University Physics with Modern Physics
Electromagnetism Notes

Chris Doble

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21 Electric Charge and Electric Field

21.1 Electric Charge

- Electrons have a much smaller mass than neutrons and protons
- Neutrons and protons have a very similar mass
- Electrons and protons have the same magnitude of charge
- The number of protons in an atom determins its **atomic number**

- If an electron is added to a neutral atom it becomes a **negative ion**, if one is removed it becomes a **positive ion** — this is called **ionisation**
- The **principle of conservation of charge** states that the algebraic sum of all the electric charges in any closed system is constant
- The electron or proton's magnitude of charge is a natural unit of charge — every observable amount of electric charge is an integer multiple of this

21.2 Conductors, Insulators, and Incuded Charges

- **Conductors** pemit easy movement of charge, **insulators** do not
- Holding a charged object near an uncharged object causes free electrons in the latter to move away/towards the former, resulting in a net charge on either side — this is called **induced charge**

21.3 Coulomb's Law

- The SI unit of charge is called one **coulomb** (1 C) and is defined such that $1.602176634 \times 10^{-19}$ C is equal to the charge of an electron or proton
- **Coulomb's law** describes the electric force between two point charges

  $$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2}$$

  where the **electric constant** $\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$, q_1 and q_2 are the magnitudes of the charges, and r is the distance between them
- The electric force is always directed along the line between the two charges, attracting opposite charges and repelling like charges
- $\frac{1}{4\pi\epsilon_0}$ can be approximated as $9.0 \times 10^9\,\mathrm{N \cdot m^2/C^2}$
- The principle of superposition of forces also applies to electric charges

21.4 Electric Field and Electric Forces

- The electric force on a charged object is exerted by the electric field created by other charged objects
- We can determine if there is an electric field at a point by placing a test charge q_0 there and seeing if it experiences an electric force — the electric field at that point (the electric force per unit charge) is then given by

  $$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

- Rearranging, the force experienced by a charge q_0 at a point is given by

$$\mathbf{F} = q_0\mathbf{E}$$

- When considering an electric field produced by a point charge, the location of the point charge is called the **source point** and the location at which we're trying to determine the field is called the **field point**
- The electric field produced by a point charge is given by

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

  where q is the charge of the point charge, r is the distance between the source and field points, and $\hat{\mathbf{r}}$ is the unit vector from the source to the field point
- Unlike Coulomb's law this equation doesn't use the absolute value of q meaning that the electric fields of positive charges point away from the charge, while those of negative charges point towards them
- In electrostatics, the electric field inside the material of a conductor (but not holes within the material) is $\mathbf{0}$

21.5 Electric-Field Calculations

- The **principle of superposition of electric fields** states that the total electric field at a point P is the vector sum of the fields at P due to each point charge in the charge distribution

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots$$

- For a line charge distribution the **linear charge density** is represented by λ (the charge per unit length, measured in C/m)
- For a surface charge distribution the **surface charge density** is represented by σ (the charge per unit area, measured in C/m^2)
- For a volume charge distribution the **volume charge density** is represented by ρ (the charge per unit volume, measured in C/m^3)
- The electric field of an infinitely long line charge along the y-axis is

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

21.6 Electric Field Lines

- An **electric field line** is a line drawn through space such that its tangent at any point is in the direction of the electric field vector at that point
- Fewer lines are drawn in areas where the electric field is weak and more lines are drawn in areas where it's strong

21.7 Electric Dipoles

- An **electric dipole** is a pair of point charges of equal magnitude q and opposite sign separated by a distance d
- The net force on an electric dipole in a uniform electric field is $\mathbf{0}$
- The **electric dipole moment p** of an electric dipole is a vector directed from the negative charge to the positive charge with magnitude qd
- The net torque on an electric dipole in a uniform electric field is $\mathbf{p} \times \mathbf{E}$ or $qEd \sin \phi$ where ϕ is the angle between the electric dipole and the electric field
- The potential energy of an electric dipole in a uniform electric field is

$$U = -\mathbf{p} \cdot \mathbf{E}$$

22 Gauss's Law

22.1 Calculating Electric Flux

- The electric flux of a uniform electric field through a flat surface A is

  $$\Phi_E = \mathbf{E} \cdot \mathbf{A}$$

  where $\mathbf{A}$ is normal to A and has a magnitude equal to its area
- The electric flux of a nonuniform electric field through a curved surface A is

  $$\Phi_E = \int \mathbf{E} \cdot \mathbf{dA}$$

22.2 Gauss's Law

- Gauss's law states that the total electric flux through a closed surface is equal to the total electric charge enclosed by the surface divided by ϵ_0

  $$\Phi_E = \oint \mathbf{E} \cdot \mathbf{dA} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

22.3 Applications of Gauss's Law

- Gauss's law can be used in two ways:
  - If we know the charge distribution and it has enough symmetry to let us evaluate the integral in Gauss's law, we can find the field
  - If we know the field, we can use Gauss's law to find the charge distribution

- Under electrostatics, excess charge always lies of the surface of a conductor
- The electric field of an infinite line charge is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}\hat{\mathbf{r}}$$

22.4 Charges on Conductors

- If there is excess charge at rest on a conductor, all of that charge must lie on the surface of the conductor and the electric field inside the conductor must be zero. If there is a cavity inside the conductor, the net charge on the cavity walls equals the amount of charge enclosed by the cavity
- Charges outside a conductor have no effect on the interior of the conductor, even if it has a cavity inside — this is why Faraday cages work
- At the surface of a conductor, the component of the electric field that is perpendicular to the surface is

$$E_\perp = \frac{\sigma}{\epsilon_0}$$

23 Electric Potential

23.1 Electric Potential Energy

- The electric potential energy of two point charges is

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

- The electric potential energy of a point charge q_0 and a collection of charges q_1, q_2, etc. is

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- For every electric field due to a static charge distribution, the force extered by that field is conservative
- The total electric potential energy of a collection of charges q_1, q_2, etc. is

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

where r_{ij} is the distance between q_i and q_j

23.2 Electric Potential

- **Potential** is potential energy per unit charge
- The unit of potential is the **volt**, equal to 1 joule per coulomb
- The potential difference between two points $V_{ab} = V_a - V_b$ is called the potential of a with respect to b and equals the amount of work done by the electric force when a unit (1 C) of charge moves from a to b
- The electric potential due to a point charge is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

- The electric potential due to a collection of point charges is

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- The electric potential due to a continuous charge distribution is

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

- The electric potential difference between two points is given by

$$V_a - V_b = \int_a^b \mathbf{E}\cdot d\mathbf{l} = \int_a^b E\cos\phi\, dl$$

- Positive charges tend to "fall" from high- to low-potential regions while negative charges do the opposite
- When a particle with charge $e = 1.602 \times 10^{-19}\,\mathrm{C}$ moves between two points with a potential difference of $1\,\mathrm{V} = 1\,\mathrm{J/C}$ the change in energy is $U_a - U_b = qV_{ab} = (1.602 \times 10^{-19}\,\mathrm{C})(1\,\mathrm{J/C}) = 1.602 \times 10^{-19}\,\mathrm{J}$ which is called 1 **electron volt**

23.4 Equipotential Surfaces

- An **equipotential surface** is a three-dimensional surface on which the electric potential is the same at every point
- Because electric potential energy doesn't change as a test charge moves over an equipotential surface, the electric field can do no work and thus **field lines and equipotential surfaces are always perpendicular**
- When all charges are at rest, the surface of a conductor is an equipotential surface
- When all charges are at rest, the entire solid volume of a conductor is at the same potential

23.5 Potential Gradient

- The relationship between $\mathbf{E}$ and V is given by

$$\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$$

- If E has a radial component E_r with respect to an axis or a point and r is the distance from that axis or point, then

$$E_r = -\frac{\partial V}{\partial r}$$