# Introduction to Electrodynamics by David J. Griffiths Problems

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## Contents

# 2 Electrostatics

## 2.1

(a) $\mathbf{0}$

(b) The same as if only the opposite charge were present — all others are cancelled out.

## 2.2

$$\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0} 2 \frac{q}{\mathscr{r}^2} \cos\theta \hat{\mathbf{x}} \\
&= \frac{1}{4\pi\epsilon_0} \frac{dq}{[(d/2)^2 + z^2]^{3/2}} \hat{\mathbf{x}}
\end{aligned}$$

## 2.3

$$
\begin{aligned}
\mathbf{r} &= z\hat{\mathbf{z}} \\
\mathbf{r}' &= x\hat{\mathbf{x}} \\
\boldsymbol{\mathscr{r}} &= z\hat{\mathbf{z}} - x\hat{\mathbf{x}} \\
\mathscr{r} &= \sqrt{x^2 + z^2} \\
\hat{\boldsymbol{\mathscr{r}}} &= \frac{z\hat{\mathbf{z}} - x\hat{\mathbf{x}}}{\sqrt{x^2 + z^2}} \\
\mathbf{E} &= \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda}{x^2 + z^2} \frac{z\hat{\mathbf{z}} - x\hat{\mathbf{x}}}{\sqrt{x^2 + z^2}}\, dx \\
&= \frac{1}{4\pi\epsilon_0}\lambda \left( z\hat{\mathbf{z}} \int_0^L \frac{1}{(x^2 + z^2)^{3/2}}\, dx - \hat{\mathbf{x}} \int_0^L \frac{x}{(x^2 + z^2)}\, dx \right) \\
&= \frac{1}{4\pi\epsilon_0}\lambda \left[ \frac{L}{z\sqrt{L^2 + z^2}}\hat{\mathbf{z}} - \left( \frac{1}{z} - \frac{1}{\sqrt{L^2 + z^2}} \right) \hat{\mathbf{x}} \right] \\
&= \frac{1}{4\pi\epsilon_0} \frac{\lambda}{z} \left[ \left( -1 + \frac{z}{\sqrt{L^2 + z^2}} \right) \hat{\mathbf{x}} + \frac{L}{\sqrt{L^2 + z^2}}\hat{\mathbf{z}} \right]
\end{aligned}
$$

## 2.4

The electric field a distance $z$ above the midpoint of a line segment of length $2L$ and uniform line charge $\lambda$ is

$$
\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z\sqrt{z^2 + L^2}}\hat{\mathbf{z}}.
$$

Applying this to the four sides of the square, the horizontal components of opposite sides cancel leaving only the vertical component.

$$
\begin{aligned}
\cos\theta &= \frac{z}{\mathscr{r}} \\
&= \frac{z}{\sqrt{(a/2)^2 + z^2}} \\
\mathbf{E} &= 4 \left( \frac{1}{4\pi\epsilon_0} \frac{\lambda a}{\sqrt{(a/2)^2 + z^2}\sqrt{(a/2)^2 + (a/2)^2 + z^2}}\hat{\mathbf{z}} \right) \cos\theta \\
&= \frac{1}{4\pi\epsilon_0} \frac{4a\lambda z}{[(a/2)^2 + z^2]\sqrt{(a^2/2) + z^2}}\hat{\mathbf{z}}
\end{aligned}
$$

## 2.5

$$
\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\frac{\lambda r}{r^2+z^2}\cos\alpha\,d\theta\,\hat{\mathbf{z}}\\
&= \frac{1}{4\pi\epsilon_0}\frac{2\pi\lambda r z}{(r^2+z^2)^{3/2}}\hat{\mathbf{z}}
\end{aligned}
$$

## 2.6

$$
\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int\frac{dq}{\mathscr{r}^2}\cos\theta\hat{\mathbf{z}}\\
&= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{R}\frac{\sigma}{r^2+z^2}\frac{z}{\sqrt{r^2+z^2}}r\,dr\,d\theta\hat{\mathbf{z}}\\
&= \frac{1}{4\pi\epsilon_0}2\pi\sigma z\int_0^{R}\frac{r}{(r^2+z^2)^{3/2}}\,dr\,\hat{\mathbf{z}}\\
&= \frac{1}{4\pi\epsilon_0}2\pi\sigma z\left(\frac{1}{z}-\frac{1}{\sqrt{R^2+z^2}}\right)\hat{\mathbf{z}}
\end{aligned}
$$

When $R\to\infty$

$$
\mathbf{E} = \frac{\sigma}{2\epsilon_0}\hat{\mathbf{z}}.
$$

## 2.7

$$
\mathbf{E} = \begin{cases}\frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\hat{\mathbf{z}} & z>R\\ \mathbf{0} & z<R\end{cases}
$$

## 2.8

$$
\mathbf{E} = \begin{cases}\frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\hat{\mathbf{z}} & z>R\\ \frac{1}{4\pi\epsilon_0}\frac{qz}{R^3}\hat{\mathbf{z}} & z<R\end{cases}
$$

## 2.9

(a)

$$
\begin{aligned}
\rho &= \epsilon_0\nabla\cdot\mathbf{E}\\
&= \epsilon_0\frac{1}{r^2}\frac{\partial}{\partial r}(kr^5)\\
&= 5\epsilon_0 k r^2
\end{aligned}
$$

(b)

$$
\begin{aligned}
Q_{\text{enc}} &= \epsilon_0 \oint \mathbf{E} \cdot d\mathbf{a} \\
&= \epsilon_0 \int_0^{2\pi} \int_0^{\pi} kR^3 R \, d\theta R \sin\theta \, d\phi \\
&= 2\pi\epsilon_0 k R^5 [-\cos\theta]_0^{\pi} \\
&= 4\pi\epsilon_0 k R^5 \\
Q_{\text{enc}} &= \int_V \rho \, d\tau \\
&= \int_0^{2\pi} \int_0^{\pi} \int_0^{R} 5\epsilon_0 k r^2 \, dr r \, d\theta r \sin\theta \, d\phi \\
&= 10\pi\epsilon_0 k \int_0^{\pi} \int_0^{R} r^4 \sin\theta \, dr \, d\theta \\
&= 2\pi\epsilon_0 k R^5 [-\cos\theta]_0^{\pi} \\
&= 4\pi\epsilon_0 k R^5
\end{aligned}
$$

## 2.10

If the charge was surrounded by 8 such cubes the total flux through all the cubes would be $q/\epsilon_0$. There are 24 outside faces to the larger cube, so the total flux through the shaded face is $q/(24\epsilon_0)$.

## 2.11

$$
\begin{aligned}
\int \mathbf{E}_{\text{inside}} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
&= 0 \\
\mathbf{E}_{\text{inside}} &= \mathbf{0} \\
\int \mathbf{E}_{\text{outside}} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
4\pi r^2 E_{\text{outside}} &= \frac{4\pi R^2 \sigma}{\epsilon_0} \\
\mathbf{E}_{\text{outside}} &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}
\end{aligned}
$$

## 2.12

$$
\begin{aligned}
\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
4\pi r^2 E &= \frac{\frac{4}{3}\pi r^3 \rho}{\epsilon_0} \\
\mathbf{E} &= \frac{r\rho}{3\epsilon_0}\hat{\mathbf{r}}
\end{aligned}
$$

## 2.13

$$
\begin{aligned}
\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
2\pi s l E &= \frac{l\lambda}{\epsilon_0} \\
\mathbf{E} &= \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}\hat{\mathbf{s}}
\end{aligned}
$$

## 2.14

$$
\begin{aligned}
Q_{\text{enc}} &= \int_V \rho \, d\tau \\
&= \int_0^{2\pi} \int_0^{\pi} \int_0^{r} k r'^3 \sin\theta \, dr' \, d\theta \, d\phi \\
&= 2\pi k \int_0^{\pi} \left[\frac{1}{4} r'^4 \sin\theta\right]_0^r \, d\theta \\
&= \frac{1}{2}\pi k r^4 [-\cos\theta]_0^{\pi} \\
&= \pi k r^4 \\
\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
4\pi r^2 E &= \frac{\pi k r^4}{\epsilon_0} \\
\mathbf{E} &= \frac{k r^2}{4\epsilon_0}\hat{\mathbf{r}}
\end{aligned}
$$

## 2.15

(a) $\mathbf{E} = \mathbf{0}$

(b)

$$\begin{aligned}
Q_{\text{enc}} &= \int_0^{2\pi}\int_0^{\pi}\int_a^{r} k\sin\theta\,dr'\,d\theta\,d\phi \\
&= 4\pi k(r-a) \\
4\pi r^2 E &= \frac{4\pi k(r-a)}{\epsilon_0} \\
\mathbf{E} &= \frac{k(r-a)}{\epsilon_0 r^2}\hat{\mathbf{r}}
\end{aligned}$$

(c) $\mathbf{E} = \frac{k(b-a)}{\epsilon_0 r^2}\hat{\mathbf{r}}$

## 2.16

(a)

$$\begin{aligned}
Q_{\text{enc}} &= \pi s^2 l\rho \\
2\pi s l E &= \frac{\pi s^2 l\rho}{\epsilon_0} \\
\mathbf{E} &= \frac{s\rho}{2\epsilon_0}\hat{\mathbf{s}}
\end{aligned}$$

(b)

$$\mathbf{E} = \frac{a^2\rho}{2\epsilon_0 s}\hat{\mathbf{s}}$$

(c)

$$\mathbf{E} = \mathbf{0}$$

## 2.17

$$\begin{aligned}
2AE_{\text{inside}} &= \frac{2Ay\rho}{\epsilon_0} \\
\mathbf{E}_{\text{inside}} &= \frac{y\rho}{\epsilon_0} \\
\mathbf{E} &= \begin{cases} \frac{d\rho}{\epsilon_0} & d < y \\ \frac{y\rho}{\epsilon_0} & 0 < y < d \\ -\frac{y\rho}{\epsilon_0} & -d < y < 0 \\ -\frac{d\rho}{\epsilon_0} & y < -d \end{cases}
\end{aligned}$$

## 2.18

The electric field inside a uniformly charged solid sphere is

$$\mathbf{E} = \frac{r\rho}{3\epsilon_0}\hat{\mathbf{r}}.$$

$$\begin{aligned}
\mathbf{d} &= \mathbf{r}_1 - \mathbf{r}_2 \\
\mathbf{E} &= \frac{r_1\rho}{3\epsilon_0}\hat{\mathbf{r}}_1 - \frac{r_2\rho}{3\epsilon_0}\hat{\mathbf{r}}_2 \\
&= \frac{\rho}{3\epsilon_0}(\mathbf{r}_1 - \mathbf{r}_2) \\
&= \frac{\rho}{3\epsilon_0}\mathbf{d}
\end{aligned}$$

## 2.20

a is impossible because its curl is nonzero.

$$\begin{aligned}
V &= -\int_0^y 2kxy'\,dy' - \int_0^z 2kyz'\,dz \\
&= -2kx\left[\frac{1}{2}y'^2\right]_0^y - 2ky\left[\frac{1}{2}z'^2\right]_0^z \\
&= -k(xy^2 + yz^2) \\
-\nabla V &= k[y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}] \\
&= \mathbf{E}
\end{aligned}$$

## 2.21

$$
\begin{aligned}
\mathbf{E} &= \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} & r > R \\ \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3} & r < R \end{cases} \\
V_{\text{outside}}(r) &= -\int_\infty^r \frac{1}{4\pi\epsilon_0}\frac{q}{r'^2}\,dr' \\
&= -\frac{1}{4\pi\epsilon_0} q \left[-\frac{1}{r'}\right]_\infty^r \\
&= \frac{1}{4\pi\epsilon_0}\frac{q}{r} \\
-\nabla V_{\text{outside}} &= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \\
&= \mathbf{E}_{\text{outside}} \\
V_{\text{inside}}(r) &= -\left(\int_\infty^R \frac{1}{4\pi\epsilon_0}\frac{q}{r'^2}\,dr' + \int_R^r \frac{1}{4\pi\epsilon_0}\frac{qr'}{R^3}\,dr'\right) \\
&= -\left(-\frac{1}{4\pi\epsilon_0}\frac{q}{R} + \frac{1}{4\pi\epsilon_0}\frac{q}{R^3}\left[\frac{1}{2}r'^2\right]_R^r\right) \\
&= \frac{1}{4\pi\epsilon_0}\frac{q}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right] \\
-\nabla V_{\text{inside}} &= \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3}\hat{\mathbf{r}} \\
&= \mathbf{E}_{\text{inside}}
\end{aligned}
$$

## 2.22

$$
\begin{aligned}
\mathbf{E} &= \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}\hat{\mathbf{s}} \\
V &= -\int_O^s \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s'}\,ds' \\
&= -\frac{1}{2\pi\epsilon_0}\lambda \ln\frac{s}{O} \\
-\nabla V &= \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}\hat{\mathbf{s}}
\end{aligned}
$$

## 2.23

$$
\begin{aligned}
\mathbf{E} &= \begin{cases} \mathbf{0} & r < a \\ \frac{k(r-a)}{\epsilon_0 r^2}\hat{\mathbf{r}} & a < r < b \\ \frac{k(b-a)}{\epsilon_0 r^2}\hat{\mathbf{r}} & b < r \end{cases} \\
V(0) &= -\int_\infty^0 E\,dr \\
&= -\left(\int_\infty^b \frac{k(b-a)}{\epsilon_0 r^2}\,dr + \int_b^a \frac{k(r-a)}{\epsilon_0 r^2}\,dr\right) \\
&= -\left(\frac{k(b-a)}{\epsilon_0}\left[-\frac{1}{r}\right]_\infty^b + \frac{k}{\epsilon_0}\left[\ln r + \frac{a}{r}\right]_b^a\right) \\
&= -\left[-\frac{k(b-a)}{\epsilon_0 b} + \frac{k}{\epsilon_0}\left(\ln a + 1 - \ln b - \frac{a}{b}\right)\right] \\
&= -\frac{k}{\epsilon_0}\left(-1 + \frac{a}{b} + \ln\frac{a}{b} + 1 - \frac{a}{b}\right) \\
&= \frac{k}{\epsilon_0}\ln\frac{b}{a}
\end{aligned}
$$

## 2.24

$$
\begin{aligned}
V(b) - V(0) &= -\int_0^b E\,dr \\
&= -\left(\int_0^a \frac{s\rho}{2\epsilon_0}\,ds + \int_a^b \frac{a^2\rho}{2\epsilon_0 s}\,ds\right) \\
&= -\left(\frac{\rho}{2\epsilon_0}\left[\frac{1}{2}s^2\right]_0^a + \frac{a^2\rho}{2\epsilon_0}\ln\frac{b}{a}\right) \\
&= -\left(\frac{a^2\rho}{4\epsilon_0} + \frac{a^2\rho}{2\epsilon_0}\ln\frac{b}{a}\right) \\
&= -\frac{a^2\rho}{4\epsilon_0}\left(1 + 2\ln\frac{a}{b}\right)
\end{aligned}
$$

## 2.25

(a)

$$
V = \frac{1}{4\pi\epsilon_0}\frac{2q}{\sqrt{(d/2)^2 + z^2}}
$$

(b)

$$
\begin{aligned}
V &= \frac{1}{4\pi\epsilon_0}\int_{-L}^{L}\frac{\lambda}{\sqrt{x^2+z^2}}\,dx \\
&= \frac{1}{4\pi\epsilon_0}\lambda\ln\left(1+\frac{2L(L+\sqrt{L^2+z^2})}{z^2}\right)
\end{aligned}
$$

(c)

$$
\begin{aligned}
V &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{R}\frac{\sigma}{\sqrt{r^2+z^2}}r\,dr\,d\theta \\
&= \frac{1}{4\pi\epsilon_0}2\pi\sigma(\sqrt{R^2+z^2}-z)
\end{aligned}
$$

## 2.26

$$
\begin{aligned}
V_{\text{bottom}} &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{h}\frac{\sqrt{2}\sigma z}{\sqrt{2}z}\,d\phi\,dz \\
&= \frac{\sigma h}{2\epsilon_0} \\
V_{\text{top}} &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{h}\frac{\sqrt{2}\sigma z}{\sqrt{z^2+(h-z)^2}}\,d\phi\,dz \\
&= \frac{\sqrt{2}\sigma}{2\epsilon_0}\int_0^{h}\frac{z}{\sqrt{z^2+(h-z)^2}}\,dz \\
&= \frac{\sigma h}{4\epsilon_0}\ln(3+2\sqrt{2}) \\
V_{\text{bottom}}-V_{\text{top}} &= \frac{\sigma h}{2\epsilon_0}\left[1-\frac{1}{2}\ln(3+2\sqrt{2})\right]
\end{aligned}
$$

## 2.28

$$
\begin{aligned}
V(r) &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^{R}\frac{\rho r'^2\sin\theta}{\sqrt{r^2+r'^2-2rr'\cos\theta}}\,dr'\,d\theta\,d\phi \\
&= \frac{\rho}{2\epsilon_0}\int_0^{\pi}\int_0^{R}\frac{r'^2\sin\theta}{\sqrt{r^2+r'^2-2rr'\cos\theta}}\,dr'\,d\theta \\
&= \frac{\rho}{2\epsilon_0}\left(R^2-\frac{r^2}{3}\right) \\
&= \frac{q}{8\pi\epsilon_0 R}\left(3-\frac{r^2}{R^2}\right)
\end{aligned}
$$

## 2.31

(a)

$$W = \frac{q^2}{4\pi\epsilon_0 a}\left(\frac{1}{\sqrt{2}} - 2\right)$$

(b)

$$\begin{aligned}
W &= \frac{1}{4\pi\epsilon_0}\left(-\frac{q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a}\right)\\
&= \frac{q^2}{2\pi\epsilon_0 a}\left(\frac{1}{\sqrt{2}} - 2\right)
\end{aligned}$$

## 2.32

$$\begin{aligned}
W &= \frac{1}{4\pi\epsilon_0}\frac{q_A q_B}{a}\\
W &= K_1 + K_2\\
\frac{1}{4\pi\epsilon_0}\frac{q_A q_B}{a} &= \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2\\
\frac{1}{2\pi\epsilon_0}\frac{q_A q_B}{a} &= m_A v_A^2 + m_B v_B^2\\
0 &= m_B v_B - m_A v_A\\
v_B &= \frac{m_A}{m_B}v_A\\
\frac{1}{2\pi\epsilon_0}\frac{q_A q_B}{a} &= m_A v_A^2 + m_B\left(\frac{m_A}{m_B}v_A\right)^2\\
&= m_A v_A^2 + \frac{m_A^2}{m_B}v_A^2\\
&= \frac{m_A(m_A + m_B)}{m_B}v_A^2\\
v_A &= \sqrt{\frac{1}{2\pi\epsilon_0}\frac{q_A q_B}{(m_A + m_B)a}\frac{m_B}{m_A}}\\
v_B &= \sqrt{\frac{1}{2\pi\epsilon_0}\frac{q_A q_B}{(m_A + m_B)a}\frac{m_A}{m_B}}
\end{aligned}$$

## 2.33

$$
\begin{aligned}
W &= \frac{1}{4\pi\epsilon_0}\left(-\frac{q^2}{a}+\frac{q^2}{2a}-\frac{q^2}{3a}+\dots\right)\\
&= \frac{1}{4\pi\epsilon_0}\frac{q^2}{a}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\\
&= -\frac{1}{4\pi\epsilon_0}\frac{q^2}{a}\ln 2
\end{aligned}
$$

## 2.34

(a)

$$
\begin{aligned}
V &= \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{2R}\left[3-\left(\frac{r}{R}\right)^2\right] & r<R \\ \frac{1}{4\pi\epsilon_0}\frac{q}{r} & r>R \end{cases}\\
W &= \frac{1}{2}\int \rho V\, d\tau\\
&= \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_0^{R}\rho\frac{1}{4\pi\epsilon_0}\frac{q}{2R}\left[3-\left(\frac{r}{R}\right)^2\right]r^2\sin\theta\, dr\, d\theta\, d\phi\\
&= \frac{q\rho}{8\epsilon_0 R}\int_0^{\pi}\int_0^{R}\left[3-\left(\frac{r}{R}\right)^2\right]r^2\sin\theta\, dr\, d\theta\\
&= \frac{q\rho R^2}{5\epsilon_0}\\
&= \frac{qR^2}{5\epsilon_0}\frac{q}{\frac{4}{3}\pi R^3}\\
&= \frac{1}{4\pi\epsilon_0}\frac{3q^2}{5R}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\mathbf{E} &= \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} & r > R \\ \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3}\hat{\mathbf{r}} & r < R \end{cases} \\
E^2 &= \begin{cases} \frac{1}{16\pi^2\epsilon_0^2}\frac{q^2}{r^4} & r > R \\ \frac{1}{16\pi^2\epsilon_0^2}\frac{q^2r^2}{R^6} & r < R \end{cases} \\
W &= \frac{\epsilon_0}{2}\int E^2\,d\tau \\
&= \frac{\epsilon_0}{2}\left(\int_0^{2\pi}\int_0^{\pi}\int_0^{R}\frac{1}{16\pi^2\epsilon_0^2}\frac{q^2r^2}{R^6}r^2\sin\theta\,dr\,d\theta\,d\phi\right. \\
&\qquad \left.+\int_0^{2\pi}\int_0^{\pi}\int_R^{\infty}\frac{1}{16\pi^2\epsilon_0^2}\frac{q^2}{r^4}r^2\sin\theta\,dr\,d\theta\,d\phi\right) \\
&= \frac{\epsilon_0}{2}\frac{1}{16\pi^2\epsilon_0^2}2\pi q^2\left(\int_0^{\pi}\int_0^{R}\frac{r^4}{R^6}\sin\theta\,dr\,d\theta+\int_0^{\pi}\int_R^{\infty}\frac{1}{r^2}\sin\theta\,dr\,d\theta\right) \\
&= \frac{1}{16\pi\epsilon_0}q^2\left(\int_0^{\pi}\int_0^{R}\frac{r^4}{R^6}\sin\theta\,dr\,d\theta+\int_0^{\pi}\int_R^{\infty}\frac{1}{r^2}\sin\theta\,dr\,d\theta\right) \\
&= \frac{1}{16\pi\epsilon_0}q^2\left(\frac{2}{5R}+\frac{2}{R}\right) \\
&= \frac{1}{4\pi\epsilon_0}\frac{3q^2}{5R}
\end{aligned}
$$

(c)

$$
\begin{aligned}
W &= \frac{\epsilon_0}{2}\left(\int_V E^2\,d\tau + \oint_S V\mathbf{E}\cdot d\mathbf{a}\right) \\
&= \frac{\epsilon_0}{2}\left(\int_0^{2\pi}\int_0^{\pi}\int_0^{R} \frac{1}{(4\pi\epsilon_0)^2}\frac{q^2r^2}{R^6}r^2\sin\theta\,dr\,d\theta\,d\phi\right. \\
&\qquad + \int_0^{2\pi}\int_0^{\pi}\int_R^{a} \frac{1}{(4\pi\epsilon_0)^2}\frac{q^2}{r^4}r^2\sin\theta\,dr\,d\theta\,d\phi \\
&\qquad \left. + \int_0^{2\pi}\int_0^{\pi} \frac{1}{4\pi\epsilon_0}\frac{q}{a}\frac{1}{4\pi\epsilon_0}\frac{q}{a^2}a^2\sin\theta\,d\theta\,d\phi\right) \\
&= \frac{\epsilon_0}{2}\frac{1}{(4\pi\epsilon_0)^2}2\pi q^2\left(\int_0^{\pi}\int_0^{R}\frac{r^4}{R^6}\sin\theta\,dr\,d\theta\right. \\
&\qquad \left. + \int_0^{\pi}\int_R^{a}\frac{1}{r^2}\sin\theta\,dr\,d\theta + \int_0^{\pi}\frac{1}{a}\sin\theta\,d\theta\right) \\
&= \frac{\epsilon_0}{2}\frac{1}{(4\pi\epsilon_0)^2}2\pi q^2\left[\frac{2}{5R} + 2\left(\frac{1}{R}-\frac{1}{a}\right)+\frac{2}{a}\right] \\
&= \frac{1}{8\pi\epsilon_0}q^2\left[\frac{1}{5R}+\frac{1}{R}\right] \\
&= \frac{1}{4\pi\epsilon_0}\frac{3q^2}{5R}
\end{aligned}
$$

## 2.36

(a)

$$
\mathbf{E} = \begin{cases} \mathbf{0} & r < a \\ \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} & a < r < b \\ \mathbf{0} & b < r \end{cases}
$$

$$
E^2 = \begin{cases} 0 & r < a \\ \frac{1}{(4\pi\epsilon_0)^2}\frac{q^2}{r^4} & a < r < b \\ 0 & b < r \end{cases}
$$

$$
\begin{aligned}
W &= \frac{\epsilon_0}{2}\int E^2\,d\tau \\
&= \frac{\epsilon_0}{2}\int_0^{2\pi}\int_0^{\pi}\int_a^{b}\frac{1}{(4\pi\epsilon_0)^2}\frac{q^2}{r^4}r^2\sin\theta\,dr\,d\theta\,d\phi \\
&= \frac{\epsilon_0}{2}\frac{1}{(4\pi\epsilon_0)^2}2\pi q^2\int_0^{\pi}\int_a^{b}\frac{\sin\theta}{r^2}\,dr\,d\theta \\
&= \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)
\end{aligned}
$$

(b)

$$
\begin{aligned}
W_{\text{shell}} &= \frac{1}{8\pi\epsilon_0}\frac{q^2}{R} \\
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \\
\mathbf{E}_1 \cdot \mathbf{E}_2 &= -\frac{1}{(4\pi\epsilon_0)^2}\frac{q^2}{r^4} \\
W_{\text{total}} &= W_1 + W_2 + \epsilon_0 \int \mathbf{E}_1 \cdot \mathbf{E}_2 \, d\tau \\
&= \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} + \frac{1}{b}\right) - \epsilon_0 \int_0^{2\pi}\int_0^{\pi}\int_b^{\infty} \frac{1}{(4\pi\epsilon_0)^2}\frac{q^2}{r^4} r^2 \sin\theta \, dr \, d\theta \, d\phi \\
&= \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} + \frac{1}{b}\right) - \frac{1}{8\pi\epsilon_0} q^2 \int_0^{\pi}\int_b^{\infty} \frac{1}{r^2} \sin\theta \, dr \, d\theta \\
&= \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} + \frac{1}{b}\right) - \frac{1}{4\pi\epsilon_0} q^2 \int_b^{\infty} \frac{1}{r^2} \, dr \\
&= \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} + \frac{1}{b}\right) - \frac{1}{4\pi\epsilon_0}\frac{q^2}{b} \\
&= \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{b}\right) \\
&= \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)
\end{aligned}
$$

## 2.37

$$\begin{aligned}
r_1 &= r \\
E_1 &= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2} \\
&= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2} \\
r_2 &= \sqrt{a^2 + r^2 - 2ar\cos\theta} \\
E_2 &= \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2^2} \\
&= \frac{1}{4\pi\epsilon_0}\frac{q_2}{a^2 + r^2 - 2ar\cos\theta} \\
\cos\alpha &= \frac{r - a\cos\theta}{\sqrt{a^2 + r^2 - 2ar\cos\theta}} \\
\mathbf{E}_1 \cdot \mathbf{E}_2 &= E_1 E_2 \cos\alpha \\
&= \frac{1}{(4\pi\epsilon_0)^2}\frac{q_1 q_2}{r^2(a^2 + r^2 - 2ar\cos\theta)}\frac{r - a\cos\theta}{\sqrt{a^2 + r^2 - 2ar\cos\theta}} \\
&= \frac{1}{(4\pi\epsilon_0)^2}\frac{q_1 q_2 (r - a\cos\theta)}{r^2(a^2 + r^2 - 2ar\cos\theta)^{3/2}} \\
\epsilon_0 \int \mathbf{E}_1 \cdot \mathbf{E}_2 \, d\tau &= \epsilon_0 \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \frac{1}{(4\pi\epsilon_0)^2}\frac{q_1 q_2 (r - a\cos\theta)}{r^2(a^2 + r^2 - 2ar\cos\theta)^{3/2}} r^2 \sin\theta \, dr\, d\theta\, d\phi \\
&= \frac{q_1 q_2}{8\pi\epsilon_0}\int_0^{\pi}\int_0^{\infty} \frac{(r - a\cos\theta)\sin\theta}{(a^2 + r^2 - 2ar\cos\theta)^{3/2}} \, dr\, d\theta
\end{aligned}$$

## 2.38

(a)

$$\begin{aligned}
\sigma_R &= \frac{q}{4\pi R^2} \\
\sigma_a &= -\frac{q}{4\pi a^2} \\
\sigma_b &= \frac{q}{4\pi b^2}
\end{aligned}$$

(b)

$$\begin{aligned}
V &= -\int_\infty^b \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\, dr - \int_a^R \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\, dr \\
&= \frac{1}{4\pi\epsilon_0} q \left(\frac{1}{b} + \frac{1}{R} - \frac{1}{a}\right)
\end{aligned}$$

(c)

$$\sigma_b = 0$$
$$V = \frac{1}{4\pi\epsilon_0} q \left( \frac{1}{R} - \frac{1}{a} \right)$$

## 2.39

(a)

$$\sigma_a = -\frac{q_a}{4\pi a^2}$$
$$\sigma_b = -\frac{q_b}{4\pi b^2}$$
$$\sigma_R = \frac{q_a + q_b}{4\pi R^2}$$

(b)

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q_a + q_b}{r^2} \hat{\mathbf{r}}$$

(c)

$$\mathbf{E}_a = \frac{1}{4\pi\epsilon_0} \frac{q_a}{r^2} \hat{\mathbf{r}}$$
$$\mathbf{E}_b = \frac{1}{4\pi\epsilon_0} \frac{q_b}{r^2} \hat{\mathbf{r}}$$

(d)

$$\mathbf{0}$$

(e) a, b

## 2.40

(a) No. If it's close to the wall it will induce a surface charge and be attracted.

(b) No. If the conductor contains a cavity containing a like charge it will be repelled.

## 2.41

By Gauss's law, the electric field of each plate is

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$$
$$2A'E = \frac{A' \frac{Q}{A}}{\epsilon_0}$$
$$\mathbf{E} = \frac{Q}{2A\epsilon_0} \hat{\mathbf{n}}$$

so the field between the plates is zero and the field outside is $Q/A\epsilon_0\hat{\mathbf{n}}$, resulting in a pressure of

$$\begin{aligned} P &= \frac{\epsilon_0}{2}E^2 \\ &= \frac{\epsilon_0}{2}\frac{Q^2}{A^2\epsilon_0^2} \\ &= \frac{Q^2}{2A^2\epsilon_0} \end{aligned}$$

## 2.42

$$\begin{aligned} \mathbf{E}_{\text{above}} &= \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \\ \mathbf{f} &= \frac{1}{2}\sigma\mathbf{E}_{\text{above}} \\ &= \frac{1}{2}\frac{Q}{4\pi R^2}\frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\hat{\mathbf{r}} \\ &= \frac{Q^2}{32\pi^2\epsilon_0 R^4}\hat{\mathbf{r}} \\ \mathbf{F} &= \int_0^{2\pi}\int_0^{\pi/2}\frac{Q^2}{32\pi^2\epsilon_0 R^4}\cos\theta R^2\sin\theta\, d\theta\, d\phi\hat{\mathbf{z}} \\ &= \frac{Q^2}{16\pi\epsilon_0 R^2}\int_0^{\pi/2}\cos\theta\sin\theta\, d\theta\hat{\mathbf{z}} \\ &= \frac{Q^2}{32\pi\epsilon_0 R^2}\hat{\mathbf{z}} \end{aligned}$$

## 2.43

$$\begin{aligned}
\oint \mathbf{E} \cdot d\mathbf{a} &= \frac{Q}{\epsilon_0} \\
2\pi s L E &= \frac{Q}{\epsilon_0} \\
\mathbf{E} &= \frac{Q}{2\pi L \epsilon_0} \frac{1}{s} \hat{\mathbf{s}} \\
V &= -\int_b^a \frac{Q}{2\pi\epsilon_0 L} \frac{1}{s} \, dr \\
&= \frac{Q}{2\pi\epsilon_0 L} \ln \frac{b}{a} \\
C &= \frac{Q}{V} \\
&= \frac{2\pi\epsilon_0 L}{\ln b/a}
\end{aligned}$$

So the capacitance per unit length is

$$C = \frac{2\pi\epsilon_0}{\ln b/a}.$$

## 2.44

(a)

$$\begin{aligned}
P &= \frac{\epsilon_0}{2} E^2 \\
W &= Fd \\
&= PA\epsilon \\
&= \frac{\epsilon_0}{2} E^2 A\epsilon
\end{aligned}$$

(b)

$$\frac{\epsilon_0}{2} E^2 A\epsilon$$