Directed Graphs

Outline

Introduction

One Way Road

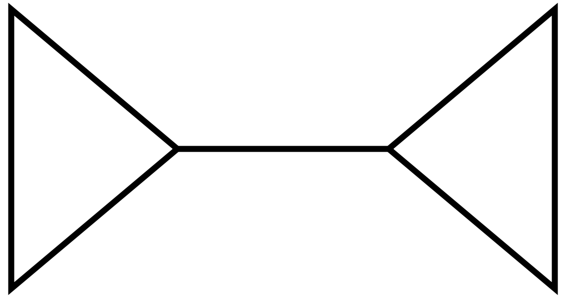
An Efficient Computer Drum

Tournament Winner

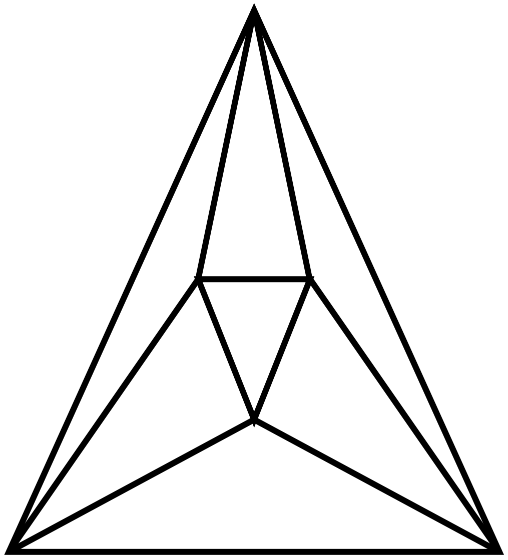
Directed Graph

A directed graph is an ordered triple $(V(D), A(D), \phi_D)$ consisting of a nonempty set $V(D)$ of vertices, a set $A(D)$ disjoint from $V(D)$ of arcs and an incidence function ϕ_D that associates each arc of D an ordered pair of vertices of D.

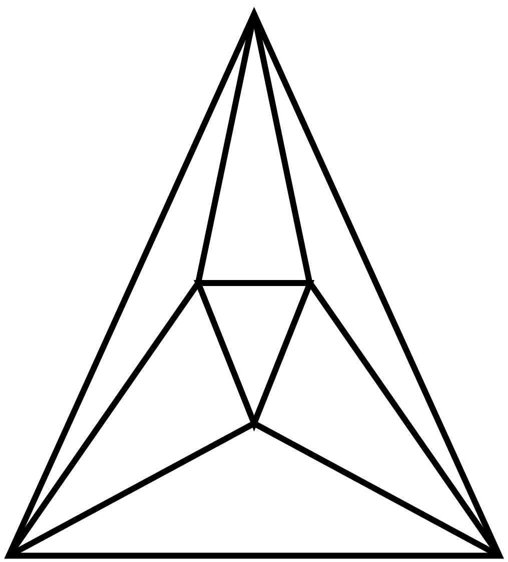
One Way Road



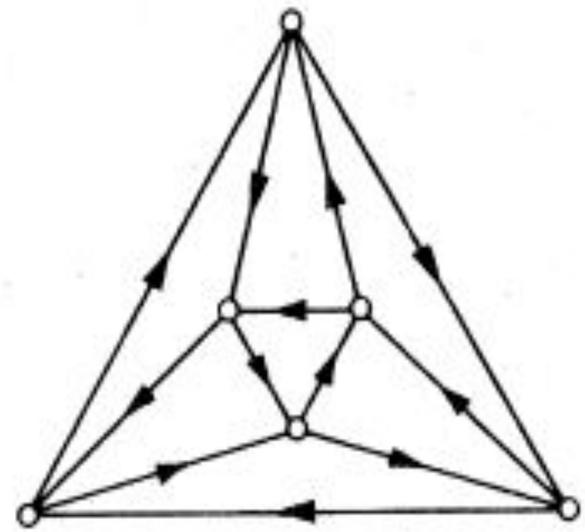
G_1



G_2



G_2



D_2

THEOREM

If G is 2-edge-connected, then G has a diconnected orientation.

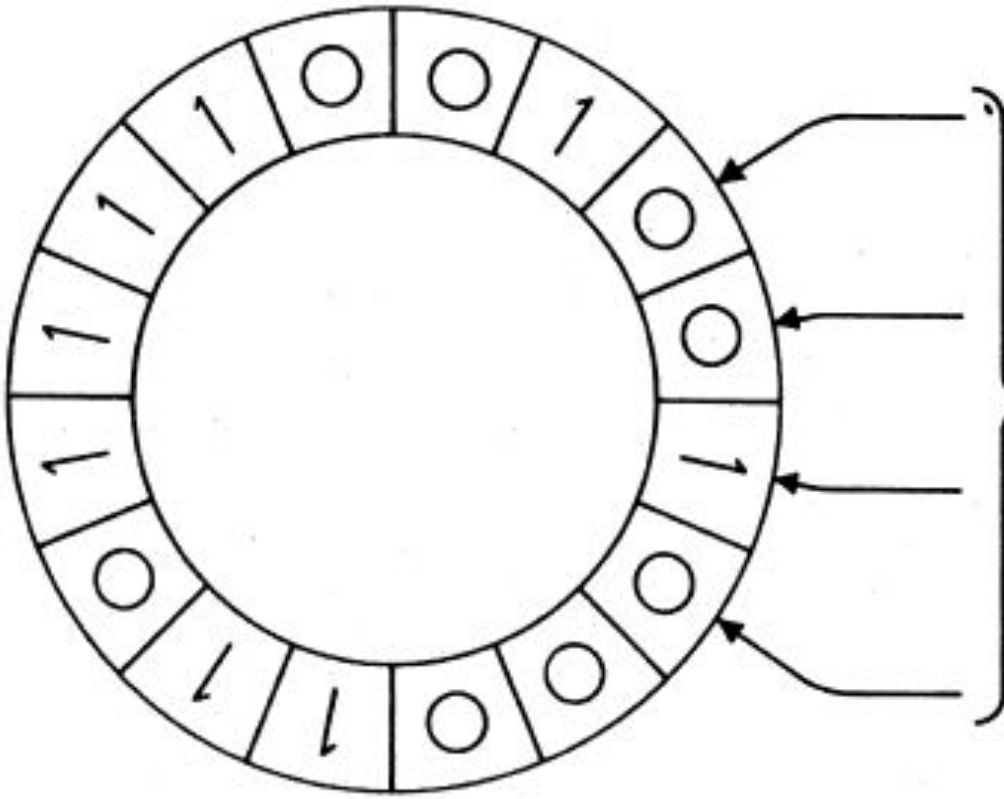
PROOF

- Let G be 2 edge-connected. Then G contains a cycle G.
- Induce a sequence of $G_1, G_2, \ldots$ of connected subgraphs of G.
- If $G_i (i=1,2,\ldots)$ is not a spanning subgraph of G, let vi be a vertex of G not in G_i. Then there exist edge-disjoint paths P_i and Q_i from v_i to G_i.

  Define, $G_{i+1} = G_i \cup P_i \cup Q_i$

- Since $v(G_{i+1})>v(G_i)$, this sequence must terminate in a spanning subgraph G_n of G
- Now orient G_n by orienting G_1 as a directed cycle, each path P_i as a directed path with origin v_i, and each path Q_i as a directed path with terminus v_i
- Clearly every G_i and hence in particular G_n, is thereby given a disconnected orientation
- Since G_n is a spanning subgraph of G it follows that, G too has a disconnected orientation

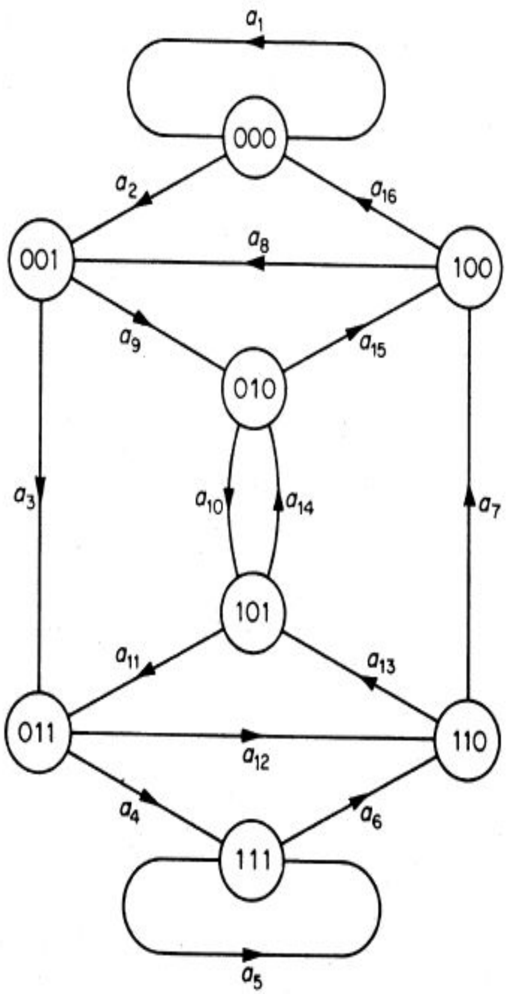
AN EFFICIENT COMPUTER DRUM



Contacts

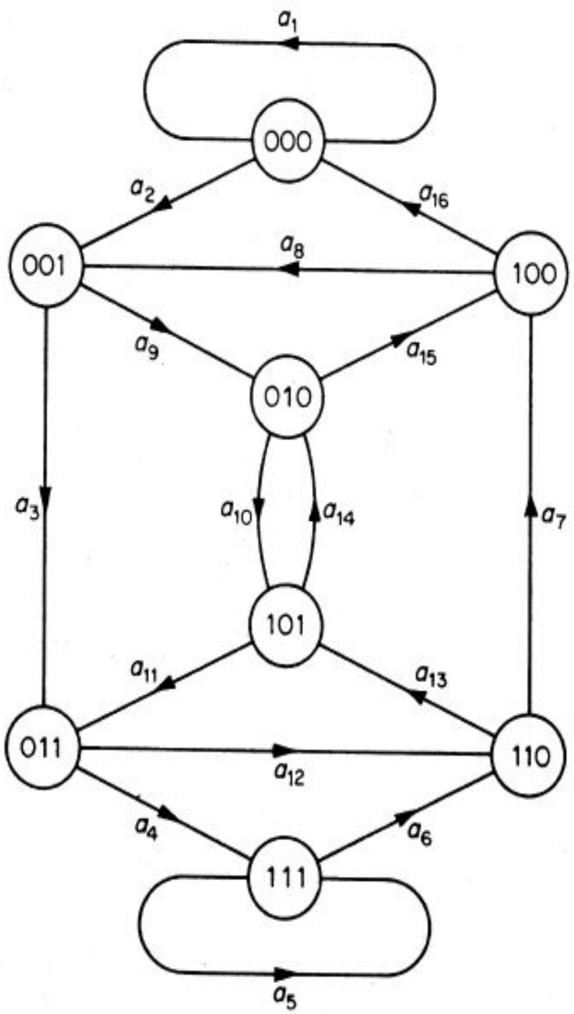
OBJECTIVE

Design the drum surface in such a way that 2^n different positions of the drum can be distinguished by k contacts placed consecutively around part of the drum. K to be as small as possible.



Arc	Label
a_1	0000
a_2	0001
a_3	0011
a_4	0111
a_5	1111
a_6	1110
a_7	1100
a_8	1001
a_9	0010
a_{10}	0101
a_{11}	1011
a_{12}	0110
a_{13}	1101
a_{14}	1010
a_{15}	0100
a_{16}	1000

- The vertices of D_n are the (n-1)-digit binary numbers $p_1p_2\ldots p_{n-1}$ with p_i=0 or 1
- There is an arc with tail p1p2…pn-1 and head $q_1q_2\ldots q_{n-1}$ if and only if p_{i+1}=qi for 1≤i ≤n-2(all arcs of the form $p_1p_2\ldots p_{n-1}, p_2p_3\ldots p_n$)
- Each arc of D_n is assigned the label $p_1p_2\ldots p_n$



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a_{13}	1101
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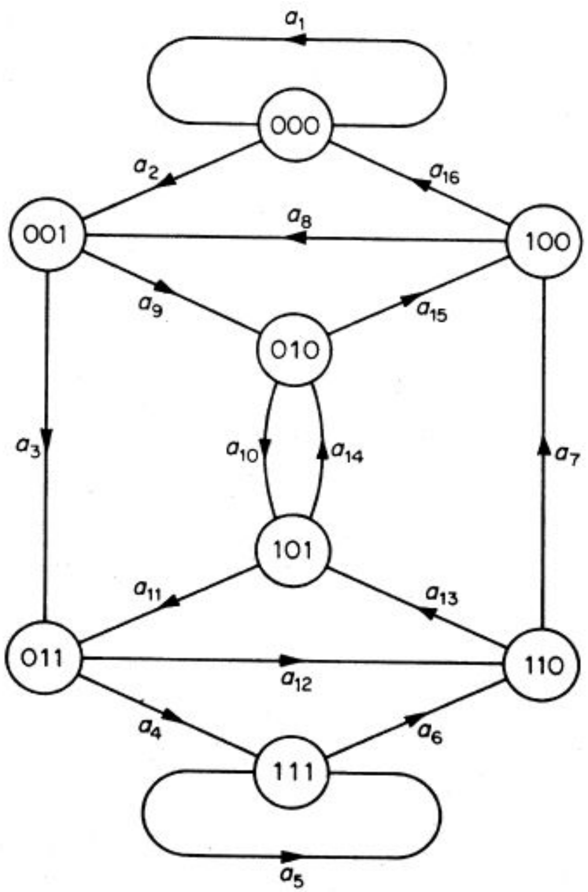
a_{15}	0100
a_{16}	1000

- D_n is connected and each vertex of D_n has indegree two and outdegree two
- Therefore D_n has a directed Euler tour
- The directed Euler tour, regarded as sequence of arcs of D_n, yields a binary sequence of length 2^n suitable for the design of drum surface



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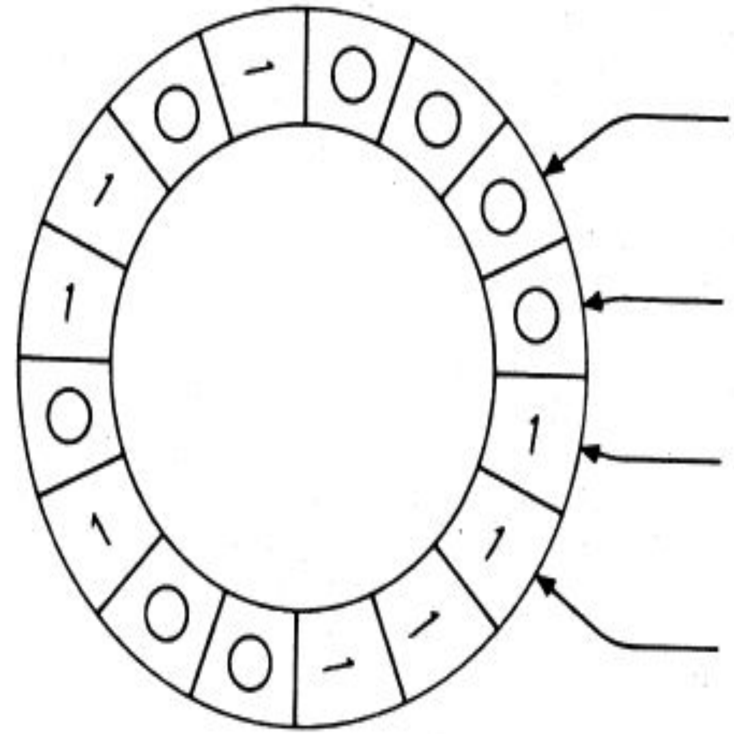
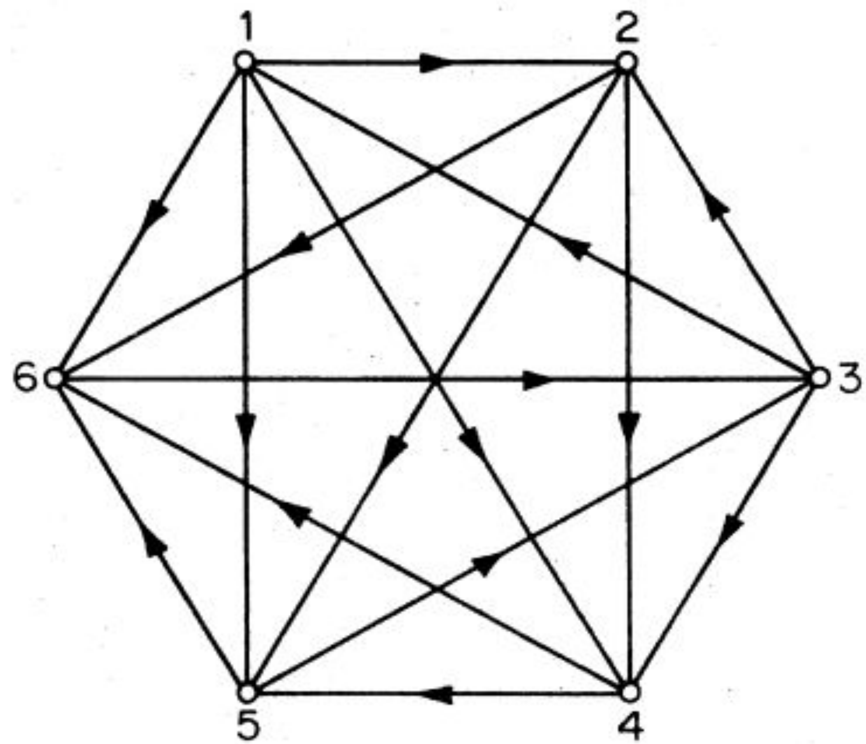


The digraph D4 in the above example has a directed Euler tour giving the 16-digit binary sequence 0000111100101101

Tournament Winner


1
2
3
4
5
6

Ranking the Participants in a Tournament

1) Finding Directed Hamiltonian Paths

2) Computing the scores of each player

Score Vector $s_1 = (4,3,3,2,2,1)$

Sum of score vectors, $s_2 = (8,5,9,3,4,3)$

Further vectors obtained are,

$s_3 = (15,10,16,7,12,9)$

$S_4 = (38,28,32,21,25,16)$

$S_5 = (90,62,87,41,48,32)$

$S_6 = (183,121,193,80,119,87)$

References

J. A. Bondy and U. S. R. Murty,Graph Theory with Applications