

Diderot: A DSL for image analysis

Diderot is a domain specific language for image analysis and scientific visualization

[CHIW PLDI]

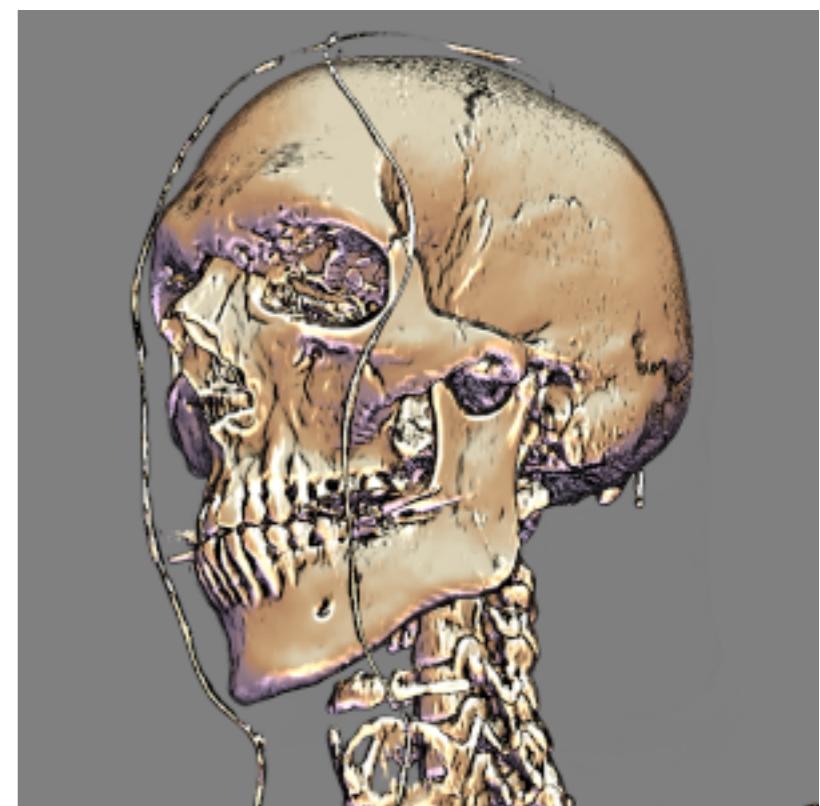
Diderot: A Parallel DSL for Image Analysis and Visualization

Charisee Chiw Gordon Kindlmann John Reppy Lamont Samuels Nick Seltzer **
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Abstract

Research scientists and medical professionals use imaging technology, such as computed tomography (CT) and magnetic resonance imaging (MRI) to measure a wide variety of biological and physical objects. The increasing sophistication of imaging technology creates demand for equally sophisticated computational techniques to analyze and visualize the image data. Analysis and visualization codes are often crafted for a specific experiment or set of images.

In this paper, we present a domain-specific language (DSL), called *Diderot*, that allows for the parallel implementation of anal-



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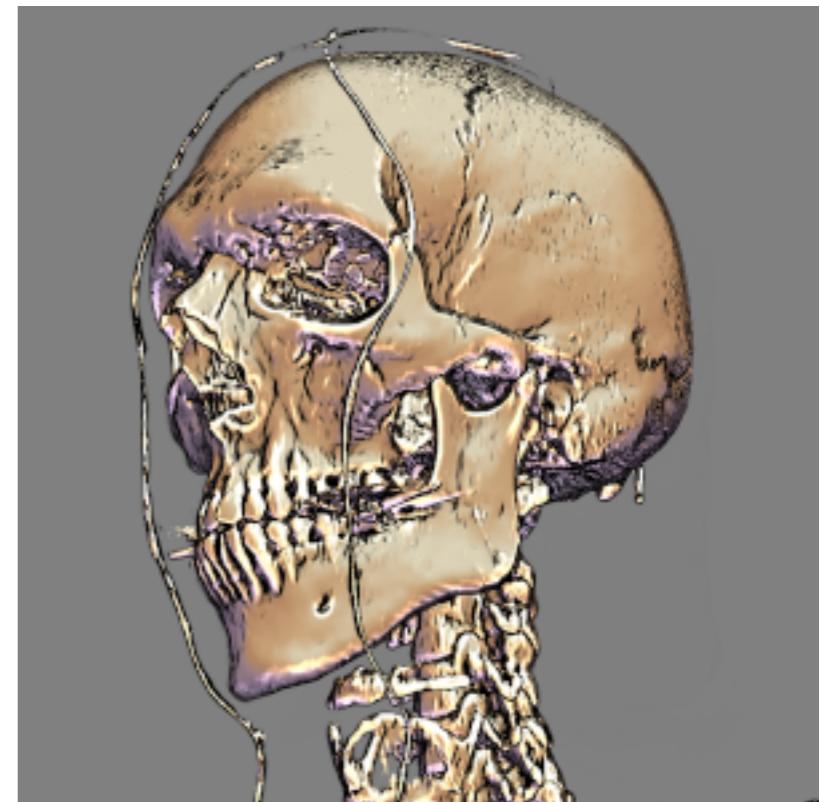
- Provide a notation can express statements in the domain space

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Diderot: A Parallel DSL for Image Analysis and Visualization

Charissee Chiw Gordon Kindlmann John Reppy Lamont Samuels Nick Seltzer ^{**}

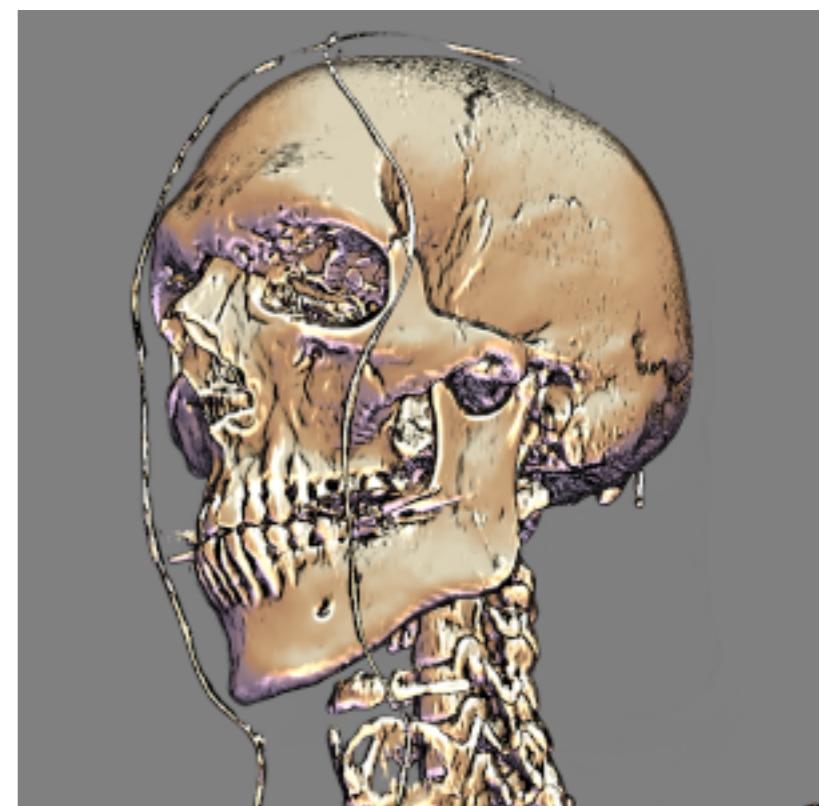
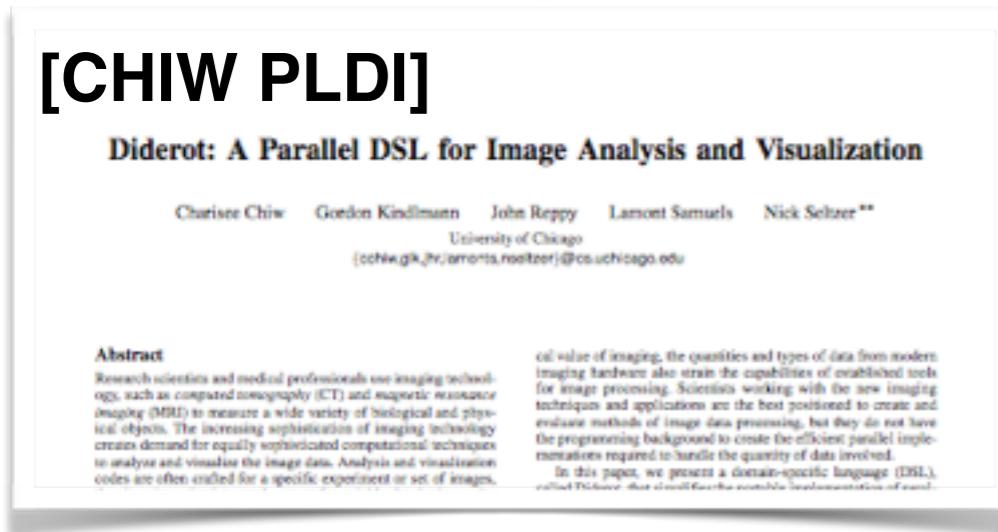
University of Chicago
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Diderot: A DSL for image analysis

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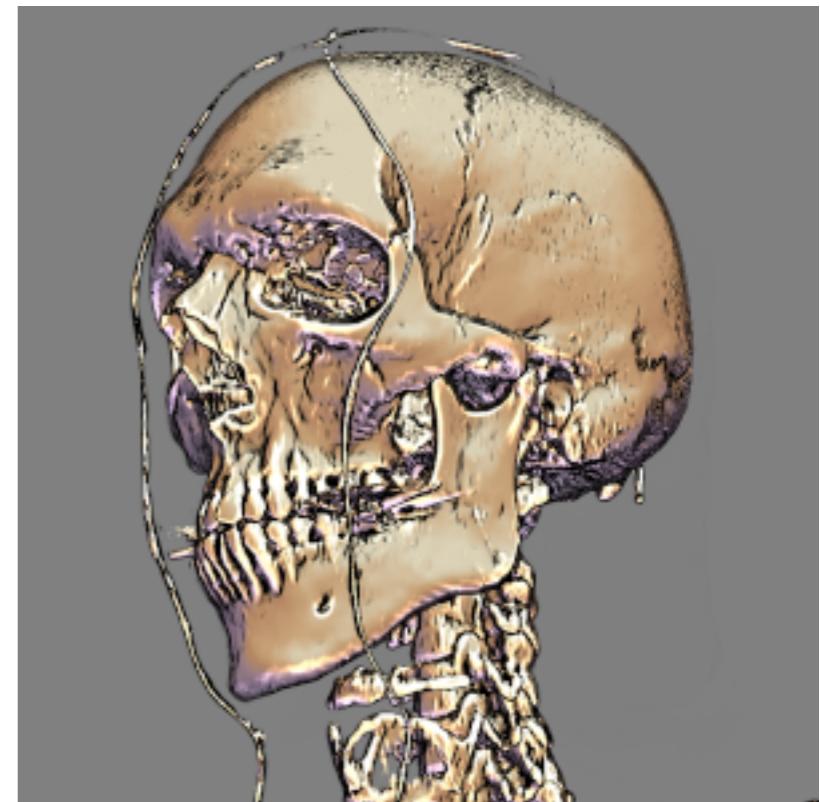
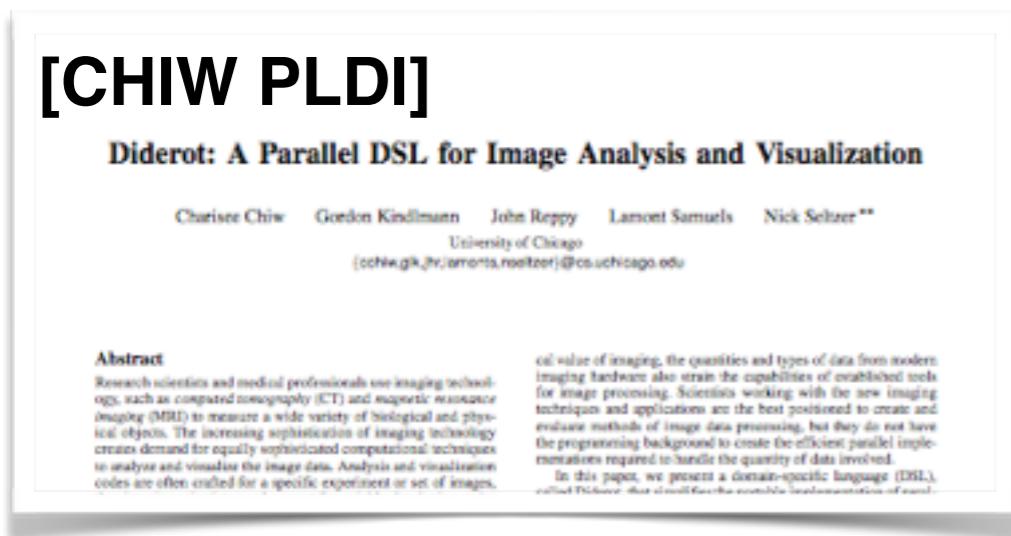
- Provide a **notation** can **express** statements in the domain space
- **Efficient** execution of expensive computations



Diderot: A DSL for image analysis

Diderot is a domain specific language for image analysis and scientific visualization

- Provide a **notation** can **express** statements in the domain space
- **Efficient** execution of expensive computations
- Enable cutting-edge research



Diderot: A DSL for image analysis

Diderot: A DSL for image analysis

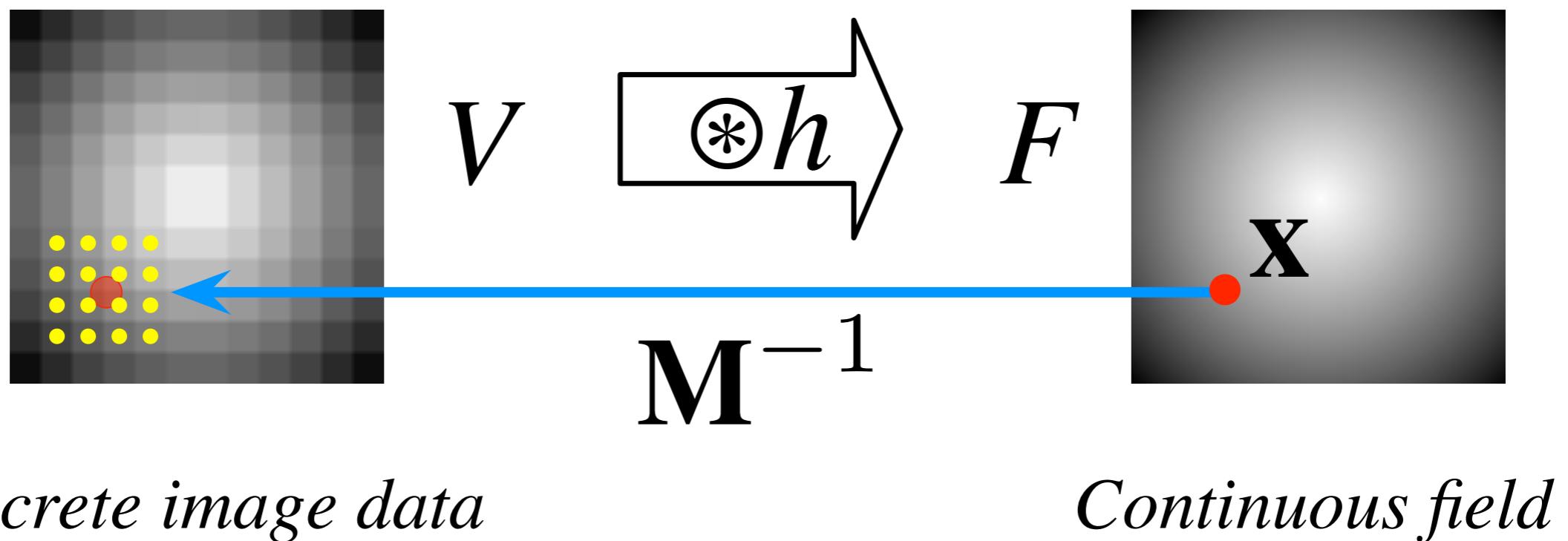
- Ease transition of ideas from whiteboard to code

Diderot: A DSL for image analysis

- Ease transition of ideas from whiteboard to code
- Tensor based IR (EIN) enables vis algorithms

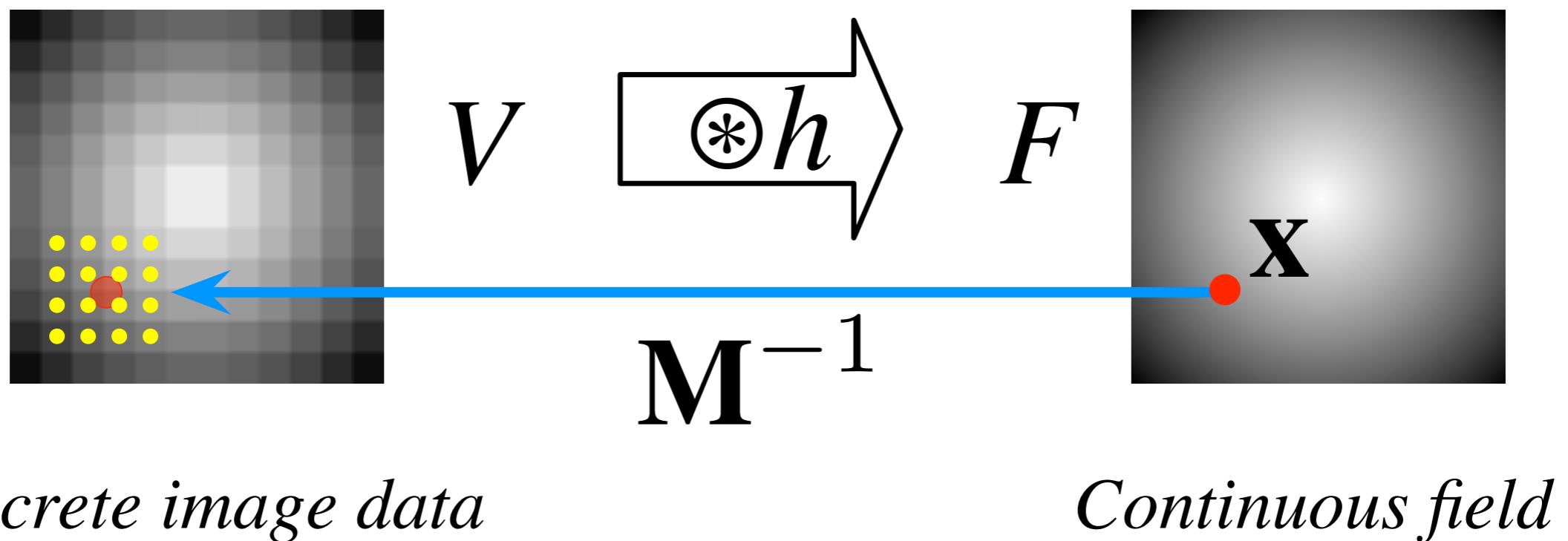
Diderot: A DSL for image analysis

- Ease transition of ideas from whiteboard to code
- Tensor based IR (EIN) enables vis algorithms
- Turn discrete data into a continuous field



Diderot: A DSL for image analysis

- Ease transition of ideas from whiteboard to code
- Tensor based IR (EIN) enables vis algorithms
- Turn discrete data into a continuous field



$$F(x) = \sum_{i=1-s}^s \sum_{j=1-s}^s V[n+ < i, j >] h(f_x - i) h(f_y - j)$$

Roadmap

Roadmap

1. Developing of the Diderot Language

Roadmap

1. Developing of the Diderot Language
2. Application of the language to a new domain: finite element data

Roadmap

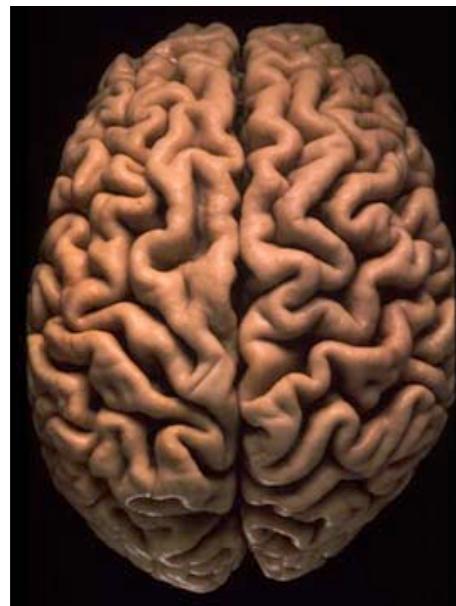
0. Image analysis and scientific visualization
1. Developing of the Diderot Language
2. Application of the language to a new domain: finite element data

Why image analysis is important

- Scientists need software tools to extract structure from many kinds of image data.
- Creating new visualization programs is part of the experimental process.
- Visualization helps explore data.

Why image analysis is important

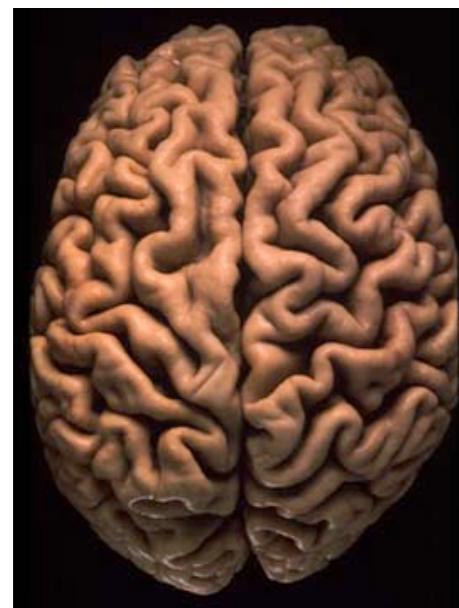
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Physical object

Why image analysis is important

- Scientists need software tools to extract structure from many kinds of image data.
- Creating new visualization programs is part of the experimental process.
- Visualization helps explore data.



Physical object

Imaging

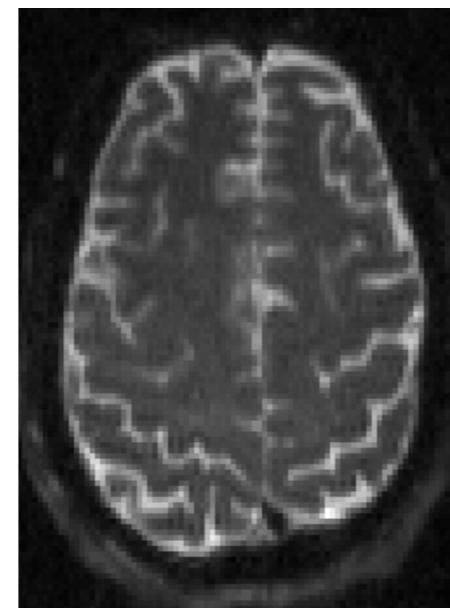
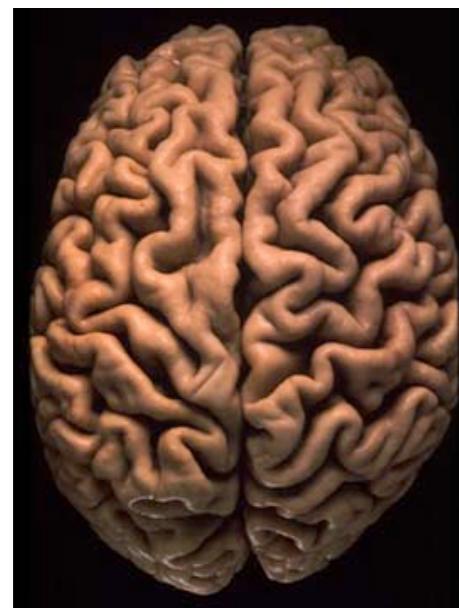


Image data

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Physical object

Imaging

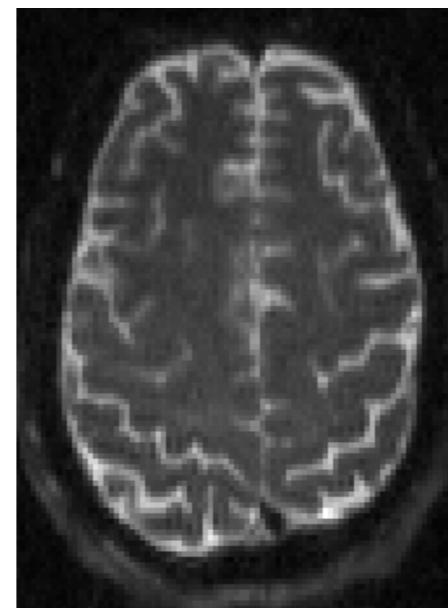
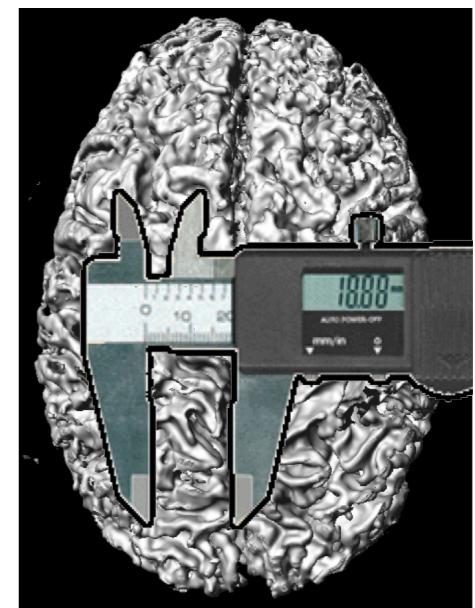


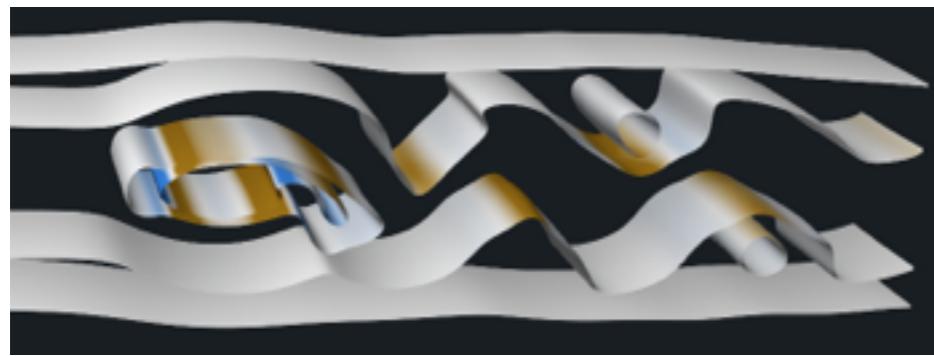
Image data

Visualization
Analysis

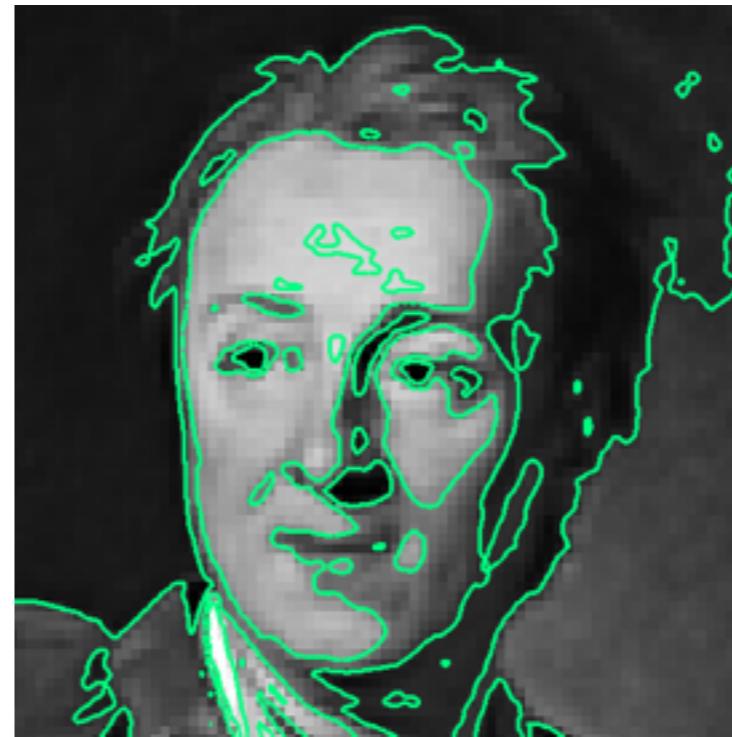


Computational
representation

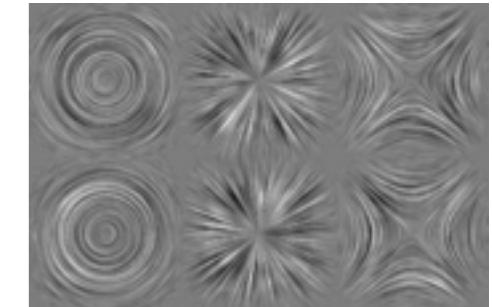
Visualization Applications



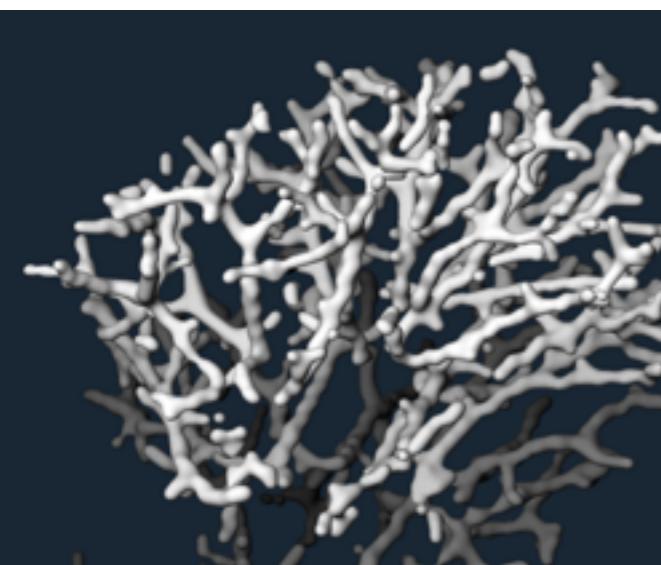
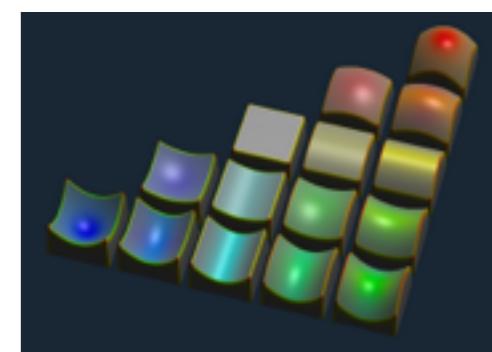
Fluid flow



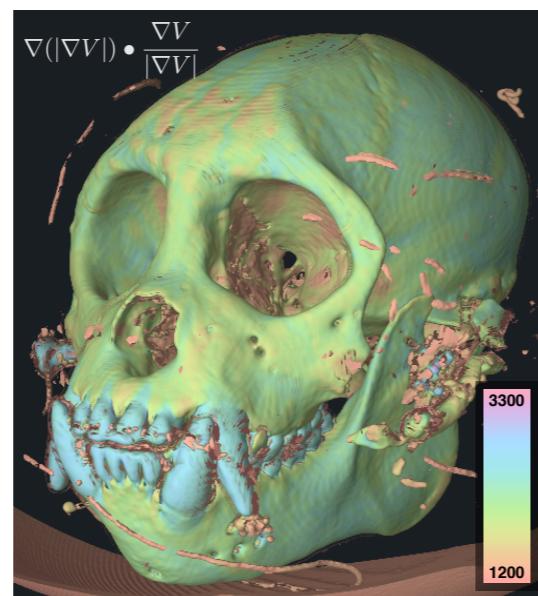
Diderot ^ The guy



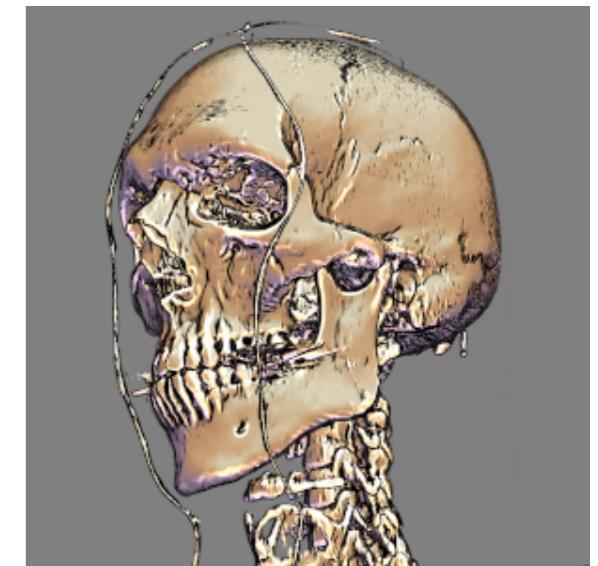
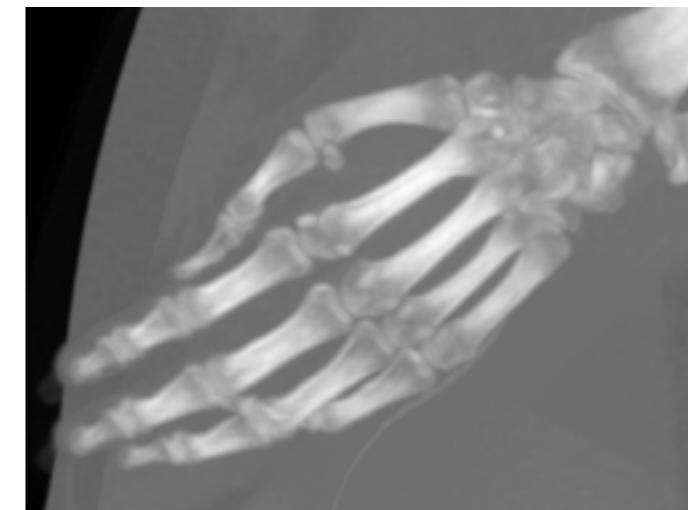
Synthetic data



Lung data



X-Ray data and CT Scans
6



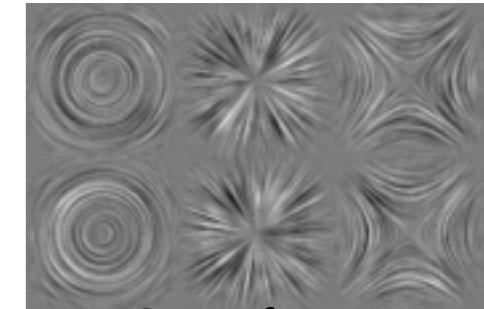
Visualization Applications

Visually explore data and compute features

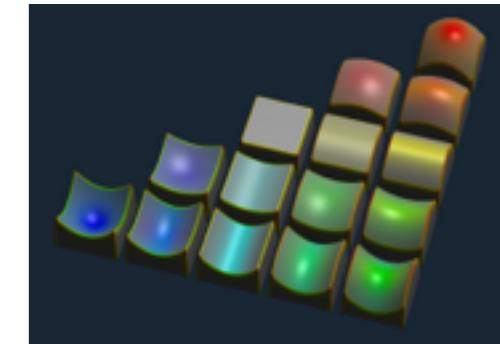
Flow Magnitude



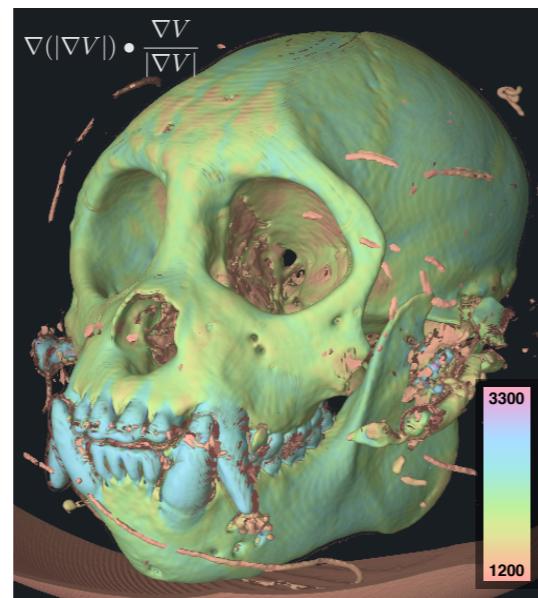
LIC



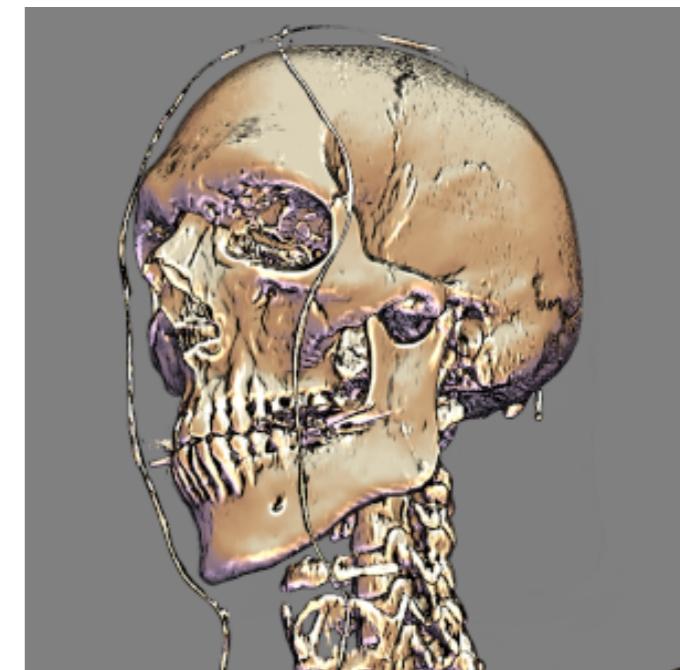
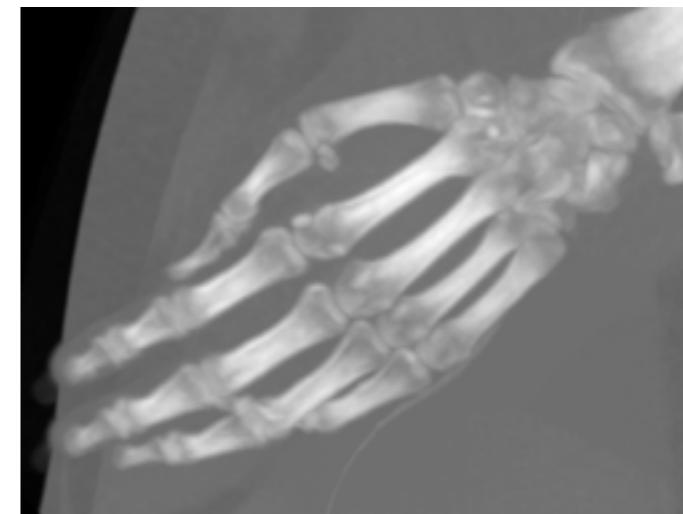
Surface
Curvature



Canny Edges



Isocontour Surface
MIP

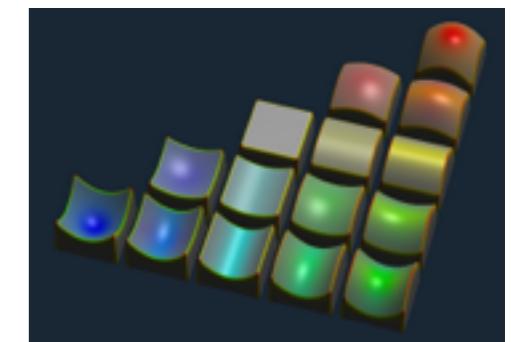
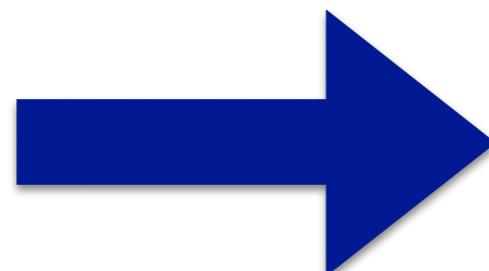
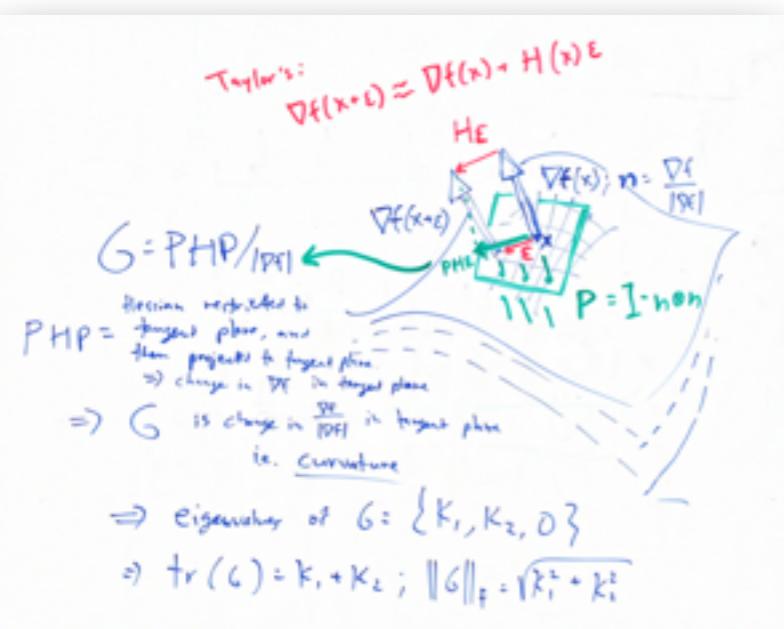


Ridge Detection

Volume rendering

Challenges

- What do I want to implement?



Visualization concept

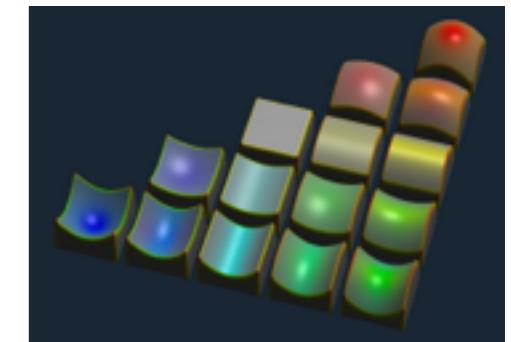
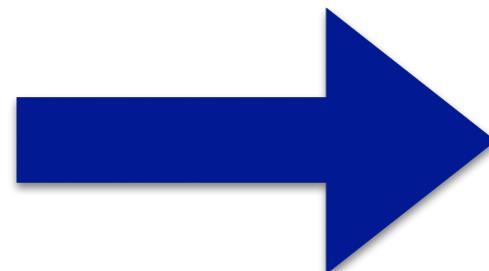
Implementation

Image

Challenges

- What do I want to implement?

```
grad = - $\nabla F$ ;  
norm = normalize(grad);  
H =  $\nabla \otimes \nabla F$ ;  
P = identity[3] - norm $\otimes$ norm;  
G = -(P $\bullet$ H $\bullet$ P)/|grad|;  
disc =  $\sqrt{2.0 * |G|^2 - \text{trace}(G)^2}$ ;  
k1 =  $\frac{\text{trace}(G) + disc}{2}$ ;  
k2 =  $\frac{\text{trace}(G) - disc}{2}$ ;
```



Visualization concept

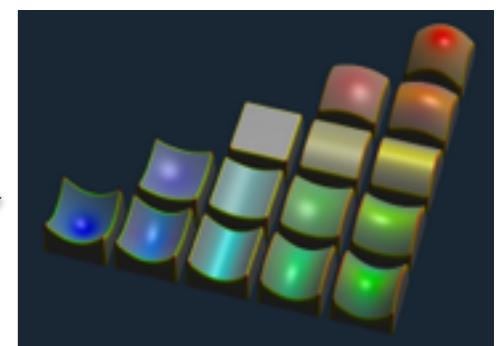
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Challenges

- What do I want to implement?
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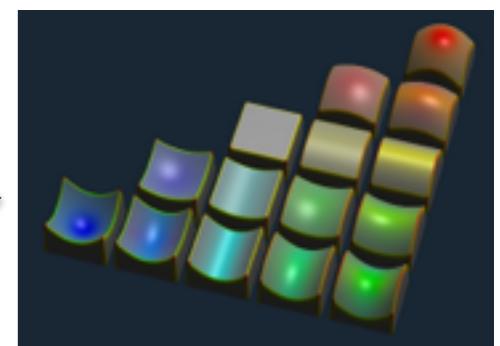
Implementation

Image

Challenges

- What do I want to implement?
- How do I implement this?
- How do I implement it efficiently?

```
grad = - $\nabla F$ ;  
norm = normalize(grad);  
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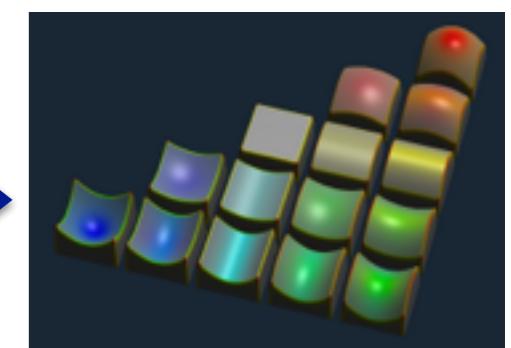
Visualization concept

Implementation

Image

Programmability: From whiteboard to code

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Visualization concept

Diderot code

Image

Programmability: From whiteboard to code

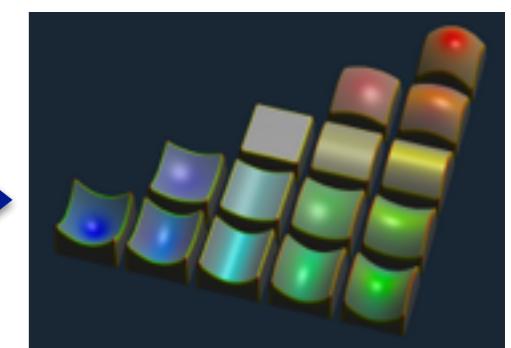
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```
vec3 grad = - $\nabla F$  (pos);
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tensor[3,3] H =  $\nabla \otimes \nabla F$  (pos);
tensor[3,3] P = identity[3] - norm  $\otimes$  norm;
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```



Surface
Curvature



Visualization concept

Diderot code

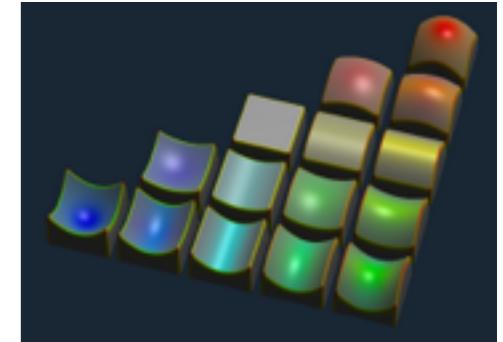
Image

Diderot program

```
field#4(3) [] F = bspln5 ⋅ image("quad-patches.nrrd");
field#0(2)[3] RGB = tent ⋅ image("2d-bow.nrrd");
...
strand RayCast (int ui, int vi) {
    ...
    update
    ...
    vec3 grad = - $\nabla$ F (pos);
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    tensor[3,3] H =  $\nabla \otimes \nabla$  F (pos);
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Surface

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Diderot program

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    real out = ...
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```

Tensor: reals, vectors, matrices,...

Tensor operators: additional, subtraction, multiplication

Diderot program

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```

Field: function from d-dimensional space to tensors

Probe: fields are probed at a position to produce a tensor

Derivative: rate of change

Diderot program

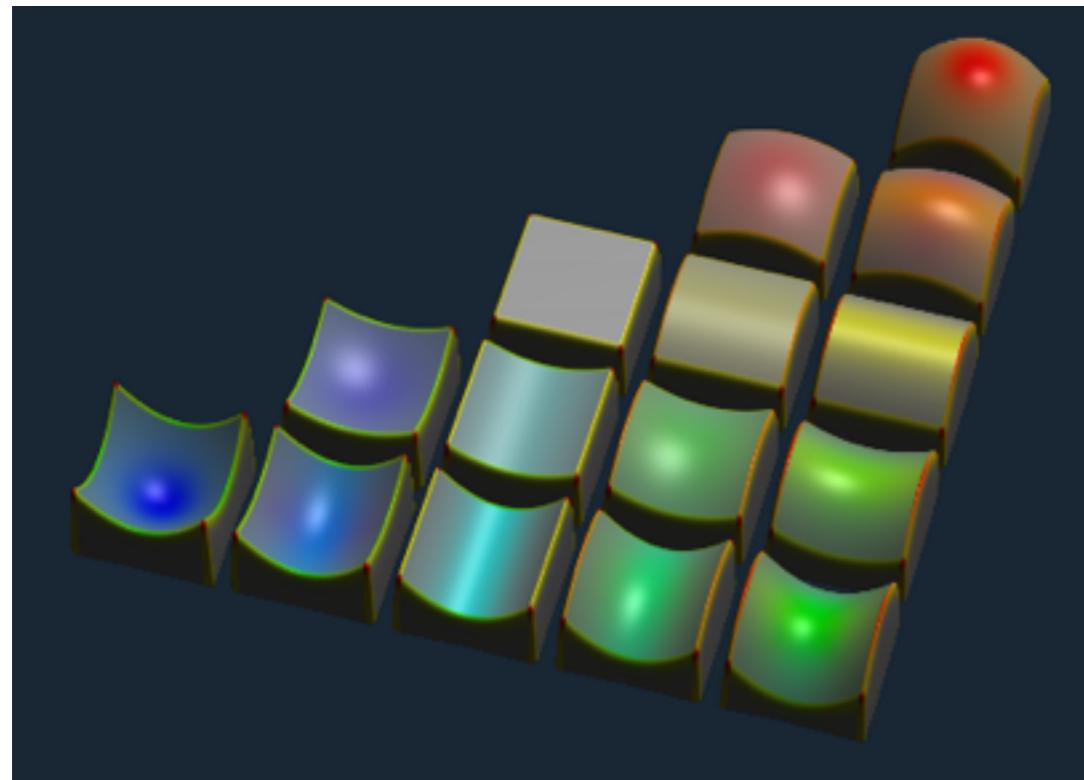
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```

Field: function from d-dimensional space to tensors

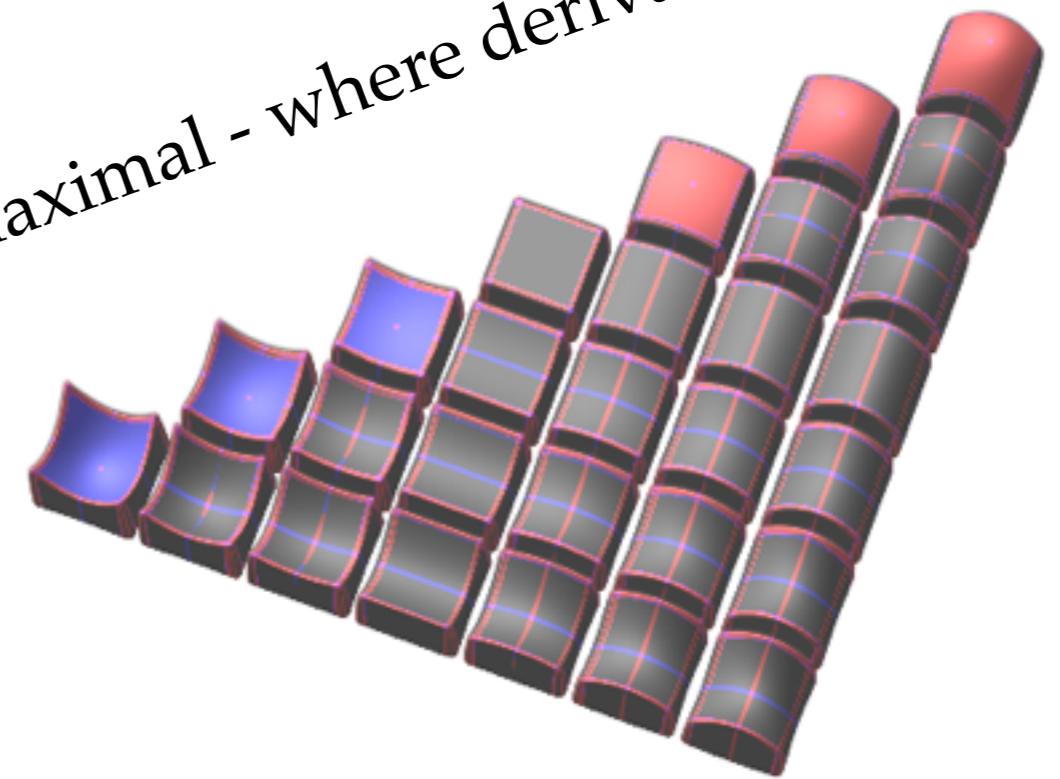
Probe: fields are probed at a position to produce a tensor

Derivative: rate of change

Motivation



maximal - where derivative is 0



Curvature

Crest Lines are places where the surface curvature is maximal along the **curvature** direction.

Motivation

```
field#4(3) [] F = bspln5 ⋘ image("quad-patches.nrrd");
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    vec3 matRGB = ...RGB(...k1, ...k2));
    real out = ... $\nabla \otimes \nabla$ k1...
}
...
}
```

Directly add line
14

Motivation

```
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    vec3 matRGB = ...RGB(...k1, ...k2]);
    real out = ... $\nabla \otimes \nabla$ k1...
}
...
}
```

Not permitted
14

Motivation

first-order	higher-order
$q = \det V(x)$	
$\det_{\text{tr}} = q[0,0,:] + q[1,1,:] + q[2,2,:]$	
$E = V(x) - \text{trace}^* \text{identity}[3]/3$	
$\text{val} = 3 \cdot \sqrt{6} \cdot \det(E / \sqrt{E:E})$	
$K = (\text{identity}[3] \otimes \det_{\text{tr}})/3$	
$\det E = \det V(x) - K$	$E = V - \text{trace}(V) \cdot \text{identity}[3]$
$N1 = \det E \cdot \sqrt{E:E}$	$F = 3 \cdot \sqrt{6} \cdot \det(E / E)$
$\text{inner} = 2 \cdot E : \det E$	$\alpha = \text{alpha}(F(x), \nabla F(x))$
$B = (\text{inner} / E:E)/2$	
$N2 = E \bullet B$	
$dQ = (N1 - N2) / E:E$	
$Q = E / \sqrt{E:E}$	
$\text{part1} = dQ[0,0,:] \cdot (Q[1,0] \cdot Q[2,2] - Q[1,2] \cdot Q[2,0])$	
$+ Q[0,0] \cdot (Q[2,2] \cdot dQ[1,1,:] + Q[1,0] \cdot dQ[2,2])$	
$- dQ[1,2] \cdot Q[2,0] + Q[1,2] \cdot dQ[2,0])$	
$\text{part2} = dQ[0,1,:] \cdot (Q[1,0] \cdot Q[2,1] - Q[1,2] \cdot Q[2,0])$	
$+ Q[0,1] \cdot (Q[2,1] \cdot dQ[1,0] + Q[1,0] \cdot dQ[2,1])$	
$- dQ[1,2] \cdot Q[2,0] + Q[1,2] \cdot dQ[2,0])$	
$\text{part3} = dQ[0,2,:] \cdot (Q[0,0] \cdot Q[2,1] - Q[1,1] \cdot Q[2,0])$	
$Q[0,2] \cdot (Q[2,1] \cdot Q[1,0] + Q[1,0] \cdot dQ[2,1])$	
$- dQ[1,1,:] \cdot Q[2,0] + Q[1,1] \cdot dQ[2,0])$	
$\det M = \text{part1} - \text{part2} + \text{part3}$	
$\text{grad} = 3 \cdot \sqrt{6} \cdot \det M$	
$\alpha = \text{alpha}(\text{val}, \text{lgrad})$	

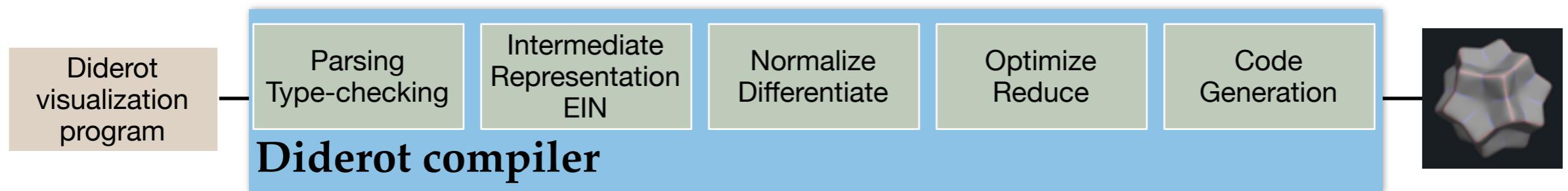
Motivation

The compiler, not the programmer, should do the work.

first-order	higher-order
$q = \det V(x)$	
$\det tr = q[0,0,:] + q[1,1,:] + q[2,2,:]$	
$E = V(x) - \text{trace}^* \text{identity}[3]/3$	
$\text{val} = 3\sqrt{6} \det(E/\sqrt{E:E})$	
$K = (\text{identity}[3] \otimes \det tr)/3$	
$\det E = \det V(x) - K$	$E = V - \text{trace}(V) \cdot \text{identity}[3]$
$N1 = \det E \cdot \sqrt{E:E}$	$F = 3\sqrt{6} \det(E/ E)$
$\text{inner} = 2 \cdot E : \det E$	$\alpha = \text{alpha}(F(x), \nabla F(x))$
$B = (\text{inner}/ E:E)/2$	
$N2 = E \bullet B$	
$dQ = (N1 - N2) / E : E$	
$Q = E/\sqrt{E:E}$	
$\text{part1} = dQ[0,0,:] \cdot (Q[1,0] \cdot Q[2,2] - Q[1,2] \cdot Q[2,0])$	
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Implementing Expressivity

- Motivated our need for higher-level notation
- Compilation of tensor calculus to C
- Our intermediate representation, EIN



EIN Intermediate Representation

- Our intermediate representation, EIN based on Einstein Index Notation
- Concise way to represent operations on and between tensors and tensor fields

EIN Intermediate Representation

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- Concise way to represent operations on and between tensors and tensor fields

Our design and implementation



[CHIW CPC]

**EIN: An Intermediate Representation
for
Compiling Tensor Calculus**

Charisee Chiw, Gordon L. Kindlman, and John Reppy

University of Chicago

Abstract. Diderot is a parallel domain-specific language for analysis and visu-

EIN Intermediate Representation

The matrix M is represented in EIN with two variable indices

$$\text{Matrix } M \quad \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad \boxed{\quad}$$

EIN Intermediate Representation

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$$\text{Matrix } M \quad \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = M_{ij} \quad \text{EIN Term}$$

EIN Intermediate Representation

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$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = M_{ij}$$
 EIN Term

Switch row and column
$$\begin{bmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{bmatrix} = M_{ji}$$
 Switch the order

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 Switch the order

Add diagonal components
$$trace(M) = \sum_i M_{ii}$$
 Summation

Single entry
$$M_{12} = M_{12}$$
 Set indices to constants

Select a column
$$\begin{bmatrix} M_{11} \\ M_{21} \end{bmatrix} = M_{i1}$$
 Mix of different indices

EIN Intermediate Representation

- Multiply two vectors together

$$\sum_i A_i B_i \quad A_i B_i \quad A_i B_j$$

EIN Intermediate Representation

- Multiply two vectors together

real	vector	matrix
$\sum_i A_i B_i$	$A_i B_i$	$A_i B_j$
dot product	component-wise multiplication	outer product

EIN Intermediate Representation

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- Express more complicated computations

$$\lambda F \left\langle \sum_{ijk} F_{0i} F_{1j} F_{2k} \epsilon_{ijk} \right\rangle$$

determinant

$$\lambda F \left\langle \frac{\sum_k F_{kk} \delta_{ij} - F_{ij}}{F_{00} F_{11} - F_{01} F_{10}} \right\rangle_{ij}$$

inverse

EIN Intermediate Representation

- Multiply two vectors together

$$\sum_i A_i B_i$$

real

dot product

vector

$$A_i B_i$$

component-wise multiplication

matrix

$$A_i B_j$$

outer product

- Express more complicated computations

An EIN term can concisely represent tensor math

$$\lambda F \left\langle \sum_{ijk} F_{0i} F_{1j} F_{2k} \epsilon_{ijk} \right\rangle$$

determinant

$$\lambda F \left\langle \frac{\sum_k F_{kk} \delta_{ij} - F_{ij}}{F_{00} F_{11} - F_{01} F_{10}} \right\rangle_{ij}$$

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EIN Intermediate Representation

Consider the computation

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$$t = \text{trace}(a \otimes b)$$

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Diderot expression is represented in compiler as

$$M = \text{Outer}(a, b)$$

$$t = \text{Trace}(M)$$

$$M = (\lambda(A, B) \langle A_i B_j \rangle_{ij})(a, b)$$

$$t = (\lambda T \langle \sum_i T_{ii} \rangle)(M)$$

EIN Intermediate Representation

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substitution of the definition of M for X yields

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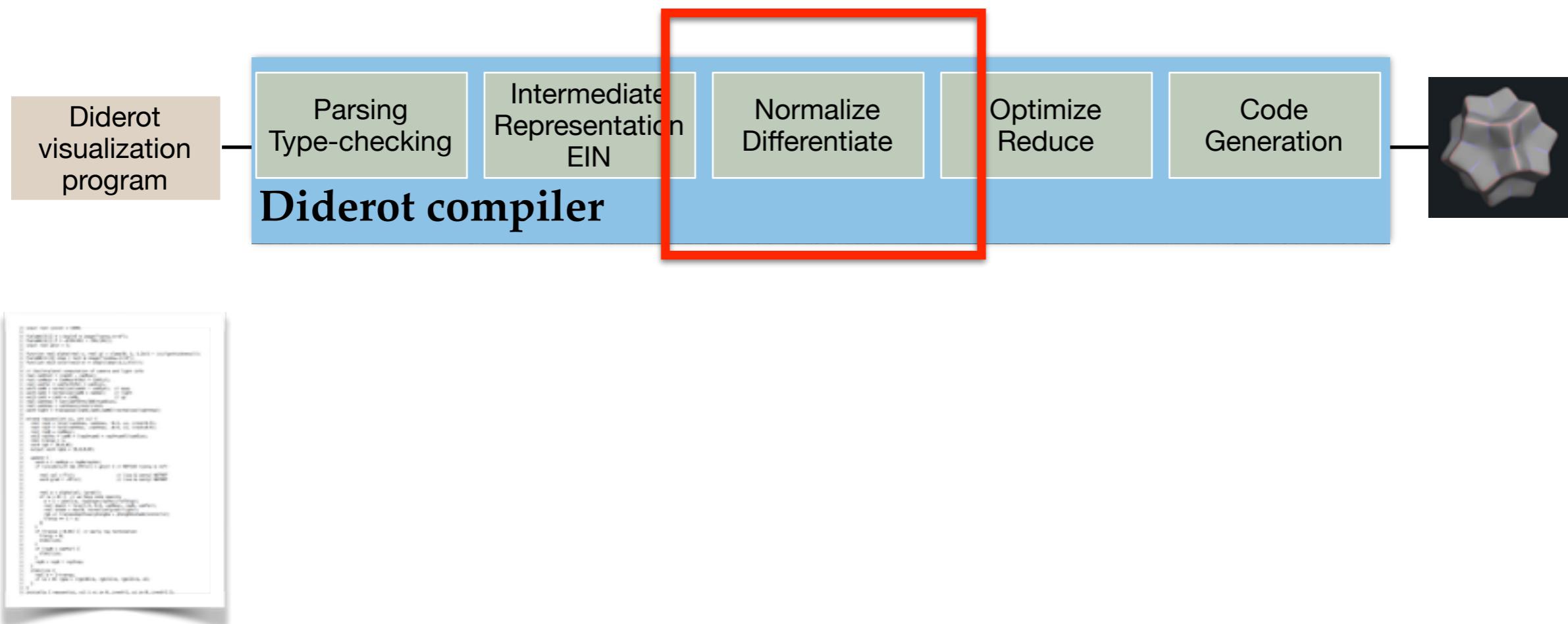
$$t = \text{Trace}(\text{Outer}(a, b)) \quad t = (\lambda(A, B) \langle \sum_i A_i B_i \rangle)(a, b)$$

Replaces the ad hoc rewrite rule:

$$\text{Trace}(\text{Outer}(a, b)) \Rightarrow \text{Dot}(a, b)$$

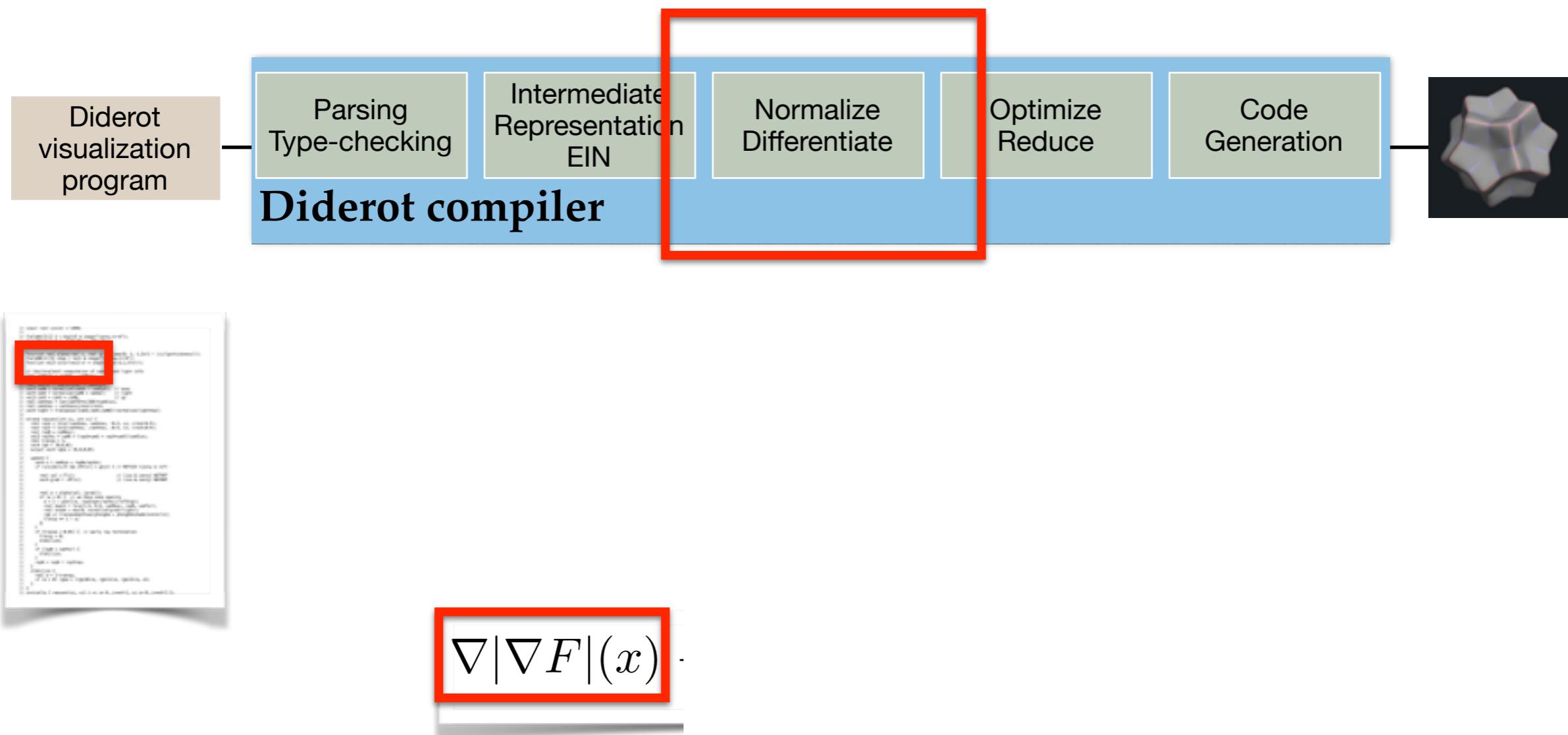
Compiling Diderot

From Tensor Calculus to C



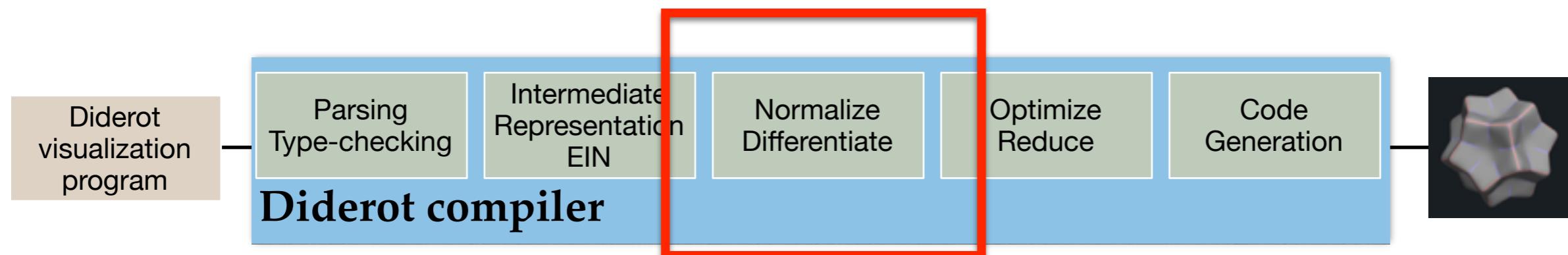
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Compiling Diderot

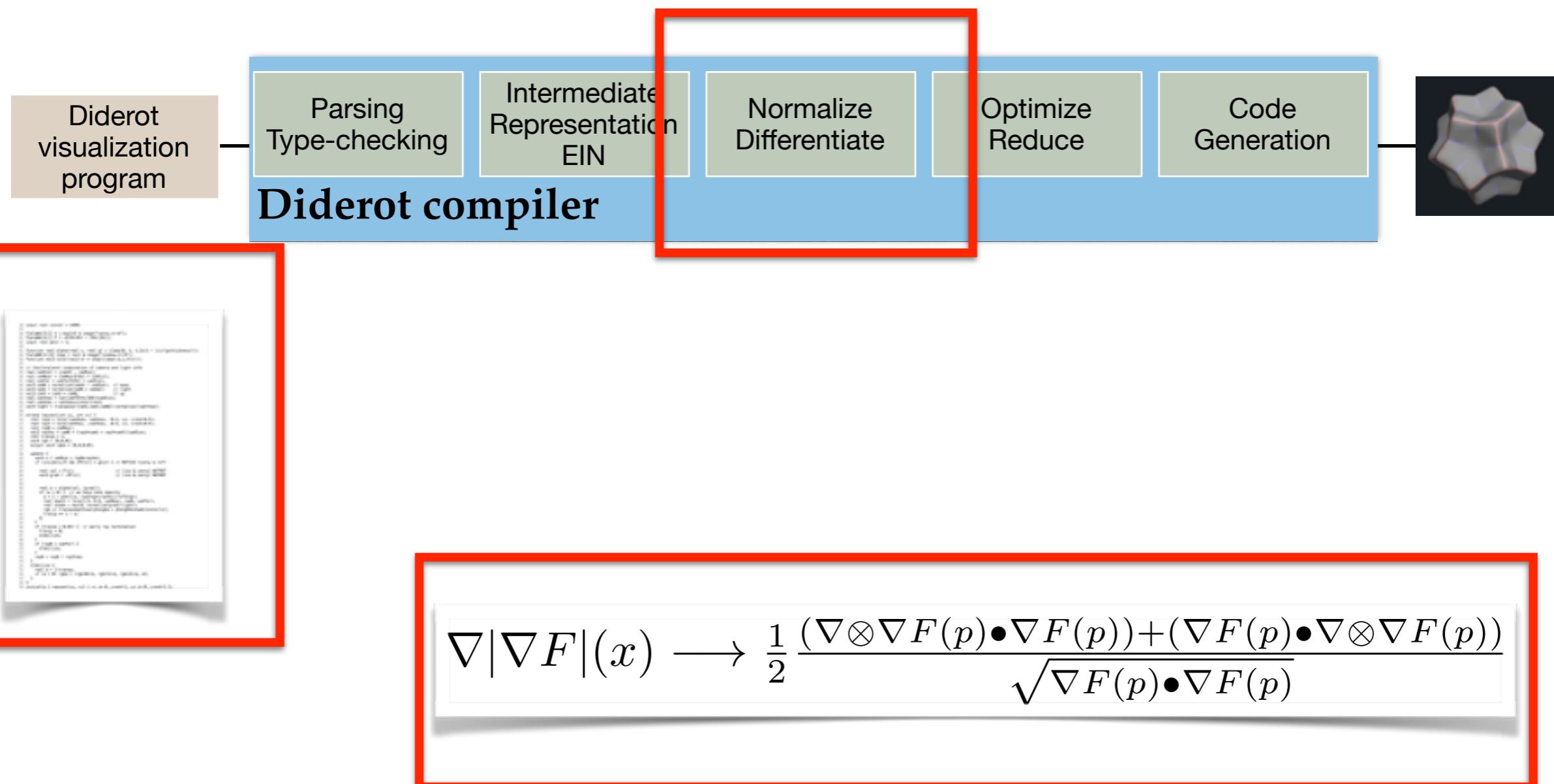
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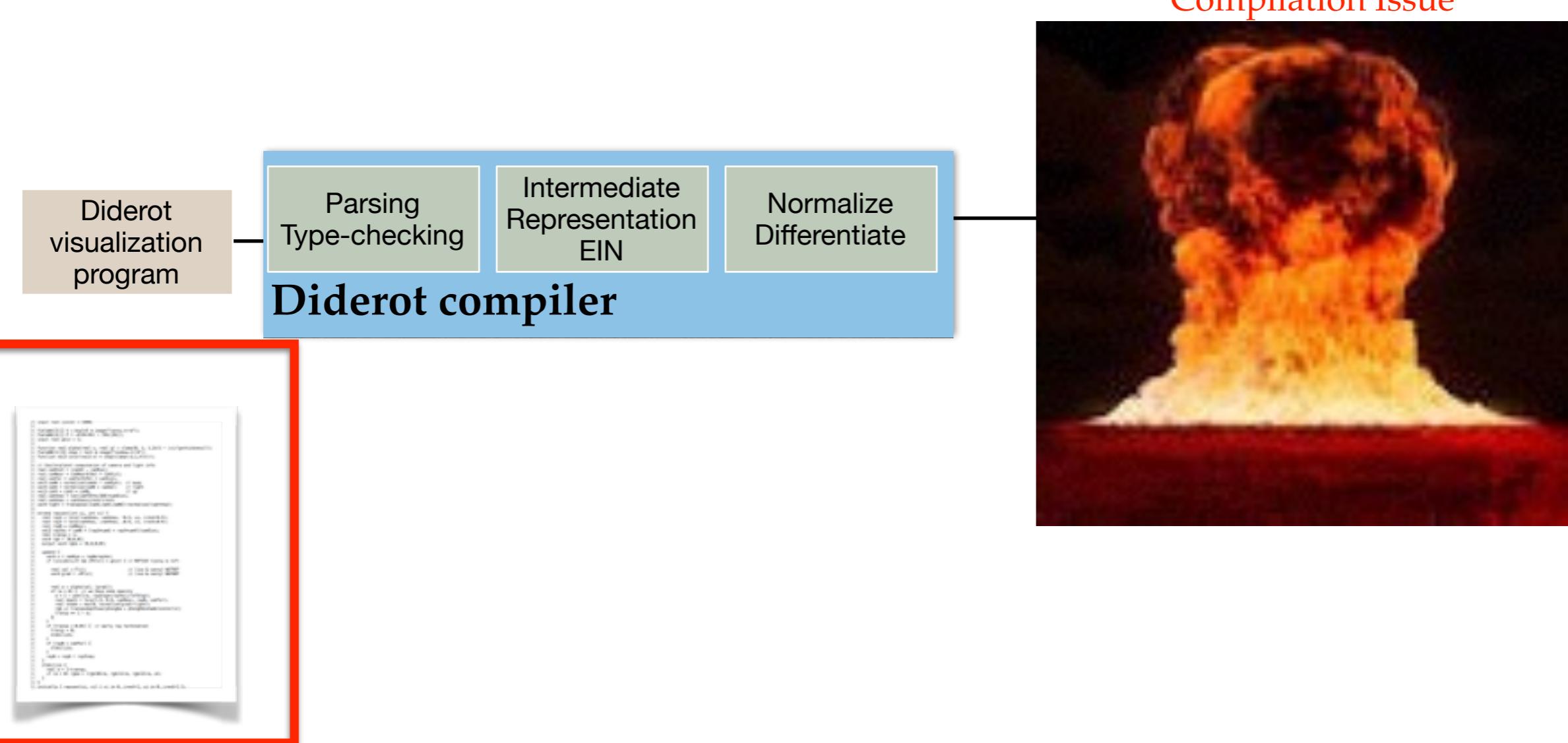
$$\nabla |\nabla F|(x) \longrightarrow \frac{1}{2} \frac{(\nabla \otimes \nabla F(p) \bullet \nabla F(p)) + (\nabla F(p) \bullet \nabla \otimes \nabla F(p))}{\sqrt{\nabla F(p) \bullet \nabla F(p)}}$$

Compiling Diderot

From Tensor Calculus to C



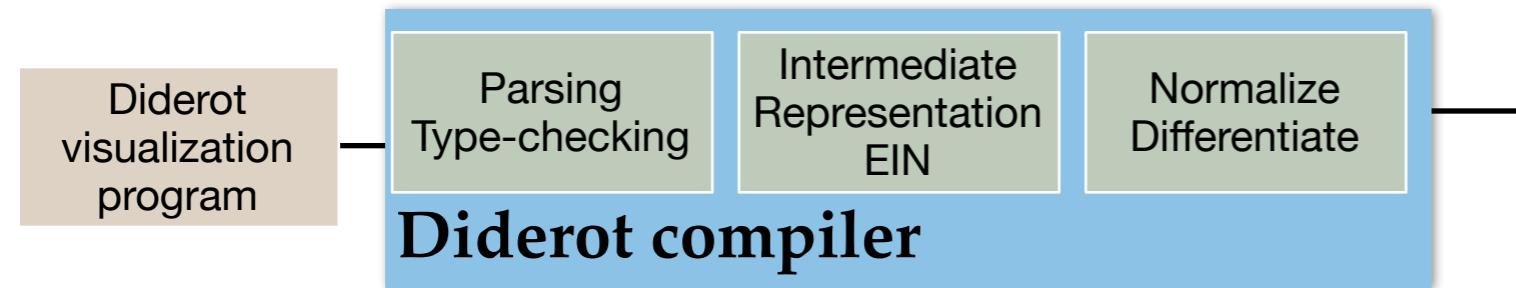
Compilation Issue



Programs could not compile or took too long to compile

Compilation Issue

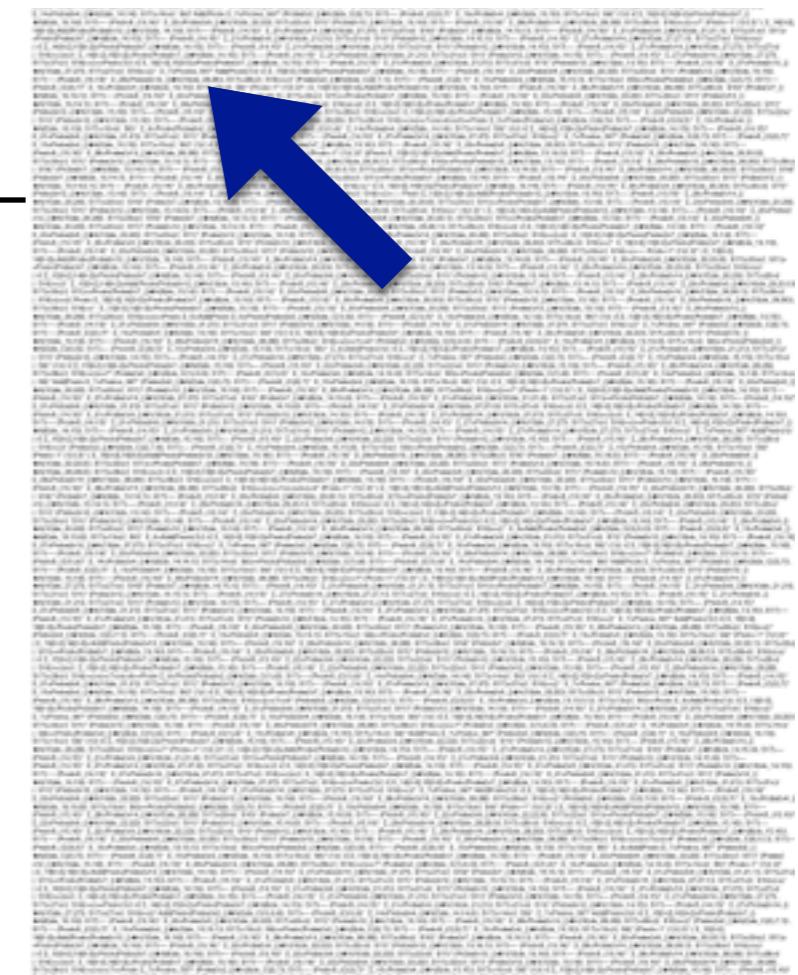
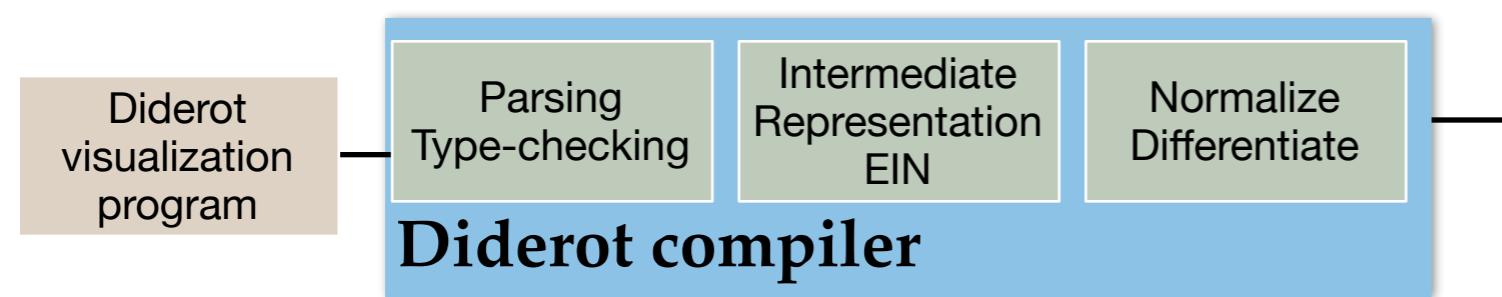
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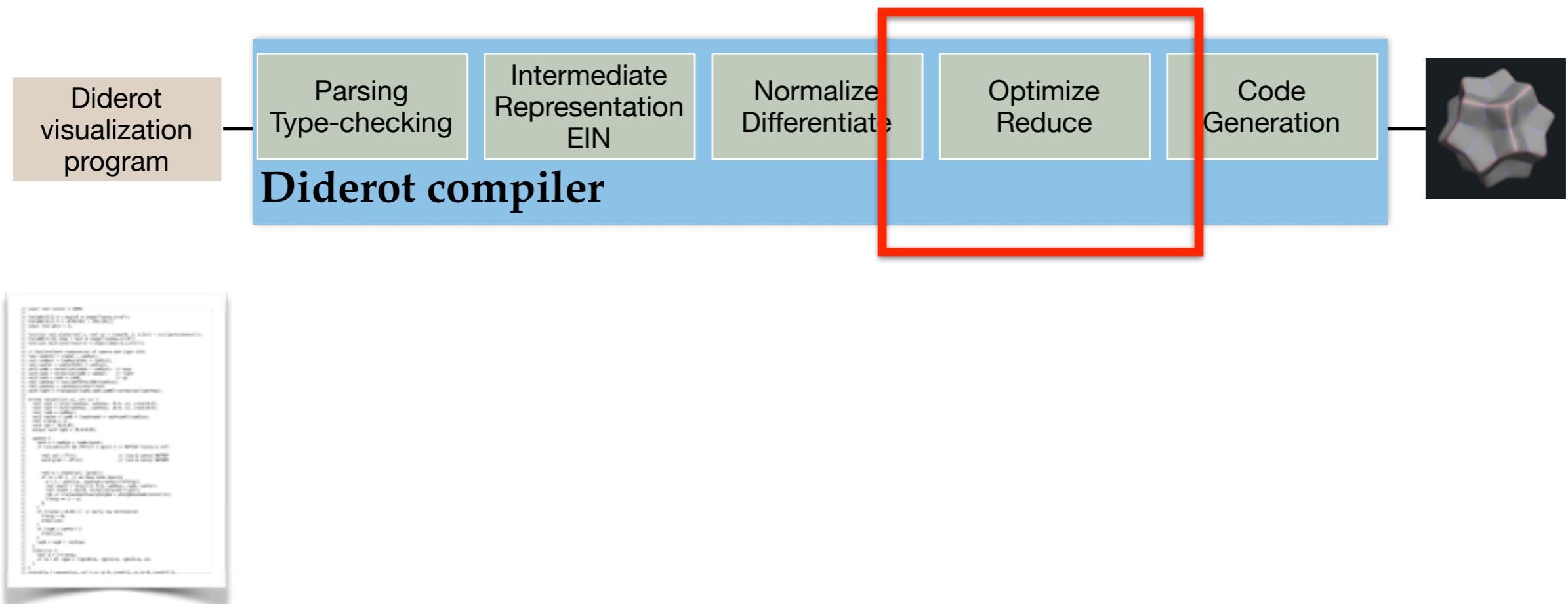
$$y = \lambda F, n \langle F_{ij}(n)F_{1k}(n) - F_{0i}(n)F_{jk}(n) + F_{ij}(n)F_{k1}(n) \rangle_{ijk} (f, n)$$



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Compilation Issue

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Phase that enables effective compilation of tensor math

Compilation Issue

Field Terms

$$y = \lambda F, n \langle F_{ij}(n)F_{1k}(n) - F_{0i}(n)F_{jk}(n) + F_{ij}(n)F_{k1}(n) \rangle_{ijk}(\text{f}, n)$$

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Split

$$v_1 = \lambda F, n \langle F_{ij}(n) \rangle_{ij} (f, n)$$

$$v_2 = \lambda F, n \langle F_{1i}(n) \rangle_i (f, n)$$

$$v_3 = \lambda F, n \langle F_{0i}(n) \rangle_i (f, n)$$

$$v_4 = \lambda F, n \langle F_{ij}(n) \rangle_{ij} (f, n)$$

$$v_5 = v_1$$

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$$y = \lambda T, B, C, D \left\langle T_{ij} B_k - C_i T_{jk} + T_{ij} D_k \right\rangle_{ijk} (v_1, v_2, v_3, v_6)$$

Write programs with higher-order operations

- Diderot is designed to represent general tensor math

Write programs with higher-order operations

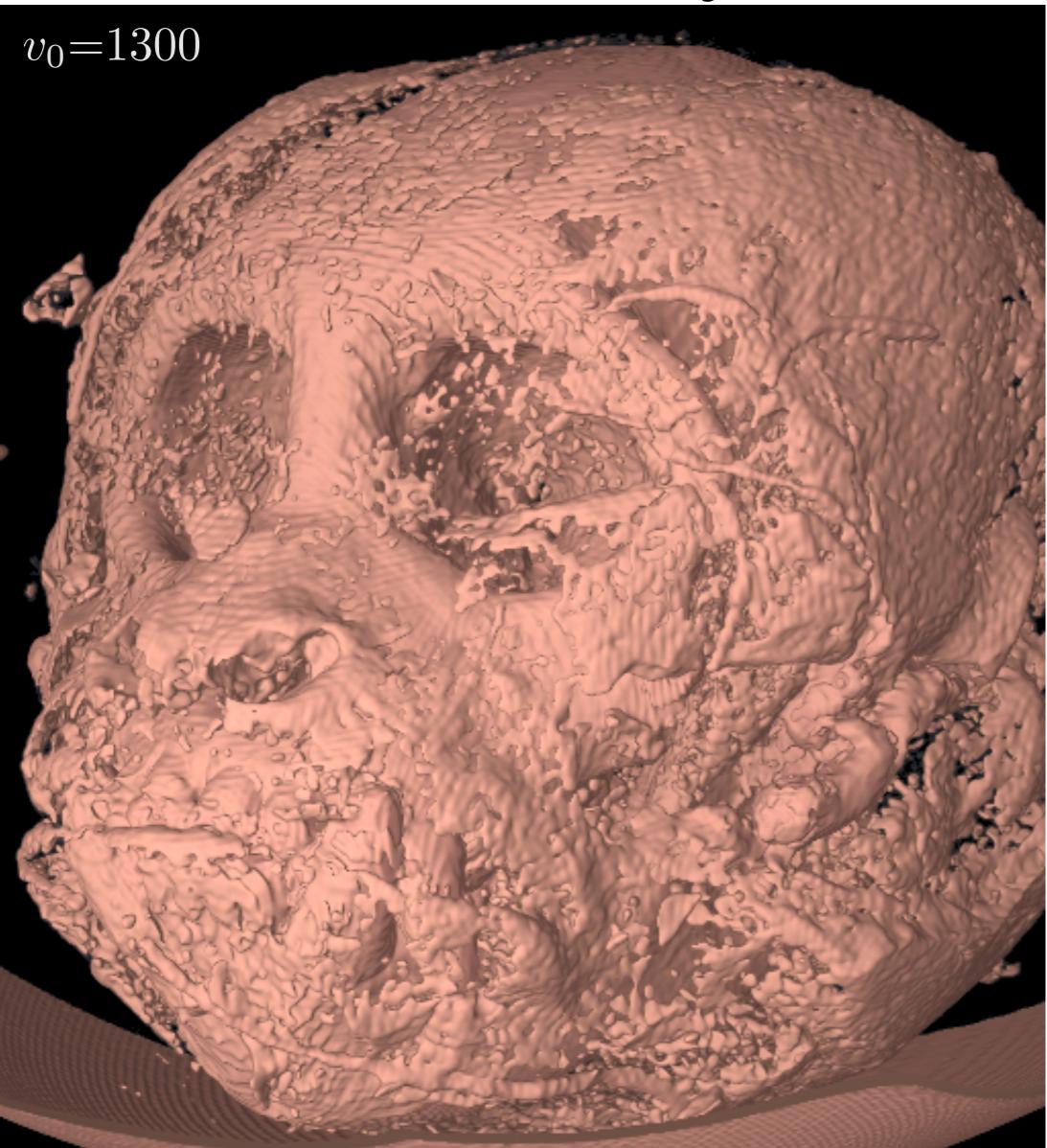
- Diderot is designed to represent general tensor math
- We rely on the **expressivity** of the language to support the implementation of visualization ideas.

Write programs with higher-order operations

- Diderot is designed to represent general tensor math
- We rely on the **expressivity** of the language to support the implementation of visualization ideas.
- Push development of compiler to enable **sophisticated visualizations**

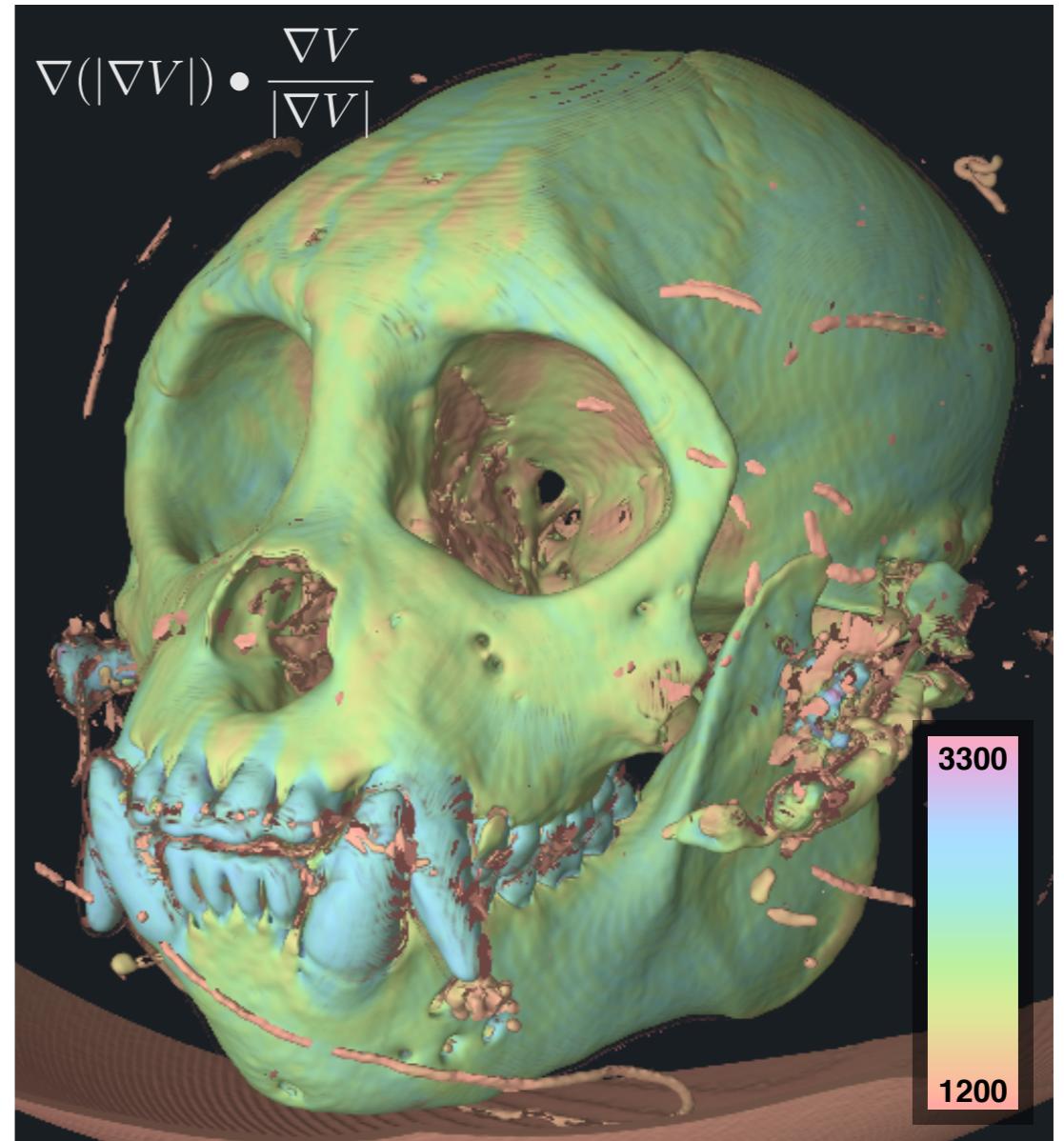
Results: Sophisticated Vis

Previously:



Iso-Surface

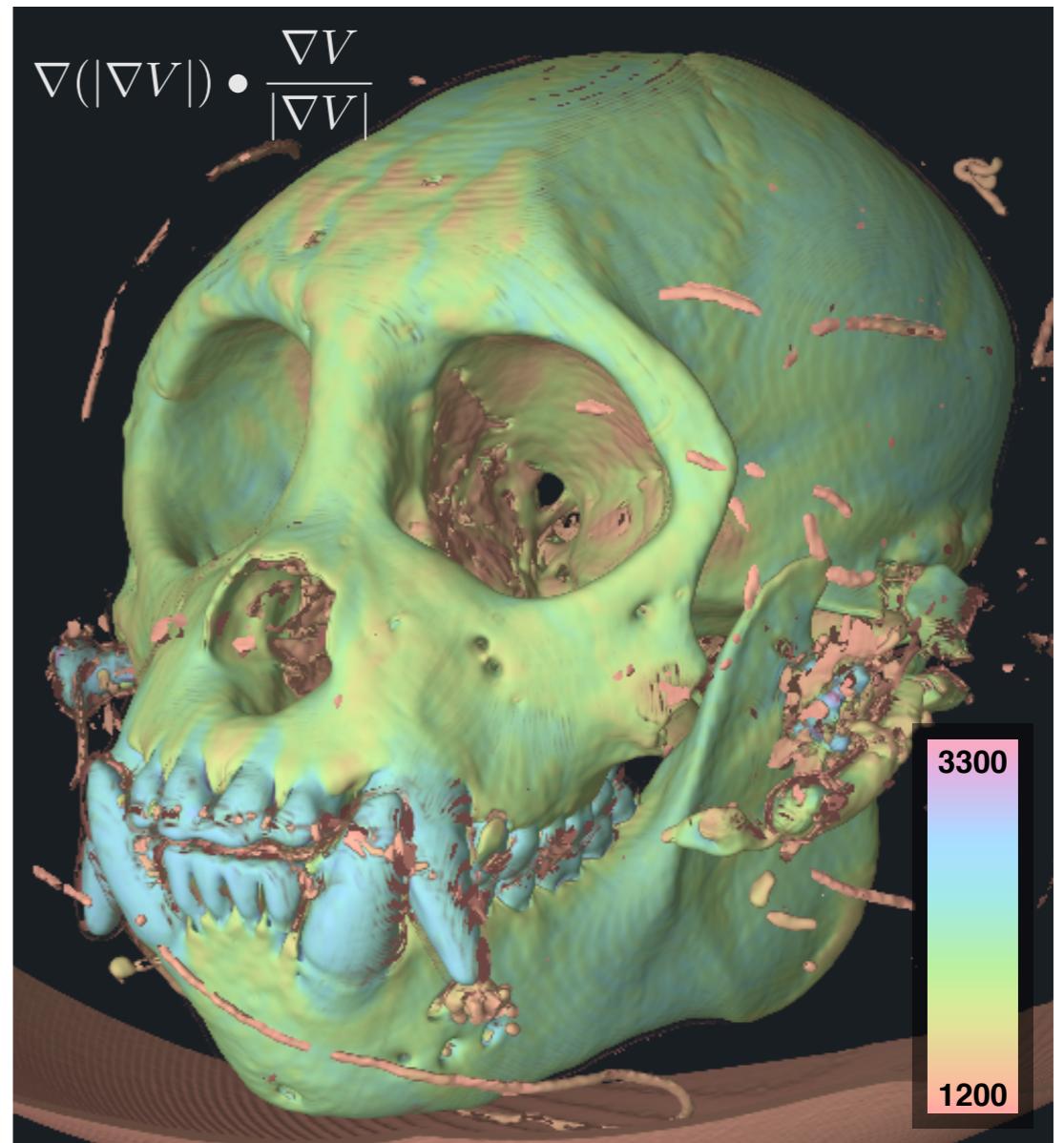
Now:



Canny Edges

Data provided by Callum Ross (U of Chicago)

Results: Sophisticated Vis

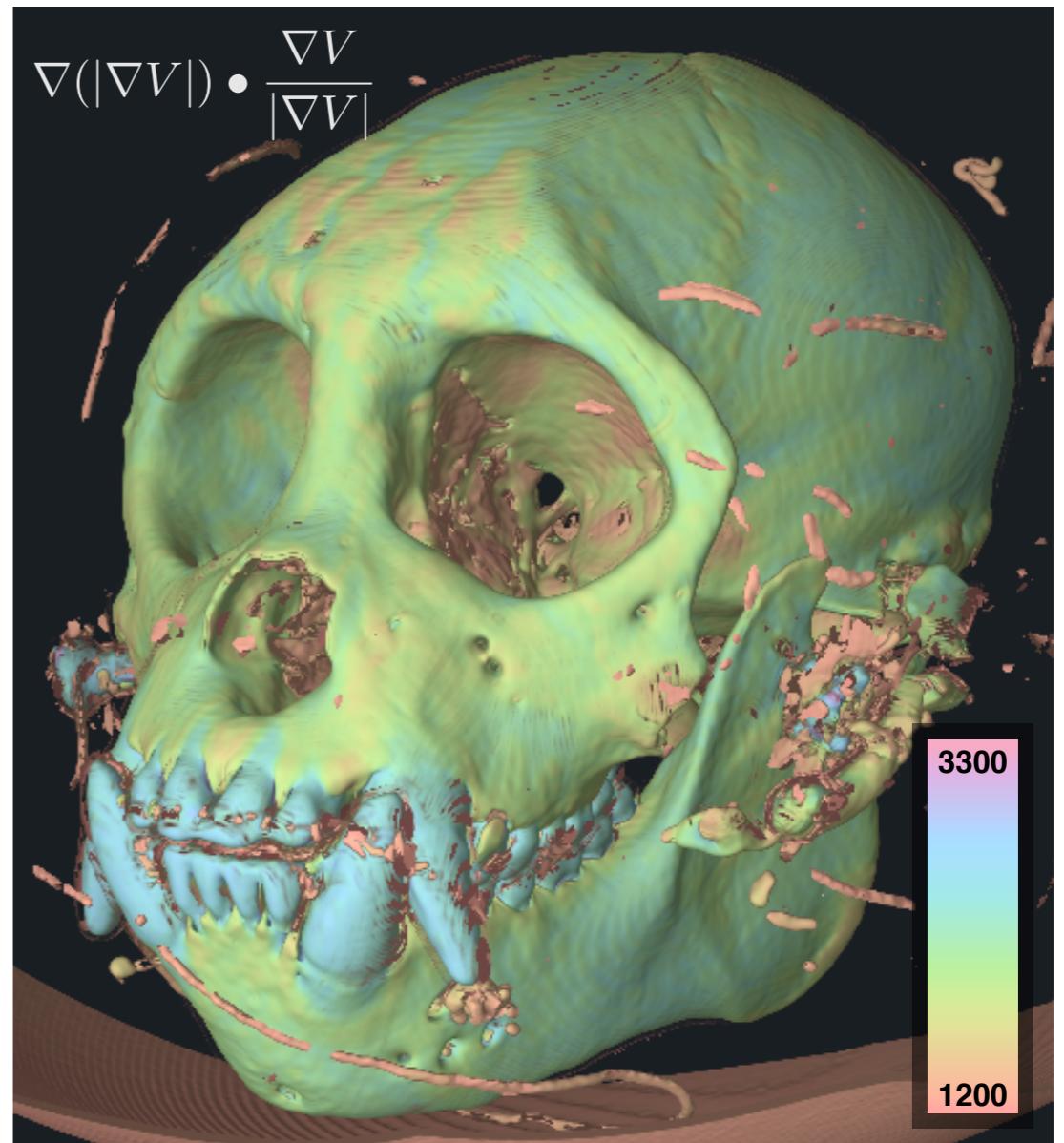


Canny Edges

Results: Sophisticated Vis

Inline high level syntax into Diderot code

```
-∇(|∇F|) • ∇F / |∇F|;
```



Canny Edges

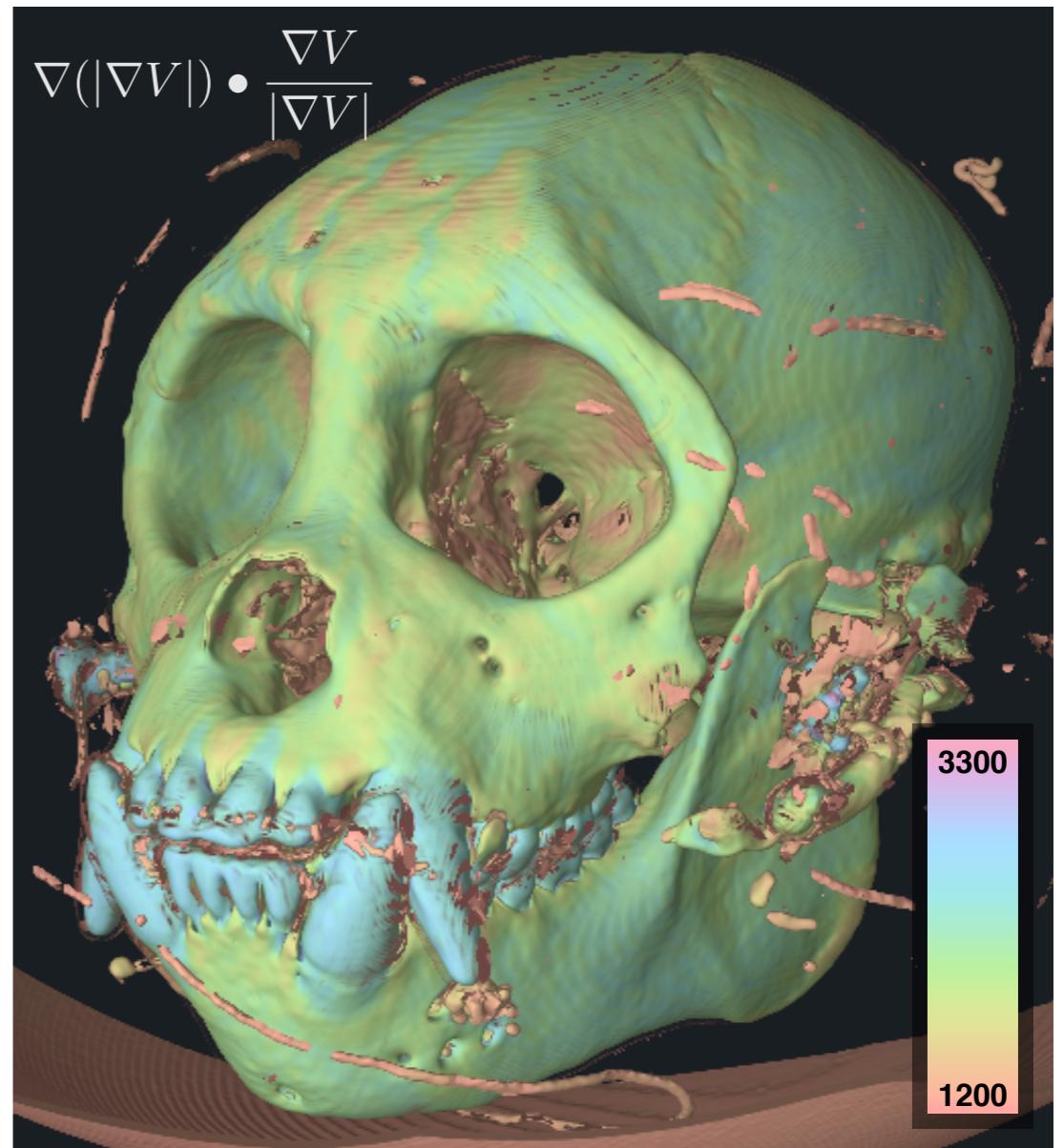
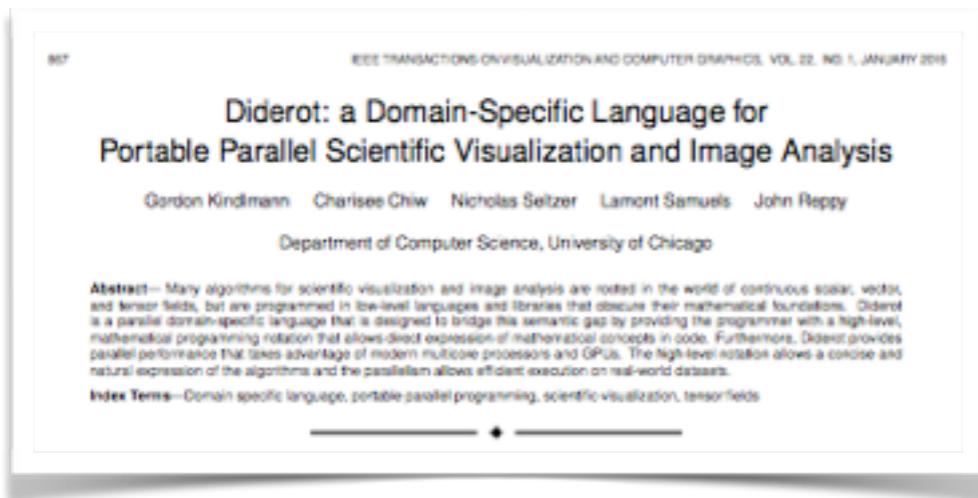
Results: Sophisticated Vis

Inline high level syntax into Diderot code

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```

Users focused on visualization

[Kindlmann VIS]



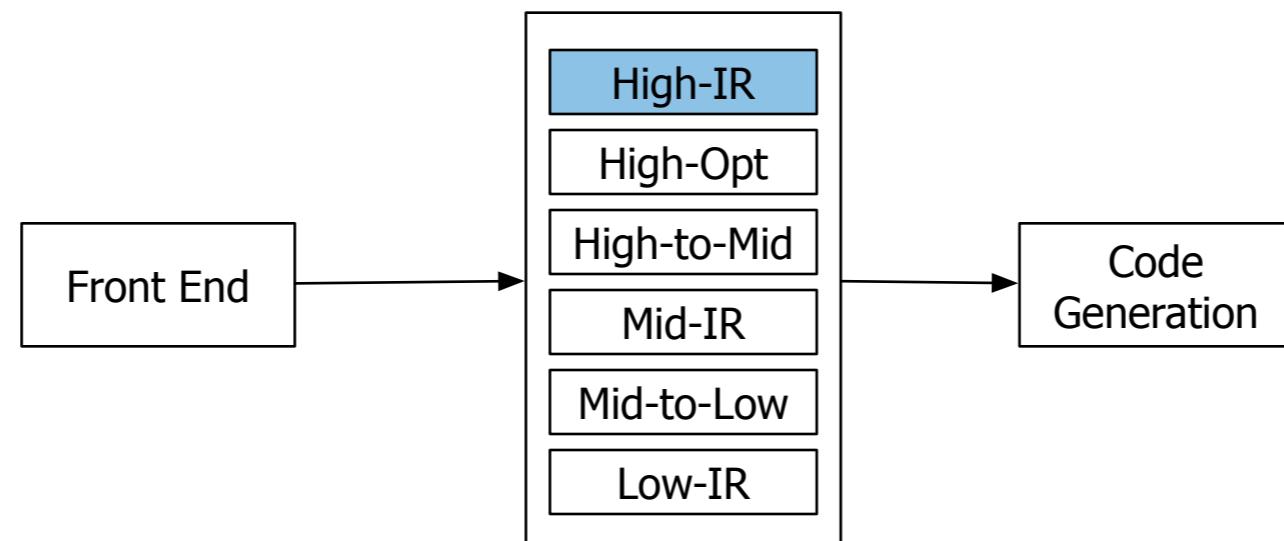
Canny Edges

Extra Slides

EIN

EIN IR

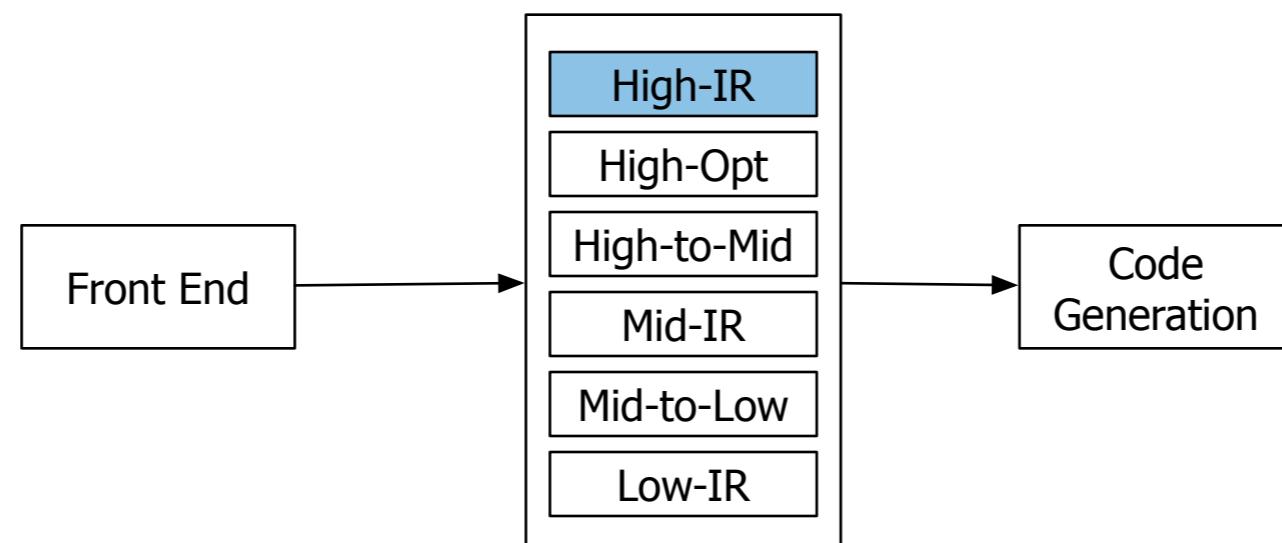
IR is in Static single assignment (SSA) form



Replace our fixed set of direct-style operators with an expressive language

EIN IR

IR is in Static single assignment (SSA) form

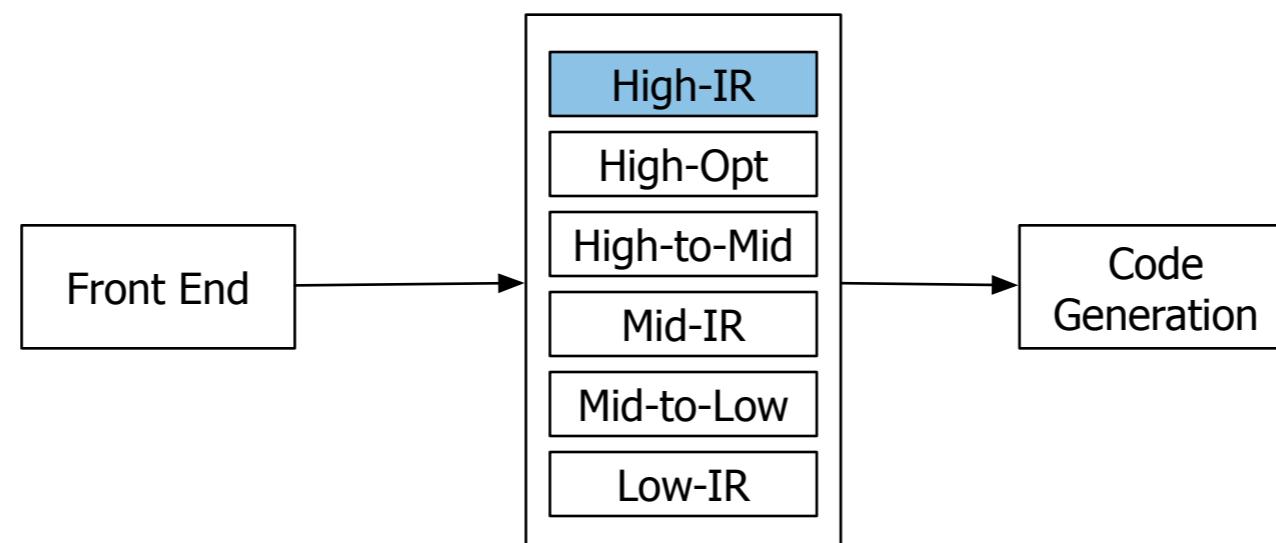


Replace our fixed set of direct-style operators with an expressive language

$$\mathbf{E} = \lambda \bar{x} \langle e \rangle_\sigma$$

EIN IR

IR is in Static single assignment (SSA) form



Replace our fixed set of direct-style operators with an expressive language

$$\mathbf{E} = \lambda \bar{x} \langle e \rangle_{\sigma}$$

whose semantics are specified by the expression e , where \bar{x} are tensor and field parameters, and σ determines the shape of the result.

EIN IR

$$E = \lambda \bar{x} \langle e \rangle_{\sigma}$$

EIN Operator

EIN IR

$$E = \lambda \bar{x} \langle e \rangle_\sigma$$

EIN Operator

- Provides a concise and expressive internal representation

EIN IR

- Provides a concise and expressive internal representation
- Terms include fields, images, permutation

$$\begin{aligned} \mathbf{E} &= \lambda \bar{x} \langle e \rangle_\sigma \\ e ::= & T_\alpha, A_\alpha, B_\alpha \\ &| F_\alpha, G_\alpha \\ &| \delta_{ij} \\ &| \mathcal{E}_{ij}, \mathcal{E}_{ijk} \end{aligned}$$

EIN Operator
Tensor
Field
Kronecker deltas
Levi-Civita tensor

EIN IR

- Provides a concise and expressive internal representation
- Terms include fields, images, permutation
- Operators include differentiation, summation, domain-specific

$\mathbf{E} = \lambda \bar{x} \langle e \rangle_\sigma$	EIN Operator
$e ::= T_\alpha, A_\alpha, B_\alpha$	Tensor
F_α, G_α	Field
δ_{ij}	Kronecker deltas
$\mathcal{E}_{ij}, \mathcal{E}_{ijk}$	Levi-Civita tensor
\sum_e	Summation
$\frac{\partial}{\partial x_\alpha} \diamond e$	Derivative of e
$e_1 @ e_2$	Probe of a field
$\text{lift}(e)$	Lift tensor e to a field
$V_\alpha \circledast H^\beta$	Convolution
$\sqrt{e}, -e, \exp(e), e^n, \sin(e)$	Unary operators
$e + e, e - e, \frac{e}{e}, ee$	Binary operators
$\sigma = \dots$	index mapping

EIN IR

- Provides a concise and expressive internal representation
- Terms include fields, images, permutation
- Operators include differentiation, summation, domain-specific
- Index is bound to variable or constant

\mathbf{E}	$= \lambda x \langle e \rangle_\sigma$	EIN Operator
$e ::=$	$T_\alpha, A_\alpha, B_\alpha$	Tensor
	$ F_\alpha, G_\alpha$	Field
	$ \delta_{ij}$	Kronecker deltas
	$ \mathcal{E}_{ij}, \mathcal{E}_{ijk}$	Levi-Civita tensor
	$ \sum e$	Summation
	$ \frac{\partial}{\partial x_\alpha} \diamond e$	Derivative of e
	$ e_1 @ e_2$	Probe of a field
	$ \text{lift}(e)$	Lift tensor e to a field
	$ V_\alpha \circledast H^\beta$	Convolution
	$ \sqrt{e}, -e, \exp(e), e^n, \sin(e)$	Unary operators
	$ e + e, e - e, \frac{e}{e}, ee$	Binary operators
i, j, k	$\in \text{INDEXVAR}$	Variable index ;
n	$\in \text{CONSTVAR} = \mathbb{N}$	Constant index
μ	$\in \text{INDEXVAR} \cup \text{CONSTVAR}$	Single index
α, β, γ	$= \bar{\mu}$	Sequence of indices
σ	$= (\mathbb{Z} \times \text{INDEXVAR} \times \mathbb{Z})^*$	index mapping

Compilation Issue

$$y = \lambda Params \langle e_{ij} e_{1k} - e_{0i} e_{jk} + e_{ij} e_{k1} \rangle_{ijk}(args)$$

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- Consider example: $e = f \otimes (a + b)$

Compilation Issue

$$y = \lambda F, A, B \left\langle \left(\sum_l F_{il}(A_{lj} + B_{lj}) \right) \left(\sum_l F_{1l}(A_{lk} + B_{lk}) \right) - \left(\sum_l F_{0l}(A_{li} + B_{li}) \right) \left(\sum_l F_{jl}(A_{lk} + B_{lk}) \right) + \left(\sum_l F_{il}(A_{lj} + B_{lj}) \right) \left(\sum_l F_{kl}(A_{l1} + B_{l1}) \right) \right\rangle_{ijk} (f, a, b)$$

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- Pull it out and create a new operator

$$z = \lambda F, A, B, x \left\langle \sum_k F_{ik}(A_{kj} + B_{kj}) \right\rangle_{ij} (f, a, b, x)$$

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- Replace subexpression with tensor term

$$y' = \lambda F, A, B, Z \left\langle Z_{ij} \left(\sum_l F_{1l}(A_{lk} + B_{lk}) \right) - \left(\sum_l F_{0l}(A_{li} + B_{li}) \right) Z_{jk} + Z_{ij} \left(\sum_l F_{kl}(A_{l1} + B_{l1}) \right) \right\rangle_{ijk} (f, a, b, z)$$

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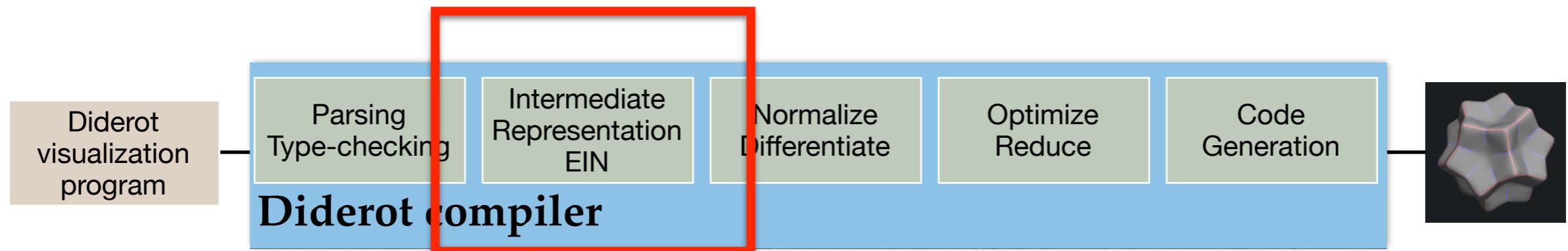
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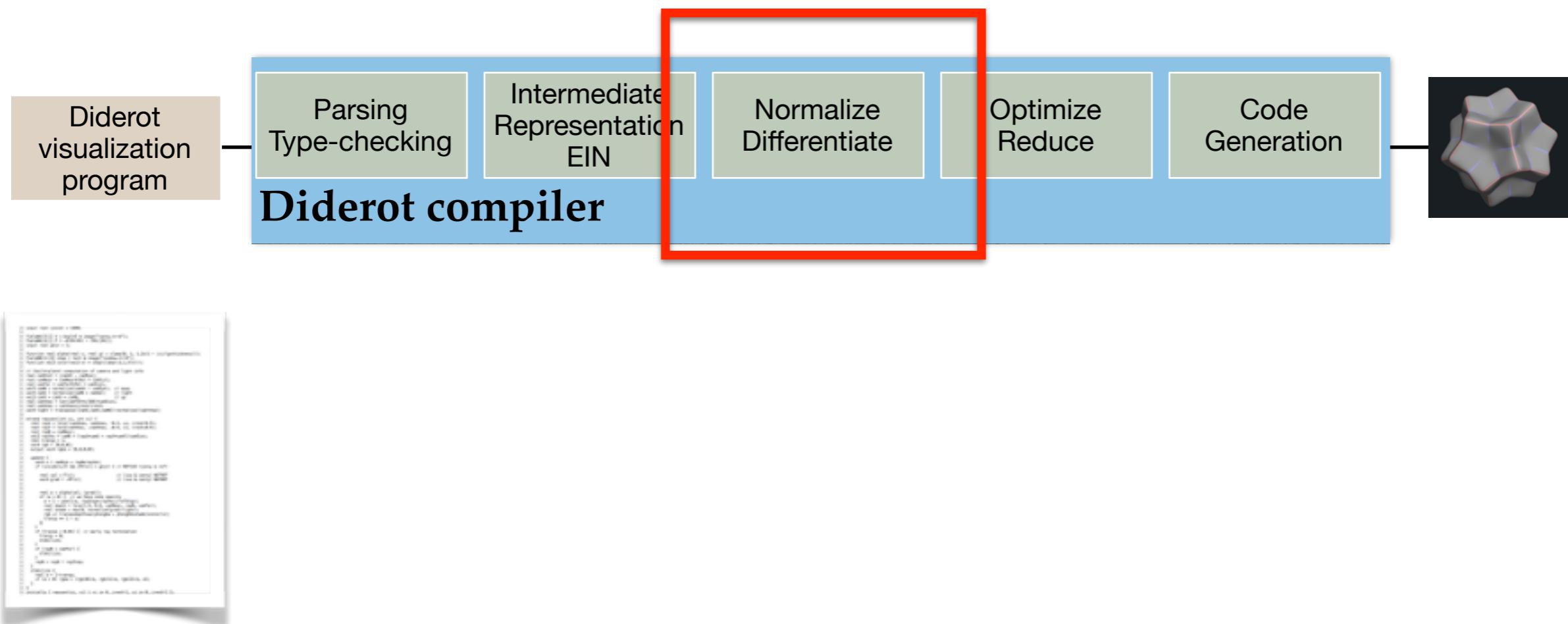
EIN Intermediate Representation



Phase that represents tensor math

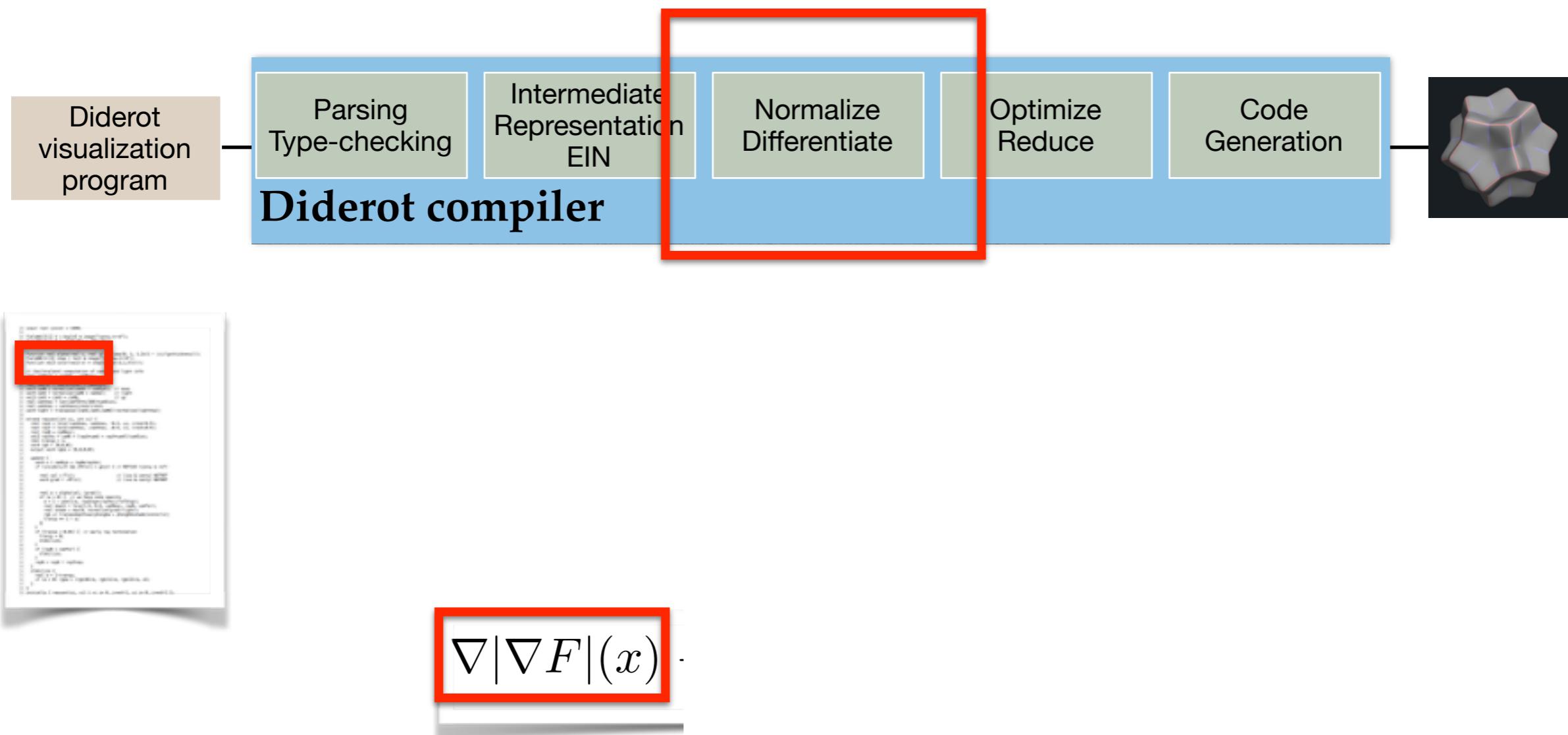
Compiling Diderot

From Tensor Calculus to C



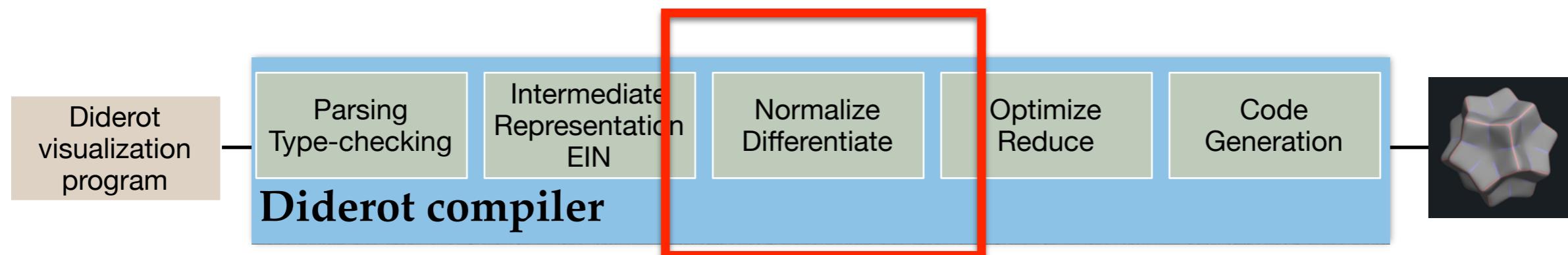
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Compiling Diderot

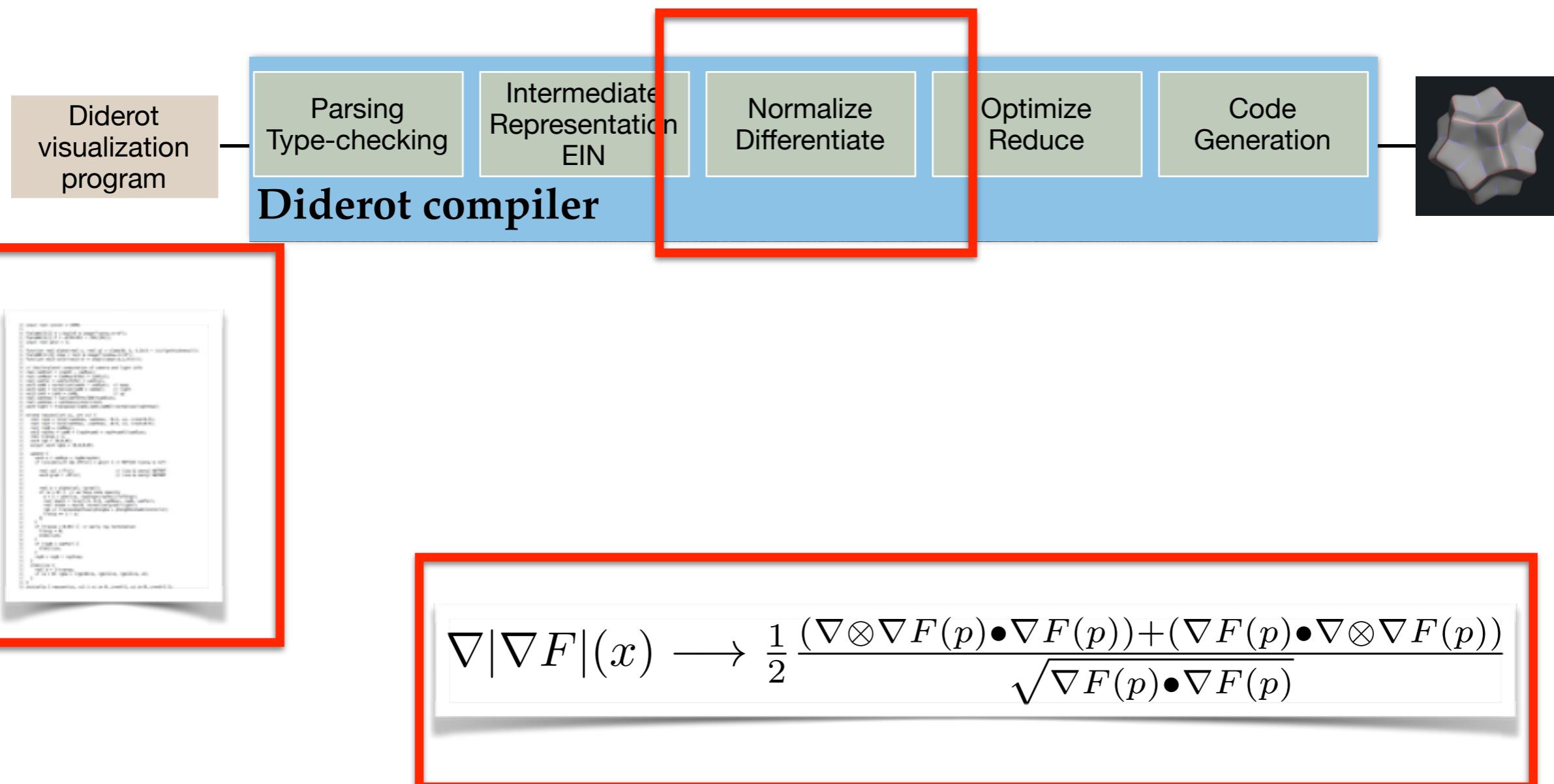
From Tensor Calculus to C



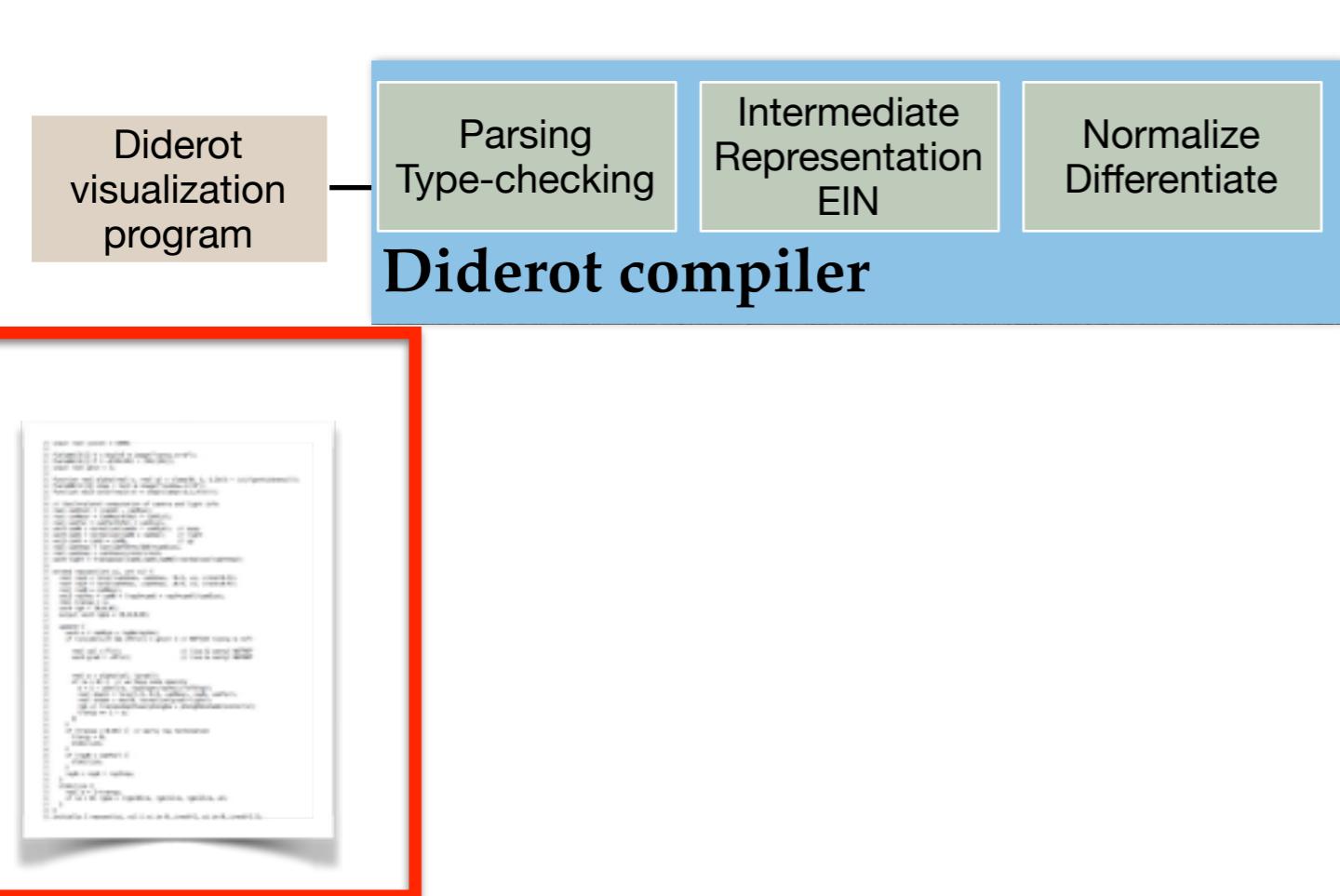
$$\nabla |\nabla F|(x) \longrightarrow \frac{1}{2} \frac{(\nabla \otimes \nabla F(p) \bullet \nabla F(p)) + (\nabla F(p) \bullet \nabla \otimes \nabla F(p))}{\sqrt{\nabla F(p) \bullet \nabla F(p)}}$$

Compiling Diderot

From Tensor Calculus to C



Compilation Issue

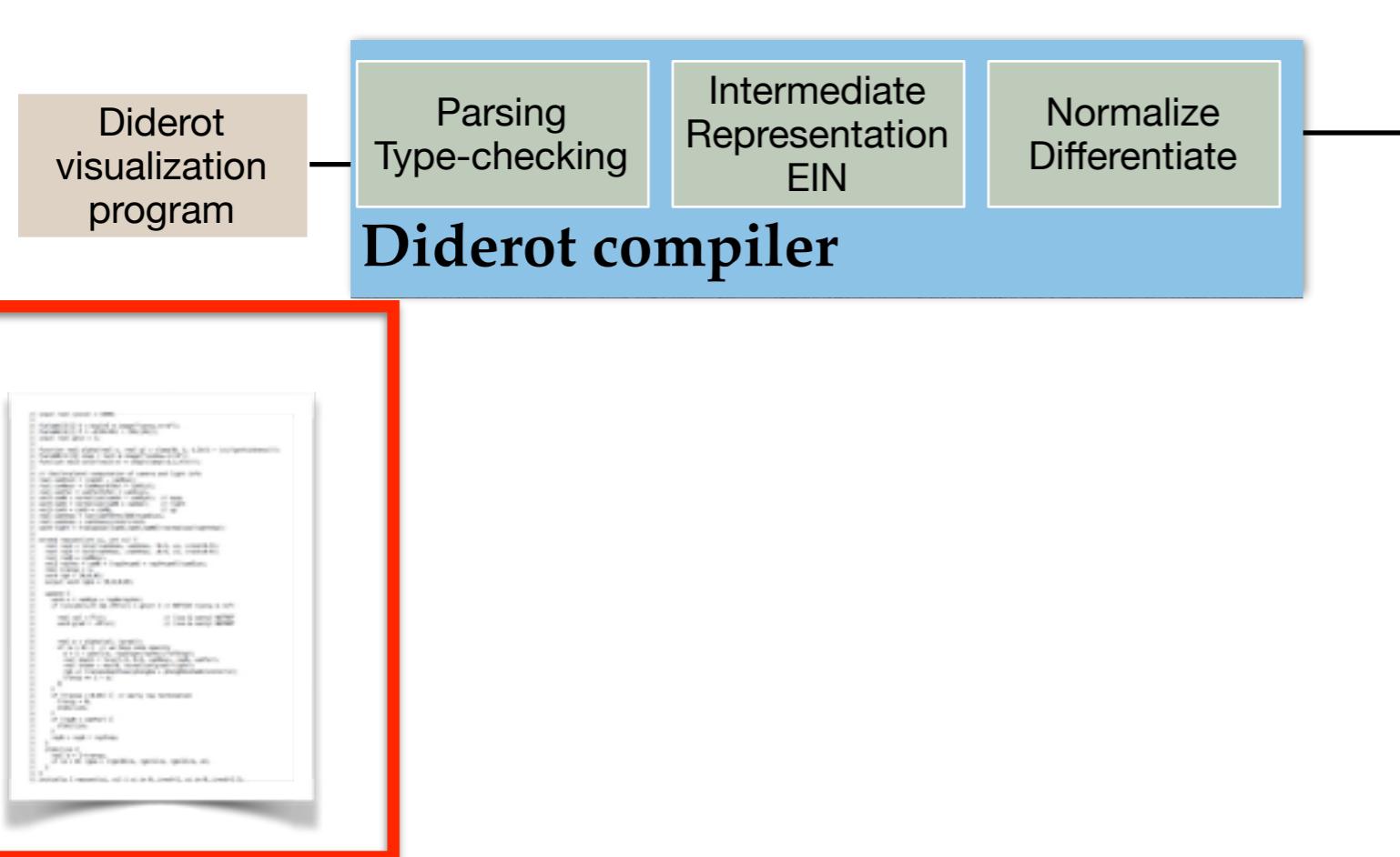


Compilation Issue



Programs could not compile or took too long to compile

Compilation Issue

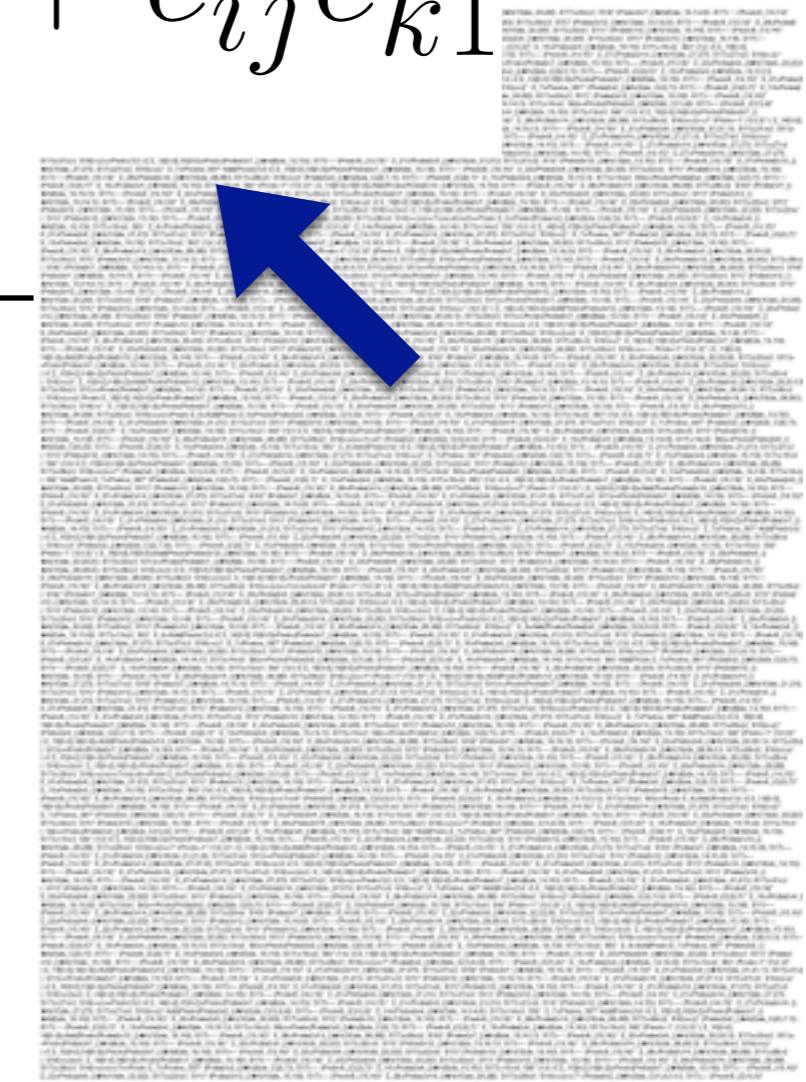
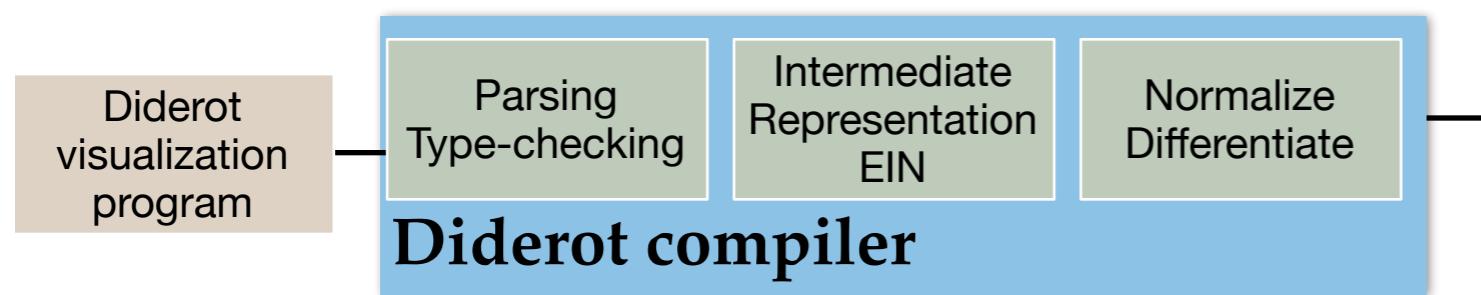


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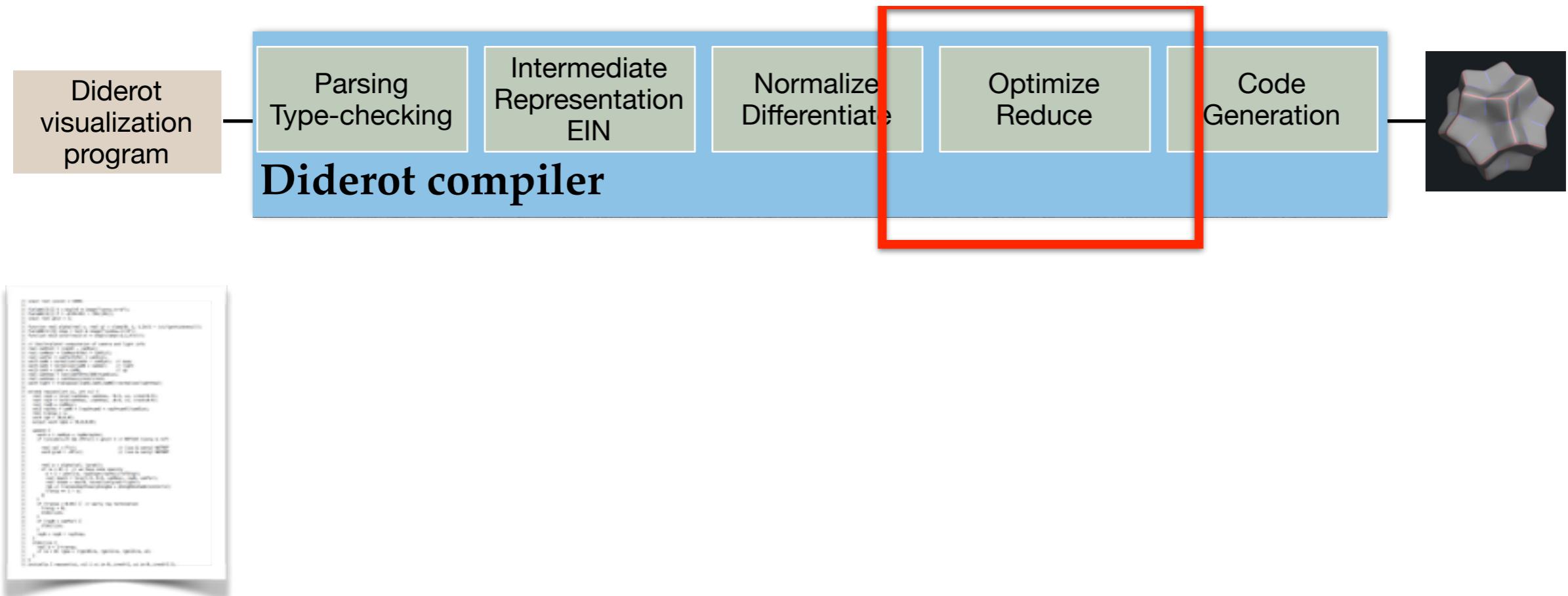
$$e_{ij}e_{1k} - e_{0i}e_{jk} + e_{ij}e_{k1}$$



Programs could not compile or took too long to compile

Compilation Issue

$$e_{ij}e_{1k} - e_{0i}e_{jk} + e_{ij}e_{k1}$$



Phase that enables effective compilation of tensor math

Compilation Issue

$$e_{ij}e_{1k} - e_{0i}e_{jk} + e_{ij}e_{k1}$$

To reduce the size of the IR we want to

Compilation Issue

$$\boxed{e_{ij}} e_{1k} - e_{0i} e_{jk} + \boxed{e_{ij}} e_{k1}$$

To reduce the size of the IR we want to

- enable traditional compiler approaches ([cse](#))

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Terms can be syntactically different even when they

- are doing the same thing

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[CHIW CPC]

**EIN: An Intermediate Representation
for
Compiling Tensor Calculus**

Charisee Chiw, Gordon L. Kindlman, and John Reppy

University of Chicago

Our design and implementation



Abstract. Diderot is a parallel domain-specific language for analysis and visu-