

Classical Mechanics by John R. Taylor Problems

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Contents

1	Newton’s Laws of Motion	1
1.1	1
1.5	2
1.11	2
1.23	2
1.25	2
1.35	3
1.37	4
1.39	5
1.41	6
1.47	6

1 Newton’s Laws of Motion

1.1

$$\begin{aligned}\mathbf{b} + \mathbf{c} &= 2\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}} \\ 5\mathbf{b} + 2\mathbf{c} &= 7\hat{\mathbf{x}} + 5\hat{\mathbf{y}} + 2\hat{\mathbf{z}} \\ \mathbf{b} \cdot \mathbf{c} &= 1 \\ \mathbf{b} \times \mathbf{c} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \\ &= \hat{\mathbf{x}} - \hat{\mathbf{y}} - \hat{\mathbf{z}}\end{aligned}$$

1.5

$$\begin{aligned}
 \mathbf{v}_{\text{body}} &= \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}} \\
 \mathbf{v}_{\text{face}} &= \hat{\mathbf{x}} + \hat{\mathbf{z}} \\
 \mathbf{v}_{\text{body}} \cdot \mathbf{v}_{\text{face}} &= v_{\text{body}} v_{\text{face}} \cos \theta \\
 2 &= \sqrt{6} \cos \theta \\
 \cos \theta &= \frac{2}{\sqrt{6}} \\
 \theta &= \arccos \frac{2}{\sqrt{6}} \\
 &= 35.26^\circ
 \end{aligned}$$

1.11

The particle moves counterclockwise in an ellipse of width $2b$ and height $2c$. The angular speed is ω .

1.23

$$\begin{aligned}
 \mathbf{v} &= v \cos \theta \frac{\mathbf{b}}{b} - v \sin \theta \frac{\mathbf{b} \times \mathbf{c}}{bc} \\
 &= \frac{\lambda}{b} \frac{\mathbf{b}}{b} - \frac{c}{b} \frac{\mathbf{b} \times \mathbf{c}}{bc} \\
 &= \frac{\lambda \mathbf{b} - \mathbf{b} \times \mathbf{c}}{b^2}
 \end{aligned}$$

1.25

$$\begin{aligned}
 \frac{df}{dt} &= -3f \\
 \frac{1}{f} \frac{df}{dt} &= -3 \\
 \ln f &= -3t + c \\
 f &= ce^{-3t}
 \end{aligned}$$

One constant.

1.35

$$\begin{aligned}
 F_x &= 0 \\
 ma_x &= 0 \\
 a_x &= 0 \\
 v_x &= c_1 \\
 &= v_o \cos \theta \\
 r_x &= v_o \cos(\theta)t + c_2 \\
 &= v_o \cos(\theta)t
 \end{aligned}$$

$$\begin{aligned}
 F_y &= 0 \\
 ma_y &= 0 \\
 a_y &= 0 \\
 v_y &= c_3 \\
 v_y &= 0 \\
 r_y &= c_4 \\
 r_y &= 0
 \end{aligned}$$

$$\begin{aligned}
 F_z &= -mg \\
 ma_z &= -mg \\
 a_z &= -g \\
 v_z &= -gt + c_5 \\
 &= v_o \sin \theta - gt \\
 r_z &= v_o \sin(\theta)t - \frac{1}{2}gt^2 + c_6 \\
 &= v_o \sin(\theta)t - \frac{1}{2}gt^2
 \end{aligned}$$

$$\begin{aligned}
 0 &= v_o \sin(\theta)t - \frac{1}{2}gt^2 \\
 t &= \frac{2 \sin(\theta)v_o}{g} \\
 r_x &= v_o \cos(\theta)t \\
 &= \frac{2 \cos(\theta) \sin(\theta)v_o^2}{g} \\
 &= \frac{\sin(2\theta)v_o^2}{g}
 \end{aligned}$$

1.37

(a)

$$\begin{aligned}F &= -mg \sin \theta \\ma &= -mg \sin \theta \\a &= -g \sin \theta \\v &= c_1 - gt \sin \theta \\&= v_o - gt \sin \theta \\x &= v_o t - \frac{1}{2}gt^2 \sin \theta\end{aligned}$$

(b)

$$t = \frac{2v_o}{g \sin \theta}$$

1.39

$$\begin{aligned}
 F_x &= -mg \sin \phi \\
 ma_x &= -mg \sin \phi \\
 a_x &= -g \sin \phi \\
 v_x &= c_1 - gt \sin \phi \\
 &= v_o \cos \theta - gt \sin \phi \\
 r_x &= v_o t \cos \theta - \frac{1}{2}gt^2 \sin \phi + c_2 \\
 &= v_o t \cos \theta - \frac{1}{2}gt^2 \sin \phi
 \end{aligned}$$

$$\begin{aligned}
 F_y &= -mg \cos \phi \\
 ma_y &= -mg \cos \phi \\
 a_y &= -g \cos \phi \\
 v_y &= c_3 - gt \cos \phi \\
 &= v_o \sin \theta - gt \cos \phi \\
 r_y &= v_o t \sin \theta - \frac{1}{2}gt^2 \cos \phi + c_4 \\
 &= v_o t \sin \theta - \frac{1}{2}gt^2 \cos \phi
 \end{aligned}$$

$$\begin{aligned}
 0 &= v_o t \sin \theta - \frac{1}{2}gt^2 \cos \phi \\
 t &= \frac{2v_o \sin \theta}{g \cos \phi}
 \end{aligned}$$

$$\begin{aligned}
 r_x &= \frac{2v_o^2 \cos \theta \sec \phi \sin \theta}{g} - \frac{2v_o^2 \sec \phi \sin^2 \theta \tan \phi}{g} \\
 &= \frac{2v_o^2 \sin \theta (\cos \theta \cos \phi - \sin \theta \sin \phi)}{g \cos^2 \phi} \\
 &= \frac{2v_o^2 \sin \theta \cos(\theta + \phi)}{g \cos^2 \phi}
 \end{aligned}$$

$$\begin{aligned}
\frac{dr_x}{d\theta} &= \frac{2v_o^2}{g \cos^2 \phi} [\cos \theta \cos(\theta + \phi) - \sin \theta \sin(\theta + \phi)] \\
&= \frac{2v_o^2 \cos(2\theta + \phi)}{g \cos^2 \phi} \\
0 &= \frac{2v_o^2 \cos(2\theta + \phi)}{g \cos^2 \phi} \\
&= \cos(2\theta + \phi) \\
2\theta + \phi &= \frac{\pi}{2} \\
\theta &= \frac{\pi}{4} - \frac{\phi}{2} \\
r_{x,\max} &= \frac{2v_o^2 \sin\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \cos\left(\frac{\pi}{4} - \frac{\phi}{2} + \phi\right)}{g \cos^2 \phi} \\
&= \frac{v_o^2(1 - \sin \phi)}{g \cos^2 \phi} \\
&= \frac{v_o^2}{g(1 + \sin \phi)}
\end{aligned}$$

1.41

$$\begin{aligned}
F &= ma \\
T &= m \frac{v^2}{R} \\
&= m \frac{(\omega R)^2}{R} \\
&= m\omega^2 R
\end{aligned}$$

1.47

(a)

$$\begin{aligned}
\rho &= \sqrt{x^2 + y^2} \\
\phi &= \arctan \frac{y}{x} \\
z &= z
\end{aligned}$$

ρ is the distance of P from the z -axis.

The use of r may be unfortunate because it suggests it's the distance of P from the origin.

- (b) $\hat{\rho}$ points away from the z -axis, $\hat{\phi}$ points counter-clockwise around the z -axis, and \hat{z} points in the positive z direction.

$$\mathbf{r} = \rho\hat{\rho} + z\hat{z} + \sqrt{x^2 + y^2}\hat{\rho} + z\hat{z}$$

(c)

$$\begin{aligned}\mathbf{v} &= \frac{d\mathbf{r}}{dt} \\ &= \dot{\rho}\hat{\rho} + \rho\frac{d\hat{\rho}}{dt} + \dot{z}\hat{z} + z\frac{d\hat{z}}{dt} \\ &= \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z} \\ \mathbf{a} &= \frac{d\mathbf{v}}{dt} \\ &= \ddot{\rho}\hat{\rho} + \dot{\rho}\frac{d\hat{\rho}}{dt} + \dot{\rho}\dot{\phi}\hat{\phi} + \rho\ddot{\phi}\hat{\phi} + \rho\dot{\phi}\frac{d\hat{\phi}}{dt} + \ddot{z}\hat{z} \\ &= \ddot{\rho}\hat{\rho} + \dot{\rho}\dot{\phi}\hat{\phi} + \dot{\rho}\dot{\phi}\hat{\phi} + \rho\ddot{\phi}\hat{\phi} - \rho\dot{\phi}^2\hat{\rho} + \ddot{z}\hat{z} \\ &= (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{\phi} + \ddot{z}\hat{z}\end{aligned}$$