Numerical Analysis HW13: Solving a Simple RK Circuit

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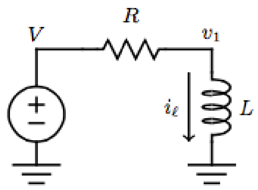
0.1 Objective

In this assignment, we are required to solve[1] simple RL circuit using three ODE methods, Forward Euler, Backward Euler, and Trapezoidal method.

The circuit is like:



For this homework, we have that R is $1\,\Omega$, L is 1×10^{-9} Henry. And at $t = 0$, $V(t) = v_1(t) = 1$ Volt and $i_\ell(t) = 1$ Ampere; while $V(t) = 0$ Volts for $t > 0$.

0.2 Implementation: Three ODE methods

For this circuit, we have

$$\frac{di(t)}{dt} = \frac{V(t) - R * i(t)}{L} = f(t, i(t))$$

as the system equation.

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$$\left|\frac{1}{2}\right| |\frac{1}{2}| \tag{1}$$

$$\frac{1}{\sqrt{2\pi}} \int_1^\infty \frac{1}{x} dx \frac{1}{\sqrt{2\pi}} \int_1^\infty \frac{1}{x}\, dx \quad \text{is divergent} \tag{2}$$

By applying Forward Euler method2, which(2) is

$$i(t + h) = i(t) + h * f(t, i(t))$$

we have

$$i(t + h) = i(t) + \frac{h(V(t) - R * i(t))}{L}$$

By applying Backward Euler method, which is

$$i(t + h) = i(t) + h * f(t + h, i(t + h))$$

we have

$$(1 + \frac{hR}{L}) * i(t + h) = i(t) + \frac{hV(t)}{L}$$

[1] eric chiu

By applying Trapezoidal method, which is

$$i(t+h) = i(t) + h * \frac{f(t, i(t)) + f(t+h, i(t+h))}{2}$$

we have

$$(1 + \frac{hR}{2L}) * i(t+h) = i(t) + \frac{hV(t)}{L} - \frac{hR * i(t)}{2L}$$

As for analytical function for $i(t)$, it can be solved as $i(t) = e^{-\frac{R}{L}t}$

0.3 Workflow

Usage: ./hw13.out δ, where δ specifies different method, 1 for Forward Euler, 2 for Backward Euler, 3 for Trapezoidal Method. For example, ./hw13.out 1

Solve: One of the three methods is applied.

Desired output: The program can generate three different files, which are "data*.csv", "*" corresponds to the three methods mentioned above. The format of the csv file is as follow: Time, value using the specific method, analytical value, error.

Notice: In the program, I am not able to print out specifically the value when t = 10^{-9} etc,. I suspect the reason is that the computer can not discern such precise accuracy.

0.4 Result and Plots

Table 1: Current

Time(s)	Forward Euler(A)	Backward Euler(A)	Trapezoidal(A)	Analytical(A)
0	1	1	1	1
1E-9	0.36417	0.371528	0.366648	0.367879
2E-9	0.13262	0.138033	0.134431	0.135335
3E-9	0.048296	0.051283	0.049289	0.049787
4E-9	0.017588	0.019053	0.018072	0.018316
5E-9	0.006405	0.007079	0.006626	0.006738

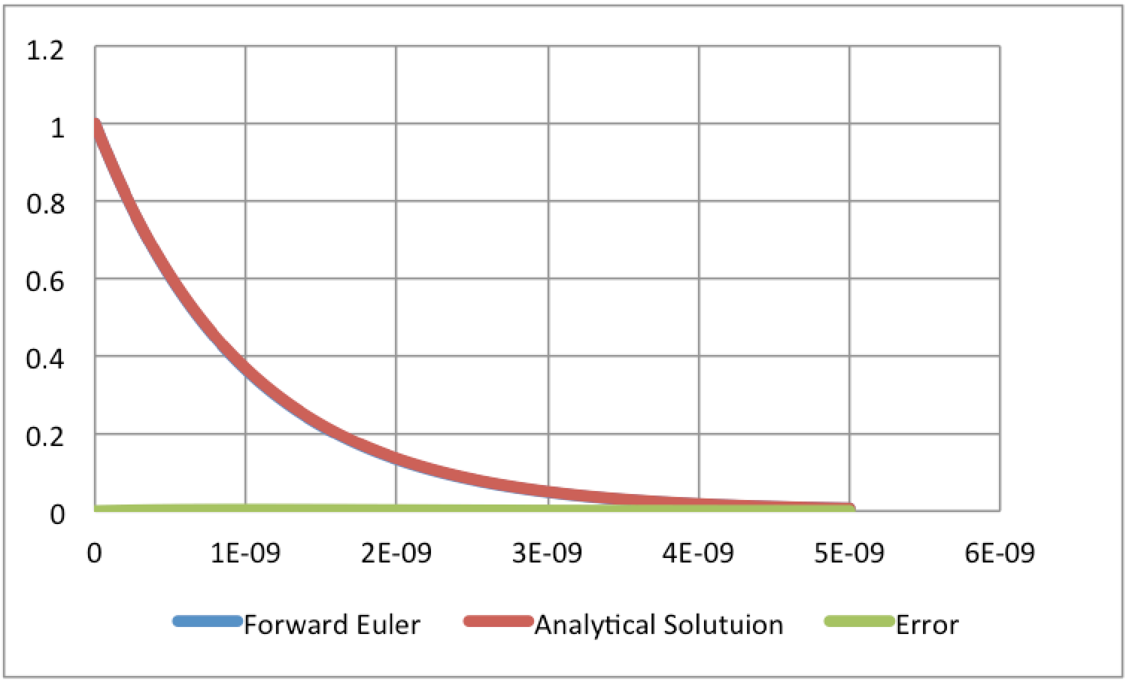

Figure 1: Forward Euler Method I(A)-t(s) Plot

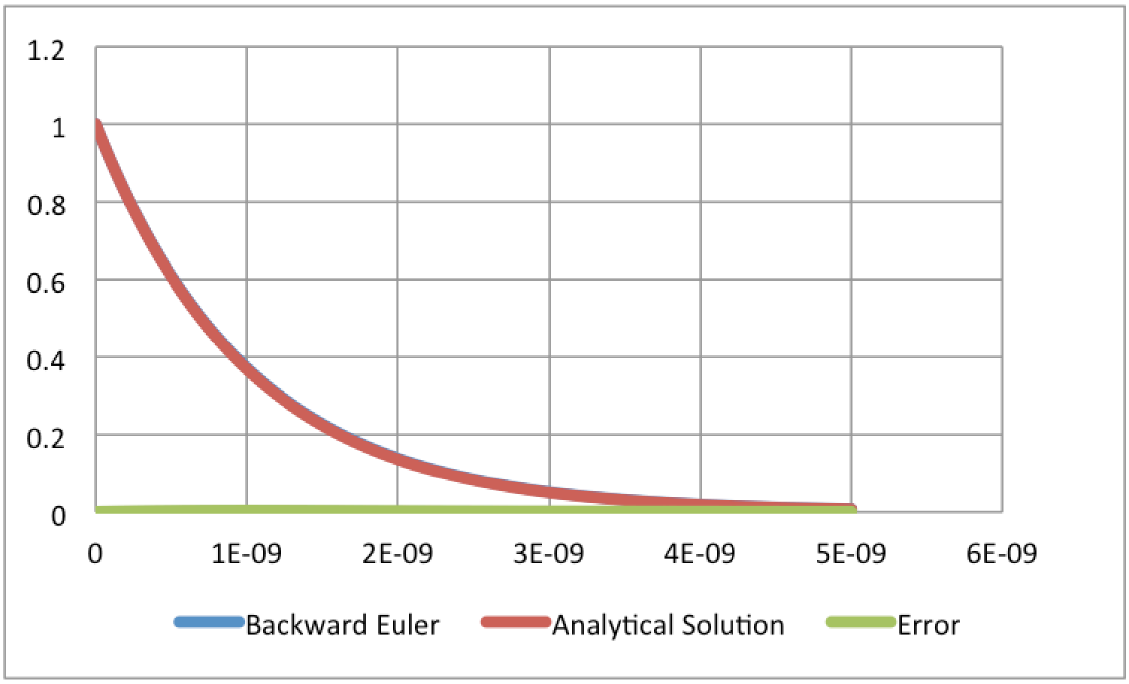

Figure 2: Backward Euler Method I(A)-t(s) Plot

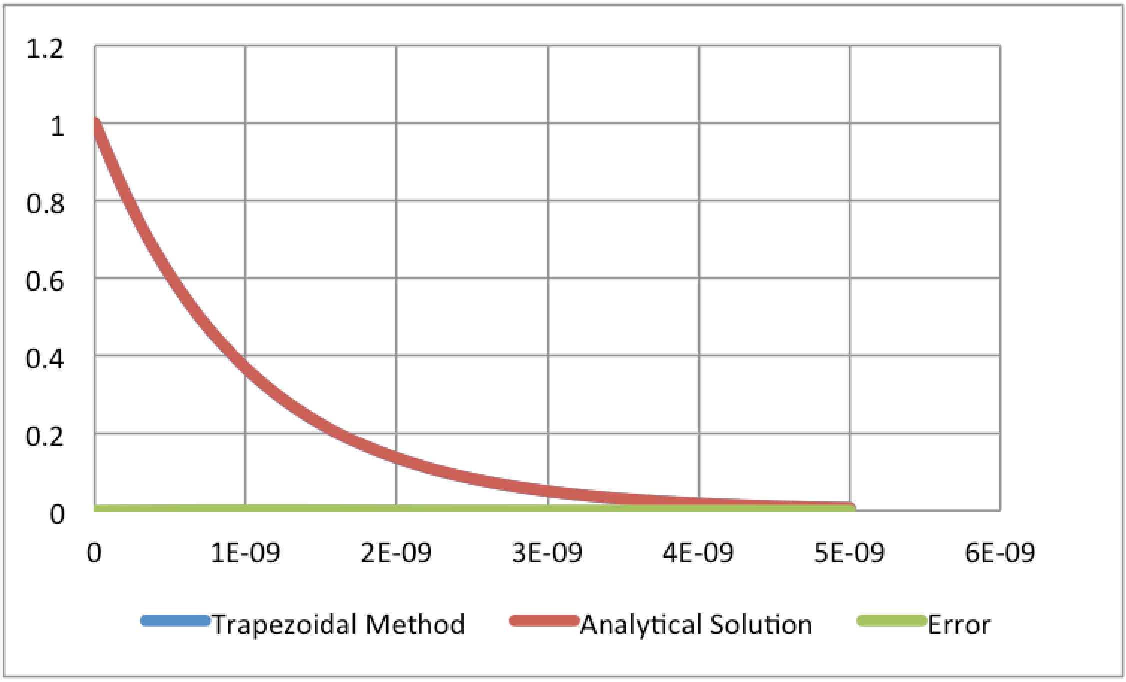

Figure 3: Trapezoidal Method I(A)-t(s) Plot

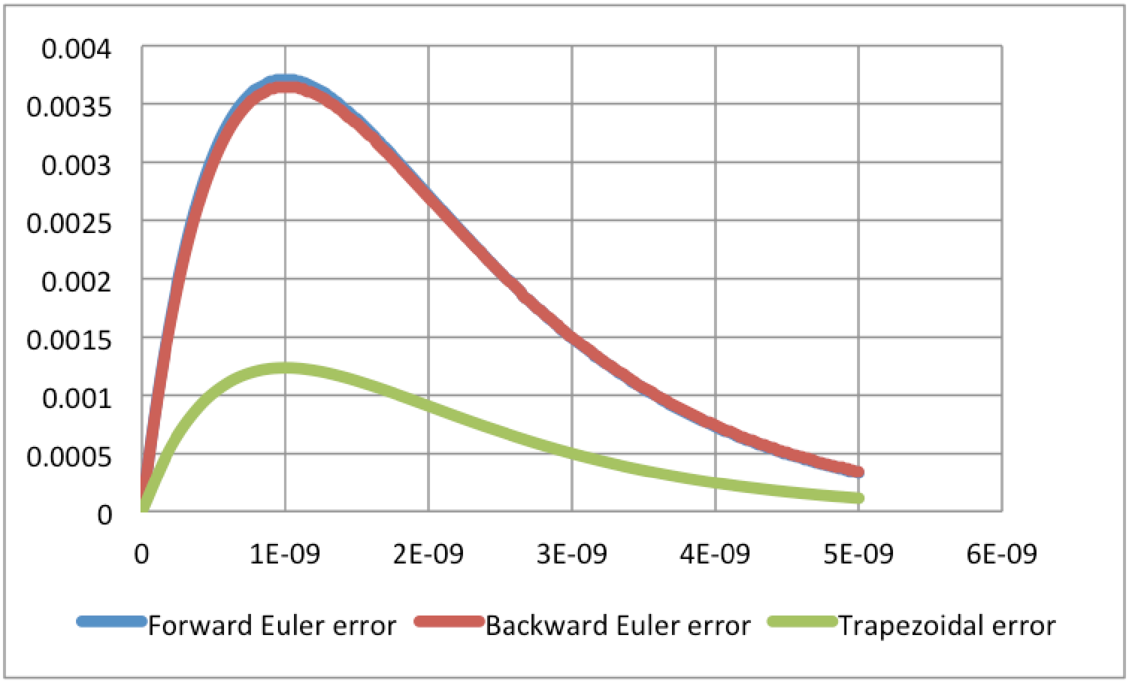

Figure 4: Error value-t(s) Plot

0.5 Observations

As mentioned in lecture note, trapezoidal method is much more accurate than both Forward and Backward Euler method. Normally, the numerical value of Forward method is less than real value, and that of Backward method is higher than the analytical value. In sum, we can see that the error tends to converge for all three methods in the end as time is after $10^{-9}(s)$