Numerical Analysis HW06: Conjugate Gradient Method

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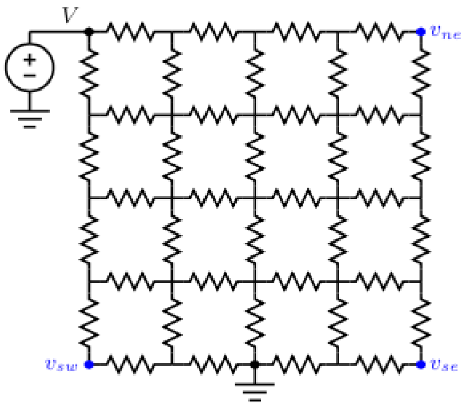
April 16, 2015

1 Objective

The objective of Conjugate Gradient Method(a.k.a. CGM) is to find the minimum solution of a linear system iteratively as efficient as possible. In this homework, we are supposed to solve multiple resistor networks like the following figure using CGM but with different resistors per side, get corner voltages and equivalent resistance, and then compare the results with the those of hw04.cpp, which is implemented with LU decomposition.



2 Workflow

Usage: For example, ./hw06.out *num*. *num* is the number of resisters per side and it could be any number, but even number is preferred.

Form matrix: For linear system $Ax = b$, first build system matrix A with Kirchoff node method.

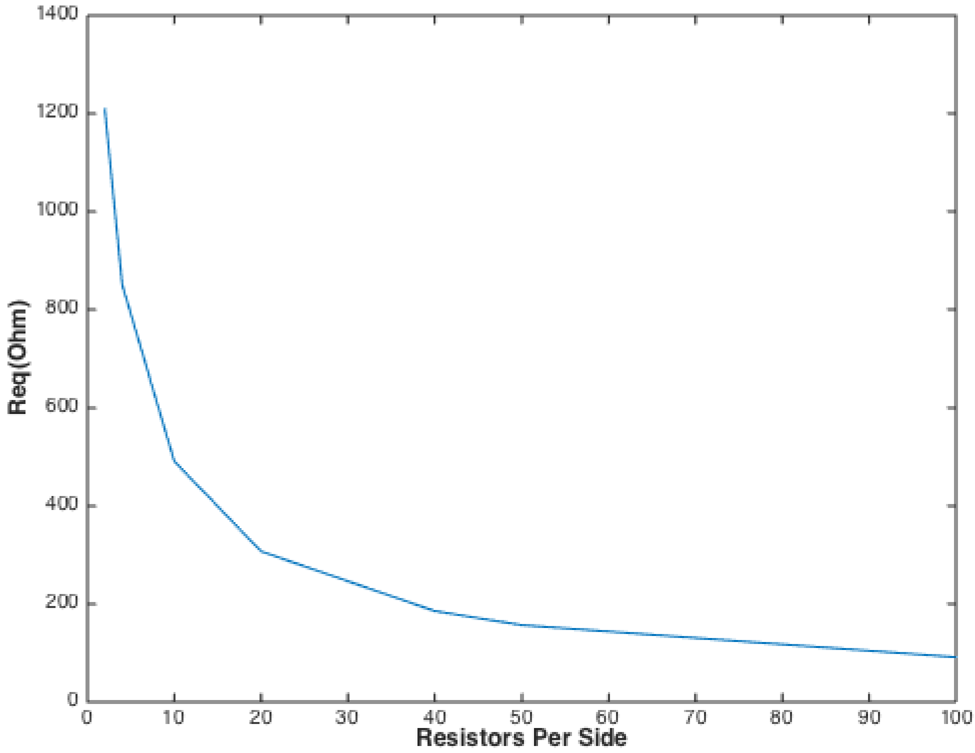
Modify matrix: In order to satisfy the limitation of CGM, A has to be positive definite matrix, so it is important to modify the system matrix to reach the this confinement.

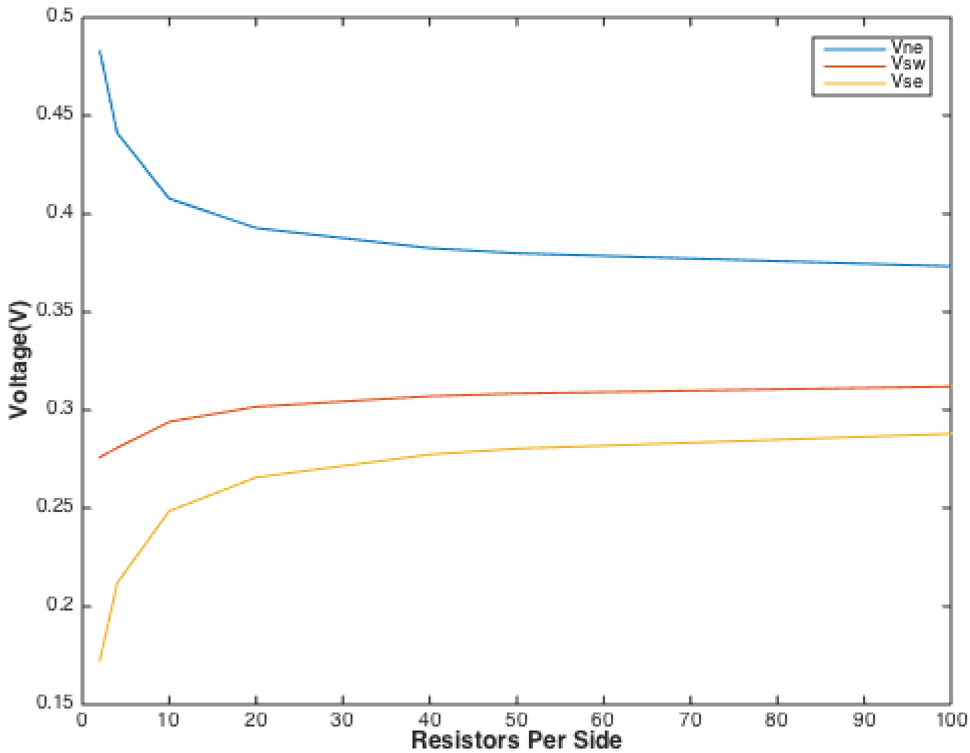
Initialize RHS: Since LHS is modified, RHS shall simultaneously be assigned accordingly so that the linear system stay equivalent to the original one.

Solve: Eventually, use CGM to solve the linear system.

3 Results

Resistors Per Side	Iteration number	$Vne(V)$	$Vsw(V)$	$Vse(V)$	Time(s)	$Req(Ohm)$
2	9	0.482759	0.275862	0.172414	0	1208.333333
2(hw4)	N/A	0.482759	0.275862	0.172414	0	1208.333333
4	20	0.441080	0.280747	0.211722	0	850.525689
4(hw4)	N/A	0.441080	0.280752	0.211722	0.002	850.530303
10	49	0.407872	0.293964	0.248430	0.017	491.202658
10(hw4)	N/A	0.407838	0.294016	0.248441	0.012	491.202388
20	96	0.392841	0.301598	0.265615	0.401	307.389238
20(hw4)	N/A	0.392824	0.301635	0.265600	0.510	307.389274
40	184	0.382571	0.307028	0.277334	19.540	185.644983
40(hw4)	N/A	0.382576	0.307032	0.277288	28.149	185.645028
50	226	0.379942	0.308437	0.280325	64.904	156.853212
50(hw4)	N/A	0.379957	0.308423	0.280271	104.032	156.853262
100	426	0.373215	0.312095	0.287907	2005.044	91.476893
100(hw4)	N/A	0.373304	0.311969	0.287845	6229.428	91.476989


1400
1200
1000
800
600
400
200
0
Req(Ohm)
0
10
20
30
40
50
60
70
80
90
100
Resistors Per Side


0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
Voltage(V)
Vne
Vsw
Vse
0
10
20
30
40
50
60
70
80
90
100
Resistors Per Side

4 Observations

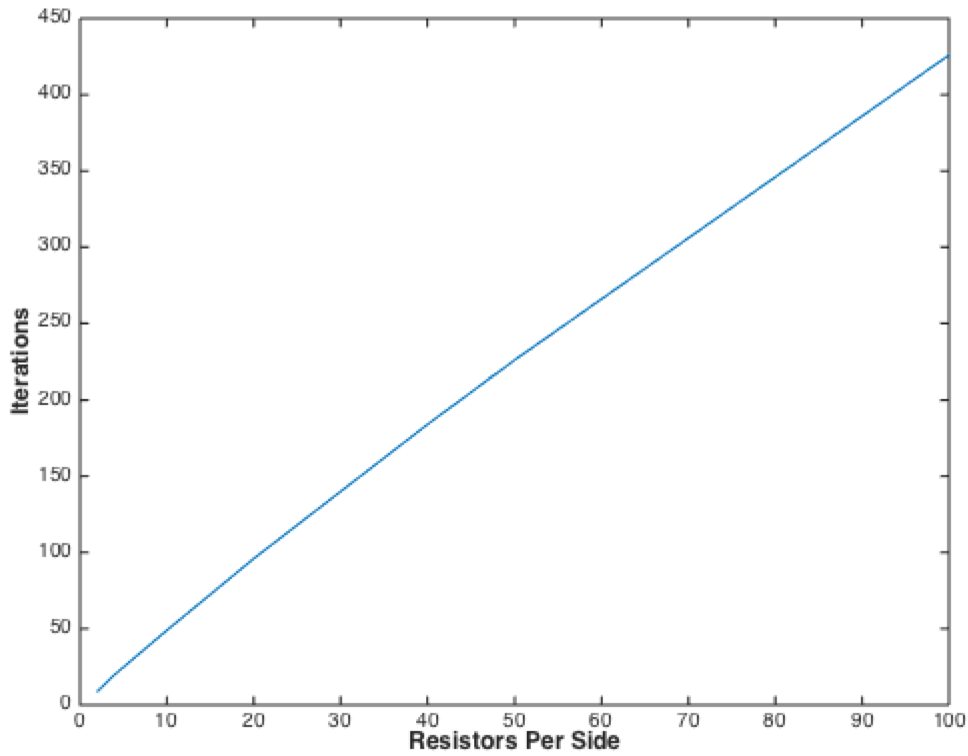
Iteration: The number of iterations of CGM for each problem is roughly linear to the resistors per side, that is, CGM is highly efficient for the iterations is linear to square root of the size of matrix A.

Node Voltage: The voltage is at least accurate to the second digit after the decimal point; for most cases, it is precise to the third digit after the decimal point. This explains the reason why people are using CGM since the precision is acceptable. For all Vne, Vsw, Vse, they tend to converge while the network size expands.

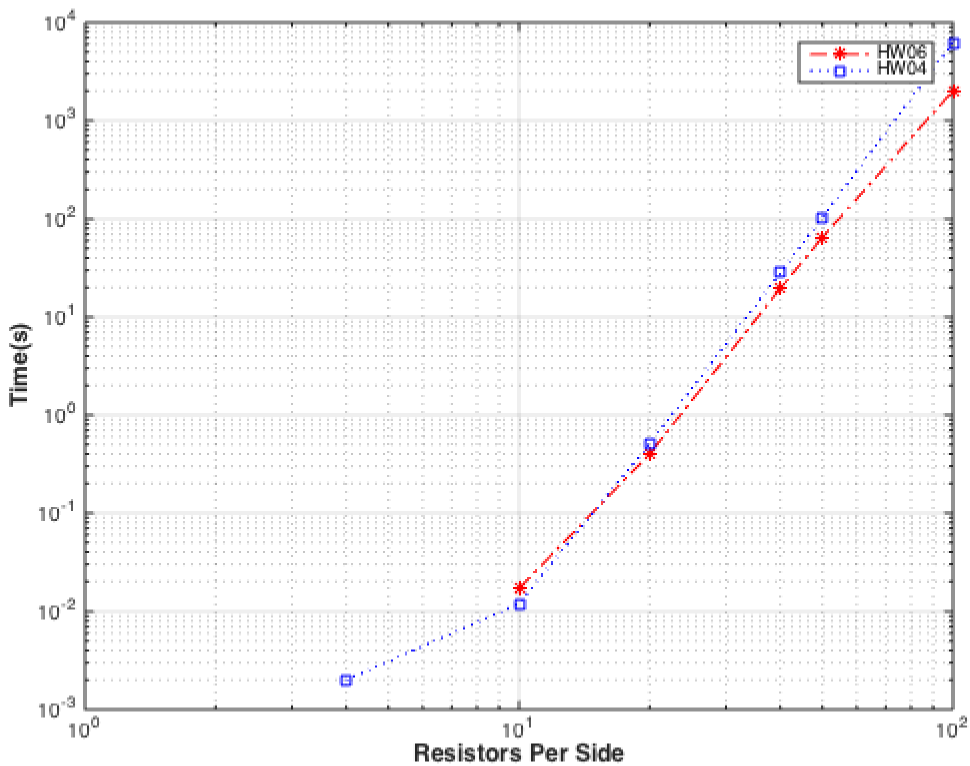
Time analysis: As I have discussed, CGM is a very fast method to converge. Using CGM is normally faster than using LU method when tackling large amounts of variables(>100) in the linear system. The complexity for HW4 is $O(n^{2.9974})$, calculated by $\frac{log(28.149)-log(0.510)}{log((40+1)^2)-log((20+1)^2)}$, and the counterpart for HW6 is not that stable but roughly $O(n^{2.9043})$, derived from $\frac{log(19.410)-log(0.396)}{log((40+1)^2)-log((20+1)^2)}$, which means CGM has similar complexity to LU decomposition but the coefficient of the time complexity is smaller.

Equivalent resistance: The equivalent resistance tend to exponentially decrease and the inclination to convergence is high.

Overall: The precision of CGM to me is accurate enough since no matter for equivalent resistance or node voltages, the differences between HW4 and HW6 are not unacceptable. In addition, the CGM is much faster when the variables to be solved are massive. Therefore, CGM is a better choice when solving this kind of problems.



(a) b



(b) a

Figure 1: abc