

# Role of Game Theory in AI/ML

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# Agenda

- **Introduction To Game Theory**
  - ▶ Motivation
  - ▶ What is Game?
  - ▶ Equilibrium: Analyzing a Game



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- **Introduction To Game Theory**
  - ▶ Motivation
  - ▶ What is Game?
  - ▶ Equilibrium: Analyzing a Game
- **Two-Player Zero Sum Games**
- **Stackelberg Games**
- **Summary**



# What is Game Theory talked here?

- AI/ML course at IIITH:



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  - ▶ **Nobel Memorial Prize for Economics:** Samuelson(1970), Arrow (1972), Selten, Nash, Harshyani (1994), Vickery (1996), Schelling, Aumann (2005), Maskin, Myerson, Hurwicz (2007), Shapley, Roth (2012)



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- What is it's significance for AI/ML?



# Example 1: ML and Game Theory

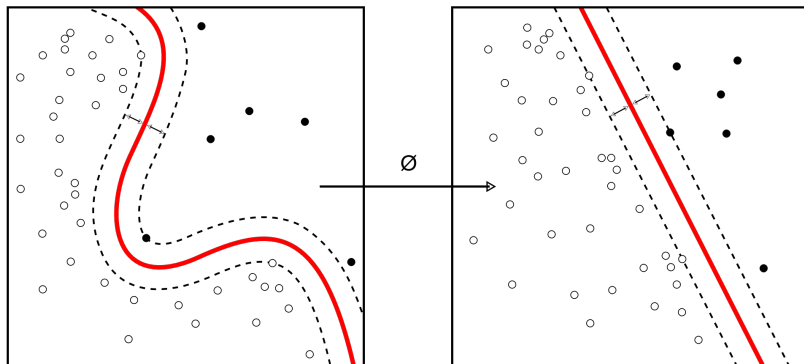
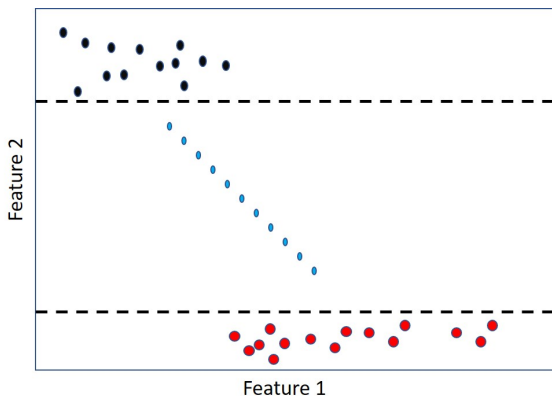


Image: Wikipedia

- Designing boosting algorithm using game theory<sup>1</sup>
- Many applications of game theory in classification problems

<sup>1</sup> Boosting Approach to Machine Learning An Overview by Robert Schapire, 2003.

# Example 1: ML and Game Theory



- Which features are important for classification/mining?<sup>1</sup>

<sup>1</sup>Feature evaluation and selection with cooperative game theory by X Sun *et.al.*, Pattern Recognition, 2012



# Example 1: ML and Game Theory



Generative Adversarial Networks (GANs) to generate synthetic content <sup>1</sup>

<sup>1</sup>Generative Adversarial Nets by Goodfellow *et.al.*, NIPS 2014



## Example 2: Game Theory for Public and Wild Life Safety



PROTECT



## Example 2: Game Theory for Public and Wild Life Safety



PROTECT



PAWS

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PROTECT

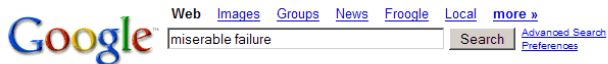


PAWS

<http://teamcore.usc.edu>



## Example 3: Google Bomb



**Web** Results 1 - 10 of about 969,000 for [miserable failure](#). (0.06 seconds)

### [Biography of President George W. Bush](#)

Biography of the president from the official White House web site.

[www.whitehouse.gov/president/gwbbio.html](#) - 29k - [Cached](#) - [Similar pages](#)

[Past Presidents](#) - [Kids Only](#) - [Current News](#) - [President](#)

[More results from www.whitehouse.gov »](#)

### [Welcome to MichaelMoore.com!](#)

Official site of the gadfly of corporations, creator of the film Roger and Me and the television show The Awful Truth. Includes mailing list, message board, ...

[www.michaelmoore.com/](#) - 35k - Sep 1, 2005 - [Cached](#) - [Similar pages](#)

### [BBC NEWS | Americas | 'Miserable failure' links to Bush](#)

Web users manipulate a popular search engine so an unflattering description leads to the president's page.

[news.bbc.co.uk/2/hi/americas/3298443.stm](#) - 31k - [Cached](#) - [Similar pages](#)

### [Google's \(and Inktomi's\) Miserable Failure](#)

A search for **miserable failure** on Google brings up the official George W.

Bush biography from the US White House web site. Dismissed by Google as not a ...

[searchenginewatch.com/sereport/article.php/3296101](#) - 45k - Sep 1, 2005 - [Cached](#) - [Similar pages](#)

image credits: wikipedia



## Example 4: Sybil Attacks

- An attack on peer-to-peer network by creating a large number of **pseudonymous** identities



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<https://arstechnica.com>

## Example 4: Sybil Attacks

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- TOR: The Onion Routing (anonymous communication)



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- An attack on peer-to-peer network by creating a large number of **pseudonymous** identities
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- Successful Sybil attack with traffic confirmation attack to gain certain user information (2014)



image credits:

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## Example 4: Sybil Attacks

- An attack on peer-to-peer network by creating a large number of **pseudonymous** identities
- TOR: The Onion Routing (anonymous communication)
- Successful Sybil attack with traffic confirmation attack to gain certain user information (2014)
- Use of Game Theory has been proved to be useful to prevent Sybil Attacks (Prof V Conitzer, Duke)

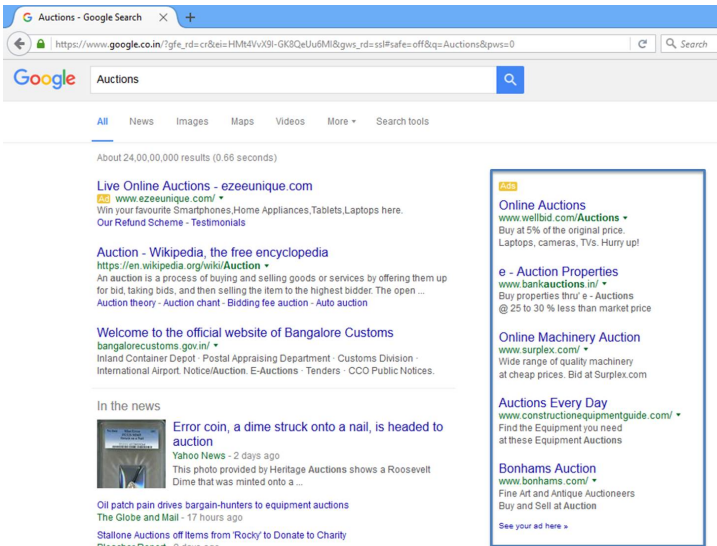


image credits:

<https://arstechnica.com>



# Example 5: Sponsored Search Auctions (Google)



The screenshot shows a Google search interface with the query 'Auctions'. The browser address bar shows the URL: [https://www.google.co.in/?gfe\\_rd=cr&ei=HMt4VvX9l-GK8QeUu6MI&gws\\_rd=ssl#safe=off&q=Auctions&pw=0](https://www.google.co.in/?gfe_rd=cr&ei=HMt4VvX9l-GK8QeUu6MI&gws_rd=ssl#safe=off&q=Auctions&pw=0). The search results are categorized by 'All', 'News', 'Images', 'Maps', 'Videos', 'More', and 'Search tools'. The results show 'About 24,00,00,000 results (0.66 seconds)'.

**Organic Search Results:**

- Live Online Auctions - ezeenique.com**  
[www.ezeenique.com/](http://www.ezeenique.com/)  
 Win your favourite Smartphones, Home Appliances, Tablets, Laptops here.  
[Our Refund Scheme - Testimonials](#)
- Auction - Wikipedia, the free encyclopedia**  
<https://en.wikipedia.org/wiki/Auction>  
 An auction is a process of buying and selling goods or services by offering them up for bid, taking bids, and then selling the item to the highest bidder. The open ...  
[Auction theory](#) - [Auction chant](#) - [Bidding fee auction](#) - [Auto auction](#)
- Welcome to the official website of Bangalore Customs**  
[bangalorecustoms.gov.in/](http://bangalorecustoms.gov.in/)  
 Inland Container Depot - Postal Appraising Department - Customs Division - International Airport. Notice/Auction. E-Auctions - Tenders - CCO Public Notices.
- In the news**
  - Error coin, a dime struck onto a nail, is headed to auction**  
[Yahoo News](#) - 2 days ago  
 This photo provided by Heritage Auctions shows a Roosevelt Dime that was minted onto a ...
  - Oil patch pain drives bargain-hunters to equipment auctions**  
[The Globe and Mail](#) - 17 hours ago
  - Stallone Auctions off items from 'Rocky' to Donate to Charity**  
[Bleacher Report](#) - 2 days ago

**Sponsored Search Results (Ads):**


- Online Auctions**  
[www.wellbid.com/Auctions](http://www.wellbid.com/Auctions)  
 Buy at 5% of the original price.  
 Laptops, cameras, TVs. Hurry up!
- e - Auction Properties**  
[www.bankauctions.in/](http://www.bankauctions.in/)  
 Buy properties thru e - Auctions  
 @ 25 to 30 % less than market price
- Online Machinery Auction**  
[www.surplex.com/](http://www.surplex.com/)  
 Wide range of quality machinery  
 at cheap prices. Bid at Surplex.com
- Auctions Every Day**  
[www.constructionequipmentguide.com/](http://www.constructionequipmentguide.com/)  
 Find the Equipment you need  
 at these Equipment Auctions
- Bonhams Auction**  
[www.bonhams.com/](http://www.bonhams.com/)  
 Fine Art and Antique Auctioneers  
 Buy and Sell at Auction  
[See your ad here >](#)


At the bottom right, there is a logo for 'IIIT, HYDERABAD' featuring a tree.


## Example 5: Display Ads (Flipkart)


← x gb x memory x card x ×

FILTER SORT BY Price - Lo...

**Transcend 4 GB MicroSD Card Class...**  
  
**Rs.185**  
★★★★★ 471 Ratings

**AD**  
**Copper 16 GB MicroSD Card Class 4 ...**  
  
**Rs.299** ~~Rs.1200~~ **75% OFF**  
★★★★★ 1 Rating

**Toshiba 4 GB MicroSD Card Class 4...**  
  
**Rs.220** ~~Rs.350~~ **37% OFF**  
★★★★★ 22 Ratings

**Kingston 8 GB MicroSD Card Class 4...**  
  
**Rs.247** ~~Rs.299~~ **17% OFF**  
★★★★★ 1115 Ratings

## Example 6: Online Auctions (e-Commerce)

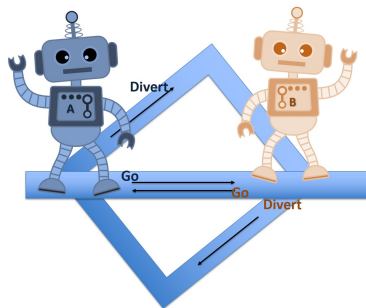
- Airline seat allocation, when agents arrive dynamically to book the tickets
- Hotel room bookings, prices to be varied according to demand
- Resource allocation such as computing power to dynamically arriving requests





## Example 7: Robot Game

- Consider two robots designed by two different companies  
Hence don't cooperate with each other
- Each robot can 'Go' straight (no extra cost) when both their paths intersect or choose 'Divert' to avoid collision
- Cost of diversion is 1
- If they collide, incur loss of 2
- How to write your agent to handle such situations?



## Example 8: Exit Polls

- Which Party will emerge as the largest party in Karnataka State Elections?



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- **Information Aggregation** over crowdsourcing



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- Strategic agents can manipulate your predictions/decisions



## Example 8: Exit Polls

- Which Party will emerge as the largest party in Karnataka State Elections?
- **Information Aggregation** over crowdsourcing
- Strategic agents can manipulate your predictions/decisions
- **Prediction Market** and **Peer Predictions** based on game theory

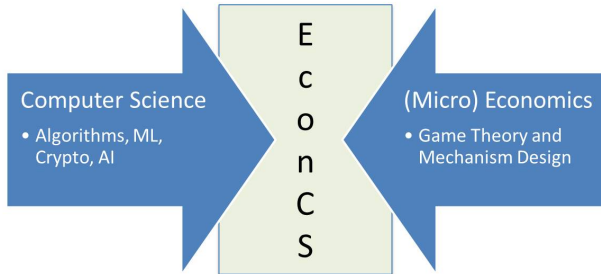


# Applications of Game Theory

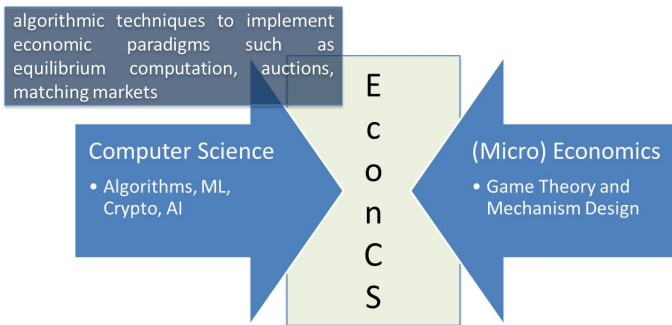
- Internet Advertising, e-commerce
- Social Network, Crowd Sourcing
- Resource Allocation, Distributed Systems
- Spectrum Allocations, Cognitive Radio
- Stock Market Software



# Game Theory Meets Computer Science

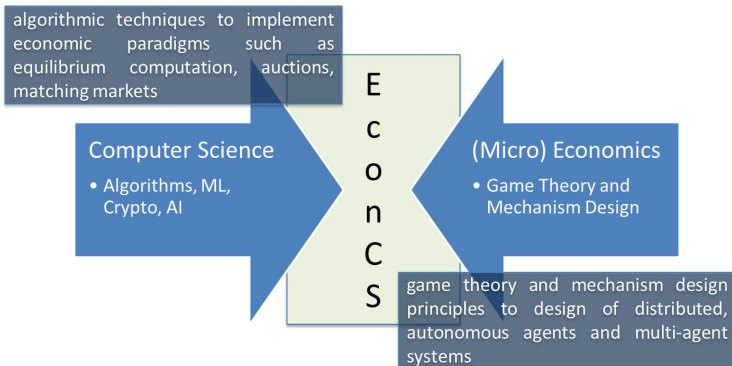


# Game Theory Meets Computer Science





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# Game Theory Meets Computer Science

- Duke University: MSEC (MS in Economics and Computation), Brown University: B.Sc. in Computer Science-Economics, University of St Andrews (UK): B.Sc. (Hons) Computer Science and Economics
- Most of universities offer courses Computational Economics, Algorithmic Game Theory, Economics and Computation, Computational Game Theory and many more variants.



# Introduction to Game Theory

# Introduction to Game Theory



# Game Theory

- Analysis of conflict
- According to Myerson

## Definition (Game Theory)

Game theory is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers.

- Analyses and predicts the behaviour of strategic agents (players) with conflicting interests
- Suggests the strategies to play



John von Neumann

Image Credits: Wikipedia



## Example 9: Prisoner's Dilemma



- Two partners in crime (A,B) are caught
- Offers A: 5 years of prison if both confess
- 1 year of prison if one confesses and other does not
- The non-confess person gets 10 yrs in prison
- In absence of confession, 2 yrs of prison each
- Goal: find out which is optimal strategy for A? For B?



## Example 10: Matching Coins

- Two Friends (Say A and B) with ₹ 10 coins
- Both independently choose **H** or **T**
- If both the coins match A keeps both the coins else B keeps both the coins

### Matching Coins without Observations



# What are Elements of A Game?

---

<sup>2</sup>Credits Prof Boi Faltings



# What are Elements of A Game?

Elements of a game<sup>2</sup>:

- **Players**: the agents playing the game
- **States** of the game
- **Actions**: that change the state of the game
- **Knowledge** (beliefs) of the state and actions
- **Outcome** of the players actions, in particular payoffs for each player
- **Payoff or Utility** That each player derives from the outcome
- **Assumptions**
  - ▶ Every player acts rationally so as to maximize its own payoff
  - ▶ Information about game is **common knowledge**



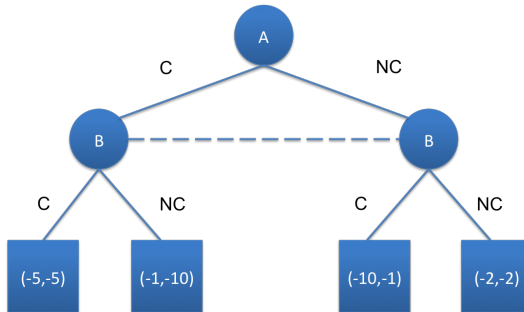
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<sup>2</sup>Credits Prof Boi Faltings



# Extensive Form Games

## Prisoner's Dilemma



# Prisoner's Dilemma as Extensive Form Game (1)

## Prisoner's Dilemma

- Who are the players?



# Prisoner's Dilemma as Extensive Form Game (1)

## Prisoner's Dilemma

- 2 players, Player A and B



# Prisoner's Dilemma as Extensive Form Game (1)

## Prisoner's Dilemma

- 2 players, Player A and B
- States:



# Prisoner's Dilemma as Extensive Form Game (1)

## Prisoner's Dilemma

- 2 players, Player A and B
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions ( $2 \times 2$ )



# Prisoner's Dilemma as Extensive Form Game (1)

## Prisoner's Dilemma

- 2 players, Player A and B
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions ( $2 \times 2$ )
- What are actions?



# Prisoner's Dilemma as Extensive Form Game (1)

## Prisoner's Dilemma

- 2 players, Player A and B
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions ( $2 \times 2$ )
- Actions: player A and B choose between C and NC



# Prisoner's Dilemma as Extensive Form Game (1)

## Prisoner's Dilemma

- 2 players, Player A and B
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions ( $2 \times 2$ )
- Actions: player A and B choose between C and NC
- Knowledge: A and B both know the game and rules. However both do not know the others choice





# Prisoner's Dilemma as Extensive Form Game (1)

## Prisoner's Dilemma

- 2 players, Player A and B
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions ( $2 \times 2$ )
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- Knowledge: A and B both know the game and rules. However both do not know the others choice
- Outcomes:



# Prisoner's Dilemma as Extensive Form Game (1)

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- States: Initial + 2 states for actions of player A + 4 states for each combination of actions ( $2 \times 2$ )
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- Knowledge: A and B both know the game and rules. However both do not know the others choice
- Outcomes: 4 possible outcomes



# Prisoner's Dilemma as Extensive Form Game (1)

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- States: Initial + 2 states for actions of player A + 4 states for each combination of actions ( $2 \times 2$ )
- Actions: player A and B choose between C and NC
- Knowledge: A and B both know the game and rules. However both do not know the others choice
- Outcomes: 4 possible outcomes
- Utilities:



# Prisoner's Dilemma as Extensive Form Game (1)

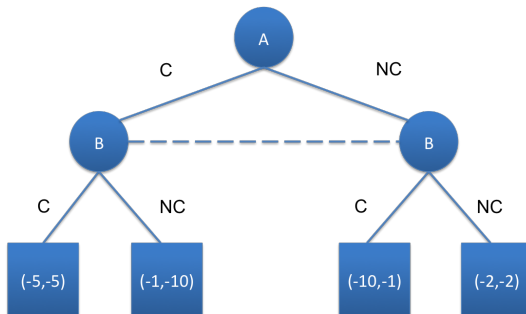
## Prisoner's Dilemma

- 2 players, Player A and B
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions ( $2 \times 2$ )
- Actions: player A and B choose between C and NC
- Knowledge: A and B both know the game and rules. However both do not know the others choice
- Outcomes: 4 possible outcomes
- Utilities: depend on the outcome

**Extensive Form** of a game is a graphical representation of the game



# Prisoner's Dilemma as Extensive Form Game (2)



- Dotted line indicates Player B does not know in which state she is.
- Such sets are called as **Information Sets**
- If all information sets are singleton, then its called a game with perfect information
- Is Chess a game with perfect information or imperfect information? What about prisoners dilemma? Cards?



# Towards Analyzing a Game

- Recall: Game theory analyses and predicts the behavior of strategic agents (players) with conflicting interests
- So far no analysis or prediction
- Strategic agents will strategize
- What is strategy?



# Strategies

- **Strategy:** is an algorithm or rule by which each player chooses an action
  - a complete contingent plan explaining what a player will do in every situation (state)



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- **Pure strategy**: for each state (or believed state), the action is chosen in a deterministic way



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# Strategies

- **Strategy**: is an algorithm or rule by which each player chooses an action
  - a complete contingent plan explaining what a player will do in every situation (state)

Time being we focus on
- **Pure strategy**: for each state (or believed state), the action is chosen in a deterministic way
  - Pure**: not mixed or adulterated with any other substance or material
- Better representation for analysis



# Strategic Form Games (Normal form Games)



$N$  : Set of players  
 $N = \{1, 2, \dots, n\}$

$S_1$  : Strategies available  
to player 1

$S_2$  : Strategies available  
to player 2

$\vdots$

$S_n$  : Strategies available  
to player  $n$

$S = S_1 \times S_2 \times \dots \times S_n$   
Strategy space of all the  
players

$$u_1 : S \rightarrow \mathbb{R}$$

$$u_2 : S \rightarrow \mathbb{R}$$

$\vdots$



$$u_n : S \rightarrow \mathbb{R}$$

Utility or Payoff  
Functions

- This is also known as **matrix form** games



# Prisoner's Dilemma As Normal Form Game

 	No Confess NC	Confess C
No Confess NC	- 2, - 2	- 10, - 1
Confess C	-1, - 10	- 5, - 5



# Matching Coins Games without Observation

	H	T
H	$(10, -10)$	$(-10, 10)$
T	$(-10, 10)$	$(10, -10)$



# Notion of Equilibrium

## Analyzing games: Equilibrium



# Equilibrium In Prisoner's Dilemma

	NC	C
NC	$(-2,-2)$	$(-10,-1)$
C	$(-1,-10)$	$(-5,-5)$

- What if the column player is playing C?





# Equilibrium In Prisoner's Dilemma

	NC	C
NC	$(-2,-2)$	$(-10,-1)$
C	$(-1,-10)$	$(-5,-5)$

- What if the column player is playing C?
- What if the row player is playing C?



# Equilibrium In Prisoner's Dilemma

	NC	C
NC	$(-2,-2)$	$(-10,-1)$
C	$(-1,-10)$	$(-5,-5)$

- What if the column player is playing C?
- What if the row player is playing C?
- Thus, playing  $(C,C)$  is an **equilibrium**



# Multiple Equilibria In A Game

Consider another game

	NC	C
NC	$(-1,-1)$	$(-10,-1)$
C	$(-1,-10)$	$(-5,-5)$

- Similar to the prisoner's dilemma, this has  $(C,C)$  as an equilibrium



# Multiple Equilibria In A Game

Consider another game

	NC	C
NC	$(-1,-1)$	$(-10,-1)$
C	$(-1,-10)$	$(-5,-5)$

- Similar to the prisoner's dilemma, this has  $(C,C)$  as an equilibrium
- We can argue that  $(NC,NC)$  is also an equilibrium



# No (Pure) Equilibrium In Matching Coins Game

	H	T
H	(10,-10)	(-10,10)
T	(-10,10)	(10,-10)

- What should the row player should play when the column player is playing H?



# No (Pure) Equilibrium In Matching Coins Game

	H	T
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# No (Pure) Equilibrium In Matching Coins Game

	H	T
H	(10,-10)	(-10,10)
T	(-10,10)	(10,-10)

- What should the row player should play when the column player is playing H?T?
- What should the column player should play when the row player is playing H?



# No (Pure) Equilibrium In Matching Coins Game

	H	T
H	(10,-10)	(-10,10)
T	(-10,10)	(10,-10)

- What should the row player should play when the column player is playing H?T?
- What should the column player should play when the row player is playing H?





# No (Pure) Equilibrium In Matching Coins Game

	H	T
H	(10,-10)	(-10,10)
T	(-10,10)	(10,-10)

- What should the row player should play when the column player is playing H?T?
- What should the column player should play when the row player is playing H?T?
- **Result:** players should always randomize their strategies



# Mixed Strategies

- Say for player  $i$ , there are  $i_k$  actions,  $a_{i_1}, a_{i_2}, \dots, a_{i_k}$
- She decides to play these actions with probabilities  $p_{i_1}, p_{i_2}, \dots, p_{i_k}$  with  $p_{i_1} + p_{i_2} + \dots + p_{i_k} = 1$
- $\sigma_i = (p_{i_1}, p_{i_2}, \dots, p_{i_k})$  is mixed strategy of the player  $i$
- For example, in matching coins game,  $(x, 1 - x)$  is mixed strategy for the row player if he decides to play H with probability  $x$



# Expected Utility

- For two-player games, we refer mixed strategies as  
 $p = (p_1, \dots, p_m), q = (q_1, \dots, q_n)^T$
- Mixed strategies leads to **Utility Theory** by Neumann and Morgenstern

- For Player 1, expected payoff

$$U_1(p, q) = \sum_{j \in \{1, 2, \dots, n\}} p_1 * q_j * U_1(a_1, b_j) + p_2 * q_j * U_1(a_2, b_j) \\ + \dots + p_m * q_j * U_1(a_m, b_j)$$



# Two Player Zero Sum Games

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# Matrix Games: Two Player Zero Sum Games

- Gain of one player = Loss to the Other  
Total sum of utilities = 0



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<sup>3</sup>This term is different than Matrix Form Games

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- For Example, Matching coins game

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# Matrix Games: Two Player Zero Sum Games

- Gain of one player = Loss to the Other  
Total sum of utilities = 0
- For Example, Matching coins game
- Consider the following game  $\Gamma^Z =$ :

	L	M	R
T	(1,-1)	(2,-2)	(2,-2)
M	(-3,3)	(-1,1)	(-2,2)
B	(-2,2)	(0,0)	(1,-1)

<sup>3</sup>This term is different than Matrix Form Games



# Matrix Games: Two Player Zero Sum Games

- Gain of one player = Loss to the Other  
Total sum of utilities = 0
- For Example, Matching coins game
- Consider the following game  $\Gamma^Z =$ :

	L	M	R
T	(1,-1)	(2,-2)	(2,-2)
M	(-3,3)	(-1,1)	(-2,2)
B	(-2,2)	(0,0)	(1,-1)

- $\Gamma = \langle \{1, 2\}, S_1, S_2, U_1, -U_1 \rangle$ . Also called as **Matrix Games**.<sup>3</sup>

<sup>3</sup>This term is different than Matrix Form Games





# Matrix Games: Two Player Zero Sum Games

- Gain of one player = Loss to the Other  
Total sum of utilities = 0
- For Example, Matching coins game
- Consider the following game  $\Gamma^Z =$ :

	L	M	R
T	(1,-1)	(2,-2)	(2,-2)
M	(-3,3)	(-1,1)	(-2,2)
B	(-2,2)	(0,0)	(1,-1)

- $\Gamma = \langle \{1, 2\}, S_1, S_2, U_1, -U_1 \rangle$ . Also called as **Matrix Games**.<sup>3</sup>
- We can represent the game by a single  $m \times n$  matrix

<sup>3</sup>This term is different than Matrix Form Games



# Equilibrium In Two Player Zero-Sum Games

- Consider another game  $\Gamma^Z =$ :

	L	M	R
T	1	2	2
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- What if the column player is playing L?
- What is the row player is playing T?
- (T,L) is an equilibrium



# Mini-Max Equilibrium and Mini-Max Strategies

- If an pure strategy equilibrium exists, then the row player is maximizing her min assured gain



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# Mini-Max Equilibrium and Mini-Max Strategies

- If an pure strategy equilibrium exists, then the row player is maximizing her min assured gain
- The column player is minimizing her worst loss (same as maximizing her min assured gain)
- Such equilibrium is called mini-max equilibrium and the strategy that achieves it is **mini-max** strategy





# Mini-Max Theorem

von Neumann and Morgenstern showed:

## Theorem (Mini-Max Theorem)

*For every  $(m \times n)$  matrix  $A$ , there is a stochastic row vector  $p^* = (p_1^*, \dots, p_n^*)$  and a stochastic column vector  $q^{*T} = (q_1^*, \dots, q_n^*)$  such that*

$$\min_{q \in \Delta(S_2)} p^* A q = \max_{p \in \Delta(S_1)} p A q^*$$

$(p^*, q^*)$  is an equilibrium



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What is equilibrium of Matching Coins without Observation game?

$$p^* = (0.5, 0.5) = q^{*T}$$



# Stackelberg Games

# Stackelberg Games



# An Entry Game

- Firm A already in the market with making profit 2



# An Entry Game

- Firm A already in the market with making profit 2
- Firm B can enter INTO the market or opt OUT



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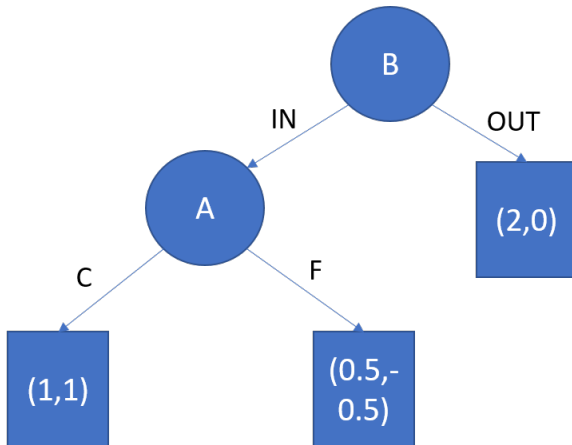


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# An Entry Game

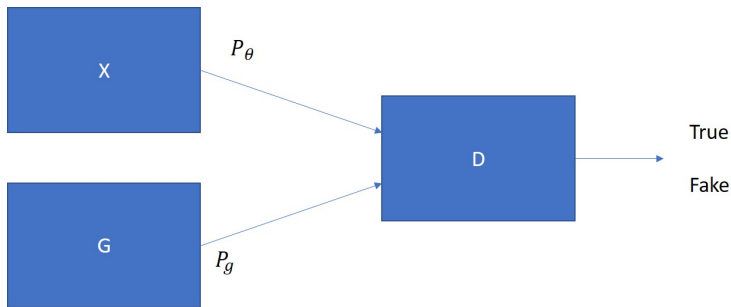


# Subgame Perfect Equilibrium

- A strategy is subgame perfect if every subgame has it as an equilibrium
- Last player will play its best response strategy
- Last but one stage, the player will take action that maximize its utility assuming the last stage action is taken in accordance with the above
- Repeat this till the first step
- This is called as **backward induction**



# GANs: Zero-sum Stackelberg Game



# Summary

We have seen

- Components of a game
- Extensive form and Normal form representation of a game
- Mixed strategies
- Two player zero sum games
- Stackelberg Games
- GANs as zero-sum/Stackelberg game



## Further Reading

- **Game Theory and Mechanism Design**, Y Narahari. World Scientific Publishing Company, 2014.
- **Multiagent systems: Algorithmic, game-theoretic, and logical foundations**, Shoham, Yoav, and Kevin Leyton-Brown. Cambridge University Press, 2008. (Free download).
- **Game Theory** by Roger Myerson. Harvard University press, 2013.
- **Algorithmic Game Theory**, edited by Noam Nisan, Tim Roughgarden, Eva Tardos and Vijay Vazirani. (Non-printable version available online).

<http://gametheory.net/>

<http://lcm.csa.iisc.ernet.in/gametheory/lecture.html>



# Thank You!!



<https://sites.google.com/site/sujitgujar/>

