

University Physics with Modern Physics - Modern Physics by Young and Freedman Problems

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17 Temperature and Heat

17.1 Guided Practice

17.1.1

(a)

$$\begin{aligned}\Delta L &= \alpha L_0 \Delta T \\ \alpha &= \frac{\Delta L}{L_0 \Delta T} \\ &= 2.0 \times 10^{-5} \text{ K}^{-1}\end{aligned}$$

(b)

$$\begin{aligned}\Delta L &= \alpha L_0 \Delta T \\ &= -0.27 \text{ mm}\end{aligned}$$

17.1.2

$$\begin{aligned}\Delta V_C &= \beta V_{C0} \Delta T \\ &= (5.1 \times 10^{-5})(250)(-70) \\ &= -0.893 \text{ cm}^3 \\ \Delta V_E &= \beta V_{E0} \Delta T \\ &= (75 \times 10^{-5})(250)(-70) \\ &= -13.1 \text{ cm}^3 \\ \Delta V_C - \Delta V_E &= 12.2 \text{ cm}^3 \\ &= 12.2 \text{ mL}\end{aligned}$$

17.1.3

$$\begin{aligned}\frac{\Delta L}{L_0} &= \alpha \Delta T \\ Y &= \frac{F/A}{\Delta L/L_0} \\ \frac{\Delta L}{L_0} &= \frac{F}{AY} \\ \alpha \Delta T + \frac{F}{AY} &= 0 \\ \frac{F}{AY} &= -\alpha \Delta T \\ F &= -\alpha AY \Delta T \\ &= -(2.0 \times 10^{-5})(\pi 0.005^2)(9.0 \times 10^{10})(-12) \\ &= 1.70 \times 10^3 \text{ N}\end{aligned}$$

Tensile

17.1.4

$$\begin{aligned}\Delta L &= \alpha_A L_A \Delta T + \alpha_B L_B \Delta T \\ \frac{\Delta L}{\Delta T} &= \alpha_A L_A + \alpha_B (L - L_A) \\ &= (\alpha_A - \alpha_B) L_A + \alpha_B L \\ L_A &= \frac{1}{\alpha_A - \alpha_B} \left(\frac{\Delta L}{\Delta T} - \alpha_B L \right)\end{aligned}$$

17.1.5

$$\begin{aligned}0 &= m_{Al} c_{Al} \Delta T_{Al} + m_W c_W \Delta T_W \\ &= m_{Al} c_{Al} (T - T_{Al}) + m_W c_W (T - T_W) \\ m_{Al} &= - \frac{m_W c_W (T - T_W)}{c_{Al} (T - T_{Al})} \\ &= 0.20 \text{ kg}\end{aligned}$$

17.1.6

$$\begin{aligned}0 &= m_I L_f + m_C c_C \Delta T \\ &= m_I L_f - m_C c_C T \\ T &= \frac{m_I L_f}{m_C c_C} \\ &= 14.0^\circ \text{C}\end{aligned}$$

17.1.7

$$\begin{aligned}0 &= m_I L_F + m_I c_I \Delta T_I + m_E c_E \Delta T_E \\ &= m_I (L_F + c_I \Delta T_I) + m_E c_E \Delta T_E \\ m_I &= - \frac{m_E c_E \Delta T_E}{L_F + c_I \Delta T_I} \\ &= 0.176 \text{ kg}\end{aligned}$$

17.1.8

Cooling the silver to 0°C would take

$$Q = mc\Delta T = 92\,137.5 \text{ J}$$

whereas melting all of the ice would take

$$Q = mL_f = 83\,500\text{ J}$$

so all of the ice will melt.

$$\begin{aligned} 0 &= m_{Ag}c_{Ag}\Delta T_{Ag} + m_IL_f + m_Ic_I\Delta T_I + m_Ic_W\Delta T_W \\ &= m_{Ag}c_{Ag}(T - T_{Ag}) + m_IL_f - m_Ic_IT_I + m_Ic_WT \\ &= (m_{Ag}c_{Ag} + m_Ic_W)T - m_{Ag}c_{Ag}T_{Ag} + m_IL_f - m_Ic_IT_I \\ T &= \frac{m_{Ag}c_{Ag}T_{Ag} + m_Ic_IT_I - m_IL_f}{m_{Ag}c_{Ag} + m_Ic_W} \\ &= 3.31\text{ }^\circ\text{C} \end{aligned}$$

17.1.9

(a)

$$\begin{aligned} H &= kA\frac{T_H - T_C}{L} \\ k &= \frac{HL}{A(T_H - T_C)} \\ &= 0.754\text{ W}/(\text{m K}) \end{aligned}$$

(b)

$$H = kA\frac{T_H - T_C}{L} = 733\text{ W}$$

17.1.10

(a)

$$\begin{aligned} L &= 0.250\text{ m} \\ A &= 2.00 \times 10^{-4}\text{ m}^2 \\ k_B &= 109.0\text{ W}/(\text{m K}) \\ k_{Pb} &= 34.7\text{ W}/(\text{m K}) \\ T &= 185\text{ }^\circ\text{C} \\ H &= 6.00\text{ W} \end{aligned}$$

$$\begin{aligned} H &= k_B A \frac{T_H - T}{L} \\ T_H &= \frac{HL}{k_B A} + T \\ &= 254\text{ }^\circ\text{C} \end{aligned}$$

(b)

$$\begin{aligned}
 H &= k_{Pb}A \frac{T - T_C}{L} \\
 T_C &= T - \frac{HL}{k_{Pb}A} \\
 &= -31.1^\circ\text{C}
 \end{aligned}$$

17.1.11

$$\begin{aligned}
 H &= 4\pi(kr_E)^2 e\sigma T^4 \\
 (kr_E)^2 &= \frac{H}{4\pi e\sigma T^4} \\
 k &= \frac{1}{r_E} \sqrt{\frac{H}{4\pi e\sigma T^4}} \\
 &= 1.70
 \end{aligned}$$

17.1.12

(a)

$$\begin{aligned}
 H &= Ae\sigma T^4 \\
 &= \pi r^2 \sigma T^4 \\
 H &= kA \frac{T_H - T_C}{L} \\
 &= k\pi r^2 \frac{T_H - T_C}{L} \\
 \pi r^2 \sigma T^4 &= k\pi r^2 \frac{T_H - T_C}{L} \\
 T_H &= \frac{L\sigma T^4}{k} + T_C \\
 &= 14.26\text{ K}
 \end{aligned}$$

(b)

$$\begin{aligned}
 H &= mL_f \\
 \pi r^2 \sigma T^4 &= mL_f \\
 m &= \frac{\pi r^2 \sigma T^4}{L_f} \\
 &= 1.19 \times 10^{-4} \text{ kg/s} \\
 &= 0.427 \text{ kg/h}
 \end{aligned}$$

17.2 Exercises and Problems

17.2.15

$$\begin{aligned}\Delta V &= \beta V_0 \Delta T \\ \frac{\Delta V}{V_0} &= \beta(T - T_0) \\ T &= T_0 + \frac{\Delta V}{\beta V_0} \\ &= 49^\circ\text{C}\end{aligned}$$

17.2.25

$$\begin{aligned}Q &= (m_{Al}c_{Al} + m_Wc_W)\Delta T \\ &= 5.55 \times 10^5 \text{ J}\end{aligned}$$

17.2.33

$$\begin{aligned}\Delta K &= \frac{1}{2}mv^2 - \frac{1}{2}mv'^2 \\ &= \frac{1}{2}m(v^2 - v'^2) \\ &= 3.47 \text{ kJ} \\ \Delta K &= mc\Delta T \\ \Delta T &= \frac{\Delta K}{mc} \\ &= 6.14 \times 10^{-2}^\circ\text{C}\end{aligned}$$

17.2.35

(a)

$$\begin{aligned}0 &= m_m c_m \Delta T_m + m_w c_w \Delta T_w \\ c_m &= -\frac{m_w c_w \Delta T_w}{m_m \Delta T_m} \\ &= 215 \text{ J}/(\text{kg K})\end{aligned}$$

(b) Water because it has a higher specific heat

(c) It would be too small

17.2.45

$$\begin{aligned}\frac{1}{2}mv^2 &= mc\Delta T + mL_F \\ v &= \sqrt{2(c\Delta T + L_F)} \\ &= 366 \text{ m/s}\end{aligned}$$

17.2.55

$$\begin{aligned}k_C A \frac{T_H - T}{L} &= k A \frac{T}{L} \\ k_C T_H - k_C T &= k T \\ k_C T_H &= (k + k_C) T \\ T &= \frac{k_C}{k + k_C} T_H \\ 0.71 &= \frac{k_C}{k + k_C} \\ 0.71(k + k_C) &= k_C \\ 0.71k + 0.71k_C &= k_C \\ 0.71k &= 0.29k_C \\ k &= \frac{0.29}{0.71} k_C \\ &\approx 157 \text{ W/(mK)}\end{aligned}$$

17.2.57

(a)

$$\begin{aligned}k_W \frac{T - T_C}{L_W} &= k_S \frac{T_H - T}{L_S} \\ \left(\frac{k_W}{L_W} + \frac{k_S}{L_S} \right) T &= \frac{k_S}{L_S} T_H + \frac{k_W}{L_W} T_C \\ T &= \frac{\frac{k_S}{L_S} T_H + \frac{k_W}{L_W} T_C}{\frac{k_W}{L_W} + \frac{k_S}{L_S}} \\ &= -0.86^\circ\text{C}\end{aligned}$$

(b)

$$\begin{aligned}H &= k_W \frac{T - T_C}{L_W} \\ &= 24.4 \text{ W/m}^2\end{aligned}$$

17.2.65

$$\begin{aligned} H &= Ae\sigma T^4 \\ A &= \frac{H}{e\sigma T^4} \\ &= 2.1 \text{ cm}^2 \end{aligned}$$

17.2.69

$$\begin{aligned} \Delta L &= (\alpha_B L_B + \alpha_S L_S) \Delta T \\ T &= T_0 + \frac{\Delta L}{\alpha_B L_B + \alpha_S L_S} \\ &= 35.0^\circ \text{C} \end{aligned}$$

17.2.71

$$\begin{aligned} Q &= mc\Delta T \\ &= \rho V c \Delta T \\ \Delta T &= \frac{Q}{\rho V c} \\ \Delta V &= \beta V \Delta T \\ &= \frac{\beta Q}{\rho c} \\ c &= \frac{\beta Q}{\rho \Delta V} \end{aligned}$$

17.2.73

(a)

$$\begin{aligned} 0.0^\circ \text{M} &= -39^\circ \text{C} \\ 100.0^\circ \text{M} &= 357^\circ \text{C} \\ T_M &= \frac{T_C + 39^\circ \text{C}}{3.96} \\ \frac{100^\circ \text{C} + 39^\circ \text{C}}{3.96} &= 35.1^\circ \text{M} \end{aligned}$$

(b)

$$10^\circ \text{M} = 10 \frac{357^\circ \text{C} - (-39^\circ \text{C})}{100} = 39.6^\circ \text{C}$$

17.2.75

$$\begin{aligned}
 Ah + \beta_G Ah(T - T_0) &= Ah' + \beta_O Ah'(T - T_0) \\
 Ah + \beta_G AhT - \beta_G AhT_0 &= Ah' + \beta_O Ah'T - \beta_O Ah'T_0 \\
 (\beta_G Ah - \beta_O Ah')T &= (Ah' - \beta_O Ah'T_0) - (Ah - \beta_G AhT_0) \\
 T &= \frac{(1 - \beta_O T_0)h' - (1 - \beta_G T_0)h}{\beta_G h - \beta_O h'} \\
 &= 69.4^\circ\text{C}
 \end{aligned}$$

17.2.79

(a)

$$\begin{aligned}
 Y &= \frac{F/A}{\Delta L/L_0} \\
 \Delta L &= \frac{FL_0}{AY} \\
 \Delta L &= \alpha L_0 \Delta T \\
 \Delta L &= \alpha L_0 \Delta T + \frac{FL_0}{AY} \\
 \frac{F}{A} &= Y \left(\frac{\Delta L}{L_0} - \alpha \Delta T \right)
 \end{aligned}$$

(b)

$$\begin{aligned}
 \Delta L_B &= \alpha_B L_{B0} \Delta T \\
 \frac{\Delta L_B}{L_{B0}} &= \alpha_B \Delta T \\
 \frac{F}{A} &= Y_S (\alpha_B - \alpha_S) \Delta T \\
 &= 1.9 \times 10^8 \text{ Pa}
 \end{aligned}$$

17.2.85

(a)

$$\begin{aligned}
 \frac{dQ}{dT} &= nk \frac{T^3}{\theta^3} \\
 Q &= \int_a^b nk \frac{T^3}{\theta^3} \\
 &= \frac{nk}{\theta^3} \left[\frac{1}{4} T^4 \right]_a^b \\
 &= \frac{nk}{4\theta^3} (b^4 - a^4) \\
 &= 83.6 \text{ J}
 \end{aligned}$$

(b)

$$\begin{aligned}Q &= nC\Delta T \\C &= \frac{Q}{n\Delta T} \\&= 1.86 \text{ J}/(\text{mol K})\end{aligned}$$

(c)

$$C = 5.60 \text{ J}/(\text{mol K})$$

17.2.95

(a)

$$\begin{aligned}0 &= m_I L_f + m_I c_W \Delta T_I + m_C c_C \Delta T_I - m_S L_v + m_S c_W \Delta T_S \\&= m_I L_f + m_I c_W T + m_C c_C T - m_S L_v + m_S c_W (T - T_S) \\T &= \frac{m_S (L_v + c_W T_S) - m_I L_f}{m_I c_W + m_C c_C + m_S c_W} \\&= 86.1^\circ \text{C}\end{aligned}$$

(b) No ice, 0.13 kg water, no steam

17.2.99

(a)

$$\begin{aligned}H &= kA \frac{T_H - T_C}{L} \\&= 94 \text{ W}\end{aligned}$$

(b)

$$\begin{aligned}H_{\text{wood}} &= 12.4 \text{ W} \\H_{\text{glass}} &= 45.0 \text{ W} \\H' &= H + (H_{\text{glass}} - H_{\text{wood}}) \\&= 126.6 \text{ W} \\\frac{H'}{H} &= 1.35\end{aligned}$$

17.2.105

(b)

$$\begin{aligned}
 \frac{dQ}{dt} &= k \frac{T_H - T_C}{L} \\
 \frac{dQ}{dL} &= \rho L_f \\
 \frac{dL}{dt} &= \frac{dL}{dQ} \frac{dQ}{dt} \\
 &= \frac{1}{\rho L_f} k \frac{T_H - T_C}{L} \\
 L \frac{dL}{dt} &= \frac{k(T_H - T_C)}{\rho L_f} \\
 \int_0^t L \frac{dL}{dt} dt &= \int_0^t \frac{k(T_H - T_C)}{\rho L_f} dt \\
 \int_0^L L' dL' &= \frac{k(T_H - T_C)}{\rho L_f} t \\
 \frac{1}{2} L^2 &= \frac{k(T_H - T_C)}{\rho L_f} t \\
 L &= \sqrt{\frac{2k(T_H - T_C)}{\rho L_f} t}
 \end{aligned}$$

(c)

$$\begin{aligned}
 t &= \frac{L^2 \rho L_f}{2k(T_H - T_C)} \\
 &= 7.5 \text{ days}
 \end{aligned}$$

(d) $t \approx 530$ years; no

17.2.107

$$\begin{aligned}
A &= 2\pi \left(\frac{d}{2}\right)^2 + 2\pi \left(\frac{d}{2}\right) h \\
&= 8.34 \times 10^{-2} \text{ m}^2 \\
H &= Ae\sigma(T^4 - T_s^4) \\
&= Ae\sigma(T^4 - T_s^4) \\
&= -3.38 \times 10^{-2} \text{ W} \\
m &= \frac{H \times 60 \times 60}{L_v} \\
&= 5.82 \times 10^{-3} \text{ kg/h} \\
&= 5.82 \text{ g/h}
\end{aligned}$$

17.2.113

$$\begin{aligned}
r(x) &= R_2 - (R_2 - R_1) \frac{x}{L} \\
A(x) &= \pi r(x)^2 \\
&= \pi \left[R_2 - (R_2 - R_1) \frac{x}{L} \right]^2 \\
H &= kA(x) \frac{dT}{dx} \\
&= k\pi \left[R_2 - (R_2 - R_1) \frac{x}{L} \right]^2 \frac{dT}{dx} \\
\frac{1}{\left[R_2 - (R_2 - R_1) \frac{x}{L} \right]^2} H dx &= k\pi dT \\
\int_0^L \frac{1}{\left[R_2 - (R_2 - R_1) \frac{x}{L} \right]^2} H dx &= \int_{T_H}^{T_C} k\pi dT \\
\frac{HL}{R_2 - R_1} \left[\frac{1}{R_2 - (R_2 - R_1) \frac{x}{L}} \right]_0^L &= k\pi(T_C - T_H) \\
\frac{HL}{R_2 - R_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) &= k\pi(T_C - T_H) \\
\frac{HL}{R_2 - R_1} \frac{R_2 - R_1}{R_1 R_2} &= k\pi(T_C - T_H) \\
H &= \frac{k\pi R_1 R_2 (T_C - T_H)}{L}
\end{aligned}$$

17.2.115

(a)

$$\begin{aligned}
 H &= k(2\pi r L) \frac{dT}{dr} \\
 \frac{1}{r} H \, dr &= 2\pi k L \, dT \\
 \int_a^b \frac{1}{r} H \, dr &= \int_{T_1}^{T_2} 2\pi k L \, dT \\
 H \ln \frac{b}{a} &= 2\pi k L (T_2 - T_1) \\
 H &= \frac{2\pi k L (T_2 - T_1)}{\ln b/a}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \frac{2\pi k L (T - T_2)}{\ln r/a} &= \frac{2\pi k L (T_2 - T_1)}{\ln b/a} \\
 \frac{T - T_2}{\ln r/a} &= \frac{T_2 - T_1}{\ln b/a} \\
 T - T_2 &= \frac{\ln r/a}{\ln b/a} (T_2 - T_1) \\
 T &= T_2 + \frac{\ln r/a}{\ln b/a} (T_2 - T_1)
 \end{aligned}$$

17.2.117

a

17.2.119

a

18 Thermal Properties of Matter

18.1 Guided Practice

18.1.1

(a)

$$\begin{aligned}pV &= nRT \\ \frac{p}{T} &= \frac{nR}{V} \\ \frac{p_1}{T_1} &= \frac{p_2}{T_2} \\ p_2 &= p_1 \frac{T_2}{T_1} \\ &= 4.67 \times 10^5 \text{ Pa}\end{aligned}$$

(b)

$$\begin{aligned}pV &= nRT \\ n &= \frac{pV}{RT} \\ &= 0.280 \text{ mol}\end{aligned}$$

18.1.2

(a)

$$\begin{aligned}pV &= nRT \\ \frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\ V_2 &= \frac{V_1 p_1 T_2}{p_2 T_1} \\ &= 1.2 \times 10^3 \text{ m}^3\end{aligned}$$

(b)

$$\begin{aligned}\frac{V_2}{V_1} &= \frac{\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi r_1^3} \\ &= \left(\frac{r_2}{r_1}\right)^3 \\ \frac{r_2}{r_1} &= \sqrt[3]{\frac{V_2}{V_1}} \\ &= 4.5\end{aligned}$$

18.1.3

(a)

$$\begin{aligned}
 pV &= nRT \\
 n &= \frac{pV}{RT} \\
 &= 2.9 \times 10^{-3} \text{ mol/m}^3
 \end{aligned}$$

(b)

$$8.0 \times 10^{-5} \text{ kg/m}^3$$

18.1.4

(a)

$$\begin{aligned}
 pV &= \frac{m_{\text{total}}}{M} RT \\
 \frac{p}{\rho T} &= \frac{R}{M} \\
 \frac{p_1}{\rho_1 T_1} &= \frac{p_2}{\rho_2 T_2} \\
 &= \frac{p_2}{\rho_1 (p_2/p_1)^{3/5} T_2} \\
 T_2 &= \left(\frac{p_2}{p_1} \right)^{2/5} T_1
 \end{aligned}$$

(b)

$$\begin{aligned}
 \frac{\rho_2}{\rho_1} &= \frac{\rho_1 (p_2/p_1)^{3/5}}{\rho_1} \\
 &= \left(\frac{\frac{1}{2} p_1}{p_1} \right)^{3/5} \\
 &= \left(\frac{1}{2} \right)^{3/5} \\
 &\approx 0.660 \\
 \frac{T_2}{T_1} &= \frac{(p_2/p_1)^{2/5} T_1}{T_1} \\
 &= \left(\frac{\frac{1}{2} p_1}{p_1} \right)^{2/5} \\
 &= \left(\frac{1}{2} \right)^{2/5} \\
 &\approx 0.758
 \end{aligned}$$

(c)

$$\frac{\rho_2}{\rho_1} = 2^{3/5}$$

$$\approx 1.52$$

$$\frac{T_2}{T_1} = 2^{2/5}$$

$$\approx 1.32$$

18.1.5

$$\sqrt{\frac{3RT}{M_{\text{H}}}} = \sqrt{\frac{3RT_{\text{N}}}{M_{\text{N}}}}$$

$$T = \frac{M_{\text{H}}}{M_{\text{N}}} T_{\text{N}}$$

$$= 41.9 \text{ K}$$

$$= -231 \text{ }^{\circ}\text{C}$$

18.1.6

(a)

$$K_{\text{tr}} = \frac{3}{2} kT = 6.21 \times 10^{-20} \text{ J}$$

(b)

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = 8.63 \times 10^3 \text{ m/s}$$

18.1.7

(a)

$$pV = \frac{N}{N_A} RT$$

$$N = \frac{N_A pV}{RT}$$

$$= 1.50 \times 10^{27}$$

(b)

$$K_{\text{tr}} = \frac{3}{2} nRT = 9.11 \times 10^6 \text{ J}$$

(c)

$$\begin{aligned}\frac{1}{2}mv^2 &= K_{\text{tr}} \\ v &= \sqrt{\frac{2K_{\text{tr}}}{m}} \\ &= 110 \text{ m/s}\end{aligned}$$

18.1.8

(a) 5.5

(b) 38.5

(c) 6.2

18.1.9

(a)

$$\lambda = \frac{kT}{4\pi\sqrt{2}r^2p} = 6.8 \times 10^{-6} \text{ m}$$

(b)

$$\begin{aligned}\lambda_{\text{Earth}} &= 5.54 \times 10^{-8} \text{ m} \\ \frac{\lambda_{\text{Mars}}}{\lambda_{\text{Earth}}} &= 1.2 \times 10^2\end{aligned}$$

18.1.10

(a)

$$\begin{aligned}\lambda &= \frac{kT}{4\pi\sqrt{2}r^2p} \\ p &= \frac{kT}{4\pi\sqrt{2}r^2\lambda} \\ &= 5.7 \times 10^{-3} \text{ Pa}\end{aligned}$$

(b)

$$\begin{aligned}pV &= nRT \\ n &= \frac{pV}{RT} \\ &= 2.3 \times 10^{-6} \text{ mol}\end{aligned}$$

18.1.11

(a)

$$\begin{aligned}
 pV &= nRT \\
 p &= \frac{nRT}{V} \\
 &= 2.0 \times 10^7 \text{ Pa} \\
 \lambda &= \frac{kT}{4\pi\sqrt{2}r^2p} \\
 &= 1.2 \times 10^{-8} \text{ m}
 \end{aligned}$$

(b)

$$\begin{aligned}
 v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\
 &= 1.4 \times 10^3 \text{ m/s} \\
 \lambda &= vt_{\text{mean}} \\
 t_{\text{mean}} &= \frac{\lambda}{v} \\
 &= 8.6 \times 10^{-12} \text{ s}
 \end{aligned}$$

18.1.12

(a)

$$\begin{aligned}
 v_{\text{rms}}t_{\text{mean}} &= \lambda \\
 \sqrt{\frac{3kT}{m}}t_{\text{mean}} &= \frac{kT}{4\pi\sqrt{2}r^2p} \\
 t_{\text{mean}} &= \frac{kT}{4\pi\sqrt{2}r^2p} \sqrt{\frac{m}{3kT}} \\
 &= \frac{1}{4\pi r^2p} \sqrt{\frac{mkT}{6}}
 \end{aligned}$$

(b) Doubling r .

18.1.13

(a)

$$\begin{aligned}v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\&= 515 \text{ m/s} \\ \frac{1}{2}mv_{\text{rms}}^2 &= mgh \\ h &= \frac{v_{\text{rms}}^2}{2g} \\&= 102 \text{ km}\end{aligned}$$

(b)

$$\begin{aligned}&\int_{2025}^{\infty} 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} dv \\&= (3.03 \times 10^{-8}) \int_{2025}^{\infty} v^2 e^{-(5.65 \times 10^{-6})v^2} dv \\&= 4.8 \times 10^{-10}\end{aligned}$$

Yes, some escape.

18.2 Exercises and Problems

18.2.7

$$\begin{aligned}pV &= nRT \\ \frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\ T_2 &= \frac{p_2 V_2 T_1}{p_1 V_1} \\&= 776 \text{ K} \\&= 503 \text{ }^\circ\text{C}\end{aligned}$$

18.2.9

$$\begin{aligned}pV &= nRT \\ \frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\ p_2 &= \frac{p_1 V_1 T_2}{T_1 V_2} \\&= 1.97 \times 10^4 \text{ Pa}\end{aligned}$$

18.2.13

$$\begin{aligned}\frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\ V_2 &= \frac{p_1 T_2}{T_1 p_2} V_1 \\ &= (5.08 \times 10^{-2}) V_1\end{aligned}$$

18.2.17

(a)

$$\begin{aligned}pV &= \frac{m_{\text{total}}}{M} RT \\ m_{\text{total}} &= \frac{pVM}{RT} \\ &= 6.91 \times 10^{-16} \text{ kg}\end{aligned}$$

(b)

$$\rho = \frac{m_{\text{total}}}{V} = 2.30 \times 10^{-13} \text{ kg/m}^3$$

18.2.21

(a)

$$\begin{aligned}pV &= \frac{N}{N_A} RT \\ N &= \frac{pV N_A}{RT} \\ &= 2.19 \times 10^6\end{aligned}$$

(b)

$$2.44 \times 10^{19}$$

18.2.23

(a)

$$\begin{aligned}
 pV &= \frac{N}{N_A} RT \\
 \frac{V}{N} &= \frac{RT}{N_A p} \\
 s &= \sqrt[3]{\frac{V}{N}} \\
 &= \sqrt[3]{\frac{RT}{N_A p}} \\
 &= 3.45 \times 10^{-9} \text{ m}
 \end{aligned}$$

18.2.25

(a)

$$\begin{aligned}
 K_{\text{tr}} &= \frac{3}{2} nRT \\
 &= \frac{3}{2} pV \\
 &= 5.82 \times 10^7 \text{ J}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \frac{1}{2} m v^2 &= K_{\text{tr}} \\
 v &= \sqrt{\frac{2K_{\text{tr}}}{m}} \\
 &= 241 \text{ m/s}
 \end{aligned}$$

18.2.27

$$\begin{aligned}
 pV &= nRT \\
 p &= \frac{nR}{V} T \\
 \frac{nR}{V} &= m \\
 n &= \frac{mV}{R} \\
 &= 1.07 \text{ mol} \\
 N &= nN_A \\
 &= 6.44 \times 10^{23}
 \end{aligned}$$

18.2.29

(a)

$$\begin{aligned}
 v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} \\
 &= 1.93 \times 10^6 \text{ m/s} \\
 &= 0.006c
 \end{aligned}$$

Not a significant fraction of c .

(b)

$$\begin{aligned}
 0.10c &= \sqrt{\frac{3kT}{m}} \\
 (0.10c)^2 &= \frac{3kT}{m} \\
 T &= \frac{(0.10c)^2 m}{3k} \\
 &= 7.26 \times 10^{10} \text{ K}
 \end{aligned}$$

18.2.31

(a)

$$\frac{3}{2}kT = 6.21 \times 10^{-21} \text{ J}$$

(b)

$$(v^2)_{\text{av}} = \frac{2}{m} \left(\frac{3}{2}kT \right) = 2.34 \times 10^5 \text{ (m/s)}^2$$

(c)

$$v_{\text{rms}} = \sqrt{(v^2)_{\text{av}}} = 484 \text{ m/s}$$

(d)

$$p = mv = \frac{M}{N_A} v = 2.57 \times 10^{-23} \text{ kg m/s}$$

(e)

$$\begin{aligned}
 \Delta P &= 2P \\
 &= 5.14 \times 10^{-23} \text{ kg m/s} \\
 \Delta t &= \frac{2l}{v} \\
 &= 4.13 \times 10^{-4} \text{ s} \\
 F_{\text{av}} &= \frac{\Delta P}{\Delta t} \\
 &= 1.24 \times 10^{-19} \text{ N}
 \end{aligned}$$

(f)

$$p_{\text{av}} = \frac{F_{\text{av}}}{A} = 1.24 \times 10^{-17} \text{ Pa}$$

(g)

$$\begin{aligned} p &= N p_{\text{av}} \\ N &= \frac{p}{p_{\text{av}}} \\ &= 8.15 \times 10^{21} \end{aligned}$$

(h)

$$\begin{aligned} pV &= \frac{N}{N_A} RT \\ N &= \frac{pV N_A}{RT} \\ &= 2.44 \times 10^{22} \end{aligned}$$

18.2.33

$$\begin{aligned} \sqrt{\frac{3RT}{M_{\text{N}}}} &= \sqrt{\frac{3RT_{\text{H}}}{M_{\text{H}}}} \\ T &= \frac{M_{\text{N}}}{M_{\text{H}}} T_{\text{H}} \\ &= 4074 \text{ K} \\ &= 3800 ^\circ\text{C} \end{aligned}$$

18.2.35

$$\begin{aligned}
 C_V &= \frac{5}{2}R \\
 v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\
 T &= \frac{Mv_{\text{rms}}^2}{3R} \\
 Q &= nC_V\Delta T \\
 \Delta T &= \frac{Q}{nC_V} \\
 v'_{\text{rms}} &= \sqrt{\frac{3R(T + \Delta T)}{M}} \\
 &= \sqrt{\frac{3R\left(\frac{Mv_{\text{rms}}^2}{3R} + \frac{Q}{nC_V}\right)}{M}} \\
 &= \sqrt{v_{\text{rms}}^2 + \frac{3RQ}{Mn\frac{5}{2}R}} \\
 &= \sqrt{v_{\text{rms}}^2 + \frac{6Q}{5Mn}} \\
 &= 1.02 \times 10^3 \text{ m/s}
 \end{aligned}$$

18.2.39

(a)

$$\begin{aligned}
 c_{V,\text{N}} &= \frac{5}{2}R \\
 &= 742 \text{ J/(kg K)} \\
 c_{V,\text{water}} &= 4190 \text{ J/(kg K)} \\
 &= 5.6c_{V,\text{N}}
 \end{aligned}$$

(b)

$$\begin{aligned}Q &= mc_{V,\text{water}}\Delta T \\&= 4.19 \times 10^4 \text{ J} \\m &= \frac{Q}{c_{V,\text{N}}\Delta T} \\&= 5.65 \text{ kg} \\pV &= \frac{m_{\text{total}}}{M}RT \\V &= \frac{m_{\text{total}}RT}{Mp} \\&= 4.87 \text{ m}^3 \\&= 4.87 \times 10^3 \text{ L}\end{aligned}$$

18.2.41

(a)

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}} = 337 \text{ m/s}$$

(b)

$$v_{\text{av}} = 380 \text{ m/s}$$

(c)

$$v_{\text{rms}} = 412 \text{ m/s}$$

18.2.43

(a)

$$\frac{v_{\text{rms}}}{v} = \sqrt{\frac{3RT}{M}} \sqrt{\frac{M}{\gamma RT}} = \sqrt{\frac{3}{\gamma}} = 1.34$$

(b)

$$\frac{v_{\text{av}}}{v} = \sqrt{\frac{8RT}{\pi M}} \sqrt{\frac{M}{\gamma RT}} = \sqrt{\frac{8}{\pi\gamma}} = 1.23$$

18.2.45

- (a) The minimum pressure is $p_1 = 611.657 \text{ Pa}$. If $p < p_1$ the ice sublimates directly to gas.
- (b) The maximum pressure is $p_2 = 2.212 \times 10^7 \text{ Pa}$. The ice melts to water, then the properties of the water gradually change to those of steam with no phase transition.

18.2.49

(a)

$$\begin{aligned}
 p' - p &= -\rho gh \\
 &= -1.18 \times 10^4 \text{ Pa}
 \end{aligned}$$

(b)

$$\begin{aligned}
 p_1 V_1 &= p_2 V_2 \\
 V_2 &= \frac{p_1}{p_2} V_1 \\
 &= 0.56 \text{ L}
 \end{aligned}$$

18.2.51

$$\begin{aligned}
 0 &= \rho_{\text{cold}} V g - \rho_{\text{hot}} V g - m g \\
 &= \rho_{\text{cold}} V - \rho_{\text{hot}} V - m \\
 \rho_{\text{hot}} &= \rho_{\text{cold}} - \frac{m}{V} \\
 \frac{Mp}{RT} &= \rho_{\text{cold}} - \frac{m}{V} \\
 T &= \frac{Mp}{R(\rho_{\text{cold}} - m/V)} \\
 &= 542 \text{ K} \\
 &= 269^\circ \text{C}
 \end{aligned}$$

18.2.53

$$\begin{aligned}
 pV &= \frac{m_{\text{total}}}{M} RT \\
 m_{\text{total}} &= \frac{pVM}{RT} \\
 &= 0.285 \text{ kg} \\
 m'_{\text{total}} &= 0.0896 \text{ kg} \\
 \Delta m &= 0.195 \text{ kg}
 \end{aligned}$$

18.2.57

(a)

$$\begin{aligned}
 0 &= \rho V g - (m_{\text{adventurer}} + m_{\text{bell}} + m_{\text{water}})g \\
 &= \rho V - m_{\text{adventurer}} - m_{\text{bell}} - m_{\text{water}} \\
 m_{\text{water}} &= \rho V - m_{\text{adventurer}} - m_{\text{bell}} \\
 &= 98 \text{ kg} \\
 V_{\text{water}} &= \frac{m_{\text{water}}}{\rho_{\text{water}}} \\
 &= 0.0956 \text{ m}^3
 \end{aligned}$$

(b)

$$\begin{aligned}
 pV &= nRT \\
 p &= \frac{nRT}{V} \\
 p &= \rho g y \\
 \rho g y &= \frac{nRT}{V} \\
 n &= \frac{\rho g V}{RT} y \\
 \frac{dn}{dt} &= \frac{\rho g V}{RT} \frac{dy}{dt} \\
 &= 18.2 \text{ mol/s}
 \end{aligned}$$

(c)

$$\begin{aligned}
 pV &= nRT \\
 n &= \frac{pV}{RT} \\
 &= 756 \text{ mol} \\
 \frac{n}{dn/dt} &= 41.5 \text{ m}
 \end{aligned}$$

18.2.59

(a)

$$\begin{aligned}pV &= nRT \\n_{\text{balloon}} &= \frac{pV}{RT} \\&= (9.11 \times 10^6) \frac{1}{T} \\n_{\text{cylinder}} &= \frac{pV}{RT} \\&= (2.97 \times 10^5) \frac{1}{T} \\\frac{n_{\text{balloon}}}{n_{\text{cylinder}}} &= 30.7\end{aligned}$$

(b)

$$\begin{aligned}0 &= \rho Vg - Mng - mg \\mg &= (\rho V - Mn)g \\&= 8420 \text{ N}\end{aligned}$$

(c)

$$mg = 7810 \text{ N}$$

18.2.67

(c)

$$U(r) = U_0 \left[\left(\frac{R_0}{r} \right)^{12} - 2 \left(\frac{R_0}{r} \right)^6 \right]$$

$$F(r) = 12 \frac{U_0}{R_0} \left[\left(\frac{R_0}{r} \right)^{13} - \left(\frac{R_0}{r} \right)^7 \right]$$

$$\begin{aligned} 0 &= U_0 \left[\left(\frac{R_0}{r_1} \right)^{12} - 2 \left(\frac{R_0}{r_1} \right)^6 \right] \\ &= \left(\frac{R_0}{r_1} \right)^{12} - 2 \left(\frac{R_0}{r_1} \right)^6 \\ &= \left(\frac{R_0}{r_1} \right)^6 - 2 \\ 2 &= \left(\frac{R_0}{r_1} \right)^6 \\ 2r_1^6 &= R_0^6 \\ r_1 &= \frac{1}{\sqrt[6]{2}} R_0 \\ &\approx 0.89 R_0 \end{aligned}$$

$$\begin{aligned} 0 &= 12 \frac{U_0}{R_0} \left[\left(\frac{R_0}{r_2} \right)^{13} - \left(\frac{R_0}{r_2} \right)^7 \right] \\ 0 &= \left(\frac{R_0}{r_2} \right)^{13} - \left(\frac{R_0}{r_2} \right)^7 \\ &= \left(\frac{R_0}{r_2} \right)^6 - 1 \\ r_2 &= R_0 \end{aligned}$$

$$\frac{r_1}{r_2} = \frac{1}{\sqrt[6]{2}}$$

(d)

$$\begin{aligned} W &= \int_{r_2}^{\infty} -F \, dr \\ &= \int_{R_0}^{\infty} -12 \frac{U_0}{R_0} \left[\left(\frac{R_0}{r} \right)^{13} - \left(\frac{R_0}{r} \right)^7 \right] \, dr \\ &= -12 \frac{U_0}{R_0} \left(-\frac{R_0}{12} \right) \\ &= U_0 \end{aligned}$$

18.2.69

(a)

$$C_V = 2R = 16.63 \text{ J/(mol K)}$$

(b) Less than because vibrational energy will play a smaller role.

18.2.71

(a)

$$\begin{aligned} \frac{1}{2}mv^2 &\geq \frac{GmM}{R_p} \\ &\geq gmR_p \end{aligned}$$

(b)

$$\begin{aligned} \frac{3}{2}kT &\geq mgR_p \\ T_N &\geq \frac{2mgR_p}{3k} \\ &\geq 1.40 \times 10^5 \text{ K} \\ T_H &\geq 1.02 \times 10^4 \text{ K} \end{aligned}$$

(c)

$$\begin{aligned} T_N &\geq 6.37 \times 10^3 \text{ K} \\ T_H &\geq 459 \text{ K} \end{aligned}$$

(d) Because it's very easy to atmospheric particles to escape.

18.2.73

$$\begin{aligned}
 \int_0^\infty v^2 f(v) dv &= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^4 e^{-mv^2/2kT} dv \\
 &= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{3}{2^3(m/2kT)^2} \sqrt{\frac{\pi}{(m/2kT)}} \\
 &= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{3}{8} \left(\frac{2kT}{m}\right)^2 \sqrt{\frac{2\pi kT}{m}} \\
 &= \frac{3kT}{m}
 \end{aligned}$$

18.2.75

(b)

$$\begin{aligned}
 v_{\text{mp}} &= \sqrt{\frac{2kT}{m}} \\
 &= 395 \text{ m/s} \\
 f(v_{\text{mp}}) &= 2.10 \times 10^{-3} \\
 \Delta N &\approx N f(v_{\text{mp}}) \Delta v \\
 &\approx (4.20 \times 10^{-2}) N
 \end{aligned}$$

(c)

$$\begin{aligned}
 7v_{\text{mp}} &= 2765 \text{ m/s} \\
 f(7v_{\text{mp}}) &= 1.43 \times 10^{-22} \\
 \Delta N &\approx (2.85 \times 10^{-21}) N
 \end{aligned}$$

18.2.77

(a)

$$\begin{aligned}
 0 &= pA - p_0A - mg \\
 p &= p_0 + \frac{mg}{A} \\
 &= p_0 + \frac{mg}{\pi r^2}
 \end{aligned}$$

(b)

$$\begin{aligned} p_1 V_1 &= p_2 V_2 \\ p_2 &= \frac{V_1}{V_2} p_1 \\ &= \frac{Ah}{A(h+y)} p_1 \\ &= \frac{h}{h+y} p_1 \\ &\approx \left(1 - \frac{y}{h}\right) p_1 \\ F &= \left(1 - \frac{y}{h}\right) p_1 \pi r^2 - p_0 \pi r^2 - mg \\ &= \left(1 - \frac{y}{h}\right) \left(p_0 + \frac{mg}{\pi r^2}\right) \pi r^2 - p_0 \pi r^2 - mg \\ &= \left(1 - \frac{y}{h}\right) (p_0 \pi r^2 + mg) - p_0 \pi r^2 - mg \\ &= -\frac{y}{h} (p_0 \pi r^2 + mg) \end{aligned}$$

(c)

$$\begin{aligned} F &= -kx \\ k &= \frac{1}{h} (p_0 \pi r^2 + mg) \\ \omega &= \sqrt{\frac{k}{m}} \\ &= \sqrt{\frac{1}{h} \left(\frac{p_0 \pi r^2}{m} + g \right)} \\ f &= \frac{\omega}{2\pi} \\ &= \frac{1}{2\pi} \sqrt{\frac{g}{h} \left(1 + \frac{p_0 \pi r^2}{gm} \right)} \end{aligned}$$

If the displacement is not small the oscillation is not simple harmonic because we can't use the approximation $\frac{h}{h+y} \approx 1 - \frac{y}{h}$.

18.2.81

(a)

$$I = 2mr^2 = 4.1 \times 10^{-46} \text{ kg m}^2$$

(b)

$$2 \left(\frac{1}{2} (2m) v_i^2 \right) = 2 \left(\frac{1}{2} (2m) v_f^2 + \frac{1}{2} I \omega^2 \right)$$

$$2m v_i^2 = 2m v_f^2 + 2m r^2 \omega^2$$

$$v_i^2 = v_f^2 + r^2 \omega^2$$

$$-2r(2m)v_i = -2I\omega$$

$$2mr v_i = 2mr^2 \omega$$

$$v_i = r\omega$$

(c)

$$\omega = \frac{v_i}{r}$$

$$v_i^2 = v_f^2 + r^2 \left(\frac{v_i}{r} \right)^2$$

$$v_f = 0$$

(d)

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

$$= 514 \text{ m/s}$$

$$\omega = 5.47 \times 10^{12} \text{ rad/s}$$

18.2.83

(a)

$$\begin{aligned} \lambda &= \frac{V}{4\pi\sqrt{2}r^2N} \\ &= 4.50 \times 10^{11} \text{ m} \end{aligned}$$

(b)

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} \\ &= 704 \text{ m/s} \end{aligned}$$

$$\begin{aligned} t_{\text{mean}} &= \frac{\lambda}{v_{\text{rms}}} \\ &= 6.39 \times 10^8 \text{ s} \\ &= 20 \text{ years} \end{aligned}$$

(c)

$$\begin{aligned}pV &= NkT \\p &= \frac{NkT}{V} \\&= 1.38 \times 10^{-14} \text{ Pa}\end{aligned}$$

(d)

$$\begin{aligned}m_{\text{total}} &= \rho V \\&= \rho \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 \\&= 2.96 \times 10^{32} \text{ kg}\end{aligned}$$

$$\begin{aligned}\frac{1}{2}mv^2 &= \frac{Gmm_{\text{total}}}{r} \\v &= \sqrt{\frac{4Gm_{\text{total}}}{d}} \\&= 640 \text{ m/s}\end{aligned}$$

It would evaporate.

(f)

$$\begin{aligned}T_{\text{ISM}} &= \frac{(N/V)_{\text{nebula}}}{(N/V)_{\text{ISM}}} T_{\text{nebula}} \\&= 2.0 \times 10^5 \text{ K}\end{aligned}$$

34 times hotter than the sun.

18.2.85

a

18.2.87

c

19 The First Law of Thermodynamics

19.1 Guided Practice

19.1.1

(a)

$$\begin{aligned}\Delta U &= Q - W \\ Q &= \Delta U + W \\ &= 5.75 \times 10^3 \text{ J}\end{aligned}$$

(b)

$$\begin{aligned}\Delta U &= Q - W \\ &= -3.2 \times 10^4 \text{ J}\end{aligned}$$

(c)

$$\begin{aligned}\Delta U &= Q - W \\ W &= Q - \Delta U \\ &= -1.85 \times 10^3 \text{ J}\end{aligned}$$

19.1.2

(a)

$$W = p(V_2 - V_1) = 155 \text{ J}$$

(b)

$$W = 0$$

(c)

$$W = p(V_2 - V_1) = -375 \text{ J}$$

(d)

$$W = \frac{1}{2}(p_1 + p_2)(V_2 - V_1) = 875 \text{ J}$$

19.1.3

(a)

$$\begin{aligned}W &= p(V_2 - V_1) \\ &= -240 \text{ J} \\ \Delta U &= Q - W \\ &= 1.80 \times 10^3 \text{ J}\end{aligned}$$

(b)

$$\begin{aligned}W &= p(V_2 - V_1) \\&= -720 \text{ J} \\ \Delta U &= Q - W \\ Q &= \Delta U + W \\&= 1.08 \times 10^3 \text{ J}\end{aligned}$$

19.1.4

(a)

$$Q = mL = 3.43 \times 10^6 \text{ J}$$

(b)

$$W = p(V_2 - V_1) = 3.43 \times 10^5 \text{ J}$$

(c)

$$\Delta U = Q - W = 3.09 \times 10^6 \text{ J}$$

19.1.5

(a)

$$\Delta U = \Delta Q = nC_V \Delta T = 998 \text{ J}$$

(b)

$$\Delta U = \Delta Q = nC_V \Delta T = 748 \text{ J}$$

(c)

$$\Delta U = \Delta Q = nC_V \Delta T = 599 \text{ J}$$

19.1.6

(a)

$$V = \frac{nRT}{p} = 5.24 \times 10^{-2} \text{ m}^3$$

(b) (i)

$$\begin{aligned}T &= 327^\circ \text{C} \\ \Delta U &= Q \\&= nC_V \Delta T \\&= 1.31 \times 10^4 \text{ J}\end{aligned}$$

(ii)

$$\begin{aligned}T &= 327^\circ\text{C} \\ \Delta U &= Q \\ &= nC_V\Delta T \\ &= 1.31 \times 10^4 \text{ J}\end{aligned}$$

(iii)

$$\begin{aligned}T &= 927^\circ\text{C} \\ \Delta U &= 3.92 \times 10^4 \text{ J}\end{aligned}$$

19.1.7

(a)

$$\begin{aligned}pV &= nRT \\ \frac{pV}{R} &= nT\end{aligned}$$

$$\begin{aligned}(2p) &= nR(2T) \\ \Delta T &= T\end{aligned}$$

$$\begin{aligned}\Delta U &= Q - W \\ &= nC_V\Delta T \\ &= C_V(nT) \\ &= \frac{3}{2}R\frac{pV}{R} \\ &= \frac{3}{2}pV \\ &= 4.50 \times 10^4 \text{ J}\end{aligned}$$

(b)

$$pV = nRT$$

$$\frac{pV}{R} = nT$$

$$pV = nRT$$

$$p\left(\frac{1}{2}V\right) = nR\left(\frac{1}{2}T\right)$$

$$\Delta T = -\frac{1}{2}T$$

$$\Delta U = nC_V\Delta T$$

$$= C_V\left(-\frac{1}{2}nT\right)$$

$$= -\frac{3}{4}R\frac{pV}{R}$$

$$= -\frac{3}{4}pV$$

$$= -2.25 \times 10^4 \text{ J}$$

(c)

$$\Delta U = 1.17 \times 10^5 \text{ J}$$

19.1.8

(a)

$$Q = nC_V\Delta T$$

$$= \frac{5}{2}nRT$$

$$W = 0$$

$$\Delta U = Q - W$$

$$= \frac{5}{2}nRT$$

(b)

$$\begin{aligned}Q &= nC_P\Delta T \\&= \frac{7}{2}nRT\end{aligned}$$

$$W = p(V_2 - V_1)$$

$$\begin{aligned}\Delta U &= \frac{7}{2}nRT - p(V_2 - V_1) \\&= \frac{7}{2}nRT - 2nRT + nRT \\&= \frac{5}{2}nRT\end{aligned}$$

(c)

$$Q = 0$$

$$\begin{aligned}W &= nC_V(T_1 - T_2) \\&= -\frac{5}{2}nRT\end{aligned}$$

$$\begin{aligned}\Delta U &= Q - W \\&= \frac{5}{2}nRT\end{aligned}$$

19.1.9

(a)

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

(b)

$$\begin{aligned}p_1 V_1^\gamma &= p_2 V_2^\gamma \\p_2 &= \left(\frac{V_1}{V_2}\right)^\gamma p_1 \\&= 6.41 \times 10^4 \text{ Pa}\end{aligned}$$

(c)

$$\begin{aligned}W &= \frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2) \\&= 623 \text{ J}\end{aligned}$$

19.1.10

(a)

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5}$$

(b)

$$\begin{aligned} T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ V_2^{\gamma-1} &= \frac{T_1}{T_2} V_1^{\gamma-1} \\ V_2 &= \left(\frac{T_1}{T_2} \right)^{1/(\gamma-1)} V_1 \\ &= 5.79 \times 10^{-4} \text{ m}^3 \end{aligned}$$

(c)

$$\begin{aligned} p_1 V_1^\gamma &= p_2 V_2^\gamma \\ p_2 &= \left(\frac{V_1}{V_2} \right)^\gamma p_1 \\ &= 2.95 \times 10^6 \text{ Pa} \end{aligned}$$

(d)

$$\begin{aligned} W &= \frac{1}{\gamma-1} (p_1 V_1 - p_2 V_2) \\ &= -2.65 \times 10^3 \text{ J} \end{aligned}$$

19.1.11

(a)

$$\begin{aligned} pV &= nRT \\ p &= \frac{nRT}{V} \\ &= 3.17 \times 10^5 \text{ Pa} \end{aligned}$$

(b)

$$\begin{aligned} p_1 V_1^\gamma &= p_2 V_2^\gamma \\ p_2 &= \left(\frac{V_1}{V_2} \right)^\gamma p_1 \\ &= 8.21 \times 10^4 \text{ Pa} \end{aligned}$$

(c)

$$\begin{aligned}T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\T_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 \\&= 178 \text{ K}\end{aligned}$$

(d)

$$\begin{aligned}W &= \frac{1}{\gamma-1}(p_1 V_1 - p_2 V_2) \\&= 7.94 \times 10^3 \text{ J}\end{aligned}$$

19.1.12

(a)

$$\begin{aligned}\left[p + \left(\frac{an^2}{V^2}\right)\right](V - nb) &= nRT \\p + \left(\frac{an^2}{V^2}\right) &= \frac{nRT}{V - nb} \\p &= \frac{nRT}{V - nb} - \frac{an^2}{V^2}\end{aligned}$$

$$\begin{aligned}W &= \int_{V_1}^{V_2} p dV \\&= \int_{V_1}^{V_2} \left(\frac{nRT}{V - nb} - \frac{an^2}{V^2}\right) dV \\&= \left[nRT \ln(V - nb) + \frac{an^2}{V}\right]_{V_1}^{V_2} \\&= nRT \ln(V_2 - nb) + \frac{an^2}{V_2} - nRT \ln(V_1 - nb) - \frac{an^2}{V_1} \\&= nRT \ln \frac{V_2 - nb}{V_1 - nb} + an^2 \frac{V_1 - V_2}{V_1 V_2}\end{aligned}$$

(b) (i)

$$W = 2.80 \times 10^3 \text{ J}$$

(ii)

$$\begin{aligned} W &= \int_{V_1}^{V_2} p \, dV \\ &= \int_{V_1}^{V_2} \frac{nRT}{V} \, dV \\ &= nRT [\ln V]_{V_1}^{V_2} \\ &= 3.11 \times 10^3 \text{ J} \end{aligned}$$

19.2 Exercises and Problems

19.2.1

(b)

$$\begin{aligned} W &= p(V_2 - V_1) \\ &= nR(T_2 - T_1) \\ &= 1.33 \times 10^3 \text{ J} \end{aligned}$$

19.2.3

(b)

$$\begin{aligned} p_1 V_1 &= nRT \\ p_2 V_2 &= nRT \\ 3p_1 V_2 &= nRT \\ V_2 &= \frac{1}{3} V_1 \\ W &= \int_{V_1}^{V_2} p \, dV \\ &= \int_{V_1}^{V_1/3} \frac{nRT}{V} \, dV \\ &= nRT \ln \frac{1}{3} \\ &= -6.18 \times 10^3 \text{ J} \end{aligned}$$

19.2.5

(a)

$$pV = nRT$$

$$V = \frac{nRT}{p}$$

$$\begin{aligned} W &= \int_{V_1}^{V_2} p \, dV \\ &= \int_{nRT/p_1}^{nRT/p_2} \frac{nRT}{V} \, dV \\ &= nRT \ln \frac{nRT/p_2}{nRT/p_1} \\ &= nRT \ln \frac{p_1}{p_2} \end{aligned}$$

$$\frac{W}{nRT} = \ln \frac{p_1}{p_2}$$

$$\begin{aligned} p_1 &= p_2 e^{W/nRT} \\ &= 1.05 \times 10^5 \text{ Pa} \\ &= 1.04 \text{ atm} \end{aligned}$$

19.2.9

(a)

$$W = p(V_2 - V_1) = 3.47 \times 10^4 \text{ J}$$

(b)

$$\Delta U = Q - W = 8.03 \times 10^4 \text{ J}$$

(c) No, because it's an isobaric process.

19.2.11

(a)

$$\begin{aligned} T_a &= \frac{pV}{nR} \\ &= 278 \text{ K} \\ T_b &= 694 \text{ K} \\ T_c &= 1250 \text{ K} \end{aligned}$$

The lowest temperature is 278 K and it occurred at point a .

(b)

$$W_{ab} = 0$$

$$W_{bc} = 162 \text{ J}$$

(c)

$$\Delta U = Q - W = 52 \text{ J}$$

19.2.13

(a)

$$T_a = \frac{pV}{nR}$$

$$= 5.35 \times 10^2 \text{ K}$$

$$T_b = 9.36 \times 10^3 \text{ K}$$

$$T_c = 1.50 \times 10^4 \text{ K}$$

(b)

$$W = 2.10 \times 10^4 \text{ J}$$

(c)

$$Q = \Delta U + W = 3.60 \times 10^4 \text{ J}$$

19.2.17

(b)

$$V_1 = \frac{nRT_1}{p_1}$$

$$= 6.18 \times 10^{-3} \text{ m}^3$$

$$V_2 = 8.23 \times 10^{-3} \text{ m}^3$$

$$W = p(V_2 - V_1)$$

$$= 207 \text{ J}$$

(c) The piston

(d)

$$C_V = \frac{5}{2}R$$

$$C_P = \frac{7}{2}R$$

$$\Delta U = nC_P\Delta T$$

$$= 727 \text{ J}$$

$$Q = \Delta U + W$$

$$= 934 \text{ J}$$

19.2.19

(a)

$$\begin{aligned}
\Delta U &= Q - W \\
&= Q - 0 \\
&= nC_V\Delta T \\
\Delta T &= \frac{\Delta U}{nC_V} \\
&= 168 \text{ K} \\
T_2 &= T_1 + \Delta T \\
&= 948 \text{ K}
\end{aligned}$$

(b)

$$\begin{aligned}
Q &= nC_P\Delta T \\
\Delta T &= \frac{Q}{nC_P} \\
&= 120 \text{ K} \\
T_2 &= T_1 + \Delta T \\
&= 900 \text{ K}
\end{aligned}$$

19.2.21

$$\begin{aligned}
C_V &= \frac{3}{2}R \\
C_P &= \frac{5}{2}R \\
Q &= nC_P\Delta T \\
&= \frac{5}{2}nR(T_2 - T_1) \\
W &= p(V_2 - V_1) \\
&= nR(T_2 - T_1) \\
\frac{W}{Q} &= \frac{2}{5}
\end{aligned}$$

19.2.23

(a)

$$\begin{aligned}
\Delta U &= Q - W \\
&= 747 \text{ J}
\end{aligned}$$

(b)

$$\begin{aligned}Q &= nC_P\Delta T \\C_P &= \frac{Q}{n\Delta T} \\&= 37.0 \text{ J}/(\text{mol K}) \\C_V &= C_P - R \\&= 28.6 \text{ J}/(\text{mol K}) \\\gamma &= \frac{C_P}{C_V} \\&= 1.29\end{aligned}$$

19.2.25

(a)

$$\begin{aligned}V_1 &= \frac{nRT}{p_1} \\&= 3.46 \times 10^{-3} \text{ m}^3 \\V_2 &= 8.64 \times 10^{-4} \text{ m}^3 \\W &= \int_{V_1}^{V_2} p dV \\&= \int_{V_1}^{V_2} \frac{nRT}{V} dV \\&= nRT \ln \frac{V_2}{V_1} \\&= -606 \text{ J}\end{aligned}$$

(b)

$$\Delta U = 0 \text{ J}$$

(c) Yes, liberate

$$Q = \Delta U + W = -606 \text{ J}$$

19.2.27

(a)

$$\begin{aligned}
C_V &= \frac{3}{2}R \\
C_P &= \frac{5}{2}R \\
\gamma &= \frac{C_P}{C_V} \\
&= \frac{5}{3} \\
p_1 V_1^\gamma &= p_2 V_2^\gamma \\
p_2 &= \left(\frac{V_1}{V_2} \right)^\gamma p_1 \\
&= 4.76 \times 10^5 \text{ Pa}
\end{aligned}$$

(b)

$$\begin{aligned}
W &= \frac{C_V}{R} (p_1 V_1 - p_2 V_2) \\
&= -1.06 \times 10^4 \text{ J}
\end{aligned}$$

(c)

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
\frac{T_2}{T_1} &= \left(\frac{V_1}{V_2} \right)^{\gamma-1} \\
&= 1.59
\end{aligned}$$

Heated

19.2.29

(b)

$$\begin{aligned}
W &= nC_V(T_1 - T_2) \\
&= 314 \text{ J}
\end{aligned}$$

(c)

$$\begin{aligned}
\Delta U &= Q - W \\
&= 0 - W \\
&= -314 \text{ J}
\end{aligned}$$

19.2.31

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_1 \left(\frac{nRT_1}{p_1} \right)^{\gamma-1} &= T_2 \left(\frac{nRT_2}{p_2} \right)^{\gamma-1} \\
T_2^\gamma &= T_1^\gamma \left(\frac{p_2}{p_1} \right)^{\gamma-1} \\
T_2 &= T_1 \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} \\
&= 285 \text{ K} \\
&= 11.6^\circ \text{C}
\end{aligned}$$

19.2.33

$$\begin{aligned}
C_V &= \frac{3}{2}R \\
C_P &= \frac{5}{2}R \\
\gamma &= \frac{5}{3} \\
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_1 \left(\frac{nRT_1}{p_1} \right)^{\gamma-1} &= 2T_1 \left(\frac{2nRT_1}{p_2} \right)^{\gamma-1} \\
\frac{1}{p_1^{\gamma-1}} &= \frac{2^\gamma}{p_2^{\gamma-1}} \\
p_1^{\gamma-1} &= \frac{p_2^{\gamma-1}}{2^\gamma} \\
p_2 &= 2^{\gamma/(\gamma-1)} p_1 \\
&= 2^{5/2} p_1 \\
&= 4\sqrt{2} p_1
\end{aligned}$$

19.2.35

(a) Increase

(b)

$$W = \frac{1}{2}(p_a + p_b)(V_B - V_A) = 4.8 \text{ kJ}$$

19.2.37

(a)

$$\begin{aligned}
 pV &= nRT \\
 n &= \frac{pV}{RT} \\
 &= 0.678 \text{ mol}
 \end{aligned}$$

(b)

$$\begin{aligned}
 pV &= nRT \\
 V &= \frac{nRT}{p} \\
 &= 3.33 \times 10^{-2} \text{ m}^3
 \end{aligned}$$

(c)

$$\begin{aligned}
 W &= \int_{V_1}^{V_2} p \, dV \\
 &= \int_{V_1}^{V_2} \frac{nRT}{V} \, dV \\
 &= nRT \ln \frac{V_2}{V_1} \\
 &= 2.22 \text{ kJ}
 \end{aligned}$$

(d)

$$\Delta U = 0$$

19.2.39

(a)

$$\begin{aligned}
 \Delta U &= Q - W \\
 &= 30.0 \text{ J} \\
 Q &= \Delta U + W \\
 &= 45.0 \text{ J}
 \end{aligned}$$

(b) Liberate

$$Q = \Delta U + W = -65.0 \text{ J}$$

(c)

$$\Delta U_{\text{ad}} = 8.0 \text{ J}$$

$$W_{\text{ad}} = 15.0 \text{ J}$$

$$\begin{aligned} Q_{\text{ad}} &= \Delta U_{\text{ad}} + W_{\text{ad}} \\ &= 23.0 \text{ J} \end{aligned}$$

$$\begin{aligned} Q_{\text{db}} &= \Delta U_{\text{ab}} - \Delta U_{\text{ad}} \\ &= 22.0 \text{ J} \end{aligned}$$

19.2.43

(a)

$$p_1 V_1 = p_2 V_2$$

$$\begin{aligned} V_2 &= \frac{p_1}{p_2} V_1 \\ &= 8.0 \times 10^{-4} \text{ m}^3 \\ &= 0.80 \text{ L} \end{aligned}$$

(b)

$$\begin{aligned} T_a &= \frac{pV}{nR} \\ &= 304 \text{ K} \end{aligned}$$

$$T_b = 1.21 \times 10^3 \text{ K}$$

$$T_c = 1.21 \times 10^3 \text{ K}$$

(c)

$$\begin{aligned}\Delta U_{ab} &= Q_{ab} - W_{ab} \\ &= Q_{ab} \\ &= nC_V\Delta T \\ &= 74.0 \text{ J into the gas}\end{aligned}$$

$$\begin{aligned}V_c &= \frac{nRT_c}{p_c} \\ &= 7.97 \times 10^{-4} \text{ m}^3 \\ \Delta U_{ca} &= Q_{ca} - W_{ca} \\ nC_V\Delta T &= Q_{ca} - p(V_a - V_c) \\ Q_{ca} &= nC_V\Delta T + p(V_a - V_c) \\ &= -104 \text{ J out of the gas}\end{aligned}$$

$$\begin{aligned}\Delta U_{bc} &= Q_{bc} - W_{bc} \\ Q_{bc} &= \Delta U_{bc} + W_{bc} \\ &= nC_V\Delta T + \int_{V_b}^{V_c} p dV \\ &= nRT \ln \frac{V_c}{V_b} \\ &= 55.6 \text{ J into the gas}\end{aligned}$$

(d)

$$\Delta U_{ab} = 74.0 \text{ J increase}$$

$$\Delta U_{bc} = 0.0 \text{ J no change}$$

$$\begin{aligned}\Delta U_{ca} &= nC_V\Delta T \\ &= -74.0 \text{ J decrease}\end{aligned}$$

19.2.47

(b)

$$V_2 = \frac{p_1}{p_2} V_1 = 6.0 \text{ L}$$

(c)

$$\begin{aligned}n &= \frac{pV}{RT} \\&= 6.01 \times 10^{-2} \text{ mol} \\W_{12} &= \int_{V_1}^{V_2} p dV \\&= nRT_1 \ln \frac{V_2}{V_1} \\&= 208 \text{ J}\end{aligned}$$

$$\begin{aligned}W_{23} &= p_2(V_3 - V_2) \\&= -113 \text{ J}\end{aligned}$$

$$W = 95.0 \text{ J}$$

(d) Heat it at constant volume

19.2.49

(a) As the wind descends it experiences greater atmospheric pressure. This compresses the wind, increasing its temperature. It's important that it be moving fast so it's an adiabatic process.

(b)

$$\begin{aligned}T_2 &= \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} T_1 \\&= 287 \text{ K} \\&= 13.9^\circ\text{C} \\\Delta T &= T_2 - T_1 \\&= 11.9^\circ\text{C}\end{aligned}$$

19.2.51

(a)

$$\begin{aligned}
p_1 V_1^\gamma &= p_2 V_2^\gamma \\
p_1 (Ah)^\gamma &= p_2 [A(h - \Delta h)]^\gamma \\
\frac{p_1}{p_2} h^\gamma &= (h - \Delta h)^\gamma \\
\left(\frac{p_1}{p_2}\right)^{1/\gamma} h &= h - \Delta h \\
\Delta h &= h \left[1 - \left(\frac{p_1}{p_2}\right)^{1/\gamma} \right] \\
&= 16.8 \text{ cm}
\end{aligned}$$

(b)

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 \\
&= \left(\frac{Ah}{A(h - \Delta h)}\right)^{\gamma-1} T_1 \\
&= \left(\frac{1}{1 - \Delta h/h}\right)^{\gamma-1} T_1 \\
&= 469 \text{ K} \\
&= 196^\circ \text{C}
\end{aligned}$$

(c)

$$W = nC_V(T_1 - T_2) = -7.03 \times 10^4 \text{ J}$$

19.2.59

(a) a is adiabatic, b is isochoric, c is isobaric

(b)

$$\begin{aligned}\Delta U &= Q_b - W_b \\ &= Q_b - 0 \\ &= Q_b\end{aligned}$$

$$\begin{aligned}\Delta U &= Q_c - W_c \\ &= Q_c - p(V_2 - V_1) \\ &= Q_c - nR(T_2 - T_1)\end{aligned}$$

$$\begin{aligned}Q_b &= Q_c - nR(T_2 - T_1) \\ T_2 &= T_1 + \frac{Q_c - Q_b}{nR} \\ &= 28.0^\circ\text{C}\end{aligned}$$

(c)

$$\begin{aligned}Q_b &= nC_V\Delta T \\ C_V &= \frac{Q_b}{n\Delta T} \\ &= 12.5\text{ J}/(\text{mol K})\end{aligned}$$

$$\begin{aligned}W_a &= nC_V(T_1 - T_2) \\ &= -30.0\text{ J}\end{aligned}$$

$$W_b = 0$$

$$\begin{aligned}\Delta U_c &= Q_c - W_c \\ W_c &= Q_c - \Delta U_c \\ &= Q_c - nC_V\Delta T \\ &= 20.0\text{ J}\end{aligned}$$

(d)

$$\begin{aligned}\gamma &= \frac{C_P}{C_V} \\ &= \frac{C_V + R}{C_V} \\ &= 1.67\end{aligned}$$

$$\begin{aligned}T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ \left(\frac{V_2}{V_1}\right)^{\gamma-1} &= \frac{T_1}{T_2} \\ \frac{V_2}{V_1} &= \left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} \\ &= 0.961\end{aligned}$$

$$\Delta V_b = 0$$

$$\begin{aligned}\frac{V_2}{V_1} &= \frac{nRT_2/p}{nRT_1/p} \\ &= \frac{T_2}{T_1} \\ &= 1.03\end{aligned}$$

a

(e) Decrease, stay the same, increase

19.2.61

(a)

$$\begin{aligned}
 r &= 1.50 \text{ cm} \\
 l_{\max} &= 30.0 \text{ cm} \\
 l_{\min} &= l_{\max}/v \\
 p &= 101 \text{ kPa} \\
 T &= 30.0^\circ\text{C} \\
 V_1 &= \pi r^2 l_{\max} \\
 &= 2.12 \times 10^{-4} \text{ m}^3 \\
 V_2 &= \pi r^2 l_{\min} \\
 &= \pi r^2 \frac{l_{\max}}{v} \\
 &= \frac{V_1}{v} \\
 n &= \frac{pV}{RT} \\
 &= 8.50 \times 10^{-3} \text{ mol} \\
 T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
 T_2 &= T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} \\
 &= T_1 v^{\gamma-1} \\
 W_{\text{adiabatic}} &= nC_V(T_1 - T_2) \\
 &= nC_V(T_1 - T_1 v^{\gamma-1}) \\
 &= nC_V T_1 (1 - v^{\gamma-1}) \\
 &= 53.5(1 - v^{0.4}) \\
 W_{\text{isothermal}} &= \int_{V_1}^{V_2} p dV \\
 &= \int_{V_1}^{V_2} \frac{nRT_2}{V} dV \\
 &= nRT_2 \ln \frac{V_2}{V_1} \\
 &= nRT_1 v^{\gamma-1} \ln v \\
 &= 21.4 v^{0.4} \ln v \\
 W &= 53.5(1 - v^{0.4}) + 21.4 v^{0.4} \ln v \\
 &= 53.5 + v^{0.40}(21.4 \ln v - 53.5)
 \end{aligned}$$

(b)

$$\begin{aligned}T_2 &\leq T_{\max} \\T_1 v^{\gamma-1} &\leq T_{\max} \\v &\leq \left(\frac{T_{\max}}{T_1}\right)^{1/(\gamma-1)} \\&\leq 7.35\end{aligned}$$

The largest integer value of v is 7.

(c) 7

(d) 7

(e)

$$\begin{aligned}T_2 &= T_1 v^{\gamma-1} \\&= 660 \text{ K} \\&= 387^\circ \text{C} \\Q &= nC_V \Delta T \\&= -63.0 \text{ J}\end{aligned}$$

19.2.63

$$\begin{aligned}\frac{p_1}{T_1} &= \frac{p_2}{T_2} \\p_2 &= \frac{T_2}{T_1} p_1 \\&= 1.27 \times 10^7 \text{ Pa} \\&= 1.84 \times 10^3 \text{ psi}\end{aligned}$$

c

19.2.65

$$\begin{aligned}p_1 V_1 &= p_2 V_2 \\V_1 &= \frac{p_2}{p_1} V_2 \\&= 6.01 \times 10^{-5} \text{ m}^3 \\&= 6.01 \times 10^{-2} \text{ L}\end{aligned}$$

d

20 The Second Law of Thermodynamics

20.1 Guided Practice

20.1.1

(a)

$$W = eQ_H \Rightarrow Q_H = \frac{W}{e} = 6.89 \times 10^4 \text{ J}$$

(b)

$$|W| = |Q_H| - |Q_C| \Rightarrow |Q_C| = |Q_H| - |W| = 5.65 \times 10^4 \text{ J}$$

20.1.2

$$e = \frac{W}{Q_H} = \frac{Q_H + Q_C}{Q_H} = 1 + \frac{Q_C}{Q_H} = 0.173 = 17.3\%$$

20.1.3

(a)

$$(1 - e)Q_H = Q_C \Rightarrow Q_H = \frac{Q_C}{1 - e} = 6.17 \times 10^8 \text{ J}$$

(b)

$$W = eQ_H = 1.21 \times 10^8 \text{ J}$$

20.1.4

(a)

$$W = 3600P = 3.96 \times 10^8 \text{ J}$$

(b)

$$Q_H = mL_c = 1.70 \times 10^9 \text{ J}$$

(c)

$$e = \frac{W}{Q_H} = 0.233 = 23.3\%$$

20.1.5

(a)

$$\begin{aligned} e_{\text{Carnot}} &= 1 - \frac{T_C}{T_H} \\ &= 1 + \frac{Q_C}{Q_H} \\ &= 1 + \frac{W - Q_H}{Q_H} \\ &= 0.21 \end{aligned}$$

(b)

$$|Q_C| = |Q_H| - |W| = 6.32 \times 10^4 \text{ J}$$

(c)

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H} \Rightarrow T_H = -\frac{Q_H}{Q_C} T_C = 377 \text{ K} = 104^\circ \text{C}$$

20.1.6

(a)

$$e = 1 - \frac{T_C}{T_H} = 0.6$$

(b)

$$n = 0.200 \text{ mol}$$

$$\gamma = 1.40$$

$$T_H = 227^\circ\text{C} = 500 \text{ K}$$

$$T_C = -73^\circ\text{C} = 200 \text{ K}$$

$$p_a = 10.0 \times 10^5 \text{ Pa}$$

$$\begin{aligned} V_a &= \frac{nRT_H}{p_a} \\ &= 8.31 \times 10^{-4} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} V_b &= 2V_a \\ &= 1.66 \times 10^{-3} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} p_b &= \frac{nRT_H}{V_b} \\ &= 5.01 \times 10^5 \text{ Pa} \end{aligned}$$

$$\begin{aligned} W_{ab} &= \int_{V_a}^{V_b} p dV \\ &= nRT_H \ln 2 \\ &= 576 \text{ J} \end{aligned}$$

$$\begin{aligned} V_c &= \left(\frac{T_H}{T_C} \right)^{1/(\gamma-1)} V_b \\ &= 1.64 \times 10^{-2} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} p_c &= \frac{nrT_C}{V_c} \\ &= 2.03 \times 10^4 \text{ Pa} \end{aligned}$$

$$\begin{aligned} W_{bc} &= \frac{1}{\gamma - 1} (p_b V_b - p_c V_c) \\ &= 1.25 \text{ kJ} \end{aligned}$$

$$\begin{aligned} V_d &= \frac{1}{2} V_c \\ &= 8.20 \times 10^{-3} \text{ m}^3 \end{aligned}$$

$$p_d = 4.06 \times 10^4 \text{ Pa}$$

$$\begin{aligned} W_{cd} &= \int_{V_c}^{V_d} p dV \\ &= nRT_C \ln \frac{1}{2} \\ &= -231 \text{ J} \end{aligned}$$

$$\begin{aligned} W_{da} &= \frac{1}{\gamma - 1} (p_d V_d - p_a V_a) \\ &= -1.25 \text{ kJ} \end{aligned}$$

20.1.7

(a)

$$K = \frac{T_C}{T_H - T_C} = 7.52$$

(b)

$$W = \frac{Q_C}{K} = 5.32 \times 10^5 \text{ J}$$

20.1.8

(a)

$$\begin{aligned} W &= \int_{V_a}^{V_b} p dV \\ &= \int_{V_a}^{2V_a} \frac{nRT_H}{V} dV \\ &= nRT_H \ln 2 \end{aligned}$$

(b)

$$\begin{aligned} W &= nC_V(T_H - T_C) \\ &= \frac{3}{2}nR(T_H - T_C) \end{aligned}$$

(c)

$$\begin{aligned} nRT_H \ln 2 &= \frac{3}{2}nR(T_H - T_C) \\ \ln 2 &= \frac{3}{2} \left(1 - \frac{T_C}{T_H} \right) \\ \frac{T_C}{T_H} &= 1 - \frac{2}{3} \ln 2 \\ e &= 1 - \frac{T_C}{T_H} \\ &= \frac{2}{3} \ln 2 \\ &= 0.462 \end{aligned}$$

20.1.9

(a)

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = 655 \text{ J/K}$$

(b)

$$\Delta S = mc \int_{159}^{351} \frac{dT}{T} = 1.92 \times 10^3 \text{ J/K}$$

(c)

$$\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = 2.43 \times 10^3 \text{ J/K}$$

20.1.10

(a)

$$\begin{aligned} n &= 5.00 \text{ mol} \\ V_1 &= 0.120 \text{ m}^3 \\ T_1 &= 20.0^\circ\text{C} \\ V_2 &= 0.360 \text{ m}^3 \\ T_2 &= 20.0^\circ\text{C} \\ \Delta U &= nC_V \Delta T \\ &= 0 \\ Q &= W \\ &= \int_{V_1}^{V_2} p dV \\ &= \int_{V_1}^{V_2} \frac{nRT}{V} dV \\ &= nRT \ln \frac{V_2}{V_1} \\ &= 1.34 \times 10^4 \text{ J} \\ \Delta S &= \frac{Q}{T} \\ &= 45.7 \text{ J/K} \end{aligned}$$

(b) Change in entropy is path independent, so $\Delta S = 45.7 \text{ J/K}$.

20.1.11

(a)

$$\Delta S = 0$$

(b)

$$\Delta S = \frac{Q}{T} = -150 \text{ J/K}$$

(c)

$$\Delta S = \frac{Q}{T} = 218 \text{ J/K}$$

(d)

$$\Delta S = 68 \text{ J/K}$$

The net entropy increases.

20.1.12

(a)

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = 1.22 \times 10^3 \text{ J/K}$$

(b)

$$\begin{aligned}\Delta S &= \int \frac{dQ}{T} \\ &= mc \int_{368}^{273} \frac{dT}{T} \\ &= -1.05 \times 10^3 \text{ J/K}\end{aligned}$$

(c)

$$\Delta S = 160 \text{ J/K}$$

The net entropy increases.

20.1.13

(a)

$$\begin{aligned}0 &= m_i L_f + m_w c(T - T_w) + m_i c(T - T_i) \\ T &= \frac{(m_w T_w + m_i T_i)c - m_i L_f}{(m_w + m_i)c} \\ &= 307 \text{ K} \\ &= 34.3^\circ \text{C}\end{aligned}$$

(b)

$$\begin{aligned}\Delta S_i &= \frac{Q}{T_1} + \int \frac{dQ}{T} \\ &= \frac{m_i L_f}{T_1} + m_i c \ln \frac{T_2}{T_1} \\ &= 101 \text{ J/K} \\ \Delta S_w &= m_w c \ln \frac{T_2}{T_1} \\ &= -86.0 \text{ J/K} \\ \Delta S &= 15.0 \text{ J/K}\end{aligned}$$

20.2 Exercises and Problems

20.2.5

(a)

$$\begin{aligned}p_1 V_1^\gamma &= p_2 V_2^\gamma \\p_1 &= \left(\frac{V_2}{V_1}\right)^\gamma p_2 \\&= 12.3 \text{ atm}\end{aligned}$$

(b) It enters during process *ca*.

$$\begin{aligned}T_a &= \frac{p_a V_a}{nR} \\&= 1.20 \times 10^3 \text{ K} \\T_c &= \frac{p_c V_c}{nR} \\&= 146 \text{ K} \\\gamma &= \frac{C_P}{C_V} \\&= \frac{C_V + R}{C_V} \\C_V \gamma &= C_V + R \\C_V(\gamma - 1) &= R \\C_V &= \frac{R}{\gamma - 1} \\Q &= nC_V \Delta T \\&= 5.48 \text{ kJ}\end{aligned}$$

(c) It leaves during *bc*.

$$\begin{aligned}T_b &= \frac{p_b V_b}{nR} \\&= 656 \text{ K} \\\Delta U &= Q - W \\Q &= \Delta U + W \\&= nC_V \Delta T + p_b(V_c - V_b) \\&= -3.71 \text{ kJ}\end{aligned}$$

(d)

$$\begin{aligned} W &= W_{ab} + W_{bc} + W_{ca} \\ &= nC_V(T_a - T_b) + p_b(V_c - V_b) + 0 \\ &= 1.77 \text{ kJ} \end{aligned}$$

(e)

$$e = \frac{W}{Q_H} = 0.323 = 32.3\%$$

20.2.7

(a)

$$\begin{aligned} e &= 1 - \frac{1}{r^{\gamma-1}} \\ r^{\gamma-1} &= \frac{1}{1-e} \\ r &= \left(\frac{1}{1-e} \right)^{1/(\gamma-1)} \\ &= 12 \end{aligned}$$

(b)

$$\begin{aligned} Q_C &= 7.4 \text{ kJ} \\ Q_H &= 20 \text{ kJ} \end{aligned}$$

20.2.9

(a)

$$e = 1 - \frac{1}{r^{\gamma-1}} = 0.581 = 58.1\%$$

(b)

$$\begin{aligned} e' &= 0.595 = 59.5\% \\ \Delta e &= 0.014 \\ &= 1.4\% \end{aligned}$$

20.2.11

$$\begin{aligned}
K &= 2.25 \\
W &= 135 \text{ W} \\
H &= KW \\
&= 304 \text{ W} \\
Q &= mc\Delta T \\
&= 1.30 \times 10^6 \text{ J} \\
\frac{Q}{H} &= 4.29 \times 10^3 \text{ s} \\
&= 1.2 \text{ h}
\end{aligned}$$

20.2.13

$$\begin{aligned}
T_H &= T_C + 72.0 \text{ C}^\circ \\
e &= 1 - \frac{T_C}{T_H} \\
&= 1 - \frac{T_H - 72.0 \text{ C}^\circ}{T_H} \\
&= 1 - 1 + \frac{72 \text{ C}^\circ}{T_H} \\
T_H &= \frac{72 \text{ C}^\circ}{e} \\
&= 576 \text{ K} \\
T_C &= 504 \text{ K}
\end{aligned}$$

20.2.21

$$\begin{aligned}
\frac{T_{CA}}{T_{HA} - T_{CA}} &= 1.16 \frac{T_{CB}}{T_{HB} - T_{CB}} \\
(T_{HB} - T_{CB}) &= 1.3(T_{HA} - T_{CA}) \\
T_{CB} &= 180 \text{ K} \\
\frac{T_{CA}}{T_{HA} - T_{CA}} &= 1.16 \frac{T_{CB}}{1.3(T_{HA} - T_{CA})} \\
T_{CA} &= \frac{1.16}{1.3} T_{CB} \\
&= 161 \text{ K}
\end{aligned}$$

20.2.29

$$\begin{aligned}
\Delta S &= S' - S \\
&= k \ln w' - k \ln w \\
&= k \ln \frac{w'}{w} \\
&= k \ln \frac{(425/0.0024)^N w}{w} \\
&= k \ln (1.77 \times 10^5)^{n N_A} \\
&= 10.0 \text{ J/K}
\end{aligned}$$

20.2.31

(a)

$$\begin{aligned}
\frac{Q_C}{Q_H} &= -\frac{T_C}{T_H} \\
Q_C &= -\frac{T_C}{T_H} Q_H \\
&= -121 \text{ J}
\end{aligned}$$

(b)

$$\begin{aligned}
n &= \frac{U}{W} \\
&= \frac{mgh}{Q_H + Q_C} \\
&= 3.80 \times 10^3
\end{aligned}$$

20.2.33

(a)

$$W = eQ_H = 90.2 \text{ J}$$

(b)

$$Q_C = Q_H - W = 320 \text{ J}$$

(c)

$$T_C = T_H(1 - e) = 318 \text{ K} = 45^\circ \text{C}$$

(d) 0

(e)

$$m = \frac{W}{gh} = 0.263 \text{ kg}$$

20.2.37

(a)

$$\begin{aligned}T_A &= \frac{p_A V_A}{nR} \\&= 241 \text{ K} \\T_B &= 241 \text{ K}\end{aligned}$$

(b) Absorbed during bc , rejected during ab and ca .

(c)

$$T_C = 481 \text{ K}$$

(d)

$$\begin{aligned}W_{AB} &= \int_{V_A}^{V_B} p dV \\&= nRT_A \ln \frac{V_B}{V_A} \\&= -1389 \text{ J} \\Q_{AB} &= -1389 \text{ J} \\W_{BC} &= p_B(V_C - V_B) \\&= 2000 \text{ J} \\\Delta U_{BC} &= Q_{BC} - W_{BC} \\Q_{BC} &= \Delta U_{BC} + W_{BC} \\&= nC_V \Delta T + W_{BC} \\&= nC_V(T_C - T_B) + W_{BC} \\&= \frac{5}{2}nR(T_C - T_B) + W_{BC} \\&= 6988 \text{ J} \\W_{CA} &= 0 \\Q_{CA} &= nC_V \Delta T \\&= nC_V(T_A - T_C) \\&= \frac{5}{2}nR(T_A - T_C) \\&= -4988 \text{ J} \\Q_{\text{net}} &= 611 \text{ J} \\W_{\text{net}} &= 611 \text{ J}\end{aligned}$$

(e)

$$e = \frac{W}{Q_H} = 0.0874 = 8.7\%$$

20.2.41

(a)

$$e = 1 - \frac{T_C}{T_H} = 7.0\%$$

(b)

$$\begin{aligned}P &= eQ_H \\Q_H &= \frac{P}{e} \\&= 3.0 \text{ MW} \\Q_C &= Q_H(1 - e) \\&= 2.8 \text{ MW}\end{aligned}$$

(c)

$$\begin{aligned}Q_C &= mc\Delta T \\m &= \frac{Q_C}{c\Delta T} \\&= 167 \text{ kg/s} \\&= 6.0 \times 10^5 \text{ kg/h} \\&= 6.0 \times 10^5 \text{ L/h}\end{aligned}$$

20.2.45

$$\begin{aligned}
e &= 1 - \frac{T_C}{T_H} \\
e' &= 1 - \frac{T'}{T_H} \\
W' &= Q_H e' \\
&= Q_H \left(1 - \frac{T'}{T_H} \right) \\
e'' &= 1 - \frac{T_C}{T'} \\
W'' &= Q'_H e'' \\
&= (Q_H - W') e'' \\
&= \left[Q_H - Q_H \left(1 - \frac{T'}{T_H} \right) \right] \left(1 - \frac{T_C}{T'} \right) \\
&= Q_H \left(1 - 1 + \frac{T'}{T_H} \right) \left(1 - \frac{T_C}{T'} \right) \\
&= Q_H \frac{T'}{T_H} \left(1 - \frac{T_C}{T'} \right) \\
e_{\text{Total}} &= \frac{W' + W''}{Q_H} \\
&= 1 - \frac{T'}{T_H} + \frac{T'}{T_H} \left(1 - \frac{T_C}{T'} \right) \\
&= 1 - \frac{T_C}{T_H}
\end{aligned}$$

The efficiency is the same.

20.2.49

$$\begin{aligned}
 T_A &= 300 \text{ K} \\
 W_{AB} &= nRT_A \ln \frac{V_B}{V_A} \\
 &= 2553 \text{ J} \\
 Q_{AB} &= 2553 \text{ J} \\
 T_B &= 300 \text{ K} \\
 T_C &= 1000 \text{ K} \\
 W_{BC} &= 0 \\
 Q_{BC} &= nC_V(T_C - T_B) \\
 &= \frac{5}{2}nR(T_C - T_B) \\
 &= 1.24 \times 10^4 \text{ J} \\
 W_{CA} &= p_A(V_A - V_C) \\
 &= -4949 \text{ J} \\
 \Delta U_{CA} &= Q_{CA} - W_{CA} \\
 Q_{CA} &= \Delta U_{CA} + W_{CA} \\
 &= nC_V(T_A - T_C) + W_{CA} \\
 &= \frac{5}{2}nR(T_A - T_C) + W_{CA} \\
 &= -1.73 \times 10^4 \text{ J} \\
 W &= -2396 \text{ J} \\
 K &= \frac{|Q_C|}{|W|} \\
 &= 6.24
 \end{aligned}$$

20.2.51

(a)

$$\begin{aligned}
 \Delta S &= \int \frac{dQ}{T} \\
 &= m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \frac{T'}{T_2} \\
 0 &= m_1 c_1 (T - T_1) + m_2 c_2 (T' - T_2) \\
 m_1 c_1 (T - T_1) &= m_2 c_2 (T_2 - T')
 \end{aligned}$$

(b)

$$\begin{aligned}T' &= T_2 - \frac{m_1 c_1}{m_2 c_2} (T - T_1) \\ \Delta S &= m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \left(\frac{T_2 - \frac{m_1 c_1}{m_2 c_2} (T - T_1)}{T_2} \right) \\ &= m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \left(1 - \frac{m_1 c_1}{m_2 c_2} \frac{T - T_1}{T_2} \right) \\ \frac{d}{dT} \Delta S &= \frac{m_1 c_1}{T} - m_2 c_2 \frac{m_1 c_1}{m_1 c_1 (T_1 - T) + m_2 c_2 T_2} \\ 0 &= \frac{1}{T} - \frac{m_2 c_2}{m_1 c_1 (T_1 - T) + m_2 c_2 T_2} \\ T &= \frac{m_1 c_1 (T_1 - T) + m_2 c_2 T_2}{m_2 c_2} \\ &= \frac{m_1 c_1}{m_2 c_2} (T_1 - T) + T_2 \\ &= T'\end{aligned}$$

20.2.55

20.2.57

(a)

$$r_b = 8.0 \text{ cm}$$

$$V_b = \frac{4}{3}\pi r_b^3$$
$$= 2.14 \times 10^{-3} \text{ m}^3$$

$$V_a = 2V_b$$
$$= 4.29 \times 10^{-3} \text{ m}^3$$

$$r_a = \sqrt[3]{\frac{3V_a}{4\pi}}$$
$$= 10 \text{ cm}$$

$$r_d = 3r_a$$
$$= 30 \text{ cm}$$

$$p_a V_a^\gamma = p_b V_b^\gamma$$
$$p_a = \left(\frac{V_b}{V_a}\right)^\gamma p_b$$
$$= 8.24 \text{ kPa}$$

$$\frac{p_b V_b}{T_b} = \frac{p_a V_a}{T_a}$$
$$T_b = \frac{p_b V_b}{p_a V_a} T_a$$
$$= 152 \text{ K}$$

(b)

$$n = \frac{p_b V_b}{RT_b}$$
$$= 3.44 \times 10^{-2} \text{ mol}$$

$$V_d = \frac{4}{3}\pi r_d^3$$
$$= 0.113 \text{ m}^3$$

$$p_d = \frac{nRT_d}{V_d}$$
$$= 311 \text{ Pa}$$

(c)

$$\begin{aligned}e &= 1 - \frac{T_C}{T_H} \\&= 0.191 \\&= 19.1\% \\W &= eQ_H \\Q_H &= \frac{W}{e} \\&= 5.24 \text{ kJ}\end{aligned}$$

(d)

$$\begin{aligned}Q_C &= (1 - e)Q_H \\&= 4.24 \text{ kJ}\end{aligned}$$

20.2.59

$$\begin{aligned}e &= 1 - \frac{T_C}{T_H} \\T_C &= (1 - e)T_H \\&= 281 \text{ K} \\&= 7.5^\circ \text{C}\end{aligned}$$

b

20.2.61

d

37 Relativity

37.1 Guided Practice

37.1.1

- (a) In the laboratory frame, $v_1 = \alpha c$ and $v_2 = -\alpha c$. In the frame of the first proton, $v'_1 = 0$ and $v'_2 = -\frac{1}{2}c$. Using the Lorentz velocity transformation:

$$\begin{aligned}
v_2' &= \frac{v_2 - v_1}{1 - v_1 v_2 / c^2} \\
-\frac{1}{2}c &= \frac{-\alpha c - \alpha c}{1 + \alpha^2 c^2 / c^2} \\
\frac{1}{2}c &= \frac{2\alpha c}{1 + \alpha^2} \\
\alpha^2 - 4\alpha + 1 &= 0 \\
\alpha &= \frac{4 \pm \sqrt{16 - 4}}{2} \\
&= 2 \pm \sqrt{3}
\end{aligned}$$

α can't be greater than 1, so $\alpha = 2 - \sqrt{3} \approx 0.268$.

(b)

$$\begin{aligned}
K &= (\gamma - 1)mc^2 \\
&= \left(\frac{1}{\sqrt{1 - (0.268c)^2 / c^2}} - 1 \right) mc^2 \\
&= 5.71 \times 10^{-12} \text{ J} \\
&= 35.6 \text{ MeV}
\end{aligned}$$

(c)

$$\begin{aligned}
K &= \left(\frac{1}{\sqrt{1 - (0.5c)^2 / c^2}} - 1 \right) mc^2 \\
&= 2.33 \times 10^{-11} \text{ J} \\
&= 145 \text{ MeV}
\end{aligned}$$

37.2 Exercises and Problems

37.2.1

$$\begin{aligned}x'_1 &= -d \\t'_1 &= 0 \\x'_2 &= d \\t'_2 &= 0 \\x_1 &= \gamma(x'_1 + ut'_1) \\&= -\gamma d \\t_1 &= \gamma(t'_1 + ux'_1/c^2) \\&= -\gamma ud/c^2 \\x_2 &= \gamma(x'_2 + ut'_2) \\&= \gamma d \\t_2 &= \gamma(t'_2 + ux'_2/c^2) \\&= \gamma ud/c^2\end{aligned}$$

The observer measures the lightning strike at point A to come first.

37.2.3

$$\begin{aligned}2 &= \gamma \\&= \frac{1}{\sqrt{1 - v^2/c^2}} \\2\sqrt{1 - v^2/c^2} &= 1 \\1 - \frac{v^2}{c^2} &= \frac{1}{4} \\\frac{v^2}{c^2} &= \frac{3}{4} \\v &= \frac{\sqrt{3}}{2}c \\&\approx 0.866c \\&\approx 2.60 \times 10^8 \text{ m/s}\end{aligned}$$

Present day jet planes don't reach this speed.

37.2.5

(a)

$$\begin{aligned}
\Delta t &= \gamma \Delta t_0 \\
\frac{\Delta t}{\Delta t_0} &= \frac{1}{\sqrt{1 - v^2/c^2}} \\
\frac{\Delta t_0}{\Delta t} &= \sqrt{1 - v^2/c^2} \\
\left(\frac{\Delta t_0}{\Delta t}\right)^2 &= 1 - \frac{v^2}{c^2} \\
\frac{v^2}{c^2} &= 1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2 \\
v &= \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} c \\
&= 0.998c
\end{aligned}$$

(b)

$$d = \Delta t v = 126 \text{ m}$$

37.2.7

$$\begin{aligned}
\Delta t - \Delta t_0 &= \Delta t \left(1 - \frac{1}{\gamma}\right) \\
&= \Delta t (1 - \sqrt{1 - v^2/c^2}) \\
&= 9.15 \text{ h}
\end{aligned}$$

The clock on the spacecraft measured a shorter elapsed time.

37.2.9

$$\begin{aligned}
l &= \frac{l_0}{\gamma} \\
l_0 &= \gamma l \\
&= 103 \text{ m}
\end{aligned}$$

37.2.11

(a)

$$d = \Delta t_0 v = 0.66 \text{ km}$$

(b)

$$\begin{aligned}\Delta t &= \gamma \Delta t_0 \\ &= 49 \mu\text{s} \\ d &= \Delta t v \\ &= 15 \text{ km}\end{aligned}$$

(c)

$$l = \frac{l_0}{\gamma} = 0.45 \text{ km}$$

37.2.13

(a)

$$l = \frac{l_0}{\gamma} = 3960 \text{ m}$$

(b)

$$t = \frac{d}{v} = 95.2 \mu\text{s}$$

(c)

$$t_0 = \frac{t}{\gamma} = 94.3 \mu\text{s}$$

37.2.15

(a)

$$\begin{aligned}v &= \frac{v' + u}{1 + uv'/c^2} \\ &= \frac{0.380c + 0.580c}{1 + (0.580c)(0.380c)/c^2} \\ &= \frac{(0.380 + 0.580)c}{1 + (0.580)(0.380)} \\ &= 0.787c\end{aligned}$$

(b)

$$v = 0.949c$$

(c)

$$v = 0.997c$$

37.2.17

(a) Toward

(b)

$$v' = \frac{v - u}{1 - uv/c^2} = \frac{0.650c - 0.830c}{1 - (0.830c)(0.650c)/c^2} = -0.391c$$

37.2.19

$$\begin{aligned}
v &= \frac{v' + u}{1 + uv'/c^2} \\
&= \frac{0.950c - 0.650c}{1 + (-0.650c)(0.950c)/c^2} \\
&= 0.784c
\end{aligned}$$

37.2.21

$$\begin{aligned}
v &= \frac{v' + u}{1 + uv'/c^2} \\
-\alpha c &= \frac{-0.890c + \alpha c}{1 + (\alpha c)(-0.890c)/c^2} \\
&= \frac{(\alpha - 0.890)c}{1 - 0.890\alpha} \\
-\alpha(1 - 0.890\alpha) &= (\alpha - 0.890) \\
0.890\alpha^2 - 2\alpha + 0.890 &= 0 \\
\alpha &= \frac{2 \pm \sqrt{4 - 3.1684}}{1.78} \\
&= 0.611
\end{aligned}$$

The particles are travelling at $0.611c$.

37.2.23

(a)

$$f_0 = 4.44 \times 10^{14} \text{ Hz}$$

$$f = 5.22 \times 10^{14} \text{ Hz}$$

$$f = \sqrt{\frac{c+u}{c-u}} f_0$$

$$\left(\frac{f}{f_0}\right)^2 = \frac{c+u}{c-u}$$

$$(c-u) \left(\frac{f}{f_0}\right)^2 = c+u$$

$$c \left[\left(\frac{f}{f_0}\right)^2 - 1 \right] = u \left[\left(\frac{f}{f_0}\right)^2 + 1 \right]$$

$$u = \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1} c$$

$$\left(\frac{f}{f_0}\right)^2 = 1.38$$

$$u = 0.160c$$

(b) \$173 million

37.2.25

$$1.20 = \sqrt{\frac{c+u}{c-u}}$$

$$1.44 = \frac{c+u}{c-u}$$

$$1.44(c-u) = c+u$$

$$c(1.44-1) = (1.44+1)u$$

$$u = \frac{0.44}{2.44} c$$

$$= 0.18c$$

Toward

37.2.27

$$\begin{aligned} \frac{\gamma_{0.800} 0.800c}{\gamma_{0.400} 0.400c} &= 2 \sqrt{\frac{1 - (0.400c)^2/c^2}{1 - (0.800c)^2/c^2}} \\ &= 3.06 \end{aligned}$$

37.2.29

(a)

$$\begin{aligned}
2 &= \gamma \\
&= \frac{1}{\sqrt{1 - v^2/c^2}} \\
1 - \frac{v^2}{c^2} &= \frac{1}{4} \\
v &= \frac{\sqrt{3}}{2}c \\
&= 0.866c
\end{aligned}$$

(b)

$$\begin{aligned}
2 &= \gamma^3 \\
&= \frac{1}{(1 - v^2/c^2)^{3/2}} \\
1 - \frac{v^2}{c^2} &= \left(\frac{1}{2}\right)^{2/3} \\
v &= \sqrt{1 - \left(\frac{1}{2}\right)^{2/3}} c \\
&= 0.608c
\end{aligned}$$

37.2.31

(a)

$$0.866c$$

(b)

$$\begin{aligned}
6 &= \gamma \\
&= \frac{1}{\sqrt{1 - v^2/c^2}} \\
\frac{1}{36} &= 1 - \frac{v^2}{c^2} \\
v &= \sqrt{1 - \frac{1}{36}} c \\
&= 0.986c
\end{aligned}$$

37.2.33

(a)

$$K = 3mc^2 = 4.51 \times 10^{-10} \text{ J}$$

(b)

$$\begin{aligned}E^2 &= (mc^2)^2 + (pc)^2 \\p &= \frac{\sqrt{E^2 - (mc^2)^2}}{c} \\&= \frac{\sqrt{(4mc^2)^2 - (mc^2)^2}}{c} \\&= \frac{\sqrt{15(mc^2)^2}}{c} \\&= \sqrt{15}mc \\&= 1.94 \times 10^{-18} \text{ kg m/s}\end{aligned}$$

(c)

$$p = \gamma mv \Rightarrow v = \frac{p}{\gamma m} = 2.90 \times 10^8 \text{ m/s} = 0.968c$$

37.2.35

(a)

$$E = mc^2 \Rightarrow m = \frac{E}{c^2} = 1.11 \times 10^3 \text{ kg}$$

(b)

$$\begin{aligned}\rho &= 7860 \text{ kg/m}^3 \\ \rho V &= m \\ \rho s^3 &= m \\ s &= \sqrt[3]{\frac{m}{\rho}} \\ &= 0.521 \text{ m}\end{aligned}$$

37.2.41

(a)

$$\begin{aligned}qV &= K \\ V &= \frac{(\gamma - 1)mc^2}{q} \\ &= 2.06 \times 10^6 \text{ V}\end{aligned}$$

(b)

$$K = 3.30 \times 10^{-13} \text{ J} = 2.06 \times 10^6 \text{ eV}$$

37.2.43

(a)

$$\begin{aligned}
l &= v\Delta t \\
&= v\gamma\Delta t_0 \\
&= \frac{v\Delta t_0}{\sqrt{1-v^2/c^2}} \\
l\sqrt{1-v^2/c^2} &= v\Delta t_0 \\
l^2(1-v^2/c^2) &= v^2\Delta t_0^2 \\
v^2 &= \frac{l^2}{\Delta t_0^2}(1-v^2/c^2) \\
v^2\left(1 + \frac{l^2}{\Delta t_0^2 c^2}\right) &= \frac{l^2}{\Delta t_0^2} \\
(1-\Delta)c &= \sqrt{\frac{l^2/\Delta t_0^2}{1+l^2/\Delta t_0^2 c^2}} \\
\Delta &= 1 - \frac{1}{c}\sqrt{\frac{l^2/\Delta t_0^2}{1+l^2/\Delta t_0^2 c^2}} \\
&= 8.43 \times 10^{-6}
\end{aligned}$$

(b)

$$\begin{aligned}
E &= \gamma mc^2 \\
&= 5.45 \times 10^{-9} \text{ J} \\
&= 3.40 \times 10^{10} \text{ eV}
\end{aligned}$$

37.2.45

$$\begin{aligned}
1.4 &= \gamma \\
&= \frac{1}{\sqrt{1-v^2/c^2}} \\
\frac{1}{1.4} &= \sqrt{1-v^2/c^2} \\
\frac{1}{1.96} &= 1 - \frac{v^2}{c^2} \\
v &= \sqrt{1 - \frac{1}{1.96}}c \\
&= 0.700c
\end{aligned}$$

37.2.47

$$\begin{aligned}
\Delta t &= 42.5 \text{ y} \\
\Delta t_0 &= \frac{\Delta t}{\gamma} \\
&= \Delta t \sqrt{1 - v^2/c^2} \\
&= 5.02 \text{ y}
\end{aligned}$$

Her biological age will be $19 + 5 = 24$.

37.2.51

$$\begin{aligned}
\Delta t - \Delta t_0 &= \Delta t - \frac{\Delta t}{\gamma} \\
&= \Delta t \left(1 - \frac{1}{\gamma} \right) \\
&= \Delta t (1 - \sqrt{1 - v^2/c^2}) \\
&= 8.41 \text{ ns}
\end{aligned}$$

The clock that was on the airliner will show a shorter elapsed time.

37.2.53

$$\begin{aligned}
v &\geq \frac{c}{n} \\
&\geq 1.97 \times 10^8 \text{ m/s} \\
K &\geq 2.70 \times 10^{-14} \text{ J} \\
&\geq 167 \text{ keV}
\end{aligned}$$

37.2.57

(a)

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.9995c + 0.7500c}{1 + (0.7500c)(0.9995c)/c^2} = 0.9999c$$

(b)

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{-0.9995c + 0.7500c}{1 + (0.7500c)(-0.9995c)/c^2} = -0.9965c$$

37.2.61

$$\begin{aligned}
 \frac{\Delta f}{f_0} &= \frac{f'' - f_0}{f_0} \\
 &= \sqrt{\frac{c+u}{c-u}} \frac{f'}{f_0} - 1 \\
 &= \frac{c+u}{c-u} - 1 \\
 (c-u) \left(1 + \frac{\Delta f}{f_0} \right) &= c+u \\
 c \frac{\Delta f}{f_0} &= u \left(2 + \frac{\Delta f}{f_0} \right) \\
 u &= \frac{\Delta f / f_0}{2 + \Delta f / f_0} c \\
 &= 43 \text{ m/s} \\
 &= 154 \text{ k/m}
 \end{aligned}$$

37.2.63

$$\begin{aligned}
 F &= \gamma m a \\
 &= \frac{1}{\sqrt{1 - v^2/c^2}} m \frac{v^2}{r} \\
 &= \frac{mv^2}{r \sqrt{1 - v^2/c^2}} \\
 &= 2.04 \times 10^{-13} \text{ N}
 \end{aligned}$$

37.2.65

(a)

$$\begin{aligned}
 \Delta t &= \gamma \Delta t_0 \\
 &= \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} \\
 \Delta t^2 &= \frac{\Delta t_0^2}{1 - u^2/c^2} \\
 \Delta t_0 &\approx 2.59 \times 10^{-8} \text{ s}
 \end{aligned}$$

(b)

$$\begin{aligned}(4\Delta t_0)^2 &= \frac{\Delta t_0^2}{1 - u^2/c^2} \\ \frac{1}{16} &= 1 - \frac{u^2}{c^2} \\ \frac{u}{c} &= \sqrt{1 - \frac{1}{16}} \\ &= 0.968\end{aligned}$$

37.2.71

c

37.2.73

b

38 Photons: Light Waves Behaving as Particles

38.1 Guided Practice

38.1.1

(a)

$$\begin{aligned}\lambda' - \lambda &= \frac{h}{mc}(1 - \cos \phi) \\ \lambda &= \lambda' - 2\frac{h}{mc} \\ &= 0.078 \text{ nm}\end{aligned}$$

(b)

$$\begin{aligned}p_p &= p_e - p'_p \\ \frac{h}{\lambda} &= p_e - \frac{h}{\lambda'} \\ p_e &= h \left(\frac{1}{\lambda} + \frac{1}{\lambda'} \right) \\ &= 1.65 \times 10^{-23} \text{ kg m/s} \\ p_e &= m_e v \\ v &= \frac{p_e}{m_e} \\ &= 1.81 \times 10^7 \text{ m/s}\end{aligned}$$

(c)

$$K = \frac{1}{2} m_e v^2 = 1.49 \times 10^{-16} \text{ J} = 933 \text{ eV}$$

38.2 Exercises and Problems

38.2.1

$$\lambda = 520 \text{ nm}$$

$$f = \frac{c}{\lambda} \\ = 5.77 \times 10^{14} \text{ Hz}$$

$$p = \frac{h}{\lambda} \\ = 1.27 \times 10^{-27} \text{ kg m/s}$$

$$E = \frac{hc}{\lambda} \\ = 3.82 \times 10^{-19} \text{ J} \\ = 2.39 \text{ eV}$$

38.2.3

(a)

$$f = \frac{c}{\lambda} = 5.36 \times 10^{14} \text{ Hz}$$

(b)

$$E = \frac{hc}{\lambda} \\ = 3.35 \times 10^{-19} \text{ J} \\ n = \frac{P}{E} \\ = 2.24 \times 10^{20}$$

(c) No

38.2.5

(a)

$$E = pc = 2.40 \times 10^{-19} \text{ J} = 1.50 \text{ eV}$$

(b)

$$\lambda = \frac{hc}{E} = 828 \text{ nm}$$

Infrared

38.2.7

$$\lambda = 206 \text{ nm}$$

$$f = \frac{c}{\lambda}$$
$$= 1.46 \times 10^{15} \text{ Hz}$$

$$\phi = 5.1 \text{ eV}$$

$$\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$$

$$v_{\text{max}} = \sqrt{\frac{2(hf - \phi)}{m}}$$
$$= 5.77 \times 10^5 \text{ m/s}$$

38.2.9

$$\lambda_1 = 400.0 \text{ nm}$$

$$f_1 = 7.50 \times 10^{14} \text{ Hz}$$

$$K_1 = 1.10 \text{ eV}$$

$$K_1 = hf - \phi$$

$$\phi = hf - K_1$$
$$= 2.94 \times 10^{-19} \text{ J}$$
$$= 1.83 \text{ eV}$$

$$\lambda_2 = 300.0 \text{ nm}$$

$$f_2 = 1.00 \times 10^{15} \text{ Hz}$$

$$K_2 = hf - \phi$$
$$= 3.67 \times 10^{-19} \text{ J}$$
$$= 2.3 \text{ eV}$$

38.2.11

(a)

$$\lambda = 254 \text{ nm}$$

$$f = 1.18 \times 10^{15} \text{ Hz}$$

$$V_0 = 0.181 \text{ V}$$

$$eV_0 = hf - \phi$$

$$\phi = hf - eV_0$$

$$= 7.53 \times 10^{-19} \text{ J}$$

$$= 4.71 \text{ eV}$$

$$E = \phi$$

$$\frac{hc}{\lambda} = \phi$$

$$\lambda = \frac{hc}{\phi}$$

$$= 264 \text{ nm}$$

(b)

$$\phi = 4.71 \text{ eV}$$

Same

38.2.13

$$K = E$$

$$eV = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{eV}$$

$$= 345 \text{ pm}$$

38.2.15

$$K = E' - E$$

$$= hc \left(\frac{1}{\lambda'} - \frac{1}{\lambda} \right)$$

$$= 3.17 \times 10^{-16} \text{ J}$$

$$= 1.98 \text{ keV}$$

38.2.17

$$\begin{aligned}\lambda' - \lambda &= \frac{h}{mc}(1 - \cos \phi) \\ \lambda' &= \lambda + \frac{h}{mc}(1 - \cos \phi) \\ \lambda'_{\max} &= \lambda + 2\frac{h}{mc} \\ &= 0.665 \text{ nm}\end{aligned}$$

Occurs at $\phi = \pi$.

38.2.19

(a)

$$\begin{aligned}\lambda &= 0.0430 \text{ nm} \\ \phi &= 32.0^\circ \\ \Delta\lambda &= \frac{h}{mc}(1 - \cos \phi) \\ &= 3.69 \times 10^{-13} \text{ m}\end{aligned}$$

(b)

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos \phi) = 0.0434 \text{ nm}$$

(c)

$$\Delta E = hc \left(\frac{1}{\lambda'} - \frac{1}{\lambda} \right) = -4.26 \times 10^{-17} = -266 \text{ eV}$$

(d)

$$-\Delta E$$

38.2.21

$$\begin{aligned}0.01\lambda &= \frac{h}{mc}(1 - \cos \phi) \\ \phi &= \arccos \left(1 - \frac{0.01\lambda}{h/mc} \right) \\ &= 51.0^\circ\end{aligned}$$

38.2.23

$$\Delta t = 7.20 \text{ fs}$$

$$\lambda = 522 \text{ nm}$$

$$p = \frac{h}{\lambda}$$

$$= 1.27 \times 10^{-27} \text{ kg m/s}$$

$$\Delta x = c\Delta t$$

$$= 2.16 \mu\text{m}$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{\hbar}{2\Delta x}$$

$$\geq 2.4 \times 10^{-29} \text{ kg m/s}$$

38.2.25

$$\lambda = 620 \text{ nm}$$

$$E = \frac{hc}{\lambda}$$

$$= 3.21 \times 10^{-19} \text{ J}$$

$$\Delta E = 3.21 \times 10^{-21} \text{ J}$$

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

$$\Delta t \geq \frac{\hbar}{2\Delta E}$$

$$\geq 1.64 \times 10^{-14} \text{ s}$$

$$\geq 16.4 \text{ fs}$$

38.2.27

(a)

$$\begin{aligned}
\lambda &= 585 \text{ nm} \\
\Delta t &= 450 \text{ } \mu\text{s} \\
c &= 4190 \text{ J/kg K} \\
L_v &= 2.256 \times 10^6 \text{ J/kg} \\
m &= 2.0 \text{ } \mu\text{g} \\
&= 2.0 \times 10^{-9} \text{ kg} \\
T_0 &= 33 \text{ }^\circ\text{C} \\
&= 306 \text{ K} \\
E &= mc(373 - T_0) + mL_v \\
&= 5.07 \text{ mJ}
\end{aligned}$$

(b)

$$P = E/\Delta t = 11.3 \text{ W}$$

(c)

$$\begin{aligned}
E_p &= \frac{hc}{\lambda} \\
&= 3.40 \times 10^{-19} \text{ J} \\
n &= \frac{E}{E_p} \\
&= 1.49 \times 10^{16}
\end{aligned}$$

38.2.29

(a)

$$\begin{aligned}
\lambda &= 0.0930 \text{ nm} \\
\phi &= 180^\circ \\
\lambda' &= \lambda + \frac{h}{mc}(1 - \cos \phi) \\
&= 0.0978 \text{ nm} \\
p &= \frac{h}{\lambda} \\
&= 6.77 \times 10^{-24} \text{ kg m/s}
\end{aligned}$$

(b)

$$\begin{aligned}K &= E - E' \\&= hc \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) \\&= 1.05 \times 10^{-16} \text{ J} \\&= 656 \text{ eV}\end{aligned}$$

38.2.31

$$\lambda = 0.1360 \text{ nm}$$

$$\phi = 60.0^\circ$$

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos \phi)$$

$$= \lambda + \frac{1}{2} \frac{h}{mc}$$

$$= 0.1372 \text{ nm}$$

$$K = E - E'$$

$$= hc \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right)$$

$$= 1.278 \times 10^{-17} \text{ J}$$

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}}$$

$$= 5.300 \times 10^6 \text{ m/s}$$

$$p = mv$$

$$= 4.823 \times 10^{-24} \text{ kg m/s}$$

$$p = p' + P_x$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'} \cos \phi + P_x$$

$$P_x = h \left(\frac{1}{\lambda} - \frac{1}{2\lambda'} \right)$$

$$= 2.457 \times 10^{-24} \text{ kg m/s}$$

$$0 = p' - P_y$$

$$= \frac{h}{\lambda'} \sin \phi - P_y$$

$$P_y = \frac{\sqrt{3}h}{2\lambda'}$$

$$= 4.182 \times 10^{-24} \text{ kg m/s}$$

$$\theta = \arctan \frac{P_y}{P_x}$$

$$= 59.57^\circ$$

38.2.33

(a)

$$\begin{aligned}
 E_1 &= 1 \text{ MeV} \\
 \lambda_1 &= \frac{hc}{E_1} \\
 &= 0.00124 \text{ nm} \\
 \lambda_2 &= 500 \text{ nm} \\
 n &= 10^{26} \\
 \Delta\lambda &= \frac{\lambda_2 - \lambda_1}{n} \\
 &= 5 \times 10^{-33} \text{ m}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \Delta\lambda &= \frac{h}{mc}(1 - \cos\phi) \\
 &= \frac{h}{mc} \frac{\phi^2}{2} \\
 \phi &= \sqrt{\frac{2\Delta\lambda}{h/mc}} \\
 &= 6.42 \times 10^{-11} \text{ rad} \\
 &= (3.68 \times 10^{-9})^\circ
 \end{aligned}$$

38.2.35

(a)

$$\begin{aligned}
 K &= E - E' \\
 &= hc \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) \\
 &= 3.11 \times 10^{-17} \text{ J} \\
 &= 195 \text{ eV}
 \end{aligned}$$

(b)

$$\begin{aligned}
 K &= E \\
 &= \frac{hc}{\lambda} \\
 \lambda &= \frac{hc}{K} \\
 &= 6.37 \text{ nm}
 \end{aligned}$$

38.2.37

(a)

$$\begin{aligned}
 eV_0 &= hf - \phi \\
 &= h \frac{c}{\lambda} - \phi \\
 V_0 &= \frac{hc}{e} \frac{1}{\lambda} - \frac{\phi}{e} \\
 \frac{hc}{e} &= 1.232 \times 10^{-6} \text{ V m} \\
 \frac{\phi}{e} &= 4.7651 \text{ V}
 \end{aligned}$$

(b)

$$\begin{aligned}
 h &= 6.57 \times 10^{-34} \text{ J s} \\
 \phi &= 4.7651 \text{ eV}
 \end{aligned}$$

(c)

$$\begin{aligned}
 \phi &\leq E \\
 &\leq \frac{hc}{\lambda} \\
 \lambda &\leq \frac{hc}{\phi} \\
 &\leq 261 \text{ nm}
 \end{aligned}$$

(d)

$$\begin{aligned}
 K &= hf - \phi \\
 &= h \frac{c}{\lambda} - \phi \\
 \lambda &= \frac{hc}{K + \phi} \\
 &= 84.1 \text{ nm}
 \end{aligned}$$

38.2.39

(a)

$$\begin{aligned}
 m &= 2.404 \text{ pm} \\
 c &= 5.211 \text{ pm}
 \end{aligned}$$

(b)

$$\lambda_C = 2.404 \text{ pm}$$

(c)

$$\lambda = 5.211 \text{ pm}$$

39 Particles Behaving as Waves

39.1 Guided Practice

39.1.1

(a)

$$\lambda_m = \frac{b}{T} = 192 \text{ nm}$$

Ultraviolet

(b)

$$\begin{aligned} E &= E_f - E_i \\ \frac{hc}{\frac{1}{2}\lambda_m} &= -\frac{hcR}{n^2} + hcR \\ \frac{2}{\lambda_m} &= R \left(1 - \frac{1}{n^2} \right) \\ \frac{1}{n^2} &= 1 - \frac{2}{\lambda_m R} \\ n &= \sqrt{\frac{\lambda_m R}{\lambda_m R - 2}} \\ &= 4.45 \end{aligned}$$

4

(c)

$$\begin{aligned} 2\pi r_2 &= 2\lambda_2 \\ \lambda_2 &= \epsilon_0 \frac{n^2 h^2}{me^2} \\ &= 0.667 \text{ nm} \end{aligned}$$

$$\begin{aligned} 2\pi r_3 &= 3\lambda_3 \\ \lambda_3 &= \frac{2}{3} \epsilon_0 \frac{n^2 h^2}{me^2} \\ &= 1.00 \text{ nm} \end{aligned}$$

39.2 Exercises and Problems

39.2.1

(a)

$$\lambda = \frac{h}{mv} = 0.162 \text{ nm}$$

(b)

$$\lambda = 8.82 \times 10^{-14} \text{ m}$$

39.2.3

(a)

$$p = \frac{h}{\lambda} = 2.37 \times 10^{-24} \text{ kg m/s}$$

(b)

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = 3.10 \times 10^{-18} \text{ J} = 19.4 \text{ eV}$$

39.2.5

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ &= 9.09 \times 10^{-11} \text{ m}\end{aligned}$$

$$\begin{aligned}v &= \frac{h}{m\lambda} \\ &= 4.36 \times 10^3 \text{ m/s}\end{aligned}$$

39.2.7

(a)

$$\begin{aligned}E &= \frac{hc}{\lambda} \\ \lambda &= \frac{hc}{E} \\ &= 62.1 \text{ nm}\end{aligned}$$

$$\begin{aligned}E &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2E}{m}} \\ &= 2.65 \times 10^6 \text{ m/s} \\ \lambda &= \frac{h}{mv} \\ &= 0.275 \text{ nm}\end{aligned}$$

(b)

$$\begin{aligned}E &= \frac{hc}{\lambda} \\&= 7.95 \times 10^{-19} \text{ J} \\&= 4.97 \text{ eV}\end{aligned}$$

$$\begin{aligned}\lambda &= \frac{h}{mv} \\v &= \frac{h}{m\lambda} \\&= 2.91 \times 10^3 \text{ m/s} \\E &= \frac{1}{2}mv^2 \\&= 3.86 \times 10^{-24} \text{ J} \\&= 2.41 \times 10^{-5} \text{ eV}\end{aligned}$$

39.2.9

$$\lambda = \frac{h}{mv} = 3.90 \times 10^{-34} \text{ nm}$$

No

39.2.11

(a)

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{2meV_{\text{ba}}}} \\\lambda^2 &= \frac{h^2}{2meV_{\text{ba}}} \\V_{\text{ba}} &= \frac{h^2}{2me\lambda^2} \\&= 23.6 \text{ mV}\end{aligned}$$

(b)

$$E = \frac{hc}{\lambda} = 2.48 \times 10^{-17} \text{ J} = 155 \text{ eV}$$

(c)

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ v &= \frac{h}{m\lambda} \\ &= 9.09 \times 10^4 \text{ m/s} \\ E &= \frac{1}{2}mv^2 \\ &= 3.77 \times 10^{-21} \text{ J}\end{aligned}$$

$$\begin{aligned}E &= \frac{hc}{\lambda} \\ \lambda &= \frac{hc}{E} \\ &= 52.7 \text{ }\mu\text{m}\end{aligned}$$

39.2.13

$$\begin{aligned}d &= 0.0910 \text{ nm} \\ m &= 1 \\ \theta &= 29.0^\circ \\ d \sin \theta &= \lambda \\ &= \frac{h}{\sqrt{2mK}} \\ (d \sin \theta)^2 &= \frac{h^2}{2mK} \\ K &= \frac{h^2}{2m(d \sin \theta)^2} \\ &= 6.75 \times 10^{-20} \text{ J} \\ &= 0.422 \text{ eV}\end{aligned}$$

39.2.15

(a)

$$d = 1.60 \mu\text{m}$$

$$v = 1.26 \times 10^4 \text{ m/s}$$

$$d \sin \theta = \lambda$$

$$= \frac{h}{mv}$$

$$\sin \theta = \frac{h}{dmv}$$

$$\theta = \arcsin \frac{h}{dmv}$$

$$= 2.07^\circ$$

$$\theta = \arcsin \frac{2h}{dmv}$$

$$= 4.14^\circ$$

(b) 1.8 cm

39.2.17

(a)

$$U = \frac{1}{4\pi\epsilon_0} \frac{(2e)(82e)}{r}$$

$$= \frac{164e^2}{4\pi\epsilon_0 r}$$

$$= 5.68 \times 10^{-13} \text{ J}$$

$$= 3.55 \text{ MeV}$$

(b)

$$K = 5.68 \times 10^{-13} \text{ J} = 3.55 \text{ MeV}$$

(c)

$$K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}} = 1.31 \times 10^7 \text{ m/s}$$

39.2.21

(a)

$$\begin{aligned}
 E_1 &= -\frac{2hcme^4}{\epsilon_0^2 h^3 c} \\
 &= -3.46 \times 10^{-17} \text{ J} \\
 &= -217 \text{ eV}
 \end{aligned}$$

16 times greater

(b)

$$-E_1 = 217 \text{ eV}$$

16 times greater

(c)

$$\begin{aligned}
 E_2 &= -\frac{hcR}{2^2} \\
 &= -\frac{16hcme^4}{32\epsilon_0^2 h^3 c} \\
 &= -\frac{me^4}{2\epsilon_0^2 h^2} \\
 &= -8.66 \times 10^{-18} \text{ J} \\
 &= -54.1 \text{ eV}
 \end{aligned}$$

$$\begin{aligned}
 E &= \frac{hc}{\lambda} \\
 \lambda &= \frac{hc}{E_2 - E_1} \\
 &= 7.63 \text{ nm}
 \end{aligned}$$

(d)

$$r_n = \epsilon_0 \frac{n^2 h^2}{4\pi m e^2}$$

 $\frac{1}{4}$ the radius**39.2.23**

(a)

$$\begin{aligned}
 v_1 &= \frac{1}{\epsilon_0} \frac{e^2}{2nh} \\
 &= 2.18 \times 10^6 \text{ m/s} \\
 v_2 &= 1.09 \times 10^6 \text{ m/s} \\
 v_5 &= 4.36 \times 10^5 \text{ m/s}
 \end{aligned}$$

(b)

$$\begin{aligned}a_0 &= \frac{\epsilon_0 h^2}{\pi m e^2} \\&= 5.31 \times 10^{-11} \text{ m} \\T_1 &= \frac{2\pi r_1}{v_1} \\&= \frac{2\pi n^2 a_0}{v_1} \\&= 1.53 \times 10^{-16} \text{ s} \\T_2 &= 1.22 \times 10^{-15} \text{ s} \\T_5 &= 1.91 \times 10^{-14} \text{ s}\end{aligned}$$

(c)

$$\frac{\Delta t}{T} = 8.20 \times 10^6$$

39.2.25

(a)

$$E = E_\infty - E_1 = -E_1 = 20 \text{ eV}$$

(b) 3 eV, 5 eV, 8 eV, 10 eV, 15 eV, or 18 eV

(c) Nothing because there's no energy level at -12 eV .

(d)

$$\begin{aligned}E_{32} &= 5 \text{ eV} \\E_{31} &= 15 \text{ eV} \\E_{43} &= 3 \text{ eV} \\3 \text{ eV} &< \phi < 5 \text{ eV}\end{aligned}$$

39.2.27

(a)

$$\begin{aligned}E_1 &= -17.50 \text{ eV} \\E_2 &= E_1 + \frac{hc}{\lambda} \\&= -4.36 \text{ eV} \\E_3 &= -1.92 \text{ eV} \\E_4 &= -1.07 \text{ eV} \\E_5 &= -0.68 \text{ eV}\end{aligned}$$

(b)

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = 378 \text{ nm}$$

39.2.29

(a)

$$E = -5.08 \text{ eV}$$

(b)

$$E = -5.68 \text{ eV}$$

39.2.31

$$\begin{aligned}\lambda &= 10.6 \mu\text{m} \\ P &= 0.100 \text{ kW} \\ n &= \frac{P}{hc/\lambda} \\ &= 5.33 \times 10^{21}\end{aligned}$$

39.2.37

(a)

$$\begin{aligned}A &= 2\pi \left(\frac{d}{2}\right)^2 + 2\pi \left(\frac{d}{2}\right) L \\ &= 3.83 \times 10^{-4} \text{ m}^3\end{aligned}$$

$$\begin{aligned}P &= Ae\sigma T^4 \\ T &= \sqrt[4]{\frac{P}{Ae\sigma}} \\ &= 2.0 \times 10^3 \text{ K} \\ &= 1.7 \times 10^3 \text{ }^\circ\text{C}\end{aligned}$$

(b)

$$\lambda_m = \frac{b}{T} = 1.46 \mu\text{m}$$

39.2.39

$$\lambda_m = \frac{b}{T} = 1.06 \times 10^{-3} \text{ m}$$

Microwave

39.2.41

(a)

$$\begin{aligned}
4\pi \left(\frac{5d}{2}\right)^2 \sigma T^4 &= 4\pi \left(\frac{d}{2}\right)^2 \sigma T'^4 \\
\frac{25}{16} d^2 T^4 &= \frac{1}{4} d^2 T'^4 \\
T' &= \sqrt[4]{\frac{25}{4}} T \\
&\approx 1.58 T
\end{aligned}$$

(b)

$$\frac{b/T'}{b/T} = \frac{T}{T'} = \sqrt[4]{\frac{4}{25}} \approx 0.63$$

39.2.45

(a)

$$\begin{aligned}
\Delta y \Delta p_y &\geq \frac{\hbar}{2} \\
\Delta v_y &\geq \frac{\hbar}{2m\Delta y} \\
&\geq 1.58 \times 10^4 \text{ m/s}
\end{aligned}$$

39.2.47

$$\begin{aligned}
\Delta x \Delta p_x &\geq \frac{\hbar}{2} \\
4.42 \times 10^{-35} \text{ kg m}^2/\text{s} &\geq 5.27 \times 10^{-35} \text{ kg m}^2/\text{s}
\end{aligned}$$

Not possible

39.2.49

$$\begin{aligned}
\Delta E \Delta t &\geq \frac{\hbar}{2} \\
\Delta E &\geq \frac{\hbar}{2\Delta t} \\
&\geq 9.25 \times 10^{-33} \text{ J} \\
&\geq 5.78 \times 10^{-14} \text{ eV}
\end{aligned}$$

39.2.51

(a)

$$m_r = \frac{m_1 m_2}{m_1 + m_2} = 1.69 \times 10^{-28} \text{ kg}$$

(b)

$$E_1 = -\frac{m_r e^4}{8\epsilon_0^2 h^2} = -4.02 \times 10^{-16} \text{ J} = -2514 \text{ eV}$$

(c)

$$\begin{aligned} E_2 &= -\frac{m_r e^4}{32\epsilon_0^2 h^2} \\ &= -629 \text{ eV} \\ \lambda &= \frac{hc}{E_2 - E_1} \\ &= 0.659 \text{ nm} \end{aligned}$$

39.2.53

(a)

$$E = 12.1 \text{ eV}$$

39.2.55

(a)

$$\begin{aligned} E_{\text{ionization}} &= -E_1 \\ &= 13.6 \text{ eV} \\ E_{\text{photon}} &= \frac{hc}{\lambda} \\ &= 2.34 \times 10^{-18} \text{ J} \\ &= 14.6 \text{ eV} \\ K_{\text{max}} &= E_{\text{photon}} - E_{\text{ionization}} \\ &= 1.0 \text{ eV} \end{aligned}$$

39.2.57

(a)

$$\begin{aligned}
A &= 4\pi \left(\frac{600d_{\text{sun}}}{2} \right)^2 \\
&= 2.19 \times 10^{24} \text{ m}^2 \\
P &= A\sigma T^4 \\
&= 1.01 \times 10^{31} \text{ W} \\
\lambda &= \frac{b}{T} \\
&= 966 \text{ nm} \\
n &= \frac{P}{hc/\lambda} \\
&= 4.91 \times 10^{49}
\end{aligned}$$

(b)

$$\frac{P_{\text{B}}}{P_{\text{S}}} = 2.58 \times 10^4$$

39.2.59

$$\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= \frac{hc}{b/T} \\
T &= \frac{bE}{hc} \\
&= 29\,741 \text{ K}
\end{aligned}$$

39.2.61

(a)

$$\begin{aligned}
I(f) &= \frac{2\pi hc^2}{(c/f)^5 (e^{hc/(c/f)kT} - 1)} \\
&= \frac{2\pi f^5 h}{c^3 (e^{hf/kT} - 1)}
\end{aligned}$$

(b)

$$\begin{aligned}
 \lambda &= \frac{c}{f} \\
 d\lambda &= -\frac{c}{f^2} df \\
 \int_0^\infty I(\lambda) d\lambda &= -\int_\infty^0 I(f) \frac{c}{f^2} df \\
 &= \int_0^\infty \frac{2\pi f^5 h}{c^3 (e^{hf/kT} - 1)} \frac{c}{f^2} df \\
 &= \frac{2\pi h}{c^2} \int_0^\infty \frac{f^3}{e^{hf/kT} - 1} df \\
 &= \frac{2\pi h}{c^2} \frac{1}{240} \left(\frac{2\pi}{h/kT} \right)^4 \\
 &= \frac{2\pi h}{240c^2} \left(\frac{2\pi kT}{h} \right)^4 \\
 &= \frac{2\pi h}{240c^2} \frac{16\pi^4 k^4 T^4}{h^4} \\
 &= \frac{2\pi^5 k^4}{15c^2 h^3} T^4
 \end{aligned}$$

(c)

$$\frac{2\pi^5 k^4}{15c^2 h^3} = 5.65 \times 10^{-8}$$

39.2.63

(a)

$$E = \frac{hc}{\lambda} = 1.10 \times 10^{-18} \text{ J} = 6.90 \text{ eV}$$

(b)

$$\begin{aligned}E &= K \\eV &= \frac{1}{2}mv^2 \\2emV &= p^2 \\p &= \sqrt{2emV} \\[1ex]\lambda &= \frac{h}{p} \\&= \frac{h}{\sqrt{2emV}} \\2emV &= \left(\frac{h}{\lambda}\right)^2 \\V &= \frac{1}{2em} \left(\frac{h}{\lambda}\right)^2 \\&= 4.65 \times 10^{-5} \text{ V}\end{aligned}$$

$$\begin{aligned}eV &= \frac{1}{2}mv^2 \\v &= \sqrt{\frac{2eV}{m}} \\&= 4.0 \text{ km/s}\end{aligned}$$

(c)

$$\begin{aligned}V &= 2.54 \times 10^{-8} \text{ V} \\v &= 2.21 \text{ m/s}\end{aligned}$$

39.2.67

(a)

$$\begin{aligned}\lambda_e &= \lambda_p \\\frac{hc}{E} &= \frac{h}{p} \\&= \frac{h}{\sqrt{2mK}} \\\frac{E}{c} &= \sqrt{2mK} \\E &= c\sqrt{2mK}\end{aligned}$$

(b) Photon

39.2.69

(a)

$$\begin{aligned}
 \lambda &= \frac{h}{p} \\
 &= \frac{h}{mv/\sqrt{1-v^2/c^2}} \\
 &= \frac{h}{mv} \sqrt{1-v^2/c^2} \\
 \lambda^2 &= \left(\frac{h}{mv}\right)^2 (1-v^2/c^2) \\
 &= \left(\frac{h}{mv}\right)^2 - \left(\frac{h}{mc}\right)^2 \\
 \left(\frac{h}{mv}\right)^2 &= \lambda^2 + \left(\frac{h}{mc}\right)^2 \\
 &= \frac{\lambda^2 m^2 c^2 + h^2}{m^2 c^2} \\
 \left(\frac{mv}{h}\right)^2 &= \frac{m^2 c^2}{\lambda^2 m^2 c^2 + h^2} \\
 v^2 &= \frac{h^2 c^2}{\lambda^2 m^2 c^2 + h^2} \\
 &= \frac{c^2}{1 + (mc\lambda/h)^2} \\
 v &= \frac{c}{\sqrt{1 + (mc\lambda/h)^2}}
 \end{aligned}$$

(b)

$$\begin{aligned}
 v &= (1 - \Delta)c \\
 &= \left[1 - \frac{1}{2} \left(\frac{\lambda}{h/mc}\right)^2\right] c \\
 \Delta &= \frac{1}{2} \left(\frac{\lambda}{h/mc}\right)^2
 \end{aligned}$$

(c)

$$\begin{aligned}
 \Delta &= 8.49 \times 10^{-8} \\
 v &= (1 - 8.49 \times 10^{-8})c
 \end{aligned}$$

39.2.71

(a)

$$\begin{aligned}
 K &= 6mc^2 \\
 E - mc^2 &= 6mc^2 \\
 E &= 7mc^2
 \end{aligned}$$

$$\begin{aligned}
 (7mc^2)^2 &= (pc)^2 + (mc^2)^2 \\
 49(mc^2)^2 &= (pc)^2 + (mc^2)^2 \\
 48(mc^2)^2 &= (pc)^2 \\
 pc &= \sqrt{48}mc^2 \\
 p &= 4\sqrt{3}mc
 \end{aligned}$$

$$\begin{aligned}
 \lambda &= \frac{h}{p} \\
 &= \frac{h}{4\sqrt{3}mc}
 \end{aligned}$$

39.2.73

(a)

$$\begin{aligned}
 \Delta x \Delta p_x &\geq \frac{\hbar}{2} \\
 \Delta p_x &\geq \frac{\hbar}{2\Delta x} \\
 &\geq 1.05 \times 10^{-20} \text{ kg m/s}
 \end{aligned}$$

(b)

$$\begin{aligned}
 K &= E - mc^2 \\
 &= \sqrt{(mc^2)^2 + (pc)^2} - mc^2 \\
 &= 3.15 \times 10^{-12} \text{ J} \\
 &= 19.7 \text{ MeV}
 \end{aligned}$$

(c)

$$\begin{aligned}
 U &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \\
 &= 4.60 \times 10^{-14} \text{ J} \\
 &= 288 \text{ keV} \\
 \frac{K}{U} &= 68.4
 \end{aligned}$$

No

39.2.75

(a)

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ v &= \frac{h}{m\lambda} \\ &= 1.10 \times 10^{-35} \text{ m/s}\end{aligned}$$

(b)

$$t = \frac{d}{v} = 2.31 \times 10^{27} \text{ years}$$

No

39.2.77

$$\begin{aligned}d \sin \theta &= \lambda \\ d &= \frac{\lambda}{\sin \theta} \\ &= 5.13 \times 10^{-11} \text{ m}\end{aligned}$$

$$\begin{aligned}E &= \frac{1}{2}mv^2 \\ 2mE &= m^2v^2 \\ &= p^2 \\ p &= \sqrt{2mE}\end{aligned}$$

$$\begin{aligned}\lambda &= \frac{h}{p} \\ &= \frac{h}{\sqrt{2mE}}\end{aligned}$$

$$\begin{aligned}d \sin \theta &= \lambda \\ \theta &= \arcsin \frac{\lambda}{d} \\ &= \arcsin \frac{h}{d\sqrt{2mE}} \\ &= 20.89^\circ\end{aligned}$$

39.2.79

(a)

$$E = \frac{hc}{\lambda} = 3.98 \times 10^{-17} \text{ J} = 248 \text{ eV}$$

(b)

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ v &= \frac{h}{m\lambda} \\ &= 1.45 \times 10^5 \text{ m/s}\end{aligned}$$

$$\begin{aligned}K &= \frac{1}{2}mv^2 \\ &= 9.64 \times 10^{-21} \text{ J} \\ &= 0.060 \text{ eV}\end{aligned}$$

39.2.81

(a)

$$\begin{aligned}F &= -\frac{dU}{dx} \\ &= -\frac{A|x|}{x}, x \neq 0\end{aligned}$$

(b)

$$\begin{aligned}E &= \frac{p^2}{2m} + A|x| \\&= \frac{h^2}{2mx^2} + A|x| \\\frac{dE}{dx} &= -\frac{h^2}{mx^3} + \frac{A|x|}{x}, x \neq 0 \\0 &= -\frac{h^2}{mx^3} + \frac{A|x|}{x} \\\frac{h^2}{mx^3} &= \frac{A|x|}{x} \\\frac{h^2}{m} &= A|x|x^2 \\|x|^3 &= \frac{h^2}{Am} \\x &= \pm \sqrt[3]{\frac{h^2}{Am}}\end{aligned}$$

$$\begin{aligned}E &= \frac{h^2}{2m} \left(\pm \sqrt[3]{\frac{Am}{h^2}} \right)^2 + A \sqrt[3]{\frac{h^2}{Am}} \\&= \frac{h^2}{2m} \left(\frac{Am}{h^2} \right)^{2/3} + A \sqrt[3]{\frac{h^2}{Am}} \\&= \frac{h^2}{2m} \sqrt[3]{\frac{A^2 m^2}{h^4}} + \sqrt[3]{\frac{A^2 h^2}{m}} \\&= \frac{1}{2} \sqrt[3]{\frac{A^2 h^2}{m}} + \sqrt[3]{\frac{A^2 h^2}{m}} \\&= \frac{3}{2} \sqrt[3]{\frac{A^2 h^2}{m}}\end{aligned}$$

39.2.83

(a)

$$\begin{aligned}
 E_2 - E_1 &= \frac{hc}{\lambda} \\
 -\frac{mZ^2e^4}{32\epsilon_0^2h^2} + \frac{mZ^2e^4}{8\epsilon_0^2h^2} &= \frac{hc}{\lambda} \\
 \frac{3mZ^2e^4}{32\epsilon_0^2h^2} &= \frac{hc}{\lambda} \\
 Z &= \sqrt{\frac{32\epsilon_0^2h^3c}{3\lambda me^4}} \\
 &= 3
 \end{aligned}$$

(b)

$$\begin{aligned}
 R' &= 9R \\
 &= 9.87 \times 10^7 \text{ m}^{-1} \\
 E_1 &= -hcR' \\
 &= -1.96 \times 10^{-17} \text{ J} \\
 &= -123 \text{ eV} \\
 E_3 &= -\frac{hcR'}{9} \\
 &= -13.6 \text{ eV} \\
 \lambda &= \frac{hc}{E} \\
 &= 11.4 \text{ nm}
 \end{aligned}$$

(c)

$$\begin{aligned}
 K &= E - E_{\text{ionization}} \\
 &= E + E_1 \\
 &= \frac{hc}{\lambda} + E_1 \\
 &= 60.2 \text{ eV}
 \end{aligned}$$

39.2.85

(a)

$$\begin{aligned}
 P_{\text{Polaris}} &= A\sigma T^4 \\
 &= 4\pi(rR_{\text{sun}})^2 T^4 \\
 &= 1.67 \times 10^{37} \text{ W} \\
 P_{\text{Vega}} &= 3.86 \times 10^{35} \text{ W} \\
 P_{\text{Antares}} &= 6.34 \times 10^{38} \text{ W} \\
 P_{\alpha \text{ Centauri B}} &= 3.49 \times 10^{33} \text{ W}
 \end{aligned}$$

Antares

(b)

$$\begin{aligned}
 \lambda_{\text{Polaris}} &= \frac{b}{T} \\
 &= 482 \text{ nm} \\
 \lambda_{\text{Vega}} &= 302 \text{ nm} \\
 \lambda_{\text{Antares}} &= 852 \text{ nm} \\
 \lambda_{\alpha \text{ Centauri B}} &= 551 \text{ nm}
 \end{aligned}$$

Polaris and α Centauri B

39.2.87

(a)

$$\begin{aligned}
 f &= \frac{1}{T} \\
 &= \frac{v_n}{2\pi r_n} \\
 &= \frac{\frac{1}{\epsilon_0} \frac{e^2}{2n\hbar}}{2\pi\epsilon_0 \frac{n^2\hbar^2}{\pi m e^2}} \\
 &= \frac{\pi m e^4}{4\pi\epsilon_0^2 n^3 \hbar^3} \\
 &= \frac{m e^4}{4\epsilon_0^2 n^3 \hbar^3}
 \end{aligned}$$

(b)

$$\begin{aligned}
 hf &= E_i - E_f \\
 &= -\frac{me^4}{8\epsilon_0^2 h^2 (n+1)^2} + \frac{me^4}{8\epsilon_0 h^2 n^2} \\
 f &= \frac{me^4}{8\epsilon_0^2 h^3} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right) \\
 &= \frac{me^4}{8\epsilon_0^2 h^3} \left(\frac{(n+1)^2 - n^2}{n^2(n+1)^2} \right) \\
 &= \frac{me^4}{8\epsilon_0^2 h^3} \left(\frac{2n+1}{n^2(n+1)^2} \right) \\
 &\approx \frac{me^4}{8\epsilon_0^2 h^3} \left(\frac{2n}{n^4} \right) \\
 &\approx \frac{me^4}{4\epsilon_0^2 n^3 h^3}
 \end{aligned}$$

39.2.89

$$\begin{aligned}
 eV &= \frac{1}{2}mv^2 \\
 v &= \sqrt{\frac{2eV}{m}}
 \end{aligned}$$

$$\begin{aligned}
 \lambda &= \frac{h}{mv} \\
 &= \frac{h}{m\sqrt{2eV/m}} \\
 \frac{2eV}{m} &= \left(\frac{h}{m\lambda} \right)^2 \\
 V &= \frac{m}{2e} \left(\frac{h}{m\lambda} \right)^2 \\
 &= 21 \text{ kV}
 \end{aligned}$$

a

39.2.91

a

40 Quantum Mechanics I: Wave Functions

40.1 Guided Practice

40.1.1

(a)

$$\begin{aligned}E_n &= \frac{n^2 \pi^2 \hbar^2}{2mL^2} \\E_2 &= 4E_1 \\ \psi_n(x) &= \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \\ \Psi(x, t) &= \frac{1}{\sqrt{2}} \psi_1(x) e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} \psi_2(x) e^{-iE_2 t/\hbar} \\ &= \frac{1}{\sqrt{L}} \sin \frac{\pi x}{L} e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{L}} \sin \frac{2\pi x}{L} e^{-iE_2 t/\hbar} \\ |\Psi(x, t)|^2 &= \Psi(x, t) \Psi^*(x, t) \\ &= \frac{1}{L} \sin^2 \frac{\pi x}{L} + \frac{1}{L} \sin^2 \frac{2\pi x}{L} \\ &\quad + \frac{1}{L} \sin \frac{\pi x}{L} \sin \frac{2\pi x}{L} e^{i3E_1 t/\hbar} \\ &\quad + \frac{1}{L} \sin \frac{\pi x}{L} \sin \frac{2\pi x}{L} e^{-i3E_1 t/\hbar} \\ &= \frac{1}{L} \left[\sin^2 \frac{\pi x}{L} + \sin^2 \frac{2\pi x}{L} \right. \\ &\quad \left. + 2 \sin \frac{\pi x}{L} \sin \frac{2\pi x}{L} \cos \frac{(E_2 - E_1)t}{\hbar} \right]\end{aligned}$$

(b) No, because $|\Psi(x, t)|^2$ depends on time.

(c)

$$\begin{aligned}
& \int_0^L |\Psi(x, t)|^2 dx \\
&= \int_0^L \frac{1}{L} \left[\sin^2 \frac{\pi x}{L} + \sin^2 \frac{2\pi x}{L} + 2 \sin \frac{\pi x}{L} \sin \frac{2\pi x}{L} \cos \frac{(E_2 - E_1)t}{\hbar} \right] dx \\
&= \frac{1}{L} \int_0^L \left[\frac{1}{2} - \frac{1}{2} \cos \frac{2\pi x}{L} + \frac{1}{2} - \frac{1}{2} \cos \frac{4\pi x}{L} \right. \\
&\quad \left. + \left(\cos \frac{\pi x}{L} - \cos \frac{3\pi x}{L} \right) \cos \frac{(E_2 - E_1)t}{\hbar} \right] dx \\
&= \frac{1}{L} \int_0^L \left[1 - \frac{1}{2} \cos \frac{2\pi x}{L} - \frac{1}{2} \cos \frac{4\pi x}{L} \right. \\
&\quad \left. + \left(\cos \frac{\pi x}{L} - \cos \frac{3\pi x}{L} \right) \cos \frac{(E_2 - E_1)t}{\hbar} \right] dx \\
&= \frac{1}{L} [x]_0^L \\
&= 1
\end{aligned}$$

(d)

$$\begin{aligned}
\omega &= \frac{E_2 - E_1}{\hbar} \\
&= \frac{3\pi^2 \hbar}{2mL^2}
\end{aligned}$$

This is the difference $\omega_2 - \omega_1$ or the beat frequency.

(e)

$$\begin{aligned}
\omega &= \frac{E_2 - E_1}{\hbar} \\
&= \frac{2.43E_{1\text{-IDW}} - 0.625E_{1\text{-IDW}}}{\hbar} \\
&= \frac{1.805E_{1\text{-IDW}}}{\hbar} \\
&= 0.903 \frac{\pi^2 \hbar}{mL^2}
\end{aligned}$$

40.2 Exercises and Problems

40.2.1

$$p = \hbar k$$

$$k = \frac{p}{\hbar}$$

$$= 4.27 \times 10^{10} \text{ rad/m}$$

$$E = \hbar \omega$$

$$\omega = \frac{E}{\hbar}$$

$$= \frac{p^2/2m}{\hbar}$$

$$= 1.06 \times 10^{17} \text{ rad/s}$$

$$\Psi(x, t) = Ae^{i(-(4.27 \times 10^{10})x - (1.06 \times 10^{17})t)}$$

40.2.3

(a)

$$|\Psi(x, t)|^2 = 2|A|^2 \{1 + \cos[2kx - (\omega_2 - \omega_1)t]\}$$

$$\hbar\omega = \frac{\hbar^2 k^2}{2m}$$

$$\omega = \frac{\hbar k^2}{2m}$$

$$\begin{aligned}\omega_2 &= \frac{\hbar(3k)^2}{2m} \\ &= \frac{9\hbar k^2}{2m}\end{aligned}$$

$$\omega_1 = \frac{\hbar k^2}{2m}$$

$$\omega_2 - \omega_1 = \frac{4\hbar k^2}{m}$$

$$\begin{aligned}|\Psi(x, t)|^2 &= 2|A|^2 \left\{ 1 + \cos \left[2kx - \frac{4\hbar k^2}{m} \frac{4\pi m}{\hbar k^2} \right] \right\} \\ &= 2|A|^2 [1 + \cos(2kx - 16\pi)]\end{aligned}$$

$$2kx = 16\pi$$

$$\begin{aligned}x &= \frac{16\pi}{2k} \\ &= \frac{8\pi}{k}\end{aligned}$$

(b)

$$\begin{aligned}v &= \frac{x}{t} \\ &= \frac{4}{k} \frac{\hbar k^2}{2m} \\ &= \frac{2\hbar k}{m}\end{aligned}$$

$$\begin{aligned}v_{\text{av}} &= \frac{\omega_2 - \omega_1}{2k} \\ &= \frac{2\hbar k}{m}\end{aligned}$$

40.2.5

(a)

$$\begin{aligned}
 \psi(x) &= A \sin \frac{2\pi}{\lambda} x \\
 |\psi(x)|^2 &= A^2 \sin^2 \frac{2\pi}{\lambda} x \\
 &= \frac{1}{2} A^2 \left(1 - \cos \frac{4\pi}{\lambda} x \right)
 \end{aligned}$$

$$\begin{aligned}
 \cos \frac{4\pi}{\lambda} x &= -1 \\
 \frac{4\pi}{\lambda} x &= \pi + 2\pi n, n \in \mathbb{Z} \\
 x &= \frac{\lambda}{4\pi} (\pi + 2\pi n), n \in \mathbb{Z} \\
 &= \lambda \left(\frac{1}{4} + \frac{n}{2} \right), n \in \mathbb{Z} \\
 &= \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots
 \end{aligned}$$

(b)

$$\begin{aligned}
 \cos \frac{4\pi}{\lambda} x &= 1 \\
 \frac{4\pi}{\lambda} x &= 2\pi n, n \in \mathbb{Z} \\
 x &= \frac{\lambda}{4\pi} 2\pi n, n \in \mathbb{Z} \\
 &= \frac{\lambda n}{2}, n \in \mathbb{Z} \\
 &= 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots
 \end{aligned}$$

40.2.9

(a)

$$\begin{aligned}
 E_1 &= \frac{\pi^2 \hbar^2}{2mL^2} \\
 &= 1.62 \times 10^{-67} \text{ J}
 \end{aligned}$$

(b)

$$\begin{aligned}K &= \frac{1}{2}mv^2 \\v &= \sqrt{\frac{2K}{m}} \\&= 1.27 \times 10^{-33} \text{ m/s} \\t &= \frac{d}{v} \\&= 3.75 \times 10^{25} \text{ years}\end{aligned}$$

(c)

$$\begin{aligned}E_2 - E_1 &= \frac{4\pi^2\hbar^2}{2mL} - \frac{\pi^2\hbar^2}{2mL} \\&= \frac{3\pi^2\hbar^2}{2mL} \\&= 3E_1 \\&= 4.87 \times 10^{-67} \text{ J}\end{aligned}$$

(d) No

40.2.11

$$\begin{aligned}E &= \frac{\pi^2\hbar^2}{2mL^2} \\L &= \sqrt{\frac{\pi^2\hbar^2}{2mE}} \\&= 0.166 \text{ nm}\end{aligned}$$

40.2.13

$$\begin{aligned}E &= E_2 - E_1 \\&= \frac{3\pi^2\hbar^2}{2mL^2} \\L &= \sqrt{\frac{3\pi^2\hbar^2}{2mE}} \\&= 0.568 \text{ nm}\end{aligned}$$

40.2.15

(a)

$$\int_{-\infty}^{\infty} |\psi(x, t)|^2 dx = 1$$

(b)

$$\begin{aligned}\int_{-\infty}^{\infty} |e^{ax}|^2 dx &= \int_{-\infty}^{\infty} e^{2ax} dx \\ &= \left[\frac{1}{2a} e^{2ax} \right]_{-\infty}^{\infty} \\ &= \infty\end{aligned}$$

No, no.

(c)

$$\begin{aligned}\int_0^{\infty} |Ae^{-bx}|^2 dx &= A^2 \int_0^{\infty} e^{-2bx} dx \\ &= -\frac{A^2}{2b} [e^{-2bx}]_0^{\infty} \\ &= \frac{A^2}{2b} \\ A &= \sqrt{2b} \text{ m}^{-1/2}\end{aligned}$$

40.2.17

(a)

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$$

$$\begin{aligned}\sin \frac{2\pi x}{L} &= 0 \\ \frac{2\pi x}{L} &= n\pi, n \in \mathbb{Z} \\ x &= \frac{Ln}{2}, n \in \mathbb{Z} \\ &= 0, \frac{L}{2}, L\end{aligned}$$

(b)

$$\begin{aligned} |\psi(x)|^2 &= \frac{2}{L} \sin^2 \frac{2\pi x}{L} \\ &= \frac{1}{L} \left(1 - \cos \frac{4\pi x}{L} \right) \\ \cos \frac{4\pi x}{L} &= -1 \\ \frac{4\pi x}{L} &= \pi + 2\pi n, n \in \mathbb{Z} \\ x &= \frac{L}{4\pi} (\pi + 2\pi n), n \in \mathbb{Z} \\ &= \frac{L}{4} (1 + 2n), n \in \mathbb{Z} \\ &= \frac{L}{4}, \frac{3L}{4} \end{aligned}$$

(c) Yes

40.2.19

(a)

$$\begin{aligned} \lambda_1 &= 2L \\ &= 6.8 \times 10^{-10} \text{ m} \\ p_1 &= \frac{h}{2L} \\ &= 9.74 \times 10^{-25} \text{ kg m/s} \end{aligned}$$

(b)

$$\begin{aligned} \lambda_2 &= 3.4 \times 10^{-10} \text{ m} \\ p_2 &= 1.95 \times 10^{-24} \text{ kg m/s} \end{aligned}$$

(c)

$$\begin{aligned} \lambda_3 &= 2.27 \times 10^{-10} \text{ m} \\ p_3 &= 2.92 \times 10^{-24} \text{ kg m/s} \end{aligned}$$

40.2.21

$$\begin{aligned}
E_1 &= 0.625E_{1\text{-IDW}} \\
&= 0.625 \frac{\pi^2 \hbar^2}{2mL^2} \\
L &= \sqrt{0.625 \frac{\pi^2 \hbar^2}{2mE_1}} \\
&= 0.313 \text{ nm}
\end{aligned}$$

40.2.23

$$\begin{aligned}
E &= hf \\
U_0 - E_1 &= \frac{hc}{\lambda} \\
6E_{1\text{-IDW}} - 0.625E_{1\text{-IDW}} &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{5.375E_{1\text{-IDW}}} \\
&= \frac{hc}{5.375 \frac{\pi^2 \hbar^2}{2mL^2}} \\
&= \frac{2mL^2 hc}{5.375\pi^2 \hbar^2} \\
&= 613 \text{ nm}
\end{aligned}$$

40.2.25

$$\begin{aligned}
E &= hf \\
E_3 - E_1 &= \frac{hc}{\lambda} \\
5.09E_{1\text{-IDW}} - 0.625E_{1\text{-IDW}} &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{4.465E_{1\text{-IDW}}} \\
&= \frac{hc}{4.465 \frac{\pi^2 \hbar^2}{2mL^2}} \\
&= \frac{2mL^2 hc}{4.465\pi^2 \hbar^2} \\
&= 27 \text{ fm}
\end{aligned}$$

40.2.27

(a)

$$G = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0} \right)$$

$$= 2.74$$

$$\kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

$$= 1.53 \times 10^{10}$$

$$T = Ge^{-2\kappa L}$$

$$= 0.0013$$

(b)

$$\kappa = 6.57 \times 10^{11}$$

$$T = 1 \times 10^{-143}$$

40.2.29

(a)

$$G = 3.99$$

$$\kappa = 1.16 \times 10^{10}$$

$$T = 1.1 \times 10^{-8}$$

(b)

$$T = 3.7 \times 10^{-4}$$

40.2.31

$$E_1 = \frac{p_1^2}{2m}$$

$$U_0 = \frac{h^2}{2m\lambda_1^2}$$

$$\lambda_1 = \sqrt{\frac{h^2}{2mU_0}}$$

$$\lambda_2 = \sqrt{\frac{h^2}{4mU_0}}$$

$$= \frac{1}{\sqrt{2}}\lambda_1$$

40.2.33

$$\begin{aligned}
E_n &= \left(n + \frac{1}{2}\right) \hbar \sqrt{\frac{1000}{3}} \\
E_0 &= 9.63 \times 10^{-34} \text{ J} \\
&= 6.02 \times 10^{-15} \text{ eV} \\
E_{n+1} - E_n &= \hbar \sqrt{\frac{1000}{3}} \\
&= 1.93 \times 10^{-33} \text{ J} \\
&= 1.2 \times 10^{-14} \text{ eV}
\end{aligned}$$

No

40.2.35

(a)

$$E = \frac{hc}{\lambda} = 3.43 \times 10^{-20} \text{ J} = 0.214 \text{ eV}$$

(b)

$$\begin{aligned}
\Delta E &= \hbar \sqrt{\frac{k}{m}} \\
k &= m \left(\frac{\Delta E}{\hbar} \right)^2 \\
&= 5.9 \times 10^3 \text{ N/m}
\end{aligned}$$

40.2.37

$$\begin{aligned}
E_0 &= \frac{1}{2} \hbar \omega \\
\omega &= \frac{2E_0}{\hbar} \\
&= 1.76 \times 10^{16} \text{ rad/s}
\end{aligned}$$

$$\begin{aligned}
E &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{E} \\
&= \frac{hc}{\hbar \omega} \\
&= \frac{2\pi c}{\omega} \\
&= 107 \text{ nm}
\end{aligned}$$

40.2.39

$$E = \left(n + \frac{1}{2}\right) \hbar \omega$$

$$\frac{1}{2} k' A^2 = \left(n + \frac{1}{2}\right) \hbar \sqrt{\frac{k'}{m}}$$

$$A = \sqrt{\frac{2\hbar}{\sqrt{k'm}} \left(n + \frac{1}{2}\right)}$$

$$\frac{1}{2} m v_{\max}^2 = \frac{1}{2} k' A^2$$

$$\begin{aligned} v_{\max} &= A \sqrt{\frac{k'}{m}} \\ &= \sqrt{\frac{2\hbar k'^{1/2}}{m^{3/2}} \left(n + \frac{1}{2}\right)} \end{aligned}$$

$$p_{\max} = \sqrt{2\hbar \sqrt{k'm} \left(n + \frac{1}{2}\right)}$$

$$\begin{aligned} \Delta x \Delta p &= \frac{A}{\sqrt{2}} \frac{p_{\max}}{\sqrt{2}} \\ &= \frac{1}{2} \sqrt{4\hbar^2 \left(n + \frac{1}{2}\right)^2} \\ &= \hbar \left(n + \frac{1}{2}\right) \end{aligned}$$

40.2.41

(a)

$$E_0 = \frac{1}{2} \hbar \omega = 9.44 \times 10^{-22} \text{ J} = 0.0059 \text{ eV}$$

(b)

$$\begin{aligned}\lambda &= \frac{hc}{E} \\ &= \frac{hc}{\hbar\omega} \\ &= \frac{2\pi c}{\omega} \\ &= 105 \mu\text{m}\end{aligned}$$

(c)

$$\Delta E = \hbar\omega = 1.89 \times 10^{-21} \text{ J} = 0.0118 \text{ eV}$$

40.2.43

(a)

$$\begin{aligned}E_n &= \frac{n^2 \pi^2 \hbar^2}{2mL^2} \\ \psi_n(x) &= \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \\ \Psi(x, t) &= \frac{1}{\sqrt{L}} \sin \frac{\pi x}{L} e^{-i\pi^2 \hbar t / 2mL^2} + \frac{1}{\sqrt{L}} \sin \frac{3\pi x}{L} e^{-i9\pi^2 \hbar t / 2mL^2} \\ \Psi(L/2, t) &= \frac{1}{\sqrt{L}} e^{-i\pi^2 \hbar t / 2mL^2} - \frac{1}{\sqrt{L}} e^{-i9\pi^2 \hbar t / 2mL^2} \\ |\Psi(L/2, t)|^2 &= \Psi(L/2, t) \Psi^*(L/2, t) \\ &= \frac{1}{L} \left(1 - e^{i8\pi^2 \hbar t / 2mL^2} - e^{-i8\pi^2 \hbar t / 2mL^2} + 1 \right) \\ &= \frac{2}{L} \left[1 - \cos \left(\frac{4\pi^2 \hbar}{mL^2} t \right) \right]\end{aligned}$$

(b)

$$\omega = \frac{4\pi^2 \hbar}{mL^2}$$

40.2.45

$$A + B = C$$

$$k_1(A - B) = k_2C$$

$$k_1(A - C + A) = k_2C$$

$$2k_1A - k_1C = k_2C$$

$$C = \frac{2k_1}{k_1 + k_2}A$$

$$A + B = \frac{2k_1}{k_1 + k_2}A$$

$$B = \frac{k_1 - k_2}{k_1 + k_2}A$$

40.2.47

(a)

$$E = hf$$

$$E_2 - E_1 = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\frac{3\pi^2\hbar^2}{2mL^2}}$$

$$= \frac{2mL^2hc}{3\pi^2\hbar^2}$$

$$= 1.94 \times 10^{-5} \text{ m}$$

$$= 19.4 \mu\text{m}$$

(b)

$$\lambda = \frac{2mL^2hc}{5\pi^2\hbar^2}$$

$$= 11.6 \mu\text{m}$$

40.2.49

(a)

$$\begin{aligned}\psi_2(x) &= \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L} \\ |\psi_2(x)|^2 &= \frac{2}{L} \sin^2 \frac{2\pi x}{L} \\ &= \frac{1}{L} \left(1 - \cos \frac{4\pi x}{L} \right) \\ |\psi_2(L/4)|^2 dx &= \frac{2}{L} dx\end{aligned}$$

(b)

$$|\psi_2(L/2)|^2 dx = 0$$

(c)

$$|\psi_2(3L/4)|^2 dx = \frac{2}{L} dx$$

40.2.51

(a)

$$\begin{aligned}\psi_1(x) &= \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L} \\ |\psi_1(x)|^2 &= \frac{2}{L} \sin^2 \frac{\pi x}{L} \\ &= \frac{1}{L} \left(1 - \cos \frac{2\pi x}{L} \right) \\ \int_{L/4}^{3L/4} |\psi_1(x)|^2 dx &= \int_{L/4}^{3L/4} \frac{1}{L} \left(1 - \cos \frac{2\pi x}{L} \right) dx \\ &= \frac{1}{L} \left[x - \frac{L}{2\pi} \sin \frac{2\pi x}{L} \right]_{L/4}^{3L/4} \\ &= \frac{1}{L} \left(\frac{3L}{4} + \frac{L}{2\pi} - \frac{L}{4} + \frac{L}{2\pi} \right) \\ &= \frac{1}{2} + \frac{1}{\pi} \\ &\approx 0.818\end{aligned}$$

(b)

$$\begin{aligned}
 |\psi_2(x)|^2 &= \frac{1}{L} \left(1 - \cos \frac{4\pi x}{L} \right) \\
 \int_{L/4}^{3L/4} |\psi_2(x)|^2 dx &= \int_{L/4}^{3L/4} \frac{1}{L} \left(1 - \cos \frac{4\pi x}{L} \right) dx \\
 &= \frac{1}{L} \left[x - \frac{L}{4\pi} \sin \frac{4\pi x}{L} \right]_{L/4}^{3L/4} \\
 &= \frac{1}{L} \left(\frac{3L}{4} - \frac{L}{4} \right) \\
 &= \frac{1}{2}
 \end{aligned}$$

(c) Yes

40.2.53

(b)

$$\begin{aligned}
 -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} &= E\psi(x) \\
 -\frac{\hbar^2\kappa^2}{2m} \psi(x) &= E\psi(x) \\
 E &= -\frac{\hbar^2\kappa^2}{2m}
 \end{aligned}$$

(c)

$$\begin{aligned}
 -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} &= E\psi(x) \\
 -\frac{\hbar^2\kappa^2}{2m} \psi(x) &= E\psi(x) \\
 E &= -\frac{\hbar^2\kappa^2}{2m}
 \end{aligned}$$

(d) Because the derivative is discontinuous at $x = 0$.

40.2.55

(a)

$$\begin{aligned}\psi_{\text{inside}}(x) &= A \cos\left(\frac{\sqrt{2mE}}{\hbar}x\right) + B \sin\left(\frac{\sqrt{2mE}}{\hbar}x\right) \\ \psi_{\text{outside}}(x) &= Ce^{\kappa x} + De^{-\kappa x}\end{aligned}$$

$$\begin{aligned}\psi_{\text{outside}}(0) &= \psi_{\text{inside}}(0) \\ C &= A\end{aligned}$$

$$\begin{aligned}\psi_{\text{outside}}(L) &= \psi_{\text{inside}}(L) \\ De^{-\kappa L} &= A \cos\left(\frac{\sqrt{2mE}}{\hbar}L\right) + B \sin\left(\frac{\sqrt{2mE}}{\hbar}L\right)\end{aligned}$$

(b)

$$\begin{aligned}\frac{d\psi_{\text{outside}}(x)}{dx} &= \kappa Ce^{\kappa x} - \kappa De^{-\kappa x} \\ \frac{d\psi_{\text{inside}}(x)}{dx} &= -kA \sin kx + kB \cos kx\end{aligned}$$

$$\begin{aligned}\frac{d\psi_{\text{outside}}(0)}{dx} &= \frac{d\psi_{\text{inside}}(0)}{dx} \\ \kappa C &= kB\end{aligned}$$

$$\begin{aligned}\frac{d\psi_{\text{outside}}(L)}{dx} &= \frac{d\psi_{\text{inside}}(L)}{dx} \\ -\kappa De^{-\kappa L} &= -kA \sin kL + kB \cos kL\end{aligned}$$

40.2.57

$$T = 0.500 \text{ s}$$

$$\begin{aligned} E_0 &= \frac{1}{2} \hbar \omega \\ &= \frac{1}{2} \frac{h}{2\pi} \frac{2\pi}{T} \\ &= \frac{h}{2T} \\ &= h \\ &= 6.626 \times 10^{-34} \text{ J} \\ &= 4.14 \times 10^{-15} \text{ eV} \end{aligned}$$

$$\begin{aligned} \Delta E &= \hbar \omega \\ &= \frac{h}{2\pi} \frac{2\pi}{T} \\ &= 2h \\ &= 1.33 \times 10^{-33} \text{ J} \\ &= 8.28 \times 10^{-15} \text{ eV} \end{aligned}$$

No

40.2.59

(a)

$$\begin{aligned} \lambda &= \frac{2L}{n} \\ p &= \frac{h}{\lambda} \\ &= \frac{nh}{2L} \\ E &= K \\ &= \frac{1}{2} mv^2 \\ &= \frac{p^2}{2m} \\ &= \left(\frac{nh}{2L} \right)^2 \frac{1}{2m} \\ &= \frac{n^2 h^2}{8mL^2} \end{aligned}$$

(b)

$$\begin{aligned} E_1 &= \frac{h^2}{8mL^2} \\ &= 2.15 \times 10^{-17} \text{ J} \\ &= 135 \text{ eV} \end{aligned}$$

40.2.61

(a)

$$\begin{aligned} E_n - E_1 &= \frac{(n^2 - 1)\pi^2 \hbar^2}{2mL^2} \\ L^2 &= \frac{(n^2 - 1)\pi^2 \hbar^2}{2mh f_n} \end{aligned}$$

$$\begin{aligned} E_{n+1} - E_1 &= \frac{(n^2 + 2n)\pi^2 \hbar^2}{2mL^2} \\ L^2 &= \frac{(n^2 + 2n)\pi^2 \hbar^2}{2mh f_{n+1}} \end{aligned}$$

$$\begin{aligned} \frac{(n^2 - 1)\pi^2 \hbar^2}{2mh f_n} &= \frac{(n^2 + 2n)\pi^2 \hbar^2}{2mh f_{n+1}} \\ f_{n+1}(n^2 - 1) &= f_n(n^2 + 2n) \\ (f_{n+1} - f_n)n^2 - 2f_n n - f_{n+1} &= 0 \\ n &= \frac{2f_n \pm \sqrt{4f_n^2 + 4(f_{n+1} - f_n)f_{n+1}}}{2(f_{n+1} - f_n)} \\ &= \frac{f_n \pm \sqrt{f_n^2 + f_{n+1}(f_{n+1} - f_n)}}{f_{n+1} - f_n} \\ &= 3 \end{aligned}$$

The two principle quantum numbers are 3 and 4.

(b)

$$\begin{aligned} E_4 - E_3 &= \frac{7\pi^2 \hbar^2}{2mL^2} \\ h(f_4 - f_3) &= \frac{7\pi^2 \hbar^2}{2mL^2} \\ L &= \sqrt{\frac{7\pi^2 \hbar^2}{2mh(f_4 - f_3)}} \\ &= 0.9 \text{ nm} \end{aligned}$$

(c)

$$\begin{aligned}E_2 - E_1 &= \frac{hc}{\lambda} \\ \lambda &= \frac{hc}{\frac{3\pi^2\hbar^2}{2mL^2}} \\ &= \frac{2mL^2hc}{3\pi^2\hbar^2} \\ &= 890 \text{ nm}\end{aligned}$$

40.2.63

$$\begin{aligned}E &= \frac{1}{2}mv^2 \\ &= \frac{p^2}{2m} \\ p &= \sqrt{2Em}\end{aligned}$$

$$\begin{aligned}\lambda &= \frac{h}{p} \\ \frac{2L}{n} &= \frac{h}{\sqrt{2Em}} \\ 2Em &= \left(\frac{hn}{2L}\right)^2 \\ E &= \left(\frac{hn}{2L}\right)^2 \frac{1}{2m}\end{aligned}$$

$$\begin{aligned}E_1 &= 12 \text{ eV} \\ E_2 &= 46 \text{ eV} \\ E_3 &= 105 \text{ eV}\end{aligned}$$

Adding 10 eV to each value gives 22 eV, 56 eV, and 115 eV.

40.2.65

(a)

$$\begin{aligned}E &= \frac{1}{2}k'A^2 \\ A &= \pm\sqrt{\frac{2E}{k'}}\end{aligned}$$

(b)

$$\begin{aligned}
 \frac{n\hbar}{2} &= \int_{-A}^A \sqrt{2m[E - \frac{1}{2}k'x^2]} dx \\
 &= 2\sqrt{k'm} \int_0^A \sqrt{\frac{2E}{k'} - x^2} dx \\
 &= \sqrt{k'm} \left[x\sqrt{\frac{2E}{k'} - x^2} + \frac{2E}{k'} \arcsin\left(\frac{x}{\sqrt{2E/k'}}\right) \right]_0^{\sqrt{2E/k'}} \\
 &= \sqrt{k'm} \left(\frac{2E}{k'} \frac{\pi}{2} \right) \\
 &= \sqrt{\frac{m}{k'}} E\pi \\
 E &= n\hbar\omega
 \end{aligned}$$

(c) Underestimate

40.2.67

c

40.2.69

a

41 Quantum Mechanics II: Atomic Structure

41.1 Guided Practice

41.1.1

(a)

$$\begin{aligned}
 l^3 &= \frac{4}{3}\pi r^3 \\
 l &= \sqrt[3]{\frac{4}{3}\pi r} \\
 &= 2.37 \times 10^{-10} \text{ m}
 \end{aligned}$$

(b) The values of (n_X, n_Y, n_Z, m_s) for the 22 electrons are:

$$\left(1, 1, 1, +\frac{1}{2}\right)$$

$$\left(1, 1, 1, -\frac{1}{2}\right)$$

$$\left(2, 1, 1, +\frac{1}{2}\right)$$

$$\left(2, 1, 1, -\frac{1}{2}\right)$$

$$\left(1, 2, 1, +\frac{1}{2}\right)$$

$$\left(1, 2, 1, -\frac{1}{2}\right)$$

$$\left(1, 1, 2, +\frac{1}{2}\right)$$

$$\left(1, 1, 2, -\frac{1}{2}\right)$$

$$\left(2, 2, 1, +\frac{1}{2}\right)$$

$$\left(2, 2, 1, -\frac{1}{2}\right)$$

$$\left(2, 1, 2, +\frac{1}{2}\right)$$

$$\left(2, 1, 2, -\frac{1}{2}\right)$$

$$\left(1, 2, 2, +\frac{1}{2}\right)$$

$$\left(1, 2, 2, -\frac{1}{2}\right)$$

$$\left(2, 2, 2, +\frac{1}{2}\right)$$

$$\left(2, 2, 2, -\frac{1}{2}\right)$$

$$\left(3, 1, 1, +\frac{1}{2}\right)$$

$$\left(3, 1, 1, -\frac{1}{2}\right)$$

$$\left(1, 3, 1, +\frac{1}{2}\right)$$

$$\left(1, 3, 1, -\frac{1}{2}\right)$$

$$\overset{146}{\left(1, 1, 3, +\frac{1}{2}\right)}$$

$$\left(1, 1, 3, -\frac{1}{2}\right)$$

(c)

$$E_{111} = 3.218 \times 10^{-18} \text{ J} = 20.1 \text{ eV}$$

$$E_{211} = 6.435 \times 10^{-18} \text{ J} = 40.2 \text{ eV}$$

$$E_{221} = 9.653 \times 10^{-18} \text{ J} = 60.3 \text{ eV}$$

$$E_{222} = 1.287 \times 10^{-17} \text{ J} = 80.4 \text{ eV}$$

$$E_{311} = 1.180 \times 10^{-17} \text{ J} = 73.8 \text{ eV}$$

(d)

$$E_{222} - E_{111} = 60.3 \text{ eV}$$

$$f = (2.48 \times 10^{15} \text{ Hz})(Z - 1)^2$$

$$= 1.094 \times 10^{18} \text{ Hz}$$

$$E = hf$$

$$= 4.529 \text{ keV}$$

41.2 Exercises and Problems

41.2.1

(a) 1

(b) 3

41.2.3

$$E_{221} = \frac{(n_X^2 + n_Y^2 + n_Z^2)\pi^2 \hbar^2}{2mL^2}$$

$$= 6.69 \times 10^{-17} \text{ J}$$

$$E_{111} = 2.23 \times 10^{-17} \text{ J}$$

$$\lambda = \frac{hc}{E}$$

$$= \frac{hc}{E_{221} - E_{111}}$$

$$= 4.46 \text{ nm}$$

41.2.5

When $n_X = 2$, $n_Y = 2$, and $n_Z = 1$, the probability distribution function is zero at $x = \frac{L}{2}$ and $y = \frac{L}{2}$.

When $n_X = 2$, $n_Y = 1$, and $n_Z = 1$, the probability distribution function is zero at $x = \frac{L}{2}$.

When $n_X = 1$, $n_Y = 1$, and $n_Z = 1$, the probability distribution function is only zero on the walls.

41.2.7

(a) $L_{\min} = 0$

(b) $L_{\max} = \sqrt{l(l+1)}\hbar = \sqrt{12}\hbar = 3.65 \times 10^{-34} \text{ kg m}^2/\text{s}$

(c) $L_{z,\max} = l\hbar = 3\hbar = 3.16 \times 10^{-34} \text{ kg m}^2/\text{s}$

(d) $S_z = m_s\hbar = \frac{1}{2}\hbar = 5.27 \times 10^{-35} \text{ kg m}^2/\text{s}$

(e) $\frac{S_z}{L_z} = \frac{1}{6}$

41.2.9

$$\begin{aligned} L &= \sqrt{l(l+1)}\hbar \\ \left(\frac{L}{\hbar}\right)^2 &= l(l+1) \\ l^2 + l - \left(\frac{L}{\hbar}\right)^2 &= 0 \\ l &= -5 \text{ or } 4 \\ l &= 4 \end{aligned}$$

41.2.11

If it's the smallest angle then m_l is maximised, i.e. $m_l = l$.

$$\begin{aligned} \cos \theta &= \frac{L_z}{L} \\ &= \frac{l\hbar}{\sqrt{l(l+1)}\hbar} \\ &= \frac{l}{\sqrt{l(l+1)}} \\ \cos^2 \theta &= \frac{l^2}{l(l+1)} \\ &= \frac{l}{l+1} \\ (l+1) \cos^2 \theta &= l \\ l(\cos^2 \theta - 1) + \cos^2 \theta &= 0 \\ l &= \cot^2 \theta \\ &= 4 \end{aligned}$$

41.2.13

$$\begin{aligned}
 L_9 &= \sqrt{l(l+1)}\hbar \\
 &= 1.00 \times 10^{-33} \text{ kg m}^2/\text{s} \\
 &= 1.054(9\hbar) \\
 L_{18} &= 1.95 \times 10^{-33} \text{ kg m}^2/\text{s} \\
 &= 1.0274(18\hbar) \\
 L_{208} &= 2.20 \times 10^{-32} \text{ kg m}^2/\text{s} \\
 &= 1.0024(208\hbar)
 \end{aligned}$$

The Bohr model becomes more accurate as n increases.

41.2.15

(a) 10

(b) $m_l = -2$, $\theta = \arccos \frac{L_z}{L} = \arccos \frac{m_l \hbar}{\sqrt{l(l+1)}\hbar} = 145^\circ$

(c) $m_l = 2$, $\theta = 35^\circ$

41.2.17

$$\begin{aligned}
 \Phi(\phi + 2\pi) &= e^{im_l(\phi+2\pi)} \\
 &= \cos m_l(\phi + 2\pi) + i \sin m_l(\phi + 2\pi) \\
 &= \cos(m_l\phi + 2m_l\pi) + i \sin(m_l\phi + 2m_l\pi)
 \end{aligned}$$

This is only equal to $\Phi(\phi)$ if $2m_l\pi$ is a multiple of 2π which only happens if $m_l \in \mathbb{Z}$.

41.2.19

(a)

$$\begin{aligned}
 \Delta E &= \mu_B B \\
 B &= \frac{\Delta E}{\mu_B} \\
 &= \frac{2m\Delta E}{e\hbar} \\
 &= 0.506 \text{ T}
 \end{aligned}$$

(b) 3

41.2.21

(a) 9

(b) $\Delta E = \mu_B B = 1.45 \times 10^{-5} \text{ eV}$

(c) $\Delta E = 8\mu_B B = 1.16 \times 10^{-4} \text{ eV}$

41.2.23

(a)

$$\begin{aligned}
L &= I\omega \\
&= \frac{2}{5}mr^2\omega \\
\omega &= \frac{5L}{2mr^2} \\
&= 7.74 \times 10^{29} \text{ rad/s}
\end{aligned}$$

(b)

$$v = r\omega = 1.39 \times 10^{13} \text{ m/s}$$

This isn't valid because it's greater than c .

41.2.25

$$\begin{aligned}
\Delta E &= 2 \times 1.00116\mu_B B_z \\
&= 2.00232 \frac{e\hbar}{2m} B_z \\
&= 2.78 \times 10^{-23} \text{ J} \\
&= 1.74 \times 10^{-4} \text{ eV}
\end{aligned}$$

$m_s = \frac{1}{2}$ has lower energy.

41.2.27

$$\left(1, 0, 0, +\frac{1}{2}\right)$$

$$\left(1, 0, 0, -\frac{1}{2}\right)$$

$$\left(2, 0, 0, +\frac{1}{2}\right)$$

$$\left(2, 0, 0, -\frac{1}{2}\right)$$

$$\left(2, 1, -1, +\frac{1}{2}\right)$$

$$\left(2, 1, -1, -\frac{1}{2}\right)$$

$$\left(2, 1, 0, +\frac{1}{2}\right)$$

$$\left(2, 1, 0, -\frac{1}{2}\right)$$

$$\left(2, 1, 1, -\frac{1}{2}\right)$$

$$\left(2, 1, 1, +\frac{1}{2}\right)$$

41.2.29

(a) $1s^2 2s^2$

(b) $1s^2 2s^2 2p^6 3s^2$, $Z = 12$ Magnesium

(c) $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$, $Z = 20$, Calcium

41.2.31

$$E_n = \frac{Z_{\text{eff}}^2}{n^2} (13.6 \text{ eV})$$

$$= 4.81 \text{ eV}$$

41.2.33

(a) $1s^2 2s^2 2p$

(b)

$$\begin{aligned} E_n &= -\frac{Z_{\text{eff}}^2}{n^2}(13.6 \text{ eV}) \\ &= -\frac{3^2}{2^2}(13.6 \text{ eV}) \\ &= -30.6 \text{ eV} \end{aligned}$$

(c) $1s^2 2s^2 2p^6 3s^2 3p$

(d)

$$\begin{aligned} E_n &= -\frac{Z_{\text{eff}}^2}{n^2}(13.6 \text{ eV}) \\ &= -\frac{3^2}{3^2}(13.6 \text{ eV}) \\ &= -13.6 \text{ eV} \end{aligned}$$

41.2.35

(a)

$$\begin{aligned} E_n &= -\frac{Z_{\text{eff}}^2}{n^2}(13.6 \text{ eV}) \\ &= -\frac{2^2}{2^2}(13.6 \text{ eV}) \\ &= -13.6 \text{ eV} \end{aligned}$$

(b)

$$E_n = -3.40 \text{ eV}$$

41.2.37

(a)

$$\begin{aligned} f &= (2.48 \times 10^{15} \text{ Hz})(Z - 1)^2 \\ &= 8.95 \times 10^{17} \text{ Hz} \\ E &= 3.71 \text{ keV} \end{aligned}$$

(b)

$$\begin{aligned} f &= 1.68 \times 10^{18} \text{ Hz} \\ E &= 6.94 \text{ keV} \end{aligned}$$

(c)

$$\begin{aligned} f &= 5.48 \times 10^{18} \text{ Hz} \\ E &= 22.7 \text{ keV} \end{aligned}$$

41.2.39

$$E_{221} = 3E_{111}$$

41.2.41

(a)

$$\frac{(L/4)^3}{L^3} = \frac{L^3/64}{L^3} = \frac{1}{64}$$

(b)

$$\begin{aligned} \int |\psi(x, y, z)|^2 dV &= |C|^2 \left(\int_0^{L/4} \sin^2 \frac{\pi x}{L} dx \right) \left(\int_0^{L/4} \sin^2 \frac{\pi y}{L} dy \right) \\ &\quad \left(\int_0^{L/4} \sin^2 \frac{\pi z}{L} dz \right) \\ &= \left(\frac{2}{L} \right)^3 \left(\frac{L(\pi - 2)}{8\pi} \right)^3 \\ &= \frac{8}{L^3} \frac{L^3(\pi - 2)^3}{512\pi^3} \\ &= \frac{(\pi - 2)^3}{64\pi^3} \\ &\approx 7.50 \times 10^{-4} \end{aligned}$$

(c)

$$\begin{aligned} \int |\psi(x, y, z)|^2 dV &= \left(\frac{2}{L} \right)^3 \frac{L}{8} \left(\frac{L(\pi - 2)}{8\pi} \right)^2 \\ &= \frac{8}{L^3} \frac{L}{8} \frac{L^2(\pi - 2)^2}{64\pi^2} \\ &= \frac{(\pi - 2)^2}{64\pi^2} \\ &\approx 2.06 \times 10^{-3} \end{aligned}$$

41.2.43

(a)

$$\begin{aligned}
 |C|^2 & \left(\int_0^{L/2} \sin^2 \frac{\pi x}{L} dx \right) \left(\int_0^L \sin^2 \frac{\pi y}{L} dy \right) \left(\int_0^L \sin^2 \frac{\pi z}{L} dz \right) \\
 &= \left(\frac{2}{L} \right)^3 \frac{L}{4} \left(\frac{L}{2} \right)^2 \\
 &= \frac{8}{L^3} \frac{L}{4} \frac{L^2}{4} \\
 &= \frac{1}{2}
 \end{aligned}$$

(b)

$$\begin{aligned}
 |C|^2 & \left(\int_{L/4}^{L/2} \sin^2 \frac{\pi x}{L} dx \right) \left(\int_0^L \sin^2 \frac{\pi y}{L} dy \right) \left(\int_0^L \sin^2 \frac{\pi z}{L} dz \right) \\
 &= \left(\frac{2}{L} \right)^3 \frac{L(\pi+2)}{8\pi} \left(\frac{L}{2} \right)^2 \\
 &= \frac{8}{L^3} \frac{L(\pi+2)}{8\pi} \frac{L^2}{4} \\
 &= \frac{\pi+2}{4\pi} \\
 &\approx 0.409
 \end{aligned}$$

41.2.47

(a)

$$\begin{aligned}
 \sum_{l=0}^{n-1} 2(2l+1) &= \sum_{l=0}^{n-1} (4l+2) \\
 &= 2n + 4 \sum_{l=0}^{n-1} l \\
 &= 2n + 4 \frac{(n-1)n}{2} \\
 &= 2n + 2(n^2 - n) \\
 &= 2n^2
 \end{aligned}$$

(b)

$$50 = 2n^2 \rightarrow n = 5$$

The O shell has 50 states.

41.2.49

(a)

$$\begin{aligned}
 E &= U \\
 \frac{1}{(4\pi\epsilon_0)^2} \frac{m_r e^4}{2\hbar^2} &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \\
 \frac{1}{4\pi\epsilon_0} \frac{m_e m_p}{m_e + m_p} \frac{e^2}{2\hbar^2} &= \frac{1}{r} \\
 r &= \frac{8\pi\epsilon_0(m_e + m_p)\hbar^2}{m_e m_p e^2} \\
 &= 1.06 \times 10^{-10} \text{ m} \\
 &= 2a
 \end{aligned}$$

(b)

$$\begin{aligned}
 \psi_{1s}(r) &= \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \\
 \int_{2a}^{\infty} |\psi_{1s}(r)|^2 dr &= \int_{2a}^{\infty} \frac{1}{\pi a^3} e^{-2r/a} dV \\
 &= \frac{1}{\pi a^3} \int_{2a}^{\infty} e^{-2r/a} 4\pi r^2 dr \\
 &= \frac{4}{a^3} \int_{2a}^{\infty} r^2 e^{-2r/a} dr \\
 &= 0.238
 \end{aligned}$$

41.2.51

$$\begin{aligned}
 P(r) &= (R_{2p})^2 r^2 \\
 &= \left(\frac{1}{\sqrt{24a^5}} r e^{-r/2a} \right)^2 r^2 \\
 &= \frac{1}{24a^5} r^2 e^{-r/a} r^2 \\
 &= \frac{r^4 e^{-r/a}}{24a^5} \\
 \frac{dP(r)}{dr} &= \frac{1}{24a^5} \left(4r^3 e^{-r/a} - \frac{1}{a} r^4 e^{-r/a} \right) \\
 0 &= \frac{1}{24a^5} \left(4r^3 e^{-r/a} - \frac{1}{a} r^4 e^{-r/a} \right) \\
 4r^3 e^{-r/a} &= \frac{1}{a} r^4 e^{-r/a} \\
 r &= 4a
 \end{aligned}$$

Under the Bohr model, $r_2 = 4a$ so they're equal.

41.2.53

(a)

$$\begin{aligned}\cos \theta &= \frac{L_z}{L} \\ \theta &= \arccos \frac{l\hbar}{\sqrt{l(l+1)}\hbar} \\ &= \arccos \frac{n-1}{\sqrt{n(n-1)}}\end{aligned}$$

(b)

$$\begin{aligned}\theta &= \arccos \frac{-(n-1)}{\sqrt{n(n-1)}} \\ &= \arccos -\sqrt{\frac{(n-1)^2}{n(n-1)}} \\ &= \arccos -\sqrt{1-1/n}\end{aligned}$$

41.2.55

$$\begin{aligned}\Delta E &= \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} \\ &= 2.64 \times 10^{-23} \text{ J} \\ &= 1.65 \times 10^{-4} \text{ eV} \\ \Delta E &= U \\ &= \mu_B B \\ B &= \frac{\Delta E}{\mu_B} \\ &= 2.85 \text{ T}\end{aligned}$$

41.2.57

(a)

$$\begin{aligned}E_1 &= -13.60 \text{ eV} \\ E &= E_1 \pm 1.00116\mu_B B \\ \frac{n_{+1/2}}{n_{-1/2}} &= e^{-\Delta E/kT} \\ &= e^{-2 \times 1.00116\mu_B B/kT} \\ &= 1\end{aligned}$$

(b) 0.996

(c) 0.969

41.2.61

(a)

$$\begin{aligned}
\Delta E &= E_i - E_f \\
&= -(13.60 \text{ eV}) \left(\frac{1}{2^2} - \frac{1}{1^2} \right) \\
&= 10.2 \text{ eV} \\
\Delta E &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{\Delta E} \\
&= 122 \text{ nm}
\end{aligned}$$

(b)

$$\begin{aligned}
\Delta E' &= E'_i - E'_f \\
&= E_i + m_l \mu_B B - E_f \\
&= -\frac{13.60 \text{ eV}}{2^2} - \mu_B B + 13.60 \text{ eV} \\
&= \frac{3}{4}(13.60 \text{ eV}) - \mu_B B \\
&= 10.2 \text{ eV} - 1.28 \times 10^{-4} \text{ eV} \\
\lambda &= \frac{hc}{\Delta E'} \\
&= 121.8 \text{ nm} \\
\Delta \lambda &= hc \left(\frac{1}{\Delta E'} - \frac{1}{\Delta E} \right) \\
&= 1.53 \text{ pm}
\end{aligned}$$

The magnetic field increases the wavelength.

41.2.65

(a)

$$\begin{aligned}
E_{\text{Li}} &= 5.40 \text{ eV} \\
E_{\text{Na}} &= 5.15 \text{ eV} \\
E_{\text{K}} &= 4.35 \text{ eV} \\
E_{\text{Rb}} &= 4.18 \text{ eV} \\
E_{\text{Cs}} &= 3.90 \text{ eV} \\
E_{\text{Fr}} &= 3.94 \text{ eV}
\end{aligned}$$

(b)

$$\begin{aligned}
 Z_{\text{Li}} &= 3 \\
 n_{\text{Li}} &= 2 \\
 Z_{\text{Na}} &= 11 \\
 n_{\text{Na}} &= 3 \\
 Z_{\text{K}} &= 19 \\
 n_{\text{K}} &= 4 \\
 Z_{\text{Rb}} &= 37 \\
 n_{\text{Rb}} &= 5 \\
 Z_{\text{Cs}} &= 55 \\
 n_{\text{Cs}} &= 6 \\
 Z_{\text{Fr}} &= 87 \\
 n_{\text{Fr}} &= 7
 \end{aligned}$$

(c)

$$\begin{aligned}
 E_n &= -\frac{Z_{\text{eff}}^2}{n^2}(13.6 \text{ eV}) \\
 Z_{\text{eff}} &= \sqrt{-\frac{E_n n^2}{13.6 \text{ eV}}} \\
 Z_{\text{eff,Li}} &= 1.26 \\
 Z_{\text{eff,Na}} &= 1.85 \\
 Z_{\text{eff,K}} &= 2.26 \\
 Z_{\text{eff,Rb}} &= 2.77 \\
 Z_{\text{eff,Cs}} &= 3.21 \\
 Z_{\text{eff,Fr}} &= 3.77
 \end{aligned}$$

(d) Increase

41.2.67

(a) $m = 2.842 \times 10^{10} \text{ Hz/T}$

(b)

$$\begin{aligned}
 \Delta E &= 2\mu_z \Delta B_z \\
 \mu_z &= \frac{1}{2} \frac{\Delta E}{\Delta B_z} \\
 &= \frac{1}{2} \hbar m \\
 &= 9.42 \times 10^{-24} \text{ J/T}
 \end{aligned}$$

(c)

$$\begin{aligned}\gamma &= \left| \frac{\mu_z}{S_z} \right| \\ &= \left| \frac{\mu_z}{m_s \hbar} \right| \\ &= \frac{2\mu_z}{\hbar} \\ &= 1.78 \times 10^{11} \\ \frac{\gamma}{e/2m} &= 2.025\end{aligned}$$

41.2.69

(a)

$$\begin{aligned}L &= \hbar \\ mvr &= \hbar\end{aligned}$$

$$\begin{aligned}\frac{1}{4\pi\epsilon_0} \frac{2e^2}{r^2} - \frac{1}{4\pi\epsilon_0} \frac{e^2}{(2r)^2} &= \frac{mv^2}{r} \\ \frac{1}{4\pi\epsilon_0} \left(\frac{2e^2}{r^2} - \frac{e^2}{4r^2} \right) &= \frac{mv^2}{r} \\ \frac{1}{4\pi\epsilon_0} \frac{7e^2}{4r^2} &= \frac{mv^2}{r} \\ \frac{1}{4\pi\epsilon_0} \frac{7e^2}{4r^2} &= \frac{m}{r} \left(\frac{\hbar}{mr} \right)^2 \\ &= \frac{\hbar^2}{mr^3} \\ \frac{1}{4\pi\epsilon_0} \frac{7e^2}{4} &= \frac{\hbar^2}{mr} \\ r &= \frac{16\pi\epsilon_0\hbar^2}{7e^2m} \\ &\approx 3.04 \times 10^{-11} \text{ m}\end{aligned}$$

$$\begin{aligned}v &= \sqrt{\frac{7e^2r}{16\pi\epsilon_0mr^2}} \\ &= 3.82 \times 10^6 \text{ m/s}\end{aligned}$$

(b)

$$K = 2\frac{1}{2}mv^2 = 1.33 \times 10^{-17} \text{ J} = 83.0 \text{ eV}$$

(c)

$$\begin{aligned}U &= -2 \frac{1}{4\pi\epsilon_0} \frac{2e^2}{r} + \frac{1}{4\pi\epsilon_0} \frac{e^2}{2r} \\&= \frac{1}{4\pi\epsilon_0} \left(\frac{e^2}{2r} - \frac{4e^2}{r} \right) \\&= -\frac{1}{4\pi\epsilon_0} \frac{7e^2}{2r} \\&= -2.65 \times 10^{-17} \text{ J} \\&= -166 \text{ eV}\end{aligned}$$

(d) 83.0 eV

41.2.71

b

41.2.73

d