Vibrations and Waves by A. P. French Notes

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Contents

1 Periodic motions

- Fouriers theorem states that any repeating signal of period T can be expressed as a sum of sin waves with periods T, $T/2$, etc.
- It's important to define the domain of a SHM equation, e.g. for what values of t is the motion defined?
- SHM can be considered a projection of uniform circular motion
- That uniform circular motion can be represented by a number in the complex plane, with the projection being its real part
- Multiplication by j can be considered a counter-clockwise rotation of $90°$ in the complex plane
- Euler's formula states

$$e^{j\theta} = \cos\theta + j\sin\theta$$

- Multiplication of a complex number z by $e^{j\theta}$ is equivalent to a counter-clockwise rotation of z by an angle of θ

2 The superposition of periodic motions

- The combination of two SHM's of the same period

$$x_1 = A_1\cos(\omega t + \alpha_1)$$
$$x_2 = A_2\cos(\omega t + \alpha_2)$$

is given by

$$x = A\cos(\omega t + \alpha)$$

where

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha_2 - \alpha_1),$$

$$A\sin\beta = A_2\sin(\alpha_2 - \alpha_1),$$

and

$$\alpha = \alpha_1 + \beta.$$

- The combination in complex representation

$$z_1 = A_1e^{j(\omega t+\alpha_1)}$$

$$z_2 = A_2e^{j(\omega t+\alpha_2)}$$

is given by

$$z = e^{j(\omega t+\alpha_1)}[A_1 + A_2e^{j(\alpha_2-\alpha_1)}]$$

- In the case where $A_1 = A_2$ if we denote $\delta = \alpha_2 - \alpha_1$ then

$$\beta = \frac{\delta}{2}$$

and

$$A = 2A_1\cos\beta = 2A_1\cos\frac{\delta}{2}$$

- The superposition of two sinusoids with different periods will itself be periodic if there exist integers n_1 and n_2 such that

$$T = n_1T_1 = n_2T_2$$

where T_1 and T_2 are the periods of the two sinusoids

- Periodic motion in two or more dimensions can be represented by extending the "projection of a rotating vector" approach, with one vector for each axis, e.g.

$$x = A_1\cos\omega t$$

$$y = A_2\cos\omega t$$

where differing amplitudes, frequencies, and phase differences product different curves called **Lissajous curves**