# Classical Mechanics by John R. Taylor Problems

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## Contents

# 1 Newton's Laws of Motion

## 1.1

$$
\begin{aligned}
\mathbf{b}+\mathbf{c} &= 2\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}} \\
5\mathbf{b}+2\mathbf{x} &= 7\hat{\mathbf{x}}+5\hat{\mathbf{y}}+2\hat{\mathbf{z}} \\
\mathbf{b}\cdot\mathbf{c} &= 1 \\
\mathbf{b}\times\mathbf{c} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \\
&= \hat{\mathbf{x}}-\hat{\mathbf{y}}-\hat{\mathbf{z}}
\end{aligned}
$$

## 1.5

$$
\begin{aligned}
\mathbf{v}_{\text{body}} &= \hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}} \\
\mathbf{v}_{\text{face}} &= \hat{\mathbf{x}}+\hat{\mathbf{z}} \\
\mathbf{v}_{\text{body}}\cdot\mathbf{v}_{\text{face}} &= v_{\text{body}}v_{\text{face}}\cos\theta \\
2 &= \sqrt{6}\cos\theta \\
\cos\theta &= \frac{2}{\sqrt{6}} \\
\theta &= \arccos\frac{2}{\sqrt{6}} \\
&= 35.26^\circ
\end{aligned}
$$

## 1.11

The particle moves counterclockwise in an ellipse of width $2b$ and height $2c$. The angular speed is $\omega$.

## 1.23

$$
\begin{aligned}
\mathbf{v} &= v\cos\theta\frac{\mathbf{b}}{b}-v\sin\theta\frac{\mathbf{b}\times\mathbf{c}}{bc} \\
&= \frac{\lambda}{b}\frac{\mathbf{b}}{b}-\frac{c}{b}\frac{\mathbf{b}\times\mathbf{c}}{bc} \\
&= \frac{\lambda\mathbf{b}-\mathbf{b}\times\mathbf{c}}{b^2}
\end{aligned}
$$

### 1.25

$$
\begin{aligned}
\frac{df}{dt} &= -3f \\
\frac{1}{f}\frac{df}{dt} &= -3 \\
\ln f &= -3t + c \\
f &= ce^{-3t}
\end{aligned}
$$

One constant.

## 1.35

$$\begin{aligned}
F_x &= 0 \\
ma_x &= 0 \\
a_x &= 0 \\
v_x &= c_1 \\
&= v_o \cos\theta \\
r_x &= v_o \cos(\theta) t + c_2 \\
&= v_o \cos(\theta) t
\end{aligned}$$

$$\begin{aligned}
F_y &= 0 \\
ma_y &= 0 \\
a_y &= 0 \\
v_y &= c_3 \\
v_y &= 0 \\
r_y &= c_4 \\
r_y &= 0
\end{aligned}$$

$$\begin{aligned}
F_z &= -mg \\
ma_z &= -mg \\
a_z &= -g \\
v_z &= -gt + c_5 \\
&= v_o \sin\theta - gt \\
r_z &= v_o \sin(\theta) t - \frac{1}{2} g t^2 + c_6 \\
&= v_o \sin(\theta) t - \frac{1}{2} g t^2
\end{aligned}$$

$$\begin{aligned}
0 &= v_o \sin(\theta) t - \frac{1}{2} g t^2 \\
t &= \frac{2\sin(\theta) v_o}{g} \\
r_x &= v_o \cos(\theta) t \\
&= \frac{2\cos(\theta)\sin(\theta) v_o^2}{g} \\
&= \frac{\sin(2\theta) v_o^2}{g}
\end{aligned}$$

## 1.37

(a)

$$\begin{aligned} F &= -mg\sin\theta \\ ma &= -mg\sin\theta \\ a &= -g\sin\theta \\ v &= c_1 - gt\sin\theta \\ &= v_o - gt\sin\theta \\ x &= v_o t - \frac{1}{2}gt^2\sin\theta \end{aligned}$$

(b)

$$t = \frac{2v_o}{g\sin\theta}$$

## 1.39

$$
\begin{aligned}
F_x &= -mg\sin\phi \\
ma_x &= -mg\sin\phi \\
a_x &= -g\sin\phi \\
v_x &= c_1 - gt\sin\phi \\
&= v_o\cos\theta - gt\sin\phi \\
r_x &= v_o t\cos\theta - \frac{1}{2}gt^2\sin\phi + c_2 \\
&= v_o t\cos\theta - \frac{1}{2}gt^2\sin\phi
\end{aligned}
$$

$$
\begin{aligned}
F_y &= -mg\cos\phi \\
ma_y &= -mg\cos\phi \\
a_y &= -g\cos\phi \\
v_y &= c_3 - gt\cos\phi \\
&= v_o\sin\theta - gt\cos\phi \\
r_y &= v_o t\sin\theta - \frac{1}{2}gt^2\cos\phi + c_4 \\
&= v_o t\sin\theta - \frac{1}{2}gt^2\cos\phi
\end{aligned}
$$

$$
\begin{aligned}
0 &= v_o t\sin\theta - \frac{1}{2}gt^2\cos\phi \\
t &= \frac{2v_o\sin\theta}{g\cos\phi}
\end{aligned}
$$

$$
\begin{aligned}
r_x &= \frac{2v_o^2\cos\theta\sec\phi\sin\theta}{g} - \frac{2v_o^2\sec\phi\sin^2\theta\tan\phi}{g} \\
&= \frac{2v_o^2\sin\theta(\cos\theta\cos\phi - \sin\theta\sin\phi)}{g\cos^2\phi} \\
&= \frac{2v_o^2\sin\theta\cos(\theta+\phi)}{g\cos^2\phi}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dr_x}{d\theta} &= \frac{2v_o^2}{g\cos^2\phi}[\cos\theta\cos(\theta+\phi) - \sin\theta\sin(\theta+\phi)] \\
&= \frac{2v_o^2\cos(2\theta+\phi)}{g\cos^2\phi} \\
0 &= \frac{2v_o^2\cos(2\theta+\phi)}{g\cos^2\phi} \\
&= \cos(2\theta+\phi) \\
2\theta+\phi &= \frac{\pi}{2} \\
\theta &= \frac{\pi}{4} - \frac{\phi}{2} \\
r_{x,\max} &= \frac{2v_o^2\sin\left(\frac{\pi}{4}-\frac{\phi}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\phi}{2}+\phi\right)}{g\cos^2\phi} \\
&= \frac{v_o^2(1-\sin\phi)}{g\cos^2\phi} \\
&= \frac{v_o^2}{g(1+\sin\phi)}
\end{aligned}
$$

## 1.41

$$
\begin{aligned}
F &= ma \\
T &= m\frac{v^2}{R} \\
&= m\frac{(\omega R)^2}{R} \\
&= m\omega^2 R
\end{aligned}
$$

## 1.47

(a)

$$
\begin{aligned}
\rho &= \sqrt{x^2+y^2} \\
\phi &= \arctan\frac{y}{x} \\
z &= z
\end{aligned}
$$

$\rho$ is the distance of $P$ from the $z$-axis.

The use of $r$ may be unfortunate because it suggests it's the distance of $P$ from the origin.

(b) $\hat{\boldsymbol{\rho}}$ points away from the $z$-axis, $\hat{\boldsymbol{\phi}}$ points counter-clockwise around the $z$-axis, and $\hat{\mathbf{z}}$ points in the positive $z$ direction.

$$\mathbf{r} = \rho\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}} + \sqrt{x^2 + y^2}\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}}$$

(c)

$$\begin{aligned}
\mathbf{v} &= \frac{d\mathbf{r}}{dt} \\
&= \dot{\rho}\hat{\boldsymbol{\rho}} + \rho\frac{d\hat{\boldsymbol{\rho}}}{dt} + \dot{z}\hat{\mathbf{z}} + z\frac{d\hat{\mathbf{z}}}{dt} \\
&= \dot{\rho}\hat{\boldsymbol{\rho}} + \rho\dot{\phi}\hat{\boldsymbol{\phi}} + \dot{z}\hat{\mathbf{z}} \\
\mathbf{a} &= \frac{d\mathbf{v}}{dt} \\
&= \ddot{\rho}\hat{\boldsymbol{\rho}} + \dot{\rho}\frac{d\hat{\boldsymbol{\rho}}}{dt} + \dot{\rho}\dot{\phi}\hat{\boldsymbol{\phi}} + \rho\ddot{\phi}\hat{\boldsymbol{\phi}} + \rho\dot{\phi}\frac{d\hat{\boldsymbol{\phi}}}{dt} + \ddot{z}\hat{\mathbf{z}} \\
&= \ddot{\rho}\hat{\boldsymbol{\rho}} + \dot{\rho}\dot{\phi}\hat{\boldsymbol{\phi}} + \dot{\rho}\dot{\phi}\hat{\boldsymbol{\phi}} + \rho\ddot{\phi}\hat{\boldsymbol{\phi}} - \rho\dot{\phi}^2\hat{\boldsymbol{\rho}} + \ddot{z}\hat{\mathbf{z}} \\
&= (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\boldsymbol{\rho}} + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{\boldsymbol{\phi}} + \ddot{z}\hat{\mathbf{z}}
\end{aligned}$$

# 2 Projectiles and Charged Particles

## 2.1

$$\begin{aligned}
1 &= (1.6 \times 10^3)Dv \\
v &= \frac{1}{(1.6 \times 10^3)D} \\
&= 8.9\,\mathrm{mm/s}
\end{aligned}$$

When $v \gg 1\,\mathrm{cm/s}$ the drag force can be treated as purely quadratic. For a beach ball this becomes $v \gg 1\,\mathrm{mm/s}$.

## 2.3

(a)

$$\begin{aligned}
\frac{f_{\text{quad}}}{f_{\text{lin}}} &= \frac{(1/4)\rho A v^2}{3\pi\eta D v} \\
&= \frac{\rho\pi\left(\frac{D}{2}\right)^2 v}{12\pi\eta D} \\
&= \frac{\rho D v}{48\eta} \\
&= \frac{R}{48}
\end{aligned}$$

(b)

$$R = \frac{Dv\rho}{\eta} \approx 0.01$$

## 2.5

$$\begin{aligned}
v_y(t) &= v_{\text{ter}} + (v_{\text{yo}} - v_{\text{ter}})e^{-t/\tau} \\
&= v_{\text{ter}} + (2v_{\text{ter}} - v_{\text{ter}})e^{-t/\tau} \\
&= v_{\text{ter}}(1 + e^{-t/\tau})
\end{aligned}$$

The velocity starts at $2v_{\text{ter}}$ and asymptotically approaches $v_{\text{ter}}$.

## 2.7

$$\begin{aligned}
F &= F(v) \\
m\dot{v} &= F(v) \\
m\frac{dv}{F(v)} &= dt \\
t &= \int_{v_\text{o}}^{v} m\frac{dv'}{F(v')}
\end{aligned}$$

$$\begin{aligned}
F &= F(v) \\
m\dot{v} &= F_\text{o} \\
v &= \frac{F_\text{o}}{m}t + c
\end{aligned}$$

## 2.11

(a)

$$
\begin{aligned}
m\dot{v} &= -mg - bv \\
\dot{v} &= -g - kv \\
\frac{1}{-g-kv}\dot{v} &= 1 \\
-\frac{1}{k}\ln(-g-kv) &= t + c \\
\ln(-g-kv) &= c - \frac{t}{\tau} \\
-g - kv &= Ae^{-t/\tau} \\
v &= \tau(-g - Ae^{-t/\tau}) \\
&= -v_{\text{ter}} - \tau Ae^{-t/\tau}
\end{aligned}
$$

$$
\begin{aligned}
v_{\text{o}} &= -v_{\text{ter}} - \tau A \\
A &= -k(v_{\text{o}} + v_{\text{ter}})
\end{aligned}
$$

$$
v = -v_{\text{ter}} + (v_{\text{o}} + v_{\text{ter}})e^{-t/\tau}
$$

$$
y = -v_{\text{ter}}t - \tau(v_{\text{o}} + v_{\text{ter}})e^{-t/\tau} + c
$$

$$
\begin{aligned}
0 &= -\tau(v_{\text{o}} + v_{\text{ter}}) + c \\
c &= \tau(v_{\text{o}} + v_{\text{ter}})
\end{aligned}
$$

$$
y = -v_{\text{ter}}t + \tau(v_{\text{o}} + v_{\text{ter}})(1 - e^{-t/\tau})
$$

(b)

$$
\begin{aligned}
0 &= -v_{\text{ter}} + (v_{\text{o}} + v_{\text{ter}})e^{-t/\tau} \\
e^{-t/\tau} &= \frac{v_{\text{ter}}}{v_{\text{o}} + v_{\text{ter}}} \\
-\frac{t}{\tau} &= \ln \frac{v_{\text{ter}}}{v_{\text{o}} + v_{\text{ter}}} \\
t &= -\tau \ln \frac{v_{\text{ter}}}{v_{\text{o}} + v_{\text{ter}}}
\end{aligned}
$$

$$
\begin{aligned}
y_{\max} &= -v_{\text{ter}} \left( -\tau \ln \frac{v_{\text{ter}}}{v_{\text{o}} + v_{\text{ter}}} \right) + \tau (v_{\text{o}} + v_{\text{ter}}) \left( 1 - \frac{v_{\text{ter}}}{v_{\text{o}} + v_{\text{ter}}} \right) \\
&= \tau v_{\text{ter}} \ln \frac{v_{\text{ter}}}{v_{\text{o}} + v_{\text{ter}}} + \tau (v_{\text{o}} + v_{\text{ter}} - v_{\text{ter}}) \\
&= \tau \left( v_{\text{o}} + v_{\text{ter}} \ln \frac{v_{\text{ter}}}{v_{\text{o}} + v_{\text{ter}}} \right) \\
&= \tau \left[ v_{\text{o}} - v_{\text{ter}} \ln \left( 1 + \frac{v_{\text{o}}}{v_{\text{ter}}} \right) \right]
\end{aligned}
$$

(c)

$$
\begin{aligned}
y_{\max} &= \tau \left[ v_{\text{o}} - v_{\text{ter}} \ln \left( 1 + \frac{v_{\text{o}}}{v_{\text{ter}}} \right) \right] \\
&= \tau \left[ v_{\text{o}} - g\tau \ln \left( 1 + \frac{v_{\text{o}}}{g\tau} \right) \right] \\
&\approx \tau \left\{ v_{\text{o}} - g\tau \left[ \frac{v_{\text{o}}}{g\tau} - \frac{1}{2} \left( \frac{v_{\text{o}}}{g\tau} \right)^2 \right] \right\} \\
&= \tau \left( v_{\text{o}} - v_{\text{o}} + \frac{1}{2} \frac{v_{\text{o}}^2}{g\tau} \right) \\
&= \frac{1}{2} \frac{v_{\text{o}}^2}{g}
\end{aligned}
$$

## 2.13

$$\begin{aligned}
v^2 &= \frac{2}{m}\int_{x_0}^{x} -kx'\,dx' \\
&= -\frac{2k}{m}\left(\frac{1}{2}x^2 - \frac{1}{2}x_0^2\right) \\
&= -\frac{k}{m}(x^2 - x_0^2) \\
v &= \sqrt{\frac{k}{m}(x_0^2 - x^2)} \\
&= \omega\sqrt{x_0^2 - x^2}
\end{aligned}$$

$$\begin{aligned}
\int_{x_0}^{x} \frac{1}{\sqrt{x_0^2 - x'^2}}\,dx' &= \int_0^t \omega\,dt \\
\arctan\frac{x}{\sqrt{x_0^2 - x^2}} - \arctan\frac{x_0}{\sqrt{x_0^2 - x_0^2}} &= \omega t \\
\arctan\frac{x}{\sqrt{x_0^2 - x^2}} &= \omega t + \frac{\pi}{2} \\
\frac{x}{\sqrt{x_0^2 - x^2}} &= \tan\left(\omega t + \frac{\pi}{2}\right) \\
&= -\cot\omega t \\
\frac{\sqrt{x_0^2 - x^2}}{x} &= -\tan\omega t \\
\sqrt{x_0^2 - x^2} &= -x\tan\omega t \\
x_0^2 - x^2 &= x^2\tan^2\omega t \\
x^2 &= \frac{x_0^2}{1 + \tan^2\omega t} \\
&= \frac{x_0^2\cos^2\omega t}{\cos^2\omega t + \sin^2\omega t} \\
&= x_0^2\cos^2\omega t \\
x &= x_0\cos\omega t
\end{aligned}$$

## 2.15

$$
\begin{aligned}
a_y &= -g \\
v_y &= v_{y0} - gt \\
y &= v_{y0}t - \frac{1}{2}gt^2 \\
0 &= v_{y0}t - \frac{1}{2}gt^2 \\
t &= \frac{2v_{y0}}{g} \\
x &= v_{x0}t \\
R &= \frac{2v_{x0}v_{y0}}{g}
\end{aligned}
$$

## 2.19

(a)

$$
\begin{aligned}
x &= v_{x0}t \\
y &= v_{y0}t - \frac{1}{2}gt^2 \\
&= \frac{v_{y0}}{v_{x0}}x - \frac{1}{2}g\left(\frac{x}{v_{x0}}\right)^2
\end{aligned}
$$

(b)

$$
\begin{aligned}
y &= \frac{v_{y0} + v_{\text{ter}}}{v_{x0}}x + v_{\text{ter}}\tau \ln\left(1 - \frac{x}{v_{x0}\tau}\right) \\
&\approx \frac{v_{y0}}{v_{x0}}x + \frac{g\tau}{v_{x0}}x - g\tau^2\left[\frac{x}{v_{x0}\tau} + \frac{1}{2}\left(\frac{x}{v_{x0}\tau}\right)^2\right] \\
&= \frac{v_{y0}}{v_{x0}}x - \frac{1}{2}g\left(\frac{x}{v_{x0}}\right)^2
\end{aligned}
$$

**2.23**

(a)

$$\begin{aligned}
v_{\text{ter}} &= \sqrt{\frac{mg}{c}} \\
&= \sqrt{\frac{mg}{\gamma D^2}} \\
&= \sqrt{\frac{mg}{0.25D^2}} \\
&= \sqrt{\frac{\rho \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 g}{0.25D^2}} \\
&= \sqrt{\frac{4\pi\rho D g}{6}} \\
&= 22\,\mathrm{m/s}
\end{aligned}$$

(b)

$$\begin{aligned}
m &= \rho V \\
&= \rho \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 \\
&= \frac{\pi\rho D^3}{6} \\
D^2 &= \left(\frac{6m}{\pi\rho}\right)^{2/3} \\
v_{\text{ter}} &= \sqrt{\frac{mg}{0.25D^2}} \\
&= \sqrt{\frac{mg}{0.25(6m/\pi\rho)^{2/3}}} \\
&= 140\,\mathrm{m/s}
\end{aligned}$$

(c)

$$v_{\text{ter}} = 107\,\mathrm{m/s}$$

## 2.27

$$
\begin{aligned}
m\dot{v} &= -mg\sin\theta - cv^2 \\
-\frac{\sqrt{m}\arctan\frac{\sqrt{c}v}{\sqrt{gm\sin\theta}}}{\sqrt{cg\sin\theta}} &= t + c_1 \\
\arctan\frac{\sqrt{c}v}{\sqrt{gm\sin\theta}} &= \sqrt{\frac{cg\sin\theta}{m}}(c_1 - t) \\
\frac{\sqrt{c}v}{\sqrt{gm\sin\theta}} &= \tan\left[\sqrt{\frac{cg\sin\theta}{m}}(c_1 - t)\right] \\
v &= \sqrt{\frac{gm\sin\theta}{c}}\tan\left[\sqrt{\frac{cg\sin\theta}{m}}(c_1 - t)\right]
\end{aligned}
$$

$$
\begin{aligned}
v_0 &= \sqrt{\frac{gm\sin\theta}{c}}\tan\left(\sqrt{\frac{cg\sin\theta}{m}}c_1\right) \\
c_1 &= \sqrt{\frac{m}{cg\sin\theta}}\arctan\left(\sqrt{\frac{c}{gm\sin\theta}}v_0\right)
\end{aligned}
$$

$$
v = \sqrt{\frac{gm\sin\theta}{c}}\tan\left[\arctan\left(\sqrt{\frac{c}{gm\sin\theta}}v_0\right) - \sqrt{\frac{cg\sin\theta}{m}}t\right]
$$

$$
\begin{aligned}
0 &= \sqrt{\frac{gm\sin\theta}{c}}\tan\left[\arctan\left(\sqrt{\frac{c}{gm\sin\theta}}v_0\right) - \sqrt{\frac{cg\sin\theta}{m}}t\right] \\
\sqrt{\frac{cg\sin\theta}{m}}t &= \arctan\left(\sqrt{\frac{c}{gm\sin\theta}}v_0\right) \\
t &= \sqrt{\frac{m}{cg\sin\theta}}\arctan\left(\sqrt{\frac{c}{gm\sin\theta}}v_0\right)
\end{aligned}
$$

## 2.29

$$
\begin{aligned}
v(t) &= v_{\text{ter}}\tanh\frac{gt}{v_{\text{ter}}} \\
v(1) &= 9.6\,\text{m/s} \\
v(5) &= 38\,\text{m/s} \\
v(10) &= 48\,\text{m/s} \\
v(20) &= 50\,\text{m/s} \\
v(30) &= 50\,\text{m/s}
\end{aligned}
$$

## 2.31

(a)

$$\begin{aligned} v_{\text{ter}} &= \sqrt{\frac{mg}{c}} \\ &= \sqrt{\frac{mg}{0.25D^2}} \\ &= 20.2\,\text{m/s} \end{aligned}$$

(b)

$$\begin{aligned} y &= -30 + \frac{v_{\text{ter}}^2}{g} \ln\left(\cosh \frac{gt}{v_{\text{ter}}}\right) \\ 0 &= -30 + \frac{v_{\text{ter}}^2}{g} \ln\left(\cosh \frac{gt}{v_{\text{ter}}}\right) \\ t &= 2.78\,\text{s} \\ v(2.78) &= 17.6\,\text{m/s} \end{aligned}$$

## 2.33

(b)

$$\begin{aligned} \cosh z &= \frac{e^z + e^{-z}}{2} \\ &= \frac{1}{2}\left[\left(1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \cdots\right) + \left(1 - z + \frac{z^2}{2} - \frac{z^3}{6} + \cdots\right)\right] \\ &= 1 + \frac{z^2}{2} + \frac{z^4}{24} + \frac{z^6}{720} + \cdots \\ \cos iz &= 1 - \frac{(iz)^2}{2} + \frac{(iz)^4}{24} - \frac{(iz)^6}{720} + \cdots \\ &= 1 + \frac{z^2}{2} + \frac{z^4}{24} + \frac{z^6}{720} + \cdots \\ &= \cosh z \\ \sinh z &= -i \sin iz \end{aligned}$$

(c)

$$\begin{aligned}
\frac{d}{dz}\cosh z &= \frac{d}{dz}\left(\frac{e^z + e^{-z}}{2}\right) \\
&= \frac{e^z - e^{-z}}{2} \\
&= \sinh z \\
\frac{d}{dz}\sinh z &= \frac{d}{dz}\left(\frac{e^z - e^{-z}}{2}\right) \\
&= \frac{e^z + e^{-z}}{2} \\
&= \cosh z
\end{aligned}$$

(d)

$$\begin{aligned}
\cosh^2 z - \sinh^2 z &= \left(\frac{e^z + e^{-z}}{2}\right)^2 - \left(\frac{e^z - e^{-z}}{2}\right)^2 \\
&= \frac{1}{4}(e^{2z} + 2 + e^{-2z} - e^{2z} + 2 - e^{-2z}) \\
&= 1
\end{aligned}$$

(e)

$$\begin{aligned}
\int \frac{1}{\sqrt{1+x^2}}\,dx &= \int \frac{\cosh z}{\sqrt{1+\sinh^2 z}}\,dz \\
&= \int 1\,dz \\
&= z \\
&= \operatorname{arcsinh} x
\end{aligned}$$

## 2.35

(a)

$$\begin{aligned}
m\dot{v} &= mg - cv^2 \\
\dot{v} &= g\left(1 - \frac{v^2}{v_{\text{ter}}^2}\right) \\
\int_0^v \frac{1}{1 - v'^2/v_{\text{ter}}^2}\,dv' &= \int_0^t g\,dt \\
v_{\text{ter}} \operatorname{arctanh} \frac{v}{v_{\text{ter}}} &= gt \\
v &= v_{\text{ter}} \tanh \frac{gt}{v_{\text{ter}}}
\end{aligned}$$

$$\begin{aligned}
y &= \int_0^t v_{\text{ter}} \tanh \frac{gt'}{v_{\text{ter}}} dt' \\
&= \frac{v_{\text{ter}}^2}{g} \ln\left[\cosh\left(\frac{gt}{v_{\text{ter}}}\right)\right]
\end{aligned}$$

(b)

$$\begin{aligned}
v &= g\tau \tanh \frac{t}{\tau} \\
y &= g\tau^2 \ln\left[\cosh\left(\frac{t}{\tau}\right)\right] \\
v(\tau) &= g\tau \tanh 1 \\
&= 0.76 v_{\text{ter}} \\
v(2\tau) &= 0.96 v_{\text{ter}} \\
v(3\tau) &= 0.99 v_{\text{ter}}
\end{aligned}$$

(c)

$$\begin{aligned}
y &= g\tau^2 \ln\left[\cosh\left(\frac{t}{\tau}\right)\right] \\
&= g\tau^2 \ln\left(\frac{e^{t/\tau} + e^{-t/\tau}}{2}\right) \\
&= g\tau^2 \ln\left(\frac{e^{t/\tau}}{2}\right) \\
&= g\tau^2 (\ln e^{t/\tau} - \ln 2) \\
&= g\tau t - g\tau^2 \ln 2 \\
&= v_{\text{ter}} t - g\tau^2 \ln 2
\end{aligned}$$

(d)

$$\begin{aligned}
y &= \frac{(v_{\text{ter}})^2}{g} \ln\left[\cosh\left(\frac{gt}{v_{\text{ter}}}\right)\right] \\
&\approx \frac{(v_{\text{ter}})^2}{g} \ln\left[1 + \frac{1}{2}\left(\frac{gt}{v_{\text{ter}}}\right)^2\right] \\
&\approx \frac{(v_{\text{ter}})^2}{g} \frac{1}{2}\left(\frac{gt}{v_{\text{ter}}}\right)^2 \\
&= \frac{1}{2} g t^2
\end{aligned}$$

## 2.39

(a)

$$\begin{aligned}
m\dot{v} &= -cv^2 - 3 \\
\int_{v_0}^{v} \frac{m}{-cv'^2 - 3}\, dv' &= \int_0^t dt' \\
\frac{m}{\sqrt{3c}}\left[\arctan\left(\sqrt{\frac{c}{3}}v_0\right) - \arctan\left(\sqrt{\frac{c}{3}}v\right)\right] &= t
\end{aligned}$$

(b)

| Speed | Time |
|---|---|
| 15 m/s | 6.34 s |
| 10 m/s | 18.4 s |
| 5 m/s | 48.3 s |
| 0 m/s | 142 s |

## 2.41

$$
\begin{aligned}
m\dot{v} &= -mg - cv^2 \\
\dot{v} &= -g\left[1 + \left(\frac{v}{v_{\text{ter}}}\right)^2\right] \\
v\frac{dv}{dy} &= -g\left[1 + \left(\frac{v}{v_{\text{ter}}}\right)^2\right] \\
\int_{v_0}^{v} \frac{v'}{1 + (v'/v_{\text{ter}})^2}\, dv' &= \int_0^y -g\, dy' \\
\frac{1}{2}v_{\text{ter}}^2[\ln(v_{\text{ter}}^2 + v^2) - \ln(v_{\text{ter}}^2 + v_0^2)] &= -gy \\
\ln\frac{v_{\text{ter}}^2 + v^2}{v_{\text{ter}}^2 + v_0^2} &= -\frac{2gy}{v_{\text{ter}}^2} \\
\frac{v_{\text{ter}}^2 + v^2}{v_{\text{ter}}^2 + v_0^2} &= e^{-2gy/v_{\text{ter}}^2} \\
v &= \sqrt{(v_{\text{ter}}^2 + v_0^2)e^{-2gy/v_{\text{ter}}^2} - v_{\text{ter}}^2}
\end{aligned}
$$

$$
\begin{aligned}
0 &= \sqrt{(v_{\text{ter}}^2 + v_0^2)e^{-2gy/v_{\text{ter}}^2} - v_{\text{ter}}^2} \\
v_{\text{ter}}^2 &= (v_{\text{ter}}^2 + v_0^2)e^{-2gy/v_{\text{ter}}^2} \\
\frac{v_{\text{ter}}^2}{v_{\text{ter}}^2 + v_0^2} &= e^{-2gy/v_{\text{ter}}^2} \\
-\frac{2gy}{v_{\text{ter}}^2} &= \ln\frac{v_{\text{ter}}^2}{v_{\text{ter}}^2 + v_0^2} \\
y &= -\frac{v_{\text{ter}}^2}{2g}\ln\frac{v_{\text{ter}}^2}{v_{\text{ter}}^2 + v_0^2} \\
&= \frac{v_{\text{ter}}^2}{2g}\ln\frac{v_{\text{ter}}^2 + v_0^2}{v_{\text{ter}}^2}
\end{aligned}
$$

$$
y_{\text{max}} = 17.6\,\text{m}
$$