Tracking Control of Under Actuated Ships

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***Abstract*—In this paper, we deal with the tracking control problem for an under actuated ship which has two control inputs namely surge force and yaw moment. The ship dynamics are modelled utilizing Fossens (1994) model and this model is discussed and assumptions are made about the values and certain physical properties in the model.The model is then implemented in Simulink and tested to determine the system behavior without a control system.After that,a Proportional Integral Derivative(PID) controller is designed. Later this PID controller is modified to use adaptive control to better compensate for changes in the physical properties of the ship. Finally results are summarized and potential for further improvement are discussed.**

I. Introduction

The world is inherently nonlinear. Often linear models can still describe the systems with a high degree of accuracy, such as Hooke's law for springs. Unfortunately some systems will not be accurately described by a linear model. This necessitates an understanding of nonlinear dynamics.One such nonlinear system is the dynamics of ships. Water, and foods in general, often act in very nonlinear ways. These effects cannot simply be ignored or linearized as they have a substantial impact on the behavior of ships.Traditionally, a nonlinear ship model has control inputs as the forces in the surge and the sway direction and the yaw moment. But economically, some ships may have two independent aft thrusters or one aft thruster and a rudder but may not have any bow or side thrusters which means there is no sway force [1] [2]. So in such situations based on [3], our inputs are just the surge force and yaw moment while controlling the velocity in the surge,sway and yaw direction. Since we have two control inputs and three control outputs, this makes the problem under actuated [1] [4].An under actuated control system implies that it has fewer number of actuator inputs than the degrees of freedom [5] [6].In other words a system is under actuated if each degree of freedom cannot be directly controlled. While all the degrees of freedom cannot be directly controlled the system as a whole can still be controllable.The degrees of freedom of the ship can be seen in figure 1.

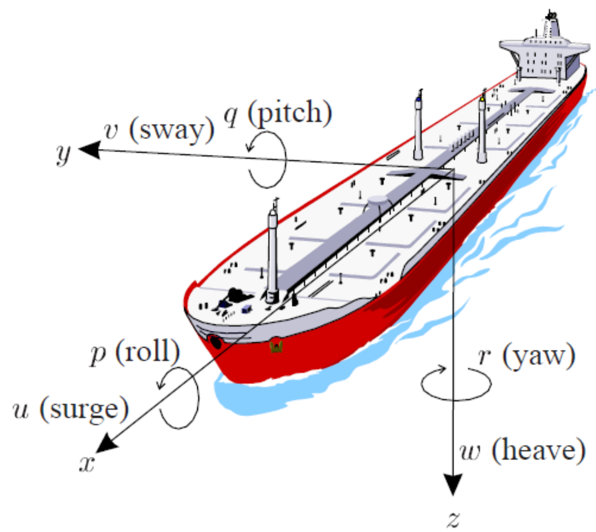


Fig. 1. Forward velocity over time from non-zero initial conditions

[1] [2] [3]

II. Problem Statement

The dynamics of ship travel are already fairly well understood. They are, however, very complicated. Beyond the nonlinearities there are a significant number of variables. Fortunately, the mass matrix can be assumed to be diagonal. This dramatically simplifies the problem.The problem can further be simplified by reducing the number of dimensions of the model. As was discussed in the introduction there are a variety of ways and dimensions the ship can move in. However, many of these dimensions such as pitch will not significantly affect ship's ability to navigate. They can therefore be ignored.

With these assumptions a mathematical model was constructed [1] [2]. The equations governing this model are shown below.

$$\dot{u} = \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{1}{m_{11}} u_1$$

$$\dot{v} = -\frac{m_{11}}{m_{22}} ur - \frac{d_{22}}{m_{22}} v$$

$$\dot{r} = \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} u_2$$

$$\dot{x} = u\cos(\theta) - v\sin(\theta)$$
$$\dot{y} = u\sin(\theta) + v\cos(\theta)$$
$$\dot{\theta} = r$$

These are all essentially acceleration equations.The variables $\dot{u}$ and $\dot{v}$ represent the forward and lateral acceleration with respect to the ship [1]. Similarly the variables $\dot{x}$ and $\dot{y}$ represent the velocity with respect to a fixed coordinate system. The ships heading is represented by θ, and the rate of change of $\dot{\theta}$ is the angular acceleration. This is represented by the variable $\dot{r}$.The parameters $m_{ii} > 0$ represent the ship inertia and added mass effects while parameters $d_{ii} > 0$ show the hydrodynamic damping. [1]

These equations are enough to fully describe the relevant motion of a ship. They are extremely difficult to analyze by hand because of the nonlinearities. Fortunately they can be numerically computed with relative ease.

III. Modelling System Dynamics

For simulating the model using Simulink, the systems dynamics equations were broken down and represented by blocks. The full Simulink model can be found in Appendix A. Simulink does not support symbolic calculation; in order to run the simulation values had to be found for all the coefficients in the dynamics equation. This is difficult to do without having a specific ship to model. Fortunately, the coefficients were all divided by elements of the mass matrix. This made it possible to make reasonable assumptions for these coefficients without knowing their exact value.

The system without any control input is predictably quite boring, especially when all values are initially zero. To get an actual sense of the dynamics of the system the simulation was run with nonzero initial conditions. The forward velocity was defined to be 5 m/s while the lateral velocity was assumed to be 1 m/s. The initial rotational velocity was set at -20 rad/s. The results of this simulation are shown in Fig 2, Fig 3 and Fig 4.

These figures show many things about the dynamics of the system. First they show that the system is asymptotically stable. More accurately they show that the velocity of the ship, both translational and rotational, is asymptotically stable. Although,the position will not be asymptotically stable.The over shoot in the forward and lateral velocities may seem odd at first.But in reality this overshoot in velocity can happen. Overshoot like this is much more common on small boats then on large ships, and it only occurs under the right circumstances, but it is a physical phenomenon. These graphs demonstrate that the dynamics have been simulated correctly.

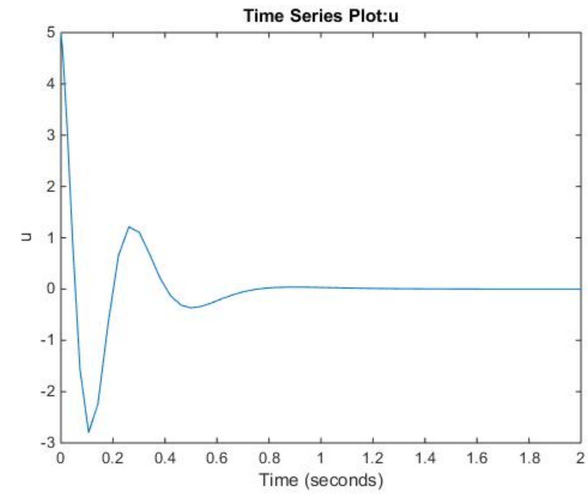


Fig. 2. Forward velocity over time from non-zero initial conditions

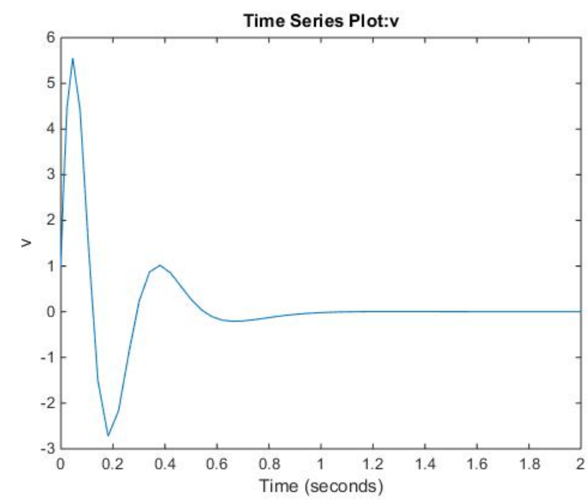


Fig. 3. Lateral velocity over time from non-zero initial conditions

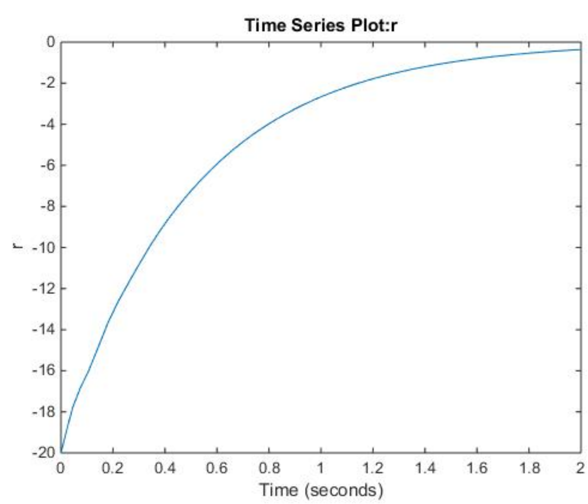


Fig. 4. Rotational velocity over time from non-zero initial conditions

IV. PID Control Design

The most primitive type of control system is a proportional control. In a proportional control there is a feedback loop which increases in size the further away a system parameter becomes from the desired. Because of the system dynamics a proportional control will actually work reasonably well in this system, particularly when it comes to controlling the heading of the ship.PID controllers, on the other hand, are more complex. There feedback loops take into account the rate of change of a system parameter. They also look at the integral of the system parameter. They provide better control of the system but are more difficult to implement and properly tune. Fortunately, Simulink makes it relatively easy to implement and to PID control systems.

To improve the quality of the control system PID controllers were implemented to control the ships heading and it's forward velocity. The full simulant model is shown in appendix B. Two pulse signals were added to test the system's ability to respond to disturbances. The results of the simulations are shown in the figures 5 and 6.

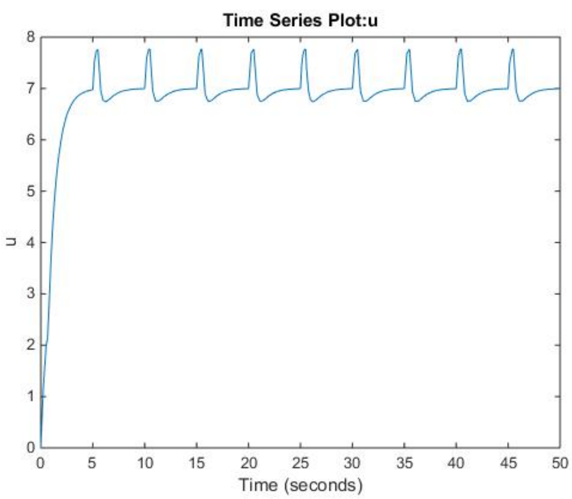


Fig. 5. The ships forward velocity with disturbances controlled by a PID controller.

In order to make the disturbances easier to see on the graphs, the periodic signals representing the disturbances were given amplitudes significantly greater then what would likely be found in nature. Therefore even though there are very noticeable disturbances in these graphs PID controllers are still very effective.

V. Adaptive Control Design

As was discussed earlier the mass of the ship can vary. This change in mass can also change the drag coefficients. This change in mass can cause the PID controller to work improperly. This can be seen by running a simulation with the original PID controller, but new values for the mass and drag coefficients. The results of this simulation are shown in figure 7.

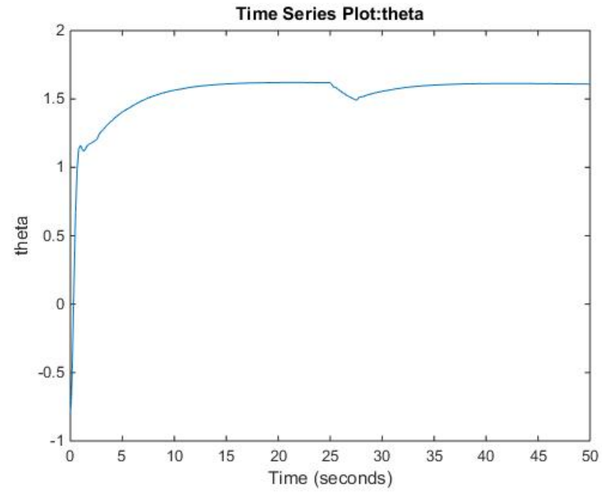


Fig. 6. The ships heading in radians with disturbances controlled by a PID controller.

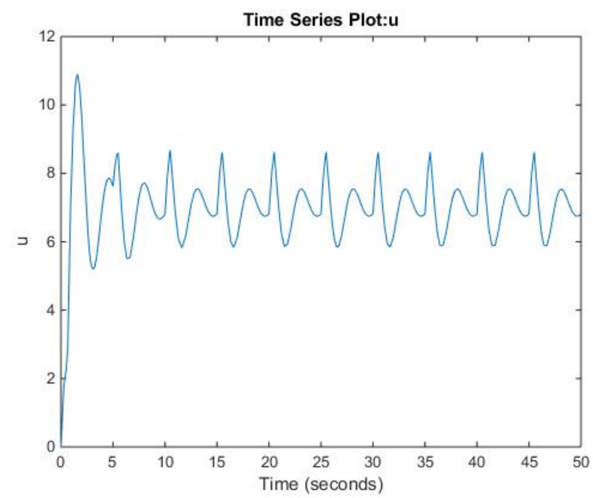


Fig. 7. Forward velocity response for new values of mass and drag with the same PID controller.

The system response in this scenario is unacceptable. Obviously it is not possible to design a new PID controller every time the mass of the ship changes. Therefore, it is necessary to implement adaptive control, to improve the quality of response of the control system to changes in mass and other physical parameters of the ship.The goal of adaptive control is to modify the gain of the PID control. The actual system response is compared to the theoretical model and in error term is calculated. This is integrated to represent taking the sum over time. Different values for Gamma are tested

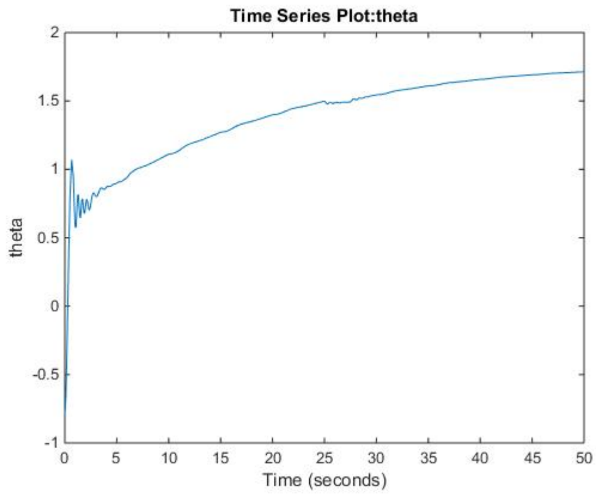


Fig. 8. Heading response for new values of mass and drag with the same PID controller.

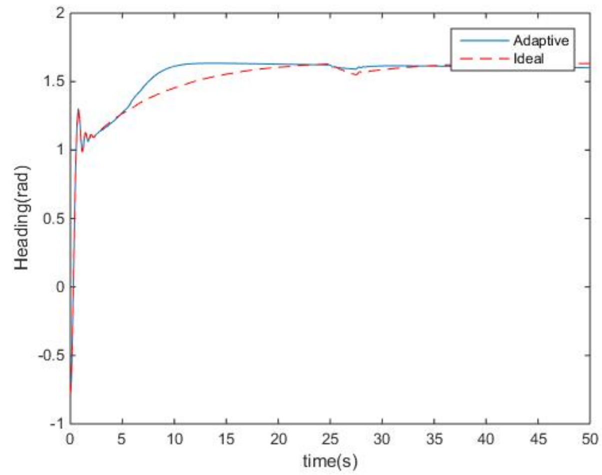


Fig. 10. Heading in adaptive and ideal cases.

to determine an effective. The adaptive control model is shown in appendix C.The system was then simulated. The system's response was compared to the response of the system the PID controller was designed to control. For simplicity this is referred to as the ideal case. The results of this test are shown in figures 9 and 10. Initially, the adaptive control does not improve the system response. However as time goes on the integrator builds up a larger value of errors and the adaptive control becomes more efficient. The response is still far from ideal but it is a significant improvement over the non-adaptive control.

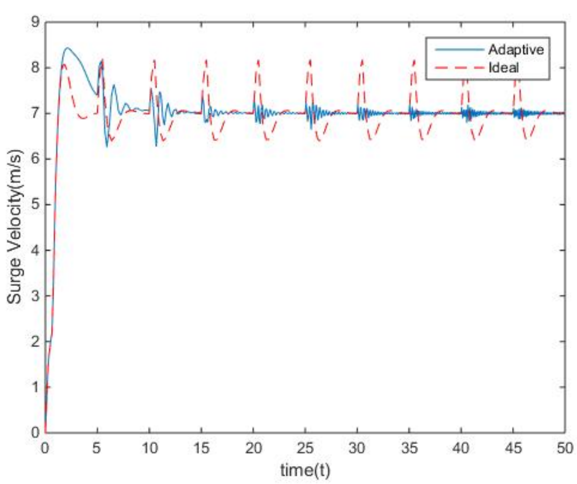


Fig. 9. Forward velocity in both adaptive and ideal cases.

VI. Conclusion

Further ways to more accurately simulate the system were discussed. The most obvious method was to use a more complex mathematical model of the system. These do exist and take into account many more parameters. It is however somewhat questionable how significantly a more complex model would affect the accuracy of the simulation in most cases.

In reality ships has a finite ability to generate thrust. This means that all the control inputs can reach saturation. This effect was not modelled, but would likely significantly impact the simulation. It is likely that the control system would need to be redesigned to take into account this effect.

A final area where the accuracy of the model could be improved is the latency between the control signal being sent and the thrust being generated. The signal propagation time will be negligible but it does take time to start firing thrusters. Similarly it is difficult to change the speed of a propeller, and it takes time to do that. Well fully accounting for this effect would improve the accuracy of the simulation, it would likely also involve redesigning the control system to use a more complex system then a simple PID control.

References

[1] A. A. J. Lefeber, *Tracking control of nonlinear mechanical systems*. Universiteit Twente, 2000.

[2] E. Lefeber, K. Y. Pettersen, and H. Nijmeijer, "Tracking control of an underactuated ship," *IEEE transactions on control systems technology*, vol. 11, no. 1, pp. 52–61, 2003.

[3] T. I. Fossen, *Guidance and control of ocean vehicles*. John Wiley & Sons Inc, 1994.

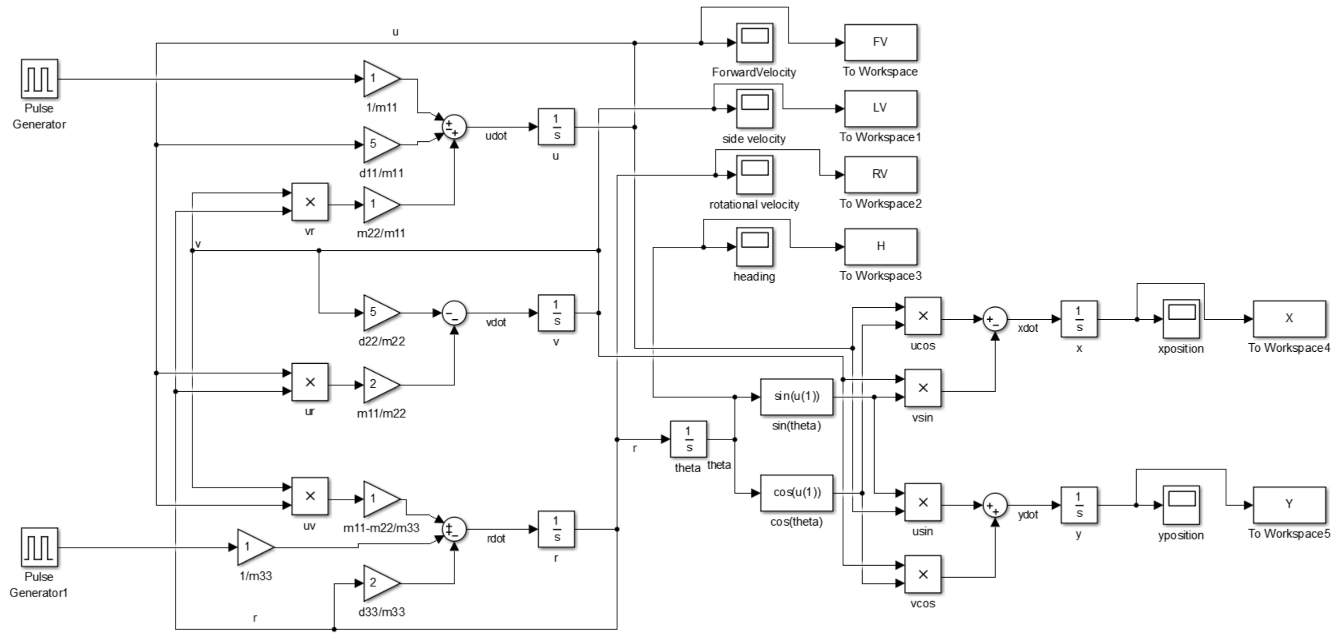
[4] K. Y. Pettersen and H. Nijmeijer, "Underactuated ship tracking control: theory and experiments," *International Journal of Control*, vol. 74, no. 14, pp. 1435–1446, 2001.

[5] https://en.wikipedia.org/wiki/Underactuation.

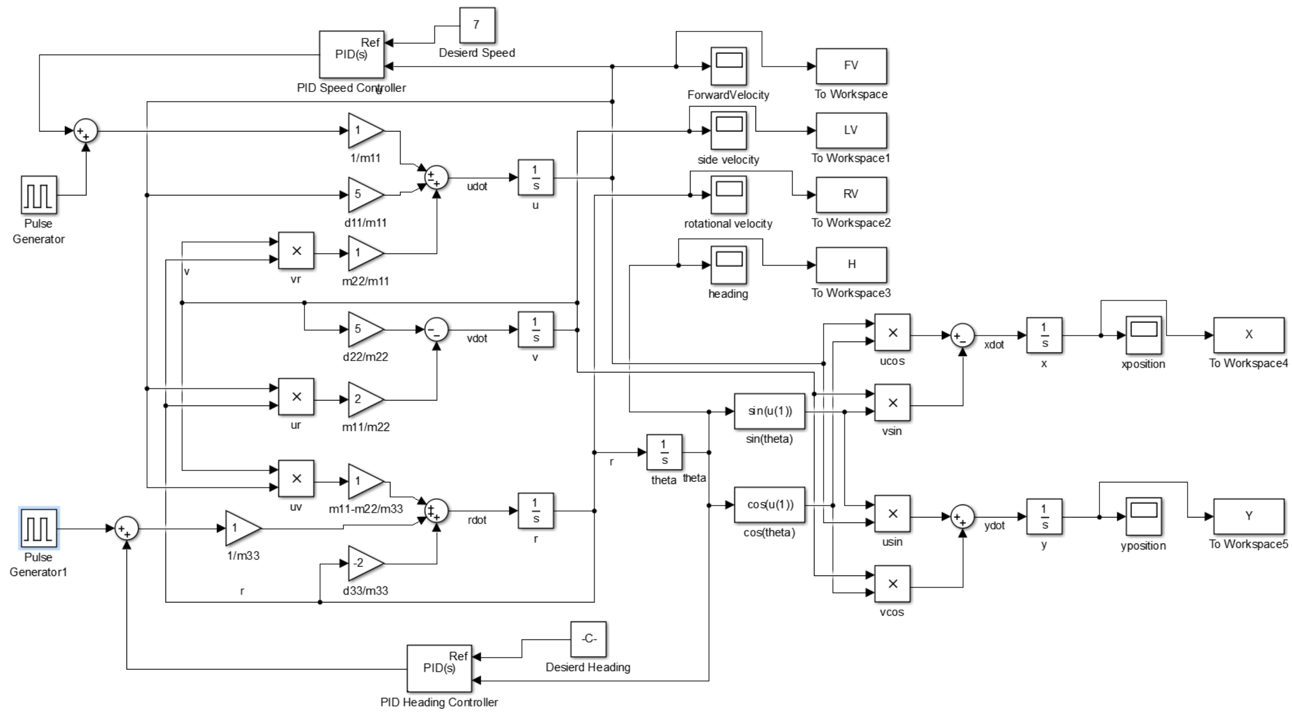
[6] Z.-P. Jiang, "Global tracking control of underactuated ships by lyapunov's direct method," *Automatica*, vol. 38, no. 2, pp. 301–309, 2002.

Appendix

Appendix A
System Dynamics Model



APPENDIX B
PID CONTROL MODEL



APPENDIX C
ADAPTIVE CONTROL MODEL