

A BAYESIAN APPROACH TO COMPUTER MODEL CALIBRATION AND
MODEL-ASSISTED DESIGN

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ABSTRACT

Computer models of phenomena that are difficult or impossible to study directly are critical for enabling research and assisting design in many areas. In order to be effective, computer models must be calibrated so that they accurately represent the modeled phenomena. There exists a rich variety of methods for computer model calibration that have been developed in recent decades. Among the desiderata of such methods is a means of quantifying remaining uncertainty after calibration regarding both the values of the calibrated model inputs and the model outputs. Bayesian approaches to calibration have met this need in recent decades. However, limitations remain. Whereas in model calibration one finds point estimates or distributions of *calibration inputs* in order to induce the model to reflect reality accurately, interest in a computer model often centers primarily on its use for model-assisted design, in which the goal is to find values for *design inputs* to induce the modeled system to approximate some target outcome. Existing Bayesian approaches are limited to the first of these two tasks. The present work develops an approach adapting Bayesian methods for model calibration for application in model-assisted design. The approach retains the benefits of Bayesian calibration in accounting for and quantifying all sources of uncertainty. It is capable of generating a comprehensive assessment of the Pareto optimal inputs for a multi-objective optimization problem. The present work shows that this approach can apply as a method for model-assisted design using a previously calibrated system, and can also serve as a method for model-assisted design using a model that still requires calibration, accomplishing both ends simultaneously.

DEDICATION

For my father, who made me see that this was possible, and for my wife, without whom it would not have been.

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