BARGAINING GAMES

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The Structure of Bargaining

A Bargaining game of Alternating offers

Consider the situation in which 2 bargainers have the opportunity to reach an agreement on an outcome in some set X and perceive that if they fail to do so then the outcome will be some fixed event D

Formal Definition

N = {1, 2}

X - The set of possible agreements

T - The set of non-negative integers denoting time

H - The set of histories

I. $\varnothing$ (the initial history), or $(x^0, R, x^1, R, \ldots, x^t, R)$

II. $(x^0, R, x^1, R, \ldots, x^t)$

III. $(x^0, R, x^1, R, \ldots, x^t, A)$

IV. $(x^0, R, x^1, R, \ldots)$

Preference relation

For each x and t, the set of all histories of type 3 for which xt = x is a member of the is denoted by (x, t) and the set of all histories of type 4 is a member of the partition, denoted by D.

The preference relation of each player i over histories is induced from a preference relation $\succsim_i$ over the set $(X \times T) \cup \{D\}$ of members of this

We refer to the extensive game with perfect information $\langle N, H, P, (\succsim_i)\rangle$ thus defined as the **bargaining game of alternating offers** $\langle X, (\succsim_i)\rangle$.

A Bargaining Game: Split-the-Pie

- Two players trying to split a desirable pie. The set of all possible agreements X is the set of all divisions of the pie, $X = \{(x1, x2) : xi \geq 0 \text{ for } i = 1, 2 \text{ and } x1 + x2 = 1\}$.

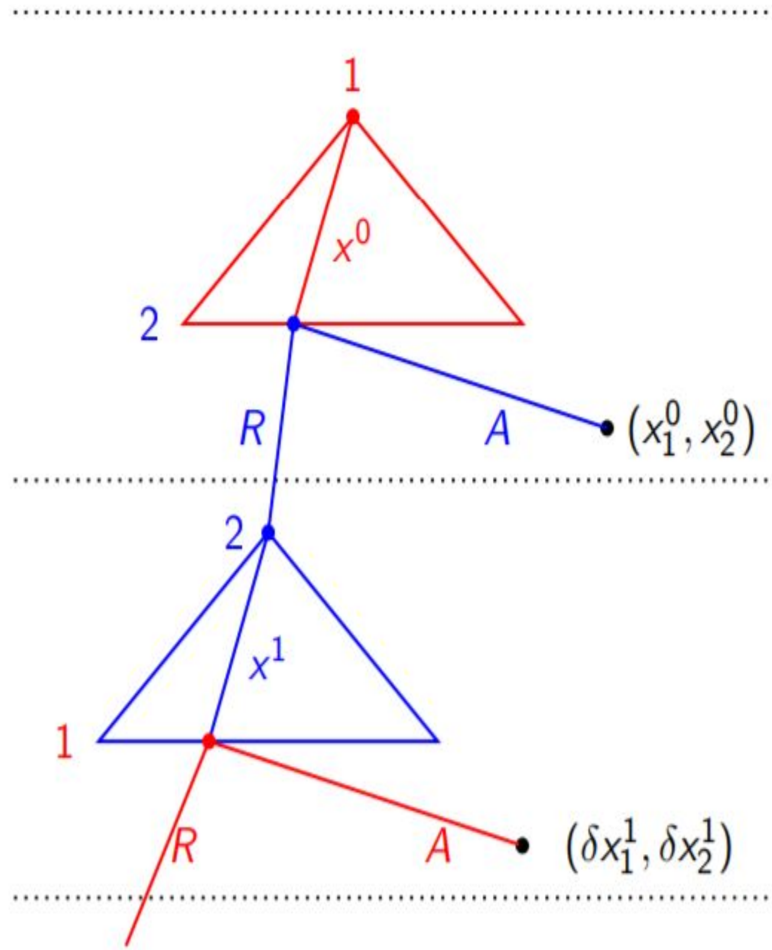
- $ui(x,t) = \delta t\ xi$ if proposal x has been accepted in period t, $\delta \in (0, 1)$.

- $ui = 0$ if no agreement has reached.

Split-the-Pie as A Perfect-Information Extensive-Form Game

Time period 0

Time period 1



Nash Equilibria of the Split-the-Pie Game

- The set of NEs is very large. For example, for any $x \in X$ there is a NE in which the players immediately agree on x.

- E.g. Player 1 always propose (0.99, 0.01) and only accepts a proposal (0.99, 0.01).

Subgame Perfect Equilibrium

Assumptions

A1 For no two agreements x and y is it the case that $(x, 0) \sim_i (y, 0)$ for both $i = 1$ and $i = 2$.

The next assumption simplifies the analysis.

A2 $(b^i, 1) \sim_j (b^i, 0) \sim_j D$ for $i = 1, 2$, $j \neq i$ where b^i is the best agreement for player i.

To state the next two assumptions we define the *Pareto frontier* of the set X of agreements to be the set of agreements x for which there is no agreement y with $(y,0) \succ_i (x,0)$ for $i = 1$, 2. We refer to a member of the Pareto frontier as an *efficient* agreement.

A3 The Pareto frontier of X is strictly monotone: if an agreement x is efficient then there is no other agreement y such that $(y,0) \succsim_i (x,0)$ for both players i.

A4 There is a unique pair (x^*, y^*) of agreements for which $(x^*, 1) \sim_1 (y^*, 0)$, $(y^*, 1) \sim_2 (x^*, 0)$, and both x^* and y^* are efficient.

Proposition

A bargaining game of alternating offers that satisfies A1 through A4 has a Subgame perfect equilibrium (SPE).

Let

(x^*, y^*) *be the unique pair of efficient agreements for which*

$$(x^*, 1) \sim_1 (y^*, 0) \qquad \textit{and} \qquad (y^*, 1) \sim_2 (x^*, 0).$$

Properties of Equilibrium

- Efficiency
- Stationarity of strategies
- First mover advantage
- Comparative statics of impatience

Variations and Extensions

One sided offers

- To model bargaining as an extensive game: we need to specify a bargaining procedure

- The structure of the bargaining procedure plays an important role in determining the outcome.

- Assume that player 1 makes all the offers (rather than the players alternating offers). Under A1 through A3 the resulting game has an essentially unique subgame perfect equilibrium, in which regardless of the players’ preferences the agreement reached is b1 , the best possible agreement for player 1.

Finite grid of possible offers

- In the certain variants of the game, neither player can force the other to choose between an agreement now and a better agreement later.
- Consider the case in which the set X of agreements contains finitely many elements.
- A player’s ability to offer an agreement today that is slightly better than the agreement that the responder expects tomorrow is limited.
- Consider a variant of a split-the-pie game in which the pie can be divided only into integral multiples of a basic indivisible unit > 0 and the preferences of each player i are represented by the function δ txi . Denote this game by Γ() and the game in which the pie is perfectly divisible by Γ(0).

Opting Out

- One or both players, at various points in the game, is allowed to “opt out”
- Is the additional “threat” induced by the ability worthy?
- This result is sometimes called the “outside option principle”. It is not robust to the assumptions about the points at which the players can exercise their outside options.

Breakdown

- Consider a modification of the model of a bargaining game of alternating offers in which at the end of each period a chance move ends the game.
- Given the presence of chance moves, we need to specify the players' preferences over the set of lotteries over terminal histories.
- We assume that the preference relation of each player i is represented by a von Neumann–Morgenstern utility function.

More Than Two Players

- $X = \{(x1, x2, x3): xi \geq 0$ for $i = 1, 2, 3$ and $x1 + x2 + x3 = 1\}$

- Player 1 initially makes a proposal. A proposal x made by player j in period t is first considered by player j + 1 (mod 3), who may accept or reject it. If he accepts it, then player j + 2 (mod 3) may accept or reject it. If both accept it, then the game ends and x is implemented. Otherwise player j + 1 (mod 3) makes the next proposal, in period t + 1.

- For every agreement x there is a subgame perfect equilibrium in which x is accepted immediately (when $0.5 \leq \delta < 1$)

References

A Course in Game Theory - *Martin J. Osborne. Ariel Rubinstein. The MIT Press.*