Role of Game Theory in AI/ML

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13 May 2018

Agenda

- **Introduction To Game Theory**
  - Motivation
  - What is Game?
  - Equilibrium: Analyzing a Game

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  - Motivation
  - What is Game?
  - Equilibrium: Analyzing a Game
- **Two-Player Zero Sum Games**
- **Stackelberg Games**
- **Summary**

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  - **Nobel Memorial Prize for Economics**: Samuelson(1970), Arrow (1972), Selten, Nash, Harshyani (1994), Vickery (1996), Schelling, Aumann (2005), Maskin, Myerson, Hurwicz (2007), Shapley, Roth (2012)

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- What is it's significance for AI/ML?

Example 1: ML and Game Theory

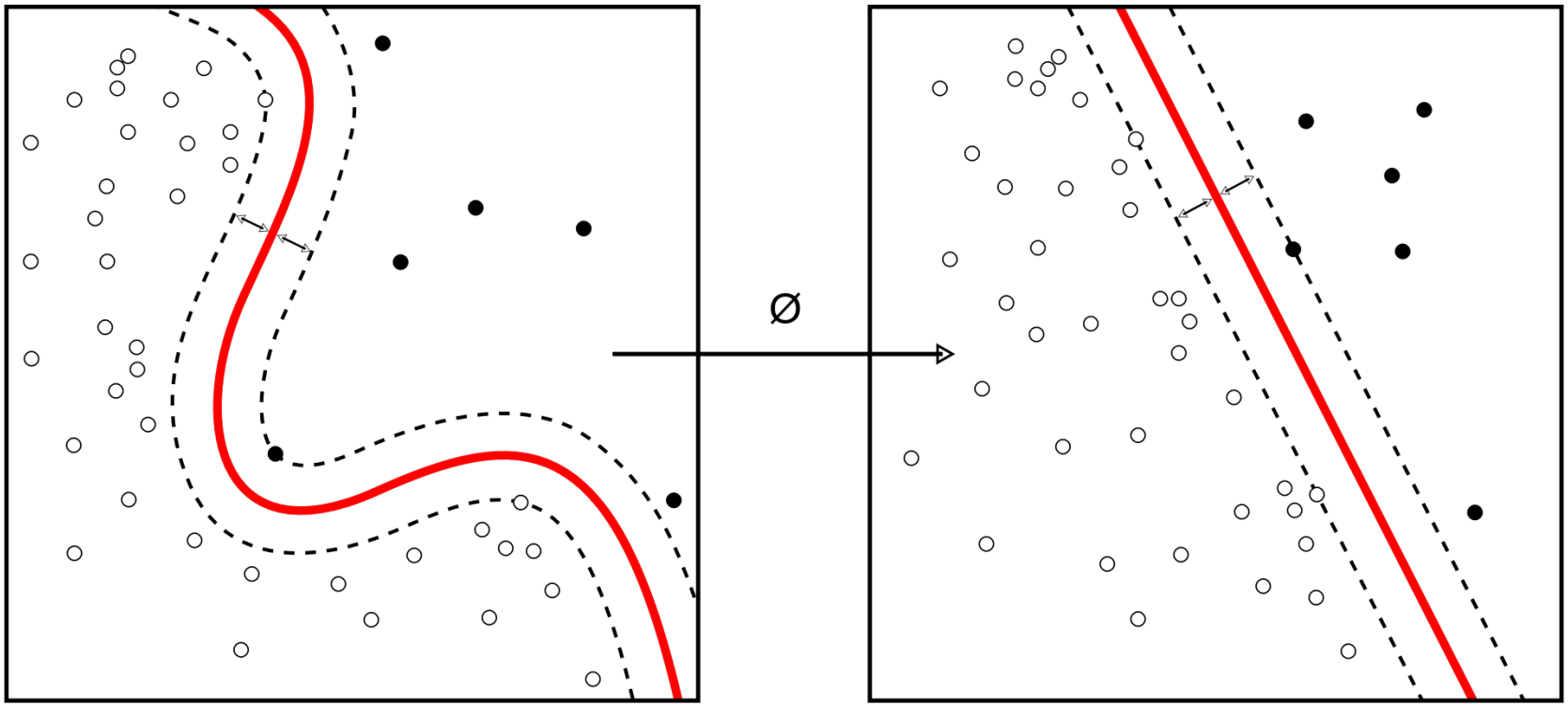
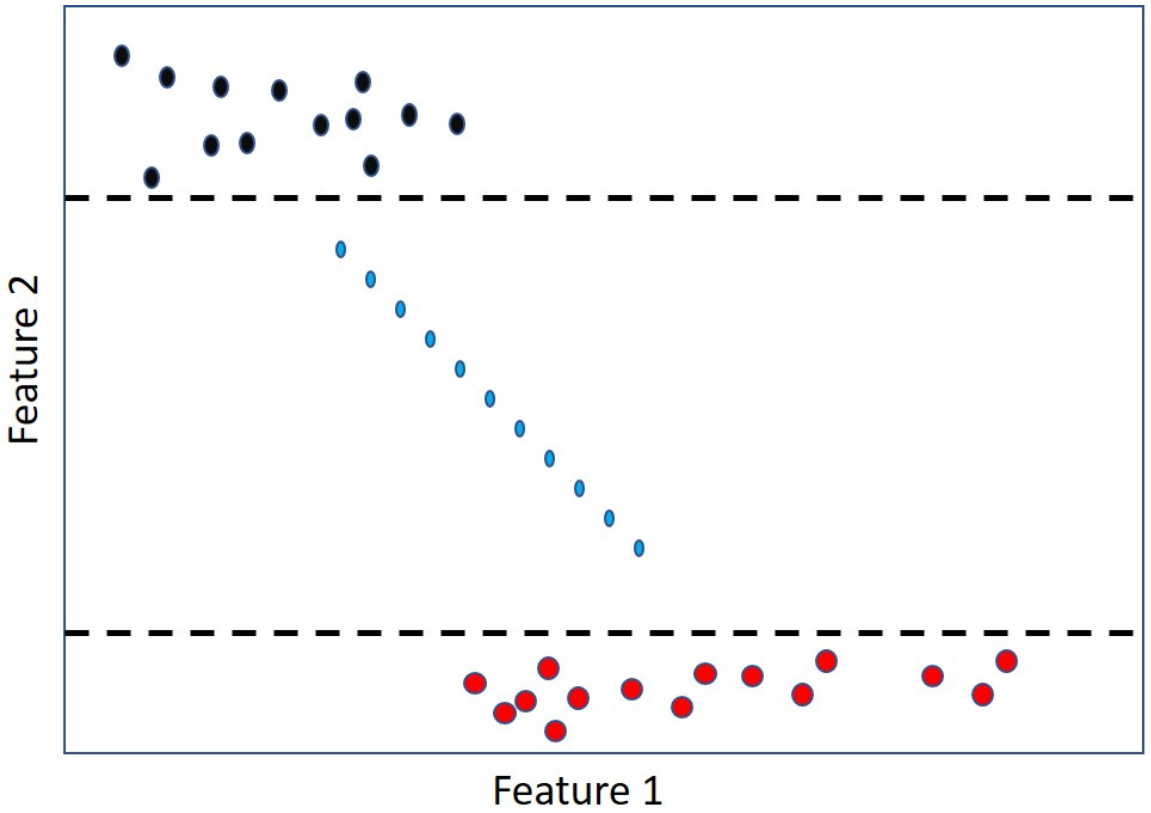


Image: Wikipedia

- Designing boosting algorithm using game theory[1]
- Many applications of game theory in classification problems

[1] Boosting Approach to Machine Learning An Overview by Robert Schapire, 2003.

Example 1: ML and Game Theory



- Which features are important for classification/mining?[1]

[1] Feature evaluation and selection with cooperative game theory by X Sun *et.al.*, Pattern Recognition, 2012

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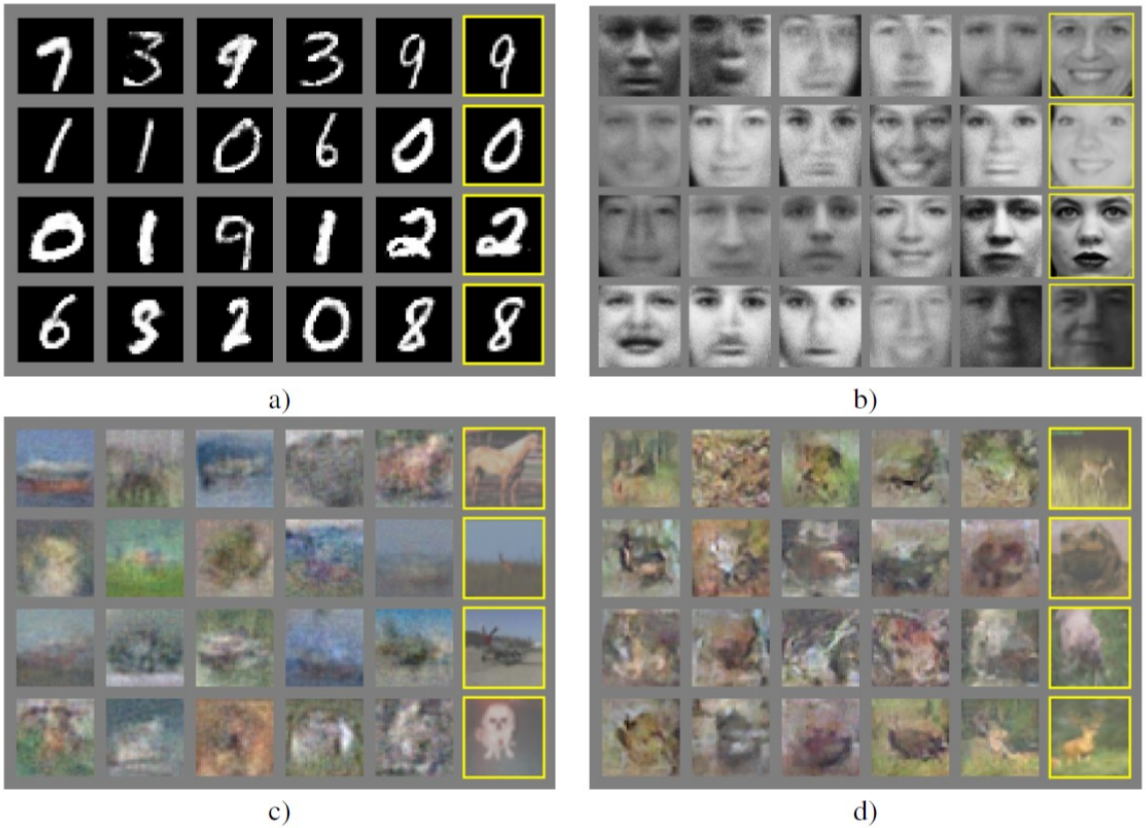

Generative Adversarial Networks (GANs) to generate synthetic content [1]

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[1] Generative Adversarial Nets by Goodfellow *et.al.*, NIPS 2014

Example 2: Game Theory for Public and Wild Life Safety

PROTECT

Example 2: Game Theory for Public and Wild Life Safety

PROTECT

PAWS

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PROTECT

PAWS

http://teamcore.usc.edu

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Example 3: Google Bomb

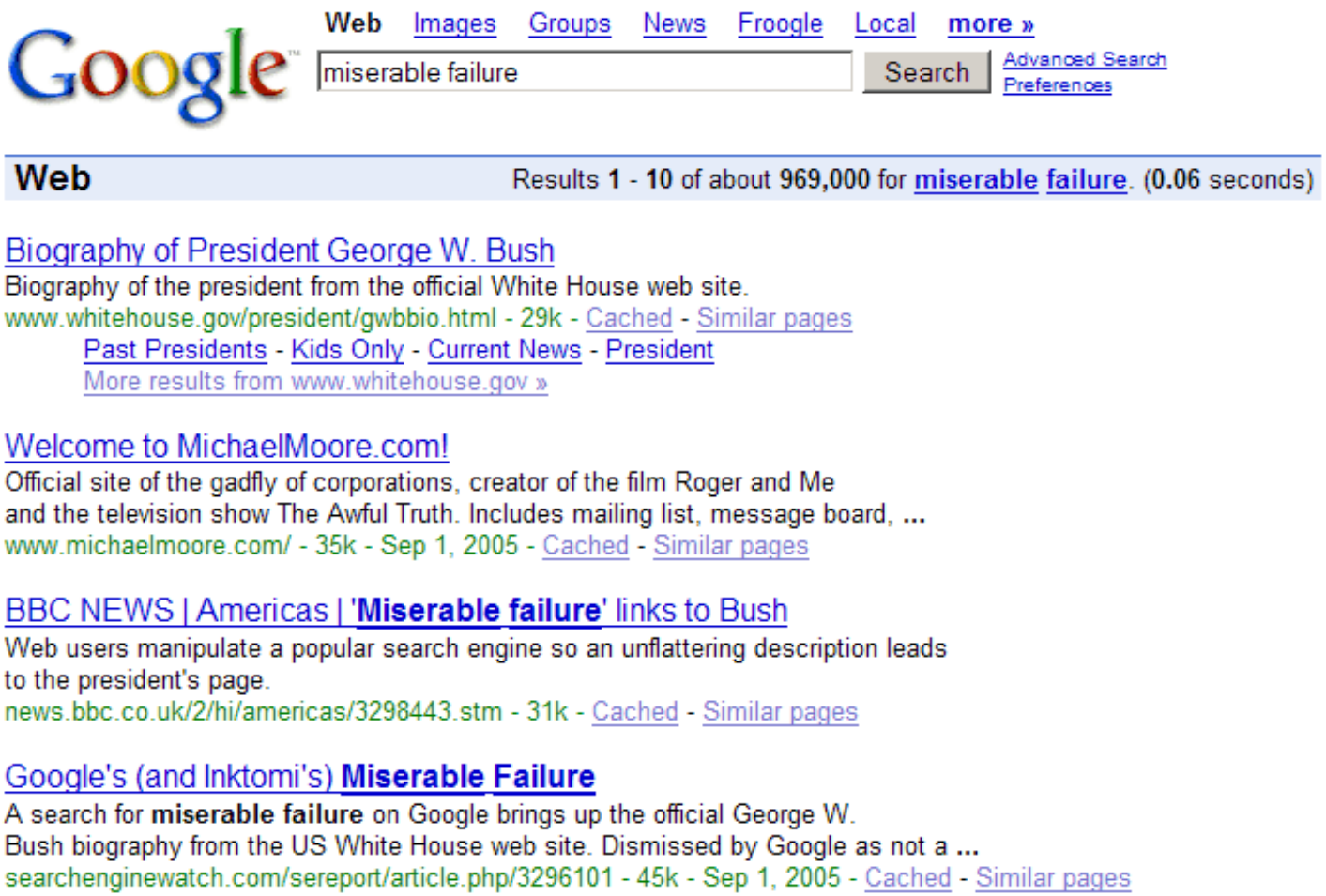

image credits: wikipedia

Example 4: Sybil Attacks

- An attack on peer-to-peer network by creating a large number of **pseudonymous** identities

image credits:
https://arstechnica.com

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- Successful Sybil attack with traffic confirmation attack to gain certain user information (2014)

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Example 4: Sybil Attacks

- An attack on peer-to-peer network by creating a large number of **pseudonymous** identities
- TOR: The Onion Routing (anonymous communication)
- Successful Sybil attack with traffic confirmation attack to gain certain user information (2014)
- Use of Game Theory has been proved to be useful to prevent Sybil Attacks (Prof V Conitzer, Duke)

image credits:
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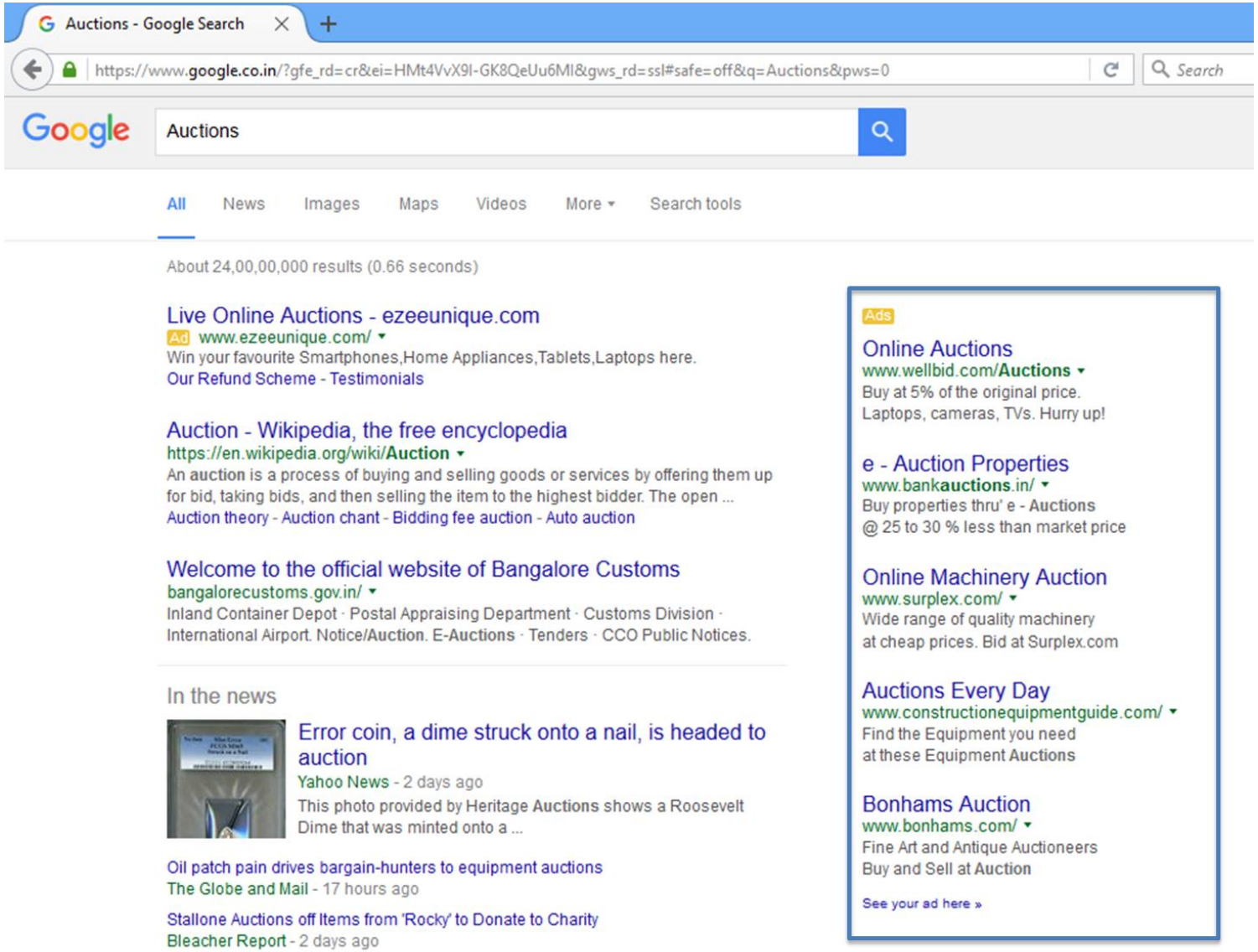
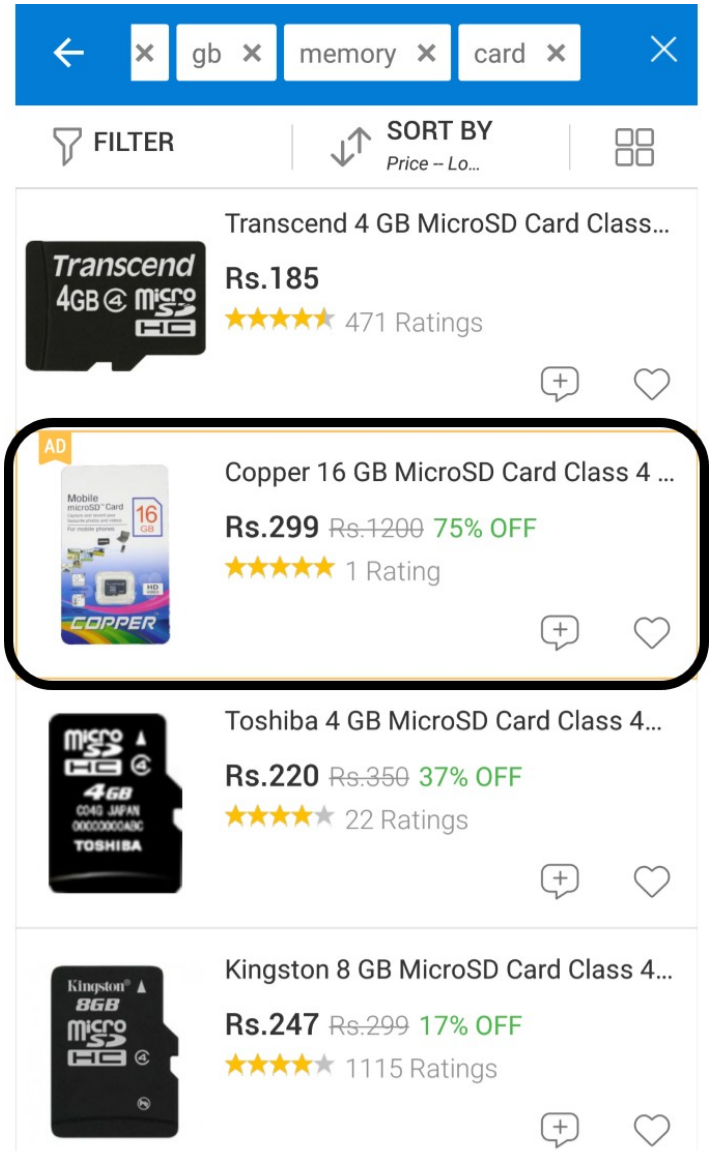
Example 5: Sponsored Search Auctions (Google)



Example 5: Display Ads (Flipkart)



Example 6: Online Auctions (e-Commerce)

- Airline seat allocation, when agents arrive dynamically to book the tickets
- Hotel room bookings, prices to be varied according to demand
- Resource allocation such as computing power to dynamically arriving requests

Example 7: Robot Game

- Consider two robots designed by two different companies

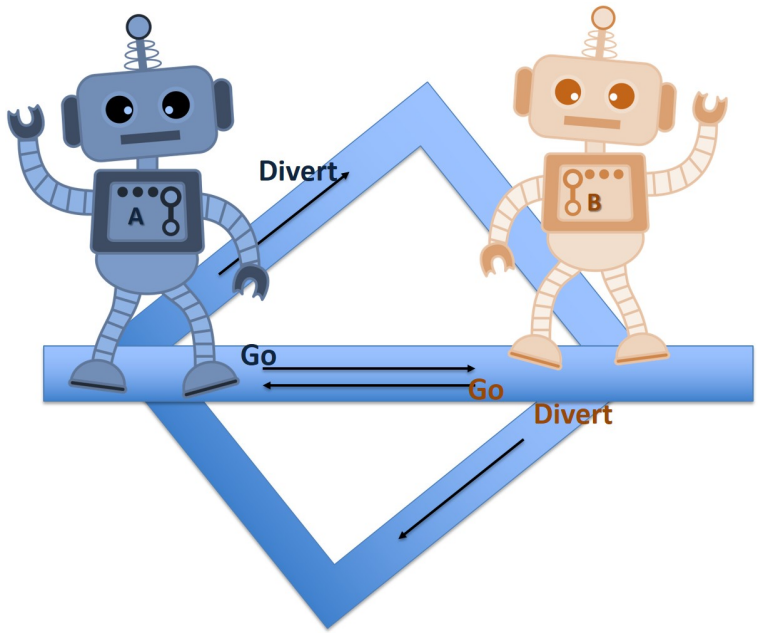
  Hence don't cooperate with each other

- Each robot can 'Go' straight (no extra cost) when both their paths intersect or choose 'Divert' to avoid collision

- Cost of diversion is 1

- If they collide, incur loss of 2

- How to write your agent to handle such situations?



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Example 8: Exit Polls

- Which Party will emerge as the largest party in Karnataka State Elections?

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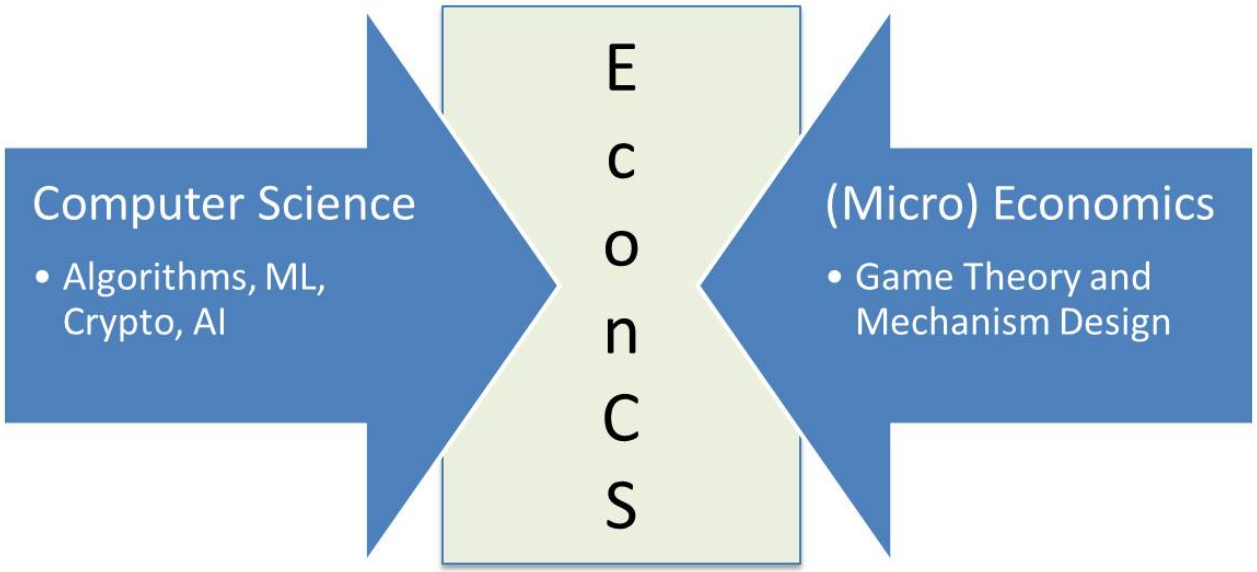
- **Information Aggregation** over crowdsourcing
- Strategic agents can manipulate your predictions/decisions
- **Prediction Market** and **Peer Predictions** based on game theory

Applications of Game Theory

- Internet Advertising, e-commerce
- Social Network, Crowd Sourcing
- Resource Allocation, Distributed Systems
- Spectrum Allocations, Cognitive Radio
- Stock Market Software

Game Theory Meets Computer Science



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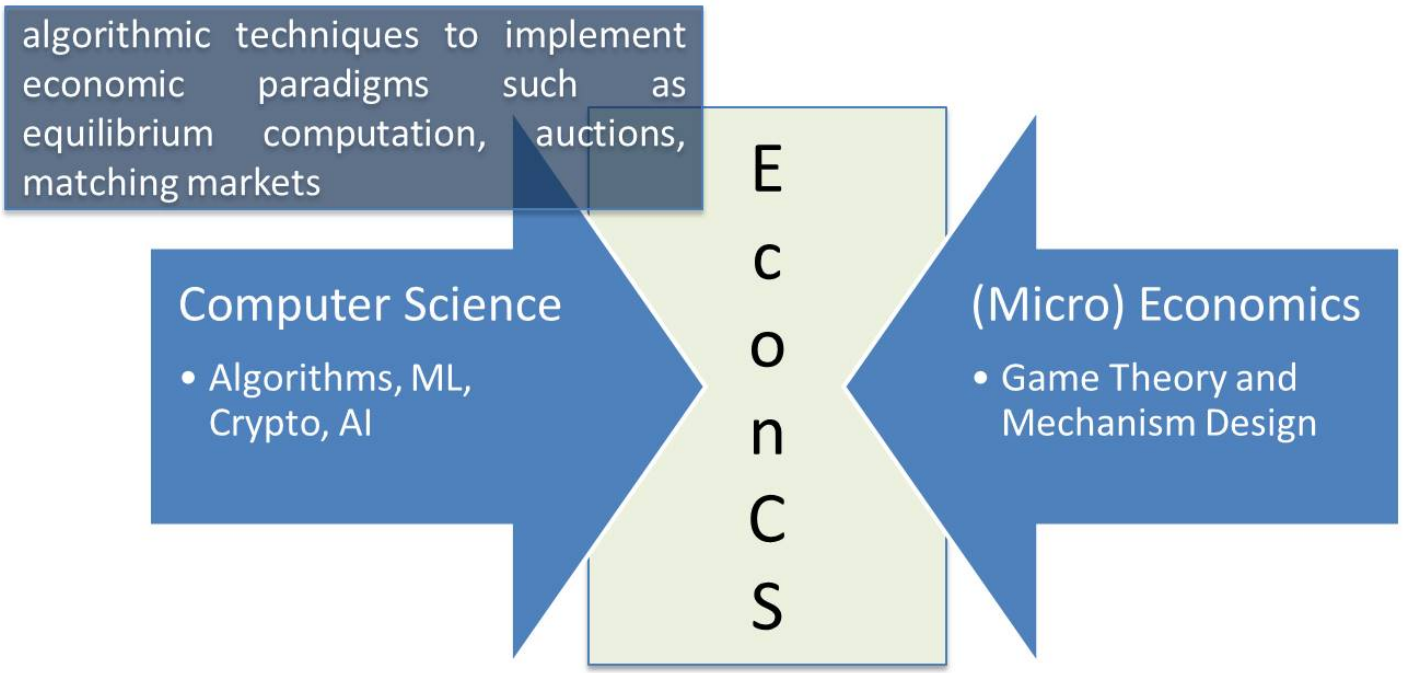

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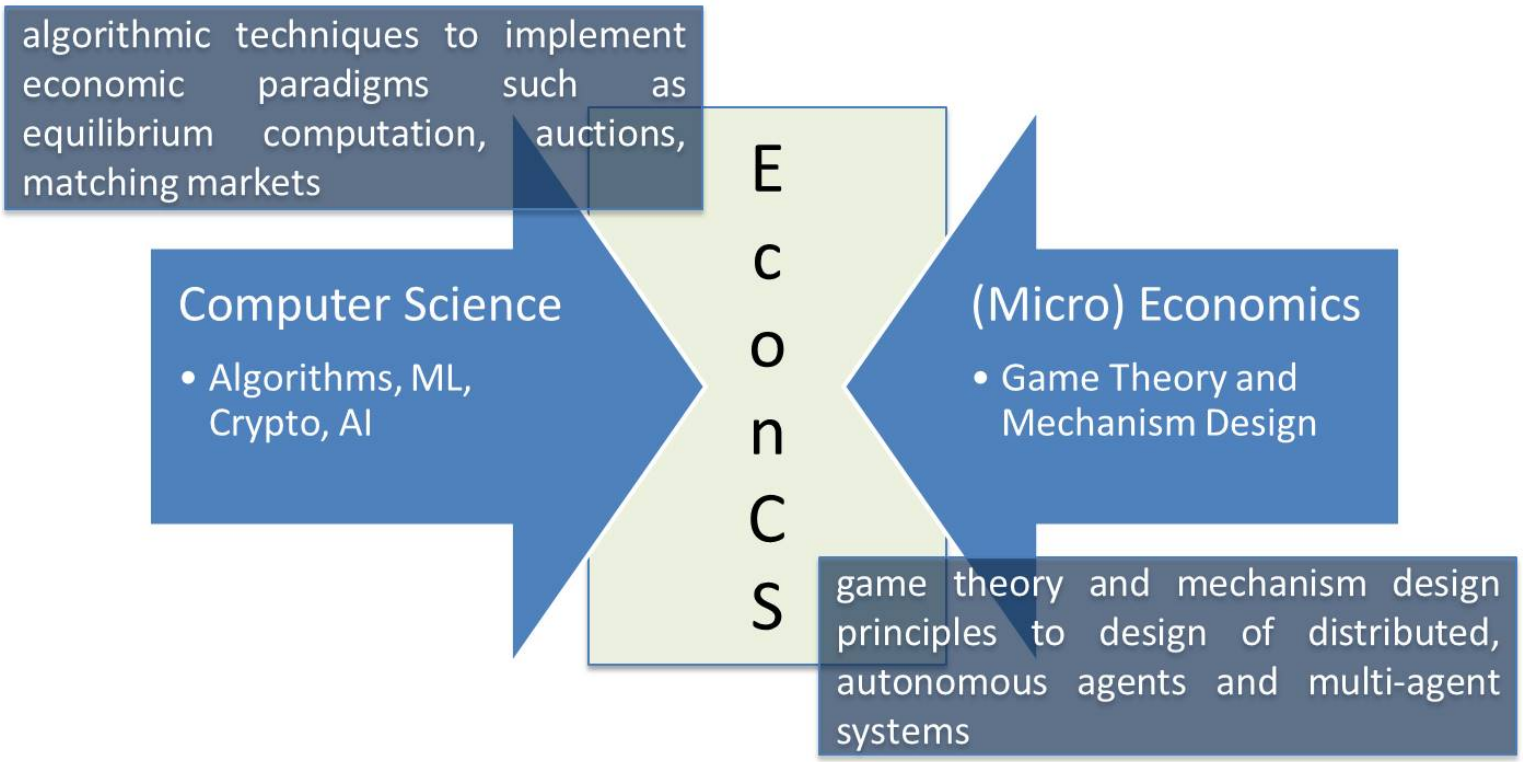

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Game Theory Meets Computer Science

- Duke University: MSEC (MS in Economics and Computation), Brown University: B.Sc. in Computer Science-Economics, University of St Andrews (UK): B.Sc. (Hons) Computer Science and Economics
- Most of universities of offer courses Computational Economics, Algorithmic Game Theory, Economics and Computation, Computational Game Theory and many more variants.

Introduction to Game Theory

Introduction to Game Theory

Game Theory

- Analysis of conflict
- According to Myerson

Definition (Game Theory)

Game theory is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers.

- Analyses and predicts the behaviour of strategic agents (players) with conflicting interests
- Suggests the strategies to play

John von Neumann

Image Credits: Wikipedia

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Example 9: Prisoner's Dilemma

- Two partners in crime (A,B) are caught
- Offers A: 5 years of prison if both confess
- 1 year of prison if one confesses and other does not
- The non-confess person gets 10 yrs in prison
- In absence of confession, 2 yrs of prison each
- Goal: find out which is optimal strategy for A?For B?

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Example 10: Matching Coins

- Two Friends (Say A and B) with ₹ 10 coins
- Both independently choose **H** or **T**
- If both the coins match A keeps both the coins else B keeps both the coins

Matching Coins without Observations

What are Elements of A Game?

[2] Credits Prof Boi Faltings

What are Elements of A Game?

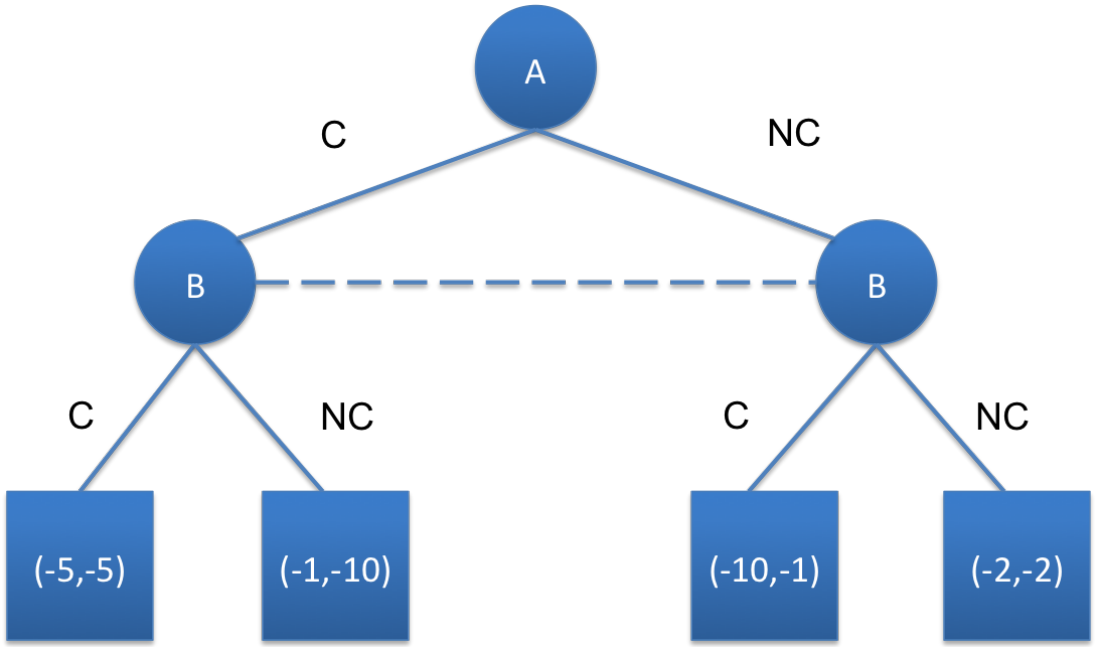
Elements of a game[2]:

- **Players**: the agents playing the game
- **States** of the game
- **Actions**: that change the state of the game
- **Knowledge** (beliefs) of the state and actions
- **Outcome** of the players actions, in particular payoffs for each player
- **Payoff or Utility** That each player derives from the outcome
- **Assumptions**
  - Every player acts rationally so as to maximize its own payoff
  - Information about game is **common knowledge**

[2] Credits Prof Boi Faltings

Extensive Form Games

Prisoner's Dilemma



Prisoner's Dilemma as Extensive Form Game (1)

Prisoner's Dilemma

- Who are the players?

Prisoner's Dilemma as Extensive Form Game (1)

Prisoner's Dilemma

- 2 players, Player A and B

Prisoner's Dilemma as Extensive Form Game (1)

Prisoner's Dilemma

- 2 players, Player A and B
- States:

Prisoner's Dilemma as Extensive Form Game (1)

Prisoner's Dilemma

- 2 players, Player A and B
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions (2×2)

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Prisoner's Dilemma

- 2 players, Player A and B
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- What are actions?

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Prisoner's Dilemma

- 2 players, Player A and B
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions (2×2)
- Actions: player A and B choose between C and NC

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- 2 players, Player A and B
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions (2×2)
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- Knowledge: A and B both know the game and rules. However both do not know the others choice

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- Outcomes: 4 possible outcomes
- Utilities:

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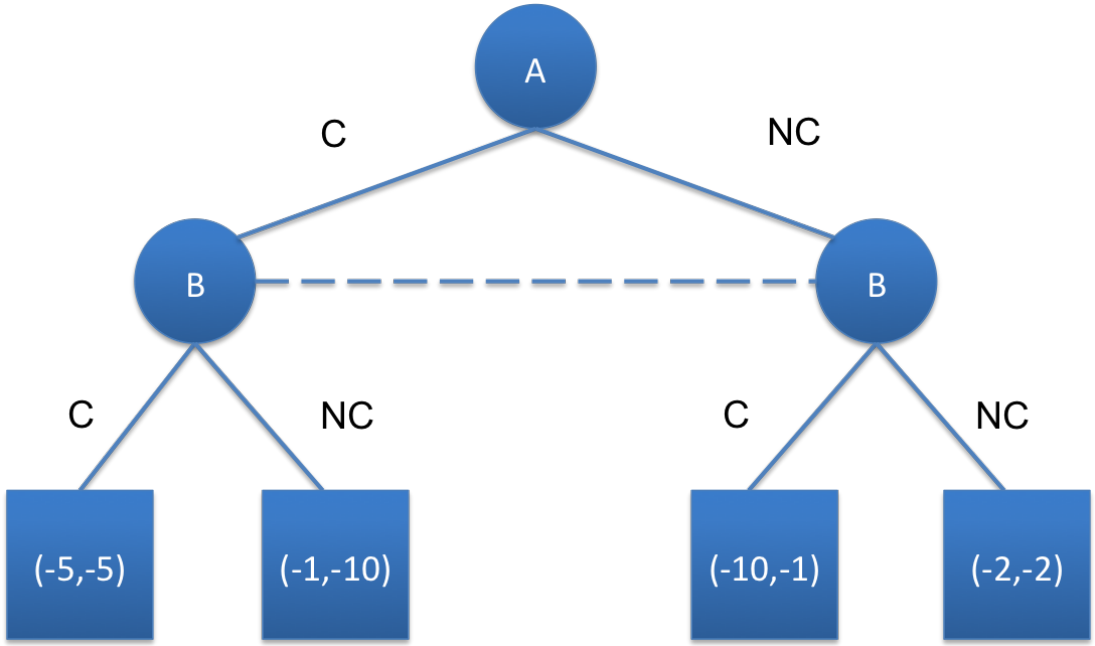
- 2 players, Player A and B
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions (2×2)
- Actions: player A and B choose between C and NC
- Knowledge: A and B both know the game and rules. However both do not know the others choice
- Outcomes: 4 possible outcomes
- Utilities: depend on the outcome

Extensive Form of a game is a graphical representation of the game

Prisoner's Dilemma as Extensive Form Game (2)



- Dotted line indicates Player B does not know in which state she is.
- Such sets are called as **Information Sets**
- If all information sets are singleton, then its called a game with perfect information
- Is Chess a game with perfect information or imperfect information? What about prisoners dilemma? Cards?

Towards Analyzing a Game

- Recall: Game theory analyses and predicts the behavior of strategic agents (players) with conflicting interests
- So far no analysis or prediction
- Strategic agents will strategize
- What is strategy?

Strategies

- **Strategy**: is an algorithm or rule by which each player chooses an action
  - a complete contingent plan explaining what a player will do in every situation (state)

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  Time being we focus on
- **Pure strategy**: for each state (or believed state), the action is chosen in a deterministic way
  Pure: not mixed or adulterated with any other substance or material
- Better representation for analysis

Strategic Form Games (Normal form Games)

N : Set of players
$N = \{1, 2, \ldots, n\}$

S_1 : Strategies available to player 1
S_2 : Strategies available to player 2
$\vdots$
S_n : Strategies available to player n
$S = S_1 \times S_2 \times \ldots \times S_n$
Strategy space of all the players

$u_1 : S \to \mathbb{R}$
$u_2 : S \to \mathbb{R}$
$\vdots$
$u_n : S \to \mathbb{R}$

Utility or Payoff Functions

- This is also known as **matrix form** games

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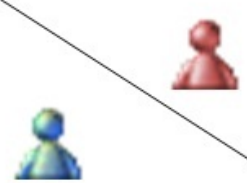
Prisoner's Dilemma As Normal Form Game

	No Confess **NC**	Confess **C**
No Confess **NC**	**- 2, - 2**	**- 10, - 1**
Confess **C**	**-1, - 10**	**- 5, - 5**

Matching Coins Games without Observation

	H	T
H	(10,-10)	(-10,10)
T	(-10,10)	(10,-10)

Notion of Equilibrium

Analyzing games: Equilibrium

Equilibrium In Prisoner's Dilemma

	NC	C
NC	(-2,-2)	(-10,-1)
C	(-1,-10)	(-5,-5)

- What if the column player is playing C?

Equilibrium In Prisoner's Dilemma

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- What if the column player is playing C?
- What if the row player is playing C?

Equilibrium In Prisoner's Dilemma

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- What if the column player is playing C?
- What if the row player is playing C?
- Thus, playing (C,C) is an **equilibrium**

Multiple Equilibria In A Game

Consider another game

	NC	C
NC	(-1,-1)	(-10,-1)
C	(-1,-10))	(-5,-5)

- Similar to the prisoner's dilemma, this has (C,C) as an equilibrium

Multiple Equilibria In A Game

Consider another game

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- Similar to the prisoner's dilemma, this has (C,C) as an equilibrium
- We can argue that (NC,NC) is also an equilibrium

No (Pure) Equilibrium In Matching Coins Game

	H	T
H	(10,-10)	(-10,10)
T	(-10,10)	(10,-10)

- What should the row player should play when the column player is playing H?

No (Pure) Equilibrium In Matching Coins Game

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No (Pure) Equilibrium In Matching Coins Game

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- What should the row player should play when the column player is playing H?T?
- What should the column player should play when the row player is playing H?

No (Pure) Equilibrium In Matching Coins Game

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No (Pure) Equilibrium In Matching Coins Game

	H	T
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- What should the row player should play when the column player is playing H?T?
- What should the column player should play when the row player is playing H?T?
- **Result**: players should always randomize their strategies

Mixed Strategies

- Say for player i, there are i_k actions, $a_{i_1}, a_{i_2}, \ldots, a_{i_k}$
- She decides to play these actions with probabilities $p_{i_1}, p_{i_2}, \ldots, p_{i_k}$ with $p_{i_1} + p_{i_2} + \ldots + p_{i_k} = 1$
- $\sigma_i = (p_{i_1}, p_{i_2}, \ldots, p_{i_k})$ is mixed strategy of the player i
- For example, in matching coins game, $(x, 1-x)$ is mixed strategy for the row player if he decides to play H with probability x

Expected Utility

- For two-player games, we refer mixed strategies as $p = (p_1, \ldots, p_m), q = (q_1, \ldots, q_n)^T$
- Mixed strategies leads to **Utility Theory** by Neumann and Morgenstern
- For Player 1, expected payoff

$$U_1(p, q) = \sum_{j \in \{1,2,\ldots,n\}} p_1 * q_j * U_1(a_1, b_j) + p_2 * q_j * U_1(a_2, b_j) + \ldots + p_m * q_j * U_1(a_m, b_j)$$

Two Player Zero Sum Games

Two Player Zero Sum Games

Matrix Games: Two Player Zero Sum Games

- Gain of one player = Loss to the Other
  Total sum of utilities = 0

[3] This term is different than Matrix Form Games

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Matrix Games: Two Player Zero Sum Games

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  Total sum of utilities = 0
- For Example, Matching coins game
- Consider the following game Γ^Z =:

	L	M	R
T	(1,-1)	(2,-2)	(2,-2)
M	(-3,3)	(-1,1)	(-2,2)
B	(-2,2)	(0,0)	(1,-1)

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- $\Gamma =< \{1,2\}, S_1, S_2, U_1, -U_1 >$. Also called as **Matrix Games**.[3]

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- $\Gamma =< \{1,2\}, S_1, S_2, U_1, -U_1 >$. Also called as **Matrix Games**.[3]
- We can represent the game by a single $m \times n$ matrix

[3] This term is different than Matrix Form Games

Equilibrium In Two Player Zero-Sum Games

- Consider another game $\Gamma^Z =$:

	L	M	R
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Equilibrium In Two Player Zero-Sum Games

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Equilibrium In Two Player Zero-Sum Games

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- What is the row player is playing T?

Equilibrium In Two Player Zero-Sum Games

- Consider another game $\Gamma^Z =$:

	L	M	R
T	1	2	2
M	0	-1	-2
B	-2	0	1

- What if the column player is playing L?
- What is the row player is playing T?
- (T,L) is an equilibrium

Mini-Max Equilibrium and Mini-Max Strategies

- If an pure strategy equilibrium exists, then the row player is maximizing her min assured gain

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- The column player is minimizing her worst loss (same as maximizing her min assured gain)

Mini-Max Equilibrium and Mini-Max Strategies

- If an pure strategy equilibrium exists, then the row player is maximizing her min assured gain
- The column player is minimizing her worst loss (same as maximizing her min assured gain)
- Such equlibrium is called mini-max equilibrium and the strategy that achieves it is **mini-max** strategy

Mini-Max Theorem

von Neumann and Morgenstern showed:

Theorem (Mini-Max Theorem)

For every $(m \times n)$ matrix A, there is a stochastic row vector $p^* = (p_1^*, \ldots, p_n^*)$ and a stochastic column vector $q^{*T} = (q_1^*, \ldots, q_n^*)$ such that

$$\min_{q \in \Delta(S_2)} p^* A q = \max_{p \in \Delta(S_1)} p A q^*$$

(p^*, q^*) is an equilibrium

Mini-Max Theorem

von Neumann and Morgenstern showed:

Theorem (Mini-Max Theorem)

For every $(m \times n)$ matrix A, there is a stochastic row vector $p^* = (p_1^*, \ldots, p_n^*)$ and a stochastic column vector $q^{*T} = (q_1^*, \ldots, q_n^*)$ such that

$$\min_{q \in \Delta(S_2)} p^* A q = \max_{p \in \Delta(S_1)} p A q^*$$

(p^*, q^*) is an equilibrium

What is equilibrium of Matching Coins without Observation game?

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$p^* = (0.5, 0.5) = q^{*T}$

Stackelberg Games

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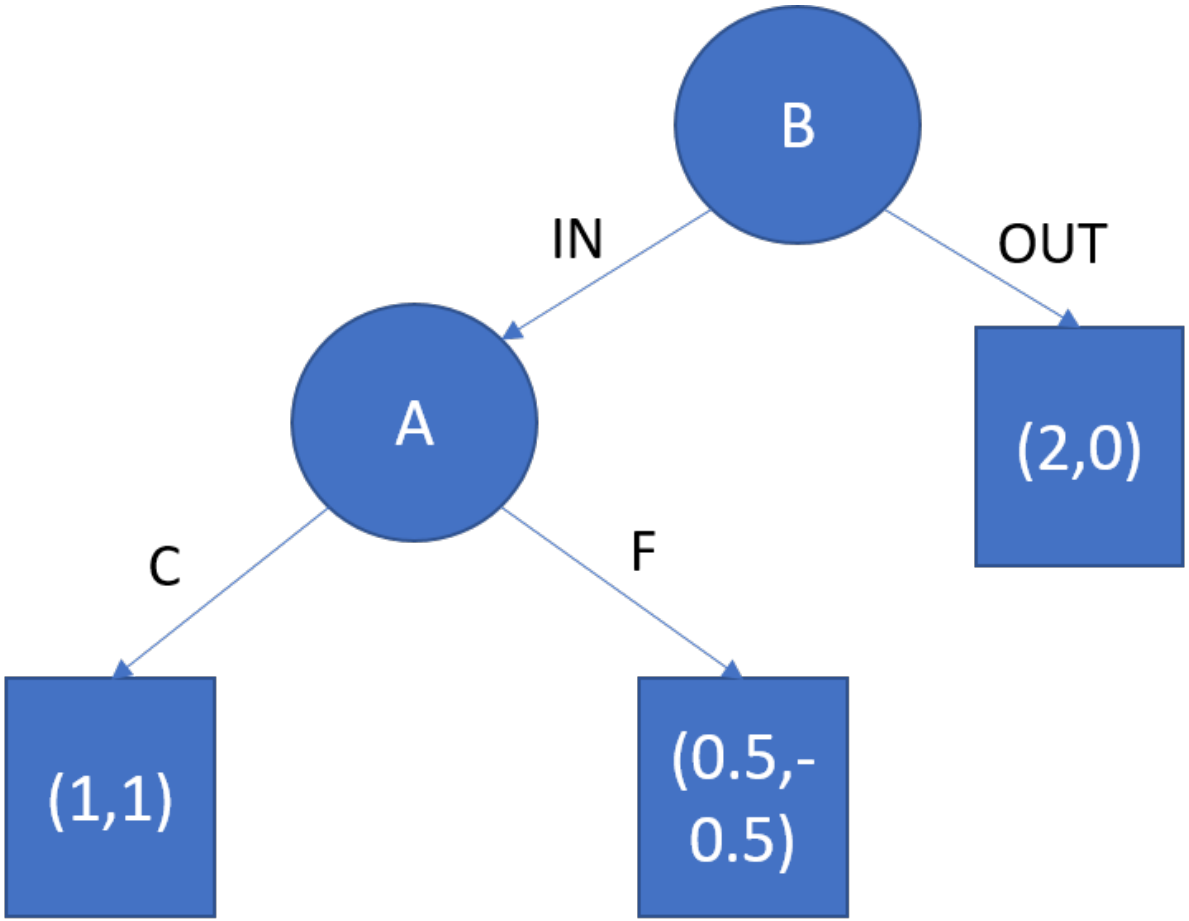
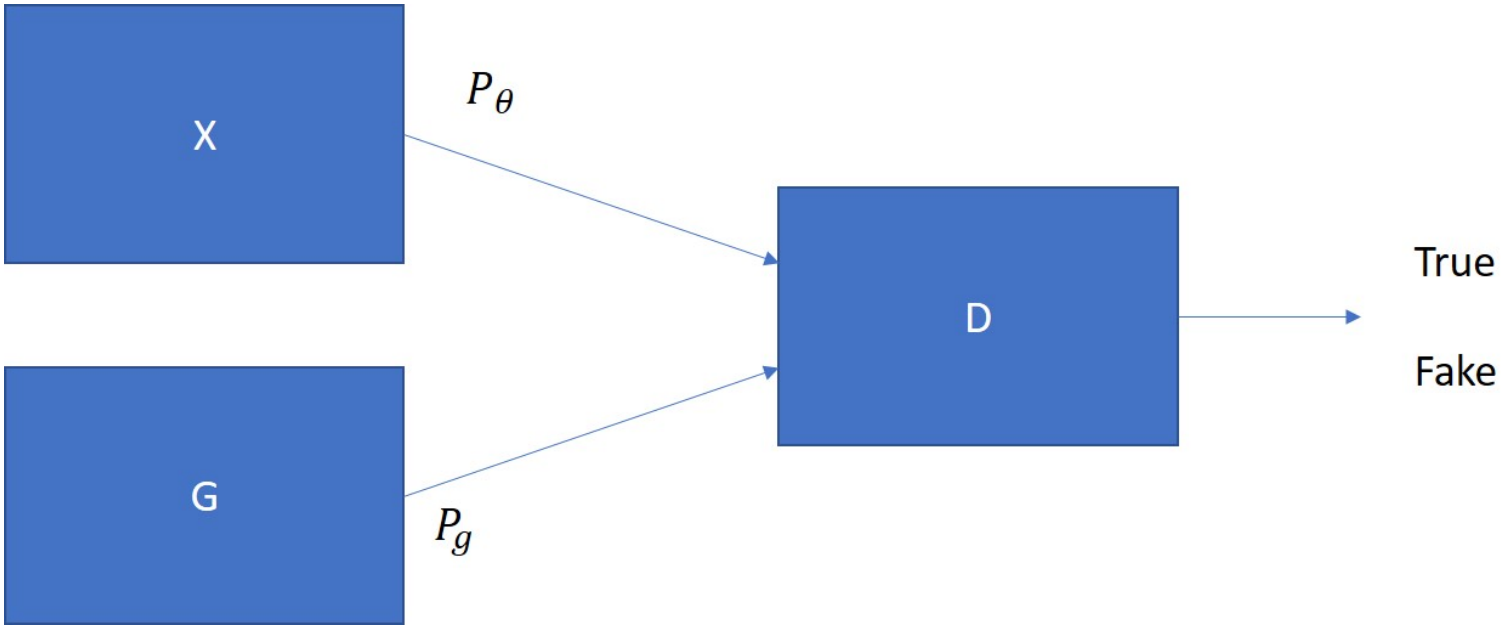


Subgame Perfect Equilibrium

- A strategy is subgame perfect if every subgame has it as an equilibrium
- Last player will play its best response strategy
- Last but one stage, the player will take action that maximize its utility assuming the last stage action is taken in accordance with the above
- Repeat this till the first step
- This is called as **backward induction**

GANs: Zero-sum Stackelberg Game



Summary

We have seen

- Components of a game
- Extensive form and Normal form representation of a game
- Mixed strategies
- Two player zero sum games
- Stackelberg Games
- GANs as zero-sum/Stackelberg game

Further Reading

- **Game Theory and Mechanism Design**, Y Narahari. World Scientific Publishing Company, 2014.
- **Multiagent systems: Algorithmic, game-theoretic, and logical foundations**, Shoham, Yoav, and Kevin Leyton-Brown. Cambridge University Press, 2008. (Free download).
- **Game Theory** by Roger Myerson. Harvard University press, 2013.
- **Algorithmic Game Theory**, edited by Noam Nisan, Tim Roughgarden, Eva Tardos and Vijay Vazerani. (Non-printable version available online).

`http://gametheory.net/`

`http://lcm.csa.iisc.ernet.in/gametheory/lecture.html`

IIIT, HYDERABAD

Thank You!!

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