Math 504

Cuong Ly

April 2022

1 Question 1

1.1 1a

l_0(x) =

$$\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-2)(x-4)}{(-1)(-3)} = \frac{(x-2)(x-4)}{3}$$

l_1(x) =

$$\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-1)(x-4)}{(1)(-2)} = \frac{(x-1)(x-4)}{-2}$$

l_2(x) =

$$\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-1)(x-2)}{(3)(2)} = \frac{(x-1)(x-2)}{6}$$

$$p(x) = y_0 * l_0(x) + y_1 * l_1(x) + y_2 * l_2(x)$$

=

$$(-1) * \frac{(x-2)(x-4)}{3} - 1 * \frac{(x-1)(x-4)}{-2} + 2 * \frac{(x-1)(x-2)}{6}$$

=

$$\frac{x^2}{2} - \frac{3x}{2}$$

1.2 1b

p(x) =

$$a_0 + a_1 * x + a_2 * x^2$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 16 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

Solve the system, we obtain

$$a_0 = 0, a_1 = \frac{-3}{2}, a_2 = \frac{1}{2}$$

Thus, the polynomial has the form

$$\frac{x^2}{2} - \frac{3x}{2}$$

2 Question 2

2.1 2a

$p_3(\mathrm{x}) = \frac{x^2}{2} - \frac{3x}{2} + a_3(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)$
$= \frac{x^2}{2} - \frac{3x}{2} + a_3(6-1)(6-2)(6-4)$
$= \frac{x^2}{2} - \frac{3x}{2} + a_3 * 40$ where $a_3 = \frac{(y_3)(P_2(6)}{40} = \frac{(1)(9)}{40} = 9/40$
$p_3(\mathrm{x}) = \frac{x^2}{2} - \frac{3x}{2} + \frac{9}{40}(x-1)(x-2)(x-4)$
$= \frac{x^2}{2} - \frac{3x}{2} + \frac{9}{40}(x^3 - 7x^2 + 14x - 8)$
$= \frac{x^2}{2} - \frac{3x}{2} + \frac{9}{40}x^3 - \frac{63}{40}x^2 + \frac{63}{20}x - \frac{9}{5}$
$= \frac{9}{40}x^3 - \frac{43}{40}x^2 + \frac{33}{20}x - \frac{9}{5}$

2.2 2b

$l_3(\mathrm{x}) =$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x-1)(x-2)(x-4)}{(5)(4)(2)} = \frac{(x-1)(x-2)(x-4)}{40}$$

$= \frac{1}{40}(x^3 - 7x^2 + 14x - 8)$
$p_3(\mathrm{x}) = y_0 * l_0(x) + y_1 * l_1(x) + y_2 * l_2(x) + y_3 * l_3(x) = \frac{x^2}{2} - \frac{3x}{2} + \frac{1}{40}(x^3 - 7x^2 + 14x - 8)$
$= \frac{1}{40}x^3 + \frac{13}{40}x^2 - \frac{23}{20}x - \frac{1}{5}$

3 Question 3

$\mathrm{p(x)} =$

$$a_0 + a_1 * x + a_2 * x^2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

Solve the system, we obtain

$$a_0 = -1, a_1 = 3, a_2 = \frac{4\sqrt{3} - 9}{3}$$

Thus, the polynomial has the form -1 + 3*Sin(x) + $\frac{4\sqrt{3}-9}{3}$*Sin(2x)

4 Question 4

4.1 4a

$$x_0 = -1, x_1 = -0.5, x_2 = 0, x_3 = 0.5, x_4 = 1$$

$$y_0 = 0.04979, y_1 = 0.22313, y_2 = 1, y_3 = 4.48169, y_4 = 20.08554$$

l_0(x) =

$$\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4}$$

=

$$\frac{(x+0.5)(x)(x-0.5)(x-1)}{(-0.5)(-1)(-1.5)(-2)}$$

=

$$\frac{(x+0.5)(x)(x-0.5)(x-1)}{1.5}$$

At x = 0.8, l_0(x) = -0.0416

l_1(x) =

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4}$$

=

$$\frac{(x+1)(x)(x-0.5)(x-1)}{(0.5)(-0.5)(-1)(-1.5)}$$

=

$$\frac{(x+1)(x)(x-0.5)(x-1)}{-0.375}$$

At x = 0.8, l_1(x) = 0.2304

l_2(x) =

$$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4}$$

=

$$\frac{(x+1)(x+0.5)(x-0.5)(x-1)}{(1)(0.5)(-0.5)(-1)}$$

=

$$\frac{(x+1)(x+0.5)(x-0.5)(x-1)}{0.25}$$

At x = 0.8, l_2(x) = -0.5616

l_3(x) =

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4}$$

=

$$\frac{(x+1)(x+0.5)(x)(x-1)}{(1.5)(1)(0.5)(-0.5)}$$

=

$$\frac{(x+1)(x+0.5)(x)(x-1)}{-0.375}$$

At x = 0.8, l_3(x) = 0.9984

l_4(x) =

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3}$$

=

$$\frac{(x+1)(x+0.5)(x)(x-0.5)}{(2)(1.5)(1)(0.5)}$$

=

$$\frac{(x+1)(x+0.5)(x)(x-0.5)}{1.5}$$

At x = 0.8, l_4(x) = 0.3744

p_4(x) =

$$y_0 * l_0 + y_1 * l_1 + y_2 * l_2 + y_3 * l_3 + y_4 * l_4$$

=

$p_4(0.8)$ = 0.04979 *(-0.0416) + 0.22313 * 0.2304 + 1 * (-0.5616) + 4.48169 * 0.9984 + 20.08554 * 0.3744 = 11.48228

f(0.8) = $e^(3 * 0.8)$ = 11.02317

The error $e_4(0.8)$ = 11.02317 - 11.48228 = -0.45911

4.2 4b

$e_4 = \frac{1}{5!}$ (0.8 + 1)(0.8 +0.5)(0.8)(0.3)(0.2) = 0.000936