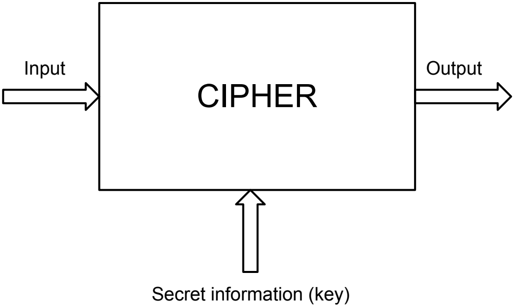
AES Timing Attacks

Hardware and Software Design for Cryptographic Applications

May 7, 2013

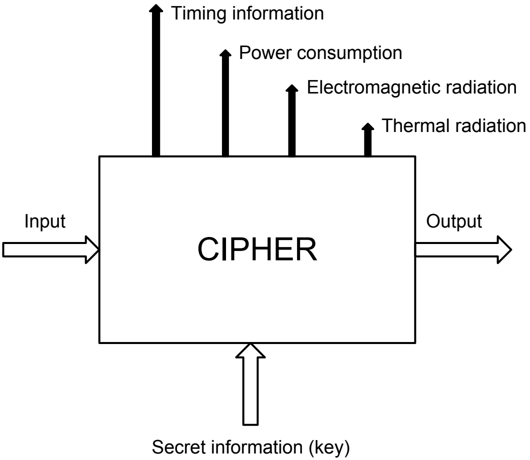
Ciphers as a Black Box

In theory, encryption (and decryption) implementations operate as black boxes.



Information Leakage

In reality, it's hard to prevent additional information from being leaked at runtime.



Side Channel Attacks

Definition: Any attack on a cryptosystem using information leaked given off as a byproduct of the physical implementation of the cryptosystem, rather than a theoretical weakness [1], is a *side channel attack*.

We focus on **timing attacks** for **software implementations** of AES.

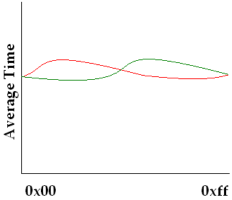
We assume the attacker can *easily* capture this timing information.

History of Timing Attacks on Cryptographic Primitives

- RSA's modular exponentiation ($c \equiv m^e \pmod{n}$)
    - Square-and-multiply algorithms for $\mathcal{O}(\log_2(n))$ complexity has branch statement whose execution depends directly on e.
- Branch statements to compute the multiplicative inverse of elements in $GF(2^8)$ [3].
- Timing attacks against OpenSSL [4].
- Cache hit ratio is predicted to be a fruitful side-channel for launching attacks [5].

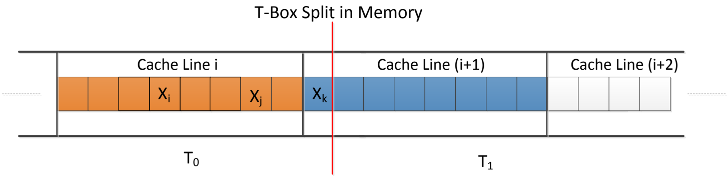
Timing Attacks on AES

- Rijndael was deemed not susceptible to timing attacks in the AES contest
- AES attacks can be based on *statistical* evidence [2].
  - Observation: The entire encryption time can be affected by the input bytes $p_i^0 \oplus k_i^0$
    - Why?
  - Step 1: Capture timing data on a reference and target machine for each value of a particular input byte $p_i^0 \oplus k_i^0 = x_i^0$
  - Step 2: Perform correlation between reference and target data
  - Step 3: Heel click.



- Or they can be more targeted:
  - Exploit relationships between secret information of the primitive and known data.
  - This is the approach of Bonneau et al., among others.

Cache Memory



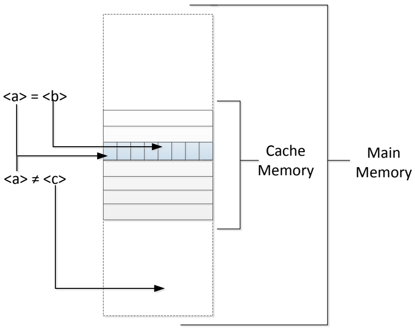
$\langle x_i \rangle = \langle x_j \rangle$ are the higher bits of the data entry.

Data is pulled into cache based on the most-significant bits in its address.

Cache Collisions

Let *a* and *b* be two memory addresses looked up in memory. Let $\langle a \rangle$ and $\langle b \rangle$ denote the MSBs of *a* and *b*, respectively.

- Cache memory is organized into *lines*
- Reads on *a* and *b* cause a collision if $\langle a \rangle = \langle b \rangle$ (assuming other memory reads have not evicted (or invalidated) *a* or *b* from the cache.
- If $\langle a \rangle \neq \langle b \rangle$ then a cache collision *might* occur.
- We cannot say for certain whether or not the lower LSBs are equivalent...



Cache Collisions Assumption

Let $T_E(K,P)$ be the encryption time for a plaintext P using key K. Let $\bar{T}_E(K) = \frac{1}{n}\sum_{i=1}^{n} T_E(K,P_i)$, where P_i is a random plaintext from $\{0|1\}^{128}$.

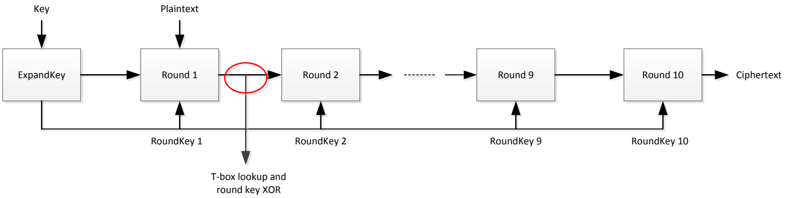
Cache-Collision Assumption [1]. For any pair of table lookups i,j, given a large enough number of *random* AES encryption that use the *same key*, $\bar{T}_E(K)$ will be lower when $\langle I_i \rangle = \langle I_j \rangle$ than when $\langle I_i \rangle \neq \langle I_j \rangle$

Note: The table lookup indices must be *independent* for random plaintexts.

Attacks from Cache Collisions

That's it! We may now build an attack based on this result.

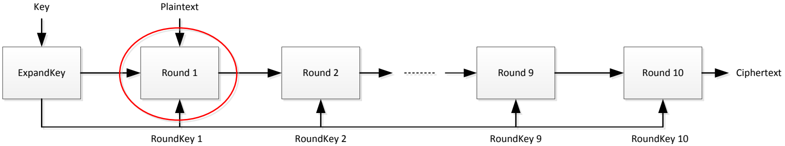
LUT-Based Implementations



Let X^i be the state of AES at round i. With the exception of $i = 10$, we have:

$$
\begin{aligned}
X^{i+1} = \{ & T_0[x_0^i] \oplus T_1[x_5^i] \oplus T_2[x_{10}^i] \oplus T_3[x_{15}^i] \oplus \{k_0^i, k_1^i, k_2^i, k_3^i\}, \\
& T_0[x_4^i] \oplus T_1[x_9^i] \oplus T_2[x_{14}^i] \oplus T_3[x_3^i] \oplus \{k_4^i, k_5^i, k_6^i, k_7^i\}, \\
& T_0[x_8^i] \oplus T_1[x_{13}^i] \oplus T_2[x_2^i] \oplus T_3[x_7^i] \oplus \{k_8^i, k_9^i, k_{10}^i, k_{11}^i\}, \\
& T_0[x_{12}^i] \oplus T_1[x_1^i] \oplus T_2[x_6^i] \oplus T_3[x_{11}^i] \oplus \{k_{12}^i, k_{13}^i, k_{14}^i, k_{15}^i\}\}
\end{aligned}
$$

First Round Attack



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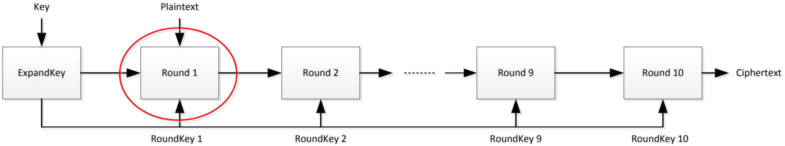
& T_0[x_{12}^i] \oplus T_1[x_1^i] \oplus T_2[x_6^i] \oplus T_3[x_{11}^i] \oplus \{k_{12}^i, k_{13}^i, k_{14}^i, k_{15}^i\}\}
\end{aligned}
$$

First Round Attack (cont'd)



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X^{i+1} = \{ & T_0[x_0^i] \oplus T_1[x_5^i] \oplus T_2[x_{10}^i] \oplus T_3[x_{15}^i] \oplus \{k_0^i, k_1^i, k_2^i, k_3^i\}, \\
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& T_0[x_{12}^i] \oplus T_1[x_1^i] \oplus T_2[x_6^i] \oplus T_3[x_{11}^i] \oplus \{k_{12}^i, k_{13}^i, k_{14}^i, k_{15}^i\}\}
\end{aligned}
$$

First Round

- First round: $x_i^0 = p_i \oplus k_i$
- With the T-box implementation, x_0^0, x_4^0, x_8^0, and x_{12}^0 are used as indices into T_0
- If we are looking for cache collisions, we must consider input bytes of the same T-box.

$$\begin{aligned} \langle x_i^0 \rangle = \langle x_j^0 \rangle &\Rightarrow \langle p_i \rangle \oplus \langle k_i \rangle = \langle p_j \rangle \oplus \langle k_j \rangle \\ &\Rightarrow \langle p_i \rangle \oplus \langle p_j \rangle = \langle k_i \rangle \oplus \langle k_j \rangle \end{aligned}$$

First Round Attack Algorithm

ALGORITHM 1: FirstRoundAttackSetup(N_s)

$n \leftarrow 2^8 - 1,\ T \leftarrow \text{array}[0 \ldots n, 1 \ldots n, 0 \ldots n]$
for $count = 0 \rightarrow N_s$ **do**
  $p \leftarrow RandomBytes(16)$
  $start \leftarrow time()$
  $c \leftarrow E_K(p)$ ▷ Time the encryption
  $end \leftarrow time()$
  $tt \leftarrow (start - end)$ ▷ Increment the time for this particular plaintext difference
  for all i, j **do** ▷ i, j are input bytes of the *same* T-box
    $T[i, j, \langle p_i \rangle \oplus \langle p_j \rangle] \leftarrow T[i, j, \langle p_i \rangle \oplus \langle p_j \rangle] + tt$
  end for
end for
$T[i, j, \Delta_{i,j}] \leftarrow T[i, j, \Delta_{i,j}] / N_s$ ▷ Compute the average time for all i, j pairs
$\{\Delta'_{0,1}, \Delta'_{0,1}, \ldots, \Delta'_{i,j}\} \leftarrow min(T)$ ▷ Find the lowest average time for each i, j pair
$\langle k_i \rangle \oplus \langle k_j \rangle \leftarrow \Delta'_{i,j}$ for all i, j pairs ▷ Guess: $\langle k_i \rangle \oplus \langle k_j \rangle = \Delta_{mi,mj}$

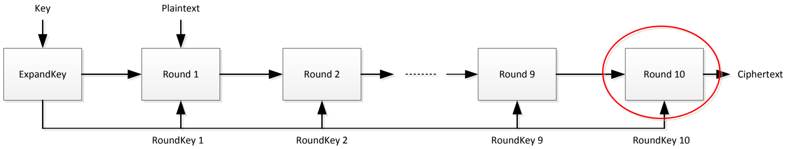
First Round Limitation

We only know that $\langle k_0 \rangle \oplus \langle k_4 \rangle = \Delta_{0,4}$, $\langle k_0 \rangle \oplus \langle k_8 \rangle = \Delta_{0,8}$, $\langle k_0 \rangle \oplus \langle k_{12} \rangle = \Delta_{0,12}$, $\langle k_4 \rangle \oplus \langle k_8 \rangle = \Delta_{4,8}$, $\langle k_4 \rangle \oplus \langle k_{12} \rangle = \Delta_{4,12}$, and $\langle k_8 \rangle \oplus \langle k_{12} \rangle = \Delta_{8,12}$.

There exists **18** other equations we can derive for T_1, T_2, and T_3.

We cannot determine the lower $\log_2$ bits of each key... What to do now?

The Last Round



When $i = 10$, the lookup table is just the S-box S. At this point, the ciphertext C is:

$$\begin{aligned} C = \{ & S[x_0^{10}] \oplus k_0^{10}, S[x_5^{10}] \oplus k_1^{10}, S[x_{10}^{10}] \oplus k_2^{10}, S[x_{15}^{10}] \oplus k_3^{10}, \\ & S[x_4^{10}] \oplus k_5^{10}, S[x_9^{10}] \oplus k_6^{10}, S[x_{14}^{10}] \oplus k_7^{10}, S[x_3^{10}] \oplus k_7^{10}, \\ & S[x_8^{10}] \oplus k_8^{10}, S[x_{13}^{10}] \oplus k_9^{10}, S[x_2^{10}] \oplus k_{10}^{10}, S[x_7^{10}] \oplus k_{11}^{10}, \\ & S[x_{12}^{10}] \oplus k_{12}^{10}, S[x_1^{10}] \oplus k_{13}^{10}, S[x_6^{10}] \oplus k_{14}^{10}, S[x_{11}^{10}] \oplus k_{15}^{10} \} \end{aligned}$$

Final Round Collisions

Let x_s and x_t be two random bytes in the last round.

We will always have that $c_i = k_i^{10} \oplus S[x_s]$ and $c_j = k_j^{10} \oplus S[x_t]$. If $x_s = x_t$, then a collision *will usually* occur and $c_i = k_i^{10} \oplus \alpha$ and $c_j = k_j^{10} \oplus \alpha$.

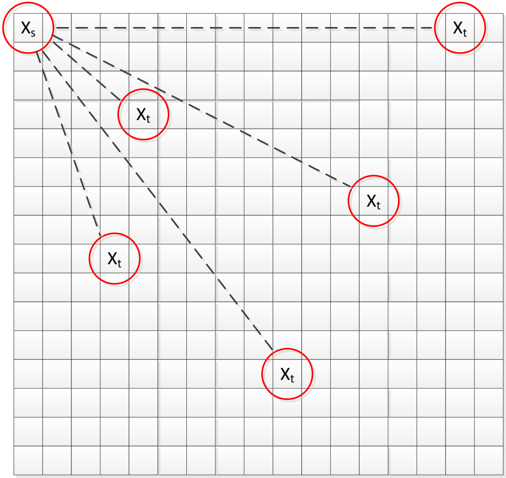
Therefore, $c_i \oplus c_j = k_i^{10} \oplus k_j^{10}$

Final Round Misses

What if $x_s \neq x_t$?

$c_i \oplus c_j \neq k_i^{10} \oplus k_j^{10} \Rightarrow$ two different values came out of the LUT!

S-Box Nonlinearity



Final Round Misses (cont'd)

The *S-box nonlinearity* means that the difference between $S[x_s]$ and $S[x_t]$ *does not* imply a fixed difference between x_s and x_t.

If $c_i \oplus c_j \neq k_i^{10} \oplus k_j^{10}$, then x_s and x_t are two *random* values.

Final Round Attack Algorithm

ALGORITHM 2: LastRoundAttackSetup(N_s)

$n \leftarrow 2^8 - 1$
$diffs \leftarrow [0,\dots,2^n-1,0,\dots,2^n-1]$
$T \leftarrow \text{array}[0\dots2^n-1,0\dots2^n-1,0\dots2^n-1]$
for $count = 0 \rightarrow N_s$ **do**
    $p \leftarrow RandomBytes(16)$
    $start \leftarrow time()$
    $c \leftarrow E_K(p)$ ▷ Time the encryption
    $end \leftarrow time()$
    $tt \leftarrow (start - end)$ ▷ Increment the time for this particular plaintext difference
    for all i,j **do** ▷ i,j are input bytes of the *same* T-box
        $T[i,j,C_i \oplus C_j] \leftarrow T[i,j,C_i \oplus C_j] + tt$
    end for
end for
$T[i,j,\Delta_{i,j}] \leftarrow T[i,j,\Delta_{i,j}]/N_s$
$\{\Delta'_{0,1},\Delta'_{0,1},\dots,\Delta'_{i,j}\} \leftarrow min(T)$
$\langle k_i\rangle \oplus \langle k_j\rangle \leftarrow \Delta'_{i,j}$ for all i,j pairs ▷ Guess: $\langle k_i\rangle \oplus \langle k_j\rangle = \Delta_{mi,mj}$

Final Round Attack Algorithm (cont'd)

With knowledge of $\Delta_{i,j}$ for all i,j such that $\Delta_{i,j} = k_i^{10} \oplus k_j^{10}$ the attacker can now make informed guesses at the key:

- Idea (1): Define a cost function $c(i,j,\Delta_{i,j})$ such that the output of this function is *low* if $\Delta_{i,j}$ is low.
- Idea (2): The correct key will yield the lowest value for the cost function when all of the input sums are considered
  - Minimize the sum of all costs for a given key K
  - $C[K] = \sum_{i,j} c(i,j,\Delta_{i,j})$
- Goal: Find the correct key guess that minimizes the total cost
- Approach: Local optimization search, where each new candidate key guess is obtained by changing one byte of the key.

Thanks to Rijndael's invertible key schedule, $[k^{10}]$ can be reverted back to $[k^0]$.

Done.

Timing Attack Countermeasures

- Avoid memory access altogether
- Use alternative lookup-tables (i.e. non T-box implementation, similar to the fourth)
- Data-independent memory access patterns
- Mask (randomize) data-dependent techniques
- Disable cache :-(
- Use processor-specific instructions for AES encryption (e.g. Intel's AES ISA)

References

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