Math 504

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1 Question 1

$$det(A - I\lambda) = \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}.$$

$= -(\lambda - 5)(\lambda - 1)^2$
Solve the equation $-(\lambda - 5)(\lambda - 1)^2 = 0$, we obtain the roots are $\lambda_1 = \lambda_2 = 1$ and $\lambda_3 = 5$
These are the eigenvalues
We now proceed to find eigenvectors
$*\lambda_1 = \lambda_2 = 1$

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}$$

$= \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

We need to solve
$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

In order to solve for x_1, x_2, x_3, we need to solve $\begin{bmatrix} 1 & 2 & 1 & | & 0 \\ 1 & 2 & 1 & | & 0 \\ 1 & 2 & 1 & | & 0 \end{bmatrix}$

The reduce form of the matrix is $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Suppose x_2 = s, x_3 = t, then x_1 = -s - 2t
$\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ t + $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ s

The null space and also the eigenvectors are $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$*\lambda_3 = 5$

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix}$$

Similar to the previous case, we need to solve for $\begin{bmatrix} 1 & 2 & 1 & | & 0 \\ 1 & 2 & 1 & | & 0 \\ 1 & 2 & 1 & | & 0 \end{bmatrix}$

The reduce form of the matrix is $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

We have $x_1 = x_2 = x_3$. Suppose $x_1 = 1$, the eigenvector is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

det(A) = 1 . 1 . 5 = 5
trace(A) = 1 + 1 + 5 = 7

2 Question 2

a) vector b is $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

Matrix A = $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

b) Matrix B = $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 2 \\ -1 & 2 & 5 \end{bmatrix}$

c) ∇f(x) = $\begin{bmatrix} 2x_1 + 2x_2 - 2x_3 + 1 \\ 2x_2 + 2x_1 + 4x_3 - 1 \\ 10x_3 - 2x_1 + 4x_2 \end{bmatrix}$