Numerical Analysis HW11: Numerical Integrations

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1 Objective

In this assignment, we are given different numbers of support points in each .dat file, with $475 \leq x \leq 775$. We are required to implement composite n'th order Newton-Cotes(N.-C.) formula. I implemented

double integ(VEC &X,VEC &Y,int n);

as my integration function, X as the x-axis coordinates, Y as the function values, $f(x_i)$, and n as the order of the integration.

2 Implementation of Composite Newton-Cotes formula

Newton-Cotes formulae are based on Lagrange interpolation with equally spaced nodes in [a,b], the integration interval. With Lagrange interpolation,

$$I_n(f) = \int_{x=a}^{b} \sum_{i=0}^{n} \prod_{k=0,k\neq i}^{n} \frac{x-x_k}{x_i-x_k} f(x_i)\,dx = \sum_{i=0}^{n} f(x_i) \int_{t=0}^{n} \prod_{k=0,k\neq i}^{n} \frac{t-k}{i-k} h dt = h \sum_{i=0}^{n} w_i f(x_i) \tag{6.1.22}$$

with

$$w_i = \int_{t=0}^{n} \prod_{k=0,k\neq i}^{n} \frac{t-k}{i-k} dt. \tag{6.1.23}$$

and

$$h = \frac{b-a}{n}, \; x_i = a + ih,$$

where w_i can be pre-calculated and $I_n(f)$ denotes the $n'th$ order integration.

As for composite Newton-Cotes:

The composite Newton-Cotes Formulas divide the integration interval $[a, b]$ into m subintervals, $[a_i, b_i]$, $i = 0, \cdots, m-1$, with $a_0 = a$; $a_{i+1} = b_i = a + (b-a)/m$, $i = 0, \cdots, m-2$; $b_{m-1} = b$.

Then carry out Newton-Cotes integration on each subinterval $[a_i, b_i]$.

The overall integration is the sum of the integrations of the subintervals.

$$I(f) = \int_a^b f(x)\,dx = \sum_{i=0}^{m-1} \int_{a_i}^{b_i} f(x)\,dx$$
$$= h \sum_{i=0}^{m-1} \sum_{k=0}^{n} w_k f(a_i + kh) \tag{6.1.37}$$

where an n-th order quadrature is assumed and $h = (b-a)/mn$.

Notice that the mn should be exactly equal the number of intervals, so for example, if the input file is f301 then number of intervals is 300 and if n is 5, then m will then be $\frac{300}{5} = 60$.

3 Workflow

Usage: ./hw11.out * Δ < f*.dat, where * could be 21 or 301, Δ could be 1, 2, 4, 5 in this assignment. For example, ./hw11.out 21 1< f21.dat

Solve: N.-C. method applied on input file.

Desired output: The program will print out the result of integration in command line.

4 Results

First order Newton-Cotes integration on f301.dat is 28007.679797

Table 1: N.-C. integration on f21.dat

Order	Value	$\lvert Error w.r.t. f301 \rvert$
1	28109.399520	101.719723
2	28193.136660	185.456863
4	28161.306744	153.626947
5	28157.922555	150.242758

5 Plot Analysis

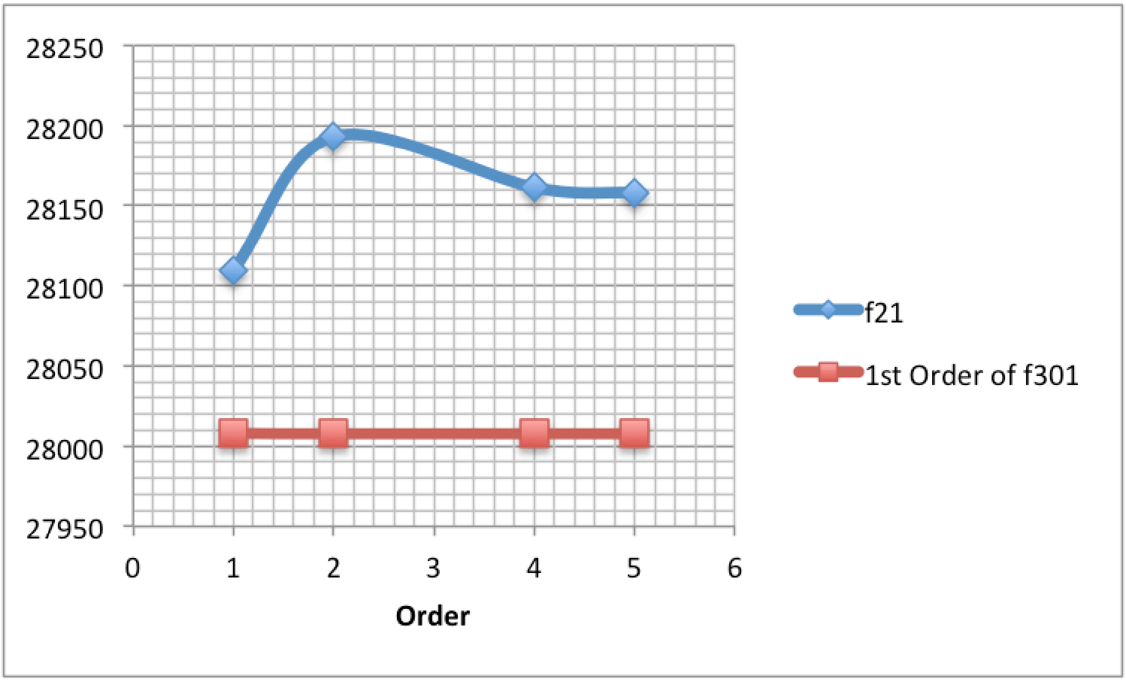

Figure 1: Integration Values

Figure 2: Error

6 Observations

The standard result should be 28007.679797, the result of first order N.-C. integration on f301, since f301 contains much more support points than f21 does. Knowing that trapezoidal and Simpson formulae are special instances of the Newton-Cotes formulas when order is 1 and 2, we can view the error plot to see how accurate they really are, and amazingly the result shows us that for this assignment, trapezoidal method would be the most accurate one. As for the trend, since I only did 4 orders, I can only say that the error will start declining after order of 2.