ID2204: Assignment 2

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1 Simple linear inequality (SUB, 2 points)

To define a propagator for the linear inequality $ax + by \leq c$, let us first rewrite the inequality under the assumption $a, b > 0$:

$$x \leq \frac{c - by}{a} \tag{1}$$

$$y \leq \frac{c - ax}{b} \tag{2}$$

Then, the propagator $p_{\leq}$ is defined as:

$$\begin{aligned} p_{\leq}(s) = \{x \rightarrow \{n \in s(x) \,|\, n \leq \frac{c - b \cdot min(s(y))}{a}\}, \\ y \rightarrow \{n \in s(y) \,|\, n \leq \frac{c - a \cdot min(s(x))}{b}\}\} \end{aligned} \tag{3}$$

With the above propagator, we can observe that both stores $s(x)$ and $s(y)$ get reduced from the higher side (informally the right side). The result of this is that unless one or both stores becomes empty when applying the propagator, the value of $min()$ function will be the same with all subsequent re-applications. This implies idempotence.

A fix-point is reached after one propagation (for all combinations of a, b). Note that all stronger stores are also fix-points because the stores are reduced from the higher side and the propagator idempotent. This is the definition of subsumption and can thus be detected as early as after one propagation.

2 Changing Propagation Order (SUB, 1 point)

Let us consider the following example:

- For the variables: $V = \{x, y\}$
- For the allowed values: $U = \{1, 2\}$
- For the store: $s = \{x \rightarrow U,\, y \rightarrow U\}$

Then, one propagator p_1 can be:

$$p_1(s) = \{x \rightarrow \{1\} \cap s(x),\, y \rightarrow s(y)\} \tag{4}$$

And, a second one p_2 can be:

$$p_2(s) = \{x \to s(x) \cap s(y),\, y \to s(x) \cap s(y)\} \tag{5}$$

With these propagators, we have the following unequal results:

$$p_1(p_2(s)) = \{x \to \{1\},\, y \to \{1, 2\}\} \tag{6}$$

$$p_2(p_1(s)) = \{x \to \{1\},\, y \to \{1\}\} \tag{7}$$

$$p_1(p_2(s)) \neq p_2(p_1(s)) \tag{8}$$

We can conclude that changing the propagation order has an impact on the final result. The statement is false.

3 Idempotent Propagators (SUB, 2 points)

The first statement can be rewritten as:

$$\forall\, s,\, there\, \exists\, n \in \mathbb{N}^+\, that\, p^{n+1}(s) = p^{n(s)} \tag{9}$$

Per definition of the general properties of a propagator it computes stronger stores, where a store is a finite contracting set. This alone implies that for a given store it is enough to re-apply a propagator a finite number of times until we reach a fix-point (or completely emptied the store). Further, this means that any propagator have an n where it is idempotent which is what was stated. This doesn't need any further proof.

We can strengthen to further encompass more generalized sets:

$$\exists\, n \in \mathbb{N}^+\, that\, \forall\, s,\, p^{n+1}(s) = p^n(s) \tag{10}$$

With similar reasoning as in the previous statement (stores are finite sets and becoming stronger), it is possible to choose a sufficiently large n to encompass all stores and thus all sets in that form. This can be shown with an induction proof.

However, for arbitrary functions on arbitrary sets, the statement doesn't hold as a propagator has to be contracting and the order on stores needs to be well defined. A simple counterexample is trying to apply some function on the infinite set $\mathbb{Z}$, it is always further reducible.

Idempotency is very interesting because if a particular propagator is idempotent, then it calculates fix points of it self which can be used to (if we know when it happens) increase the general propagation loop speed.