DEPARTMENT OF INFORMATION TECHNOLOGY, NITK SURATHKAL

MID-SEMESTER EXAMINATION, SEPTEMBER 2023

IT300: DESIGN AND ANALYSIS OF ALGORITHMS

Class: Vth SEM B.TECH. (IT) — **Time: 1:30 Hrs.**
Date: 25/09/2023 — **Marks: 25**

Register No. | 2 | 1 | 1 | 0 | 5 | 0 | 1 | |

NOTE: 1. Answer all questions

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1. Write Dynamic Programming algorithms for Longest Common Sub-sequence (LCS). Find the LCS between two sequences using dynamic programming, Where S1="ABACB" and S2="BADCABC". Give final solution. **4+4+2 Marks**

2. Compute the remaining values in the given table for Matrix Chain Multiplication problem. Where Matrices dimension are A: 10x30, B: 30x5, C: 5x60, D: 60x10, E: 10x20, F: 20x5. **5Marks**

	A	B	C	D	E	F
A	0	1500	4500	9000	13500	
B		0	900	3900	7900	
C			0	3000	9000	16500
D				0	6000	13000
E					0	2000
F						0

3. Provide a pseudo code or algorithm for implementing Kruskal's greedy algorithm using the Find-Union Disjoint-Set data structure to find the minimum spanning tree in a given graph. Include the implementations of the following operations: make_set, find with path compression, and union by rank. **5 + 5 Marks**

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NATIONAL INSTITUTE OF TECHNOLOGY KARNATAKA, SURATHKAL

Department of Information Technology

B.Tech V Semester-Test

Course Name: **Design and Analysis of Algorithms** Course Code: **IT300**

Date: **November 22, 2023** Time: **01:00 PM**

Duration: **1 Hour** Max. Marks: **20**

Note:

- Your arguments should be convincing. Unnecessary details attracts penalty.
- Any notations used should be precisely defined.
- You are required to carry out thorough Mathematical analysis to convince your arguments.
- The question paper is of *one* page.

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1. a) Write a pseudocode for a divide-and-conquer algorithm for finding the position of the largest element in an array of n numbers.**(5)**

b) Set up and solve a recurrence relation for the number of key comparisons made by your algorithm. **Note:** You are not supposed to use Master method. **(4)**

c) How does this algorithm compare with the brute-force algorithm for this problem?**(1)**

2. a) Write pseudocode for a divide-and-conquer algorithm for finding values of both the largest and smallest elements in an array of n numbers.**(5)**

b) Set up and solve (for $n = 2^k$) a recurrence relation for the number of key comparisons made by your algorithm. **Note:** You are not supposed to use Master method.**(4)**

c) How does this algorithm compare with the brute-force algorithm for this problem?**(1)**

*************************************All the Best************************************

Roll No	2	1	1	I	T	0	1	7

NATIONAL INSTITUTE OF TECHNOLOGY KARNATAKA, SURATHKAL

Department of Information Technology

B.Tech (IT) V Semester-End Semester Examination, November 2023

Course Name: **Design and Analysis of Algorithms**
Course Code: **IT300**
Date: **November 28, 2023**
Time: **09:00 AM**
Duration: **3 Hours**
Max. Marks: **100**

Note:

- Your arguments should be convincing. Unnecessary details attracts penalty.
- Any notations used should be precisely defined.
- You are required to carry out thorough Mathematical analysis to convince your arguments. Mere guesses for the multiple choice questions may not be awarded complete marls.
- The question paper is of *three* pages.

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1. Show that, if $T(n) = 2^{10n}$, then $T(n)$ is not $O(2^n)$. **(4)**

2. Consider an array of n numbers, in arbitrary order. The goal is to output array of the same numbers, sorted from smallest to largest. Use the divide and conquer technique to solve the problem. Analyze the complexity of the algorithm using recursion tree approach. **(10)**

3. Give a divide-and-conquer algorithm to find the average of all numbers in an array $A[1 \dots n]$ of integers, where n is a power of 2. The algorithm should start by dividing the input elements into approximately two halves. Analyze the time complexity of the algorithm. Compare it with Brute-force algorithm. **(6)**

4. a) What is the minimum number of comparisons required to find the minimum and the maximum of 100 numbers? Justify. **(3)**

   100 146

   A) 148 B) 147 C) 146 D) 140

   b) The recurrence equation

$$T(1) = 1, \quad T(n) = 2T(n-1) + n, n \geq 2$$

   evaluates to **(4)**

   A) $2^{n+1} - n - 2$
   B) $2^n - n$
   C) $2^{n+1} - 2n - 2$
   D) $2^{n+1} - n$

5. Given a sorted array of distinct integers $A[1, \dots, n]$, you are supposed to find out whether there is an index i for which $A[i] = i$. Give a divide-and-conquer algorithm that runs in time $O(log\, n)$. You should show that the complexity of your algorithm is $O(log\, n)$. **(6)**

6. Let $A[0..n-1]$ be an array of n real numbers. A pair (i, j) is said to be an inversion, if these numbers are out of order, i.e., $i < j$ but $A[i] > A[j]$. Give the bruteforce algorithm for computing the number of inversions. Analyze its complexity. Design an efficient algorithm for counting the number of inversions. Analyze the algorithm for it's complexity. The algorithm should be better than the Brute-force algorithm for solving the counting inversion problem. Compare with the Brute-force algorithm for computing the number of inversions. **(10)**

7. Consider the problem of Minimizing the Sum of Weighted Completion Times.

Input: A set of n jobs with positive lengths $l_1, l_2, \ldots, l_n$ and positive weights $w_1, w_2, \ldots, w_n$.

Output: A job sequence that minimizes the following sum of weighted completion time.

$$\min_{\sigma} \sum_{j=1}^{n} w_j C_j(\sigma)$$

Note: $C_j(\sigma)$ is the completion time of job j in the schedule σ.

a) Design an efficient algorithm to solve the above problem.(4)

b) Prove the correctness of the algorithm and analyze the complexity of the algorithm. Compare the running time with the Brute-force algorithm's running time. (8)

8. (a) Consider multiplying two $n - digit$ numbers. I know that traditional method requires $O(n^2)$ basic operations. As an algorithm designer my mantra is "Can I do better?". Design an efficient **integer multiplication algorithm** to multiply two $n - digit$ numbers. Analyze the algorithm for its complexity.(8)

(b) Modify the **integer multiplication algorithm** to divide each integer into i) three, and ii) four pieces. What are the complexities of these algorithms? **(6)**

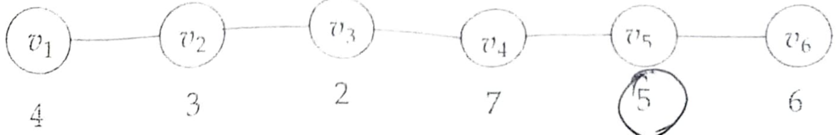
9. a) Consider the following Weighted Independent Set (WIS) problem.

Input: An undirected graph $G = (V, E)$ and a non-negative weight w_v for each vertex $v \in V$.

Output: An independent set $S \subseteq V$ of G with the maximum-possible sum $\sum_{v \in S} w_v$ of vertex weights.

Design an algorithm to compute a maximum-weight independent set (MWIS). You can consider the graph as path graph while designing the algorithm. Note that, your algorithm not only should output maximum sum but also should output subset S. Analyze the complexity of the algorithm. **(8)**

b) What is the MWIS of the following path graph? All the entries in the array should be provided. Also, you should output total weight and the subset of vertices of an optimal solution.(7)



10. Suppose you are given an array A with $2n$ numbers. The numbers in odd positions are sorted in ascending order, that is $A[1] \leq A[3] \leq \ldots \leq A[2n-1]$; the numbers in the even positions are sorted in descending order, that is, $A[2] \geq A[4] \geq \ldots \geq A[2n]$. What is the method you would recommend for determining if a given number is in the array? Justify. **(4)**

1. Sort the array using quick-sort and then use binary search.
2. Merge the sorted lists and perform binary search.
3. Perform a single binary search on the entire array.
4. Perform separate binary searches on the odd positions and the even positions.
5. Search sequentially from the end of the array.

11. In the following array, which of the following options uses the least number of comparisons among the array elements to sort the array in the ascending order? Justify. **(4)**

23	32	45	69	72	73	89	97

A) Selection sort

B) Mergesort

C) Insertion sort

D) Bubble sort

12. There is an unsorted list of n integers. You are given 3 distinct integers and you have to check if all 3 integers are present in the list or not. The only operation that you are allowed to perform is a comparison. Let A be an algorithm for this task that performs the least number of comparisons. Let c be the number of comparisons done by A. Then, **(4)**

A) $c = 3n$

B) $c = 2n + 5$

C) $c \geq 3n - 1$

D) $c \leq n$

E) $c \leq 2n + 3$

13. Given the following pseudocode for function `printx()` below, how many times is x is printed if we execute `printx(5)`? **(4)**

```
void printx(int n) {
    if (n==0) {
        printf("x");
    }
    for (int i=0;i<=n-1;++i){
        printx(n-1);
    }
}
```

5

A) 625

B) 256

C) 120

D) 24

E) 5

**************************************All the Best**************************************

Roll No	2	1	1	I	T	0	7	4

NATIONAL INSTITUTE OF TECHNOLOGY KARNATAKA, SURATHKAL

Department of Information Technology

B.Tech V Semester-Test

Course Name: **Design and Analysis of Algorithms**

Date: **November 08, 2023**

Duration: **1 Hour**

Course Code: **IT300**

Time: **02:00 PM**

Max. Marks: **30**

Note:

- Your arguments should be convincing. Unnecessary details attracts penalty.
- Any notations used should be precisely defined.
- You are required to carry out thorough Mathematical analysis to convince your arguments.
- The question paper is of *two* pages.

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1. Consider the following recursive algorithm.**(3+3)**

```
ALGORITHM Q(n)
//Input: A positive integer n
if n = 1 return 1
else return Q(n - 1) + 2 * n - 1
```

   a) Set up a recurrence relation for this function's values and solve it to determine what this algorithm computes.

   b) Set up a recurrence relation for the number of multiplications made by this algorithm and solve it.

2. Let us call the problem as River crossing problem: There are four people who want to cross a rickety bridge. They all begin on the same side. You have 17 minutes to get them all across to the other side. It is night, and they have one flashlight. A maximum of two people can cross the bridge at one time. Any party that crosses, either one or two people, must have the flashlight with them. The flashlight must be walked back and forth; it cannot be thrown, for example. Person 1 takes 1 minute to cross the bridge, person 2 takes 2 minutes, person 3 takes 5 minutes, and person 4 takes 10 minutes. A pair must walk together at the rate of the slower person's pace. Solve the problem. **(4)**

3. Consider the following algorithm.

```
ALGORITHM GE(A[0..n - 1, 0..n])
//Input: An n X (n + 1) matrix A[0..n - 1, 0..n] of real numbers
for i <- 0 to n - 2 do
        for j <- i + 1 to n - 1 do
            for k <- i to n do
                A[j,k] <- A[j,k] - A[i,k] * A[j,i] / A[i,i]
```

a) Find the efficiency class of this algorithm.(4)

b) What glaring inefficiency does this pseudocode contain and how can it be eliminated to speed the algorithm up? (5)

4. a) Design a recursive algorithm for computing 2^n for any nonnegative integer n that is based on the formula $2^n = 2^{n-1} + 2^{n-1}$.(4)

b) Set up a recurrence relation for the number of additions made by the algorithm and solve it.(3)

c) Draw a tree of recursive calls for this algorithm and count the number of calls made by the algorithm.(3)

d) Is it a good algorithm for solving this problem?(1)

***********************************All the Best***********************************