Economics 4

Cheat sheet · Spring 2021

Carlos Lezama ECO - 21104

ITAM

1 General Equilibrium

1.1 Pure Exchange Economies

Initial Assumptions

· There are m consumers such that $\mathcal{I} = \{1, \ldots, m\}$.

· There are n goods such that $\mathcal{L} = \{1, \ldots, n\}$.

· The preferences of each consumer are given by a utility function $u_i : \mathbb{R}^{\mathcal{L}} \to \mathbb{R}$.

· Each consumer can consume goods in $x_i \in \mathbb{R}_+^{\mathcal{L}}$.

· Each consumer has an initial endowment of $w_i \in \mathbb{R}_+^{\mathcal{L}}$.

· The ordered pair (u_i, w_i) describes each consumer.

· The utility functions represent neoclassical preferences.

Proposition 1.1

If $x \succ_i y$, then $u_i(x) > u_i(y)$.

Definition 1.1 (Exchange Economy). *A **pure exchange economy** is:*

$$\mathcal{E} = \langle \mathcal{I}, (u_i, w_i)_{i \in \mathcal{I}} \rangle,$$

where $\mathcal{I}$ is the set of agents; u_i and w_i are the utility function and initial endowment of the i-th consumer, respectively.

Definition 1.2 (Total Endowment).

$$\Omega = \sum_{i \in \mathcal{I}} w_i.$$

Definition 1.3 (Resource Allocation). *The **resource allocation** is denoted by $X = (x_1, x_2, \ldots, x_m)$, where $x_i \in \mathbb{R}_+^{\mathcal{L}}$.*

Definition 1.4 (Feasible Allocation). *The **feasible allocation** $\mathcal{F}$ of an economy $\mathcal{E}$ is defined by:*

$$\mathcal{F} = \left\{ X = (x_1, x_2, \ldots, x_m) \ : \ x_i \in \mathbb{R}_+^{\mathcal{L}}, \sum_{i \in \mathcal{I}} x_i = \sum_{i \in \mathcal{I}} w_i \right\}.$$

Definition 1.5 (Pareto-Efficiency). *Let $\mathcal{E}$ be an exchange economy. A feasible allocation of resources $X = (x_1, x_2, \ldots, x_m)$ is said to be **Pareto-efficient** if and only if there is no other feasible allocation $\hat{X} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_m)$ such that, for every agent in $\mathcal{I}$, $u_i(\hat{x}_i) \geq u_i(x_i)$ and, for at least one agent j, $u_j(\hat{x}_j) > u_j(x_j)$.*

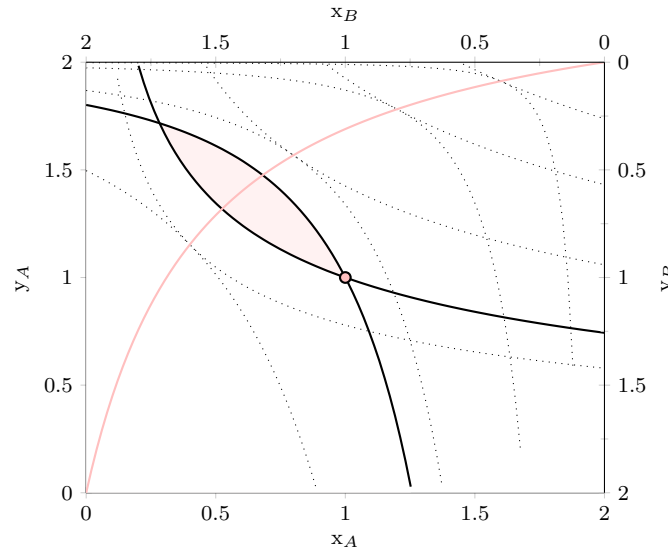**Definition 1.6** (Pareto-Dominance). *Let X and $\hat{X}$ be two feasible allocations. We say that $\hat{X}$ **Pareto-dominates** X if and only if:*

$$u_i(\hat{x}_{i,1}, \ldots, \hat{x}_{i,n}) \geq u_i(x_{i,1}, \ldots, x_{i,n}), \quad \forall i \in \mathcal{I},$$

and there is at least one consumer j such that:

$$u_j(\hat{x}_{j,1}, \ldots, \hat{x}_{j,n}) > u_j(x_{j,1}, \ldots, x_{j,n}).$$

Definition 1.7 (Contract Curve). *The set of all Pareto-allocations is known as the **contract curve**.*

Edgeworth Box

General Case

$$\max_{\{(x_{1,1}, \ldots, x_{1,n}), \ldots, (x_{m,1}, \ldots, x_{m,n})\}} u_1(x_{1,1}, \ldots, x_{1,n})$$

$$\text{subject to} \quad u_2(x_{2,1}, \ldots, x_{2,n}) \geq \bar{u}_2,$$

$$\vdots$$

$$u_m(x_{m,1}, \ldots, x_{m,n}) \geq \bar{u}_m;$$

$$\sum_{i \in \mathcal{I}} x_{i,1} \leq w_1,$$

$$\vdots$$

$$\sum_{i \in \mathcal{I}} x_{i,n} \leq w_n.$$

Theorem 1.1

Let all utility functions be strictly increasing and quasi-concave, and $((\hat{x}_{1,1}, \ldots, \hat{x}_{1,n}), \ldots, (\hat{x}_{m,1}, \ldots, \hat{x}_{m,n}))$ be a feasible interior allocation. Then $((\hat{x}_{1,1}, \ldots, \hat{x}_{1,n}), \ldots, (\hat{x}_{m,1}, \ldots, \hat{x}_{m,n}))$ is **Pareto-efficient** if and only if $((\hat{x}_{1,1}, \ldots, \hat{x}_{1,n}), \ldots, (\hat{x}_{m,1}, \ldots, \hat{x}_{m,n}))$ exhausts all resources and, for all pairs of goods (ℓ, ℓ'):

$$\text{MRS}(\ell, \ell')(\hat{x}_{1,1}, \ldots, \hat{x}_{1,n}) = \cdots = \text{MRS}(\ell, \ell')(\hat{x}_{m,1}, \ldots, \hat{x}_{m,n}).$$

1.2 Competitive Equilibrium

Initial Assumptions

· There is a market for each good.

· Every agent can access the market without any cost.

· There is a single price for each good.

· All consumers know the price.

· Each consumer can sell her initial endowment in the market and use the income to buy goods and services.

· Consumers seek to maximize their utility given their budget restriction, independently of what everyone else is doing.

· There is no centralized mechanism.

· People may not know others' preferences or endowments.

· There is perfect competition (namely, everyone is a price-taker).

· Prices are the only source of information of agents.

Definition 1.8 (Competitive Equilibrium). *An ordered pair of an allocation and a price vector, $(X^*, p = (p_1, \ldots, p_n))$, is called a **competitive equilibrium** if the following conditions hold:*

(i) $\forall i \in \mathcal{I}$, $x_i^* = (x_{i,1}^*, \ldots, x_{i,n}^*)$ *solves the following maximization problem:*

$$\max_{\{x_i\}} \quad u_i(x_i)$$

$$\text{subject to} \quad wh + \langle p, x_i \rangle \leq \langle p, w_i \rangle = \sum_{\ell \in \mathcal{L}} p_\ell w_{i,\ell}.$$

(ii) *Markets clear, i.e.* $\sum_{i \in \mathcal{I}} x_{i,\ell}^* = \sum_{i \in \mathcal{I}} w_{i,\ell}, \; \forall \ell \in \mathcal{L}.$

Proposition 1.2

Given, at least, one consumer with strictly monotone preferences. Then, if (X^*, p) is a competitive equilibrium, $p_1, p_2, \ldots, p_n > 0$.

Proposition 1.3

Given, at least, one consumer with weakly monotone preferences. Then, if (X^*, p) is a competitive equilibrium, for at least one ℓ, $p_\ell > 0$.

Proposition 1.4

Let (X^*, p) be a competitive equilibrium. Then, (X^*, cp) is also a competitive equilibrium, $\forall c \in \mathbb{R}_+$.

Theorem 1.2: Walras' Law

If the consumer i has weakly monotone preferences and also $\hat{x}_i \in x_i^*(p)$, then:

$$\langle p, \hat{x}_i \rangle = \sum_{\ell \in \mathcal{L}} p_\ell \hat{x}_{i,\ell} = \sum_{\ell \in \mathcal{L}} p_\ell w_{i,\ell} = \langle p, w_i \rangle.$$

Corollary 1.1: Walras' Law

Given weakly monotonic utility functions and $p = (p_1, \ldots, p_n)$ such that $p_\ell > 0$. If any (X^*, p) in which maximization condition holds, $\forall i \in \mathcal{I}$, and markets clear $\forall \ell = 1, 2, \ldots, n-1$; then, the market clearing condition holds for commodity n as well.

Theorem 1.3: Fixed Point

For any continuous function $f : \triangle^{n-1} \to \triangle^{n-1}$, there exists a point $p^* = (p_1^*, p_2^*, \ldots, p_n^*)$ such that $f(p^*) = p^*$, where:

$$\triangle^{n-1} = \left\{ (p_1, p_2, \ldots, p_n) \in \mathbb{R}_+^{\mathcal{L}} : \sum_{\ell \in \mathcal{L}} p_\ell = 1 \right\}.$$

Definition 1.9 (Shortage). *We define **shortage** or **excess demand** as follows:*

$$Z(p) = (z_1(p), z_2(p), \ldots, z_n(p)) = \sum_{i \in \mathcal{I}} x_i^*(p) - \sum_{i \in \mathcal{I}} w_i.$$

Proposition 1.5

p is a competitive equilibrium if and only if $Z(p) = 0$.

Excess Demand Properties

(i) Continuous in p.

(ii) Homogeneous of degree zero.

(iii) $\langle p, Z(p) \rangle = 0$.

Proposition 1.6

The equilibrium is not unique.

Theorem 1.4: Welfare I

Given any pure exchange economy such that all consumers have weakly monotonic utility functions. If (X^*, p) is a competitive equilibrium, then X^* is a Pareto-efficient allocation.

Theorem 1.5: Welfare II

Given an economy $\mathcal{E} = \langle \mathcal{I}, (u_i, w_i)_{i \in \mathcal{I}} \rangle$ where all consumers have weakly monotonic quasi-concave utility functions.
If $(x_1, x_2, \ldots, x_m)$ is a Pareto-optimal allocation; then, there exists a redistribution of resources $(\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_m)$ and some prices $p = (p_1, p_2, \ldots, p_n)$ such that:

(i) $\sum_{i \in \mathcal{I}} \hat{w}_i = \sum_{i \in \mathcal{I}} w_i.$

(ii) $(p, (x_1, x_2, \ldots, x_m))$ is a competitive equilibrium of the economy $\mathcal{E}$.

1.3 Production

Initial Assumptions

· There are k firms such that $\mathcal{J} = \{1, \ldots, k\}$

· The production of firm j for good l is described by the function $f_{j,l}$ such that $f_j(z_{\ell,l}) = f_j(z_{1,l}, \ldots, z_{n,l})$. Namely, the firm j uses z_ℓ units of commodities $\ell \in \mathcal{L}$ to produce commodity l.

· The firms are owned by consumers in society.

· The firms' ownership is exogenous.

· θ_{ij} represents the *fraction* of the firm j owned by the consumer i.

· The firms do not have endowments.

Remark 1.1

$$\sum_{i \in \mathcal{I}} \theta_{ij} = \theta_{1j} + \theta_{2j} + \cdots + \theta_{mj} = 1.$$

Definition 1.10 (Competitive equilibrium). $((X^*, Z^*), \mathrm{p})$ *is called a* **competitive equilibrium** *if the following conditions hold:*

(i) $\forall j \in \mathcal{J}$, $z_j^* = ((z_{j,1,1'}, \ldots, z_{j,1,n'}), \ldots, (z_{j,n,1'}, \ldots, z_{j,n,n'}))$ *solves the following maximization problem:*

$$\pi_j^* \stackrel{\text{def}}{=} \max_{\{z_j\}} \quad p_l f_{j,l}(z_\ell) - \sum_{\ell \in \mathcal{L}} p_\ell z_{j,\ell}.$$

(ii) $\forall i \in \mathcal{I}$, $x_i^* = (x_{i,1}^*, \ldots, x_{i,n}^*)$ *solves the following maximization problem:*

$$\max_{\{x_i\}} \quad u_i(x_i)$$

$$\text{subject to} \quad \langle p, x_i \rangle \leq \langle p, w_i \rangle + \sum_{j \in \mathcal{J}} \theta_{ij} \pi_j^*.$$

(iii) *Markets clear, namely:*

$$\sum_{i \in \mathcal{I}} x_{i,\ell}^* + \sum_{j \in \mathcal{J}} \sum_{\ell \in \mathcal{L}} z_{j,\ell}^* = \sum_{i \in \mathcal{I}} w_{i,\ell} + \sum_{j \in \mathcal{J}} f_{j,l}(z_\ell^*).$$

Proposition 1.7

Walras' law and welfare theorems hold.

Proposition 1.8

Let $((X^*, Z^*), \mathrm{p})$ be a competitive equilibrium.
Then, $((X^*, Z^*), c\mathrm{p})$ is also a competitive equilibrium, $\forall c \in \mathbb{R}_+$.

Note: with production, Edgeworth box illustrations are no longer helpful.

General Case

$$\max_{\{X,Z\}} \quad u_1(x_1)$$

$$\text{subject to} \quad u_2(x_2) \geq \bar{u}_2,$$

$$\vdots$$

$$u_m(x_m) \geq \bar{u}_m;$$

$$\sum_{i \in \mathcal{I}} x_{i,1} + \sum_{j \in \mathcal{J}} z_{j,1} \leq \sum_{j \in \mathcal{J}} f_{j,1}(z_\ell) + \sum_{i \in \mathcal{I}} w_{i,1},$$

$$\vdots$$

$$\sum_{i \in \mathcal{I}} x_{i,n} + \sum_{j \in \mathcal{J}} z_{j,n} \leq \sum_{j \in \mathcal{J}} f_{j,n}(z_\ell) + \sum_{i \in \mathcal{I}} w_{i,n}.$$

Theorem 1.6

Let all utility functions be strictly increasing and quasi-concave, and $(\hat{X}, \hat{Z})$ be a feasible interior allocation. Then, $(\hat{X}, \hat{Z})$ is **Pareto-efficient** if and only if all of the following equalities hold for all pairs of goods (ℓ, ℓ'):

(i) Marginal rates of substitution are equal across consumers for any pair of commodities,

$$\text{i.e.} \quad \text{MRS}(\ell, \ell')(\hat{x}_1) = \cdots = \text{MRS}(\ell, \ell')(\hat{x}_m).$$

(ii) Marginal rates of technical substitution are equal across firms for any pair of commodities,

$$\text{i.e.} \quad \text{MRTS}(\ell, \ell')(\hat{z}_1) = \cdots = \text{MRTS}(\ell, \ell')(\hat{z}_k).$$

(iii) Marginal rates of transformation are equal to the marginal rates of substitution,

$$\text{i.e.} \quad \text{MRT}(j, j')(\hat{z}_{\ell''}) = \text{MRS}(\ell, \ell')(\hat{x}_i), \quad \forall i \in \mathcal{I}.$$

Definition 1.11 (Production possibility set). *We define the* **production possibility set** *as the set of all non-negative outputs of goods that the firms can produce using the economy's available factor inputs. The output combinations on the* frontier *of this set correspond to the* Pareto-efficient allocation *of factor inputs, i.e. the allocation in which it is not possible, given the total factor endowment, to increase the production of one good without decreasing the production of some other good.*

Remark 1.2: A friendly reminder on marginal rates

Marginal rate of substitution (MRS)

· It is the rate at which a consumer is willing to trade one good for another to maintain a constant level of utility.

· It is the slope of an *indifference curve*.

· It focuses on demand side of consumer theory.

Marginal rate of transformation (MRT)

· It is the amount of one good that must be given up to produce an additional unit of another good.

· It is the slope of *production possibility frontier*.

· It focuses on supply side of a commodity.

Marginal rate of technical substitution (MRTS)

· It is the amount by which the quantity of one input has to be reduced in order to use another input.

· It is the slope of an *isoquant curve*.

· It focuses on production side of economic theory.

2 Monopoly and Monopsony

Definition 2.1 (Monopoly). *A **monopoly** is a single supplier of a market. This firm may choose to produce at any point on the market demand curve. For such reason, there is* no monopoly supply curve.

Remark 2.1: Price-taking

We say that a firm has a *price-taking* behavior if it cannot dictate prices in the market.

Firms' problem

$$\max_{\{x\}} \quad p(x)x - c(x),$$

or

$$\max_{\{p\}} \quad pq(p) - c(q(p));$$

where $p(x)$ is the **demand** function and $q(p)$ is the **inverse demand** function.

Definition 2.2 (Revenue). *We define the **revenue** of the firm as follows:*

$$R(q) = p(q)q.$$

Proposition 2.1

$$\frac{dR}{dq} = p(q) + q\frac{dp}{dq}(q) = p(q)\left(1 + \frac{1}{\varepsilon_{q,p}}\right).$$

Note: $\varepsilon_{q,p}$ is the elasticity of demand with respect to price.

Proposition 2.2

We say that the demand is *inelastic* if $\varepsilon_{q,p} \in (-1, 0)$, thus:

· An increase in price leads to a small decrease in demand.

· An increase in quantity leads to a big decrease in price.

Likewise, we say that the demand is *elastic* if $\varepsilon_{q,p} < -1$, thus:

· An increase in price leads to a big decrease in demand.

· An increase in quantity leads to a small decrease in price.

Proposition 2.3

Whenever the demand function has constant elasticity κ, namely: $\varepsilon_{q,p} = \kappa < 0$. Then:

$$q(p) = Ap^\kappa \iff p(q) = \left(\frac{q}{A}\right)^{1/\kappa}, \quad \forall A = e^C > 0.$$

Remark 2.2

A monopoly firm always produces at a point such that demand is *elastic*. Which lead us to:

$$p = \frac{1}{1 + (1/\varepsilon_{q,p})}\frac{dc}{dq} > \frac{dc}{dq}.$$

Remark 2.3

The quantity produced q is below the quantity where $p = \dfrac{dc}{dq}$.

Proposition 2.4

Whenever the demand function has constant elasticity $\kappa \geq -1$, the firm will always prefer to increase the price and there is no solution.

2.1 Price discrimination

Definition 2.3 (Price discrimination). *A monopoly engages in **price discrimination** if it is able to sell otherwise identical units of output at different prices.*

Definition 2.4 (First-degree price discrimination). ***First-degree price discrimination** occurs when each buyer can be separately identified by a monopolist, then it may be possible to charge each maximum price they would willingly pay for the good. Namely, this strategy would then extract all available consumer surplus, leaving demanders as a group indefferent between buying the monopolist's good or doing without it. This is also called **perfect price discrimination** or **one-to-one marketing**.*

Proposition 2.5

Given a monopoly with a **perfect price discrimination** strategy, all the followin conditions hold:

· Consumer surplus equals zero.

· The firm will price at $p(q)$ for the q-th unit and continue to produce until $p(q^*) = c'(q^*)$.

· The firm's profit is:

$$\pi = \int_0^{q^*} p(q)dq - c(q^*).$$

· There is no deadweight loss.

3 Game Theory