1 Introduction

$$\mathbf{y}_c \sim \mathrm{N}(\mathbf{m}_c, \mathbf{C}_c)$$

- $(\mathbf{x_s}, \mathbf{t_{1s}}, \mathbf{t_{2s}})$ is the design matrix for the simulation runs
- $\mathbf{y_s}$ is the simulator output
- $\mathbf{y_r}$ is the real observations at $\mathbf{x_r}, \mathbf{t_{2r}}$
- $\mathbf{y}_c = (\mathbf{y_s}, \mathbf{y_r})^T$
- $m_0()$ and $C_0()$ are GP prior mean and covariance of the emulator
- $m_1()$ and $C_1()$ are GP prior mean and covariance of the observation discrepancy

$$\mathbf{m}_c = \begin{pmatrix} m_0(\mathbf{x_s}, \mathbf{t_{1s}}, \mathbf{t_{2s}}) \\ m_0(\mathbf{x_r}, \mathbf{1}\theta_1, \mathbf{t_{2r}}) + m_1(\mathbf{x_1}, \mathbf{t_{2r}}) \end{pmatrix},$$

$$\mathbf{C}_c = \begin{pmatrix} \mathbf{C_{11}} & \mathbf{C_{12}} \\ \mathbf{C_{21}} & \mathbf{C_{22}} \end{pmatrix},$$

$$\begin{aligned}
\mathbf{C_{11}} &= C_0\left((\mathbf{x_s}, \mathbf{t_{1s}}, \mathbf{t_{2s}}), (\mathbf{x_s}, \mathbf{t_{1s}}, \mathbf{t_{2s}})\right) \\
\mathbf{C_{21}} &= C_0\left((\mathbf{x_s}, \mathbf{t_{1s}}, \mathbf{t_{2s}}), (\mathbf{x_r}, \mathbf{1}\theta_1, \mathbf{t_{2r}})\right) \\
\mathbf{C_{12}} &= \mathbf{C_{21}}^T \\
\mathbf{C_{22}} &= C_0\left((\mathbf{x_r}, \mathbf{1}\theta_1, \mathbf{t_{2r}}), (\mathbf{x_r}, \mathbf{1}\theta_1, \mathbf{t_{2r}})\right) + \\
&\quad C_1\left((\mathbf{x_r}, \mathbf{t_{2r}}), (\mathbf{x_r}, \mathbf{t_{2r}})\right) + \sigma_2 \mathbf{I}
\end{aligned}$$

$$\mathbf{y}_e \sim \mathrm{N}(\mathbf{m}_e, \mathbf{C}_e)$$

- $(\mathbf{x_s}, \mathbf{t_{1s}}, \mathbf{t_{2s}})$ is the design matrix for the simulation runs
- $\mathbf{y_s}$ is the simulator output
- $\mathbf{y_d}$ is the target outcomes at control settings $\mathbf{x_d}$.
- $\mathbf{y}_e = (\mathbf{y_s}, \mathbf{y_d})^T$
- $m_0()$ and $C_0()$ are GP prior mean and covariance of the emulator
- $m_1^{t_1}()$ and $C_1^{t_1}()$ are posterior GP mean and covariance of the observation discrepancy, using current calibration draw t_1
- $m_2()$ and $C_2()$ are GP prior mean and covariance of the target discrepancy

$$\mathbf{m}_e = \begin{pmatrix} m_0(\mathbf{x_s}, \mathbf{t_{1s}}, \mathbf{t_{2s}}) \\ m_0(\mathbf{x_d}, \mathbf{1}\theta_1, \mathbf{1}\theta_2) + m_1^{t_1}(\mathbf{x_d}, \mathbf{1}\theta_2) + m_2(\mathbf{x_d}) \end{pmatrix},$$

$$\mathbf{C}_e = \begin{pmatrix} \mathbf{C_{11}} & \mathbf{C_{12}} \\ \mathbf{C_{21}} & \mathbf{C_{22}} \end{pmatrix},$$

$$\begin{aligned}
\mathbf{C_{11}} &= C_0\left((\mathbf{x_s}, \mathbf{t_{1s}}, \mathbf{t_{2s}}), (\mathbf{x_s}, \mathbf{t_{1s}}, \mathbf{t_{2s}})\right) \\
\mathbf{C_{21}} &= C_0\left((\mathbf{x_s}, \mathbf{t_{1s}}, \mathbf{t_{2s}}), (\mathbf{x_d}, \mathbf{1}\theta_1, \mathbf{1}\theta_2)\right) \\
\mathbf{C_{12}} &= \mathbf{C_{21}}^T \\
\mathbf{C_{22}} &= C_0\left((\mathbf{x_d}, \mathbf{1}\theta_1, \mathbf{1}\theta_2), (\mathbf{x_d}, \mathbf{1}\theta_1, \mathbf{1}\theta_2)\right) + \\
&\quad C_1^{t_1}\left((\mathbf{x_d}, \mathbf{1}\theta_2), (\mathbf{x_d}, \mathbf{1}\theta_2)\right) + C_2\left(\mathbf{x}_d, \mathbf{x}_d\right)
\end{aligned}$$