

Numerical Analysis

Homework 8. Matrix Eigenvalues

Due: April 28, 2015

The power method and inverse power method, and their variation with shifting, are simple ways to find the extreme values of matrix spectrum. However, some applications need to find most or all eigenvalues. In this case, the QR iteration and shifted QR iteration methods should be better choices. Please implement the following functions for QR and shifted QR iterations.

```
int EVqr(MAT &A,double tol,int maxiter);
int EVqrShifted(MAT &A,double mu,double tol,int maxiter);
```

Both functions have the following input arguments:

MAT A: the matrix of which the eigenvalues are to be found.

double tol: the tolerance for convergence check.

int maxiter: the maximum number of iterations to be executed.

In addition, the `EVqrShifted()` function takes one more argument, **mu** for the shifted value.

Both functions return the number of iterations if they find the error smaller than **tol**, or they return **maxiter** if such condition cannot be met. If matrix $\mathbf{A} = [a_{i,j}], 1 \leq i, j \leq n$, the error for each iteration is defined as

$$\text{error} = \max_{2 \leq i \leq n} |a_{i-1,i}| \quad (8.1)$$

It is the largest magnitude of the elements just below the diagonal. For this homework, the tolerance is set to be 10^{-9} .

1. Given six matrices, **m3.dat**, **m4.dat**, **m5.dat**, **m6.dat**, **m7.dat**, and **m8.dat**, please find the three largest eigenvalues and the three smallest eigenvalues using QR iterations. Please also record the number of iterations and CPU time for each matrix.
2. For those six matrices, please find their three largest and three smallest eigenvalues using shifted QR iterations with **mu=0.5**. Please also record the number of iterations and CPU time for each matrix.
3. Please state your observations on the answers you found for the two questions above.

Notes.

1. For this homework you need to turn in a set of **C++** source codes. That includes **hw08.cpp**, which finds the answers for question 2, **MAT.h**, the new header file, **MAT.cpp**, which implements the necessary QR iteration methods, and the **VEC.h** and **VEC.cpp**. The compiled program should be executed in the same way as **hw01**. Unix input redirection should be used to redirect the standard input to a specific file as

```
$ time .\a.out < m3.dat
```

And the program outputs the requested eigenvalues and the number of iterations. The **time** command reports the CPU time used.

2. A pdf file is also needed. Please name this file `hw08a.pdf`.
3. Submit your files on EE workstations. Please use the following command to submit your homework 8.

```
$ ~ee407002/bin/submit hw08 hw08a.pdf hw08.cpp MAT.h MAT.cpp VEC.h VEC.cpp
```

where `hw08` indicates homework 8.

4. Your report should be clearly written such that I can understand it. The writing, including English grammar, is part of the grading criteria.