University Physics with Modern Physics Electromagnetism Notes

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21 Electric Charge and Electric Field

21.1 Electric Charge

- Electrons have a much smaller mass than neutrons and protons
- Neutrons and protons have a very similar mass
- Electrons and protons have the same magnitude of charge
- The number of protons in an atom determins its **atomic number**
- If an electron is added to a neutral atom it becomes a **negative ion**, if one is removed it becomes a **positive ion** — this is called **ionisation**
- The **principle of conservation of charge** states that the algebraic sum of all the electric charges in any closed system is constant
- The electron or proton's magnitude of charge is a natural unit of charge — every observable amount of electric charge is an integer multiple of this

21.2 Conductors, Insulators, and Incuded Charges

- **Conductors** pemit easy movement of charge, **insulators** do not
- Holding a charged object near an uncharged object causes free electrons in the latter to move away/towards the former, resulting in a net charge on either side — this is called **induced charge**

21.3 Coulomb's Law

- The SI unit of charge is called one **coulomb** (1 C) and is defined such that $1.602176634 \times 10^{-19}$ C is equal to the charge of an electron or proton

- **Coulomb's law** describes the electric force between two point charges

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

where the **electric constant** $\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2$, q_1 and q_2 are the magnitudes of the charges, and r is the distance between them

- The electric force is always directed along the line between the two charges, attracting opposite charges and repelling like charges
- $\frac{1}{4\pi\epsilon_0}$ can be approximated as $9.0 \times 10^9\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2$
- The principle of superposition of forces also applies to electric charges

21.4 Electric Field and Electric Forces

- The electric force on a charged object is exerted by the electric field created by other charged objects
- We can determine if there is an electric field at a point by placing a test charge q_0 there and seeing if it experiences an electric force — the electric field at that point (the electric force per unit charge) is then given by

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

- Rearranging, the force experienced by a charge q_0 at a point is given by

$$\mathbf{F} = q_0\mathbf{E}$$

- When considering an electric field produced by a point charge, the location of the point charge is called the **source point** and the location at which we're trying to determine the field is called the **field point**
- The electric field produced by a point charge is given by

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

where q is the charge of the point charge, r is the distance between the source and field points, and $\hat{\mathbf{r}}$ is the unit vector from the source to the field point

- Unlike Coulomb's law this equation doesn't use the absolute value of q meaning that the electric fields of positive charges point away from the charge, while those of negative charges point towards them
- In electrostatics, the electric field inside the material of a conductor (but not holes within the material) is $\mathbf{0}$

21.5 Electric-Field Calculations

- The **principle of superposition of electric fields** states that the total electric field at a point P is the vector sum of the fields at P due to each point charge in the charge distribution

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots$$

- For a line charge distribution the **linear charge density** is represented by λ (the charge per unit length, measured in C/m)
- For a surface charge distribution the **surface charge density** is represented by σ (the charge per unit area, measured in C/m^2)
- For a volume charge distribution the **volume charge density** is represented by ρ (the charge per unit volume, measured in C/m^3)
- The electric field of an infinitely long line charge along the y-axis is

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

21.6 Electric Field Lines

- An **electric field line** is a line drawn through space such that its tangent at any point is in the direction of the electric field vector at that point
- Fewer lines are drawn in areas where the electric field is weak and more lines are drawn in areas where it's strong

21.7 Electric Dipoles

- An **electric dipole** is a pair of point charges of equal magnitude q and opposite sign separated by a distance d
- The net force on an electric dipole in a uniform electric field is $\mathbf{0}$
- The **electric dipole moment p** of an electric dipole is a vector directed from the negative charge to the positive charge with magnitude qd
- The net torque on an electric dipole in a uniform electric field is $\mathbf{p} \times \mathbf{E}$ or $qEd \sin \phi$ where ϕ is the angle between the electric dipole and the electric field
- The potential energy of an electric dipole in a uniform electric field is

$$U = -\mathbf{p} \cdot \mathbf{E}$$

22 Gauss's Law

22.1 Calculating Electric Flux

- The electric flux of a uniform electric field through a flat surface A is

$$\Phi_E = \mathbf{E} \cdot \mathbf{A}$$

  where $\mathbf{A}$ is normal to A and has a magnitude equal to its area

- The electric flux of a nonuniform electric field through a curved surface A is

$$\Phi_E = \int \mathbf{E} \cdot \mathbf{dA}$$

22.2 Gauss's Law

- Gauss's law states that the total electric flux through a closed surface is equal to the total electric charge enclosed by the surface divided by ϵ_0

$$\Phi_E = \oint \mathbf{E} \cdot \mathbf{dA} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

22.3 Applications of Gauss's Law

- Gauss's law can be used in two ways:
  - If we know the charge distribution and it has enough symmetry to let us evaluate the integral in Gauss's law, we can find the field
  - If we know the field, we can use Gauss's law to find the charge distribution
- Under electrostatics, excess charge always lies of the surface of a conductor
- The electric field of an infinite line charge is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r} \hat{\mathbf{r}}$$

22.4 Charges on Conductors

- If there is excess charge at rest on a conductor, all of that charge must lie on the surface of the conductor and the electric field inside the conductor must be zero. If there is a cavity inside the conductor, the net charge on the cavity walls equals the amount of charge enclosed by the cavity
- Charges outside a conductor have no effect on the interior of the conductor, even if it has a cavity inside — this is why Faraday cages work
- At the surface of a conductor, the component of the electric field that is perpendicular to the surface is

$$E_\perp = \frac{\sigma}{\epsilon_0}$$

23 Electric Potential

23.1 Electric Potential Energy

- The electric potential energy of two point charges is

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

- The electric potential energy of a point charge q_0 and a collection of charges q_1, q_2, etc. is

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- For every electric field due to a static charge distribution, the force exterted by that field is conservative

- The total electric potential energy of a collection of charges q_1, q_2, etc. is

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

  where r_{ij} is the distance between q_i and q_j

23.2 Electric Potential

- **Potential** is potential energy per unit charge

- The unit of potential is the **volt**, equal to 1 joule per coulomb

- The potential difference between two points $V_{ab} = V_a - V_b$ is called the potential of a with respect to b and equals the amount of work done by the electric force when a unit (1 C) of charge moves from a to b

- The electric potential due to a point charge is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

- The electric potential due to a collection of point charges is

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- The electric potential due to a continuous charge distribution is

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

- The electric potential difference between two points is given by

$$V_a - V_b = \int_a^b \mathbf{E} \cdot d\mathbf{l} = \int_a^b E \cos\phi \, dl$$

- Positive charges tend to "fall" from high- to low-potential regions while negative charges do the opposite

- When a particle with charge $e = 1.602 \times 10^{-19}\,\mathrm{C}$ moves between two points with a potential difference of $1\,\mathrm{V} = 1\,\mathrm{J/C}$ the change in energy is $U_a - U_b = qV_{ab} = (1.602 \times 10^{-19}\,\mathrm{C})(1\,\mathrm{J/C}) = 1.602 \times 10^{-19}\,\mathrm{J}$ which is called 1 **electron volt**

23.4 Equipotential Surfaces

- An **equipotential surface** is a three-dimensional surface on which the electric potential is the same at every point

- Because electric potential energy doesn't change as a test charge moves over an equipotential surface, the electric field can do no work and thus **field lines and equipotential surfaces are always perpendicular**

- When all charges are at rest, the surface of a conductor is an equipotential surface

- When all charges are at rest, the entire solid volume of a conductor is at the same potential

23.5 Potential Gradient

- The relationship between $\mathbf{E}$ and V is given by

$$\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$$

- If E has a radial component E_r with respect to an axis or a point and r is the distance from that axis or point, then

$$E_r = -\frac{\partial V}{\partial r}$$

24 Capacitance and Dielectrics

24.1 Capacitors and Capacitance

- Any two conductors separated by an insulator (or a vacuum) form a **capacitor**

- The **capacitance** of a capacitor measures its ability to store charge

$$C = \frac{Q}{V_{AB}}$$

- Capacitance is measured in **farads** where

$$1\,\mathrm{F} = 1\,\mathrm{C/V}$$

- The capacitance of a parallel plate capacitor in a vacuum is

$$C = \epsilon_0 \frac{A}{d}$$

24.2 Capacitors in Series and Parallel

- In a series connection, the magnitude of charge on all plates is the same
- The **equivalent capacitance** of a combination of capacitors is the capacitance of a single capacitor that would have equivalent behaviour
- In a series connection, the reciprocal of the equivalent capacitance equals the sum of the reciprocals of the individual capacitances

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

  meaning the equivalent capacitance is always less than any individual capacitance
- In a parallel connection, the potential difference is the same for all capacitors
- In a parallel connection, the equivalent capacitance equals the sum of the individual capacitances

$$C_{\mathrm{eq}} = C_1 + C_2 + \cdots$$

  meaning the equivalent capacitance is always greater than any individual capacitance

24.3 Energy Storage in Capacitors and Electric-Field Energy

- The potential energy stored in a capacitor is

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- The **energy density** of a parallel plate capacitor is its energy per unit volume

$$u = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\epsilon_0 E^2$$

24.4 Dielectrics

- **Dielectrics** are nonconducting materials
- Most capacitors have a dielectric material between their plates because
  1. It preserves the distance between the plates
  2. It increases the maximum potential difference between the plates by avoiding **dielectric breakdown** when the material between the plates becomes ionized and becomes conductive — this happens more easily for air
  3. It increases the capacitance by decreasing the potential difference for a given charge
- The **dielectric constant** of a material is defined as

  $$K = \frac{C}{C_0}$$

  where C_0 is the capacitance of a capacitor with vacuum between the plates and C is the capacitance of the same capacitor with the material between the plates
- If E_0 is the magnitude of the electric field between the plates of a parallel plate capacitor when separated by a vacuum and E is the magnitude when separated by a dielectric then

  $$E = \frac{E_0}{K}$$

- The electric field (and electric potential) are reduced because the dielectric becomes **polarized** and an induced surface charge appears of magnitude

  $$\sigma_i = \sigma \left(1 - \frac{1}{K}\right)$$

- The **permittivity** of a dielectric is defined as

  $$\epsilon = K\epsilon_0$$

- The capacitance of a parallel plate capacitor with dielectric between the plates is thus

  $$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

  and the electric energy density is

  $$u = \frac{1}{2} K\epsilon_0 E^2 = \frac{1}{2} \epsilon E^2$$

- The maximum electric-field magnitude that a material can withstand without the occurence of breakdown is called its **dielectric strength** and is denoted E_m

24.5 Molecular Model of Induced Charge

- If a material is comprised of polar molecules where the net charge of the molecule is 0 but the charge isn't distributed equally, electric fields cause the molecules to rotate which induces a charge
- Even if a material isn't comprised of polar molecules, electric fields cause molecules' positive and negative charges to separate slightly resulting in a dipole which again experiences a torque
- The charges in conductors are free to move so they're known as **free charges** while the charges in dielectrics aren't so they're known as **bound charges**

24.6 Gauss's Law in Dielectrics

- Gauss's Law in a dielectric material relates the flux of $K\mathbf{E}$ through the surface to the amount of free (not bound) charge enclosed by the surface

$$\oint K\mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0}$$

- This shows that filling a volume with a dielectric with relative permittivity K reduces the magnitude of the electric field by a factor of $1/K$

25 Current, Resistance, and Electromotive Force

25.1 Current

- A **current** is any motion of charge from one region to another
- The **drift velocity $\mathbf{v_d}$** of a current is the velocity of its particles
- While a current may come about through the movement of negative and/or positive charges, **conventional current** dictates that by convention we describe currents as if they were carried by positive charges
- The unit of current is the **ampere** which is defined to be one coulomb per second

$$1\,\text{A} = 1\,\text{C/s}$$

- The **charge concentration** n is the number of moving charged particles per unit volume
- The current through an area is given by

$$I = \frac{dQ}{dt} = n|q|v_d A$$

- The **current density** is the current per unit cross-sectional area

$$\mathbf{J} = nq\mathbf{v_d}$$

25.2 Resistivity

- The **resistivity** ρ of a material is defined by **Ohm's law**

$$\rho = \frac{E}{J}$$

- The unit of resistivity is ohm-meters (Ωm)
- The reciprocal of resistivity is **conductivity**
- Materials that obey Ohm's law are called **ohmic** or **linear** conductors
- Materials that don't obey Ohm's law are called **nonohmic** or **nonlinear** conductors
- The resistivity of a metallic conductor nearly always increases with increasing temperature

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

  where ρ_0 is the resistivity at reference temperature T_0 and α is the **temperature coefficient of resistivity**
- The resistivity of semiconductors decreases with increasing temperature
- Some materials exhibit **superconductivity** where their resistivity drops to 0 below a critical temperature

25.3 Resistance

- The ratio of the voltage and current in a conductor is called its **resistance**

$$R = \frac{V}{I} = \frac{\rho L}{A}$$

  where ρ is the resistivity of the conductor, L is its length, and A is its cross-sectional area
- If ρ is constant (as in ohmic materials), then R is also constant
- The unit of resistance is the ohm

$$1\,\Omega = 1\,\text{V/A}$$

- Because the resistivity of a material varies with temperature, so too does the resistance of a specific conductor

$$R(T) = R_0[1 + \alpha(T - T_0)]$$

- A device made to have a specific resistance is called a **resistor**

25.4 Electromotive Force and Circuits

- When a charge goes around a complete circuit and returns to its starting position its electric potential energy must be the same, but it experienced losses due to resistance along the way
- **Electromotive force** or **emf** $\mathcal{E}$ is the influence that makes current flow from lower to higher potential in a circuit and restores its original potential energy
- A device that provides emf is called a **source of emf**
- The SI unit of emf is the volt
- In an **ideal source of emf**
  - The potential difference between its terminals is constant regardless of the current passing through it
  - $\mathcal{E} = V = IR$
- Real sources of emf have **internal resistance** r that reduce the **terminal voltage**
$$V_{ab} = \mathcal{E} - Ir$$
- Real sources of emf can be modelled as an ideal source of emf $\mathcal{E}$ in series with a resistor r

25.5 Energy and Power in Electric Circuits

- **Power** is the time rate change of energy transfer
$$P = VI$$
where V is the voltage across a circuit element and I is the current in it
- The SI unit of power is the watt
$$1\,\mathrm{W} = 1\,\mathrm{J/s}$$
- If the circuit element is a resistor then $V = IR$ and
$$P = VI = I^2R = \frac{V^2}{R}$$
- If the circuit element is a source of emf outputting power then
$$P = VI = (\mathcal{E} - Ir)I = \mathcal{E}I - I^2r$$
where the $\mathcal{E}I$ term is the power generated by the element and the I^2r term is the power dissipated by its internal resistance
- If the circuit element is a source of emf consuming power (charging) then
$$P = VI = (\mathcal{E} + Ir)I = \mathcal{E}I + I^2r$$
where the terms are the same as above

25.6 Theory of Metallic Conduction

- The average time between collisions of an electron and positive ions is called the **mean free time** τ
- The resistivity of a metal can be approximated as

$$\rho = \frac{m}{ne^2\tau}$$

  where m is the mass of an electron, n is the number of free electrons per unit volume, e is the charge of an electron, and τ is the mean free time

26 Direct-Current Circuits

26.1 Resistors in Series and Parallel

- Circuit elements connected one after another with a single current path between them are said to be connected in **series**
- The current is the same for all circuit elements connected in series
- Circuit elements connected such there is an alternate current path for each element are said to be connected in **parallel**
- The potential difference / voltage is the same for all circuit elements connected in parallel
- For any combination of resistors we can always find a single resistor that could replace the combination and result in the same current and potential difference — the resistance of this resistor is called the **equivalent resistance**
- The equivalent resistance of a series combination of resistors equals the sum of the individual resistances

$$R_{\text{eq}} = R_1 + R_2 + \cdots$$

- The reciprocal of the equivalent resistance of a parallel combination of resistors equals the sum of the reciprocals of the individual resistances

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$