Assignment: Random Variables Concepts

Objective:
These challenging problems cover various advanced concepts related to random variables and will test your understanding of probability theory and statistical concepts.

Submission:
Submit your assignment with detailed explanations, calculations, and any code or visualizations as required.

Write a Python code to calculate the following:
(i) Theory: Write the procedure how to calculate following (10M)
(ii) Coding: Write the code to calculate following (10M)

Problem 1: Joint Probability Distribution (2M)

Consider two random variables, X and Y, with the following joint probability distribution:

X\Y	1	2	3
1	0.1	0.2	0.1
2	0.2	0.3	0.0
3	0.0	0.1	0.0

a) Calculate the marginal probability distribution of X and Y.
b) Calculate the conditional probability P(X = 2 Y = 1).
c) Determine if X and Y are independent.

Problem 2: Continuous Random Variables (2M)
You have a continuous random variable X with probability density function (PDF):

f(x) = 3x^2 for 0 ≤ x ≤ 1
f(x) = 0 otherwise

a) Calculate the cumulative distribution function (CDF) of X.
b) Determine the expected value (mean) of X.
c) Find the variance of X.

Problem 3: Transformation of Random Variables (2M)

Let X be a random variable with a uniform distribution on the interval [0, 1]. Consider a new random variable Y = X^2.

a) Find the probability density function (PDF) of Y.
b) Calculate the expected value (mean) of Y.
c) Determine the variance of Y.

Problem 4: Bivariate Random Variables (2M)

You have two random variables X and Y with the following joint probability distribution:

X\Y	1	2	3
1	0.2	0.1	0.1
2	0.1	0.2	0.1
3	0.1	0.1	0

a) Calculate the covariance between X and Y.
b) Determine whether X and Y are positively or negatively correlated.
c) Calculate the correlation coefficient between X and Y.

Problem 5: Stock Price Random Variable (2M)

Consider a random variable X representing the daily percentage change in the stock price of a certain company. The probability distribution of X is given by:

X (%)	-2	-1	0	1	2
P(X)	0.1	0.2	0.3	0.25	0.15

Calculate the following:

a) The expected value (mean) of the random variable X.
b) The variance of the random variable X.
c) The standard deviation of the random variable X.
d) The probability that the stock price increases by at least 1%.

e) The probability that the stock price changes (increases or decreases) by less than 2%.