Advanced Engineering Mathematics Vectors, Matrices, and Vector Calculus by Dennis G. Zill Problems

Chris Doble

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Contents

1 Vectors

1.1 Vectors in 2-Space

1.1.1

(a) $3\mathbf{a} = 6\mathbf{i} + 12\mathbf{j}$

(b) $\mathbf{a} + \mathbf{b} = \mathbf{i} + 8\mathbf{j}$

(c) $\mathbf{a} - \mathbf{b} = 3\mathbf{i}$

(d) $||\mathbf{a} + \mathbf{b}|| = \sqrt{1 + 8^2} = \sqrt{65}$

(e) $||\mathbf{a} - \mathbf{b}|| = 3$

1.1.9

(a) $4\mathbf{a} - 2\mathbf{b} = \langle 6, -14 \rangle$

(b) $-3\mathbf{a} - 5\mathbf{b} = \langle 2, 4 \rangle$

1.1.15

$\overrightarrow{P_1 P_2} = \langle 2, 5 \rangle$

1.1.19

$(1, 18)$

1.1.21

(a) Yes

(b) Yes

(c) Yes

(d) No

(e) Yes

(f) Yes

1.1.25

(a) $\frac{\mathbf{a}}{||\mathbf{a}||} = \frac{\langle 2,2 \rangle}{\sqrt{2^2+2^2}} = \frac{1}{2\sqrt{2}} \langle 2, 2 \rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

(b) $\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$

1.1.31

$2\frac{\mathbf{a}}{||\mathbf{a}||} = 2\frac{\langle 3,7 \rangle}{\sqrt{3^2+7^2}} = \frac{2}{\sqrt{58}} \langle 3, 7 \rangle = \langle \frac{6}{\sqrt{58}}, \frac{14}{\sqrt{58}} \rangle$

1.1.37

$\mathbf{x} = -(\mathbf{a}+\mathbf{b})$

1.1.41

$$\begin{aligned}
\mathbf{a} &= 2\mathbf{i}+3\mathbf{j}\\
\mathbf{b} &= \mathbf{i}+\mathbf{j}\\
\mathbf{c} &= \mathbf{i}-\mathbf{j}
\end{aligned}$$

$$\begin{aligned}
\mathbf{i} &= \frac{1}{2}(\mathbf{b}+\mathbf{c})\\
\mathbf{j} &= \frac{1}{2}(\mathbf{b}-\mathbf{c})\\
\mathbf{a} &= 2\left(\frac{1}{2}(\mathbf{b}+\mathbf{c})\right)+3\left(\frac{1}{2}(\mathbf{b}-\mathbf{c})\right)\\
&= \mathbf{b}+\mathbf{c}+\frac{3}{2}\mathbf{b}-\frac{3}{2}\mathbf{c}\\
&= \frac{5}{2}\mathbf{b}-\frac{1}{2}\mathbf{c}
\end{aligned}$$

1.1.43

$$\begin{aligned}
y &= \frac{1}{4}x^2+1\\
y(2) &= 2\\
y' &= \frac{1}{2}x\\
y'(2) &= 1\\
\mathbf{v} &= \pm\langle\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\rangle
\end{aligned}$$

1.1.45

(a)

$$\begin{aligned}
\mathbf{F}_n &= \mathbf{F}\cos\theta\\
\mathbf{F}_g &= \mathbf{F}\sin\theta\\
||\mathbf{F}_f|| &= \mu||\mathbf{F}_n||\\
||-\mathbf{F}_g|| &= \mu||\mathbf{F}_n||\\
||-\mathbf{F}\sin\theta|| &= \mu||\mathbf{F}\cos\theta||\\
||\mathbf{F}||\sin\theta &= \mu||\mathbf{F}||\cos\theta\\
\tan\theta &= \mu
\end{aligned}$$

(b) $\theta = \arctan \mu \approx 31^\circ$

1.1.47

$$\begin{aligned}
F_x &= \frac{qQ}{4\pi\epsilon_0} \int_{-a}^{a} \frac{L\,dy}{2a(L^2+y^2)^{3/2}} \\
&= \frac{LqQ}{8\pi\epsilon_0} \int_{-a}^{a} (L^2+y^2)^{-3/2}\,dy \\
&= \frac{LqQ}{8\pi\epsilon_0} \frac{2a}{L^2\sqrt{a^2+L^2}} \\
&= \frac{aqQ}{4\pi\epsilon_0 L\sqrt{a^2+L^2}} \\
F_y &= -\frac{qQ}{4\pi\epsilon_0} \int_{-a}^{a} \frac{y\,dy}{2a(L^2+y^2)^{3/2}} \\
&= 0 \\
\mathbf{F} &= \langle \frac{1}{4\pi\epsilon_0} \frac{qQ}{L\sqrt{a^2+L^2}}, 0\rangle
\end{aligned}$$

1.1.49

Let the three sides of the triangle be vectors **a**, **b**, and **c**. The triangle is closed so it must be the case that

$$\mathbf{a}+\mathbf{b}+\mathbf{c} = 0.$$

This gives

$$\mathbf{c} = -(\mathbf{a}+\mathbf{b}).$$

The vector from the midpoint of side **a** to the midpoint of side **b** is

$$\left(\mathbf{a} + \frac{1}{2}\mathbf{b}\right) - \frac{1}{2}\mathbf{a} = \frac{1}{2}(\mathbf{a}+\mathbf{b})$$

which is parallel with **c** and half its length.

1.2 Vectors in 3-Space

1.2.7

A plane at $z = 5$ parellel with the x-y plane.

1.2.9

A line parallel to the z axis at $x = 2$ and $y = 3$.

1.2.13

(a) $(0, 5, 4)$, $(-2, 0, 4)$, $(-2, 5, 0)$

(b) $(-2, 5, -2)$

(c) $(3, 5, 4)$

1.2.15

The planes $x = 0$, $y = 0$, and $z = 0$.

1.2.17

$(-1, 2, -3)$

1.2.19

The planes $z = \pm 5$.

1.2.21

$\sqrt{(6-3)^2 + (4+1)^2 + (8-2)^2} = \sqrt{9 + 25 + 36} = \sqrt{70}$

1.2.31

$$\begin{aligned}
\sqrt{(2-x)^2 + (1-2)^2 + (1-3)^2} &= \sqrt{21} \\
(2-x)^2 + 1 + 4 &= 21 \\
(2-x)^2 &= 16 \\
2 - x &= \pm 4 \\
x &= 2 \pm 4 \\
&= -2 \text{ or } 6
\end{aligned}$$

1.2.33

$\left(4, \frac{1}{2}, \frac{3}{2}\right)$

1.2.37

$(-3, -6, 1)$