eVaR Optimisation

Problem: minimise eVaR of p/f loss (negative return) subject to leverage and short-selling constraints.

risk measure (eVaR)

p/f loss, i.e. negative return

a compact set representing user-specified requirements (in our case: no further constraints. $D = \mathbb{R}^n$)

$$\min \rho(G(w, R)) \quad \text{s.t.} \quad w^T \mathbb{1} = 1, \quad w \geq 0, \quad w \in D$$

w: p/f weights

R: vector of k risk factors needed to specify p/f return (e.g. indiv asset returns, e.g. PCs)

assm: mgf $< \infty$ for all $\theta \in \mathbb{R}$

Defn of eVaR: $\rho(X) = \text{EVaR}_{1-\alpha}(X) = \inf_{\theta > 0} \{ \theta^{-1} \ln (M_X(\theta) / \alpha) \}$

$$= \inf_{\theta>0} \{ \tfrac{1}{\theta} \ln [E(e^{\theta X})] - \tfrac{1}{\theta} \ln \alpha \}$$

$$\theta = \tfrac{1}{t} \Rightarrow \rho(G(w,R)) = \inf_{t>0} \{ t \ln (\underbrace{E[e^{t^{-1} G(w,R)}]}_{= \sum_{j=1}^{N} p_j e^{t^{-1} G(w, a_j)}}) - t \ln \alpha \}$$

** sample-based approach

a_j: a realisation of r.v. R

p_j: prob of each sample usu set $p_j = \frac{1}{N}$ (equal prob)

$$= \sum_{j=1}^{N} e^{t^{-1} G(w, a_j) + \ln(p_j)}$$

Rewrite in matrix notation

$$\min \left[t \ln \left(\sum_{j=1}^{N} e^{t^{-1} G(w, a_j) + \ln(p_j)} \right) - t \ln \alpha \right] \Leftrightarrow \min \left[t \, \text{lse} \left(-\frac{R^s w}{t} + \ln p \right) - t \ln \alpha \right]$$

$R^s = \begin{pmatrix} a^{1T} \\ \vdots \\ a^{NT} \end{pmatrix}$

$\text{lse}(\underline{v}) = \ln(e^{v_1} + \dots + e^{v_k})$

$\ln p = \begin{pmatrix} \ln p_1 \\ \vdots \\ \ln p_N \end{pmatrix}$

Algorithm (Assuming $D = \mathbb{R}^n$, i.e. no add. constraints)

- Inputs
  - n: no. of instruments
  - $R^s = \begin{pmatrix} a_1^T \\ \vdots \\ a_N^T \end{pmatrix}$, realisations of our risk factors; $a_j^T = (a_1^j \; \dots \; a_k^j)$
  - $p = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$, probability of each realisation
- Set $w_0 = \frac{1}{n} \mathbb{1}$
- Let $\underline{x} = \begin{bmatrix} w^0 \\ t^0 \end{bmatrix}$. Update $\underline{x}$ by applying PD algorithm.

  * have yet to figure this out

  Set $\underline{w} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$