EM 314

ASSIGNMENT 3

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Theory

(01). $f(x) = \ln x \quad ; \quad x \in [1, 4]$

x	$f(x)$
1	0
2	0.693
3	1.099
4	1.386

$$l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$$l_0(x) = \left(\frac{x-2}{1-2}\right)\left(\frac{x-3}{1-3}\right)\left(\frac{x-4}{1-4}\right) = -\frac{1}{6}(x-2)(x-3)(x-4)$$

$$l_1(x) = \left(\frac{x-1}{2-1}\right)\left(\frac{x-3}{2-3}\right)\left(\frac{x-4}{2-4}\right) = \frac{1}{2}(x^3-8x^2+19x-12)$$

$$l_2(x) = \left(\frac{x-1}{3-1}\right)\left(\frac{x-2}{3-2}\right)\left(\frac{x-4}{3-4}\right) = -\frac{1}{2}(x^3-7x^2+14x-8)$$

$$l_3(x) = \left(\frac{x-1}{4-1}\right)\left(\frac{x-2}{4-2}\right)\left(\frac{x-3}{4-3}\right) = \frac{1}{6}(x^3-6x^2+11x-6)$$

$$p(x) = \sum_{i=0}^{n} y_i \, l_i(x)$$

$$= 0 . l_0(x) + 0.693 \times l_1(x) + 1.099 \times l_2(x) + 1.386 \times l_3(x)$$

$$= 0 + 0.3465(x^3-8x^2+19x-12) + (-0.5495)(x^3-7x^2 + 14x-8) + 0.231(x^3-6x^2+11x-6)$$

$$= 0.028x^3 - 0.3115x^2 + 1.4315x - 1.148$$

(2). $l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \left(\frac{x - x_j}{x_i - x_j} \right)$; $i = 0, 1, \ldots, n$

Let the fn to be interpolate , $f(x) = 1$

Then the lagrange form of the interpolation polynomial is,

$$p(x) = \sum_{i=1}^{n} f(x_i) . l_i(x)$$

$$= \sum_{i=1}^{n} l_i(x) \quad ; \quad f(x_i) = 1 \ \forall \ x_i$$

For all x , data can be interpolated by the zeroth order polynomial, $f(x) = p(x) = 1$

$$\therefore \quad p(x) = \sum_{i=1}^{n} l_i(x)$$

$$= 1$$

//

IMPLEMENTATION

3. (a) LagrangeInterpolant.m

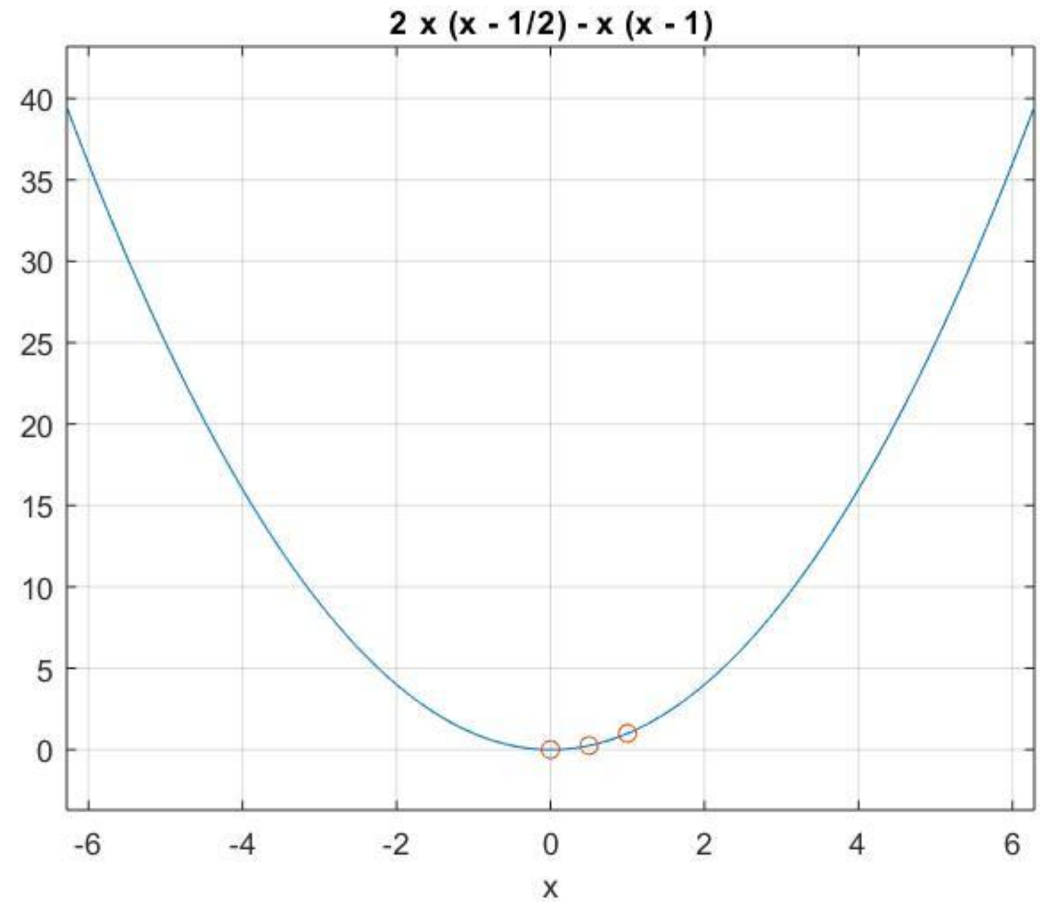
```
function LagrangeInterpolant(l,m,size)

syms p(x)
    p(x)=0;

for i=1:size
    temp=1;
    for j=1:size
        if i==j
            continue
        else
           temp=temp*(x-l(j))/(l(i)-l(j));
        end
    end

    p=p+temp*m(i);

end

ezplot(p)
hold;
grid on;
plot(l,m,'o');
```

(b) q3.m

```
x=[0 0.5 1];
y=[0 0.25 1];
size=length(x);

LagrangeInterpolant(x,y,size)
```

Output

```
>> q3

ans =

x^2
```



Expected answer is $p(x)=x^2$

Therefore, expected answer is obtained.

APPLICATION

4. (a)

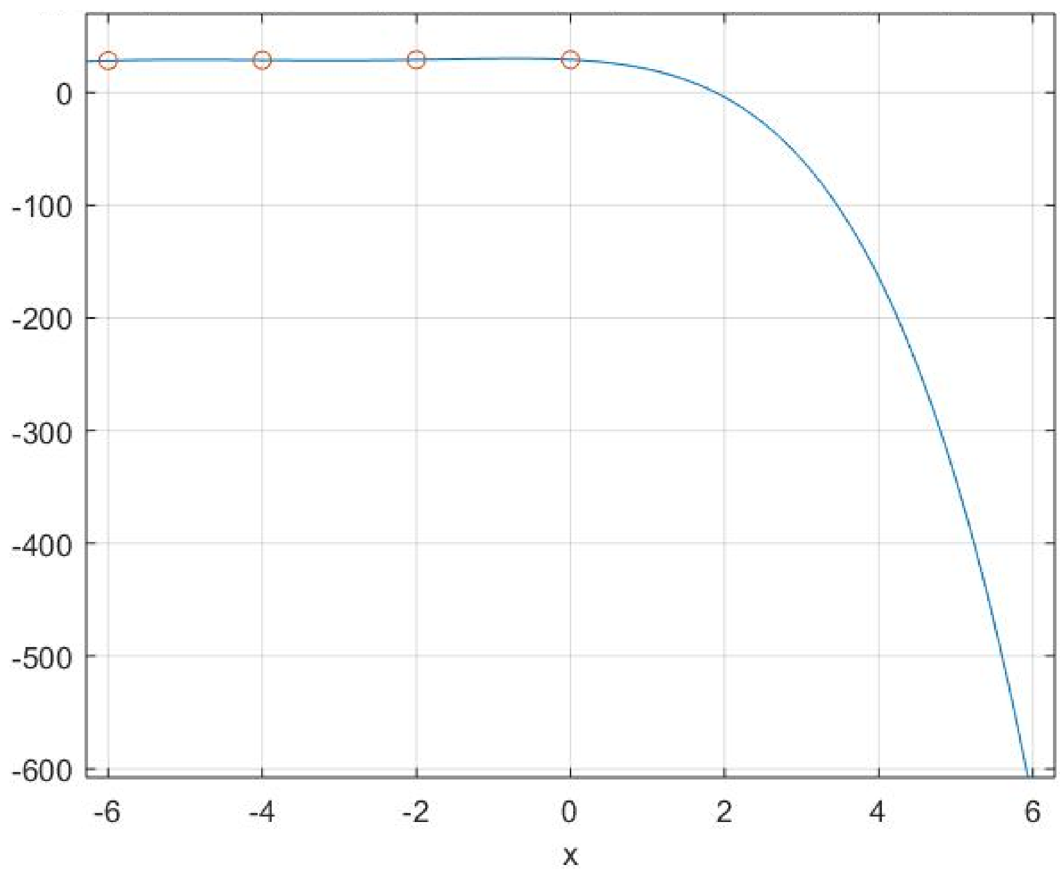
```
function LagrangeInterpolant(l,m,size)

syms p(x)
    p(x)=0;

for i=1:size
    temp=1;
    for j=1:size
        if i==j
            continue
        else
           temp=temp*(x-l(j))/(l(i)-l(j));
        end
    end

    p=p+temp*m(i);

end

ezplot(p)
hold;
grid on;
plot(l,m,'o');
```

q4.m

```
x=[0 -2 -4 -6 -8 -10];
y=[29.1 29 28.7 28.2 20.7 19.1];
size=length(x);

LagrangeInterpolant(x,y,size)
```

Output

```
>> q4
Current plot held

ans(x) =

- (2263*x)/320 - (299*x^2)/160 - (53*x^3)/384 - 1711/240
```



(b).

```
function LagrangeInterpolant(l,m,size)

syms p(x)
    p(x)=0;

for i=1:size
    temp=1;
    for j=1:size
        if i==j
            continue
        else
           temp=temp*(x-l(j))/(l(i)-l(j));
        end
    end

    p=p+temp*m(i);

end

ezplot(simplify(p))
grid on;
depth_at_7=p(-7);
fprintf('%f\n', depth_at_7);
```

Output

```
>> q4
25.291016
```

According to the table value should be between 28.2 and 20.7

Therefore the answer is valid.

(c).

```
function LagrangeInterpolant(l,m,size)

syms p(x)
    p(x)=0;

for i=1:size
    temp=1;
    for j=1:size
        if i==j
            continue
        else
           temp=temp*(x-l(j))/(l(i)-l(j));
        end
    end

    p=p+temp*m(i);

end

d1=diff(p);
d2=diff(d1);
simplify(d2)

r=vpasolve(d2);
disp('Roots of diffrential eqn');
disp(r);
val=d1(r);
disp('Corresponding values');
disp(val);
```

Output

```
>> q4

ans(x) =

- (2263*x)/320 - (299*x^2)/160 - (53*x^3)/384 - 1711/240

Roots of diffrential eqn
  -7.851926551198844377101861952963
 -4.072305533852631206202495067232 7
 -1.615390556457958378959793923200 5

Corresponding values
    4.870512524420261208456890351743
 -0.52832644870974851598873665705211
   1.1802519306638769043130818218765
```

Maximizer of the T’(z) is at -7.85192… according to the output.

Hence, thermocline exists at 7.8519 m deep.