Group Coding

Decoding

Outline

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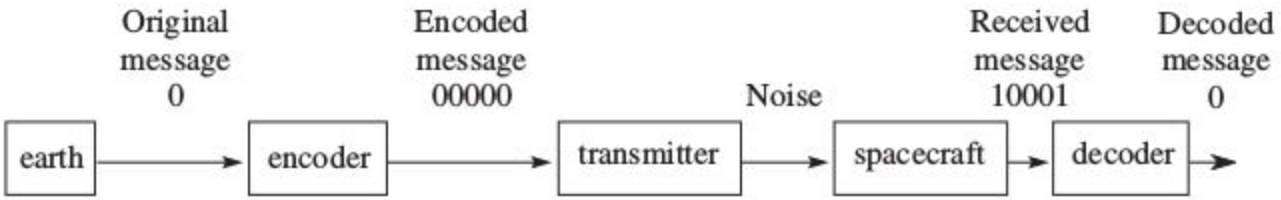
Motivation


Original
message
0
Encoded
message
00000
Noise
Received
message
10001
Decoded
message
0
earth
encoder
transmitter
spacecraft
decoder

Properties of Distance Function

$\delta(x,y) = \delta(y,x)$

$\delta(x,y) \geq 0$

$\delta(x,y) = 0$ if and only if $x=y$

$\delta(x,y) \leq \delta(x,z)+\delta(z,y)$

Group Code

An encoding function $e : B^m \to B^n$ is called a **group code** if

$e(B^m) = \{e(b) \mid b \in B^m\}$ is a **subgroup** of B^n.

Decoding and Error Correction

An onto function d: B^m -> B^n is called an (n,m) decoding function associated with e if

$$d \circ e = 1_B^m$$

Maximum Likelihood Technique

Since B^m has 2^m elements, there are 2^m codewords in B^n.

If received word is r = e(b), we choose the first codeword s among the keywords such that d(r,s) is minimum.

Maximum Likelihood decoding function d associated with e :

$$d(r) = b$$

Parity Check Matrix Decoding Procedure

1. For any received word w, compute wH.
2. If wH is the zero vector, assume that no error was made.
3. If there is exactly one instance of a nonzero element $s \in F$ and a row i of H such that wH is s times row i, assume that the sent word was $w - (0 \ldots s \ldots 0)$, where s occurs in the ith component. If there is more than one such instance, do not decode.

3′. When the code is binary, category 3 reduces to the following. If wH is the ith row of H for exactly one i, assume that an error was made in the ith component of w. If wH is more than one row of H, do not decode.

4. If wH does not fit into either category 2 or category 3, we know that at least two errors occurred in transmission and we do not decode.

To describe this method, suppose that V is a systematic linear code over the field F given by the standard generator matrix $G = [I_k \mid A]$, where I_k represents the $k \times k$ identity matrix and A is the $k \times (n - k)$ matrix obtained from G by deleting the first k columns of G. Then, the $n \times (n - k)$ matrix

$$H = \left[\frac{-A}{I_{n-k}}\right],$$

where $-A$ is the negative of A and I_{n-k} is the $(n - k) \times (n - k)$ identity matrix, is called the *parity-check matrix* for V. (In the literature, the transpose of H is called the parity-check matrix, but H is much more convenient for our purposes.) The decoding procedure is:

Coset Decoding

We construct a table, called a standard array. The first row of the table is the set C of code words, beginning in column 1 with the identity 0...0.

To form additional rows of the table, choose an element v of $V=F^n$ not listed in the table thus far.

Among all the elements of the coset v+C, choose one of minimum weight, say, v'.

Complete the next row of the table by placing under the column headed by the code word c the vector v'+C.

Continue this process until all the vectors in V have been listed in the table.

Example

C = {000000, 100110, 010101, 001011, 110011, 101101, 011110, 111000}

Coset Leaders	Words						
000000	100110	010101	001011	110011	101101	011110	111000
100000	000110	110101	101011	010011	001101	111110	011000
010000	110110	000101	011011	100011	111101	001110	101000
001000	101110	011101	000011	111011	100101	010110	110000
000100	100010	010001	001111	110111	101001	011010	111100
000010	100100	010111	001001	110001	101111	011100	111010
000001	100111	010100	001010	110010	101100	011111	111001
100001	000111	110100	101010	010010	001100	111111	011001

Syndrome

If an (n, k) linear code over F has parity-check matrix H, then, for any vector u in Fn, the vector uH is called the syndrome of u.

Coset Decoding

- Determine all left cosets of $N = e(B^m)$ in B^n.
- For each coset, find a coset leader and compute the syndrome of all leaders.
- If y is received, compute the syndrome of y and find the coset leader l having the same syndrome.
- Then $y \pm l = x$ is the codeword

Example

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Coset leader	000000	100000	010000	001000	000100	000010	000001	100001
Syndromes	000	110	101	011	100	010	001	111

References

Joseph A Gallian, Contemporary Abstract Algebra

Bernard Kolman,Robert C.Busby,Sharon Ross, Discrete Mathematical Structures