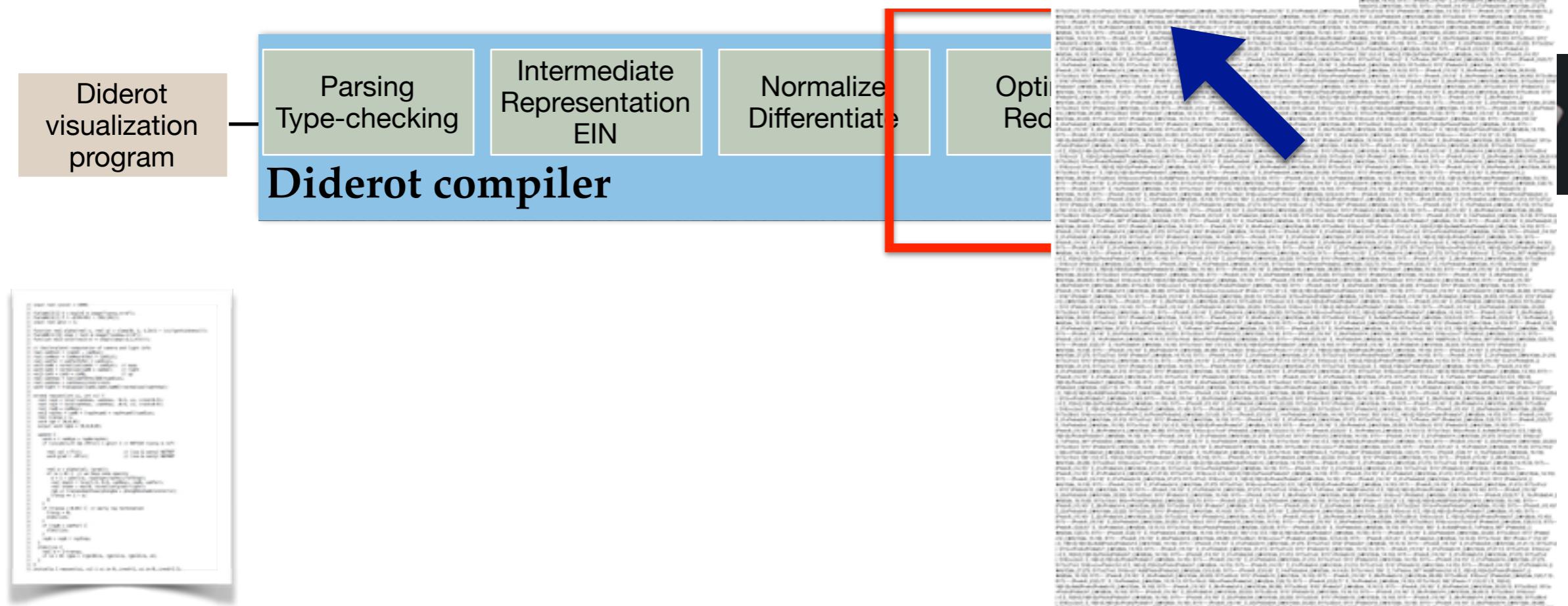
# Compilation Issue



Phase that enables effective compilation of tensor math

# Compilation Issue

$$e_{ij}e_{1k} - e_{0i}e_{jk} + e_{ij}e_{k1}$$



Phase that enables effective compilation of tensor math

# Compilation Issue

$$e_{ij}e_{1k} - e_{0i}e_{jk} + e_{ij}e_{k1}$$

To reduce the size of the IR we want to

# Compilation Issue

$$\boxed{e_{ij}} e_{1k} - e_{0i} e_{jk} + \boxed{e_{ij}} e_{k1}$$

To reduce the size of the IR we want to

- enable traditional compiler approaches ([cse](#))

# Compilation Issue

$$\boxed{e_{ij}}e_{1k} - \boxed{e_{0i}}e_{jk} + e_{ij}e_{k1}$$

To reduce the size of the IR we want to

- enable traditional compiler approaches ([cse](#))
- maintain mathematical integrity

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Terms can be syntactically different even when they

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Terms can be syntactically different even when they

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[CHIW CPC]

**EIN: An Intermediate Representation  
for  
Compiling Tensor Calculus**

Charisee Chiw, Gordon L. Kindlman, and John Reppy

University of Chicago

Our design and implementation



# Write programs with higher-order operations

- Diderot is designed to represent general tensor math

# Write programs with higher-order operations

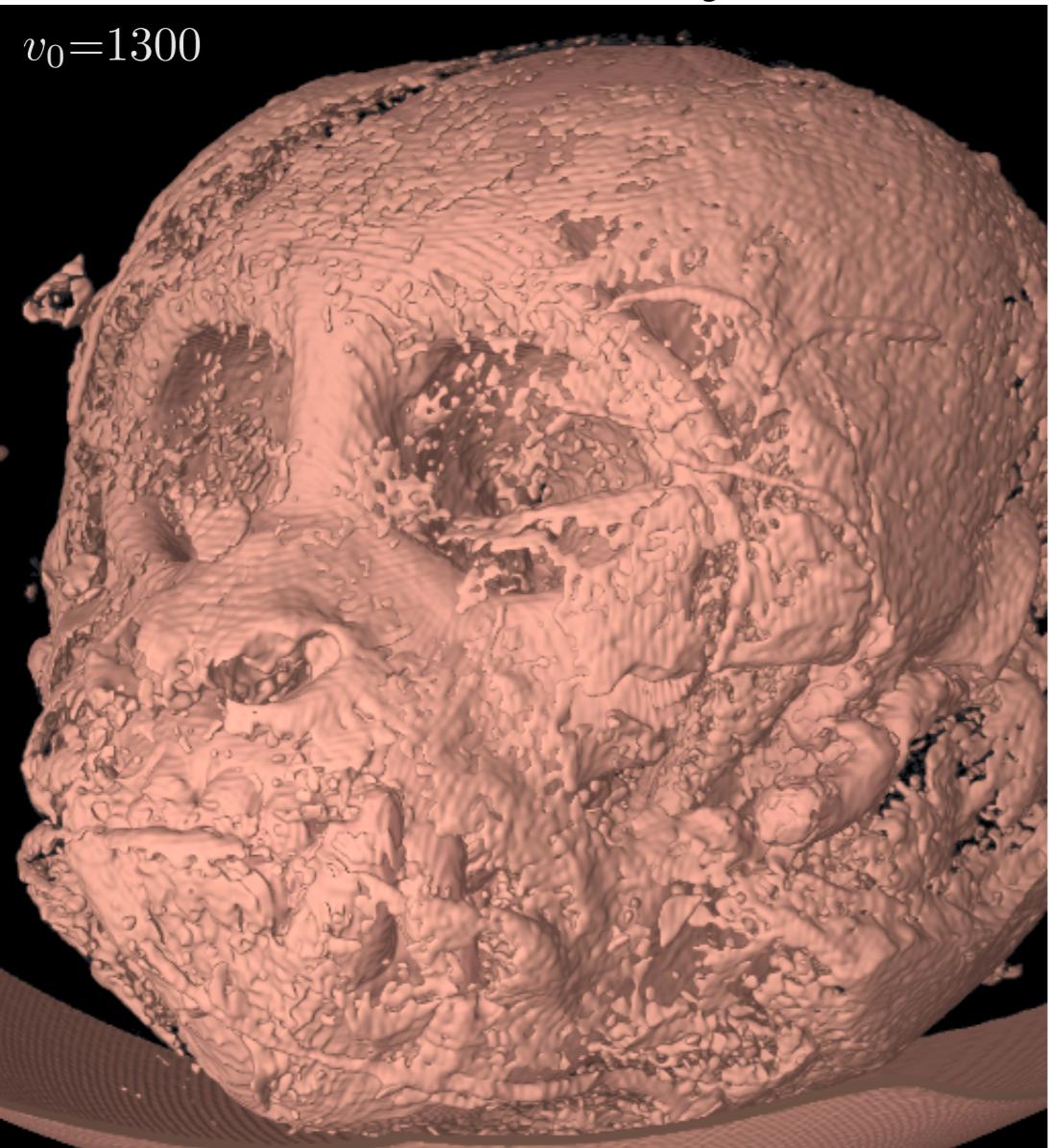
- Diderot is designed to represent general tensor math
- We rely on the **expressivity** of the language to support the implementation of visualization ideas.

# Write programs with higher-order operations

- Diderot is designed to represent general tensor math
- We rely on the **expressivity** of the language to support the implementation of visualization ideas.
- Push development of compiler to enable **sophisticated visualizations**

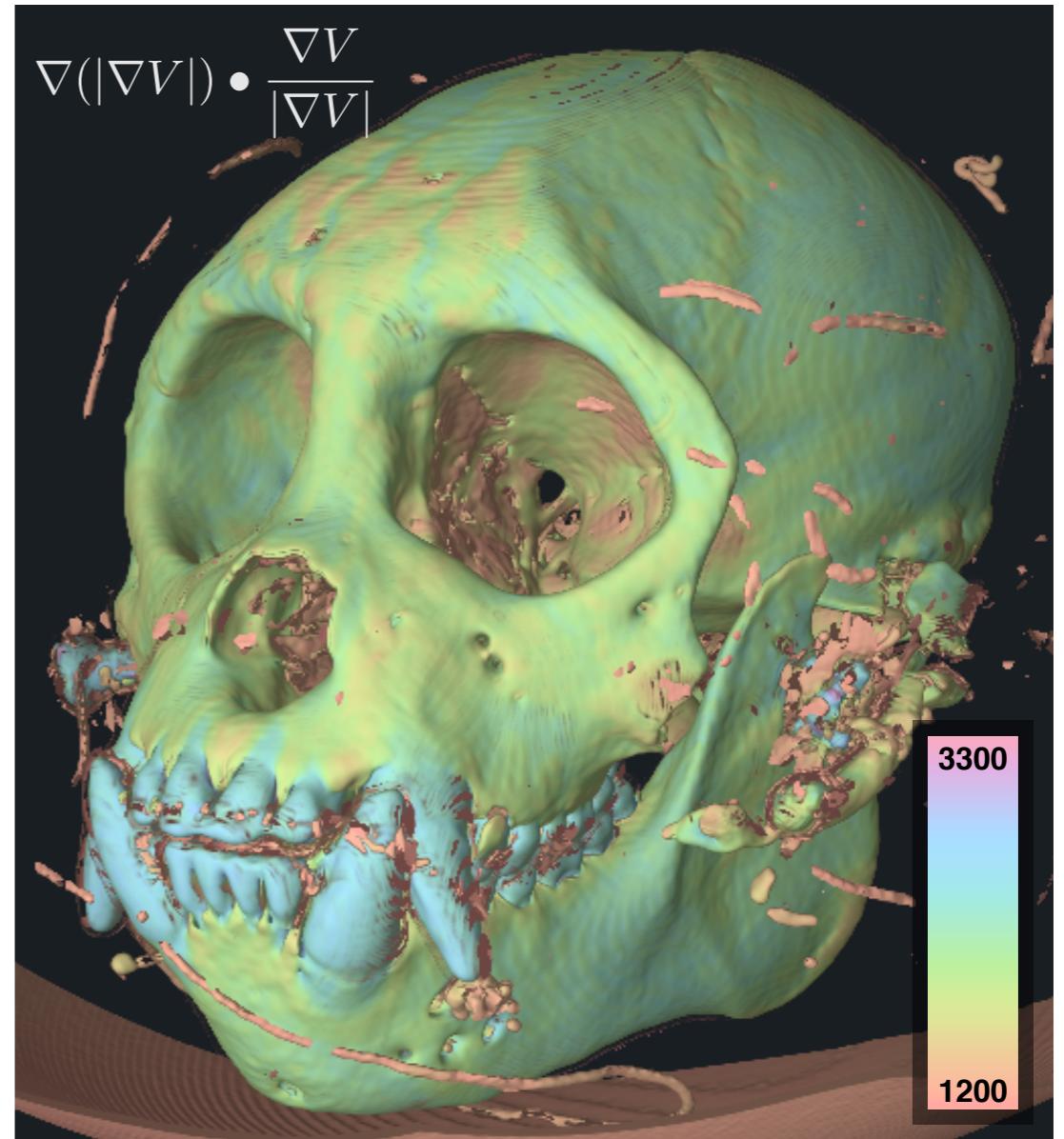
# Results: Sophisticated Vis

Previously:



Iso-Surface

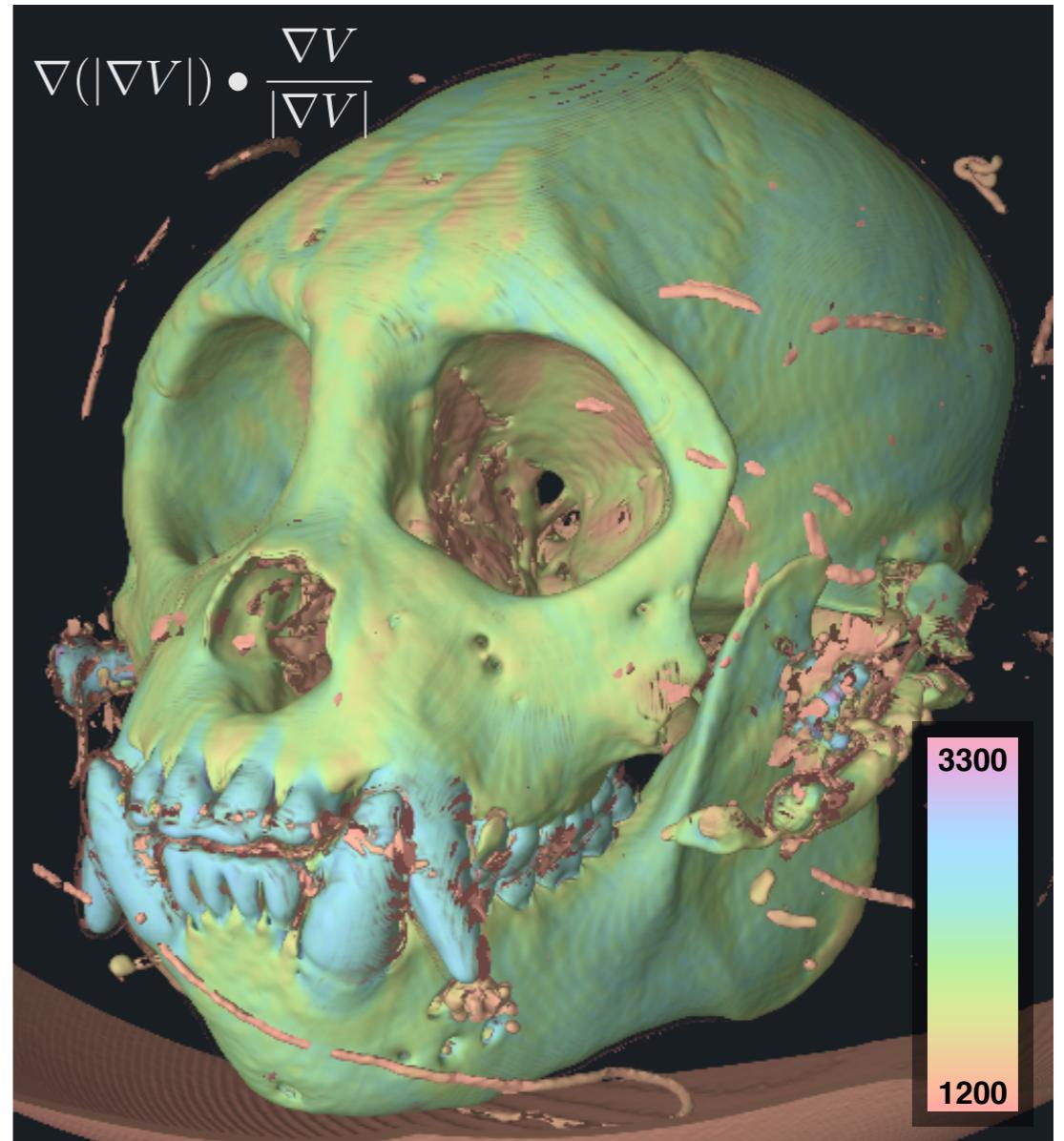
Now:



Canny Edges

Data provided by Callum Ross (U of Chicago)

# Results: Sophisticated Vis

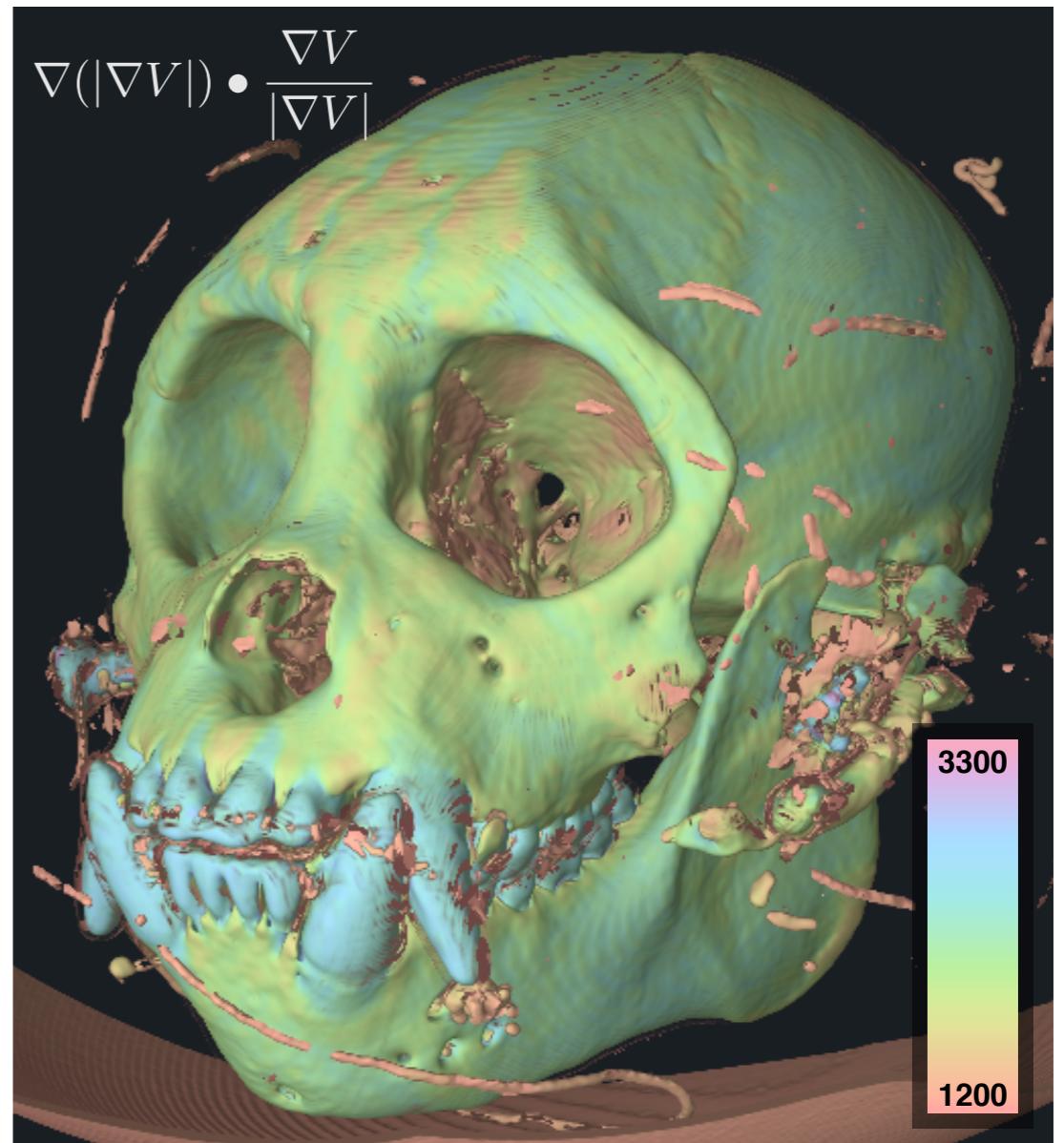


Canny Edges

# Results: Sophisticated Vis

Inline high level syntax into Diderot code

```
-∇(|∇F|) • ∇F / |∇F|;
```



Canny Edges

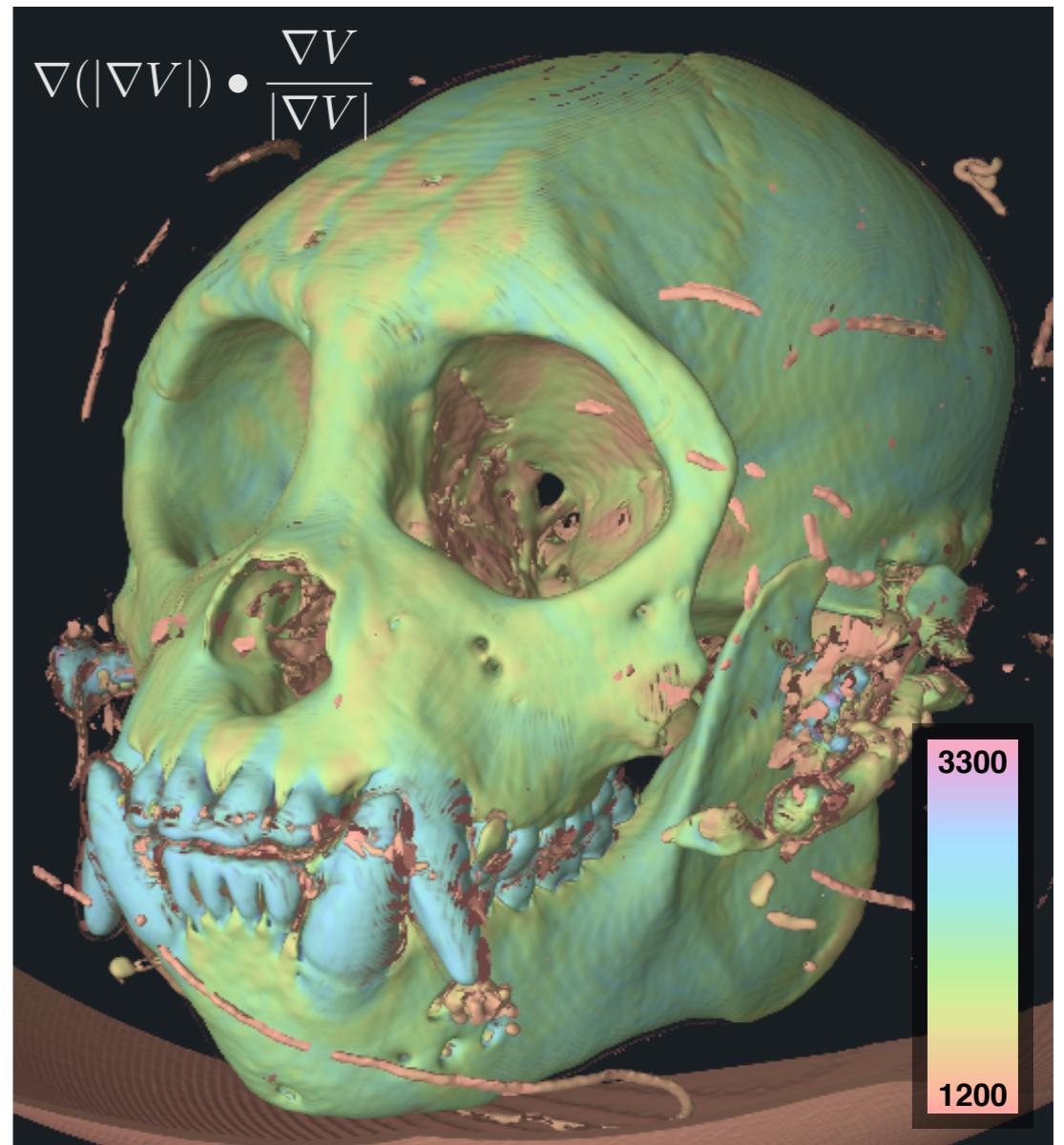
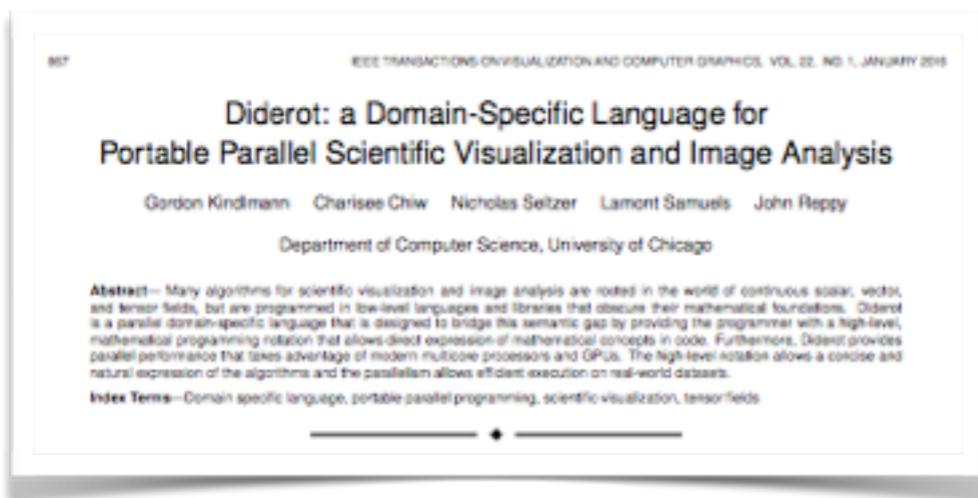
# Results: Sophisticated Vis

Inline high level syntax into Diderot code

```
-∇(|∇F|) • ∇F / |∇F|;
```

Users focused on visualization

## [Kindlmann VIS]



Canny Edges

# Domain Specific Languages and Tools

- Notation can **express** statements in the domain space
- **Efficient** execution of expensive computations
- Enable cutting-edge research



Chemistry



Cryptography

Unified Form Language (UFL)

Solve PDEs



Digital Signal Processing



Web design



Microsoft®  
SQL Server™

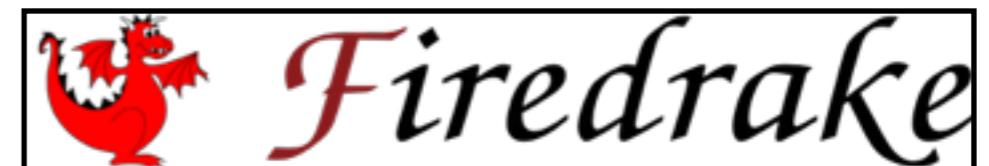
Databases



Diderot Image analysis

# Domain Specific Languages and Tools

- Notation can **express** statements in the domain space
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Big Matrix computations



Cryptography: assurance and security

Unified Form Language (UFL)

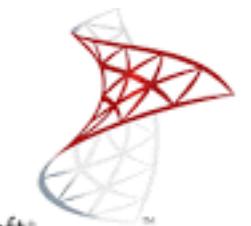
Hands over to form compiler



Digital Signal Processing  
Transformation



Web design



Microsoft®  
SQL Server™

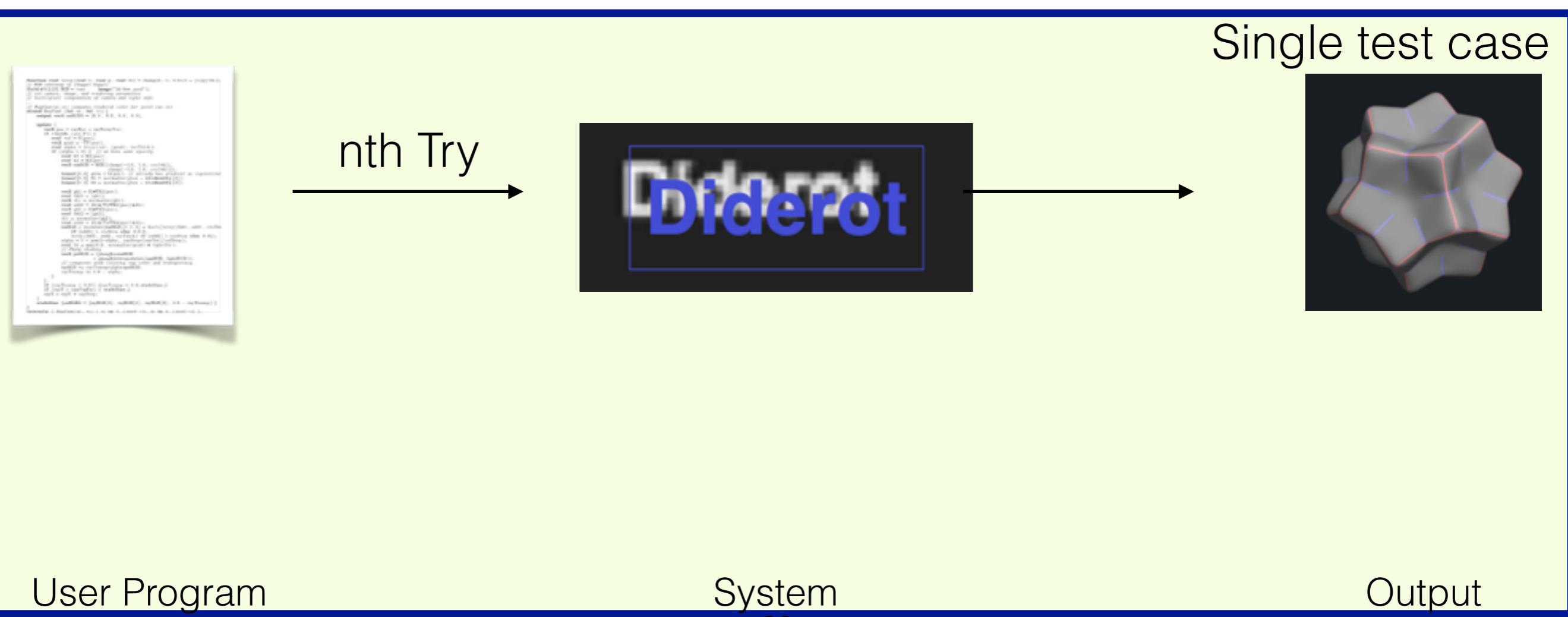
Databases

Diderot

Vis and general tensor math

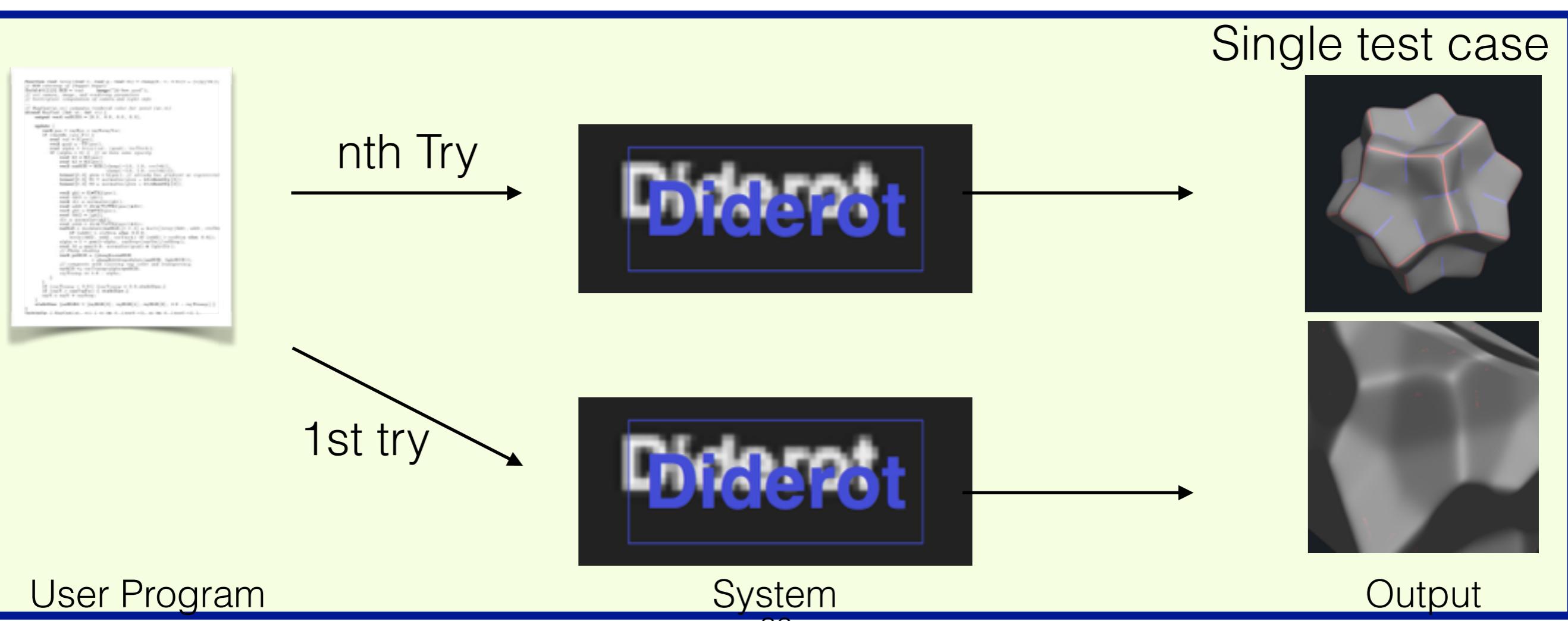
# Return to Crest Line example

We need to test our language implementation



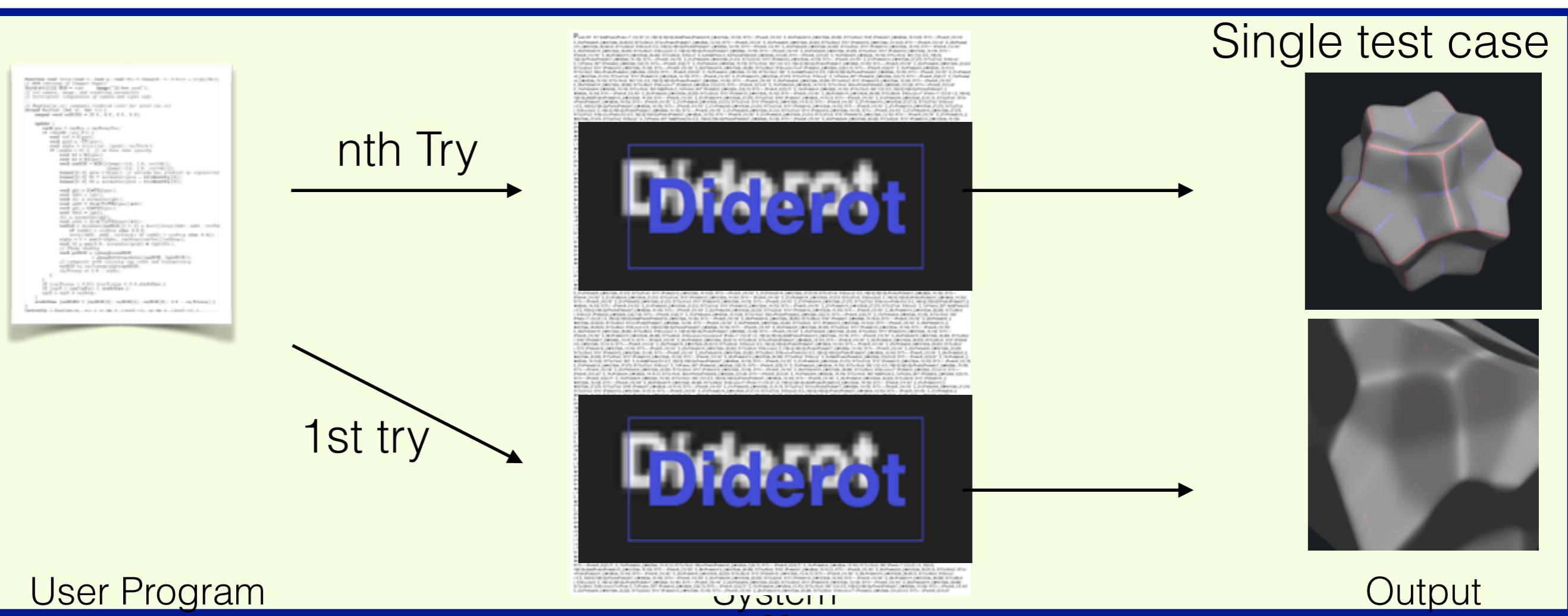
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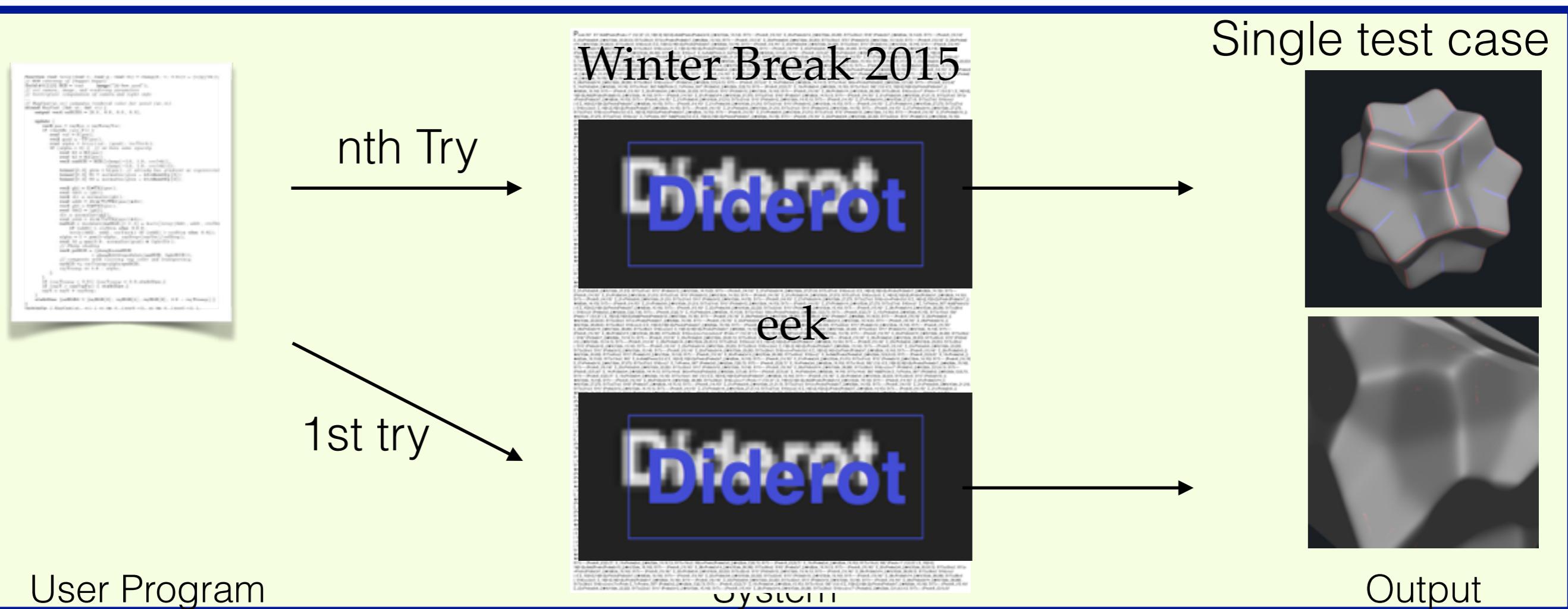
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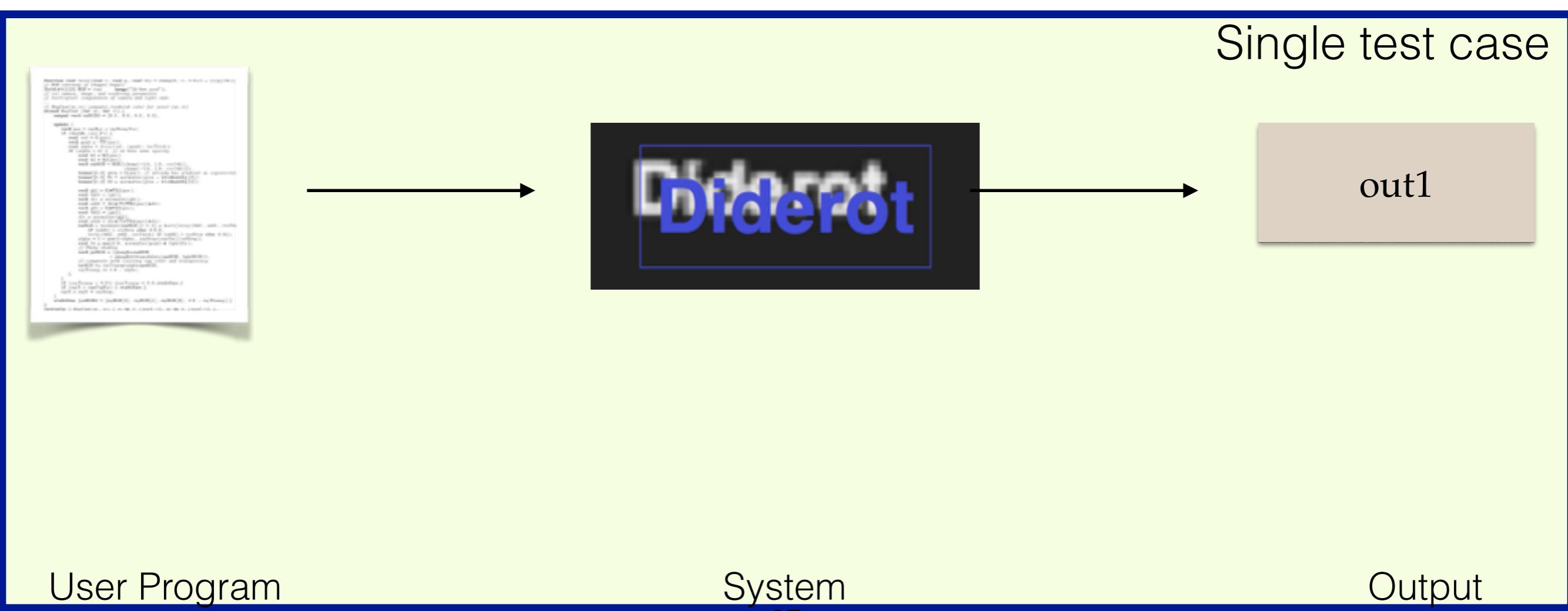
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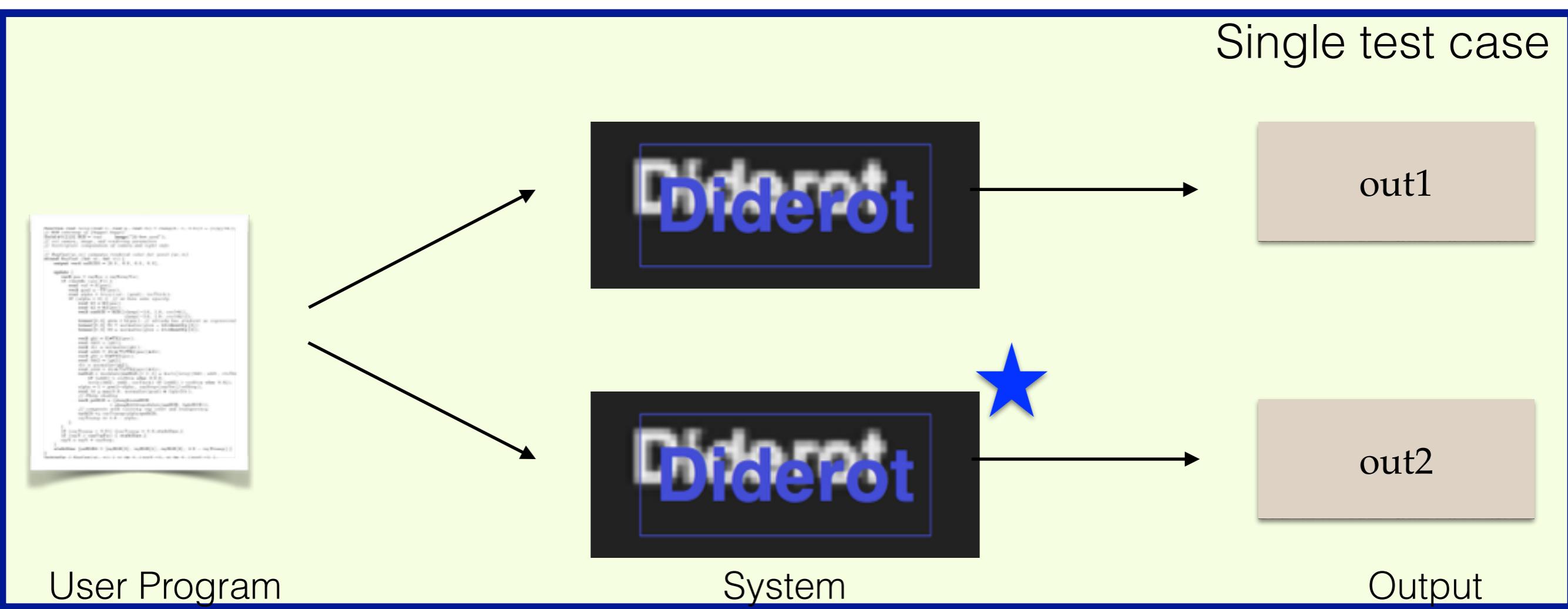
# Software testing

- Different Optimization Levels [Chen]



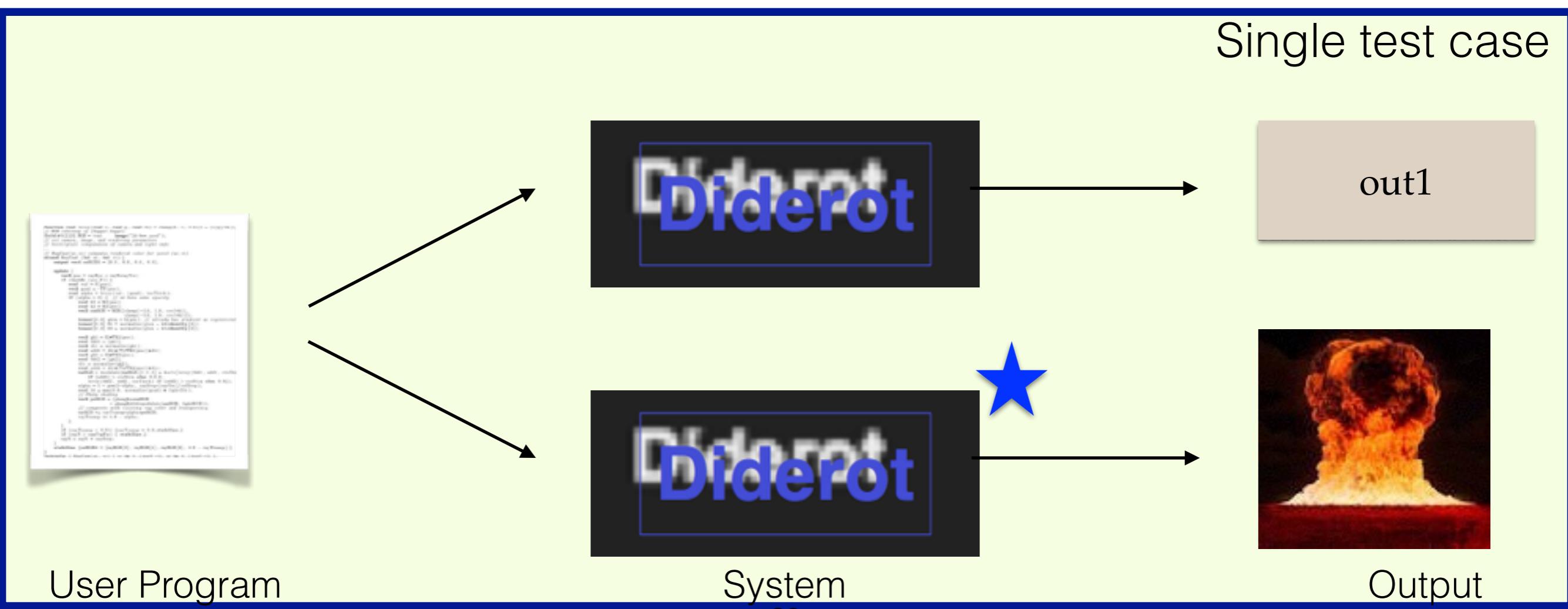
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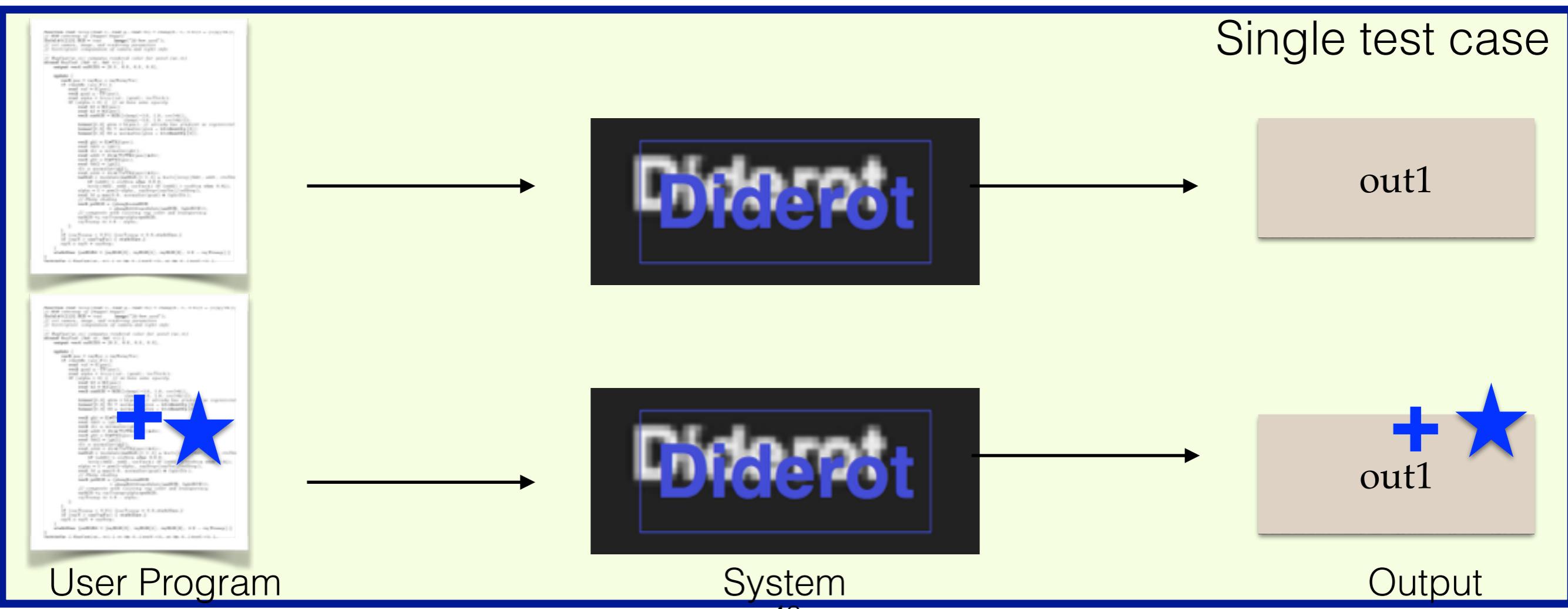
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# Software testing

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User Program

System

Output

# Software testing

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- **Differential Testing** [Yang et.al'11, Lindig'05, McKeeman'98]

Single test case



User Program



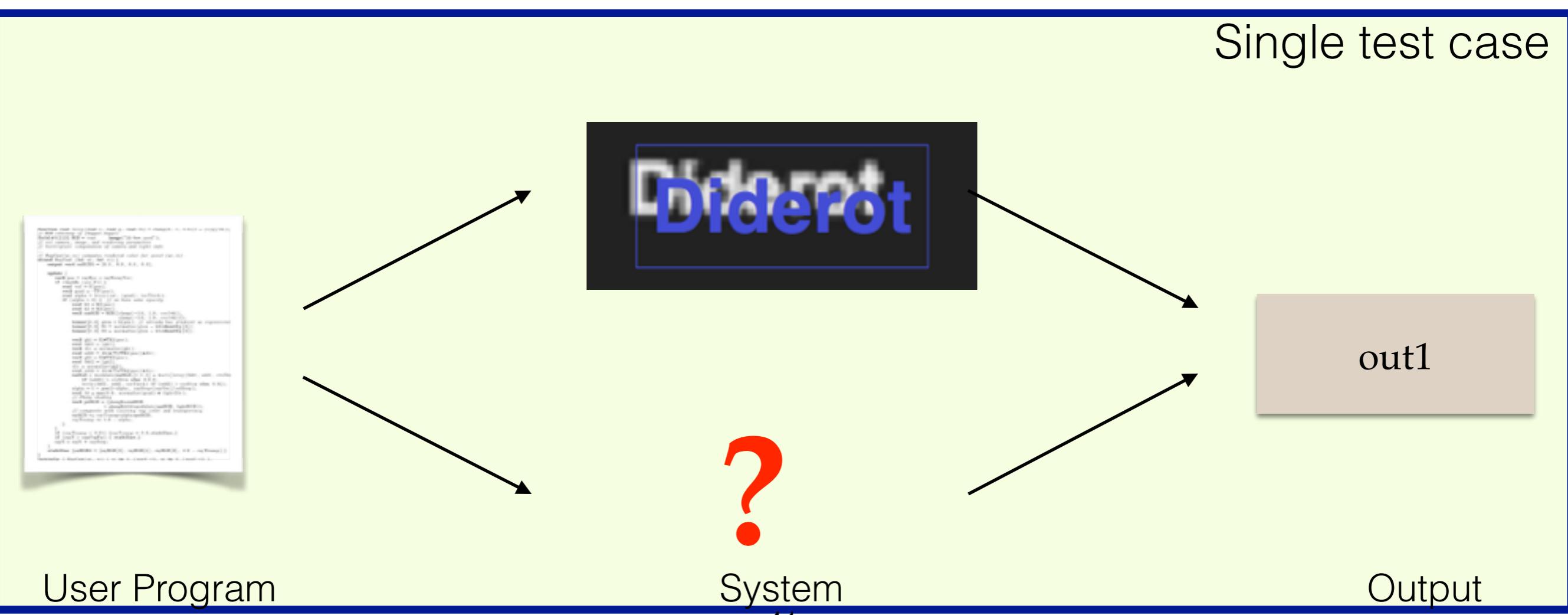
System



Output

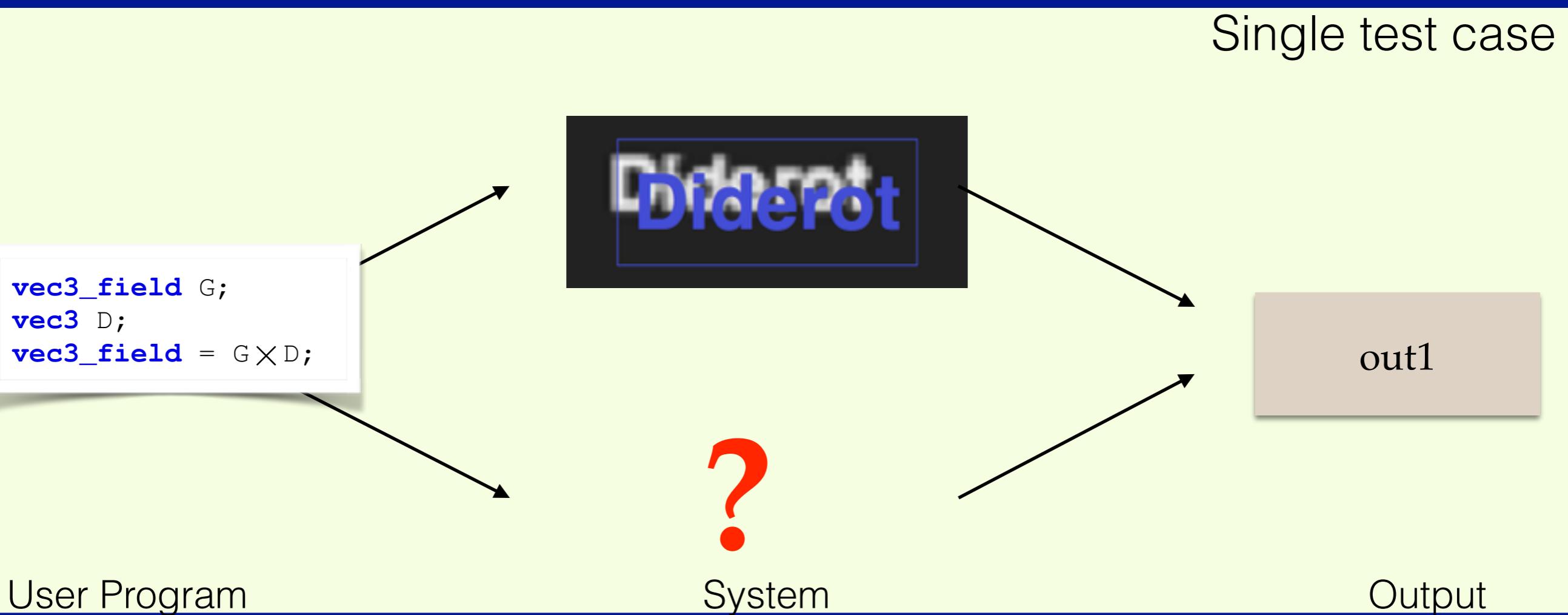
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# Software testing

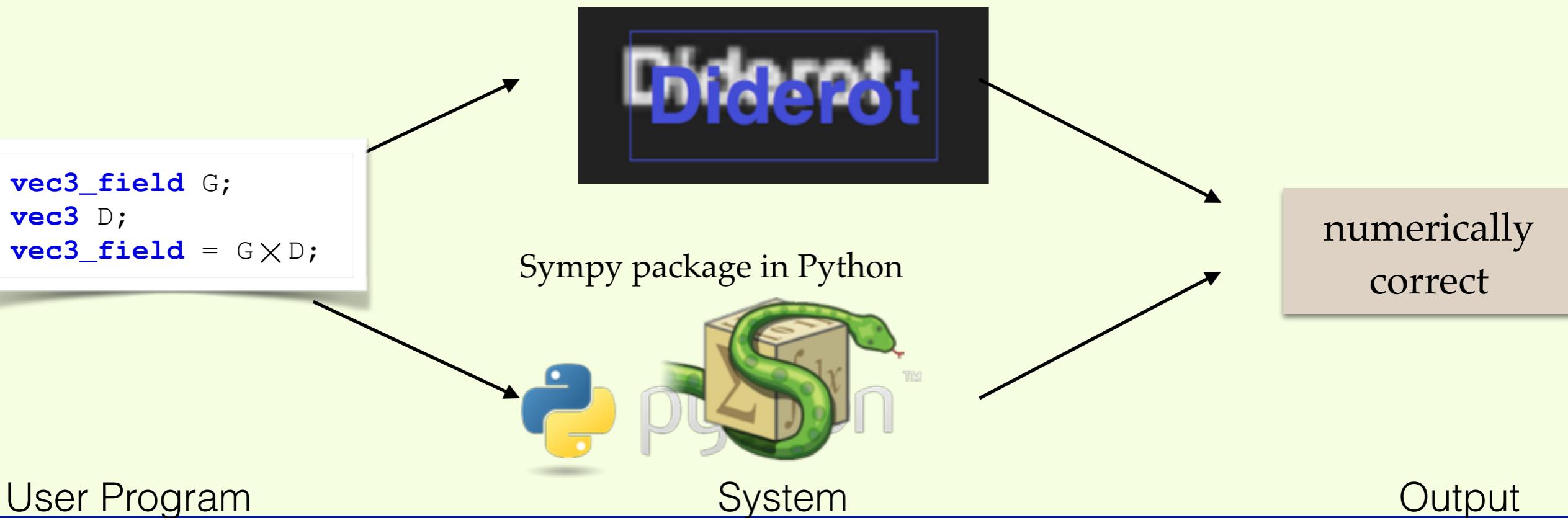
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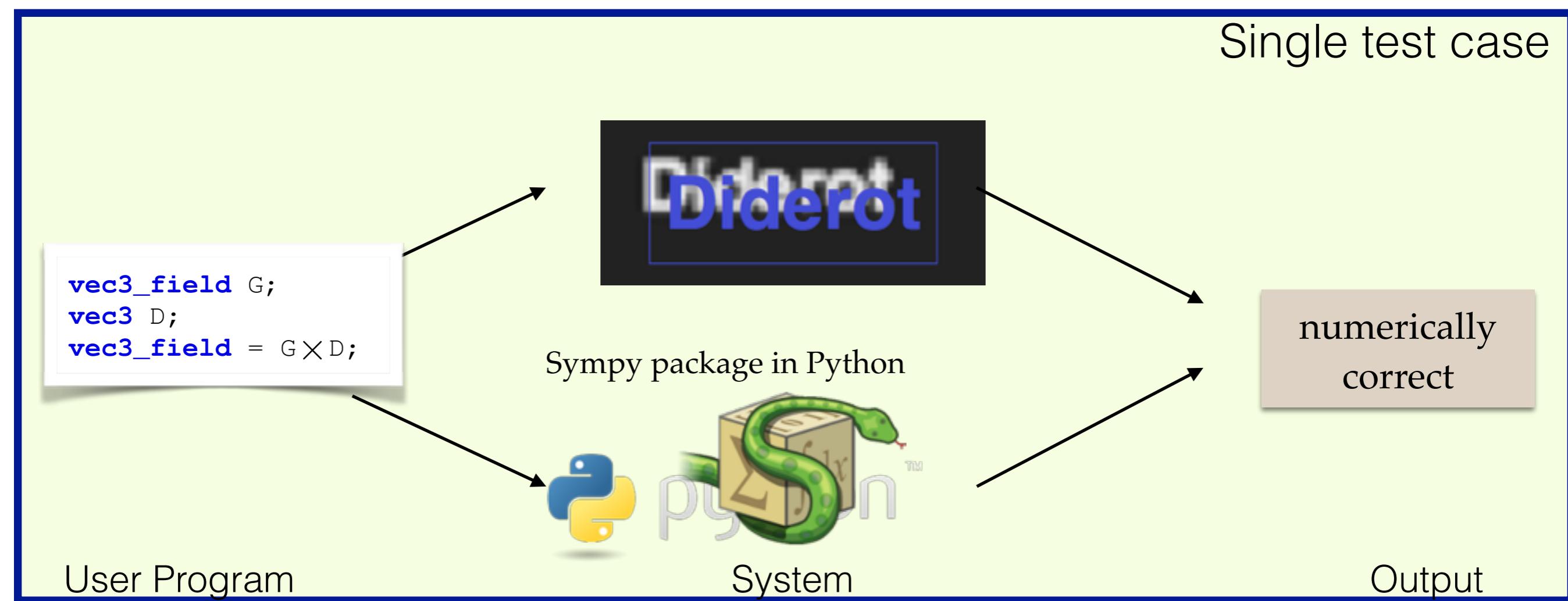
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- Differential Testing [Yang et.al'11, Lindig'05, McKeeman'98]
- DATm: Diderot's Automated Testing model [Chiw '17]

Single test case



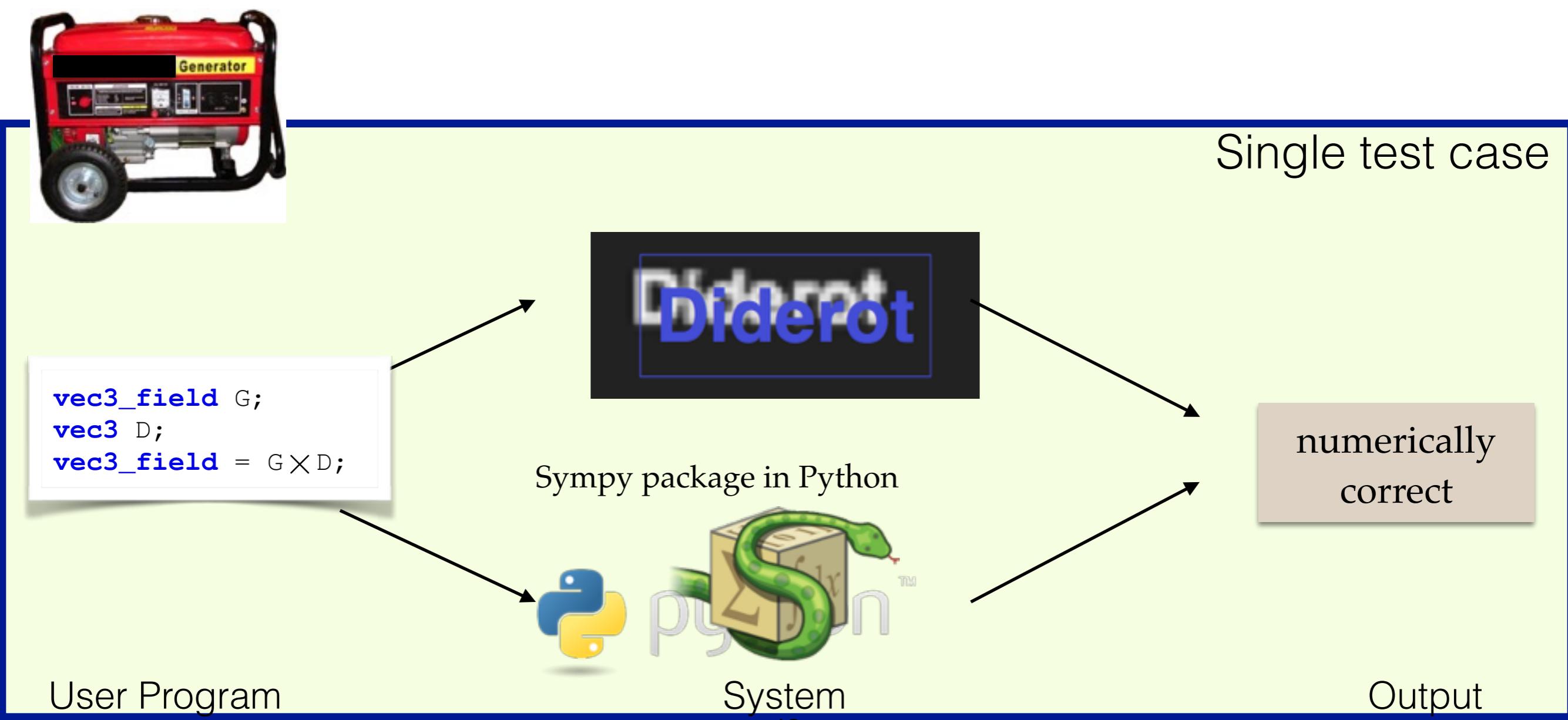
# DATm: Diderot's Automated Testing model

- Manual construction of test infeasible



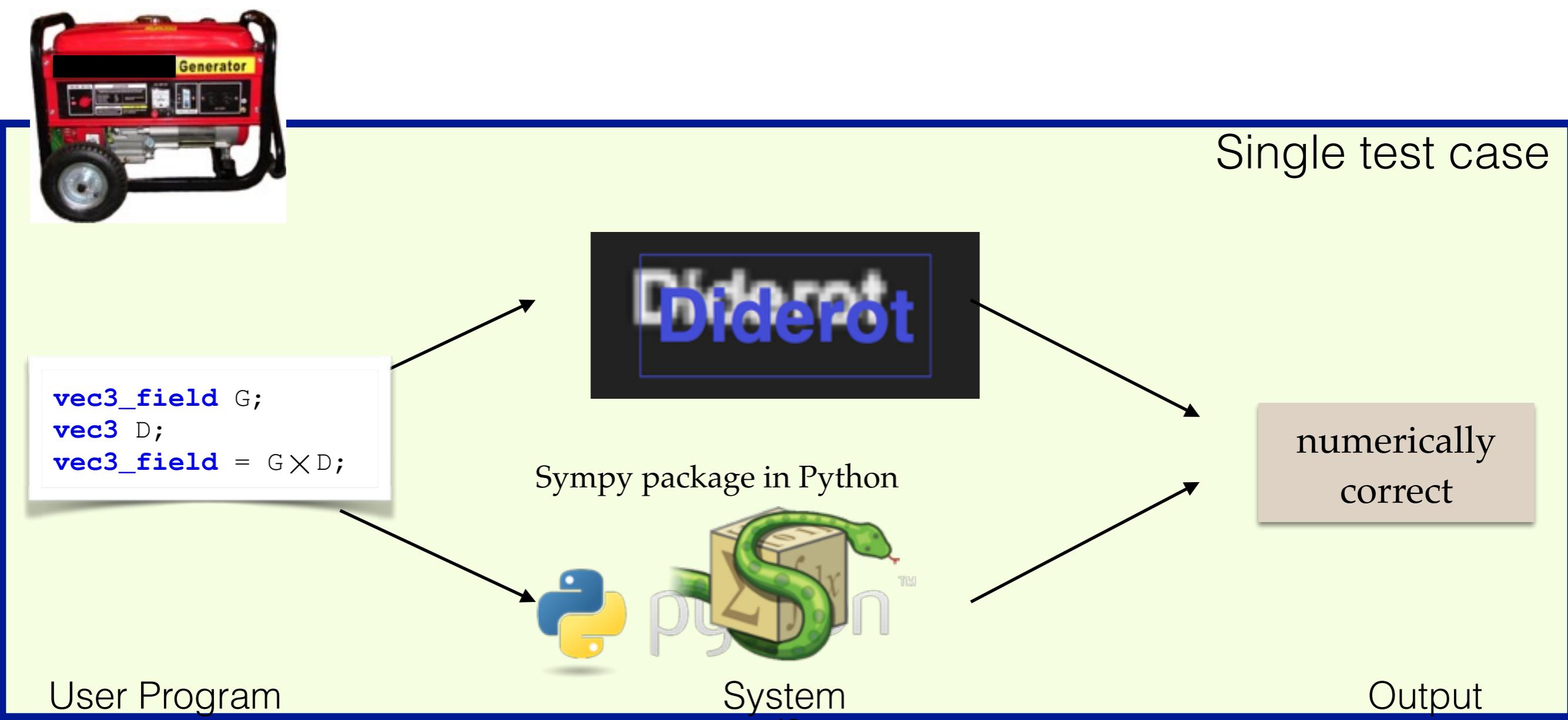
# DATm: Diderot's Automated Testing model

- Manual construction of test infeasible
- DATm automatically creates hundred of thousands of tests



# DATm: Diderot's Automated Testing model

- Manual construction of test infeasible
  - DATm automatically creates hundred of thousands of tests
  - Successful at finding bugs



# DATm: Diderot's Automated Testing model

Generate test cases



Check operator “x” with  
different argument types

- OK
- Type error
- Compiler error
- Numerical error

# DATm: Diderot's Automated Testing model

Generate test cases



```
vec2 A;  
vec2 B;  
vec2 out = A × B;
```

```
vec3 C;  
vec3 D;  
vec3 out = C × D;
```

```
vec2_field E;  
vec2_field F;  
vec2_field = E × F;
```

```
vec3_field G;  
vec3_field H;  
vec3_field = G × H;
```

```
vec2 A;  
vec2_field F;  
vec2_field = A × F;
```

```
vec3_field G;  
vec3 D;  
vec3_field = G × D;
```

```
vec2_field E;  
vec2 B;  
vec2_field = E × B;
```

```
vec3 C;  
vec3_field H;  
vec3_field = C × H;
```

Check operator “x” with different argument types

- OK
- Type error
- Compiler error
- Numerical error

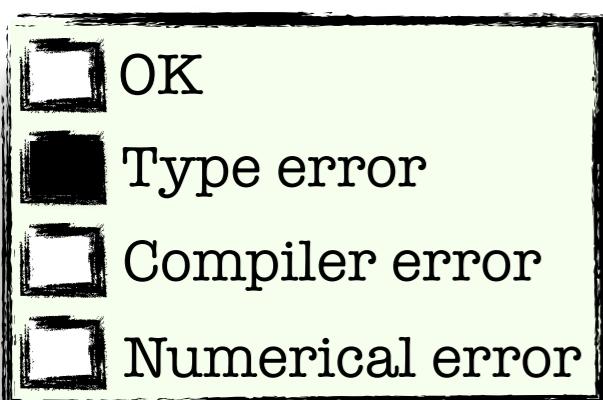
# DATm: Diderot's Automated Testing model

Generate test cases



Check operator “ $\times$ ” with different argument types

<code>vec2 A;</code>	<code>vec2 B;</code>	<code>vec2 out = A <math>\times</math> B;</code>	
<code>vec2_field E;</code>	<code>vec2_field F;</code>	<code>vec2_field = E <math>\times</math> F;</code>	
<code>vec2 A;</code>	<code>vec2_field F;</code>	<code>vec2_field = A <math>\times</math> F;</code>	
<code>vec3_field G;</code>	<code>vec3 D;</code>	<code>vec3_field = G <math>\times</math> D;</code>	
<code>vec2_field E;</code>	<code>vec2 B;</code>	<code>vec2_field = E <math>\times</math> B;</code>	
<code>vec3 C;</code>	<code>vec3_field H;</code>	<code>vec3_field = C <math>\times</math> H;</code>	



# DATm: Diderot's Automated Testing model

Generate test cases



Check combination of operators

- OK
- Type error
- Compiler error
- Numerical error

# DATm: Diderot's Automated Testing model

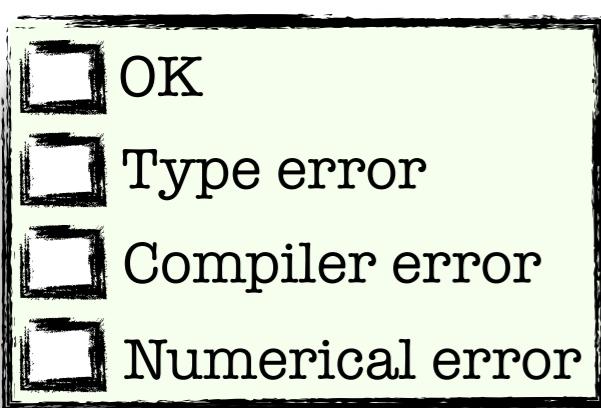
Generate test cases



```
mat3_field F;  
mat3_field G;  
mat3_field out modulate (F, G);
```

Check combination of  
operators

```
mat3_field F;  
mat3_field G;  
real_field out = trace ( modulate (F, G) );
```



```
mat3_field F;  
mat3_field G;  
real_field out = trace (-modulate (F, G));
```

# DATm: Diderot's Automated Testing model

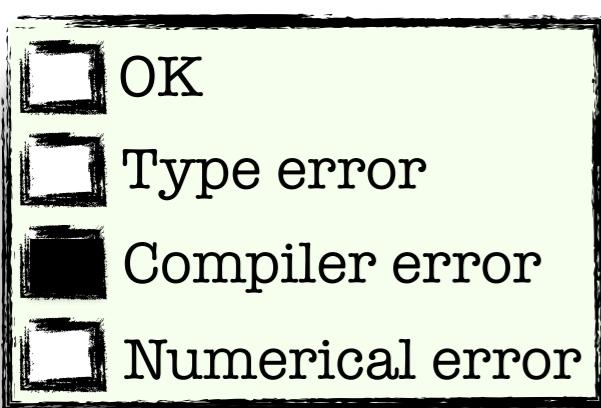
Generate test cases



```
mat3_field F;  
mat3_field G;  
mat3_field out modulate (F, G);
```



Check combination of operators



```
mat3_field F;  
mat3_field G;  
real_field out = trace ( modulate (F, G));
```



```
mat3_field F;  
mat3_field G;  
real_field out = trace (-modulate (F, G));
```



# DATm: Diderot's Automated Testing model

Generate test cases



```
mat3_field F;  
mat3_field G;  
mat3_field out modulate (F, G);
```



Check combination of operators

2017 IEEE/ACM 12th International Workshop on Automation of Software Testing

**DATm: Diderot's Automated Testing Model**

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Department of Computer Science, University of Chicago Email: {cciw,gk,jr}@cs.uchicago.edu

**Abstract**—Diderot is a parallel domain-specific language for the analysis and visualization of multidimensional scientific images, such as those produced by CT and MRI scanners [6], [14], [5]. Diderot is designed to support algorithms that are based on differential tensor calculus and produces a higher-order mathematical model which allows direct manipulation of tensor fields. One of the main challenges of the Diderot implementation is bridging this semantic gap by effectively translating high-level mathematical notation of tensor calculus into efficient low-level code in the target language.

A key question for a high-level language, such as Diderot, is how do we know that the implementation is correct. We have previously presented and defended a core set of rewriting rules, but the full translation from source to executable requires much more work. In this paper, we present DATm, Diderot's automated

combinations is too large for manual exploration. Thus, as in previous work, it is vital that we build a testing tool that can automatically generate test cases that provide good coverage of the features of the language [16], [8].

There is extensive research in compiler testing. Differential testing relies on comparing various versions and implementations of the compiler [17], [25]. Equivalence Modulo Inputs [15] creates a family of programs that can be used for differential testing. Alone, these approaches did not seem sufficient in the case of Diderot. Most of the substantial transformations that occur during compilation are necessary and not reasonable to disable. Additionally, earlier versions of the compiler are

```
mat3_field F;  
mat3_field G;  
real_field out = trace ( modulate (F, G));
```



```
mat3_field F;  
mat3_field G;  
real_field out = trace (-modulate (F, G));
```



# What's next?

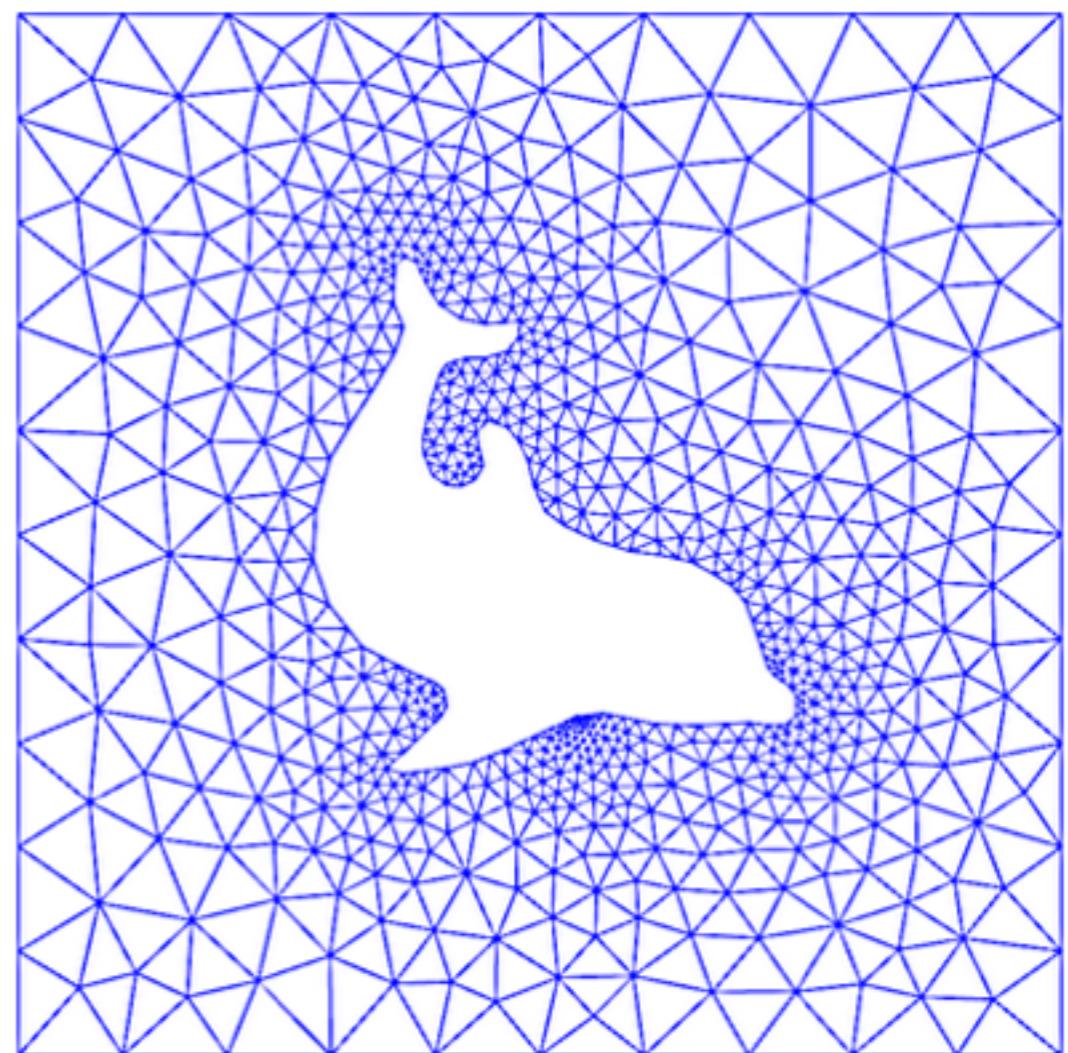
- Shown that we can address domain specific needs

# What's next?

- Shown that we can address domain specific needs
- New domain focus: scientific computing

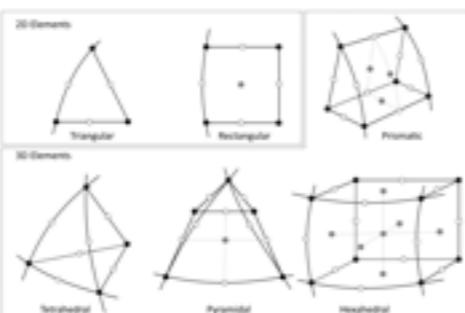
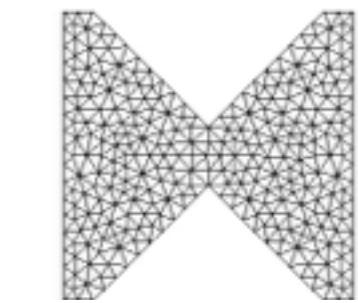
# Scientific computing domain

- Partial differential equations (PDEs) model a range of problems.

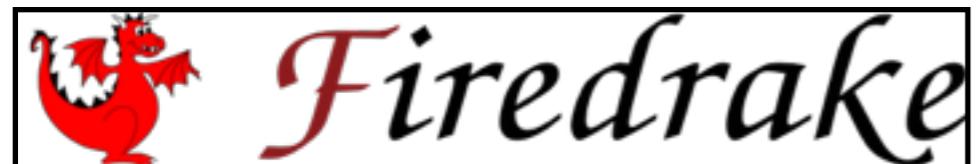
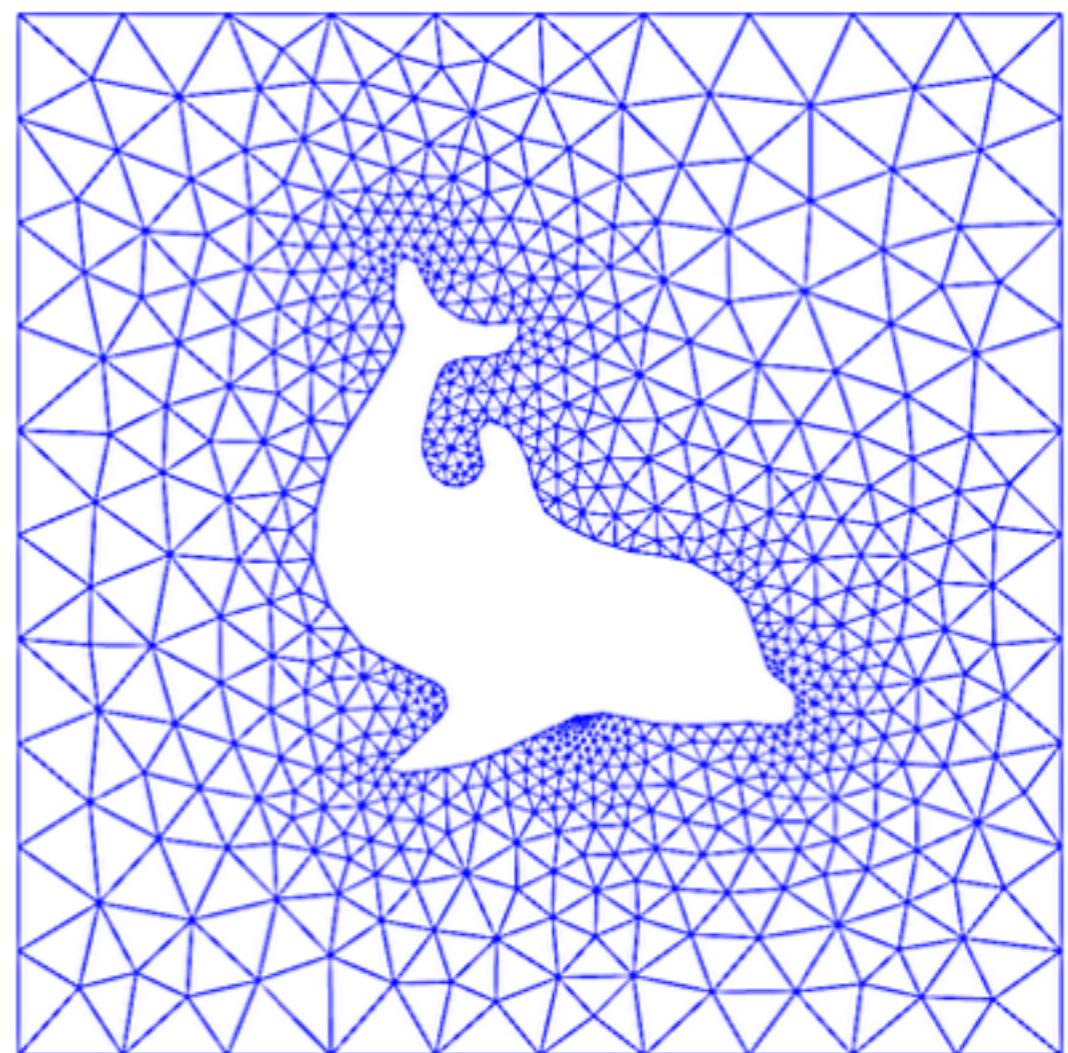


# Scientific computing domain

- Partial differential equations (PDEs) model a range of problems.
- Various software tools are used solve PDEs

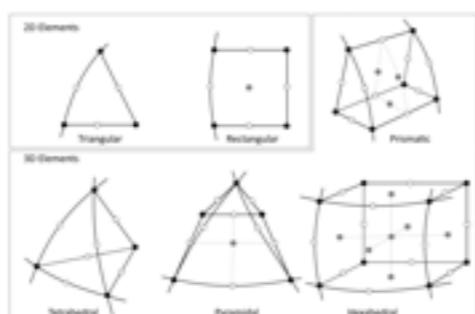
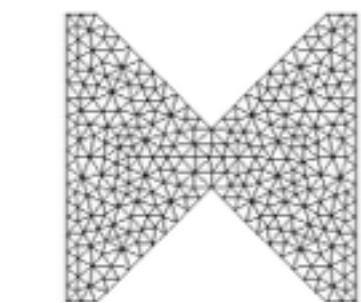


FENICS  
PROJECT

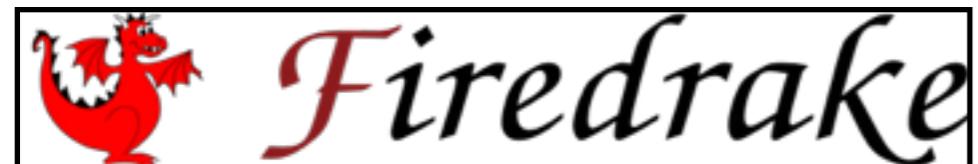


# Scientific computing domain

- Partial differential equations (PDEs) model a range of problems.
- Various software tools are used solve PDEs
- Tools know PDEs, not Vis.

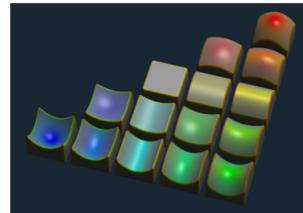


FENICS  
PROJECT

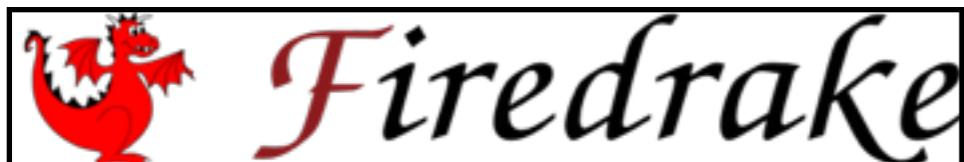


# First approach

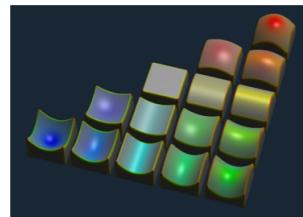
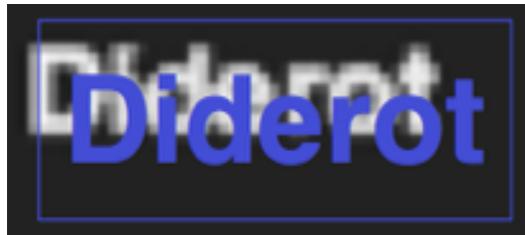
Connect Diderot and Firedrake with call back functions



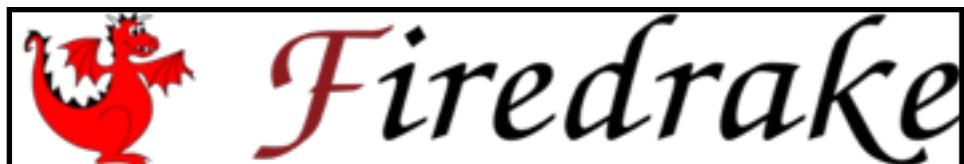
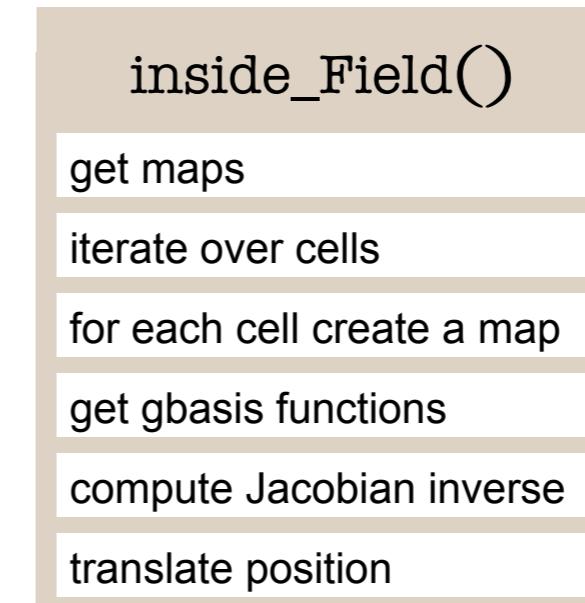
```
input fem#k(d) [] F;
field#k(d)[d] G = - $\nabla$ F;
field#k(d)[d,d] H = - $\nabla \otimes \nabla$ F;
tensor[d] pos;
...
if (inside(pos,F)){
    real out = F(pos);
    tensor[d] grad = G(pos);
    tensor[d,d] hess = H(pos);
}
```



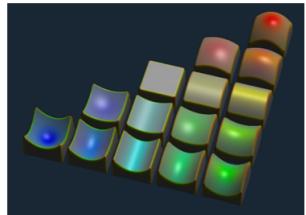
# First approach



```
input fem#k(d) [] F;
field#k(d)[d] G = - $\nabla$ F;
field#k(d)[d,d] H = - $\nabla \otimes \nabla$ F;
tensor[d] pos;
...
if (inside(pos,F)){
    real out = r(pos);
    tensor[d] grad = G(pos);
    tensor[d,d] hess = H(pos);
}
```



# First approach



```
input fem#k(d) [] F;
field#k(d)[d] G = - $\nabla$ F;
field#k(d)[d,d] H = - $\nabla \otimes \nabla$ F;
tensor[d] pos;
...
if (inside(pos,r)){
    real out = F(pos);
    tensor[d] grad = G(pos);
    tensor[d,d] hess = H(pos);
}
```

inside\_Field()

get maps

iterate over cells

for each cell create a map

get gbasis functions

compute Jacobian inverse

translate position

evaluate\_Field()

get maps

iterate over cells

for each cell create a map

get gbasis functions

compute Jacobian inverse

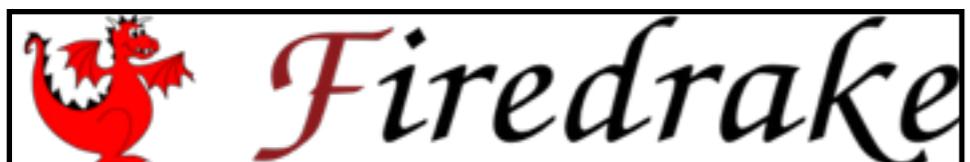
translate position

get data coordinates

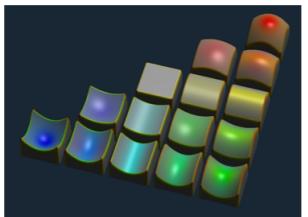
compute weights

get sbasis function

evaluate basis functions



# First approach



```
input fem#k(d) [] F;
field#k(d)[d] G = - $\nabla$ F;
field#k(d)[d,d] H = - $\nabla \otimes \nabla$ F;
tensor[d] pos;
...
if (inside(pos,F)){
    real out = F(pos),
    tensor[d] grad = G(pos);
    tensor[d,d] hess = H(pos);
}
```



inside\_Field()

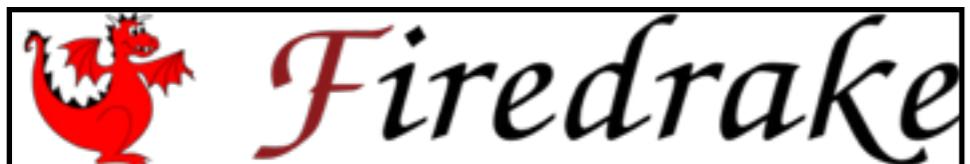
- get maps
- iterate over cells
- for each cell create a map
- get gbasis functions
- compute Jacobian inverse
- translate position

evaluate\_Field()

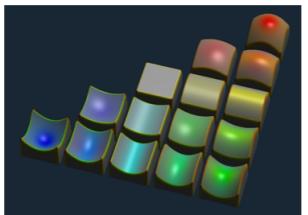
- get maps
- iterate over cells
- for each cell create a map
- get gbasis functions
- compute Jacobian inverse
- translate position
- get data coordinates
- compute weights
- get derivative basis function values
- evaluate basis functions

gradient\_Field()

- get maps
- iterate over cells
- for each cell create a map
- get gbasis functions
- compute Jacobian inverse
- translate position
- check cell
- use maps
- get basis Jacobian
- compute Jacobian inverse
- get data coordinates
- compute weights
- get derivative basis function values
- evaluate basis functions



# First approach



```
input fem#k(d) [] F;
field#k(d) [d] G = - $\nabla$ F;
field#k(d) [d,d] H = - $\nabla \otimes \nabla$ F;
tensor[d] pos;
...
if (inside(pos,F)){
    real out = F(pos);
    tensor[d] grad = C(pos);
    tensor[d,d] hess = H(pos);
}
```



inside\_Field()

get maps  
iterate over cells  
for each cell create a map  
get gbasis functions  
compute Jacobian inverse  
translate position

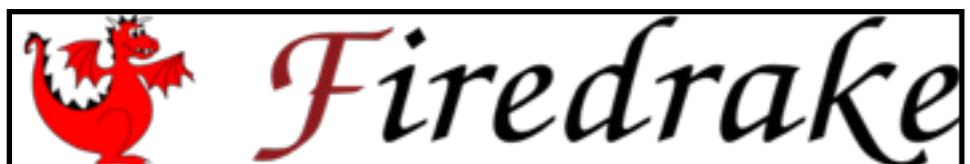
evaluate\_Field()

get maps  
iterate over cells  
for each cell create a map  
get gbasis functions  
compute Jacobian inverse  
translate position  
get data coordinates  
compute weights  
get derivative basis function values  
evaluate basis functions

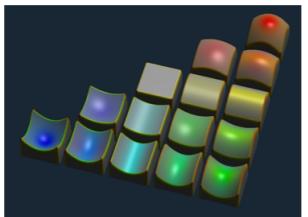
gradient\_Field()

get maps  
iterate over cells  
for each cell create a map  
get gbasis functions  
compute Jacobian inverse  
translate position  
check cell  
use maps  
get basis Jacobian  
compute Jacobian inverse  
get data coordinates  
compute weights  
get derivative basis function values  
evaluate basis functions

H(pos)



# First approach



```
input fem#k(d) [] F;
field#k(d) [d] G = - $\nabla$ F;
field#k(d) [d,d] H = - $\nabla \otimes \nabla$ F;
tensor[d] pos;
...
if (inside(pos,F)){
    real out = F(pos);
    tensor[d] grad = C(pos);
    tensor[d,d] hess = H(pos);
}
```



inside\_Field()

get maps  
iterate over cells  
for each cell create a map  
get gbasis functions  
compute Jacobian inverse  
translate position

evaluate\_Field()

get maps  
iterate over cells  
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gradient\_Field()

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translate position  
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compute Jacobian inverse  
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compute weights  
get derivative basis function values  
evaluate basis functions

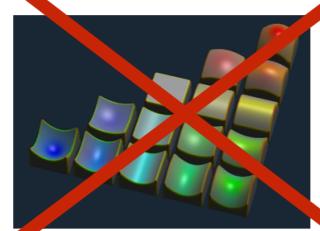
H(pos)

?

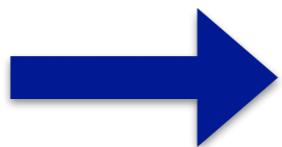


# First approach

can't apply visualization



```
input fem#k(d) [] F;
field#k(d)[d] G = - $\nabla$ F;
field#k(d)[d,d] H = - $\nabla \otimes \nabla$ F;
tensor[d] pos;
...
if (inside(pos,F)){
    real out = F(pos);
    tensor[d] grad = G(pos);
    tensor[d,d] hess = H(pos);
}
```



inside\_Field()

get maps  
iterate over cells  
for each cell create a map  
get gbasis functions  
compute Jacobian inverse  
translate position

evaluate\_Field()

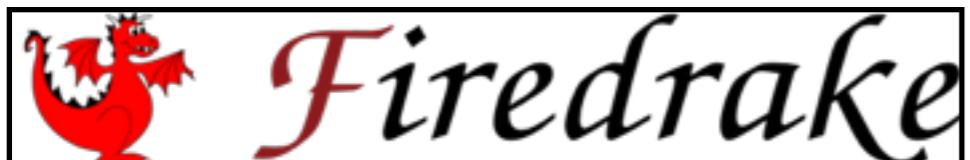
get maps  
iterate over cells  
for each cell create a map  
get gbasis functions  
compute Jacobian inverse  
translate position  
get data coordinates  
compute weights  
get derivative basis function values  
evaluate basis functions

gradient\_Field()

get maps  
iterate over cells  
for each cell create a map  
get gbasis functions  
compute Jacobian inverse  
translate position  
check cell  
use maps  
get basis Jacobian  
compute Jacobian inverse  
get data coordinates  
compute weights  
get derivative basis function values  
evaluate basis functions

H(pos)

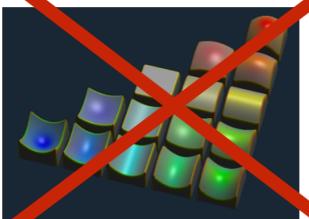
?



# First approach

## redundancy and missing optimizations

can't apply visualization



```
input fem#k(d) [] F;
field#k(d)[d] G = - $\nabla$ F;
field#k(d)[d,d] H = - $\nabla \otimes \nabla$ F;
tensor[d] pos;
...
if (inside(pos,F)){
    real out = F(pos);
    tensor[d] grad = G(pos);
    tensor[d,d] hess = H(pos);
}
```



*Firedrake*



inside\_Field()

```
get maps
iterate over cells
for each cell create a map
get gbasis functions
compute Jacobian inverse
translate position
```

gradient\_Field()

```
get maps
iterate over cells
for each cell create a map
get gbasis functions
compute Jacobian inverse
translate position
```

evaluate Field()

```
get maps
iterate over cells
for each cell create a map
get gbasis functions
compute Jacobian inverse
translate position
```

```
check cell
use maps
get basis Jacobian
compute Jacobian inverse
get data coordinates
compute weights
get derivative basis
function values
evaluate basis functions
```

H(pos)

?

# The long term plan

- We want to expand the number of Diderot users by enhancing our understand of fields
- Build collaborations with experts in this domain
- Enable new informed visualization
- Domain-specific optimizations to improve run time

# Conclusion

My work has been around enabling the expressivity in the Diderot language so that the user

- can write intuitive code
- compile programs with complicated tensor math
- believe in the correctness in the compiler
- and visualize different types of data

Thank you

Github Page

<https://github.com/Diderot-Language/examples>

Diderot

<http://diderot-language.cs.uchicago.edu/>

Acknowledgement: This material is based upon work supported by the NSF under Grants CCF-1446412 and CCF-1564298

# Questions?