P346 Computer Lab

End-Semester examination, 2021

NISER, Bhubaneswar

Full marks: 20 Time: 3 hours

Marks are given in bold along with the questions. Attempt all.

1. Consider the `van der Waals` equation of state

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

Use Newton's method to compute volume V to an accuracy of 10^{-4} of Cl_2 at a temperature of $T = 300\,\mathrm{K}$, given $P = 5.95$ atm, $R = 0.0821$, $a = 6.254$ and $b = 0.05422$ (all in appropriate units). Use Ideal Gas Law to estimate the initial volume V_0. **[3]**

2. A wire of length $L = 4$ units carries a charge density $\lambda(x) = \exp(-x^2/L^2)$. The potential at a height d units above a point l units distant away from $x = 0$ end of the wire is given by

$$\phi = \int_{-l}^{L-l} \frac{k\lambda(x)}{\sqrt{x^2 + d^2}}\, dx$$

Find the potential at a height 1.5m above the point 1m away from $x = 0$ end of the wire In natural unit take $k = 1$. Use Simpson's method with $N = 12$ for the number of intervals. **[3]**.

3. In an annealing experiment, the conductivity of a novel material was measured as its temperature was raised. The temperature (T) versus conductivity (σ) (in appropriate units) data are given in the file `esem_fit1.dat`. Use least square method to fit the data with the functions *(i)* $\sigma = \sigma_0\, e^{\alpha T}$ and *(ii)* $\sigma = \sigma_0\, T^{\alpha}$. Determine σ_0 and α and hence calculate the *Pearson's r* to compare the quality of fit. **[5]**

4. The distance (r) versus height (h) of the trajectory of a test missile is given in the datafile `esem_fit2.dat`. Try a quadratic fit of the form $h = a_0 + a_1 r + a_2 r^2$ and determine the highest point reached by the missile. **[5]**

5. Equation for heat conduction in a thin, un-insulated rod of length $L = 10$ m is

$$\frac{d^2T}{dx^2} + \alpha(T_a - T) = 0$$

where the heat transfer coefficient $\alpha = 0.01\,\mathrm{m}^{-2}$ parameterizes heat dissipated to the surrounding air and $T_a = 20^o\mathrm{C}$ is the ambient temperature. If $T(x = 0) = 40^o\mathrm{C}$ and $T(x = L) = 200^o\mathrm{C}$, solve the boundary value problem using *Shooting Method* with *RK4* integrator and determine at what x the temperature $T = 100^o\mathrm{C}$. **[4]**