IRT workshop

Christopher David Desjardins

2 April 2014

Overview

1. Review
2. MIRT

Review

- Differential item functioning
- Mantel-Haenzel
- Logistic regression
- Ordinal & multinomial logistic regression
- Today: Multidimensional IRT (MIRT)

Proposed topics

MIRT (2/4) | Linking/Equating & Presentations (16/4)

What is MIRT?

- Multidimensional IRT
- Extension to the IRT model to explore the underlying dimensionality the model
- Unlike confirmatory factor analysis, MIRT doesn't depend on CTT

- Recall that IRT depends crticially on unidimensionality
  - This is important for linking participants, adaptive testing (such as CAT), and get ability scores

MIRT

- Many psychological constructs (and instruments) are inherently multidimensional
- Unobservable constructs may consist of subscales that are nested with a general construct (e.g. bifactor model)
- Many unidimensional IRT models should probably be MIRT
- This models are less studied and developed because they are quite computationally intensive (lots of integration required)
- This is active and hot area in IRT

MIRT vs. CFA

- IRT, estimates of difficulties and latent abilities are independent of each other
- In CTT, item difficulty is a function of the abilities of the sample, and the abilities of respondents are assessed as a function of item difficulty
- MIRT more readily handles ordinal data, making parameters more interpretable and realistic (whereas in CFA they are often ignored)
- Differ in interpretation of standard error of measurement
- MIRT differs from most factor analyses in that the varying characteristics of the input variables (the items), such as difficulty level and discrimination, are considered to be of important and not nuisance

MIRT vs. CFA - con't

- Differ in estimation of items and assessment of fit (e.g. in MIRT used infit and outfit statistics)
- CFA assume relationship between latent factor and indicator is linear, in MIRT this can be nonlinear
- CFA uses the factor loading to represent the relationship between the indicator and the latent variable across all levels of the latent variable
- MIRT, the relationship between indicator and latent variable is given across the range of possible values for the latent variable
- MIRT, all the IRT tools/graphs are available
- MIRT requires more data than CFA and IRT, in general
- However, the factor model can be rewritten as the IRT model

The MIRT 3-PL model

$$P(x_{ij} = 1|\boldsymbol{\theta}_i, \boldsymbol{\alpha}_j, d_j, \gamma_j) = \gamma_j + \frac{1 - \gamma_j}{1 + \exp[-D(\boldsymbol{\alpha}_j^T \boldsymbol{\theta}_i + d_j)]}$$

Where there are i $= 1, \ldots, N$ participants, j $= 1, \ldots, n$ test items, there are m latent factors $\boldsymbol{\theta}_\mathbf{i} = (\theta_{i1}, \ldots, \theta_{im})$ with associated item slopes $\boldsymbol{\alpha}_\mathbf{j} = (\alpha_1, \ldots, \alpha_m)$, γ_j is the guessing parameter, d_j is the item intercept, and D is a scaling adjustment.

Samejima's multidimensional ordinal model response model

$$
\begin{aligned}
\Phi(x_{ij} \geq 0 | \boldsymbol{\theta}_{\mathbf{i}}, \boldsymbol{\alpha}_{\mathbf{i}}, \mathbf{d}_j) &= 1 \\
\Phi(x_{ij} \geq 1 | \boldsymbol{\alpha}_{\mathbf{i}}, \boldsymbol{\alpha}_{\mathbf{i}}, \mathbf{d}_j) &= \frac{1}{1 + \exp[-D(\boldsymbol{\alpha}_{\mathbf{i}})^T \boldsymbol{\alpha}_{\mathbf{i}} + d_1]} \\
\Phi(x_{ij} \geq 2 | \boldsymbol{\alpha}_{\mathbf{i}}, \boldsymbol{\alpha}_{\mathbf{i}}, \mathbf{d}_j) &= \frac{1}{1 + \exp[-D(\boldsymbol{\alpha}_{\mathbf{i}})^T \boldsymbol{\alpha}_{\mathbf{i}} + d_2]} \\
&\vdots \\
\Phi(x_{ij} \geq C | \boldsymbol{\alpha}_{\mathbf{i}}, \boldsymbol{\alpha}_{\mathbf{i}}, \mathbf{d}_j) &= 0
\end{aligned}
$$

Where there are C_j unique categories with $\mathbf{d}_j = (d_1, \ldots, d_{(C_j - 1)})$ and the probability for the response $x_{ij} = k$ is

$$
\Phi(x_{ij} \geq k | \boldsymbol{\theta}_{\mathbf{i}}, \boldsymbol{\alpha}_{\mathbf{i}}, \mathbf{d}_j) = \Phi(x_{ij} \geq k | \boldsymbol{\theta}_{\mathbf{i}}, \boldsymbol{\alpha}_{\mathbf{i}}, \mathbf{d}_j) - \Phi(x_{ij} \geq k + 1 | \boldsymbol{\theta}_{\mathbf{i}}, \boldsymbol{\alpha}_{\mathbf{i}}, \mathbf{d}_j)
$$

Applications of MIRT

- Can be used in an exploratory manner where the number of dimensions are not assumed (or known beforehand)
- Models can then be selected with LRT
- Can also rotate solutions
- Bifactor model - a single factor is believed to be present in all items, but with additional clusters of local dependencies formed by other independent specific factors

MIRT in `R`

- To do MIRT in `R`, will use the `mirt` package
- `bfactor()` function can be used to run the bifactor model
- `mirt()` function can be used for both confirmatory and exploratory IRT models
- Can include covariates person and item level in the `mixedmirt()` function

Bifactor model

- Reise et al. (2007) argue that the bifactor model
  - Allows for examination of the distortion that may occur when unidimensional IRT models are fit to multidimensional data
  - Allows researchers to empirically examine the utility of forming subscales
  - Provides an alternative to non-hierarchical multidimensional IRT models for scaling individual differences

Bifactor model using MIRT

- Will use the SAT12 dataset
- 32 items to a science assessment test measuring chemistry, biology, physics

```
library(mirt)

## Loading required package:  stats4
## Loading required package:  lattice

data("SAT12")
data <- key2binary(SAT12, key = c(1, 4, 5, 2, 3, 1, 2, 1, 3, 1, 2, 4, 2, 1,
5, 3, 4, 4, 1, 4, 3, 3, 4, 1, 3, 5, 1, 3, 1, 5, 4, 5))
```

```r
specific <- c(2, 3, 2, 3, 3, 2, 1, 2, 1, 1, 1, 3, 1, 3, 1, 2, 1, 1, 3, 3, 1
    1, 3, 1, 3, 3, 1, 3, 2, 3, 1, 2)
mod1 <- bfactor(data, specific, verbose = FALSE)

# summary(mod1) # ---- Factor loadings

# coef(mod1) # ---- Item difficulties
```

Bifactor model - con't

```
# fscores(mod1) # ----- scores for the unique and general factors
itemplot(mod1, 1, type = "info")

## Error:  Can not plot high dimensional models
```

Exploratory MIRT

```
# Examine 3 factors
exp_1f <- mirt(data, 1, verbose = FALSE)
exp_2f <- mirt(data, 2, verbose = FALSE)
exp_3f <- mirt(data, 3, TOL = 0.001, verbose = FALSE)
```

```r
anova(exp_1f, exp_2f)

##
## Model 1: mirt(data = data, model = 1, verbose = FALSE)
## Model 2: mirt(data = data, model = 2, verbose = FALSE)
##     AIC  AICc SABIC   BIC logLik     X2 df p
## 1 19106 19121 19184 19387  -9489
## 2 19074 19110 19190 19492  -9442 93.903 31 0

anova(exp_2f, exp_3f)

##
## Model 1: mirt(data = data, model = 2, verbose = FALSE)
## Model 2: mirt(data = data, model = 3, TOL = 0.001, verbose = FALSE)
##     AIC  AICc SABIC   BIC logLik     X2 df     p
## 1 19074 19110 19190 19492  -9442
## 2 19082 19149 19235 19632  -9416 51.893 30 0.008

# Can rotate for interpretability ------- coef(exp_2f, rotate = 'varimax')
# coef(exp_2f)
```

Confirmatory MIRT

```
model.combo <- mirt.model("F1 = 1-16
                           F2 = 17-32")
mod.combo <- mirt(data, model.combo, verbose = FALSE)
```

More in MIRT

- Can include predictors and do mixed effects modeling with `mirt`
- Can impute missing values
- Can use different estimation algorithms