P452 - Computational Physics
Assignment 3

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Question 3

Data

Grade categories: [A, B, C, D, E]
Observed frequencies: [77, 150, 210, 125, 38]
Degrees of freedom: n-1 = 4

Calculation of Expected frequencies

Standard normal distribution function is given as

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2}\right)$$

where μ is the mean grade. From this, we obtain the expected frequencies as
Expected frequencies: [32.395, 145.182, 239.365, 145.182, 32.394]

χ^2-statistic

The chi-squared statistic is calculated using the formula:

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where O is the observed frequency and E is the expected frequency.

Plugging in the values:

$$\begin{aligned}
\chi^2 =& \frac{(77-32.395)^2}{32.395} + \frac{(150-145.182)^2}{145.182} + \frac{(210-239.365)^2}{239.365} \\
&+ \frac{(125-145.182)^2}{145.182} + \frac{(38-32.394)^2}{32.394} \\
\chi^2 &= 68.957
\end{aligned}$$

Critical Values: Using a χ^2 distribution table with $df = 4$, the critical values are:

- Critical value (5% significance level): $\chi^2_{0.05} = 9.488$
- Critical value (10% significance level) : $\chi^2_{0.10} = 7.779$

Conclusion: Since the χ^2 statistic is lower for both 5% and 10% significance levels, both the null-hypothesis are rejected.

Question 4

Data

Shipment A: [4.65, 4.84, 4.59, 4.75, 4.63, 4.75, 4.58, 4.82, 4.86, 4.60, 4.77, 4.65, 4.80]
Shipment B: [4.75, 4.79, 4.74, 4.74, 4.77, 4.58, 4.81, 4.80]

The sample mean and variances can be calculated as

$$\bar{x}_1 = \frac{\sum_{i=1}^{n_1} x_{1i}}{n_1} = 4.7146 \qquad s_1^2 = \frac{\sum_{i=1}^{12} (x_{1i} - \bar{x}_1)^2}{12 - 1} = 0.0973$$

$$\bar{x}_2 = \frac{\sum_{i=1}^{n_2} x_{2i}}{n_2} = 4.74 \qquad s_2^2 = \frac{\sum_{i=1}^{8} (x_{2i} - \bar{x}_2)^2}{8 - 1} = 0.0697$$

F-test

The F-statistic is calculated as $F = \frac{s_1^2}{s_2^2} = 1.9499$

Critical Value:

- For significance level $\alpha = 0.05$ and degrees of freedom $df_1 = 12$ and $df_2 = 7$, the critical value of F-statistic = 3.9999
- For significance level $\alpha = 0.10$ and degrees of freedom $df_1 = 12$ and $df_2 = 7$, the critical value of F-statistic = 2.9047

Conclusion: The obtained f value is less than the critical value for both 5% and 10% so, we fail to reject the null hypothesis and thus the variance are close and not very different.

T-test

Given two sets of data, x_1 and x_2, representing shipments A and B respectively, we want to test if the means of the two populations are significantly different.

We already have calculated the mean and variance for both the datasets. We will now calculate the pooled standard deviation.

Pooled Standard Deviation: Compute the pooled standard deviation to estimate the common standard deviation of the two populations:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = 0.089$$

t-Statistic: Compute the t-statistic, which measures the difference between the sample means in terms of the pooled standard deviation:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p} = -0.285$$

Critical Value:

- For significance level $\alpha = 0.05$ and degrees of freedom $df_1 = 12$ and $df_2 = 7$, the critical value of F-statistic = 1.734
- For significance level $\alpha = 0.10$ and degrees of freedom $df_1 = 12$ and $df_2 = 7$, the critical value of F-statistic = 1.3304

Conclusion: The obtained t value is less than the critical value for both 5% and 10% so, we fail to reject the null hypothesis and thus the means are close and not very different.

Final conclusion The lenses are from the same population in the two sets.