Economics 4

Cheat sheet · Spring 2021

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ITAM

1 General Equilibrium

1.1 Pure Exchange Economies

Initial Assumptions

- There are m consumers such that $\mathcal{I} = \{1, \ldots, m\}$.
- There are n goods such that $\mathcal{L} = \{1, \ldots, n\}$.
- The preferences of each consumer are given by a utility function $u_i : \mathbb{R}^{\mathcal{L}} \to \mathbb{R}$.
- Each consumer can consume goods in $x_i \in \mathbb{R}^{\mathcal{L}}_{+}$.
- Each consumer has an initial endowment of $\omega_i \in \mathbb{R}^{\mathcal{L}}_{+}$.
- The ordered pair (u_i, ω_i) describes each consumer.
- The utility functions represent neoclassical preferences.

Proposition 1.1

If $x \succ_i y$, then $u_i(x) > u_i(y)$.

Definition 1.1 (Exchange Economy). *A **pure exchange economy** is:*

$$\mathcal{E} = \langle \mathcal{I}, (u_i, \omega_i)_{i \in \mathcal{I}} \rangle ,$$

where $\mathcal{I}$ is the set of agents; u_i and ω_i are the utility function and initial endowment of the i-th consumer, respectively.

Definition 1.2 (Total Endowment).

$$\Omega = \sum_{i \in \mathcal{I}} \omega_i .$$

Definition 1.3 (Resource Allocation). *The **resource allocation** is denoted by $X = (x_1, x_2, \ldots, x_m)$, where $x_i \in \mathbb{R}^{\mathcal{L}}_{+}$.*

Definition 1.4 (Feasible Allocation). *The **feasible allocation** $\mathcal{F}$ of an economy $\mathcal{E}$ is defined by:*

$$\mathcal{F} = \left\{ X = (x_1, x_2, \ldots, x_m) \ : \ x_i \in \mathbb{R}^{\mathcal{L}}_{+}, \ \sum_{i \in \mathcal{I}} x_i = \sum_{i \in \mathcal{I}} w_i \right\}.$$

Definition 1.5 (Pareto-Efficiency). *Let $\mathcal{E}$ be an exchange economy. A* feasible allocation of resources $X = (x_1, x_2, \ldots, x_m)$ *is said to be **Pareto-efficient** if and only if there is no other feasible allocation $\hat{X} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_m)$ such that, for every agent in $\mathcal{I}$, $u_i(\hat{x}_i) \geq u_i(x_i)$ and, for at least one agent j, $u_j(\hat{x}_j) > u_j(x_j)$.*

Definition 1.6 (Pareto-Dominance). *Let X and $\hat{X}$ be two feasible allocations. We say that $\hat{X}$ **Pareto-dominates** X if and only if:*

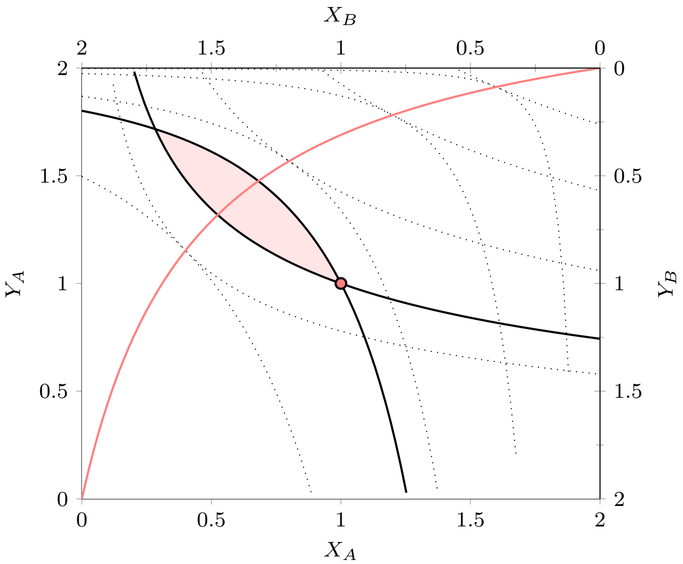
$$u_i(\hat{x}_{i,1}, \ldots, \hat{x}_{i,n}) \geq u_i(x_{i,1}, \ldots, x_{i,n}), \quad \forall i \in \mathcal{I},$$

and there is at least one consumer j such that:

$$u_j(\hat{x}_{j,1}, \ldots, \hat{x}_{j,n}) > u_j(x_{j,1}, \ldots, x_{j,n}).$$

Definition 1.7 (Contract Curve). *The set of all* Pareto-allocations *is known as the **contract curve**.*

Edgeworth Box



General Case

$$\begin{aligned}
\max_{\{(x_{1,1},\ldots,x_{1,n}),\ldots,(x_{m,1},\ldots,x_{m,n})\}} \quad & u_1(x_{1,1}, \ldots, x_{1,n}) \\
\text{subject to} \quad & u_2(x_{2,1}, \ldots, x_{2,n}) \geq \bar{u}_2 \\
& \vdots \\
& u_m(x_{m,1}, \ldots, x_{m,n}) \geq \bar{u}_m \\
& \sum_{i \in \mathcal{I}} x_{i,1} \leq \omega_1 \\
& \vdots \\
& \sum_{i \in \mathcal{I}} x_{i,n} \leq \omega_n
\end{aligned}$$

Theorem 1.1

Let all utility functions be strictly increasing and quasi-concave, and $((\hat{x}_{1,1}, \ldots, \hat{x}_{1,n}), \ldots, (\hat{x}_{m,1}, \ldots, \hat{x}_{m,n}))$ be a feasible interior allocation. Then $((\hat{x}_{1,1}, \ldots, \hat{x}_{1,n}), \ldots, (\hat{x}_{m,1}, \ldots, \hat{x}_{m,n}))$ is **Pareto-efficient** if and only if $((\hat{x}_{1,1}, \ldots, \hat{x}_{1,n}), \ldots, (\hat{x}_{m,1}, \ldots, \hat{x}_{m,n}))$ exhausts all resources and, for all pairs of goods ℓ, ℓ':

$$\mathrm{MRS}(\ell, \ell')(\hat{x}_{1,1}, \ldots, \hat{x}_{1,n}) = \cdots = \mathrm{MRS}(\ell, \ell')(\hat{x}_{m,1}, \ldots, \hat{x}_{m,n}).$$

1.2 Competitive Equilibrium

Initial Assumptions

- There is a market for each good.
- Every agent can access the market without any cost.
- There is a unique price for each good and all consumers know the price.
- Each consumer can sell her initial endowment in the market and use the income to buy goods and services.
- Consumers seek to maximize their utility given their budget restriction, independently of what everyone else is doing.
- There is no centralized mechanism.
- People may not know others preferences or endowments.
- There is perfect competition (i.e. everyone is a price-taker).
- The only source of information agents are prices.

Definition 1.8 (Competitive Equilibrium). *An ordered pair of an allocation and a price vector, $(x^*, p = (p_1, \ldots, p_n))$, is called a **competitive equilibrium** if the following conditions hold:*

(i) $\forall i \in \mathcal{I}$, $x_i^* = (x_{i,1}^*, \ldots, x_{i,n}^*)$ *solves the following maximization problem:*

$$\max_{\{x_i\}} \ u_i(x_i)$$
$$\text{subject to} \quad wh + p \cdot x_i \leq p \cdot \omega_i = \sum_{\ell \in \mathcal{L}} p_\ell \omega_{i,\ell}.$$

(ii) *Markets clear, i.e.* $\sum_{i \in \mathcal{I}} x_{i,\ell}^* = \sum_{i \in \mathcal{I}} \omega_{i,\ell}, \ \forall \ell \in \mathcal{L}.$

Proposition 1.2

Given, at least, one consumer with strictly monotone preferences. Then, if (x^*, p) is a competitive equilibrium, $p_1, p_2, \ldots, p_n > 0$.

Proposition 1.3

Given, at least, one consumer with weakly monotone preferences. Then, if (x^*, p) is a competitive equilibrium, for at least one ℓ, $p_\ell > 0$.

Proposition 1.4

Let (x^*, p) be a competitive equilibrium. Then, (x^*, cp) is also a competitive equilibrium, $\forall c \in \mathbb{R}_+$.

Theorem 1.2: Walras' Law

If the consumer i has weakly monotone preferences and also $\hat{x}_i \in x_i^*(p)$, then:

$$p \cdot \hat{x}_i = \sum_{\ell \in \mathcal{L}} p_\ell \hat{x}_{i,\ell} = \sum_{\ell \in \mathcal{L}} p_\ell \omega_{i,\ell} = p \cdot \omega_i.$$

Corollary 1.1: Walras' Law

Given weakly monotonic utility functions and $p = (p_1, \dots, p_n)$ such that $p_n > 0$. If any (x^*, p) in which maximization condition holds, $\forall i \in \mathcal{I}$, and markets clear $\forall \ell = 1, 2, \dots, n-1$; then, the market clearing condition holds for commodity n as well.

Theorem 1.3: Fixed Point

For any continuous function $f : \triangle^{n-1} \to \triangle^{n-1}$, there exists a point $p^* = (p_1^*, p_2^*, \dots, p_n^*)$ such that $f(p^*) = p^*$, where $\triangle^{n-1} = \left\{ (p_1, p_2, \dots, p_n) \in \mathbb{R}_+^{\mathcal{L}} : \sum_{\ell \in \mathcal{L}} p_\ell = 1 \right\}$.

Definition 1.9 (Shortage)**.** *We define* ***shortage*** *or* ***excess demand*** *as follows:*

$$Z(p) = (z_1(p), z_2(p), \dots, z_n(p)) = \sum_{i \in \mathcal{I}} x_i^*(p) - \sum_{i \in \mathcal{I}} \omega_i.$$

Proposition 1.5

p is a competitive equilibrium if and only if $Z(p) = 0$.

Excess Demand Properties

(i) Continuous in p.
(ii) Homogeneous of degree zero.
(iii) $p \cdot Z(p) = 0$.

Proposition 1.6

The equilibrium is not unique.

Theorem 1.4: Welfare - I

Given any pure exchange economy such that all consumers have weakly monotonic utility functions. Then, if (x^*, p) is a competitive equilibrium, x^* is a Pareto-efficient allocation.

Theorem 1.5: Welfare - II

Given an economy $\mathcal{E} = \langle \mathcal{I}, (u_i, w_i)_{i \in \mathcal{I}} \rangle$ where all consumers have weakly monotonic, quasi-concave utility functions. If $(x_1, x_2, \dots, x_m)$ is a Pareto-optimal allocation; then, there exists a redistribution of resources $(\hat{w}_1, \hat{w}_2, \dots, \hat{w}_m)$ and some prices $p = (p_1, p_2, \dots, p_n)$ such that:

(i) $\sum_{i \in \mathcal{I}} \hat{w}_i = \sum_{i \in \mathcal{I}} w_i$.
(ii) $(p, (x_1, x_2, \dots, x_m))$ is a competitive equilibrium of the economy $\mathcal{E}$.

2 Monopoly and Monopsony

3 Game Theory