University Physics with Modern Physics - Modern Physics by Young and Freedman Problems

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Contents

17 Temperature and Heat

17.1 Guided Practice

17.1.1

(a)

$$\begin{aligned}\Delta L &= \alpha L_0 \Delta T \\ \alpha &= \frac{\Delta L}{L_0 \Delta T} \\ &= 2.0 \times 10^{-5}\,\mathrm{K}^{-1}\end{aligned}$$

(b)

$$\begin{aligned}\Delta L &= \alpha L_0 \Delta T \\ &= -0.27\,\mathrm{mm}\end{aligned}$$

17.1.2

$$\begin{aligned}\Delta V_C &= \beta V_{C0} \Delta T \\ &= (5.1 \times 10^{-5})(250)(-70) \\ &= -0.893\,\mathrm{cm}^3 \\ \Delta V_E &= \beta V_{E0} \Delta T \\ &= (75 \times 10^{-5}\,)(250)(-70) \\ &= -13.1\,\mathrm{cm}^3 \\ \Delta V_C - \Delta V_E &= 12.2\,\mathrm{cm}^3 \\ &= 12.2\,\mathrm{mL}\end{aligned}$$

17.1.3

$$\begin{aligned}\frac{\Delta L}{L_0} &= \alpha \Delta T \\ Y &= \frac{F/A}{\Delta L / L_0} \\ \frac{\Delta L}{L_0} &= \frac{F}{AY} \\ \alpha \Delta T + \frac{F}{AY} &= 0 \\ \frac{F}{AY} &= -\alpha \Delta T \\ F &= -\alpha A Y \Delta T \\ &= -(2.0 \times 10^{-5})(\pi 0.005^2)(9.0 \times 10^{10})(-12) \\ &= 1.70 \times 10^3\,\mathrm{N}\end{aligned}$$

Tensile

17.1.4

$$\begin{aligned}\Delta L &= \alpha_A L_A \Delta T + \alpha_B L_B \Delta T \\ \frac{\Delta L}{\Delta T} &= \alpha_A L_A + \alpha_B (L - L_A) \\ &= (\alpha_A - \alpha_B) L_A + \alpha_B L \\ L_A &= \frac{1}{\alpha_A - \alpha_B} \left(\frac{\Delta L}{\Delta T} - \alpha_B L \right)\end{aligned}$$

17.1.5

$$\begin{aligned}
0 &= m_{Al}c_{Al}\Delta T_{Al} + m_W c_W \Delta T_W \\
&= m_{Al}c_{Al}(T - T_{Al}) + m_W c_W (T - T_W) \\
m_{Al} &= -\frac{m_W c_W (T - T_W)}{c_{Al}(T - T_{Al})} \\
&= 0.20\,\mathrm{kg}
\end{aligned}$$

17.1.6

$$\begin{aligned}
0 &= m_I L_f + m_C c_C \Delta T \\
&= m_I L_f - m_C c_C T \\
T &= \frac{m_I L_f}{m_C c_C} \\
&= 14.0\,^\circ\mathrm{C}
\end{aligned}$$

17.1.7

$$\begin{aligned}
0 &= m_I L_F + m_I c_I \Delta T_I + m_E c_E \Delta T_E \\
&= m_I (L_F + c_I \Delta T_I) + m_E c_E \Delta T_E \\
m_I &= -\frac{m_E c_E \Delta T_E}{L_F + c_I \Delta T_I} \\
&= 0.176\,\mathrm{kg}
\end{aligned}$$

17.1.8

Cooling the silver to $0\,^\circ\mathrm{C}$ would take

$$Q = mc\Delta T = 92\,137.5\,\mathrm{J}$$

whereas melting all of the ice would take

$$Q = mL_f = 83\,500\,\mathrm{J}$$

so all of the ice will melt.

$$\begin{aligned}
0 &= m_{Ag}c_{Ag}\Delta T_{Ag} + m_I L_f + m_I c_I \Delta T_I + m_I c_W \Delta T_W \\
&= m_{Ag}c_{Ag}(T - T_{Ag}) + m_I L_F - m_I c_I T_I + m_I c_W T \\
&= (m_{Ag}c_{Ag} + m_I c_W)T - m_{Ag}c_{Ag}T_{Ag} + m_I L_F - m_I c_I T_I \\
T &= \frac{m_{Ag}c_{Ag}T_{Ag} + m_I c_I T_I - m_I L_F}{m_{Ag}c_{Ag} + m_I c_W} \\
&= 3.31\,^\circ\mathrm{C}
\end{aligned}$$

17.1.9

(a)

$$
\begin{aligned}
H &= kA\frac{T_H - T_C}{L} \\
k &= \frac{HL}{A(T_H - T_C)} \\
&= 0.754\,\mathrm{W/(m\,K)}
\end{aligned}
$$

(b)

$$
H = kA\frac{T_H - T_C}{L} = 733\,\mathrm{W}
$$

17.1.10

(a)

$$
\begin{aligned}
L &= 0.250\,\mathrm{m} \\
A &= 2.00 \times 10^{-4}\,\mathrm{m^2} \\
k_B &= 109.0\,\mathrm{W/(m\,K)} \\
k_{Pb} &= 34.7\,\mathrm{W/(m\,K)} \\
T &= 185\,^\circ\mathrm{C} \\
H &= 6.00\,\mathrm{W}
\end{aligned}
$$

$$
\begin{aligned}
H &= k_B A\frac{T_H - T}{L} \\
T_H &= \frac{HL}{k_B A} + T \\
&= 254\,^\circ\mathrm{C}
\end{aligned}
$$

(b)

$$
\begin{aligned}
H &= k_{Pb} A\frac{T - T_C}{L} \\
T_C &= T - \frac{HL}{k_{Pb} A} \\
&= -31.1\,^\circ\mathrm{C}
\end{aligned}
$$

17.1.11

$$
\begin{aligned}
H &= 4\pi(kr_E)^2 e\sigma T^4 \\
(kr_E)^2 &= \frac{H}{4\pi e\sigma T^4} \\
k &= \frac{1}{r_E}\sqrt{\frac{H}{4\pi e\sigma T^4}} \\
&= 1.70
\end{aligned}
$$

17.1.12

(a)

$$
\begin{aligned}
H &= Ae\sigma T^4 \\
&= \pi r^2 \sigma T^4 \\
H &= kA\frac{T_H - T_C}{L} \\
&= k\pi r^2 \frac{T_H - T_C}{L} \\
\pi r^2 \sigma T^4 &= k\pi r^2 \frac{T_H - T_C}{L} \\
T_H &= \frac{L\sigma T^4}{k} + T_C \\
&= 14.26\,\mathrm{K}
\end{aligned}
$$

(b)

$$
\begin{aligned}
H &= mL_f \\
\pi r^2 \sigma T^4 &= mL_f \\
m &= \frac{\pi r^2 \sigma T^4}{L_f} \\
&= 1.19 \times 10^{-4}\,\mathrm{kg/s} \\
&= 0.427\,\mathrm{kg/h}
\end{aligned}
$$

17.2 Exercises and Problems

17.2.15

$$
\begin{aligned}
\Delta V &= \beta V_0 \Delta T \\
\frac{\Delta V}{V_0} &= \beta(T - T_0) \\
T &= T_0 + \frac{\Delta V}{\beta V_0} \\
&= 49\,^\circ\mathrm{C}
\end{aligned}
$$

17.2.25

$$
\begin{aligned}
Q &= (m_{Al}c_{Al} + m_W c_W)\Delta T \\
&= 5.55 \times 10^5\,\mathrm{J}
\end{aligned}
$$

17.2.33

$$
\begin{aligned}
\Delta K &= \frac{1}{2}mv^2 - \frac{1}{2}mv'^2 \\
&= \frac{1}{2}m(v^2 - v'^2) \\
&= 3.47\,\mathrm{kJ} \\
\Delta K &= mc\Delta T \\
\Delta T &= \frac{\Delta K}{mc} \\
&= 6.14 \times 10^{-2}\,{}^\circ\mathrm{C}
\end{aligned}
$$

17.2.35

(a)

$$
\begin{aligned}
0 &= m_m c_m \Delta T_m + m_w c_w \Delta T_w \\
c_m &= -\frac{m_w c_w \Delta T_w}{m_m \Delta T_m} \\
&= 215\,\mathrm{J/(kg\,K)}
\end{aligned}
$$

(b) Water because it has a higher specific heat

(c) It would be too small

17.2.45

$$
\begin{aligned}
\frac{1}{2}mv^2 &= mc\Delta T + mL_F \\
v &= \sqrt{2(c\Delta T + L_F)} \\
&= 366\,\mathrm{m/s}
\end{aligned}
$$

17.2.55

$$\begin{aligned}
k_C A\frac{T_H - T}{L} &= kA\frac{T}{L} \\
k_C T_H - k_C T &= kT \\
k_C T_H &= (k + k_C)T \\
T &= \frac{k_C}{k + k_C}T_H \\
0.71 &= \frac{k_C}{k + k_C} \\
0.71(k + k_C) &= k_C \\
0.71k + 0.71k_C &= k_C \\
0.71k &= 0.29k_C \\
k &= \frac{0.29}{0.71}k_C \\
&\approx 157\,\mathrm{W/(m\,K)}
\end{aligned}$$

17.2.57

(a)

$$\begin{aligned}
k_W\frac{T - T_C}{L_W} &= k_S\frac{T_H - T}{L_S} \\
\left(\frac{k_W}{L_W} + \frac{k_S}{L_S}\right)T &= \frac{k_S}{L_S}T_H + \frac{k_W}{L_W}T_C \\
T &= \frac{\frac{k_S}{L_S}T_H + \frac{k_W}{L_W}T_C}{\frac{k_W}{L_W} + \frac{k_S}{L_S}} \\
&= -0.86\,^\circ\mathrm{C}
\end{aligned}$$

(b)

$$\begin{aligned}
H &= k_W\frac{T - T_C}{L_W} \\
&= 24.4\,\mathrm{W/m^2}
\end{aligned}$$

17.2.65

$$\begin{aligned}
H &= Ae\sigma T^4 \\
A &= \frac{H}{e\sigma T^4} \\
&= 2.1\,\mathrm{cm^2}
\end{aligned}$$

17.2.69

$$\begin{aligned}\Delta L &= (\alpha_B L_B + \alpha_S L_S)\Delta T\\ T &= T_0 + \frac{\Delta L}{\alpha_B L_B + \alpha_S L_S}\\ &= 35.0\,^\circ\mathrm{C}\end{aligned}$$

17.2.71

$$\begin{aligned}Q &= mc\Delta T\\ &= \rho V c\Delta T\\ \Delta T &= \frac{Q}{\rho V c}\\ \Delta V &= \beta V \Delta T\\ &= \frac{\beta Q}{\rho c}\\ c &= \frac{\beta Q}{\rho \Delta V}\end{aligned}$$

17.2.73

(a)

$$\begin{aligned}0.0\,^\circ\mathrm{M} &= -39\,^\circ\mathrm{C}\\ 100.0\,^\circ\mathrm{M} &= 357\,^\circ\mathrm{C}\\ T_M &= \frac{T_C + 39\,^\circ\mathrm{C}}{3.96}\\ \frac{100\,^\circ\mathrm{C} + 39\,^\circ\mathrm{C}}{3.96} &= 35.1\,^\circ\mathrm{M}\end{aligned}$$

(b)

$$10\,\mathrm{M}^\circ = 10\frac{357\,^\circ\mathrm{C} - (-39\,^\circ\mathrm{C})}{100} = 39.6\,\mathrm{C}^\circ$$

17.2.75

$$\begin{aligned}Ah + \beta_G Ah(T - T_0) &= Ah' + \beta_O Ah'(T - T_0)\\ Ah + \beta_G AhT - \beta_G AhT_0 &= Ah' + \beta_O Ah'T - \beta_O Ah'T_0\\ (\beta_G Ah - \beta_O Ah')T &= (Ah' - \beta_O Ah'T_0) - (Ah - \beta_G AhT_0)\\ T &= \frac{(1 - \beta_O T_0)h' - (1 - \beta_G T_0)h}{\beta_G h - \beta_O h'}\\ &= 69.4\,^\circ\mathrm{C}\end{aligned}$$

17.2.79

(a)

$$\begin{aligned}
Y &= \frac{F/A}{\Delta L/L_0} \\
\Delta L &= \frac{FL_0}{AY} \\
\Delta L &= \alpha L_0 \Delta T \\
\Delta L &= \alpha L_0 \Delta T + \frac{FL_0}{AY} \\
\frac{F}{A} &= Y\left(\frac{\Delta L}{L_0} - \alpha \Delta T\right)
\end{aligned}$$

(b)

$$\begin{aligned}
\Delta L_B &= \alpha_B L_{B0} \Delta T \\
\frac{\Delta L_B}{L_{B0}} &= \alpha_B \Delta T \\
\frac{F}{A} &= Y_S(\alpha_B - \alpha_S)\Delta T \\
&= 1.9 \times 10^8\,\mathrm{Pa}
\end{aligned}$$

17.2.85

(a)

$$\begin{aligned}
\frac{dQ}{dT} &= nk\frac{T^3}{\theta^3} \\
Q &= \int_a^b nk\frac{T^3}{\theta^3} \\
&= \frac{nk}{\theta^3}\left[\frac{1}{4}T^4\right]_a^b \\
&= \frac{nk}{4\theta^3}(b^4 - a^4) \\
&= 83.6\,\mathrm{J}
\end{aligned}$$

(b)

$$\begin{aligned}
Q &= nC\Delta T \\
C &= \frac{Q}{n\Delta T} \\
&= 1.86\,\mathrm{J/(mol\,K)}
\end{aligned}$$

(c)

$$C = 5.60\,\mathrm{J/(mol\,K)}$$

17.2.95

(a)

$$\begin{aligned}
0 &= m_I L_f + m_I c_W \Delta T_I + m_C c_C \Delta T_I - m_S L_v + m_S c_W \Delta T_S \\
&= m_I L_f + m_I c_W T + m_C c_C T - m_S L_v + m_S c_W (T - T_S) \\
T &= \frac{m_S(L_v + c_W T_S) - m_I L_f}{m_I c_W + m_C c_C + m_S c_W} \\
&= 86.1\,^{\circ}\mathrm{C}
\end{aligned}$$

(b) No ice, 0.13 kg water, no steam

17.2.99

(a)

$$\begin{aligned}
H &= kA\frac{T_H - T_C}{L} \\
&= 94\,\mathrm{W}
\end{aligned}$$

(b)

$$\begin{aligned}
H_{\text{wood}} &= 12.4\,\mathrm{W} \\
H_{\text{glass}} &= 45.0\,\mathrm{W} \\
H' &= H + (H_{\text{glass}} - H_{\text{wood}}) \\
&= 126.6\,\mathrm{W} \\
\frac{H'}{H} &= 1.35
\end{aligned}$$

17.2.105

(b)

$$
\begin{aligned}
\frac{dQ}{dt} &= k\frac{T_H - T_C}{L} \\
\frac{dQ}{dL} &= \rho L_f \\
\frac{dL}{dt} &= \frac{dL}{dQ}\frac{dQ}{dt} \\
&= \frac{1}{\rho L_f} k \frac{T_H - T_C}{L} \\
L\frac{dL}{dt} &= \frac{k(T_H - T_C)}{\rho L_f} \\
\int_0^t L\frac{dL}{dt}\,dt &= \int_0^t \frac{k(T_H - T_C)}{\rho L_f}\,dt \\
\int_0^L L'\,dL' &= \frac{k(T_H - T_C)}{\rho L_f}t \\
\frac{1}{2}L^2 &= \frac{k(T_H - T_C)}{\rho L_f}t \\
L &= \sqrt{\frac{2k(T_H - T_C)}{\rho L_f}t}
\end{aligned}
$$

(c)

$$
\begin{aligned}
t &= \frac{L^2 \rho L_f}{2k(T_H - T_C)} \\
&= 7.5\,\text{days}
\end{aligned}
$$

(d) $t \approx 530\,\text{years}$; no

17.2.107

$$
\begin{aligned}
A &= 2\pi \left(\frac{d}{2}\right)^2 + 2\pi \left(\frac{d}{2}\right) h \\
&= 8.34 \times 10^{-2}\,\mathrm{m}^2 \\
H &= Ae\sigma(T^4 - T_s^4) \\
&= Ae\sigma(T^4 - T_s^4) \\
&= -3.38 \times 10^{-2}\,\mathrm{W} \\
m &= \frac{H \times 60 \times 60}{L_v} \\
&= 5.82 \times 10^{-3}\,\mathrm{kg/h} \\
&= 5.82\,\mathrm{g/h}
\end{aligned}
$$

17.2.113

$$
\begin{aligned}
r(x) &= R_2 - (R_2 - R_1)\frac{x}{L} \\
A(x) &= \pi r(x)^2 \\
&= \pi \left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2 \\
H &= kA(x)\frac{dT}{dx} \\
&= k\pi \left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2 \frac{dT}{dx} \\
\frac{1}{\left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2} H\,dx &= k\pi\,dT \\
\int_0^L \frac{1}{\left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2} H\,dx &= \int_{T_H}^{T_C} k\pi\,dT \\
\frac{HL}{R_2 - R_1}\left[\frac{1}{R_2 - (R_2 - R_1)\frac{x}{L}}\right]_0^L &= k\pi(T_C - T_H) \\
\frac{HL}{R_2 - R_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) &= k\pi(T_C - T_H) \\
\frac{HL}{R_2 - R_1}\frac{R_2 - R_1}{R_1 R_2} &= k\pi(T_C - T_H) \\
H &= \frac{k\pi R_1 R_2 (T_C - T_H)}{L}
\end{aligned}
$$

17.2.115

(a)

$$\begin{aligned}
H &= k(2\pi rL)\frac{dT}{dr} \\
\frac{1}{r}H\,dr &= 2\pi kL\,dT \\
\int_a^b \frac{1}{r}H\,dr &= \int_{T_1}^{T_2} 2\pi kL\,dT \\
H\ln\frac{b}{a} &= 2\pi kL(T_2 - T_1) \\
H &= \frac{2\pi kL(T_2 - T_1)}{\ln b/a}
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{2\pi kL(T - T_2)}{\ln r/a} &= \frac{2\pi kL(T_2 - T_1)}{\ln b/a} \\
\frac{T - T_2}{\ln r/a} &= \frac{T_2 - T_1}{\ln b/a} \\
T - T_2 &= \frac{\ln r/a}{\ln b/a}(T_2 - T_1) \\
T &= T_2 + \frac{\ln r/a}{\ln b/a}(T_2 - T_1)
\end{aligned}$$

17.2.117

a

17.2.119

a

18 Thermal Properties of Matter

18.1 Guided Practice

18.1.1

(a)

$$\begin{aligned} pV &= nRT \\ \frac{p}{T} &= \frac{nR}{V} \\ \frac{p_1}{T_1} &= \frac{p_2}{T_2} \\ p_2 &= p_1 \frac{T_2}{T_1} \\ &= 4.67 \times 10^5\,\mathrm{Pa} \end{aligned}$$

(b)

$$\begin{aligned} pV &= nRT \\ n &= \frac{pV}{RT} \\ &= 0.280\,\mathrm{mol} \end{aligned}$$

18.1.2

(a)

$$\begin{aligned} pV &= nRT \\ \frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\ V_2 &= \frac{V_1 p_1 T_2}{p_2 T_1} \\ &= 1.2 \times 10^3\,\mathrm{m}^3 \end{aligned}$$

(b)

$$\begin{aligned} \frac{V_2}{V_1} &= \frac{\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi r_1^3} \\ &= \left(\frac{r_2}{r_1}\right)^3 \\ \frac{r_2}{r_1} &= \sqrt[3]{\frac{V_2}{V_1}} \\ &= 4.5 \end{aligned}$$

18.1.3

(a)

$$\begin{aligned} pV &= nRT \\ n &= \frac{pV}{RT} \\ &= 2.9 \times 10^{-3}\,\text{mol/m}^3 \end{aligned}$$

(b)

$$8.0 \times 10^{-5}\,\text{kg/m}^3$$

18.1.4

(a)

$$\begin{aligned} pV &= \frac{m_{\text{total}}}{M} RT \\ \frac{p}{\rho T} &= \frac{R}{M} \\ \frac{p_1}{\rho_1 T_1} &= \frac{p_2}{\rho_2 T_2} \\ &= \frac{p_2}{\rho_1 (p_2/p_1)^{3/5} T_2} \\ T_2 &= \left(\frac{p_2}{p_1}\right)^{2/5} T_1 \end{aligned}$$

(b)

$$\begin{aligned} \frac{\rho_2}{\rho_1} &= \frac{\rho_1 (p_2/p_1)^{3/5}}{\rho_1} \\ &= \left(\frac{\frac{1}{2}p_1}{p_1}\right)^{3/5} \\ &= \left(\frac{1}{2}\right)^{3/5} \\ &\approx 0.660 \\ \frac{T_2}{T_1} &= \frac{(p_2/p_1)^{2/5} T_1}{T_1} \\ &= \left(\frac{\frac{1}{2}p_1}{p_1}\right)^{2/5} \\ &= \left(\frac{1}{2}\right)^{2/5} \\ &\approx 0.758 \end{aligned}$$

(c)

$$\begin{aligned}\frac{\rho_2}{\rho_1} &= 2^{3/5} \\ &\approx 1.52 \\ \frac{T_2}{T_1} &= 2^{2/5} \\ &\approx 1.32\end{aligned}$$

18.1.5

$$\begin{aligned}\sqrt{\frac{3RT}{M_\mathrm{H}}} &= \sqrt{\frac{3RT_\mathrm{N}}{M_\mathrm{N}}} \\ T &= \frac{M_\mathrm{H}}{M_\mathrm{N}} T_\mathrm{N} \\ &= 41.9\,\mathrm{K} \\ &= -231\,^\circ\mathrm{C}\end{aligned}$$

18.1.6

(a)

$$K_\mathrm{tr} = \frac{3}{2} kT = 6.21 \times 10^{-20}\,\mathrm{J}$$

(b)

$$v_\mathrm{rms} = \sqrt{\frac{3RT}{M}} = 8.63 \times 10^{3}\,\mathrm{m/s}$$

18.1.7

(a)

$$\begin{aligned}pV &= \frac{N}{N_A} RT \\ N &= \frac{N_A pV}{RT} \\ &= 1.50 \times 10^{27}\end{aligned}$$

(b)

$$K_\mathrm{tr} = \frac{3}{2} nRT = 9.11 \times 10^{6}\,\mathrm{J}$$

(c)

$$
\begin{aligned}
\frac{1}{2}mv^2 &= K_{\text{tr}} \\
v &= \sqrt{\frac{2K_{\text{tr}}}{m}} \\
&= 110\,\text{m/s}
\end{aligned}
$$

18.1.8

(a) 5.5

(b) 38.5

(c) 6.2

18.1.9

(a)

$$
\lambda = \frac{kT}{4\pi\sqrt{2}r^2p} = 6.8 \times 10^{-6}\,\text{m}
$$

(b)

$$
\begin{aligned}
\lambda_{\text{Earth}} &= 5.54 \times 10^{-8}\,\text{m} \\
\frac{\lambda_{\text{Mars}}}{\lambda_{\text{Earth}}} &= 1.2 \times 10^{2}
\end{aligned}
$$

18.1.10

(a)

$$
\begin{aligned}
\lambda &= \frac{kT}{4\pi\sqrt{2}r^2p} \\
p &= \frac{kT}{4\pi\sqrt{2}r^2\lambda} \\
&= 5.7 \times 10^{-3}\,\text{Pa}
\end{aligned}
$$

(b)

$$
\begin{aligned}
pV &= nRT \\
n &= \frac{pV}{RT} \\
&= 2.3 \times 10^{-6}\,\text{mol}
\end{aligned}
$$

18.1.11

(a)

$$\begin{aligned}
pV &= nRT \\
p &= \frac{nRT}{V} \\
&= 2.0 \times 10^{7}\,\text{Pa} \\
\lambda &= \frac{kT}{4\pi\sqrt{2}r^2 p} \\
&= 1.2 \times 10^{-8}\,\text{m}
\end{aligned}$$

(b)

$$\begin{aligned}
v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\
&= 1.4 \times 10^{3}\,\text{m/s} \\
\lambda &= v t_{\text{mean}} \\
t_{\text{mean}} &= \frac{\lambda}{v} \\
&= 8.6 \times 10^{-12}\,\text{s}
\end{aligned}$$

18.1.12

(a)

$$\begin{aligned}
v_{\text{rms}} t_{\text{mean}} &= \lambda \\
\sqrt{\frac{3kT}{m}} t_{\text{mean}} &= \frac{kT}{4\pi\sqrt{2}r^2 p} \\
t_{\text{mean}} &= \frac{kT}{4\pi\sqrt{2}r^2 p}\sqrt{\frac{m}{3kT}} \\
&= \frac{1}{4\pi r^2 p}\sqrt{\frac{mkT}{6}}
\end{aligned}$$

(b) Doubling r.

18.1.13

(a)

$$
\begin{aligned}
v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\
&= 515\,\text{m/s} \\
\frac{1}{2}mv_{\text{rms}}^2 &= mgh \\
h &= \frac{v_{\text{rms}}^2}{2g} \\
&= 102\,\text{km}
\end{aligned}
$$

(b)

$$
\begin{aligned}
&\int_{2025}^{\infty} 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}\,dv \\
&= (3.03\times10^{-8})\int_{2025}^{\infty} v^2 e^{-(5.65\times10^{-6})v^2}\,dv \\
&= 4.8\times10^{-10}
\end{aligned}
$$

Yes, some escape.

18.2 Exercises and Problems

18.2.7

$$
\begin{aligned}
pV &= nRT \\
\frac{p_1V_1}{T_1} &= \frac{p_2V_2}{T_2} \\
T_2 &= \frac{p_2V_2T_1}{p_1V_1} \\
&= 776\,\text{K} \\
&= 503\,^\circ\text{C}
\end{aligned}
$$

18.2.9

$$
\begin{aligned}
pV &= nRT \\
\frac{p_1V_1}{T_1} &= \frac{p_2V_2}{T_2} \\
p_2 &= \frac{p_1V_1T_2}{T_1V_2} \\
&= 1.97\times10^4\,\text{Pa}
\end{aligned}
$$

18.2.13

$$\begin{aligned}\frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2}\\ V_2 &= \frac{p_1 T_2}{T_1 p_2} V_1\\ &= (5.08 \times 10^{-2}) V_1\end{aligned}$$

18.2.17

(a)

$$\begin{aligned}pV &= \frac{m_{\text{total}}}{M} RT\\ m_{\text{total}} &= \frac{pVM}{RT}\\ &= 6.91 \times 10^{-16}\,\text{kg}\end{aligned}$$

(b)

$$\rho = \frac{m_{\text{total}}}{V} = 2.30 \times 10^{-13}\,\text{kg/m}^3$$

18.2.21

(a)

$$\begin{aligned}pV &= \frac{N}{N_A} RT\\ N &= \frac{pVN_A}{RT}\\ &= 2.19 \times 10^{6}\end{aligned}$$

(b)

$$2.44 \times 10^{19}$$

18.2.23

(a)

$$
\begin{aligned}
pV &= \frac{N}{N_A}RT \\
\frac{V}{N} &= \frac{RT}{N_A p} \\
s &= \sqrt[3]{\frac{V}{N}} \\
&= \sqrt[3]{\frac{RT}{N_A p}} \\
&= 3.45 \times 10^{-9}\,\mathrm{m}
\end{aligned}
$$

18.2.25

(a)

$$
\begin{aligned}
K_{\mathrm{tr}} &= \frac{3}{2}nRT \\
&= \frac{3}{2}pV \\
&= 5.82 \times 10^{7}\,\mathrm{J}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{1}{2}mv^2 &= K_{\mathrm{tr}} \\
v &= \sqrt{\frac{2K_{\mathrm{tr}}}{m}} \\
&= 241\,\mathrm{m/s}
\end{aligned}
$$

18.2.27

$$
\begin{aligned}
pV &= nRT \\
p &= \frac{nR}{V}T \\
\frac{nR}{V} &= m \\
n &= \frac{mV}{R} \\
&= 1.07\,\mathrm{mol} \\
N &= nN_A \\
&= 6.44 \times 10^{23}
\end{aligned}
$$

18.2.29

(a)

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} \\ &= 1.93 \times 10^6 \, \text{m/s} \\ &= 0.006c \end{aligned}$$

Not a significant fraction of c.

(b)

$$\begin{aligned} 0.10c &= \sqrt{\frac{3kT}{m}} \\ (0.10c)^2 &= \frac{3kT}{m} \\ T &= \frac{(0.10c)^2 m}{3k} \\ &= 7.26 \times 10^{10} \, \text{K} \end{aligned}$$

18.2.31

(a)

$$\frac{3}{2}kT = 6.21 \times 10^{-21} \, \text{J}$$

(b)

$$(v^2)_{\text{av}} = \frac{2}{m}\left(\frac{3}{2}kT\right) = 2.34 \times 10^5 \, (\text{m/s})^2$$

(c)

$$v_{\text{rms}} = \sqrt{(v^2)_{\text{av}}} = 484 \, \text{m/s}$$

(d)

$$p = mv = \frac{M}{N_A} v = 2.57 \times 10^{-23} \, \text{kg m/s}$$

(e)

$$\begin{aligned} \Delta P &= 2P \\ &= 5.14 \times 10^{-23} \, \text{kg m/s} \\ \Delta t &= \frac{2l}{v} \\ &= 4.13 \times 10^{-4} \, \text{s} \\ F_{\text{av}} &= \frac{\Delta P}{\Delta t} \\ &= 1.24 \times 10^{-19} \, \text{N} \end{aligned}$$

(f)

$$p_{\mathrm{av}} = \frac{F_{\mathrm{av}}}{A} = 1.24 \times 10^{-17}\,\mathrm{Pa}$$

(g)

$$\begin{aligned} p &= N p_{\mathrm{av}} \\ N &= \frac{p}{p_{\mathrm{av}}} \\ &= 8.15 \times 10^{21} \end{aligned}$$

(h)

$$\begin{aligned} pV &= \frac{N}{N_A} RT \\ N &= \frac{pVN_A}{RT} \\ &= 2.44 \times 10^{22} \end{aligned}$$

18.2.33

$$\begin{aligned} \sqrt{\frac{3RT}{M_{\mathrm{N}}}} &= \sqrt{\frac{3RT_{\mathrm{H}}}{M_{\mathrm{H}}}} \\ T &= \frac{M_{\mathrm{N}}}{M_{\mathrm{H}}} T_{\mathrm{H}} \\ &= 4074\,\mathrm{K} \\ &= 3800\,^{\circ}\mathrm{C} \end{aligned}$$

18.2.35

$$
\begin{aligned}
C_V &= \frac{5}{2}R \\
v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\
T &= \frac{Mv_{\text{rms}}^2}{3R} \\
Q &= nC_V\Delta T \\
\Delta T &= \frac{Q}{nC_V} \\
v'_{\text{rms}} &= \sqrt{\frac{3R(T+\Delta T)}{M}} \\
&= \sqrt{\frac{3R\left(\frac{Mv_{\text{rms}}^2}{3R} + \frac{Q}{nC_V}\right)}{M}} \\
&= \sqrt{v_{\text{rms}}^2 + \frac{3RQ}{Mn\frac{5}{2}R}} \\
&= \sqrt{v_{\text{rms}}^2 + \frac{6Q}{5Mn}} \\
&= 1.02 \times 10^3\,\text{m/s}
\end{aligned}
$$

18.2.39

(a)

$$
\begin{aligned}
c_{V,\text{N}} &= \frac{5}{2}R \\
&= 742\,\text{J/(kg K)} \\
c_{V,\text{water}} &= 4190\,\text{J/(kg K)} \\
&= 5.6C_{V,\text{N}}
\end{aligned}
$$

(b)

$$\begin{aligned}
Q &= mc_{V,\text{water}}\Delta T \\
&= 4.19 \times 10^4\,\text{J} \\
m &= \frac{Q}{c_{V,\text{N}}\Delta T} \\
&= 5.65\,\text{kg} \\
pV &= \frac{m_{\text{total}}}{M}RT \\
V &= \frac{m_{\text{total}}RT}{Mp} \\
&= 4.87\,\text{m}^3 \\
&= 4.87 \times 10^3\,\text{L}
\end{aligned}$$

18.2.41

(a)

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}} = 337\,\text{m/s}$$

(b)

$$v_{\text{av}} = 380\,\text{m/s}$$

(c)

$$v_{\text{rms}} = 412\,\text{m/s}$$

18.2.43

(a)

$$\frac{v_{\text{rms}}}{v} = \sqrt{\frac{3RT}{M}}\sqrt{\frac{M}{\gamma RT}} = \sqrt{\frac{3}{\gamma}} = 1.34$$

(b)

$$\frac{v_{\text{av}}}{v} = \sqrt{\frac{8RT}{\pi M}}\sqrt{\frac{M}{\gamma RT}} = \sqrt{\frac{8}{\pi\gamma}} = 1.23$$

18.2.45

(a) The minimum pressure is $p_1 = 611.657\,\text{Pa}$. If $p < p_1$ the ice sublimates directly to gas.

(b) The maximum pressure is $p_2 = 2.212 \times 10^7\,\text{Pa}$. The ice melts to water, then the properties of the water gradually change to those of steam with no phase transition.

18.2.49

(a)

$$\begin{aligned} p' - p &= -\rho g h \\ &= -1.18 \times 10^4\,\mathrm{Pa} \end{aligned}$$

(b)

$$\begin{aligned} p_1 V_1 &= p_2 V_2 \\ V_2 &= \frac{p_1}{p_2} V_1 \\ &= 0.56\,\mathrm{L} \end{aligned}$$

18.2.51

$$\begin{aligned} 0 &= \rho_{\mathrm{cold}} V g - \rho_{\mathrm{hot}} V g - m g \\ &= \rho_{\mathrm{cold}} V - \rho_{\mathrm{hot}} V - m \\ \rho_{\mathrm{hot}} &= \rho_{\mathrm{cold}} - \frac{m}{V} \\ \frac{Mp}{RT} &= \rho_{\mathrm{cold}} - \frac{m}{V} \\ T &= \frac{Mp}{R(\rho_{\mathrm{cold}} - m/V)} \\ &= 542\,\mathrm{K} \\ &= 269\,^{\circ}\mathrm{C} \end{aligned}$$

18.2.53

$$\begin{aligned} pV &= \frac{m_{\mathrm{total}}}{M} RT \\ m_{\mathrm{total}} &= \frac{pVM}{RT} \\ &= 0.285\,\mathrm{kg} \\ m'_{\mathrm{total}} &= 0.0896\,\mathrm{kg} \\ \Delta m &= 0.195\,\mathrm{kg} \end{aligned}$$

18.2.57

(a)

$$
\begin{aligned}
0 &= \rho V g - (m_{\text{adventurer}} + m_{\text{bell}} + m_{\text{water}})g \\
&= \rho V - m_{\text{adventurer}} - m_{\text{bell}} - m_{\text{water}} \\
m_{\text{water}} &= \rho V - m_{\text{adventurer}} - m_{\text{bell}} \\
&= 98\,\text{kg} \\
V_{\text{water}} &= \frac{m_{\text{water}}}{\rho_{\text{water}}} \\
&= 0.0956\,\text{m}^3
\end{aligned}
$$

(b)

$$
\begin{aligned}
pV &= nRT \\
p &= \frac{nRT}{V} \\
p &= \rho g y \\
\rho g y &= \frac{nRT}{V} \\
n &= \frac{\rho g V}{RT} y \\
\frac{dn}{dt} &= \frac{\rho g V}{RT}\frac{dy}{dt} \\
&= 18.2\,\text{mol/s}
\end{aligned}
$$

(c)

$$
\begin{aligned}
pV &= nRT \\
n &= \frac{pV}{RT} \\
&= 756\,\text{mol} \\
\frac{n}{dn/dt} &= 41.5\,\text{m}
\end{aligned}
$$

18.2.59

(a)

$$\begin{aligned}
pV &= nRT \\
n_{\text{balloon}} &= \frac{pV}{RT} \\
&= (9.11 \times 10^6)\frac{1}{T} \\
n_{\text{cylinder}} &= \frac{pV}{RT} \\
&= (2.97 \times 10^5)\frac{1}{T} \\
\frac{n_{\text{balloon}}}{n_{\text{cylinder}}} &= 30.7
\end{aligned}$$

(b)

$$\begin{aligned}
0 &= \rho V g - Mng - mg \\
mg &= (\rho V - Mn)g \\
&= 8420\,\mathrm{N}
\end{aligned}$$

(c)

$$mg = 7810\,\mathrm{N}$$

18.2.67

(c)

$$\begin{aligned} U(r) &= U_0\left[\left(\frac{R_0}{r}\right)^{12} - 2\left(\frac{R_0}{r}\right)^{6}\right] \\ F(r) &= 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{r}\right)^{13} - \left(\frac{R_0}{r}\right)^{7}\right] \end{aligned}$$

$$\begin{aligned} 0 &= U_0\left[\left(\frac{R_0}{r_1}\right)^{12} - 2\left(\frac{R_0}{r_1}\right)^{6}\right] \\ &= \left(\frac{R_0}{r_1}\right)^{12} - 2\left(\frac{R_0}{r_1}\right)^{6} \\ &= \left(\frac{R_0}{r_1}\right)^{6} - 2 \\ 2 &= \left(\frac{R_0}{r_1}\right)^{6} \\ 2r_1^6 &= R_0^6 \\ r_1 &= \frac{1}{\sqrt[6]{2}}R_0 \\ &\approx 0.89R_0 \end{aligned}$$

$$\begin{aligned} 0 &= 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{r_2}\right)^{13} - \left(\frac{R_0}{r_2}\right)^{7}\right] \\ 0 &= \left(\frac{R_0}{r_2}\right)^{13} - \left(\frac{R_0}{r_2}\right)^{7} \\ &= \left(\frac{R_0}{r_2}\right)^{6} - 1 \\ r_2 &= R_0 \end{aligned}$$

$$\frac{r_1}{r_2} = \frac{1}{\sqrt[6]{2}}$$

(d)

$$
\begin{aligned}
W &= \int_{r_2}^{\infty} -F\,dr \\
&= \int_{R_0}^{\infty} -12\frac{U_0}{R_0}\left[\left(\frac{R_0}{r}\right)^{13} - \left(\frac{R_0}{r}\right)^{7}\right] dr \\
&= -12\frac{U_0}{R_0}\left(-\frac{R_0}{12}\right) \\
&= U_0
\end{aligned}
$$

18.2.69

(a)

$$C_V = 2R = 16.63\,\mathrm{J/(mol\,K)}$$

(b) Less than because vibrational energy will play a smaller role.

18.2.71

(a)

$$
\begin{aligned}
\frac{1}{2}mv^2 &\geq \frac{GmM}{R_p} \\
&\geq gmR_p
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{3}{2}kT &\geq mgR_p \\
T_\mathrm{N} &\geq \frac{2mgR_p}{3k} \\
&\geq 1.40 \times 10^5\,\mathrm{K} \\
T_\mathrm{H} &\geq 1.02 \times 10^4\,\mathrm{K}
\end{aligned}
$$

(c)

$$
\begin{aligned}
T_\mathrm{N} &\geq 6.37 \times 10^3\,\mathrm{K} \\
T_\mathrm{H} &\geq 459\,\mathrm{K}
\end{aligned}
$$

(d) Because it's very easy to atmospheric particles to escape.

18.2.73

$$\begin{aligned}
\int_0^\infty v^2 f(v)\,dv &= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^4 e^{-mv^2/2kT}\,dv \\
&= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\frac{3}{2^3(m/2kT)^2}\sqrt{\frac{\pi}{(m/2kT)}} \\
&= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\frac{3}{8}\left(\frac{2kT}{m}\right)^2\sqrt{\frac{2\pi kT}{m}} \\
&= \frac{3kT}{m}
\end{aligned}$$

18.2.75

(b)

$$\begin{aligned}
v_{\text{mp}} &= \sqrt{\frac{2kT}{m}} \\
&= 395\,\text{m/s} \\
f(v_{\text{mp}}) &= 2.10\times 10^{-3} \\
\Delta N &\approx N f(v_{\text{mp}})\Delta v \\
&\approx (4.20\times 10^{-2})N
\end{aligned}$$

(c)

$$\begin{aligned}
7v_{\text{mp}} &= 2765\,\text{m/s} \\
f(7v_{\text{mp}}) &= 1.43\times 10^{-22} \\
\Delta N &\approx (2.85\times 10^{-21})N
\end{aligned}$$

18.2.77

(a)

$$\begin{aligned}
0 &= pA - p_0A - mg \\
p &= p_0 + \frac{mg}{A} \\
&= p_0 + \frac{mg}{\pi r^2}
\end{aligned}$$

(b)

$$\begin{aligned}
p_1 V_1 &= p_2 V_2 \\
p_2 &= \frac{V_1}{V_2} p_1 \\
&= \frac{Ah}{A(h+y)} p_1 \\
&= \frac{h}{h+y} p_1 \\
&\approx \left(1 - \frac{y}{h}\right) p_1 \\
F &= \left(1 - \frac{y}{h}\right) p_1 \pi r^2 - p_0 \pi r^2 - mg \\
&= \left(1 - \frac{y}{h}\right) \left(p_0 + \frac{mg}{\pi r^2}\right) \pi r^2 - p_0 \pi r^2 - mg \\
&= \left(1 - \frac{y}{h}\right) (p_0 \pi r^2 + mg) - p_0 \pi r^2 - mg \\
&= -\frac{y}{h} (p_0 \pi r^2 + mg)
\end{aligned}$$

(c)

$$\begin{aligned}
F &= -kx \\
k &= \frac{1}{h} (p_0 \pi r^2 + mg) \\
\omega &= \sqrt{\frac{k}{m}} \\
&= \sqrt{\frac{1}{h} \left(\frac{p_0 \pi r^2}{m} + g\right)} \\
f &= \frac{\omega}{2\pi} \\
&= \frac{1}{2\pi} \sqrt{\frac{g}{h} \left(1 + \frac{p_0 \pi r^2}{gm}\right)}
\end{aligned}$$

If the displacement is not small the oscillation is not simple harmonic because we can't use the approximation $\frac{h}{h+y} \approx 1 - \frac{y}{h}$.

18.2.81

(a)

$$I = 2mr^2 = 4.1 \times 10^{-46}\,\mathrm{kg\,m^2}$$

(b)

$$\begin{aligned} 2\left(\frac{1}{2}(2m)v_i^2\right) &= 2\left(\frac{1}{2}(2m)v_f^2 + \frac{1}{2}I\omega^2\right) \\ 2mv_i^2 &= 2mv_f^2 + 2mr^2\omega^2 \\ v_i^2 &= v_f^2 + r^2\omega^2 \end{aligned}$$

$$\begin{aligned} -2r(2m)v_i &= -2I\omega \\ 2mrv_i &= 2mr^2\omega \\ v_i &= r\omega \end{aligned}$$

(c)

$$\omega = \frac{v_i}{r}$$

$$\begin{aligned} v_i^2 &= v_f^2 + r^2\left(\frac{v_i}{r}\right)^2 \\ v_f &= 0 \end{aligned}$$

(d)

$$\begin{aligned} v_{\mathrm{rms}} &= \sqrt{\frac{3kT}{m}} \\ &= 514\,\mathrm{m/s} \\ \omega &= 5.47 \times 10^{12}\,\mathrm{rad/s} \end{aligned}$$

18.2.83

(a)

$$\begin{aligned} \lambda &= \frac{V}{4\pi\sqrt{2}r^2N} \\ &= 4.50 \times 10^{11}\,\mathrm{m} \end{aligned}$$

(b)

$$\begin{aligned} v_{\mathrm{rms}} &= \sqrt{\frac{3kT}{m}} \\ &= 704\,\mathrm{m/s} \\ t_{\mathrm{mean}} &= \frac{\lambda}{v_{\mathrm{rms}}} \\ &= 6.39 \times 10^{8}\,\mathrm{s} \\ &= 20\,\mathrm{years} \end{aligned}$$

(c)

$$
\begin{aligned}
pV &= NkT \\
p &= \frac{NkT}{V} \\
&= 1.38 \times 10^{-14}\,\mathrm{Pa}
\end{aligned}
$$

(d)

$$
\begin{aligned}
m_{\text{total}} &= \rho V \\
&= \rho \frac{4}{3}\pi \left(\frac{d}{2}\right)^3 \\
&= 2.96 \times 10^{32}\,\mathrm{kg}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{2}mv^2 &= \frac{Gmm_{\text{total}}}{r} \\
v &= \sqrt{\frac{4Gm_{\text{total}}}{d}} \\
&= 640\,\mathrm{m/s}
\end{aligned}
$$

It would evaporate.

(f)

$$
\begin{aligned}
T_{\text{ISM}} &= \frac{(N/V)_{\text{nebula}}}{(N/V)_{\text{ISM}}} T_{\text{nebula}} \\
&= 2.0 \times 10^5\,\mathrm{K}
\end{aligned}
$$

34 times hotter than the sun.

18.2.85

a

18.2.87

c

19 The First Law of Thermodynamics

19.1 Guided Practice

19.1.1

(a)

$$\begin{aligned}\Delta U &= Q - W \\ Q &= \Delta U + W \\ &= 5.75 \times 10^3\,\mathrm{J}\end{aligned}$$

(b)

$$\begin{aligned}\Delta U &= Q - W \\ &= -3.2 \times 10^4\,\mathrm{J}\end{aligned}$$

(c)

$$\begin{aligned}\Delta U &= Q - W \\ W &= Q - \Delta U \\ &= -1.85 \times 10^3\,\mathrm{J}\end{aligned}$$

19.1.2

(a)

$$W = p(V_2 - V_1) = 155\,\mathrm{J}$$

(b)

$$W = 0$$

(c)

$$W = p(V_2 - V_1) = -375\,\mathrm{J}$$

(d)

$$W = \frac{1}{2}(p_1 + p_2)(V_2 - V_1) = 875\,\mathrm{J}$$

19.1.3

(a)

$$\begin{aligned}W &= p(V_2 - V_1) \\ &= -240\,\mathrm{J} \\ \Delta U &= Q - W \\ &= 1.80 \times 10^3\,\mathrm{J}\end{aligned}$$

(b)

$$\begin{aligned}
W &= p(V_2 - V_1) \\
&= -720\,\mathrm{J} \\
\Delta U &= Q - W \\
Q &= \Delta U + W \\
&= 1.08 \times 10^3\,\mathrm{J}
\end{aligned}$$

19.1.4

(a)

$$Q = mL = 3.43 \times 10^6\,\mathrm{J}$$

(b)

$$W = p(V_2 - V_1) = 3.43 \times 10^5\,\mathrm{J}$$

(c)

$$\Delta U = Q - W = 3.09 \times 10^6\,\mathrm{J}$$

19.1.5

(a)

$$\Delta U = \Delta Q = nC_V \Delta T = 998\,\mathrm{J}$$

(b)

$$\Delta U = \Delta Q = nC_V \Delta T = 748\,\mathrm{J}$$

(c)

$$\Delta U = \Delta Q = nC_V \Delta T = 599\,\mathrm{J}$$

19.1.6

(a)

$$V = \frac{nRT}{p} = 5.24 \times 10^{-2}\,\mathrm{m}^3$$

(b) (i)

$$\begin{aligned}
T &= 327\,^\circ\mathrm{C} \\
\Delta U &= Q \\
&= nC_V \Delta T \\
&= 1.31 \times 10^4\,\mathrm{J}
\end{aligned}$$

(ii)

$$\begin{aligned}
T &= 327\,^\circ\mathrm{C} \\
\Delta U &= Q \\
&= nC_V \Delta T \\
&= 1.31 \times 10^4\,\mathrm{J}
\end{aligned}$$

(iii)

$$\begin{aligned}
T &= 927\,^\circ\mathrm{C} \\
\Delta U &= 3.92 \times 10^4\,\mathrm{J}
\end{aligned}$$

19.1.7

(a)

$$\begin{aligned}
pV &= nRT \\
\frac{pV}{R} &= nT
\end{aligned}$$

$$\begin{aligned}
(2p) &= nR(2T) \\
\Delta T &= T
\end{aligned}$$

$$\begin{aligned}
\Delta U &= Q - W \\
&= nC_V \Delta T \\
&= C_V (nT) \\
&= \frac{3}{2} R \frac{pV}{R} \\
&= \frac{3}{2} pV \\
&= 4.50 \times 10^4\,\mathrm{J}
\end{aligned}$$

(b)

$$\begin{aligned}
pV &= nRT \\
\frac{pV}{R} &= nT
\end{aligned}$$

$$\begin{aligned}
pV &= nRT \\
p\left(\frac{1}{2}V\right) &= nR\left(\frac{1}{2}T\right) \\
\Delta T &= -\frac{1}{2}T
\end{aligned}$$

$$\begin{aligned}
\Delta U &= nC_V\Delta T \\
&= C_V\left(-\frac{1}{2}nT\right) \\
&= -\frac{3}{4}R\frac{pV}{R} \\
&= -\frac{3}{4}pV \\
&= -2.25 \times 10^4 \text{ J}
\end{aligned}$$

(c)

$$\Delta U = 1.17 \times 10^5 \text{ J}$$

19.1.8

(a)

$$\begin{aligned}
Q &= nC_V\Delta T \\
&= \frac{5}{2}nRT
\end{aligned}$$

$$W = 0$$

$$\begin{aligned}
\Delta U &= Q - W \\
&= \frac{5}{2}nRT
\end{aligned}$$

(b)

$$\begin{aligned} Q &= nC_P \Delta T \\ &= \frac{7}{2} nRT \end{aligned}$$

$$W = p(V_2 - V_1)$$

$$\begin{aligned} \Delta U &= \frac{7}{2} nRT - p(V_2 - V_1) \\ &= \frac{7}{2} nRT - 2nRT + nRT \\ &= \frac{5}{2} nRT \end{aligned}$$

(c)

$$Q = 0$$

$$\begin{aligned} W &= nC_V(T_1 - T_2) \\ &= -\frac{5}{2} nRT \end{aligned}$$

$$\begin{aligned} \Delta U &= Q - W \\ &= \frac{5}{2} nRT \end{aligned}$$

19.1.9

(a)

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

(b)

$$\begin{aligned} p_1 V_1^\gamma &= p_2 V_2^\gamma \\ p_2 &= \left(\frac{V_1}{V_2}\right)^\gamma p_1 \\ &= 6.41 \times 10^4 \, \mathrm{Pa} \end{aligned}$$

(c)

$$\begin{aligned} W &= \frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2) \\ &= 623 \, \mathrm{J} \end{aligned}$$

19.1.10

(a)

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5}$$

(b)

$$\begin{aligned} T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ V_2^{\gamma-1} &= \frac{T_1}{T_2} V_1^{\gamma-1} \\ V_2 &= \left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} V_1 \\ &= 5.79 \times 10^{-4}\,\mathrm{m}^3 \end{aligned}$$

(c)

$$\begin{aligned} p_1 V_1^{\gamma} &= p_2 V_2^{\gamma} \\ p_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma} p_1 \\ &= 2.95 \times 10^{6}\,\mathrm{Pa} \end{aligned}$$

(d)

$$\begin{aligned} W &= \frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2) \\ &= -2.65 \times 10^{3}\,\mathrm{J} \end{aligned}$$

19.1.11

(a)

$$\begin{aligned} pV &= nRT \\ p &= \frac{nRT}{V} \\ &= 3.17 \times 10^{5}\,\mathrm{Pa} \end{aligned}$$

(b)

$$\begin{aligned} p_1 V_1^{\gamma} &= p_2 V_2^{\gamma} \\ p_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma} p_1 \\ &= 8.21 \times 10^{4}\,\mathrm{Pa} \end{aligned}$$

(c)

$$
\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 \\
&= 178\,\mathrm{K}
\end{aligned}
$$

(d)

$$
\begin{aligned}
W &= \frac{1}{\gamma-1}(p_1 V_1 - p_2 V_2) \\
&= 7.94 \times 10^3\,\mathrm{J}
\end{aligned}
$$

19.1.12

(a)

$$
\begin{aligned}
\left[p + \left(\frac{an^2}{V^2}\right)\right](V - nb) &= nRT \\
p + \left(\frac{an^2}{V^2}\right) &= \frac{nRT}{V - nb} \\
p &= \frac{nRT}{V - nb} - \frac{an^2}{V^2}
\end{aligned}
$$

$$
\begin{aligned}
W &= \int_{V_1}^{V_2} p\, dV \\
&= \int_{V_1}^{V_2} \left(\frac{nRT}{V - nb} - \frac{an^2}{V^2}\right) dV \\
&= \left[nRT \ln(V - nb) + \frac{an^2}{V}\right]_{V_1}^{V_2} \\
&= nRT \ln(V_2 - nb) + \frac{an^2}{V_2} - nRT \ln(V_1 - nb) - \frac{an^2}{V_1} \\
&= nRT \ln \frac{V_2 - nb}{V_1 - nb} + an^2 \frac{V_1 - V_2}{V_1 V_2}
\end{aligned}
$$

(b) (i)

$$
W = 2.80 \times 10^3\,\mathrm{J}
$$

(ii)

$$\begin{aligned}W &= \int_{V_1}^{V_2} p\,dV \\ &= \int_{V_1}^{V_2} \frac{nRT}{V}\,dV \\ &= nRT[\ln V]_{V_1}^{V_2} \\ &= 3.11 \times 10^3\,\mathrm{J}\end{aligned}$$

19.2 Exercises and Problems

19.2.1

(b)

$$\begin{aligned}W &= p(V_2 - V_1) \\ &= nR(T_2 - T_1) \\ &= 1.33 \times 10^3\,\mathrm{J}\end{aligned}$$

19.2.3

(b)

$$\begin{aligned}p_1V_1 &= nRT \\ p_2V_2 &= nRT \\ 3p_1V_2 &= nRT \\ V_2 &= \frac{1}{3}V_1 \\ W &= \int_{V_1}^{V_2} p\,dV \\ &= \int_{V_1}^{V_1/3} \frac{nRT}{V}\,dV \\ &= nRT \ln\frac{1}{3} \\ &= -6.18 \times 10^3\,\mathrm{J}\end{aligned}$$

19.2.5

(a)

$$\begin{aligned}
pV &= nRT \\
V &= \frac{nRT}{p}
\end{aligned}$$

$$\begin{aligned}
W &= \int_{V_1}^{V_2} p\,dV \\
&= \int_{nRT/p_1}^{nRT/p_2} \frac{nRT}{V}\,dV \\
&= nRT \ln \frac{nRT/p_2}{nRT/p_1} \\
&= nRT \ln \frac{p_1}{p_2} \\
\frac{W}{nRT} &= \ln \frac{p_1}{p_2} \\
p_1 &= p_2 e^{W/nRT} \\
&= 1.05 \times 10^5\,\mathrm{Pa} \\
&= 1.04\,\mathrm{atm}
\end{aligned}$$

19.2.9

(a)

$$W = p(V_2 - V_1) = 3.47 \times 10^4\,\mathrm{J}$$

(b)

$$\Delta U = Q - W = 8.03 \times 10^4\,\mathrm{J}$$

(c) No, because it's an isobaric process.

19.2.11

(a)

$$\begin{aligned}
T_a &= \frac{pV}{nR} \\
&= 278\,\mathrm{K} \\
T_b &= 694\,\mathrm{K} \\
T_c &= 1250\,\mathrm{K}
\end{aligned}$$

The lowest temperature is 278 K and it occured at point a.

(b)

$$\begin{aligned} W_{\text{ab}} &= 0 \\ W_{\text{bc}} &= 162\,\text{J} \end{aligned}$$

(c)

$$\Delta U = Q - W = 52\,\text{J}$$

19.2.13

(a)

$$\begin{aligned} T_a &= \frac{pV}{nR} \\ &= 5.35 \times 10^2\,\text{K} \\ T_b &= 9.36 \times 10^3\,\text{K} \\ T_c &= 1.50 \times 10^4\,\text{K} \end{aligned}$$

(b)

$$W = 2.10 \times 10^4\,\text{J}$$

(c)

$$Q = \Delta U + W = 3.60 \times 10^4\,\text{J}$$

19.2.17

(b)

$$\begin{aligned} V_1 &= \frac{nRT_1}{p_1} \\ &= 6.18 \times 10^{-3}\,\text{m}^3 \\ V_2 &= 8.23 \times 10^{-3}\,\text{m}^3 \\ W &= p(V_2 - V_1) \\ &= 207\,\text{J} \end{aligned}$$

(c) The piston

(d)

$$\begin{aligned} C_V &= \frac{5}{2}R \\ C_P &= \frac{7}{2}R \\ \Delta U &= nC_P\Delta T \\ &= 727\,\text{J} \\ Q &= \Delta U + W \\ &= 934\,\text{J} \end{aligned}$$

19.2.19

(a)

$$\begin{aligned}\Delta U &= Q - W\\ &= Q - 0\\ &= nC_V\Delta T\\ \Delta T &= \frac{\Delta U}{nC_V}\\ &= 168\,\mathrm{K}\\ T_2 &= T_1 + \Delta T\\ &= 948\,\mathrm{K}\end{aligned}$$

(b)

$$\begin{aligned}Q &= nC_P\Delta T\\ \Delta T &= \frac{Q}{nC_P}\\ &= 120\,\mathrm{K}\\ T_2 &= T_1 + \Delta T\\ &= 900\,\mathrm{K}\end{aligned}$$

19.2.21

$$\begin{aligned}C_V &= \frac{3}{2}R\\ C_P &= \frac{5}{2}R\\ Q &= nC_P\Delta T\\ &= \frac{5}{2}nR(T_2 - T_1)\\ W &= p(V_2 - V_1)\\ &= nR(T_2 - T_1)\\ \frac{W}{Q} &= \frac{2}{5}\end{aligned}$$

19.2.23

(a)

$$\begin{aligned}\Delta U &= Q - W\\ &= 747\,\mathrm{J}\end{aligned}$$

(b)

$$
\begin{aligned}
Q &= nC_P\Delta T \\
C_P &= \frac{Q}{n\Delta T} \\
&= 37.0\,\mathrm{J/(mol\,k)} \\
C_V &= C_P - R \\
&= 28.6\,\mathrm{J/(mol\,K)} \\
\gamma &= \frac{C_P}{C_V} \\
&= 1.29
\end{aligned}
$$

19.2.25

(a)

$$
\begin{aligned}
V_1 &= \frac{nRT}{p_1} \\
&= 3.46 \times 10^{-3}\,\mathrm{m}^3 \\
V_2 &= 8.64 \times 10^{-4}\,\mathrm{m}^3 \\
W &= \int_{V_1}^{V_2} p\,dV \\
&= \int_{V_1}^{V_2} \frac{nRT}{V}\,dV \\
&= nRT \ln \frac{V_2}{V_1} \\
&= -606\,\mathrm{J}
\end{aligned}
$$

(b)

$$\Delta U = 0\,\mathrm{J}$$

(c) Yes, liberate

$$Q = \Delta U + W = -606\,\mathrm{J}$$

19.2.27

(a)

$$\begin{aligned} C_V &= \frac{3}{2}R \\ C_P &= \frac{5}{2}R \\ \gamma &= \frac{C_P}{C_V} \\ &= \frac{5}{3} \\ p_1V_1^\gamma &= p_2V_2^\gamma \\ p_2 &= \left(\frac{V_1}{V_2}\right)^\gamma p_1 \\ &= 4.76 \times 10^5\,\mathrm{Pa} \end{aligned}$$

(b)

$$\begin{aligned} W &= \frac{C_V}{R}(p_1V_1 - p_2V_2) \\ &= -1.06 \times 10^4\,\mathrm{J} \end{aligned}$$

(c)

$$\begin{aligned} T_1V_1^{\gamma-1} &= T_2V_2^{\gamma-1} \\ \frac{T_2}{T_1} &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} \\ &= 1.59 \end{aligned}$$

Heated

19.2.29

(b)

$$\begin{aligned} W &= nC_V(T_1 - T_2) \\ &= 314\,\mathrm{J} \end{aligned}$$

(c)

$$\begin{aligned} \Delta U &= Q - W \\ &= 0 - W \\ &= -314\,\mathrm{J} \end{aligned}$$

19.2.31

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_1 \left(\frac{nRT_1}{p_1}\right)^{\gamma-1} &= T_2 \left(\frac{nRT_2}{p_2}\right)^{\gamma-1} \\
T_2^{\gamma} &= T_1^{\gamma} \left(\frac{p_2}{p_1}\right)^{\gamma-1} \\
T_2 &= T_1 \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} \\
&= 285\,\mathrm{K} \\
&= 11.6\,^{\circ}\mathrm{C}
\end{aligned}$$

19.2.33

$$\begin{aligned}
C_V &= \frac{3}{2}R \\
C_P &= \frac{5}{2}R \\
\gamma &= \frac{5}{3} \\
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_1 \left(\frac{nRT_1}{p_1}\right)^{\gamma-1} &= 2T_1 \left(\frac{2nRT_1}{p_2}\right)^{\gamma-1} \\
\frac{1}{p_1^{\gamma-1}} &= \frac{2^{\gamma}}{p_2^{\gamma-1}} \\
p_1^{\gamma-1} &= \frac{p_2^{\gamma-1}}{2^{\gamma}} \\
p_2 &= 2^{\gamma/(\gamma-1)} p_1 \\
&= 2^{5/2} p_1 \\
&= 4\sqrt{2} p_1
\end{aligned}$$

19.2.35

(a) Increase

(b)

$$W = \frac{1}{2}(p_a + p_b)(V_B - V_A) = 4.8\,\mathrm{kJ}$$

19.2.37

(a)

$$\begin{aligned} pV &= nRT \\ n &= \frac{pV}{RT} \\ &= 0.678\,\mathrm{mol} \end{aligned}$$

(b)

$$\begin{aligned} pV &= nRT \\ V &= \frac{nRT}{p} \\ &= 3.33 \times 10^{-2}\,\mathrm{m}^3 \end{aligned}$$

(c)

$$\begin{aligned} W &= \int_{V_1}^{V_2} p\,dV \\ &= \int_{V_1}^{V_2} \frac{nRT}{V}\,dV \\ &= nRT \ln \frac{V_2}{V_1} \\ &= 2.22\,\mathrm{kJ} \end{aligned}$$

(d)

$$\Delta U = 0$$

19.2.39

(a)

$$\begin{aligned} \Delta U &= Q - W \\ &= 30.0\,\mathrm{J} \\ Q &= \Delta U + W \\ &= 45.0\,\mathrm{J} \end{aligned}$$

(b) Liberate

$$Q = \Delta U + W = -65.0\,\mathrm{J}$$

(c)

$$\begin{aligned}
\Delta U_{\mathrm{ad}} &= 8.0\,\mathrm{J} \\
W_{\mathrm{ad}} &= 15.0\,\mathrm{J} \\
Q_{\mathrm{ad}} &= \Delta U_{\mathrm{ad}} + W_{\mathrm{ad}} \\
&= 23.0\,\mathrm{J} \\
Q_{\mathrm{db}} &= \Delta U_{\mathrm{ab}} - \Delta U_{\mathrm{ad}} \\
&= 22.0\,\mathrm{J}
\end{aligned}$$

19.2.43

(a)

$$\begin{aligned}
p_1 V_1 &= p_2 V_2 \\
V_2 &= \frac{p_1}{p_2} V_1 \\
&= 8.0 \times 10^{-4}\,\mathrm{m}^3 \\
&= 0.80\,\mathrm{L}
\end{aligned}$$

(b)

$$\begin{aligned}
T_a &= \frac{pV}{nR} \\
&= 304\,\mathrm{K} \\
T_b &= 1.21 \times 10^3\,\mathrm{K} \\
T_c &= 1.21 \times 10^3\,\mathrm{K}
\end{aligned}$$

(c)

$$
\begin{aligned}
\Delta U_{\text{ab}} &= Q_{\text{ab}} - W_{\text{ab}} \\
&= Q_{\text{ab}} \\
&= nC_V \Delta T \\
&= 74.0\,\text{J into the gas}
\end{aligned}
$$

$$
\begin{aligned}
V_c &= \frac{nRT_c}{p_c} \\
&= 7.97 \times 10^{-4}\,\text{m}^3 \\
\Delta U_{\text{ca}} &= Q_{\text{ca}} - W_{\text{ca}} \\
nC_V \Delta T &= Q_{\text{ca}} - p(V_a - V_c) \\
Q_{\text{ca}} &= nC_V \Delta T + p(V_a - V_c) \\
&= -104\,\text{J out of the gas}
\end{aligned}
$$

$$
\begin{aligned}
\Delta U_{\text{bc}} &= Q_{\text{bc}} - W_{\text{bc}} \\
Q_{\text{bc}} &= \Delta U_{\text{bc}} + W_{\text{bc}} \\
&= nC_V \Delta T + \int_{V_b}^{V_c} p\,dV \\
&= nRT \ln \frac{V_c}{V_b} \\
&= 55.6\,\text{J into the gas}
\end{aligned}
$$

(d)

$$
\Delta U_{\text{ab}} = 74.0\,\text{J increase}
$$

$$
\Delta U_{\text{bc}} = 0.0\,\text{J no change}
$$

$$
\begin{aligned}
\Delta U_{\text{ca}} &= nC_V \Delta T \\
&= -74.0\,\text{J decrease}
\end{aligned}
$$

19.2.47

(b)

$$
V_2 = \frac{p_1}{p_2} V_1 = 6.0\,\text{L}
$$

(c)

$$
\begin{aligned}
n &= \frac{pV}{RT} \\
&= 6.01 \times 10^{-2}\,\mathrm{mol} \\
W_{12} &= \int_{V_1}^{V_2} p\,dV \\
&= nRT_1 \ln \frac{V_2}{V_1} \\
&= 208\,\mathrm{J}
\end{aligned}
$$

$$
\begin{aligned}
W_{23} &= p_2(V_3 - V_2) \\
&= -113\,\mathrm{J}
\end{aligned}
$$

$$
W = 95.0\,\mathrm{J}
$$

(d) Heat it at constant volume

19.2.49

(a) As the wind descends it experiences greater atmospheric pressure. This compresses the wind, increasing its temperature. It's important that it be moving fast so it's an adiabatic process.

(b)

$$
\begin{aligned}
T_2 &= \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} T_1 \\
&= 287\,\mathrm{K} \\
&= 13.9\,^\circ\mathrm{C} \\
\Delta T &= T_2 - T_1 \\
&= 11.9\,\mathrm{C}^\circ
\end{aligned}
$$

19.2.51

(a)

$$\begin{aligned}
p_1 V_1^\gamma &= p_2 V_2^\gamma \\
p_1 (Ah)^\gamma &= p_2 [A(h - \Delta h)]^\gamma \\
\frac{p_1}{p_2} h^\gamma &= (h - \Delta h)^\gamma \\
\left(\frac{p_1}{p_2}\right)^{1/\gamma} h &= h - \Delta h \\
\Delta h &= h \left[1 - \left(\frac{p_1}{p_2}\right)^{1/\gamma}\right] \\
&= 16.8\,\text{cm}
\end{aligned}$$

(b)

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 \\
&= \left(\frac{Ah}{A(h - \Delta h)}\right)^{\gamma-1} T_1 \\
&= \left(\frac{1}{1 - \Delta h/h}\right)^{\gamma-1} T_1 \\
&= 469\,\text{K} \\
&= 196\,^\circ\text{C}
\end{aligned}$$

(c)

$$W = nC_V(T_1 - T_2) = -7.03 \times 10^4\,\text{J}$$

19.2.59

(a) a is abiabatic, b is isochoric, c is isobaric

(b)

$$\begin{aligned}\Delta U &= Q_b - W_b \\ &= Q_b - 0 \\ &= Q_b\end{aligned}$$

$$\begin{aligned}\Delta U &= Q_c - W_c \\ &= Q_c - p(V_2 - V_1) \\ &= Q_c - nR(T_2 - T_1)\end{aligned}$$

$$\begin{aligned}Q_b &= Q_c - nR(T_2 - T_1) \\ T_2 &= T_1 + \frac{Q_c - Q_b}{nR} \\ &= 28.0\,^\circ\mathrm{C}\end{aligned}$$

(c)

$$\begin{aligned}Q_b &= nC_V\Delta T \\ C_V &= \frac{Q_b}{n\Delta T} \\ &= 12.5\,\mathrm{J/(mol\,K)}\end{aligned}$$

$$\begin{aligned}W_a &= nC_V(T_1 - T_2) \\ &= -30.0\,\mathrm{J}\end{aligned}$$

$$W_b = 0$$

$$\begin{aligned}\Delta U_c &= Q_c - W_c \\ W_c &= Q_c - \Delta U_c \\ &= Q_c - nC_V\Delta T \\ &= 20.0\,\mathrm{J}\end{aligned}$$

(d)

$$\begin{aligned}\gamma &= \frac{C_P}{C_V} \\ &= \frac{C_V + R}{C_V} \\ &= 1.67\end{aligned}$$

$$\begin{aligned}T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ \left(\frac{V_2}{V_1}\right)^{\gamma-1} &= \frac{T_1}{T_2} \\ \frac{V_2}{V_1} &= \left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} \\ &= 0.961\end{aligned}$$

$$\Delta V_b = 0$$

$$\begin{aligned}\frac{V_2}{V_1} &= \frac{nRT_2/p}{nRT_1/p} \\ &= \frac{T_2}{T_1} \\ &= 1.03\end{aligned}$$

a

(e) Decrease, stay the same, increase

19.2.61

(a)

$$
\begin{aligned}
r &= 1.50\,\mathrm{cm} \\
l_{\max} &= 30.0\,\mathrm{cm} \\
l_{\min} &= l_{\max}/v \\
p &= 101\,\mathrm{kPa} \\
T &= 30.0\,^{\circ}\mathrm{C} \\
V_1 &= \pi r^2 l_{\max} \\
&= 2.12 \times 10^{-4}\,\mathrm{m}^3 \\
V_2 &= \pi r^2 l_{\min} \\
&= \pi r^2 \frac{l_{\max}}{v} \\
&= \frac{V_1}{v} \\
n &= \frac{pV}{RT} \\
&= 8.50 \times 10^{-3}\,\mathrm{mol} \\
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_2 &= T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} \\
&= T_1 v^{\gamma-1} \\
W_{\text{adiabatic}} &= nC_V(T_1 - T_2) \\
&= nC_V(T_1 - T_1 v^{\gamma-1}) \\
&= nC_V T_1 (1 - v^{\gamma-1}) \\
&= 53.5(1 - v^{0.4}) \\
W_{\text{isothermal}} &= \int_{V_1}^{V_2} p\,dV \\
&= \int_{V_1}^{V_2} \frac{nRT_2}{V}\,dV \\
&= nRT_2 \ln\frac{V_2}{V_1} \\
&= nRT_1 v^{\gamma-1} \ln v \\
&= 21.4 v^{0.4} \ln v \\
W &= 53.5(1 - v^{0.4}) + 21.4 v^{0.4} \ln v \\
&= 53.5 + v^{0.40}(21.4 \ln v - 53.5)
\end{aligned}
$$

(b)

$$\begin{aligned}
T_2 &\le T_{\max} \\
T_1 v^{\gamma-1} &\le T_{\max} \\
v &\le \left(\frac{T_{\max}}{T_1}\right)^{1/(\gamma-1)} \\
&\le 7.35
\end{aligned}$$

The largest integer value of v is 7.

(c) 7

(d) 7

(e)

$$\begin{aligned}
T_2 &= T_1 v^{\gamma-1} \\
&= 660\,\mathrm{K} \\
&= 387\,^\circ\mathrm{C} \\
Q &= nC_V\Delta T \\
&= -63.0\,\mathrm{J}
\end{aligned}$$

19.2.63

$$\begin{aligned}
\frac{p_1}{T_1} &= \frac{p_2}{T_2} \\
p_2 &= \frac{T_2}{T_1}p_1 \\
&= 1.27\times 10^7\,\mathrm{Pa} \\
&= 1.84\times 10^3\,\mathrm{psi}
\end{aligned}$$

c

19.2.65

$$\begin{aligned}
p_1V_1 &= p_2V_2 \\
V_1 &= \frac{p_2}{p_1}V_2 \\
&= 6.01\times 10^{-5}\,\mathrm{m}^3 \\
&= 6.01\times 10^{-2}\,\mathrm{L}
\end{aligned}$$

d

20 The Second Law of Thermodynamics

20.1 Guided Practice

20.1.1

(a)

$$W = eQ_H \Rightarrow Q_H = \frac{W}{e} = 6.89 \times 10^4\,\mathrm{J}$$

(b)

$$|W| = |Q_H| - |Q_C| \Rightarrow |Q_C| = |Q_H| - |W| = 5.65 \times 10^4\,\mathrm{J}$$

20.1.2

$$e = \frac{W}{Q_H} = \frac{Q_H + Q_C}{Q_H} = 1 + \frac{Q_C}{Q_H} = 0.173 = 17.3\%$$

20.1.3

(a)

$$(1-e)Q_H = Q_C \Rightarrow Q_H = \frac{Q_C}{1-e} = 6.17 \times 10^8\,\mathrm{J}$$

(b)

$$W = eQ_H = 1.21 \times 10^8\,\mathrm{J}$$

20.1.4

(a)

$$W = 3600P = 3.96 \times 10^8\,\mathrm{J}$$

(b)

$$Q_H = mL_c = 1.70 \times 10^9\,\mathrm{J}$$

(c)

$$e = \frac{W}{Q_H} = 0.233 = 23.3\%$$

20.1.5

(a)

$$\begin{aligned} e_{\text{Carnot}} &= 1 - \frac{T_C}{T_H} \\ &= 1 + \frac{Q_C}{Q_H} \\ &= 1 + \frac{W - Q_H}{Q_H} \\ &= 0.21 \end{aligned}$$

(b)

$$|Q_C| = |Q_H| - |W| = 6.32 \times 10^4\,\mathrm{J}$$

(c)

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H} \Rightarrow T_H = -\frac{Q_H}{Q_C} T_C = 377\,\mathrm{K} = 104\,^\circ\mathrm{C}$$

20.1.6

(a)

$$e = 1 - \frac{T_C}{T_H} = 0.6$$

(b)

$$
\begin{aligned}
n &= 0.200\,\mathrm{mol} \\
\gamma &= 1.40 \\
T_H &= 227\,^\circ\mathrm{C} = 500\,\mathrm{K} \\
T_C &= -73\,^\circ\mathrm{C} = 200\,\mathrm{K} \\
p_a &= 10.0 \times 10^5\,\mathrm{Pa} \\
V_a &= \frac{nRT_H}{p_a} \\
&= 8.31 \times 10^{-4}\,\mathrm{m}^3 \\
V_b &= 2V_a \\
&= 1.66 \times 10^{-3}\,\mathrm{m}^3 \\
p_b &= \frac{nRT_H}{V_b} \\
&= 5.01 \times 10^5\,\mathrm{Pa} \\
W_{\mathrm{ab}} &= \int_{V_a}^{V_b} p\,dV \\
&= nRT_H \ln 2 \\
&= 576\,\mathrm{J} \\
V_c &= \left(\frac{T_H}{T_C}\right)^{1/(\gamma-1)} V_b \\
&= 1.64 \times 10^{-2}\,\mathrm{m}^3 \\
p_c &= \frac{nrT_C}{V_c} \\
&= 2.03 \times 10^4\,\mathrm{Pa} \\
W_{\mathrm{bc}} &= \frac{1}{\gamma-1}(p_bV_b - p_cV_c) \\
&= 1.25\,\mathrm{kJ} \\
V_d &= \frac{1}{2}V_c \\
&= 8.20 \times 10^{-3}\,\mathrm{m}^3 \\
p_d &= 4.06 \times 10^4\,\mathrm{Pa} \\
W_{\mathrm{cd}} &= \int_{V_c}^{V_d} p\,dV \\
&= nRT_C \ln \frac{1}{2} \\
&= -231\,\mathrm{J} \\
W_{\mathrm{da}} &= \frac{1}{\gamma-1}(p_dV_d - p_aV_a) \\
&= -1.25\,\mathrm{kJ}
\end{aligned}
$$

20.1.7

(a)

$$K = \frac{T_C}{T_H - T_C} = 7.52$$

(b)

$$W = \frac{Q_C}{K} = 5.32 \times 10^5\,\mathrm{J}$$

20.1.8

(a)

$$\begin{aligned} W &= \int_{V_a}^{V_b} p\,dV \\ &= \int_{V_a}^{2V_a} \frac{nRT_H}{V}\,dV \\ &= nRT_H \ln 2 \end{aligned}$$

(b)

$$\begin{aligned} W &= nC_V(T_H - T_C) \\ &= \frac{3}{2}nR(T_H - T_C) \end{aligned}$$

(c)

$$\begin{aligned} nRT_H \ln 2 &= \frac{3}{2}nR(T_H - T_C) \\ \ln 2 &= \frac{3}{2}\left(1 - \frac{T_C}{T_H}\right) \\ \frac{T_C}{T_H} &= 1 - \frac{2}{3}\ln 2 \\ e &= 1 - \frac{T_C}{T_H} \\ &= \frac{2}{3}\ln 2 \\ &= 0.462 \end{aligned}$$

20.1.9

(a)

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = 655\,\mathrm{J/K}$$

(b)

$$\Delta S = mc\int_{159}^{351} \frac{dT}{T} = 1.92 \times 10^3\,\mathrm{J/K}$$

(c)

$$\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = 2.43 \times 10^3\,\mathrm{J/K}$$

20.1.10

(a)

$$\begin{aligned}
n &= 5.00\,\mathrm{mol}\\
V_1 &= 0.120\,\mathrm{m}^3\\
T_1 &= 20.0\,{}^\circ\mathrm{C}\\
V_2 &= 0.360\,\mathrm{m}^3\\
T_2 &= 20.0\,{}^\circ\mathrm{C}\\
\Delta U &= nC_V\Delta T\\
&= 0\\
Q &= W\\
&= \int_{V_1}^{V_2} p\,dV\\
&= \int_{V_1}^{V_2} \frac{nRT}{V}\,dV\\
&= nRT\ln\frac{V_2}{V_1}\\
&= 1.34 \times 10^4\,\mathrm{J}\\
\Delta S &= \frac{Q}{T}\\
&= 45.7\,\mathrm{J/K}
\end{aligned}$$

(b) Change in entropy is path independent, so $\Delta S = 45.7\,\mathrm{J/K}$.

20.1.11

(a)

$$\Delta S = 0$$

(b)

$$\Delta S = \frac{Q}{T} = -150\,\mathrm{J/K}$$

(c)

$$\Delta S = \frac{Q}{T} = 218\,\mathrm{J/K}$$

(d)

$$\Delta S = 68\,\mathrm{J/K}$$

The net entropy increases.

20.1.12

(a)

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = 1.22 \times 10^3\,\mathrm{J/K}$$

(b)

$$\begin{aligned}\Delta S &= \int \frac{dQ}{T} \\ &= mc \int_{368}^{273} \frac{dT}{T} \\ &= -1.05 \times 10^3\,\mathrm{J/K}\end{aligned}$$

(c)

$$\Delta S = 160\,\mathrm{J/K}$$

The net entropy increases.

20.1.13

(a)

$$\begin{aligned}0 &= m_i L_f + m_w c(T - T_w) + m_i c(T - T_i) \\ T &= \frac{(m_w T_w + m_i T_i)c - m_i L_f}{(m_w + m_i)c} \\ &= 307\,\mathrm{K} \\ &= 34.3\,^\circ\mathrm{C}\end{aligned}$$

(b)

$$\begin{aligned}\Delta S_i &= \frac{Q}{T_1} + \int \frac{dQ}{T} \\ &= \frac{m_i L_f}{T_1} + m_i c \ln \frac{T_2}{T_1} \\ &= 101\,\mathrm{J/K} \\ \Delta S_w &= m_w c \ln \frac{T_2}{T_1} \\ &= -86.0\,\mathrm{J/K} \\ \Delta S &= 15.0\,\mathrm{J/K}\end{aligned}$$

20.2 Exercises and Problems

20.2.5

(a)

$$
\begin{aligned}
p_1 V_1^\gamma &= p_2 V_2^\gamma \\
p_1 &= \left(\frac{V_2}{V_1}\right)^\gamma p_2 \\
&= 12.3\,\mathrm{atm}
\end{aligned}
$$

(b) It enters during process ca.

$$
\begin{aligned}
T_a &= \frac{p_a V_a}{nR} \\
&= 1.20 \times 10^3\,\mathrm{K} \\
T_c &= \frac{p_c V_c}{nR} \\
&= 146\,\mathrm{K} \\
\gamma &= \frac{C_P}{C_V} \\
&= \frac{C_V + R}{C_V} \\
C_V \gamma &= C_V + R \\
C_V(\gamma - 1) &= R \\
C_V &= \frac{R}{\gamma - 1} \\
Q &= n C_V \Delta T \\
&= 5.48\,\mathrm{kJ}
\end{aligned}
$$

(c) It leaves during bc.

$$
\begin{aligned}
T_b &= \frac{p_b V_b}{nR} \\
&= 656\,\mathrm{K} \\
\Delta U &= Q - W \\
Q &= \Delta U + W \\
&= n C_V \Delta T + p_b(V_c - V_b) \\
&= -3.71\,\mathrm{kJ}
\end{aligned}
$$

(d)

$$\begin{aligned}
W &= W_{\text{ab}} + W_{\text{bc}} + W_{\text{ca}} \\
&= nC_V(T_a - T_b) + p_b(V_c - V_b) + 0 \\
&= 1.77\,\text{kJ}
\end{aligned}$$

(e)

$$e = \frac{W}{Q_H} = 0.323 = 32.3\%$$

20.2.7

(a)

$$\begin{aligned}
e &= 1 - \frac{1}{r^{\gamma-1}} \\
r^{\gamma-1} &= \frac{1}{1-e} \\
r &= \left(\frac{1}{1-e}\right)^{1/(\gamma-1)} \\
&= 12
\end{aligned}$$

(b)

$$\begin{aligned}
Q_C &= 7.4\,\text{kJ} \\
Q_H &= 20\,\text{kJ}
\end{aligned}$$

20.2.9

(a)

$$e = 1 - \frac{1}{r^{\gamma-1}} = 0.581 = 58.1\%$$

(b)

$$\begin{aligned}
e' &= 0.595 = 59.5\% \\
\Delta e &= 0.014 \\
&= 1.4\%
\end{aligned}$$

20.2.11

$$\begin{aligned}
K &= 2.25 \\
W &= 135\,\mathrm{W} \\
H &= KW \\
&= 304\,\mathrm{W} \\
Q &= mc\Delta T \\
&= 1.30 \times 10^{6}\,\mathrm{J} \\
\frac{Q}{H} &= 4.29 \times 10^{3}\,\mathrm{s} \\
&= 1.2\,\mathrm{h}
\end{aligned}$$

20.2.13

$$\begin{aligned}
T_H &= T_C + 72.0\,\mathrm{C}^\circ \\
e &= 1 - \frac{T_C}{T_H} \\
&= 1 - \frac{T_H - 72.0\,\mathrm{C}^\circ}{T_H} \\
&= 1 - 1 + \frac{72\,\mathrm{C}^\circ}{T_H} \\
T_H &= \frac{72\,\mathrm{C}^\circ}{e} \\
&= 576\,\mathrm{K} \\
T_C &= 504\,\mathrm{K}
\end{aligned}$$

20.2.21

$$\begin{aligned}
\frac{T_{CA}}{T_{HA} - T_{CA}} &= 1.16 \frac{T_{CB}}{T_{HB} - T_{CB}} \\
(T_{HB} - T_{CB}) &= 1.3(T_{HA} - T_{CA}) \\
T_{CB} &= 180\,\mathrm{K} \\
\frac{T_{CA}}{T_{HA} - T_{CA}} &= 1.16 \frac{T_{CB}}{1.3(T_{HA} - T_{CA})} \\
T_{CA} &= \frac{1.16}{1.3} T_{CB} \\
&= 161\,\mathrm{K}
\end{aligned}$$

20.2.29

$$\begin{aligned}
\Delta S &= S' - S \\
&= k \ln w' - k \ln w \\
&= k \ln \frac{w'}{w} \\
&= k \ln \frac{(425/0.0024)^N w}{w} \\
&= k \ln (1.77 \times 10^5)^{nN_A} \\
&= 10.0\,\mathrm{J/K}
\end{aligned}$$

20.2.31

(a)

$$\begin{aligned}
\frac{Q_C}{Q_H} &= -\frac{T_C}{T_H} \\
Q_C &= -\frac{T_C}{T_H} Q_H \\
&= -121\,\mathrm{J}
\end{aligned}$$

(b)

$$\begin{aligned}
n &= \frac{U}{W} \\
&= \frac{mgh}{Q_H + Q_C} \\
&= 3.80 \times 10^3
\end{aligned}$$

20.2.33

(a)

$$W = eQ_H = 90.2\,\mathrm{J}$$

(b)

$$Q_C = Q_H - W = 320\,\mathrm{J}$$

(c)

$$T_C = T_H(1 - e) = 318\,\mathrm{K} = 45\,^\circ\mathrm{C}$$

(d) 0

(e)

$$m = \frac{W}{gh} = 0.263\,\mathrm{kg}$$

20.2.37

(a)

$$\begin{aligned} T_A &= \frac{p_A V_A}{nR} \\ &= 241\,\mathrm{K} \\ T_B &= 241\,\mathrm{K} \end{aligned}$$

(b) Absorbed during bc, rejected during ab and ca.

(c)

$$T_C = 481\,\mathrm{K}$$

(d)

$$\begin{aligned} W_{AB} &= \int_{V_A}^{V_B} p\,dV \\ &= nRT_A \ln \frac{V_B}{V_A} \\ &= -1389\,\mathrm{J} \\ Q_{AB} &= -1389\,\mathrm{J} \\ W_{BC} &= p_B(V_C - V_B) \\ &= 2000\,\mathrm{J} \\ \Delta U_{BC} &= Q_{BC} - W_{BC} \\ Q_{BC} &= \Delta U_{BC} + W_{BC} \\ &= nC_V \Delta T + W_{BC} \\ &= nC_V(T_C - T_B) + W_{BC} \\ &= \frac{5}{2} nR(T_C - T_B) + W_{BC} \\ &= 6988\,\mathrm{J} \\ W_{CA} &= 0 \\ Q_{CA} &= nC_V \Delta T \\ &= nC_V(T_A - T_C) \\ &= \frac{5}{2} nR(T_A - T_C) \\ &= -4988\,\mathrm{J} \\ Q_{\text{net}} &= 611\,\mathrm{J} \\ W_{\text{net}} &= 611\,\mathrm{J} \end{aligned}$$

(e)

$$e = \frac{W}{Q_H} = 0.0874 = 8.7\%$$

20.2.41

(a)

$$e = 1 - \frac{T_C}{T_H} = 7.0\%$$

(b)

$$\begin{aligned} P &= eQ_H \\ Q_H &= \frac{P}{e} \\ &= 3.0\,\mathrm{MW} \\ Q_C &= Q_H(1-e) \\ &= 2.8\,\mathrm{MW} \end{aligned}$$

(c)

$$\begin{aligned} Q_C &= mc\Delta T \\ m &= \frac{Q_C}{c\Delta T} \\ &= 167\,\mathrm{kg/s} \\ &= 6.0 \times 10^5\,\mathrm{kg/h} \\ &= 6.0 \times 10^5\,\mathrm{L/h} \end{aligned}$$

20.2.45

$$
\begin{aligned}
e &= 1 - \frac{T_C}{T_H} \\
e' &= 1 - \frac{T'}{T_H} \\
W' &= Q_H e' \\
&= Q_H \left(1 - \frac{T'}{T_H}\right) \\
e'' &= 1 - \frac{T_C}{T'} \\
W'' &= Q'_H e'' \\
&= (Q_H - W')e'' \\
&= \left[Q_H - Q_H \left(1 - \frac{T'}{T_H}\right)\right] \left(1 - \frac{T_C}{T'}\right) \\
&= Q_H \left(1 - 1 + \frac{T'}{T_H}\right) \left(1 - \frac{T_C}{T'}\right) \\
&= Q_H \frac{T'}{T_H} \left(1 - \frac{T_C}{T'}\right) \\
e_{\text{Total}} &= \frac{W' + W''}{Q_H} \\
&= 1 - \frac{T'}{T_H} + \frac{T'}{T_H} \left(1 - \frac{T_C}{T'}\right) \\
&= 1 - \frac{T_C}{T_H}
\end{aligned}
$$

The efficiency is the same.

20.2.49

$$
\begin{aligned}
T_A &= 300\,\mathrm{K} \\
W_{AB} &= nRT_A \ln \frac{V_B}{V_A} \\
&= 2553\,\mathrm{J} \\
Q_{AB} &= 2553\,\mathrm{J} \\
T_B &= 300\,\mathrm{K} \\
T_C &= 1000\,\mathrm{K} \\
W_{BC} &= 0 \\
Q_{BC} &= nC_V(T_C - T_B) \\
&= \frac{5}{2}nR(T_C - T_B) \\
&= 1.24 \times 10^4\,\mathrm{J} \\
W_{CA} &= p_A(V_A - V_C) \\
&= -4949\,\mathrm{J} \\
\Delta U_{CA} &= Q_{CA} - W_{CA} \\
Q_{CA} &= \Delta U_{CA} + W_{CA} \\
&= nC_V(T_A - T_C) + W_{CA} \\
&= \frac{5}{2}nR(T_A - T_C) + W_{CA} \\
&= -1.73 \times 10^4\,\mathrm{J} \\
W &= -2396\,\mathrm{J} \\
K &= \frac{|Q_C|}{|W|} \\
&= 6.24
\end{aligned}
$$

20.2.51

(a)

$$
\begin{aligned}
\Delta S &= \int \frac{dQ}{T} \\
&= m_1c_1 \ln \frac{T}{T_1} + m_2c_2 \ln \frac{T'}{T_2} \\
0 &= m_1c_1(T - T_1) + m_2c_2(T' - T_2) \\
m_1c_1(T - T_1) &= m_2c_2(T_2 - T')
\end{aligned}
$$

(b)

$$
\begin{aligned}
T' &= T_2 - \frac{m_1 c_1}{m_2 c_2}(T - T_1) \\
\Delta S &= m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \left(\frac{T_2 - \frac{m_1 c_1}{m_2 c_2}(T - T_1)}{T_2} \right) \\
&= m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \left(1 - \frac{m_1 c_1}{m_2 c_2} \frac{T - T_1}{T_2} \right) \\
\frac{d}{dT}\Delta S &= \frac{m_1 c_1}{T} - m_2 c_2 \frac{m_1 c_1}{m_1 c_1 (T_1 - T) + m_2 c_2 T_2} \\
0 &= \frac{1}{T} - \frac{m_2 c_2}{m_1 c_1 (T_1 - T) + m_2 c_2 T_2} \\
T &= \frac{m_1 c_1 (T_1 - T) + m_2 c_2 T_2}{m_2 c_2} \\
&= \frac{m_1 c_1}{m_2 c_2}(T_1 - T) + T_2 \\
&= T'
\end{aligned}
$$

20.2.55

20.2.57

(a)

$$
\begin{aligned}
r_b &= 8.0\,\mathrm{cm} \\
V_b &= \frac{4}{3}\pi r_b^3 \\
&= 2.14 \times 10^{-3}\,\mathrm{m}^3 \\
V_a &= 2V_b \\
&= 4.29 \times 10^{-3}\,\mathrm{m}^3 \\
r_a &= \sqrt[3]{\frac{3V_a}{4\pi}} \\
&= 10\,\mathrm{cm} \\
r_d &= 3r_a \\
&= 30\,\mathrm{cm} \\
p_a V_a^\gamma &= p_b V_b^\gamma \\
p_a &= \left(\frac{V_b}{V_a}\right)^\gamma p_b \\
&= 8.24\,\mathrm{kPa} \\
\frac{p_b V_b}{T_b} &= \frac{p_a V_a}{T_a} \\
T_b &= \frac{p_b V_b}{p_a V_a} T_a \\
&= 152\,\mathrm{K}
\end{aligned}
$$

(b)

$$
\begin{aligned}
n &= \frac{p_b V_b}{R T_b} \\
&= 3.44 \times 10^{-2}\,\mathrm{mol} \\
V_d &= \frac{4}{3}\pi r_d^3 \\
&= 0.113\,\mathrm{m}^3 \\
p_d &= \frac{n R T_d}{V_d} \\
&= 311\,\mathrm{Pa}
\end{aligned}
$$

(c)

$$
\begin{aligned}
e &= 1 - \frac{T_C}{T_H} \\
&= 0.191 \\
&= 19.1\% \\
W &= eQ_H \\
Q_H &= \frac{W}{e} \\
&= 5.24\,\mathrm{kJ}
\end{aligned}
$$

(d)

$$
\begin{aligned}
Q_C &= (1-e)Q_H \\
&= 4.24\,\mathrm{kJ}
\end{aligned}
$$

20.2.59

$$
\begin{aligned}
e &= 1 - \frac{T_C}{T_H} \\
T_C &= (1-e)T_H \\
&= 281\,\mathrm{K} \\
&= 7.5\,^{\circ}\mathrm{C}
\end{aligned}
$$

b

20.2.61

d

21 Relativity

21.1 Guided Practice

21.1.1

(a) In the laboratory frame, $v_1 = \alpha c$ and $v_2 = -\alpha c$. In the frame of th first proton, $v_1' = 0$ and $v_2' = -\frac{1}{2}c$. Using the Lorentz velocity transformation:

$$
\begin{aligned}
v_2' &= \frac{v_2 - v_1}{1 - v_1 v_2/c^2} \\
-\frac{1}{2}c &= \frac{-\alpha c - \alpha c}{1 + \alpha^2 c^2/c^2} \\
\frac{1}{2}c &= \frac{2\alpha c}{1 + \alpha^2} \\
\alpha^2 - 4\alpha + 1 &= 0 \\
\alpha &= \frac{4 \pm \sqrt{16 - 4}}{2} \\
&= 2 \pm \sqrt{3}
\end{aligned}
$$

α can't be greater than 1, so $\alpha = 2 - \sqrt{3} \approx 0.268$.

(b)

$$
\begin{aligned}
K &= (\gamma - 1)mc^2 \\
&= \left(\frac{1}{\sqrt{1 - (0.268c)^2/c^2}} - 1 \right) mc^2 \\
&= 5.71 \times 10^{-12}\,\mathrm{J} \\
&= 35.6\,\mathrm{MeV}
\end{aligned}
$$

(c)

$$
\begin{aligned}
K &= \left(\frac{1}{\sqrt{1 - (0.5c)^2/c^2}} - 1 \right) mc^2 \\
&= 2.33 \times 10^{-11}\,\mathrm{J} \\
&= 145\,\mathrm{MeV}
\end{aligned}
$$

21.2 Exercises and Problems

21.2.1

$$
\begin{aligned}
x_1' &= -d \\
t_1' &= 0 \\
x_2' &= d \\
t_2' &= 0 \\
x_1 &= \gamma(x_1' + ut_1') \\
&= -\gamma d \\
t_1 &= \gamma(t_1' + ux_1'/c^2) \\
&= -\gamma ud/c^2 \\
x_2 &= \gamma(x_2' + ut_2') \\
&= \gamma d \\
t_2 &= \gamma(t_2' + ux_2'/c^2) \\
&= \gamma ud/c^2
\end{aligned}
$$

The observer measures the lightning strike at point A to come first.

21.2.3

$$
\begin{aligned}
2 &= \gamma \\
&= \frac{1}{\sqrt{1 - v^2/c^2}} \\
2\sqrt{1 - v^2/c^2} &= 1 \\
1 - \frac{v^2}{c^2} &= \frac{1}{4} \\
\frac{v^2}{c^2} &= \frac{3}{4} \\
v &= \frac{\sqrt{3}}{2}c \\
&\approx 0.866c \\
&\approx 2.60 \times 10^8 \text{ m/s}
\end{aligned}
$$

Present day jet planes don't reach this speed.

21.2.5

(a)

$$\begin{aligned}
\Delta t &= \gamma \Delta t_0 \\
\frac{\Delta t}{\Delta t_0} &= \frac{1}{\sqrt{1 - v^2/c^2}} \\
\frac{\Delta t_0}{\Delta t} &= \sqrt{1 - v^2/c^2} \\
\left(\frac{\Delta t_0}{\Delta t}\right)^2 &= 1 - \frac{v^2}{c^2} \\
\frac{v^2}{c^2} &= 1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2 \\
v &= \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2}\, c \\
&= 0.998c
\end{aligned}$$

(b)

$$d = \Delta t v = 126\,\mathrm{m}$$

21.2.7

$$\begin{aligned}
\Delta t - \Delta t_0 &= \Delta t \left(1 - \frac{1}{\gamma}\right) \\
&= \Delta t(1 - \sqrt{1 - v^2/c^2}) \\
&= 9.15\,\mathrm{h}
\end{aligned}$$

The clock on the spacecraft measured a shorter elapsed time.

21.2.9

$$\begin{aligned}
l &= \frac{l_0}{\gamma} \\
l_0 &= \gamma l \\
&= 103\,\mathrm{m}
\end{aligned}$$

21.2.11

(a)

$$d = \Delta t_0 v = 0.66\,\mathrm{km}$$

(b)

$$\begin{aligned}\Delta t &= \gamma \Delta t_0 \\ &= 49\,\mu\text{s} \\ d &= \Delta t v \\ &= 15\,\text{km}\end{aligned}$$

(c)

$$l = \frac{l_0}{\gamma} = 0.45\,\text{km}$$

21.2.13

(a)

$$l = \frac{l_0}{\gamma} = 3960\,\text{m}$$

(b)

$$t = \frac{d}{v} = 95.2\,\mu\text{s}$$

(c)

$$t_0 = \frac{t}{\gamma} = 94.3\,\mu\text{s}$$

21.2.15

(a)

$$\begin{aligned}v &= \frac{v' + u}{1 + uv'/c^2} \\ &= \frac{0.380c + 0.580c}{1 + (0.580c)(0.380c)/c^2} \\ &= \frac{(0.380 + 0.580)c}{1 + (0.580)(0.380)} \\ &= 0.787c\end{aligned}$$

(b)

$$v = 0.949c$$

(c)

$$v = 0.997c$$

21.2.17

(a) Toward

(b)

$$v' = \frac{v - u}{1 - uv/c^2} = \frac{0.650c - 0.830c}{1 - (0.830c)(0.650c)/c^2} = -0.391c$$

21.2.19

$$\begin{aligned}
v &= \frac{v' + u}{1 + uv'/c^2} \\
&= \frac{0.950c - 0.650c}{1 + (-0.650c)(0.950c)/c^2} \\
&= 0.784c
\end{aligned}$$

21.2.21

$$\begin{aligned}
v &= \frac{v' + u}{1 + uv'/c^2} \\
-\alpha c &= \frac{-0.890c + \alpha c}{1 + (\alpha c)(-0.890c)/c^2} \\
&= \frac{(\alpha - 0.890)c}{1 - 0.890\alpha} \\
-\alpha(1 - 0.890\alpha) &= (\alpha - 0.890) \\
0.890\alpha^2 - 2\alpha + 0.890 &= 0 \\
\alpha &= \frac{2 \pm \sqrt{4 - 3.1684}}{1.78} \\
&= 0.611
\end{aligned}$$

The particles are travelling at $0.611c$.

21.2.23

(a)

$$
\begin{aligned}
f_0 &= 4.44 \times 10^{14}\,\text{Hz} \\
f &= 5.22 \times 10^{14}\,\text{Hz} \\
f &= \sqrt{\frac{c+u}{c-u}} f_0 \\
\left(\frac{f}{f_0}\right)^2 &= \frac{c+u}{c-u} \\
(c-u)\left(\frac{f}{f_0}\right)^2 &= c+u \\
c\left[\left(\frac{f}{f_0}\right)^2 - 1\right] &= u\left[\left(\frac{f}{f_0}\right)^2 + 1\right] \\
u &= \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1} c \\
\left(\frac{f}{f_0}\right)^2 &= 1.38 \\
u &= 0.160c
\end{aligned}
$$

(b) \$173 million

21.2.25

$$
\begin{aligned}
1.20 &= \sqrt{\frac{c+u}{c-u}} \\
1.44 &= \frac{c+u}{c-u} \\
1.44(c-u) &= c+u \\
c(1.44-1) &= (1.44+1)u \\
u &= \frac{0.44}{2.44} c \\
&= 0.18c
\end{aligned}
$$

Toward

21.2.27

$$
\begin{aligned}
\frac{\gamma_{0.800} 0.800c}{\gamma_{0.400} 0.400c} &= 2\sqrt{\frac{1-(0.400c)^2/c^2}{1-(0.800c)^2/c^2}} \\
&= 3.06
\end{aligned}
$$

21.2.29

(a)

$$\begin{aligned}2 &= \gamma \\ &= \frac{1}{\sqrt{1 - v^2/c^2}} \\ 1 - \frac{v^2}{c^2} &= \frac{1}{4} \\ v &= \frac{\sqrt{3}}{2}c \\ &= 0.866c\end{aligned}$$

(b)

$$\begin{aligned}2 &= \gamma^3 \\ &= \frac{1}{(1 - v^2/c^2)^{3/2}} \\ 1 - \frac{v^2}{c^2} &= \left(\frac{1}{2}\right)^{2/3} \\ v &= \sqrt{1 - \left(\frac{1}{2}\right)^{2/3}}c \\ &= 0.608c\end{aligned}$$

21.2.31

(a)

$$0.866c$$

(b)

$$\begin{aligned}6 &= \gamma \\ &= \frac{1}{\sqrt{1 - v^2/c^2}} \\ \frac{1}{36} &= 1 - \frac{v^2}{c^2} \\ v &= \sqrt{1 - \frac{1}{36}}c \\ &= 0.986c\end{aligned}$$

21.2.33

(a)

$$K = 3mc^2 = 4.51 \times 10^{-10}\,\mathrm{J}$$

(b)

$$
\begin{aligned}
E^2 &= (mc^2)^2 + (pc)^2 \\
p &= \frac{\sqrt{E^2 - (mc^2)^2}}{c} \\
&= \frac{\sqrt{(4mc^2)^2 - (mc^2)^2}}{c} \\
&= \frac{\sqrt{15(mc^2)^2}}{c} \\
&= \sqrt{15}mc \\
&= 1.94 \times 10^{-18}\,\mathrm{kg\,m/s}
\end{aligned}
$$

(c)

$$p = \gamma mv \Rightarrow v = \frac{p}{\gamma m} = 2.90 \times 10^8\,\mathrm{m/s} = 0.968c$$

21.2.35

(a)

$$E = mc^2 \Rightarrow m = \frac{E}{c^2} = 1.11 \times 10^3\,\mathrm{kg}$$

(b)

$$
\begin{aligned}
\rho &= 7860\,\mathrm{kg/m^3} \\
\rho V &= m \\
\rho s^3 &= m \\
s &= \sqrt[3]{\frac{m}{\rho}} \\
&= 0.521\,\mathrm{m}
\end{aligned}
$$

21.2.41

(a)

$$
\begin{aligned}
qV &= K \\
V &= \frac{(\gamma - 1)mc^2}{q} \\
&= 2.06 \times 10^6\,\mathrm{V}
\end{aligned}
$$

(b)

$$K = 3.30 \times 10^{-13}\,\mathrm{J} = 2.06 \times 10^6\,\mathrm{eV}$$

21.2.43

(a)

$$
\begin{aligned}
l &= v\Delta t \\
&= v\gamma\Delta t_0 \\
&= \frac{v\Delta t_0}{\sqrt{1-v^2/c^2}} \\
l\sqrt{1-v^2/c^2} &= v\Delta t_0 \\
l^2(1-v^2/c^2) &= v^2\Delta t_0^2 \\
v^2 &= \frac{l^2}{\Delta t_0^2}(1-v^2/c^2) \\
v^2\left(1+\frac{l^2}{\Delta t_0^2 c^2}\right) &= \frac{l^2}{\Delta t_0^2} \\
(1-\Delta)c &= \sqrt{\frac{l^2/\Delta t_0^2}{1+l^2/\Delta t_0^2 c^2}} \\
\Delta &= 1-\frac{1}{c}\sqrt{\frac{l^2/\Delta t_0^2}{1+l^2/\Delta t_0^2 c^2}} \\
&= 8.43\times 10^{-6}
\end{aligned}
$$

(b)

$$
\begin{aligned}
E &= \gamma mc^2 \\
&= 5.45\times 10^{-9}\,\mathrm{J} \\
&= 3.40\times 10^{10}\,\mathrm{eV}
\end{aligned}
$$

21.2.45

$$
\begin{aligned}
1.4 &= \gamma \\
&= \frac{1}{\sqrt{1-v^2/c^2}} \\
\frac{1}{1.4} &= \sqrt{1-v^2/c^2} \\
\frac{1}{1.96} &= 1-\frac{v^2}{c^2} \\
v &= \sqrt{1-\frac{1}{1.96}}c \\
&= 0.700c
\end{aligned}
$$

21.2.47

$$\begin{aligned}\Delta t &= 42.5\,\mathrm{y}\\ \Delta t_0 &= \frac{\Delta t}{\gamma}\\ &= \Delta t\sqrt{1-v^2/c^2}\\ &= 5.02\,\mathrm{y}\end{aligned}$$

Her biological age will be $19 + 5 = 24$.

21.2.51

$$\begin{aligned}\Delta t - \Delta t_0 &= \Delta t - \frac{\Delta t}{\gamma}\\ &= \Delta t\left(1-\frac{1}{\gamma}\right)\\ &= \Delta t(1-\sqrt{1-v^2/c^2})\\ &= 8.41\,\mathrm{ns}\end{aligned}$$

The clock that was on the airliner will show a shorter elapsed time.

21.2.53

$$\begin{aligned}v &\geq \frac{c}{n}\\ &\geq 1.97\times 10^8\,\mathrm{m/s}\\ K &\geq 2.70\times 10^{-14}\,\mathrm{J}\\ &\geq 167\,\mathrm{keV}\end{aligned}$$

21.2.57

(a)

$$v = \frac{v'+u}{1+uv'/c^2} = \frac{0.9995c + 0.7500c}{1+(0.7500c)(0.9995c)/c^2} = 0.9999c$$

(b)

$$v = \frac{v'+u}{1+uv'/c^2} = \frac{-0.9995c + 0.7500c}{1+(0.7500c)(-0.9995c)/c^2} = -0.9965c$$

21.2.61

$$\begin{aligned}
\frac{\Delta f}{f_0} &= \frac{f'' - f_0}{f_0} \\
&= \sqrt{\frac{c+u}{c-u}}\frac{f'}{f_0} - 1 \\
&= \frac{c+u}{c-u} - 1 \\
(c-u)\left(1 + \frac{\Delta f}{f_0}\right) &= c + u \\
c\frac{\Delta f}{f_0} &= u\left(2 + \frac{\Delta f}{f_0}\right) \\
u &= \frac{\Delta f/f_0}{2 + \Delta f/f_0}c \\
&= 43\,\mathrm{m/s} \\
&= 154\,\mathrm{k/m}
\end{aligned}$$

21.2.63

$$\begin{aligned}
F &= \gamma m a \\
&= \frac{1}{\sqrt{1 - v^2/c^2}} m \frac{v^2}{r} \\
&= \frac{mv^2}{r\sqrt{1 - v^2/c^2}} \\
&= 2.04 \times 10^{-13}\,\mathrm{N}
\end{aligned}$$

21.2.65

(a)

$$\begin{aligned}
\Delta t &= \gamma \Delta t_0 \\
&= \frac{\Delta t_0}{\sqrt{1 - u^2/v^2}} \\
\Delta t^2 &= \frac{\Delta t_0^2}{1 - u^2/c^2} \\
\Delta t_0 &\approx 2.59 \times 10^{-8}\,\mathrm{s}
\end{aligned}$$

(b)

$$\begin{aligned}(4\Delta t_0)^2 &= \frac{\Delta t_0^2}{1-u^2/c^2}\\ \frac{1}{16} &= 1-\frac{u^2}{c^2}\\ \frac{u}{c} &= \sqrt{1-\frac{1}{16}}\\ &= 0.968\end{aligned}$$

21.2.71

c

21.2.73

b