DEPARTMENT OF INFORMATION TECHNOLOGY, NITK SURATHKAL

V Semester B.Tech (IT) END SEM EXAMINATION, November 2023

IT 302: PROBABILITY AND STATISTICS

Max Marks: 40

Time: 3 Hrs.
Date: 30/11/2023

Register No. | 2 | 1 | 1 | 0 | 5 | 0 | 1 | |

Note: 1. Answer all the Questions using relevant mathematical expressions.
2. Avoid writing irrelevant answers.

1 (a)	**Suppose you are investigating a rare medical condition, Condition-X, that affects a small percentage of the population. A new diagnostic test for Condition-X has been developed, and its accuracy is known.** **The prevalence of Condition-X in the general population is very low, estimated to be 0.1%. The diagnostic test for Condition-X has a sensitivity of 95% (true positive rate) and a specificity of 90% (true negative rate).** **Given this information, the probability that a randomly selected individual tests positive for Condition-X is 0.02.** **Then find** **(i) the probability that an individual actually has Condition-X given a positive test result** **(ii) the probability that an individual does not have Condition-X given a negative test result** (2M + 2M)	**4M**
1 (b)	**Suppose you are creating a password for a new online account. The password must be 8 characters long, consisting of letters (both uppercase and lowercase), digits, and special characters. Each character can be chosen from a set of 62 possibilities (26 uppercase letters + 26 lowercase letters + 10 digits).** **Calculate the probability of randomly guessing the correct password on the second**	**2M**

	attempt.	
2 (a)	Consider a random experiment involving a continuous random variable X and a discrete random variable Y. The joint probability distribution of X and Y is given by $P(X=x, Y=y) = \frac{1}{4} e^{-x} \cdot \frac{1}{2} \left(\frac{1}{3}\right)^{y}$ where $x>0$ and y is a non-negative integer. Then (i) Determine the marginal probability distribution of X and Y. (2M) (ii) Find the conditional probability $P(X>2 \mid Y=1)$. (2M)	4M
2 (b)	The expected no. of arrivals in the first 10 mins of a Poisson's process is $\frac{1}{6}$ per hour. Then find: (i) the probability that exactly 4 arrivals in the first 10 mins of an hour (ii) the probability of 4 or more arrivals in the first 10 mins of an hour (iii) the probability of 35 or more arrivals in an hour given that 8 or more arrivals in the first 10 mins of an hour (2M + 2M + 2M)	6M
3 (a)	Let's discuss a Markov chain that includes 4 states: "Sunny," "Cloudy," "Rainy," and "Stormy." The transition matrix P is given by $P = [0.5\ 0.2\ 0.1\ 0.2;\ 0.3\ 0.4\ 0.2\ 0.2;\ 0.1\ 0.2\ 0.5\ 0.2;\ 0.1\ 0.2\ 0.2\ 0.4]^{T};$ Then (i) If the weather is currently stormy, calculate the probability that it will be rainy two days from now. (ii) Determine the steady-state probabilities for the weather system. (2M + 2M)	4M

3 (b)	A professor is interested in understanding the performance of students in a class. He decides to randomly sample 25 exam scores from the class dataset. The exam scores (out of 100) are as follows: [57,23,86,42,11,79,94,68,30,52,5,48,89,17,63,91,74,36,60,3,98,27,46,14,70] i) Calculate the mean, median, mode and standard deviation of the sampled exam scores. (1M + 0.5M + 0.5M + 1M)=(3M) ii) The professor decides to add an extra score of 100 to the dataset. Recalculate the mean, median, mode, and standard deviation for the updated dataset. ~~(2M)~~ iii) Discuss how the addition of an outlier (the score of 100) influences the various statistical measures. ~~(2M)~~	6M (1.5) (1.5)
4 (a)	M/M/1 Queuing System with parameters: Arrival rate 5 customers per hour and Service rate 8 customers per hour. Then i) Determine the probability that all servers are busy ii) Calculate the average number of customers in the system. iii) Find the average time a customer spends in the system. (1M + 1M + 1M)	3M
4 (b)	A service center has a queuing system with a single queue and 3 identical servers. The average arrival rate is 4 customers per hour, and the average service rate is 2 customers per hour. i) Determine the probability that all servers are busy ii) Determine the probability that the customers are served without waiting in the queue. (iii) Find the expected number of idle servers at any specified time (1M + 1M + 1M)	6M

		5M
5	Suppose a pharmaceutical company claims that a new drug increases the average time it takes for patients to fall asleep. The current average time is known to be 15 minutes. To test this claim, a sample of 25 patients is selected, and their average time to fall asleep is found to be 17 minutes with a standard deviation of 3 minutes. Formulate the null (H0) and alternative (H1) hypotheses, and determine whether there is enough evidence to reject the null hypothesis at a 5% level of significance.	

******************** End of the paper ********************