Numerical Analysis

Homework 7. Matrix Condition Numbers

Due: April 21, 2015

The matrix condition number plays an important role in the error analysis of a linear system. It has been shown that for a symmetric and positive definite matrix, $\mathbf{A}$, the matrix condition number is

$$\kappa_2(\mathbf{A}) = \frac{\lambda_{max}}{\lambda_{min}}. \tag{7.1}$$

The power method and inverse power method, and their variation with shifting, are simple ways to find the extreme values of matrix spectrum. Please implement necessary functions in your `MAT` class to solve the following questions to the accuracy of 10^{-9}.

1. Please construct the symmetric matrix for Q1 of `hw04` and find its condition number. Please record the CPU time as well.
2. Please construct the symmetric matrix for Q2 of `hw04` and find its condition number. Please record the CPU time as well.
3. Please construct the symmetric matrix for Q3 of `hw04` and find its condition number. Please record the CPU time as well.
4. Please construct the symmetric matrix for Q4 of `hw04` and find its condition number. Please record the CPU time as well.
5. Continue from the last question. The resistor mesh now has 40 resistor per side with 50 Ω resistance each. Please construct the symmetric matrix and find its condition number. Please record the CPU time as well.
6. Please state your observations on the answers you found for the five questions above.

Notes.

1. For this homework you need to turn in a set of C++ source codes. That includes `hw07.cpp`, which finds all four matrix condition numbers. `MAT.h`, the new header file, `MAT.cpp`, which implements the necessary power methods, and the `VEC.h` and `VEC.cpp`. The compiled program should be executed in the same way as `hw04`. The command line argument is to be used to specified the number of resistor per side. Thus, for question 1 one needs to type the following command to run.

   ```
   $ .\a.out 2
   ```

   And the program outputs the matrix condition number.
2. A `pdf` file is also needed. Please name this file `hw07a.pdf`.
3. Submit your files on EE workstations. Please use the following command to submit your homework 7.

   ```
   $ ~ee407002/bin/submit hw07 hw07a.pdf hw07.cpp MAT.h MAT.cpp VEC.h VEC.cpp
   ```

   where `hw07` indicates homework 7.
4. Your report should be clearly written such that I can understand it. The writing, including English grammar, is part of the grading criteria.