

各種謬誤：積分篇

例.

- $\int \frac{dx}{x^2 - a^2} = - \int \frac{dx}{a^2 - x^2} = - \int \frac{dx}{a^2 + (-x^2)} = -\tan^{-1}(-x)$
- $\int \frac{dx}{x^2 - a^2}$: 令 $x = a \sec \theta$, $dx = a \sec \theta \tan \theta d\theta$ 。則 $\int \frac{dx}{x^2 - a^2} = \int \frac{a \sec \theta \tan \theta d\theta}{a^2 \tan^2 \theta} = \frac{1}{a} \int \frac{1}{\sin \theta} d\theta = \frac{1}{a} \ln |\sin \theta|$
- $\int \frac{x}{\sqrt{1-x^2}} dx$: 令 $x^2 = u$, $2x dx = du \Rightarrow x dx = \frac{1}{2} du$ 。則 $\int \frac{x}{\sqrt{1-x^2}} dx = \int \frac{\frac{1}{2} du}{\sqrt{1-u}} = \frac{1}{2} \int \frac{du}{\sqrt{1-u}} = \frac{1}{2} \int \frac{du}{\sqrt{1-(u^{\frac{1}{2}})^2}} = \frac{1}{2} \sin^{-1} u^{\frac{1}{2}}$
- $\int \cos(\ln x) dx$: 令 $\ln x = u$, $\frac{1}{x} dx = du \Rightarrow dx = x du$ 。則 $\int \cos(\ln x) dx = \int \cos u \cdot x du = x \sin(\ln x)$
- $\int \frac{x}{\sqrt{1-x^2}} dx = \int x \cdot \frac{1}{\sqrt{1-x^2}} dx = \int x dx \cdot \int \frac{1}{\sqrt{1-x^2}} dx = \frac{x^2}{2} \cdot \sin^{-1} x$

例. 微積分基本能力檢核

- $(x^\pi)' = \pi x^{\pi-1}$
- $(\pi^x)' = \pi^x \ln \pi$
- $(xe^{\pi x})' = e^{\pi x} + \pi x e^{\pi x}$
- $(e^{\pi x^2})' = e^{\pi x^2} 2\pi x$
- $(x^x)' = x^x (\ln x + 1)$
- $\int x^\pi dx = \frac{x^{\pi+1}}{\pi+1}$
- $\int \pi^x dx = \frac{\pi^x}{\ln \pi}$
- $\int \ln x dx = x \ln x - x$
- $\int x e^{\pi x^2} dx = \frac{e^{\pi x^2}}{2\pi}$
- $\int x^2 e^{\pi x} dx = e^{\pi x} \left(\frac{x^2}{\pi} - \frac{2x}{\pi^2} + \frac{2}{\pi^3} \right)$

錯誤答案例：

- $(e^{\pi x})' = x e^\pi$
- $(\pi^x)' = x \pi$
- $(e^{-x^2})' = 2e^{-x}$
- $\int x^\pi dx = \frac{1}{2} \pi^{x^2}$
- $\int \ln x dx = \frac{1}{2} \ln x$
- $\int e^{\pi x} dx = \frac{1}{4} e^{\pi^2 x^2}$
- $\int x^2 e^x dx = \frac{1}{3} x^3 \cdot \frac{1}{2} e^{x^2}$
- $\int x e^{-x^2} dx = \frac{1}{2} x^2 \cdot \frac{1}{3} e^{-x^3}$