

Math 504

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April 2022

1 Question 1

The Newton's Method Iteration is given by $x^1 = x^0 - F(x^0)^{-1} \cdot g(x^0)$

$$g(x^0) = (2, 0)^T$$

$$F(x) = \begin{bmatrix} Fx1x1 & Fx1x2 \\ Fx2x1 & Fx2x2 \end{bmatrix} = \begin{bmatrix} -400(x2 - 3x1^2) - 2 & -400x1 \\ -400x1 & 200 \end{bmatrix}$$

$$\text{Thus, } F(x^0) = \begin{bmatrix} -2 & 0 \\ 0 & 200 \end{bmatrix}$$

$$x^1 = (0 \ 0)^T - \begin{bmatrix} -2 & 0 \\ 0 & 200 \end{bmatrix}^{-1} * (2 \ 0)^T = (0 \ 0)^T - \begin{bmatrix} -1/2 & 0 \\ 0 & 1/200 \end{bmatrix}^{-1} * (2 \ 0)^T$$

$$= (0 \ 0)^T - (-1 \ 0)^T = (1 \ 0)^T$$

$$\text{Now, } x^2 = x^1 - F(x^1)^{-1} \cdot g(x^1) = (1 \ 0)^T - \begin{bmatrix} 1198 & -400 \\ -400 & 200 \end{bmatrix}^{-1} * (400 \ -200)^T$$

$$= (1 \ 0)^T - \begin{bmatrix} 1/398 & -1/199 \\ 1/199 & 599/39800 \end{bmatrix}^{-1} * (400 \ -200)^T = (1 \ 0)^T - (0 \ -1)^T = (1 \ 1)^T$$

2 Question 2

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + 0.5 * f''(x_0)(x-x_0)^2$$

The formula is generalised $x_t = x_t - f(x_0)/f'(x_0)$

2.1 Question 2b

Put initial guess = 0.01

Iteration 1 $x_1 = 0.009999$

Iteration 2

$$x_2 = 0.009998$$

Iteration 3

$$x_3 = 0.00999$$

Iteration 4

$$x_4 = 0.009999$$

Put initial guess 0.001

$$x_0 = 0.001$$

Iteration 1

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x1 = 9.999E-4
Iteration 2
x2 = 9.999E-4
Iteration 3
x3 =9.999E-4
Iteration 4
x4 = 9.999E-4
This shows that if the starting point is not zero it is not converge to zero

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3 Question 3

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clear
clc
t=linspace(0,4*pi,500);
A=input('Enter A:');
w=input('Enter omega: ');
ph=input('Enter phase: ');
y=@(t)A*sin(w*t+ph);
y1=y(t);
m=max(y1);
dt=t(2)-t(1);
figure
for i=1:499
    p=(y(t(i)+dt)-y(t(i)))/dt;
    plot(t,y1,'b');
    hold on
    axis([0 4*pi -(m*A+4) m*A+4])
    y11=p*((t(i)-1)-t(i))+y(t(i));
    y12=p*((t(i)+1)-t(i))+y(t(i));
    plot([t(i)-1 t(i)+1],[y11 y12],'r-*')
    scatter(t(i),y(t(i)),'b','filled')
    figure_title = ['The derivative at t=', num2str(t(i)), ' is: ', num2str(p)];
    title(figure_title)
    legend('sin(t)','tangent line','point on sin(t)')
    hold off
end

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