IRT workshop

Christopher David Desjardins

5 March 2014

Overview

1. Review
2. Polytomous IRT models
   - Rating scale model
   - Graded response model
   - Nominal Polytmous Model

Review

- 1-PL and Rasch model , 2-PL, and 3-PL models
- IRF, IIF, TRF, and TIF
- Person and item estimation
- Ran `R` code
- Today: Polytomous IRT models

Proposed topics

Polytomous models (5/3)	Multidimensional models (2/4)
Differential Item Function (19/3)	LCA & Presentations (16/4)

Polytomous IRT models

- Ordinal data
  - Partial credit model
  - Graded response model
- Nominal data
  - Nominal response model

Ordinal type

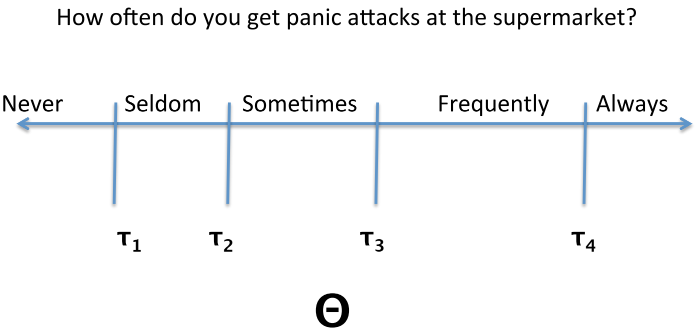
- In many situations, our data have more than two response categories
- However, the data have a natural ordering to them
- For example,
  - Judge rating a person's performance on a rating scale
  - Likert scale
  - Assign codes that reflect an answer's degree of correctness

Motivation for the polytomous Rasch

- An item asks "How often do you get panic attacks at the supermarket?"
- Responses to the item are: Never, Seldom, Sometimes, Frequently, Always
- There are scored: 0, 1, 2, 3, 4
- For this item there are 5 possible scores, *category scores*
- Higher scores indicate higher anxiety
- How should we model this?
    - One approach
        - Model them as a series of ordered pairs of adjacent categories and apply the Rasch model to each pair

Polytomous Rasch models

- Assume ordered categories correspond to *transition location* that fall on the latent variable continuum and separate adjacent response categories



- We have 4 transition points.
- The location of these points can be used to calculate the probability crossing the location

PCM

- In order to get a category score of 2 ("Sometimes"), an individual would have to pass through "Never" and "Seldom"
- The probability of getting a 0 is e^0, the probability of getting a 1 is $e^0 + e^{(\theta-\delta_1)}$ and the probability of getting a 2 is $e^0 + e^{(\theta-\delta_1)} + e^{(\theta-\delta_2)}$ after dividing by some denominator ψ
- Where ψ is the sum of the mutually exclusive outcomes (there would be 4 in this example).

$$p(x_j|\theta,\delta_{jh}) = \frac{exp\left[\sum_{h=0}^{x_j}(\theta-\delta_{jh})\right]}{\sum_{k=0}^{m_j} exp\left[\sum_{h=0}^{x_j}(\theta-\delta_{jh})\right]}$$

- Where δ_{jh} is known as the transation location, which reflects the relative difficult in endorsing category h over category $h-1$
- Note, m_j means the number of categories may vary by item

PCM continued

- Could be used on tests with both polytomous and dichotomous items (unlike other models, e.g. rating scale)
- Transition locations can only be interpretated with regards to a specific item
- This is a Rasch, so discrimination is set to 1
- This can be relaxed as a generalized partial credit model could be considered
- Should examine option response functions (like IRFs)

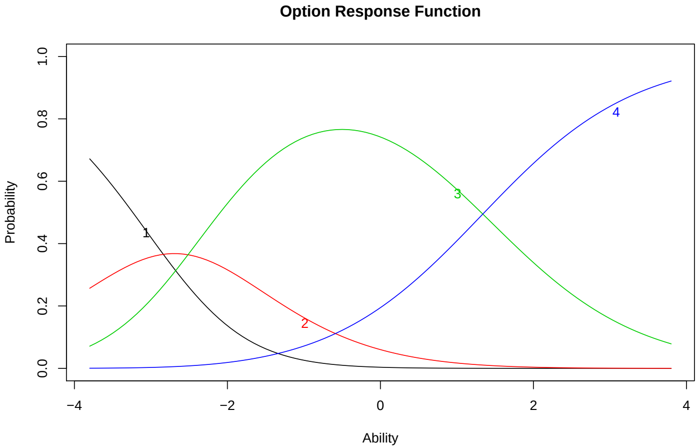
Modeling in R

- The partial credit models can be most easily fit with `ltm`

```
> library(ltm)
> part.cred <- gpcm(data = Science[,1:3], constraint="rasch")
> coef(part.cred)
```

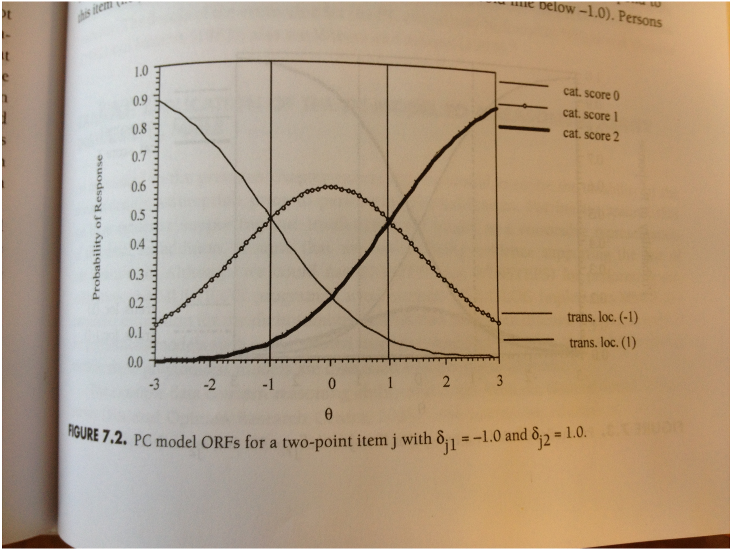
```
            Catgr.1 Catgr.2 Catgr.3 Dscrmn
Comfort      -2.838  -2.515   1.338      1
Environment  -1.882  -0.737   0.408      1
Work         -1.766  -0.888   1.774      1
```

```
> plot(part.cred,items = 1,
+ main = "Option Response Function")
```



```
> factor.scores(part.cred)$score.dat[1:5,c(1:3,6:7)]

  Comfort Environment Work         z1     se.z1
1       1           1    1 -2.2972293 0.6353661
2       1           2    3 -1.1469383 0.6134133
3       1           2    4 -0.7674760 0.6197560
4       1           3    4 -0.3758666 0.6328601
5       1           4    1 -1.1469383 0.6134133
```

How to score $(6/3) + 2 = ?$



Nothing guarantees that the transition locations are in order!

Graded response model

- Like the polytomous Rasch, the graded response data consists of a score that is on the ordinal scale
- Partially correct educational data, Likert items, scales, etc.
- The graded response model (GRM) is an extension of the 2-PL model for ordinal data.
- However, rather than calculating probabilities as a series of dichotomous choices we use cumulative comparsions and cumulative probabilities.
- For example, model the probability of Sometimes or higher vs. Seldom or lower

GRM

$$P^*_{x_j}(\theta_i) = \frac{e^{a_j(\theta - \delta_{x_j})}}{1 + e^{a_j(\theta - \delta_{x_j})}}$$

$$p_k = P^*_{x_j} - P^*_{x_j+1}(\theta_i)$$

- a_j corresponds to the discrimination parameter, and δ_{x_j} are the category boundary locations.
- Similar to the PCM, the number of categories may vary across items.
- Unlike the PCM, the δx_js are always in increasing order. Consider $(6/3) + 2$ again.
- Second line, probability of being in group k corresponds to the different in cumulative probabilities

GRM continued

- For example, to calculate $p_{sometimes}$ or p_2

$$p_2 = P_2^* - P_3^* = \frac{e^{a_j(\theta-\delta_2)}}{1+e^{a_j(\theta-\delta_2)}} - \frac{e^{a_j(\theta-\delta_3)}}{1+e^{a_j(\theta-\delta_3)}}$$

- For example, to calculate p_{never} or p_0

$$p_0 = P_0^* - P_1^* = 1 - \frac{e^{a_j(\theta-\delta_1)}}{1+e^{a_j(\theta-\delta_1)}}$$

GRM in R

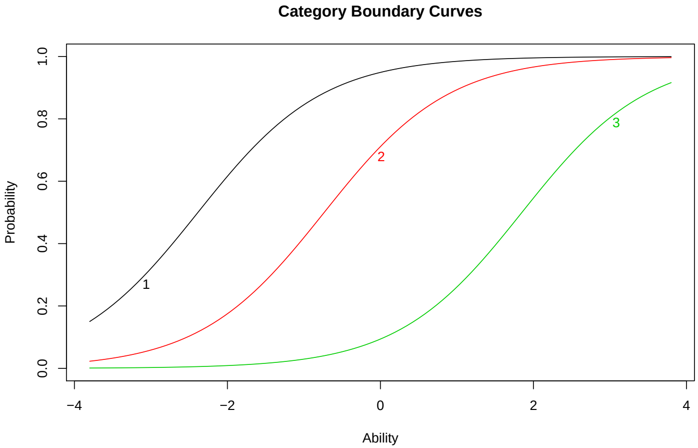
- This again is most easily fit with the `ltm` model

```
> graded <- grm(data = mirt::Science)
> coef(graded)
```

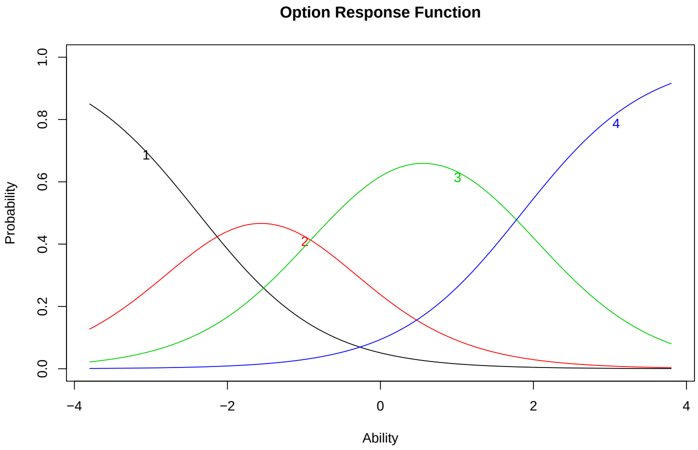
	Extrmt1	Extrmt2	Extrmt3	Dscrmn
Comfort	-4.672	-2.536	1.408	1.041
Work	-2.385	-0.735	1.849	1.226
Future	-2.281	-0.965	0.856	2.299
Benefit	-3.060	-0.906	1.543	1.094

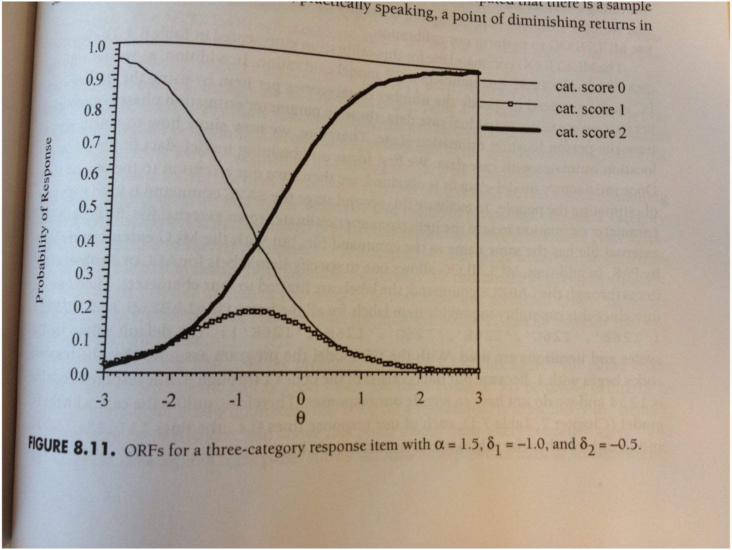
```
> factor.scores(graded)$score.dat[1:5,c(1:3,6:7)]

  Comfort Work Future         Exp         z1
1       1    1      1 0.124034907 -2.7023453
2       1    3      2 0.067021024 -1.4228786
3       1    4      2 0.019388358 -0.7639417
4       1    4      3 0.006189575 -0.4608101
5       2    1      1 0.459763605 -2.5237021
```

```
> plot(graded,items = 2,type="OCCu",
+ main="Category Boundary Curves")
```



```
> plot(graded,items = 2,main = "Option Response Function")
```





These are in order $\delta_1 = -1.0$ and $\delta_2 = -0.5$. But what does it say about responding in category "1"?

Nominal Polytmous Model

- When there is no inherent order to a categorical variable one option is Bock's nominal response model

$$p_j(x = k|\theta, \boldsymbol{\alpha}, \boldsymbol{\gamma}) = \frac{e^{\gamma_{jk} + \alpha_{jk}\theta}}{\sum_{h=1}^{m_j} e^{\gamma_{jk} + \alpha_{jk}\theta}}$$

- α_{jk} and γ_{jk} are the slope and the intercept parameters of the response function for the kth indexed category of item j.
- γ_{jk} represent's an individual's propensity to use response category k
- α_{jk} is the option's discrimination capacity

Running the Nominal Response Model

- `mirt` provides the nominal response model by specifying "nominal" as itemtype.
- However, `mirt` was not used for the other models presented today as it appeared to give non-sense answers
- More investigation into this!