Introduction to Quantum Mechanics by David J. Griffiths Problems

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Contents

Part I
Theory

1 The Wave Function

1.1

(a)

$$
\begin{aligned}
\langle j^2 \rangle &= \sum j^2 P(j) \\
&= 14^2 \frac{1}{14} + 15^2 \frac{1}{14} + 16^2 \frac{3}{14} + 22^2 \frac{2}{14} + 24^2 \frac{2}{14} + 25^2 \frac{5}{14} \\
&= \frac{3217}{7} \\
&\approx 459.571 \\
\langle j \rangle^2 &= \left(\sum j P(j) \right)^2 \\
&= 441
\end{aligned}
$$

(b)

$$\begin{aligned}
\Delta j_{14} &= -7 \\
\Delta j_{15} &= -6 \\
\Delta j_{16} &= -5 \\
\Delta j_{22} &= 1 \\
\Delta j_{24} &= 3 \\
\Delta j_{25} &= 4 \\
\sigma^2 &= \sum (\Delta j)^2 P(j) \\
&= \frac{130}{7} \\
&\approx 18.571
\end{aligned}$$

(c)

$$\sigma^2 = \sqrt{\langle j^2 \rangle - \langle j \rangle^2} = 18.571$$

1.2

(a)

$$\begin{aligned}
\langle x^2 \rangle &= \int_0^h x^2 \rho(x)\, dx \\
&= \int_0^h \frac{x^{3/2}}{2\sqrt{h}}\, dx \\
&= \frac{1}{2\sqrt{h}} \left[\frac{2}{5} x^{5/2} \right]_0^h \\
&= \frac{h^2}{5} \\
\langle x \rangle^2 &= \frac{h^2}{9} \\
\sigma &= \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \\
&= \sqrt{\frac{h^2}{5} - \frac{h^2}{9}} \\
&= h\sqrt{\frac{4}{45}} \\
&= \frac{2}{3\sqrt{5}} h
\end{aligned}$$

(b)

$$
\begin{aligned}
1 - \int_{\langle x\rangle-\sigma}^{\langle x\rangle+\sigma} \rho(x)\,dx &= 1 - \frac{1}{2\sqrt{h}}[2\sqrt{x}]_{\langle x\rangle-\sigma}^{\langle x\rangle+\sigma} \\
&= 1 - \frac{1}{\sqrt{h}}\left(\sqrt{\frac{1}{3}h + \frac{2}{3\sqrt{5}}h} - \sqrt{\frac{1}{3}h - \frac{2}{3\sqrt{5}}h}\right) \\
&= 1 - \left(\sqrt{\frac{1}{3} + \frac{2}{3\sqrt{5}}} - \sqrt{\frac{1}{3} - \frac{2}{3\sqrt{5}}}\right) \\
&\approx 0.393
\end{aligned}
$$

1.3

(a)

$$
\begin{aligned}
\rho(x) &= Ae^{-\lambda(x-a)^2} \\
1 &= \int_{-\infty}^{\infty} \rho(x)\,dx \\
&= A\int_{-\infty}^{\infty} e^{-\lambda(x-a)^2}\,dx \\
&= A\sqrt{\frac{\pi}{\lambda}} \\
A &= \sqrt{\frac{\lambda}{\pi}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\langle x\rangle &= \sqrt{\frac{\lambda}{\pi}}\int_{-\infty}^{\infty} xe^{-\lambda(x-a)^2}\,dx \\
&= a \\
\langle x^2\rangle &= \sqrt{\frac{\lambda}{\pi}}\int_{-\infty}^{\infty} x^2e^{-\lambda(x-a)^2}\,dx \\
&= a^2 + \frac{1}{2\lambda} \\
\sigma &= \sqrt{\langle x^2\rangle - \langle x\rangle^2} \\
&= \sqrt{a^2 + \frac{1}{2\lambda} - a^2} \\
&= \frac{1}{\sqrt{2\lambda}}
\end{aligned}
$$