Advanced Engineering Mathematics Vectors, Matrices, and Vector Calculus by Dennis G. Zill Problems

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Contents

1 Vectors

1.1 Vectors in 2-Space

1.1.1

(a) $3\mathbf{a} = 6\mathbf{i} + 12\mathbf{j}$

(b) $\mathbf{a} + \mathbf{b} = \mathbf{i} + 8\mathbf{j}$

(c) $\mathbf{a} - \mathbf{b} = 3\mathbf{i}$

(d) $||\mathbf{a} + \mathbf{b}|| = \sqrt{1 + 8^2} = \sqrt{65}$

(e) $||\mathbf{a} - \mathbf{b}|| = 3$

1.1.9

(a) $4\mathbf{a} - 2\mathbf{b} = \langle 6, -14 \rangle$

(b) $-3\mathbf{a} - 5\mathbf{b} = \langle 2, 4 \rangle$

1.1.15

$\overrightarrow{P_1 P_2} = \langle 2, 5 \rangle$

1.1.19

$(1, 18)$

1.1.21

(a) Yes

(b) Yes

(c) Yes

(d) No

(e) Yes

(f) Yes

1.1.25

(a) $\frac{\mathbf{a}}{||\mathbf{a}||} = \frac{\langle 2,2 \rangle}{\sqrt{2^2+2^2}} = \frac{1}{2\sqrt{2}} \langle 2, 2 \rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

(b) $\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$

1.1.31

$2\frac{\mathbf{a}}{||\mathbf{a}||} = 2\frac{\langle 3,7 \rangle}{\sqrt{3^2+7^2}} = \frac{2}{\sqrt{58}} \langle 3, 7 \rangle = \langle \frac{6}{\sqrt{58}}, \frac{14}{\sqrt{58}} \rangle$

1.1.37

$\mathbf{x} = -(\mathbf{a} + \mathbf{b})$

1.1.41

$$\begin{aligned}
\mathbf{a} &= 2\mathbf{i} + 3\mathbf{j} \\
\mathbf{b} &= \mathbf{i} + \mathbf{j} \\
\mathbf{c} &= \mathbf{i} - \mathbf{j}
\end{aligned}$$

$$\begin{aligned}
\mathbf{i} &= \frac{1}{2}(\mathbf{b} + \mathbf{c}) \\
\mathbf{j} &= \frac{1}{2}(\mathbf{b} - \mathbf{c}) \\
\mathbf{a} &= 2\left(\frac{1}{2}(\mathbf{b} + \mathbf{c})\right) + 3\left(\frac{1}{2}(\mathbf{b} - \mathbf{c})\right) \\
&= \mathbf{b} + \mathbf{c} + \frac{3}{2}\mathbf{b} - \frac{3}{2}\mathbf{c} \\
&= \frac{5}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}
\end{aligned}$$

1.1.43

$$\begin{aligned}
y &= \frac{1}{4}x^2 + 1 \\
y(2) &= 2 \\
y' &= \frac{1}{2}x \\
y'(2) &= 1 \\
\mathbf{v} &= \pm\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle
\end{aligned}$$

1.1.45

(a)

$$\begin{aligned}
\mathbf{F}_n &= \mathbf{F}\cos\theta \\
\mathbf{F}_g &= \mathbf{F}\sin\theta \\
||\mathbf{F}_f|| &= \mu||\mathbf{F}_n|| \\
||-\mathbf{F}_g|| &= \mu||\mathbf{F}_n|| \\
||-\mathbf{F}\sin\theta|| &= \mu||\mathbf{F}\cos\theta|| \\
||\mathbf{F}||\sin\theta &= \mu||\mathbf{F}||\cos\theta \\
\tan\theta &= \mu
\end{aligned}$$

(b) $\theta = \arctan \mu \approx 31°$

1.1.47

$$\begin{aligned}
F_x &= \frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{L\,dy}{2a(L^2+y^2)^{3/2}} \\
&= \frac{LqQ}{8\pi\epsilon_0}\int_{-a}^{a}(L^2+y^2)^{-3/2}\,dy \\
&= \frac{LqQ}{8\pi\epsilon_0}\frac{2a}{L^2\sqrt{a^2+L^2}} \\
&= \frac{aqQ}{4\pi\epsilon_0 L\sqrt{a^2+L^2}} \\
F_y &= -\frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{y\,dy}{2a(L^2+y^2)^{3/2}} \\
&= 0 \\
\mathbf{F} &= \langle\frac{1}{4\pi\epsilon_0}\frac{qQ}{L\sqrt{a^2+L^2}}, 0\rangle
\end{aligned}$$

1.1.49

Let the three sides of the triangle be vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. The triangle is closed so it must be the case that

$$\mathbf{a}+\mathbf{b}+\mathbf{c} = 0.$$

This gives

$$\mathbf{c} = -(\mathbf{a}+\mathbf{b}).$$

The vector from the midpoint of side $\mathbf{a}$ to the midpoint of side $\mathbf{b}$ is

$$\left(\mathbf{a}+\frac{1}{2}\mathbf{b}\right) - \frac{1}{2}\mathbf{a} = \frac{1}{2}(\mathbf{a}+\mathbf{b})$$

which is parallel with $\mathbf{c}$ and half its length.

1.2 Vectors in 3-Space

1.2.7

A plane at $z = 5$ parellel with the x-y plane.

1.2.9

A line parallel to the z axis at $x = 2$ and $y = 3$.

1.2.13

(a) $(0, 5, 4)$, $(-2, 0, 4)$, $(-2, 5, 0)$

(b) $(-2, 5, -2)$

(c) $(3, 5, 4)$

1.2.15

The planes $x = 0$, $y = 0$, and $z = 0$.

1.2.17

$(-1, 2, -3)$

1.2.19

The planes $z = \pm 5$.

1.2.21

$\sqrt{(6-3)^2 + (4+1)^2 + (8-2)^2} = \sqrt{9 + 25 + 36} = \sqrt{70}$

1.2.31

$$\begin{aligned}
\sqrt{(2-x)^2 + (1-2)^2 + (1-3)^2} &= \sqrt{21} \\
(2-x)^2 + 1 + 4 &= 21 \\
(2-x)^2 &= 16 \\
2 - x &= \pm 4 \\
x &= 2 \pm 4 \\
&= -2 \text{ or } 6
\end{aligned}$$

1.2.33

$\left(4, \frac{1}{2}, \frac{3}{2}\right)$

1.2.37

$(-3, -6, 1)$

1.3 Dot Product

1.3.1

$\mathbf{a} \cdot \mathbf{b} = 12$

1.3.11

$\left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b} = \frac{12}{30}\mathbf{b} = \langle -\frac{2}{5}, \frac{4}{5}, 2\rangle$

1.3.13

$\mathbf{a}\cdot\mathbf{b} = ||\mathbf{a}||||\mathbf{b}||\cos\theta = 25\sqrt{2}$

1.3.17

$$\begin{aligned}
\mathbf{a}\cdot\mathbf{v} &= 0 \\
3x_1 + y_1 - 1 &= 0 \\
\\
\mathbf{b}\cdot\mathbf{v} &= 0 \\
-3x_1 + 2y_2 + 2 &= 0 \\
\\
3y_2 + 1 &= 0 \\
y_2 &= -\frac{1}{3} \\
\\
3x_1 - \frac{1}{3} - 1 &= 0 \\
x_1 &= \frac{4}{9} \\
\\
\mathbf{v} &= \langle \frac{4}{9}, -\frac{1}{3}, 1\rangle
\end{aligned}$$

1.3.19

$$\begin{aligned}
\mathbf{a}\cdot\mathbf{c} &= \mathbf{a}\cdot\left(\mathbf{b} - \frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{a}||^2}\mathbf{a}\right) \\
&= \mathbf{a}\cdot\mathbf{b} - \frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{a}||^2}\mathbf{a}\cdot\mathbf{a} \\
&= 0
\end{aligned}$$

1.3.21

$$\begin{aligned}
||\mathbf{a}|| &= \sqrt{3^2 + 1^2} \\
&= \sqrt{10} \\
||\mathbf{b}|| &= \sqrt{2^2 + 2^2} \\
&= \sqrt{8} \\
&= 2\sqrt{2} \\
\mathbf{a} \cdot \mathbf{b} &= 4 \\
\theta &= \arccos \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||||\mathbf{b}||} \\
&= \arccos \frac{4}{(\sqrt{10})(2\sqrt{2})} \\
&= \arccos \frac{1}{\sqrt{5}} \\
&\approx 63^\circ
\end{aligned}$$

1.3.25

$$\begin{aligned}
||\mathbf{a}|| &= \sqrt{1^2 + 2^2 + 3^3} \\
&= \sqrt{14} \\
\cos\alpha &= \frac{1}{\sqrt{14}} \\
\alpha &\approx 75^\circ \\
\cos\beta &= \frac{2}{\sqrt{14}} \\
\beta &\approx 58^\circ \\
\cos\gamma &= \frac{3}{\sqrt{14}} \\
\gamma &\approx 37^\circ
\end{aligned}$$

1.3.29

$$\begin{aligned}
\overrightarrow{AD} &= \langle s, -s, s\rangle \\
||\overrightarrow{AD}|| &= s\sqrt{3} \\
\overrightarrow{AB} &= \langle s, 0, 0\rangle \\
||\overrightarrow{AB}|| &= s \\
\theta &= \arccos \frac{\overrightarrow{AD} \cdot \overrightarrow{AB}}{||\overrightarrow{AD}||||\overrightarrow{AB}||} \\
&= \arccos \frac{s^2}{s^2\sqrt{3}} \\
&= \arccos \frac{1}{\sqrt{3}} \\
&\approx 55^\circ
\end{aligned}$$

1.3.33

$$\begin{aligned}
\text{comp}_{\mathbf{b}}\mathbf{a} &= \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{b}||} \\
&= \frac{5}{7}
\end{aligned}$$

1.3.37

$$\begin{aligned}
\text{comp}_{\overrightarrow{OP}}\mathbf{a} &= \frac{\mathbf{a} \cdot \overrightarrow{OP}}{||\overrightarrow{OP}||} \\
&= \frac{72}{\sqrt{109}}
\end{aligned}$$

1.3.39

$$\begin{aligned}
\text{proj}_{\mathbf{b}}\mathbf{a} &= \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{b} \cdot \mathbf{b}}\right)\mathbf{b} \\
&= \frac{35}{25}\mathbf{b} \\
&= \langle -\frac{21}{5}, \frac{28}{5}\rangle
\end{aligned}$$

1.3.43

$$\begin{aligned}
\mathbf{a}+\mathbf{b} &= \langle 3,4\rangle \\
\text{proj}_{\mathbf{a}+\mathbf{b}}\mathbf{a} &= \left(\frac{\mathbf{a}\cdot(\mathbf{a}+\mathbf{b})}{(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})}\right)(\mathbf{a}+\mathbf{b}) \\
&= \frac{24}{25}(\mathbf{a}+\mathbf{b}) \\
&= \langle\frac{72}{25},\frac{96}{25}\rangle
\end{aligned}$$

1.3.45

$W = \mathbf{F}\cdot\mathbf{d} = Fd\cos\theta = 1000$

1.3.47

(a) $W = 0$

(b)

$$\begin{aligned}
||\mathbf{d}|| &= \sqrt{4^2+3^2} \\
&= 5 \\
\mathbf{F} &= F\hat{\mathbf{d}} \\
&= F\frac{\mathbf{d}}{||\mathbf{d}||} \\
&= F\langle\frac{4}{5},\frac{3}{5}\rangle \\
&= \langle 24, 18\rangle \\
W &= \mathbf{F}\cdot\mathbf{d} \\
&= 150\,\mathrm{J}
\end{aligned}$$

1.4 Cross Product

1.4.1

$$\begin{aligned}
\mathbf{a}\times\mathbf{b} &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & -1 & 0\\ 0 & 3 & 5\end{vmatrix} \\
&= -5\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}
\end{aligned}$$

1.4.11

$$\begin{aligned}\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 2 & -4 \\ -3 & 1 & 1 \end{vmatrix} \\ &= 6\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}\end{aligned}$$

1.4.17

(a)

$$\begin{aligned}\mathbf{b} \times \mathbf{c} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix} \\ &= \mathbf{j} - \mathbf{k} \\ \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 0 & 1 & -1 \end{vmatrix} \\ &= -\mathbf{i} + \mathbf{j} + \mathbf{k}\end{aligned}$$

1.4.19

$2\mathbf{k}$

1.4.21

$$\begin{aligned}\mathbf{k} \times (2\mathbf{i} - \mathbf{j}) &= (\mathbf{k} \times 2\mathbf{i}) - (\mathbf{k} \times \mathbf{j}) \\ &= \mathbf{i} + 2\mathbf{j}\end{aligned}$$

1.4.23

$$\begin{aligned}[(2\mathbf{k}) \times (3\mathbf{j})] \times (4\mathbf{j}) &= (-6\mathbf{i}) \times (4\mathbf{j}) \\ &= -24\mathbf{k}\end{aligned}$$

1.4.37

$12\mathbf{i} - 9\mathbf{j} + 18\mathbf{k}$

1.4.53

$$\begin{aligned}
\mathbf{b}\times\mathbf{c} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 6 & -6 \\ \frac{5}{2} & 3 & \frac{1}{2} \end{vmatrix} \\
&= 21\mathbf{i} - 14\mathbf{j} - 21\mathbf{k} \\
\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) &= 4\times 21 + 6\times(-14) \\
&= 0
\end{aligned}$$

They are coplanar.

1.5 Lines and Planes in 3-Space

1.5.1

$\mathbf{r} = \langle 1, 2, 1\rangle + t\langle 2, 3, -3\rangle$

1.5.7

$$\begin{aligned}
x &= 2 + 4t \\
y &= 3 - 4t \\
z &= 5 + 3t
\end{aligned}$$

1.5.13

$$\begin{aligned}
x &= 1 + 9t \\
y &= 4 + 10t \\
z &= -9 + 7t \\
\frac{x-1}{9} = \frac{y-4}{10} &= \frac{z+9}{7}
\end{aligned}$$

1.5.19

$$\begin{aligned}
x &= 4 + 3t \\
y &= 6 + \frac{1}{2}t \\
z &= -7 - \frac{3}{2}t \\
\frac{x-4}{3} = \frac{y-6}{1/2} &= -\frac{z+7}{3/2}
\end{aligned}$$

1.5.23

$$\begin{aligned} x &= 6 + 2t \\ y &= 4 - 3t \\ z &= -2 + 6t \end{aligned}$$

1.5.25

$$\begin{aligned} x &= 2 + t \\ y &= -2 \\ z &= 15 \end{aligned}$$

1.5.29

$(0, 5, 15)$, $(5, 0, \frac{15}{2})$, $(10, -5, 0)$

1.5.31

$$\begin{aligned} 4 + t_x &= 6 + 2t_x \\ t_x &= -2 \\ \\ 5 + t_y &= 11 + 4t_y \\ t_y &= -2 \\ \\ -1 + 2t_z &= -3 + t_z \\ t_z &= -2 \end{aligned}$$

$(2, 3, -5)$

1.5.35

$$\begin{aligned} \mathbf{a} &= \langle -1, 2, -2 \rangle \\ ||\mathbf{a}|| &= 3 \\ \mathbf{b} &= \langle 2, 3, -6 \rangle \\ ||\mathbf{b}|| &= 7 \\ \theta &= \arccos \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||||\mathbf{b}||} \\ &\approx 40.37^\circ \end{aligned}$$

1.5.37

$$\begin{aligned}
\mathbf{a} &= \langle 1,1,1\rangle \\
\mathbf{b} &= \langle -2,1,-5\rangle \\
\mathbf{a}\times\mathbf{b} &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ -2 & 1 & -5 \end{vmatrix} \\
&= \langle -6,3,3\rangle \\
x &= 4-6t \\
y &= 1+3t \\
z &= 6+3t
\end{aligned}$$

1.5.39

$$\begin{aligned}
\langle 2,-3,4\rangle\cdot(\mathbf{r}-\langle 5,1,3\rangle) &= 0 \\
2(x-5)-3(y-1)+4(z-3) &= 0 \\
2x-3y+4z-19 &= 0
\end{aligned}$$

1.5.45

$$\begin{aligned}
\mathbf{a} &= \langle 3,5,2\rangle \\
\mathbf{b} &= \langle 2,3,1\rangle \\
\mathbf{c} &= \langle -1,-1,4\rangle \\
\mathbf{a}-\mathbf{c} &= \langle 4,6,-2\rangle \\
\mathbf{b}-\mathbf{c} &= \langle 3,4,-3\rangle \\
(\mathbf{a}-\mathbf{c})\times(\mathbf{b}-\mathbf{c}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 6 & -2 \\ 3 & 4 & -3 \end{vmatrix} \\
&= \langle -10,6,-2\rangle \\
\mathbf{n}\cdot(\mathbf{r}-\mathbf{c}) &= 0 \\
\langle -10,6,-2\rangle\cdot(\langle x,y,z\rangle-\langle -1,-1,4\rangle) &= 0 \\
-10(x+1)+6(y+1)-2(z-4) &= 0 \\
-10x+6y-2z+4 &= 0
\end{aligned}$$

1.5.51

$$\begin{aligned}
\langle 1,1,-4\rangle\cdot(\mathbf{r}-\langle 2,3,-5\rangle) &= 0 \\
(x-2)+(y-3)-4(z+5) &= 0 \\
x+y-4z &= 25
\end{aligned}$$

1.5.63

(a) Not perpendicular

(b) Not perpendicular

(c) Perpendicular

(d) Perpendicular

1.5.65

$$\begin{aligned} 5x - 4y - 9t &= 8 \\ x + 4y + 3t &= 4 \end{aligned}$$

$$\begin{aligned} 6x - 6t &= 12 \\ x &= 2 + t \end{aligned}$$

$$y = \frac{1}{2} - t$$

$$z = t$$

1.5.69

$$\begin{aligned} 2(1 + 2t) - 3(2 - t) + 2(-3t) &= -7 \\ t &= -3 \\ x &= -5 \\ y &= 5 \\ z &= 9 \end{aligned}$$

1.5.73

$$\begin{aligned}x+y-4t&=2\\2x-y+t&=10\end{aligned}$$

$$\begin{aligned}3x-3t&=12\\x&=4+t\end{aligned}$$

$$\begin{aligned}2(4+t)-y+t&=10\\8+2t-y+t&=10\\y&=-2+3t\end{aligned}$$

$$z=t$$

$$\begin{aligned}x&=5+t\\y&=6+3t\\z&=-12+t\end{aligned}$$

1.5.75

$$\begin{aligned}\mathbf{n}&=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\3&-1&5\\1&1&1\end{vmatrix}\\&=\langle -6,2,4\rangle\\\mathbf{n}\cdot(\mathbf{r}-\langle 4,0,1\rangle)&=0\\-6(x-4)+2y+4(z-1)&=0\\-6x+2y+4z&=-20\\3x-y-2z&=10\end{aligned}$$