The following problems are to be solved numerically using LU or Cholesky decomposition as demanded. All the necessary functions will go in your own library (file) while the main code will simply read the input *i.e.* initialize, call relevant functions and write down the solutions.

1. Use LU decomposition, both Crout's and Doolittle's method, to solve for x_i 's and hence compare your answers. [2+2]

$$x_1 + x_3 + 2x_4 = 6$$

$$x_2 - 2x_3 = -3$$

$$x_1 + 2x_2 - x_3 = -2$$

$$2x_1 + x_2 + 3x_3 - 2x_4 = 0$$

2. Check whether the inverse of the following matrix exists. If yes, find the inverse and verify. [3]

$$\left(\begin{array}{cccc}
0 & 2 & 8 & 6 \\
0 & 0 & 1 & 2 \\
0 & 1 & 0 & 1 \\
3 & 7 & 1 & 0
\end{array}\right)$$

3. Use Cholesky decomposition to solve the equation $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$ where, [3]

$$\mathbf{A} = \begin{pmatrix} 10.0 & 1.0 & 0.0 & 2.5 \\ 1.0 & 12.0 & -0.3 & 1.1 \\ 0.0 & -0.3 & 9.5 & 0.0 \\ 2.5 & 1.1 & 0.0 & 6.0 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 2.20 \\ 2.85 \\ 2.79 \\ 2.87 \end{pmatrix}$$