Assignment 6.   Marks 10

Posted on 04.11.2021 and due on 12.11.2021 midnight

The following problems involving integration are to be solved numerically using Midpoint, Trapezoidal and Simpson method. Code these algorithms in your library and call them as required by passing appropriate variables, like integration limit, functions, interval length etc. For step number, you have to work out the maximum error by hand.

1. Numerically integrate the following using Midpoint, Trapezoidal and Simpson techniques for $N = 8$, 16 and 24 and compare the result (in tabular format) with the actual analytical result. **[2.5]**

$$\int_1^4 \sqrt{1 + \frac{1}{x}}\, dx$$

2. Numerically integrate the following using Midpoint, Trapezoidal and Simpson techniques with maximum error of 0.001. **[2.5]**

$$\int_0^1 x\sqrt{1 + x}\, dx$$

3. Use Monte Carlo to estimate the value of π from the following integral

$$\int_0^1 \frac{4}{1 + x^2}\, dx = \pi$$

Use system built random number generator and sample N starting from 10 and keep increasing in multiple of 10. Go as far as possible within a reasonable time. Plot π vs. N. **[3]**

4. A beam 2 meter long has linear mass density $\lambda(x) = x^2$, where x is measured from one of the ends. Find the center of mass of the beam numerically. **[2]**