Introduction to Electrodynamics by David J. Griffiths Problems

Chris Doble

December 2023

Contents

2 Electrostatics

2.1

(a) $\mathbf{0}$

(b) The same as if only the opposite charge were present — all others are cancelled out.

2.2

$$\begin{aligned}\mathbf{E} &= \frac{1}{4\pi\epsilon_0}2\frac{q}{\mathscr{r}^2}\cos\theta\hat{\mathbf{x}}\\ &= \frac{1}{4\pi\epsilon_0}\frac{dq}{[(d/2)^2+z^2]^{3/2}}\hat{\mathbf{x}}\end{aligned}$$

2.3

$$\begin{aligned}\mathbf{r} &= z\hat{\mathbf{z}}\\ \mathbf{r}' &= x\hat{\mathbf{x}}\\ \boldsymbol{\mathscr{r}} &= z\hat{\mathbf{z}} - x\hat{\mathbf{x}}\\ \mathscr{r} &= \sqrt{x^2+z^2}\\ \hat{\boldsymbol{\mathscr{r}}} &= \frac{z\hat{\mathbf{z}} - x\hat{\mathbf{x}}}{\sqrt{x^2+z^2}}\\ \mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda}{x^2+z^2}\frac{z\hat{\mathbf{z}} - x\hat{\mathbf{x}}}{\sqrt{x^2+z^2}}\,dx\\ &= \frac{1}{4\pi\epsilon_0}\lambda\left(z\hat{\mathbf{z}}\int_0^L \frac{1}{(x^2+z^2)^{3/2}}\,dx - \hat{\mathbf{x}}\int_0^L \frac{x}{(x^2+z^2)}\,dx\right)\\ &= \frac{1}{4\pi\epsilon_0}\lambda\left[\frac{L}{z\sqrt{L^2+z^2}}\hat{\mathbf{z}} - \left(\frac{1}{z} - \frac{1}{\sqrt{L^2+z^2}}\right)\hat{\mathbf{x}}\right]\\ &= \frac{1}{4\pi\epsilon_0}\frac{\lambda}{z}\left[\left(-1+\frac{z}{\sqrt{L^2+z^2}}\right)\hat{\mathbf{x}} + \frac{L}{\sqrt{L^2+z^2}}\hat{\mathbf{z}}\right]\end{aligned}$$

2.4

The electric field a distance z above the midpoint of a line segment of length $2L$ and uniform line charge λ is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}}\hat{\mathbf{z}}.$$

Applying this to the four sides of the square, the horizontal components of opposite sides cancel leaving only the vertical component.

$$\begin{aligned}\cos\theta &= \frac{z}{\mathscr{r}}\\ &= \frac{z}{\sqrt{(a/2)^2+z^2}}\\ \mathbf{E} &= 4\left(\frac{1}{4\pi\epsilon_0}\frac{\lambda a}{\sqrt{(a/2)^2+z^2}\sqrt{(a/2)^2+(a/2)^2+z^2}}\hat{\mathbf{z}}\right)\cos\theta\\ &= \frac{1}{4\pi\epsilon_0}\frac{4a\lambda z}{[(a/2)^2+z^2]\sqrt{(a^2/2)+z^2}}\hat{\mathbf{z}}\end{aligned}$$

2.5

$$\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\frac{\lambda r}{r^2+z^2}\cos\alpha\, d\theta\,\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}\frac{2\pi\lambda r z}{(r^2+z^2)^{3/2}}\hat{\mathbf{z}}
\end{aligned}$$

2.6

$$\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int\frac{dq}{\mathscr{r}^2}\cos\theta\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{R}\frac{\sigma}{r^2+z^2}\frac{z}{\sqrt{r^2+z^2}}r\,dr\,d\theta\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}2\pi\sigma z\int_0^{R}\frac{r}{(r^2+z^2)^{3/2}}\,dr\,\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}2\pi\sigma z\left(\frac{1}{z}-\frac{1}{\sqrt{R^2+z^2}}\right)\hat{\mathbf{z}}
\end{aligned}$$

When $R\to\infty$

$$\mathbf{E} = \frac{\sigma}{2\epsilon_0}\hat{\mathbf{z}}.$$

2.7

$$\mathbf{E} = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\hat{\mathbf{z}} & z > R \\ \mathbf{0} & z < R \end{cases}$$

2.8

$$\mathbf{E} = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\hat{\mathbf{z}} & z > R \\ \frac{1}{4\pi\epsilon_0}\frac{qz}{R^3}\hat{\mathbf{z}} & z < R \end{cases}$$

2.9

(a)

$$\begin{aligned}
\rho &= \epsilon_0\nabla\cdot\mathbf{E} \\
&= \epsilon_0\frac{1}{r^2}\frac{\partial}{\partial r}(kr^5) \\
&= 5\epsilon_0 k r^2
\end{aligned}$$

(b)

$$\begin{aligned}
Q_{\text{enc}} &= \epsilon_0 \oint \mathbf{E} \cdot d\mathbf{a} \\
&= \epsilon_0 \int_0^{2\pi} \int_0^{\pi} kR^3 R \, d\theta R \sin\theta \, d\phi \\
&= 2\pi\epsilon_0 k R^5 [-\cos\theta]_0^{\pi} \\
&= 4\pi\epsilon_0 k R^5 \\
Q_{\text{enc}} &= \int_V \rho \, d\tau \\
&= \int_0^{2\pi} \int_0^{\pi} \int_0^{R} 5\epsilon_0 k r^2 \, dr r \, d\theta r \sin\theta \, d\phi \\
&= 10\pi\epsilon_0 k \int_0^{\pi} \int_0^{R} r^4 \sin\theta \, dr \, d\theta \\
&= 2\pi\epsilon_0 k R^5 [-\cos\theta]_0^{\pi} \\
&= 4\pi\epsilon_0 k R^5
\end{aligned}$$

2.10

If the charge was surrounded by 8 such cubes the total flux through all the cubes would be q/ϵ_0. There are 24 outside faces to the larger cube, so the total flux through the shaded face is $q/(24\epsilon_0)$.

2.11

$$\begin{aligned}
\int \mathbf{E}_{\text{inside}} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
&= 0 \\
\mathbf{E}_{\text{inside}} &= \mathbf{0} \\
\int \mathbf{E}_{\text{outside}} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
4\pi r^2 E_{\text{outside}} &= \frac{4\pi R^2 \sigma}{\epsilon_0} \\
\mathbf{E}_{\text{outside}} &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}
\end{aligned}$$

2.12

$$\begin{aligned}
\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
4\pi r^2 E &= \frac{\frac{4}{3}\pi r^3 \rho}{\epsilon_0} \\
\mathbf{E} &= \frac{r\rho}{3\epsilon_0}\hat{\mathbf{r}}
\end{aligned}$$

2.13

$$\begin{aligned}
\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
2\pi s l E &= \frac{l\lambda}{\epsilon_0} \\
\mathbf{E} &= \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}\hat{\mathbf{s}}
\end{aligned}$$

2.14

$$\begin{aligned}
Q_{\text{enc}} &= \int_V \rho \, d\tau \\
&= \int_0^{2\pi} \int_0^{\pi} \int_0^{r} k r'^3 \sin\theta \, dr' \, d\theta \, d\phi \\
&= 2\pi k \int_0^{\pi} \left[\frac{1}{4} r'^4 \sin\theta\right]_0^r d\theta \\
&= \frac{1}{2}\pi k r^4 [-\cos\theta]_0^{\pi} \\
&= \pi k r^4 \\
\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
4\pi r^2 E &= \frac{\pi k r^4}{\epsilon_0} \\
\mathbf{E} &= \frac{k r^2}{4\epsilon_0}\hat{\mathbf{r}}
\end{aligned}$$

2.15

(a) $\mathbf{E} = \mathbf{0}$

(b)

$$
\begin{aligned}
Q_{\text{enc}} &= \int_0^{2\pi}\int_0^{\pi}\int_a^{r} k\sin\theta\, dr'\, d\theta\, d\phi \\
&= 4\pi k(r-a) \\
4\pi r^2 E &= \frac{4\pi k(r-a)}{\epsilon_0} \\
\mathbf{E} &= \frac{k(r-a)}{\epsilon_0 r^2}\hat{\mathbf{r}}
\end{aligned}
$$

(c) $\mathbf{E} = \frac{k(b-a)}{\epsilon_0 r^2}\hat{\mathbf{r}}$

2.16

(a)

$$
\begin{aligned}
Q_{\text{enc}} &= \pi s^2 l\rho \\
2\pi s l E &= \frac{\pi s^2 l\rho}{\epsilon_0} \\
\mathbf{E} &= \frac{s\rho}{2\epsilon_0}\hat{\mathbf{s}}
\end{aligned}
$$

(b)

$$
\mathbf{E} = \frac{a^2\rho}{2\epsilon_0 s}\hat{\mathbf{s}}
$$

(c)

$$
\mathbf{E} = \mathbf{0}
$$

2.17

$$
\begin{aligned}
2AE_{\text{inside}} &= \frac{2Ay\rho}{\epsilon_0} \\
\mathbf{E}_{\text{inside}} &= \frac{y\rho}{\epsilon_0} \\
\mathbf{E} &= \begin{cases} \frac{d\rho}{\epsilon_0} & d < y \\ \frac{y\rho}{\epsilon_0} & 0 < y < d \\ -\frac{y\rho}{\epsilon_0} & -d < y < 0 \\ -\frac{d\rho}{\epsilon_0} & y < -d \end{cases}
\end{aligned}
$$

2.18

The electric field inside a uniformly charged solid sphere is

$$\mathbf{E} = \frac{r\rho}{3\epsilon_0}\hat{\mathbf{r}}.$$

$$\begin{aligned}
\mathbf{d} &= \mathbf{r}_1 - \mathbf{r}_2 \\
\mathbf{E} &= \frac{r_1\rho}{3\epsilon_0}\hat{\mathbf{r}}_1 - \frac{r_2\rho}{3\epsilon_0}\hat{\mathbf{r}}_2 \\
&= \frac{\rho}{3\epsilon_0}(\mathbf{r}_1 - \mathbf{r}_2) \\
&= \frac{\rho}{3\epsilon_0}\mathbf{d}
\end{aligned}$$