P452 Computational Physics
Mid-Semester examination, 2024
NISER, Bhubaneswar

Full marks: 20 Time: 2 hours

Each question carries 4 marks. Attempt all.

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Append your answers/outputs at the end of the codes under comments.
Supplement your answer(s) with plots where ever you find appropriate.
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1. Solve the following equation to an accuracy of 10^{-6}, starting from an initial guess interval $[1.5, 2.5]$,
$$\log(x/2) - \sin(5x/2) = 0.$$
using Regula-falsi and Newton-Raphson and compare the two with respect to convergence.

2. Equation for heat conduction in a thin, un-insulated rod of length $L = 10$ m is
$$\frac{d^2T}{dx^2} + \alpha(T_a - T) = 0$$
where the heat transfer coefficient $\alpha = 0.01\,\mathrm{m}^{-2}$ parameterizes heat dissipated to the surrounding air and $T_a = 20^o$ C is the ambient temperature. If $T(x = 0) = 40^o$ C and $T(x = L) = 200^o$ C, solve the boundary value problem using *Shooting Method* with *RK4* integrator and determine at what x the temperature is $T = 100^o$ C.

3. Solve the 1-dimensional heat equation $u_{xx} = u_t$ over a conducting bar, of length 2 units, kept at 0^o C but is heated to 300^o C at its center at time $t = 0$. Choose your Δx and Δt with care such that $\Delta t/(\Delta x)^2 \ll 0.5$.

4. A 2 meter long beam has a linear mass density $\lambda(x) = x^2$, where x is measured from one its ends. Find the center of mass of the beam numerically using method of your choice. Report answer correct up to 4 decimal places.

5. Given below is a system of linear equations. Use LU decomposition to solve it.
$$\begin{aligned}
19 &= a_1 - a_2 + 4a_3 + 2a_5 + 9a_6 \\
2 &= 5a_2 - 2a_3 + 7a_4 + 8a_5 + 4a_6 \\
13 &= a_1 + 5a_3 + 7a_4 + 3a_5 - 2a_6 \\
-7 &= 6a_1 - a_2 + 2a_3 + 3a_4 + 8a_6 \\
-9 &= -4a_1 + 2a_2 + 5a_4 - 5a_5 + 3a_6 \\
2 &= 7a_2 - a_3 + 5a_4 + 4a_5 - 2a_6
\end{aligned}$$