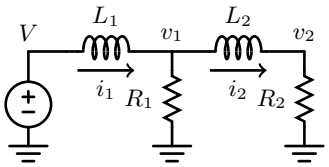
Numerical Analysis

Homework 14. Solving an LR Ladder Circuit

Due: June 16, 2015

In this homework, a stiff dynamic system is solved using different solution methods. The LR ladder circuit is shown below.



Where we have $L_1 = 1 \times 10^{-12}$ Henry, $L_2 = 1 \times 10^{-9}$ Henry, and $R_1 = R_2 = 1\ \Omega$. At $t = 0$, $V(t) = v_1(t) = v_2(t) = 0$ Volts, and $i_1(t) = i_2(t) = 0$ Amperes; when $t > 0$, $V(t) = 1$ Volts.

1. Assuming the system variables are $i_1(t)$ and $i_2(t)$, formulate the system equation for $t > 0$ in the following form, where $f_1(t)$ and $f_2(t)$ are functions of $i_1(t)$, $i_2(t)$ and $V(t)$.

$$L_1 \frac{\mathrm{d}i_1(t)}{\mathrm{d}t} = f_1(i_1, i_2, V), \tag{14.1}$$

$$L_2 \frac{\mathrm{d}i_2(t)}{\mathrm{d}t} = f_2(i_1, i_2, V). \tag{14.2}$$

2. Using trapezoidal rule method with time step of $h = 1 \times 10^{-13}$, solve for $i_1(t)$ and $i_2(t)$, $0 <= t <= 5 \times 10^{-8}$. Print out $i_1(t)$ and $i_2(t)$ for $t = 1 \times 10^{-9}, 2 \times 10^{-9}, \cdots, 4.9 \times 10^{-8}, 5 \times 10^{-8}$. Use this solutions as the base line to compare the solutions of the following questions.

3. Using trapezoidal rule method with time step of $h = 1 \times 10^{-11}$, solve for $i_1(t)$ and $i_2(t)$, $0 <= t <= 5 \times 10^{-8}$. Print out $i_1(t)$ and $i_2(t)$ for $t = 1 \times 10^{-9}, 2 \times 10^{-9}, \cdots, 4.9 \times 10^{-8}, 5 \times 10^{-8}$.

4. Using trapezoidal rule method with time step of $h = 1 \times 10^{-9}$, solve for $i_1(t)$ and $i_2(t)$, $0 <= t <= 5 \times 10^{-8}$. Print out $i_1(t)$ and $i_2(t)$ for $t = 1 \times 10^{-9}, 2 \times 10^{-9}, \cdots, 4.9 \times 10^{-8}, 5 \times 10^{-8}$.

5. Using 2nd order forward integration method with time step of $h = 1 \times 10^{-11}$, solve for $i_1(t)$ and $i_2(t)$, $0 <= t <= 5 \times 10^{-8}$. Print out $i_1(t)$ and $i_2(t)$ for $t = 1 \times 10^{-9}, 2 \times 10^{-9}, \cdots, 4.9 \times 10^{-8}, 5 \times 10^{-8}$.

6. Using 2nd order forward integration method with time step of $h = 1 \times 10^{-9}$, solve for $i_1(t)$ and $i_2(t)$, $0 <= t <= 5 \times 10^{-8}$. Print out $i_1(t)$ and $i_2(t)$ for $t = 1 \times 10^{-9}, 2 \times 10^{-9}, \cdots, 4.9 \times 10^{-8}, 5 \times 10^{-8}$.

7. Using 2nd order Gear's integration method with time step of $h = 1 \times 10^{-11}$, solve for $i_1(t)$ and $i_2(t)$, $0 <= t <= 5 \times 10^{-8}$. Print out $i_1(t)$ and $i_2(t)$ for $t = 1 \times 10^{-9}, 2 \times 10^{-9}, \cdots, 4.9 \times 10^{-8}, 5 \times 10^{-8}$.

8. Using 2nd order Gear's integration method with time step of $h = 1 \times 10^{-9}$, solve for $i_1(t)$ and $i_2(t)$, $0 <= t <= 5 \times 10^{-8}$. Print out $i_1(t)$ and $i_2(t)$ for $t = 1 \times 10^{-9}, 2 \times 10^{-9}, \cdots, 4.9 \times 10^{-8}, 5 \times 10^{-8}$.

9. Please compare the solutions obtained in the previous questions and state your observations.

Notes.

1. For this homework you need to turn in a set of C++ source codes. That includes `hw14.cpp`, which solves question 8 above, `MAT.h`, `MAT.cpp`, `VEC.h` and `VEC.cpp` files.
2. A `pdf` report file is also needed. Please name this file `hw14a.pdf`.
3. Submit your files on EE workstations. Please use the following command to submit your homework 14.

   ```
   $ ~ee407002/bin/submit hw14 hw14a.pdf hw14.cpp MAT.h MAT.cpp VEC.h VEC.cpp
   ```

   where `hw14` indicates homework 14.
4. Your report should be clearly written such that I can understand it. The writing, including English grammar, is part of the grading criteria.