

# IRT workshop

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1 Review

2 One-Parameter Model

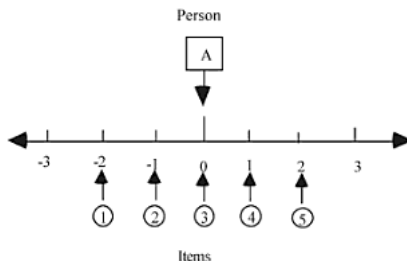
- Structure of seminar and course assignments (i.e. short paper and presentation)
- Quickly and conceptually introduce IRT
- Quickly introduced R
- Today: One-parameter and Rasch models

# Proposed topics

1-PL and Rasch model (5/2)	Differential Item Function (19/3)
2-PL and 3-PL models (19/2)	Latent class analysis (2/4)
Polytomous models (5/3)	Scaling, linking & Presentations (16/4)

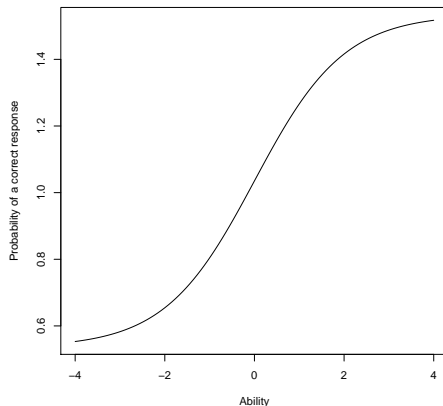
Remaining classes are from 1300 - 1500 in Aragata 14.

# Conceptually



- Assume we are measuring **reading proficiency**
- The **items** are located at: -2, 1, 1, 2. ( $\delta_j$ )
- **Person A** is located at 0. ( $\theta_A$ )
- Which items would we expect Person A to respond correctly to?  
Which should be the easiest? Which the hardest?
- Which items would we have a more difficult time of knowing whether Person A would get them correct or not?

# Conceptually (cont.)



This is known as an item characteristic curve, an item curve, or an **item response function (IRF)**. What kind of model should we fit?

- The logistic model

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- The Rasch model

$$p(x_j = 1|\theta, \delta_j) = \frac{e^{\theta - \delta_j}}{1 + e^{\theta - \delta_j}}, \text{ so } z = \theta - \delta_j$$

- The probability of responding correctly to item  $j$  is a function of the **distance between the person and the location of item  $j$  (i.e. the item's difficulty)!**

# Toy example

Assume a person with an ability score of 1 and encounters an item with a difficulty of 1.5. What is the probability they would get it correct if we assume the Rasch model is true?

```
rasch <- function(person, item) {  
  exp(person - item)/(1 + exp(person - item))  
}  
rasch(person = 1, item = 1.5)  
  
## [1] 0.3775
```

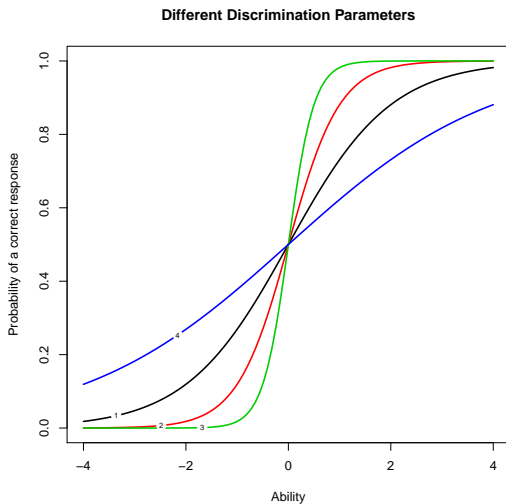
# The One-Parameter model

- The One-Parameter model

$$p(x_j = 1|\theta, \delta_j) = \frac{e^{\alpha(\theta - \delta_j)}}{1 + e^{\alpha(\theta - \delta_j)}}$$

- This parameter  $\alpha$  is known as item discrimination and this affects the slope of the IRF.
- Higher  $\alpha$  means we are doing a better job discriminating among respondents located at different abilities
- There is no item subscript in the one-parameter model!
- What does  $\alpha$  equal in the Rasch model?

# Varying $\alpha$



# Rasch vs. One-PL and Assumptions

- Many consider the Rasch model to be the true model and should be the only model to construct an instrument.
- Reality, Rasch is just the One-PL where  $\alpha = 1$ .

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- Many consider the Rasch model to be the true model and should be the only model to construct an instrument.
- Reality, Rasch is just the One-PL where  $\alpha = 1$ .
- Assumptions
  - Unidimensionality - i.e. one underlying trait relating items.
  - Local independence - i.e. only thing that relates the items is the underlying construct.
  - The data follow either the model, data are S-shaped and appropriate, i.e. functional-form is correct.
  - *Consequence*: These models assume  $\alpha$  is the same for all items and  $c$  the guessing parameter, is set to 0.

# Ability estimates

- 1 First, calculate probability of each response for a respondent.
- 2 Second, determine probability of the response pattern (this is the product of #1 bc of local independence).
- 3 Third, repeat #1 and #2 for  $\theta$  -3 to 3 (or -4 to 4).
- 4 Fourth, select the  $\theta$  with the highest likelihood of producing the pattern.

## Example: Assume that $\theta = 2.5$

- Assume the pattern of 011, where  $\delta_1 = 2$ ,  $\delta_2 = 1.2$ , and  $\delta_3 = 2.5$ .
- Step 1 - Calculate the probabilities

$$p(x_1 = 0 | \theta = 2.5, \delta_1 = 2) = 1 - \frac{e^{2.5-2}}{1+e^{2.5-2}} = 1 - 0.622 = 0.378$$

$$p(x_2 = 1 | \theta = 2.5, \delta_2 = 1.2) = \frac{e^{2.5-1.2}}{1+e^{2.5-1.2}} = 0.786$$

$$p(x_3 = 1 | \theta = 2.5, \delta_3 = 2.5) = \frac{e^{2.5-2.5}}{1+e^{2.5-2.5}} = 0.500$$

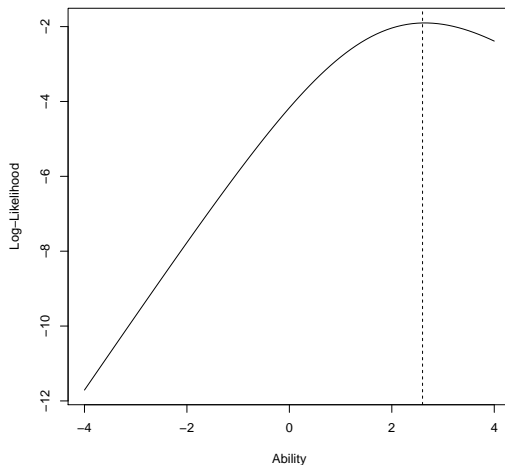
- Step 2 - Find the likelihood

$$p(x_1 = 0 | \theta = 2.5, \delta_1 = 2) * p(x_2 = 1 | \theta = 2.5, \delta_2 = 1.2) * p(x_3 = 1 | \theta = 2.5, \delta_3 = 2.5) = 0.378 * 0.786 * .500 = \mathbf{0.148}.$$

- Typically report, the log of the likelihood =  $\log(0.148) = -1.91$



# Log-likelihood function for 011 for $\theta = -4$ to 4



# Few more things about the log-likelihood function

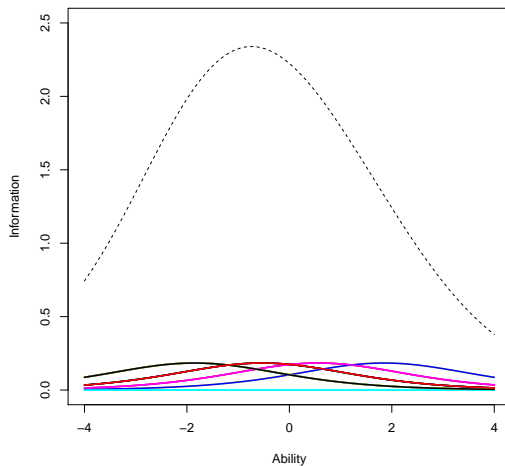
- We want the maximum of this function, i.e. the maximum likelihood estimate (MLEs).
- MLEs doesn't exist for the pattern 000 or 111, therefore need to use Bayesian estimation.

# Standard errors of estimate

- The standard error of estimate (SEE) represents our degree of uncertainty about the location of a person.
- The larger the SEE, the more uncertain we are.
- This is the same thing as a standard error in statistics and you could use it to create 95% confidence intervals around person and item parameters.  
 $\hat{\theta} \pm 1.96 * SE(\hat{\theta})$  is a 95% confidence interval for  $\theta$ .  
Recall how to interpret this!

- Information (precision) is the inverse of the SEE.
- The smaller the SEE, the more information we have about a person's location.
- This tell us, essentially, where we can get precise estimates of person location.
- With the 1-PL model
  - Item maximum information at it's location.
  - Item information function is unimodal and symmetric at  $\delta$
  - All items on the instrument provide the maximum amount of information of  $\alpha^2 0.25$ .
- Because items are independent, the total information for a test (or instrument) is the sum of the item information functions!
- This is what we'll use to build a test or an instrument.

# Information Function - Test and Item



# Next time

- Please look for a data set that you could use for your class project. If you need help, please ask!
- Please read chapters 3 & 4 in Baker's IRT book.
- Optional: Chapters 5 & 6 in de Ayala. Read 3 & 4, if you're interested in the different estimation techniques.