

# **Economics 4**

Cheat sheet · Spring 2021

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# General Equilibrium

## 1.1 Pure Exchange Economies

### Initial Assumptions

- There are  $m$  consumers such that  $\mathcal{I} = \{1, \dots, m\}$ .
- There are  $n$  goods such that  $\mathcal{L} = \{1, \dots, n\}$ .
- The preferences of each consumer are given by a utility function  $u_i : \mathbb{R}^{\mathcal{L}} \rightarrow \mathbb{R}$ .
- Each consumer can consume goods in  $x_i \in \mathbb{R}_+^{\mathcal{L}}$ .
- Each consumer has an initial endowment of  $\omega_i \in \mathbb{R}_+^{\mathcal{L}}$ .
- The ordered pair  $(u_i, \omega_i)$  describes each consumer.
- The utility functions represent neoclassical preferences.

### Claim 1.1

If  $x \succ_i y$ , then  $u_i(x) > u_i(y)$ .

**Definition 1.1** (Exchange Economy). A *pure exchange economy* is:

$$\mathcal{E} = \langle \mathcal{I}, (u_i, \omega_i)_{i \in \mathcal{I}} \rangle,$$

where  $\mathcal{I}$  is the set of agents;  $u_i$  and  $\omega_i$  are the utility function and initial endowment of the  $i$ -th consumer, respectively.

**Definition 1.2** (Total Endowment).

$$\Omega = \sum_{i \in \mathcal{I}} \omega_i.$$

**Definition 1.3** (Resource Allocation). The *resource allocation* is denoted by  $X = (x_1, x_2, \dots, x_m)$ , where  $x_i \in \mathbb{R}_+^{\mathcal{L}}$ .

**Definition 1.4** (Feasible Allocation). The *feasible allocation*  $\mathcal{F}$  of an economy  $\mathcal{E}$  is defined by:

$$\mathcal{F} = \left\{ X = (x_1, x_2, \dots, x_m) : x_i \in \mathbb{R}_+^{\mathcal{L}}, \sum_{i \in \mathcal{I}} x_i = \sum_{i \in \mathcal{I}} \omega_i \right\}.$$

**Definition 1.5** (Pareto-Efficiency). Let  $\mathcal{E}$  be an exchange economy. A feasible allocation of resources  $X = (x_1, x_2, \dots, x_m)$  is said to be **Pareto-efficient** if and only if there is no other feasible allocation  $\hat{X} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_m)$  such that, for every agent in  $\mathcal{I}$ ,  $u_i(\hat{x}_i) \geq u_i(x_i)$  and, for at least one agent  $j$ ,  $u_j(\hat{x}_j) > u_j(x_j)$ .

**Definition 1.6** (Pareto-Dominance). Let  $X$  and  $\hat{X}$  be two feasible allocations. We say that  $\hat{X}$  **Pareto-dominates**  $X$  if and only if:

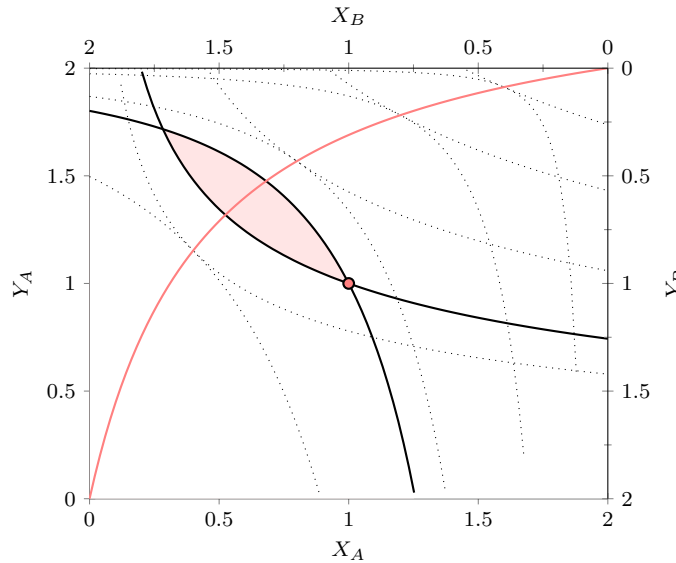
$$u_i(\hat{x}_{i,1}, \dots, \hat{x}_{i,n}) \geq u_i(x_{i,1}, \dots, x_{i,n}), \quad \forall i \in \mathcal{I},$$

and there is at least one consumer  $j$  such that:

$$u_j(\hat{x}_{j,1}, \dots, \hat{x}_{j,n}) > u_j(x_{j,1}, \dots, x_{j,n}).$$

**Definition 1.7** (Contract Curve). The set of all Pareto allocations is known as the **contract curve**.

### Edgeworth Box



### General Case

$$\begin{aligned} & \max_{\{(x_{1,1}, \dots, x_{1,n}), \dots, (x_{m,1}, \dots, x_{m,n})\}} u_1(x_{1,1}, \dots, x_{1,n}) \\ & \text{subject to } u_2(x_{2,1}, \dots, x_{2,n}) \geq \bar{u}_2 \\ & \quad \vdots \\ & u_m(x_{m,1}, \dots, x_{m,n}) \geq \bar{u}_m \\ & \sum_{i \in \mathcal{I}} x_{i,1} \leq \omega_1 \\ & \quad \vdots \\ & \sum_{i \in \mathcal{I}} x_{i,n} \leq \omega_n \end{aligned}$$

### Theorem 1.1

Let all utility functions be strictly increasing and quasi-concave, and  $((\hat{x}_{1,1}, \dots, \hat{x}_{1,n}), \dots, (\hat{x}_{m,1}, \dots, \hat{x}_{m,n}))$  be a feasible interior allocation. Then  $((\hat{x}_{1,1}, \dots, \hat{x}_{1,n}), \dots, (\hat{x}_{m,1}, \dots, \hat{x}_{m,n}))$  is **Pareto-efficient** if and only if  $((\hat{x}_{1,1}, \dots, \hat{x}_{1,n}), \dots, (\hat{x}_{m,1}, \dots, \hat{x}_{m,n}))$  exhausts all resources and, for all pairs of goods  $\ell, \ell'$ :

$$\text{MRS}(\ell, \ell')(\hat{x}_{1,1}, \dots, \hat{x}_{1,n}) = \dots = \text{MRS}(\ell, \ell')(\hat{x}_{m,1}, \dots, \hat{x}_{m,n}).$$

## 1.2 Competitive Equilibrium

### Initial Assumptions

- There is a market for each good.
- Every agent can access the market without any cost.
- There is a unique price for each good and all consumers know the price.
- Each consumer can sell her initial endowment in the market and use the income to buy goods and services.
- Consumers seek to maximize their utility given their budget restriction, independently of what everyone else is doing.
- There is no centralized mechanism.
- People may not know others preferences or endowments.
- There is perfect competition (i.e. everyone is a price-taker).
- The only source of information agents are prices.

**Definition 1.8** (Competitive Equilibrium). An ordered pair of an allocation and a price vector,  $(x^*, p = (p_1, \dots, p_n))$ , is called a **competitive equilibrium** if the following conditions hold:

(i)  $\forall i \in \mathcal{I}$ ,  $x_i^* = (x_{i,1}^*, \dots, x_{i,n}^*)$  solves the following maximization problem:

$$\begin{aligned} & \max_{\{x_i\}} u_i(x_i) \\ & \text{subject to } wh + p \cdot x_i \leq p \cdot \omega_i = \sum_{\ell \in \mathcal{L}} p_\ell \omega_{i,\ell}. \end{aligned}$$

(ii) Markets clear, i.e.  $\sum_{i \in \mathcal{I}} x_{i,\ell}^* = \sum_{i \in \mathcal{I}} \omega_{i,\ell}$ ,  $\forall \ell \in \mathcal{L}$ .

### Claim 1.2

Given, at least, one consumer with strictly monotone preferences. Then, if  $(x^*, p)$  is a competitive equilibrium,  $p_1, p_2, \dots, p_n > 0$ .

**Claim 1.3**

Given, at least, one consumer with weakly monotone preferences. Then, if  $(x^*, p)$  is a competitive equilibrium, for at least one  $\ell$ ,  $p_\ell > 0$ .

**Claim 1.4**

Let  $(x^*, p)$  be a competitive equilibrium. Then,  $(x^*, cp)$  is also a competitive equilibrium,  $\forall c \in \mathbb{R}_+$ .

**Theorem 1.2: Walras' Law**

If the consumer  $i$  has weakly monotone preferences and also  $\hat{x}_i \in x_i^*(p)$ , then:

$$p \cdot \hat{x}_i = \sum_{\ell \in \mathcal{L}} p_\ell \hat{x}_{i,\ell} = \sum_{\ell \in \mathcal{L}} p_\ell \omega_{i,\ell} = p \cdot \omega_i.$$

**Corollary 1.1: Walras' Law**

Given weakly monotonic utility functions and  $p = (p_1, \dots, p_n)$  such that  $p_n > 0$ . If any  $(x^*, p)$  in which maximization condition holds,  $\forall i \in \mathcal{I}$ , and markets clear  $\forall \ell = 1, 2, \dots, n-1$ ; then, the market clearing condition holds for commodity  $n$  as well.

**Theorem 1.3: Fixed Point**

For any continuous function  $f : \Delta^{n-1} \rightarrow \Delta^{n-1}$ , there exists a point  $p^* = (p_1^*, p_2^*, \dots, p_n^*)$  such that  $f(p^*) = p^*$ , where

$$\Delta^{n-1} = \left\{ (p_1, p_2, \dots, p_n) \in \mathbb{R}_+^n : \sum_{\ell \in \mathcal{L}} p_\ell = 1 \right\}.$$

**Definition 1.9** (Shortage). We define *shortage* or *excess demand* as follows:

$$Z(p) = (z_1(p), z_2(p), \dots, z_n(p)) = \sum_{i \in \mathcal{I}} x_i^*(p) - \sum_{i \in \mathcal{I}} \omega_i.$$

**Claim 1.5**

$p$  is a competitive equilibrium if and only if  $Z(p) = 0$ .

**Claim 1.6**

The equilibrium is not unique.

**Theorem 1.4: Welfare - I**

Given any pure exchange economy such that all consumers have weakly monotonic utility functions. Then, if  $(x^*, p)$  is a competitive equilibrium,  $x^*$  is a Pareto-efficient allocation.

**Theorem 1.5: Welfare - II**

Given an economy  $\mathcal{E} = \langle \mathcal{I}, (u_i, w_i)_{i \in \mathcal{I}} \rangle$  where all consumers have weakly monotonic, quasi-concave utility functions. If  $(x_1, x_2, \dots, x_m)$  is a Pareto-optimal allocation; then, there exists a redistribution of resources  $(\hat{w}_1, \hat{w}_2, \dots, \hat{w}_m)$  and some prices  $p = (p_1, p_2, \dots, p_n)$  such that:

- (i)  $\sum_{i \in \mathcal{I}} \hat{w}_i = \sum_{i \in \mathcal{I}} w_i$ .
- (ii)  $(p, (x_1, x_2, \dots, x_m))$  is a competitive equilibrium of the economy  $\mathcal{E}$ .

**Excess Demand Properties**

- (i) Continuous in  $p$ .
- (ii) Homogeneous of degree zero.
- (iii)  $p \cdot Z(p) = 0$ .

## 2 Monopoly and Monopsony

### 3 Game Theory