

# University Physics with Modern Physics

## Electromagnetism Problems

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December 2022

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## 21 Electric Charge and Electric Field

### 21.3 Coulomb's Law

#### 21.3.1 Example 21.1

The magnitude of electric repulsion between two  $\alpha$  particles is given by

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$$

and the magnitude of gravitational attraction is given by

$$F_g = \frac{Gm^2}{r^2}$$

. The ratio of the two values is

$$\begin{aligned} \frac{F_e}{F_g} &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \frac{r^2}{Gm^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{Gm^2} \\ &= 3.1 \times 10^{35} \end{aligned}$$

showing that the electric repulsion is significantly stronger than the gravitational attraction.

#### 21.3.2 Example 21.2

a) The magnitude of the force that  $q_1$  exerts on  $q_2$  is

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \\ &= (9.0 \times 10^9) \frac{|(25 \times 10^{-9})(-75 \times 10^{-9})|}{0.030^2} \\ &= 1.9 \times 10^{-2} \text{ N.} \end{aligned}$$

Since  $q_1$  and  $q_2$  have opposite charge, the force is attractive (from  $q_2$  to  $q_1$ ).

b) The magnitude of the force that  $q_2$  exerts on  $q_1$  is the same as in part a, but the direction is reversed (from  $q_1$  to  $q_2$ ).

### 21.3.3 Example 21.3

By the principle of superposition of forces, the net force exerted on  $q_3$  is equal to the vector sum of the forces exerted on it by  $q_1$  and  $q_2$  separately.

Both  $q_1$  and  $q_3$  have positive charge so they repel each other.  $q_1$  is to the right of  $q_3$  so  $q_3$  experiences a force to the left of magnitude

$$\begin{aligned} F_{1 \text{ on } 3} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_3|}{r^2} \\ &= (9.0 \times 10^9) \frac{|(1.0 \times 10^{-9})(5.0 \times 10^{-9})|}{0.020^2} \\ &= 1.1 \times 10^{-4} \text{ N.} \end{aligned}$$

However  $q_2$  has a negative charge so it attracts  $q_3$ . It is also to the right of  $q_3$  so  $q_3$  experiences a force to the right of magnitude

$$\begin{aligned} F_{2 \text{ on } 3} &= \frac{1}{4\pi\epsilon_0} \frac{|q_2 q_3|}{r^2} \\ &= (9.0 \times 10^9) \frac{|(-3.0 \times 10^{-9})(5.0 \times 10^{-9})|}{0.040^2} \\ &= 8.4 \times 10^{-5} \text{ N.} \end{aligned}$$

The net force experienced by  $q_3$  is therefore

$$\begin{aligned} F &= -F_{1 \text{ on } 3} + F_{2 \text{ on } 3} \\ &= -1.1 \times 10^{-4} + 8.4 \times 10^{-5} \\ &= -2.6 \times 10^{-5} \text{ N.} \end{aligned}$$

### 21.3.4 Example 21.4

Since  $q_1$  and  $q_2$  are of equal charge and are symmetric about the x axis on which  $Q$  lies, the vertical components of their forces cancel leaving only the horizontal.

The horizontal component of  $q_1$ 's force on  $Q$  is given by

$$\begin{aligned} F_{1 \text{ on } Q, x} &= \frac{1}{4\pi\epsilon_0} \frac{q_1 Q}{r_{1,Q}^2} \cos \alpha \\ &= (9.0 \times 10^9) \frac{(2.0 \times 10^{-6})(4.0 \times 10^{-6})}{\sqrt{0.30^2 + 0.40^2}^2} \frac{0.40}{0.50} \\ &= 0.23 \text{ N.} \end{aligned}$$

Again, since  $q_1$  and  $q_2$  are of equal charge and symmetric about the x axis,  $F_{1 \text{ on } Q, x} = F_{2 \text{ on } Q, x}$  and the total force experienced by  $Q$  is in the positive x direction of magnitude

$$F = 2F_{1 \text{ on } Q, x} = 0.46 \text{ N.}$$

## 21.4 Electric Field and Electric Forces

### 21.4.1 Example 21.5

The magnitude of the electric field vector is given by

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \\ &= (9.0 \times 10^9) \frac{|4.0 \times 10^{-9}|}{2.0^2} \\ &= 9.0 \text{ N/C}. \end{aligned}$$

### 21.4.2 Example 21.6

The magnitude of the electric field vector is given by

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \\ &= (9.0 \times 10^9) \frac{|-8.0 \times 10^{-9}|}{1.2^2 + 1.6^2} \\ &= 18 \text{ N/C} \end{aligned}$$

and it is directed towards the origin. If  $\theta$  is the angle between the positive x axis and  $\hat{\mathbf{r}}$  then the component form of  $\mathbf{E}$  is

$$\begin{aligned} E &= -E (\cos \theta \hat{\mathbf{i}} + \sin \theta \hat{\mathbf{j}}) \\ &= -E \left( \frac{x}{r} \hat{\mathbf{i}} + \frac{-y}{r} \hat{\mathbf{j}} \right) \\ &= \frac{-18}{\sqrt{1.2^2 + 1.6^2}} (1.2\hat{\mathbf{i}} + 1.6\hat{\mathbf{j}}) \\ &= (-11 \text{ N/C})\hat{\mathbf{i}} - (14 \text{ N/C})\hat{\mathbf{j}}. \end{aligned}$$

### 21.4.3 Example 21.7

- a) Electrons have a negative charge and the electric field is directed upwards, so the electron will move downwards. The magnitude of its acceleration is

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{eE}{m} \\ &= \frac{(1.60 \times 10^{-19})(1.00 \times 10^4)}{9.11 \times 10^{-31}} \\ &= 1.76 \times 10^{15} \text{ m/s}^2. \end{aligned}$$

b) Its acceleration is constant between the plates, so its final speed is

$$\begin{aligned}
 v^2 &= v_0^2 + 2a(x - x_0) \\
 &= 2ax \\
 v &= \sqrt{2ax} \\
 &= \sqrt{2(1.76 \times 10^{15})(0.01)} \\
 &= 5.9 \times 10^6 \text{ m/s}
 \end{aligned}$$

and thus its final kinetic energy is

$$\begin{aligned}
 K &= \frac{1}{2}mv^2 \\
 &= \frac{1}{2}(9.11 \times 10^{-31})(5.9 \times 10^6)^2 \\
 &= 1.6 \times 10^{-17} \text{ J.}
 \end{aligned}$$

c) We can find the time it takes for the electron to travel this distance by rearranging the kinematic equation

$$v = v_0 + at$$

to

$$\begin{aligned}
 t &= \frac{v - v_0}{a} \\
 &= \frac{5.9 \times 10^6}{1.76 \times 10^{15}} \\
 &= 3.4 \times 10^{-9} \text{ s.}
 \end{aligned}$$

## 21.5 Electric-Field Calculations

### 21.5.1 Example 21.8

a) At point  $a$  the electric field caused by  $q_1$  points to the right and has magnitude

$$\begin{aligned}
 E_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r^2} \\
 &= (9.0 \times 10^9) \frac{12 \times 10^{-9}}{(0.060)^2} \\
 &= 3.0 \times 10^4 \text{ N/C.}
 \end{aligned}$$



The electric field caused by  $q_2$  also points to the right and it has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r^2} \\ &= (9.0 \times 10^9) \frac{|-12 \times 10^{-9}|}{(0.040)^2} \\ &= 6.8 \times 10^4 \text{ N/C}. \end{aligned}$$

Thus the total field points to the right and has magnitude

$$E = E_1 + E_2 = 9.8 \times 10^4 \text{ N/C}.$$

b) At point  $b$  the electric field caused by  $q_1$  points to the left and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r^2} \\ &= (9.0 \times 10^9) \frac{12 \times 10^{-9}}{(0.040)^2} \\ &= 6.8 \times 10^4 \text{ N/C}. \end{aligned}$$

The electric field caused by  $q_2$  points to the right and has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r^2} \\ &= (9.0 \times 10^9) \frac{|-12 \times 10^{-9}|}{(0.140)^2} \\ &= 0.55 \times 10^4 \text{ N/C}. \end{aligned}$$

Thus the total electric field points to the left and has magnitude

$$E = E_1 - E_2 = 6.3 \times 10^4 \text{ N/C}.$$

c) At point  $c$  the electric field caused by  $q_1$  points from  $q_1$  to  $c$  and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r^2} \\ &= (9.0 \times 10^9) \frac{12 \times 10^{-9}}{0.130^2} \\ &= 6.4 \times 10^3 \text{ N/C}. \end{aligned}$$

The electric field caused by  $q_2$  points from  $c$  to  $q_2$  and has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r^2} \\ &= (9.0 \times 10^9) \frac{|-12 \times 10^{-9}|}{0.130^2} \\ &= 6.4 \times 10^3 \text{ N/C} \\ &= E_1. \end{aligned}$$

The vertical components of  $\mathbf{E}_1$  and  $\mathbf{E}_2$  cancel, leaving only a horizontal component pointing to the right of magnitude

$$\begin{aligned} E &= 2E_1 \cos \alpha \\ &= 2(6.4 \times 10^3) \frac{0.050}{0.130} \\ &= 4.9 \times 10^3 \text{ N/C}. \end{aligned}$$

### 21.5.2 Example 21.9

By symmetry, each point on the ring has a corresponding point on the opposite side. The components of their electric fields perpendicular to the axis of the ring cancel, leaving only a component parallel to the axis of the ring. Thus the total electric field at  $P$  is parallel to the axis of the ring and can be calculated as

$$\begin{aligned} E &= \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r^2} \cos \alpha \, d\theta \\ &= \frac{1}{4\pi\epsilon_0} \frac{Qx}{2\pi(a^2 + x^2)^{3/2}} \int_0^{2\pi} d\theta \\ &= \frac{1}{4\pi\epsilon_0} \frac{Qx}{(a^2 + x^2)^{3/2}}. \end{aligned}$$

### 21.5.3 Example 21.10

By symmetry, each point on the line has a corresponding point on the opposite side of the  $x$ -axis. The  $y$  components of their electric fields cancel, leaving only the  $x$  components. Thus the total electric field at  $P$  only has an  $x$  component and can be calculated as

$$\begin{aligned}
E &= \int_{-a}^a \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r^2} \cos \alpha \, dy \\
&= \frac{1}{4\pi\epsilon_0} \frac{Qx}{2a} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} \, dy \\
&= \frac{1}{4\pi\epsilon_0} \frac{Qx}{2a} \left[ \frac{y}{x^2 \sqrt{x^2 + y^2}} \right]_{-a}^a \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{2ax} \left( \frac{a}{\sqrt{x^2 + a^2}} + \frac{a}{\sqrt{x^2 + (-a)^2}} \right) \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2 + a^2}}.
\end{aligned}$$

#### 21.5.4 Example 21.11

By symmetry, each point on the disk has a corresponding point 180° rotation around the  $x$ -axis. The  $y$  and  $z$  components of their electric fields cancel, leaving only the  $x$  components. Thus the total magnetic field at  $P$  only has an  $x$  component and can be calculated as

$$\begin{aligned}
E &= \int_0^R \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{\sigma}{r^2} s \cos \alpha \, d\theta \, ds \\
&= \frac{\sigma}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{s}{s^2 + x^2} \frac{x}{\sqrt{s^2 + x^2}} \, d\theta \, ds \\
&= \frac{\sigma x}{2\epsilon_0} \int_0^R \frac{s}{(s^2 + x^2)^{3/2}} \, ds \\
&= \frac{\sigma x}{2\epsilon_0} \left[ -\frac{1}{\sqrt{s^2 + x^2}} \right]_0^R \\
&= \frac{\sigma x}{2\epsilon_0} \left( -\frac{1}{\sqrt{R^2 + x^2}} + \frac{1}{x} \right) \\
&= \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{1}{\sqrt{(R/x)^2 + 1}} \right).
\end{aligned}$$

#### 21.5.5 Example 21.12

From Example 21.11 we know that the electric field produced by an infinite plane sheet of charge is

$$E = \frac{\sigma}{2\epsilon_0}.$$

Therefore the electric field outside the sheets is  $\mathbf{0}$  and between the sheets is  $\sigma/\epsilon_0$  towards the negative sheet.

## 21.7 Electric Dipoles

### 21.7.1 Example 21.13

- a) The electric field is uniform so the net force exerted on the dipole is  $\mathbf{0}$
- b) The electric dipole moment is directed from the negative charge to the positive charge and has magnitude

$$p = qd = (1.6 \times 10^{-19})(0.125 \times 10^{-9}) = 2.0 \times 10^{-29} \text{ C} \cdot \text{m}$$

- c) The torque aligns the electric dipole moment with the electric field so it is directed out of the page and has magnitude

$$\tau = qEd \sin \phi = (1.6 \times 10^{-19})(5.0 \times 10^5)(0.125 \times 10^{-9}) \sin 35 = 5.7 \times 10^{-24} \text{ N} \cdot \text{m}$$

- d) The potential energy of an electric dipole in a uniform electric field is given by

$$U = -qEd \cos \phi = (2.0 \times 10^{-29})(5.0 \times 10^5) \cos 35 = 8.2 \times 10^{-24} \text{ J}$$

### 21.7.2 Example 21.14

As  $P$  is on the  $y$ -axis, the electric fields of the electric dipole's point charges have no  $x$  component and thus the net electric field is directed along the  $y$ -axis.

By the principle of superposition of electric fields, the magnitude of the electric field at  $P$  is

$$\begin{aligned} E &= E_- + E_+ \\ &= \frac{1}{4\pi\epsilon_0} \frac{-q}{(y - (-d/2))^2} + \frac{1}{4\pi\epsilon_0} \frac{q}{(y - d/2)^2} \\ &= \frac{1}{4\pi\epsilon_0} q \left( \frac{1}{(y - d/2)^2} - \frac{1}{(y + d/2)^2} \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{y^2} \left( \left(1 - \frac{d}{2y}\right)^{-2} - \left(1 + \frac{d}{2y}\right)^{-2} \right) \\ &\approx \frac{1}{4\pi\epsilon_0} \frac{q}{y^2} \left( 1 + \frac{d}{y} - 1 + \frac{d}{y} \right) \\ &= \frac{qd}{2\pi\epsilon_0 y^3} \\ &= \frac{p}{2\pi\epsilon_0 y^3}. \end{aligned}$$

## 21.8 Guided Practice

### 21.8.1 VP21.4.1

$q_1$  attracts  $q_3$  to the left with magnitude

$$\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_3|}{r^2} \\
&= (9.0 \times 10^9) \frac{|(4.00 \times 10^{-9})(-2.00 \times 10^{-9})|}{0.0400^2} \\
&= 4.5 \times 10^{-5} \text{ N}.
\end{aligned}$$

$q_2$  repels  $q_3$  to the left with magnitude

$$\begin{aligned}
F_2 &= \frac{1}{4\pi\epsilon_0} \frac{|q_2 q_3|}{r^2} \\
&= (9.0 \times 10^9) \frac{|(-1.20 \times 10^{-9})(-2.00 \times 10^{-9})|}{(0.0600 - 0.0400)^2} \\
&= 5.4 \times 10^{-5} \text{ N}.
\end{aligned}$$

By the principle of superposition of forces, the net force on  $q_3$  is

$$\mathbf{F} = (-F_1 - F_2)\hat{\mathbf{i}} = (-9.9 \times 10^{-5} \text{ N})\hat{\mathbf{i}}.$$

### 21.8.2 VP21.4.2

a)  $q_1$  repels  $q_2$  in the positive  $x$  direction with magnitude

$$\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} \\
&= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{|(3.60 \times 10^{-9})(2.00 \times 10^{-9})|}{0.0400^2} \\
&= 40.4 \mu\text{N}.
\end{aligned}$$

b) By the superposition of forces

$$\begin{aligned}
F &= F_1 + F_2 \\
F_2 &= F - F_1 \\
&= 54.0 - 40.4 \\
&= 13.6 \mu\text{N}
\end{aligned}$$

in the positive  $x$  direction.

c)  $q_2$  repels  $q_3$  so it must also have a positive charge of magnitude

$$\begin{aligned}
F &= \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r^2} \\
q_2 &= \frac{4\pi\epsilon_0 F r^2}{q_3} \\
&= \frac{4\pi(8.854 \times 10^{-12})(1.36 \times 10^{-5})(0.0800)^2}{2.00 \times 10^{-9}} \\
&= 4.84 \times 10^{-9} \text{ C}.
\end{aligned}$$

### 21.8.3 VP21.4.3

By symmetry the  $x$  components of  $q_1$  and  $q_2$ 's electric fields cancel leaving only their  $y$  components which are directed in the negative  $y$  direction and equal.  $q_3$  is negative and thus experiences a net force in the positive  $y$  direction of magnitude

$$\begin{aligned}
F &= 2 \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r^2} \sin \alpha \\
&= 2 \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{(6.00 \times 10^{-9})(2.50 \times 10^{-9})}{0.150^2 + 0.200^2} \frac{0.200}{\sqrt{0.150^2 + 0.200^2}} \\
&= 3.45 \times 10^{-6} \text{ N}.
\end{aligned}$$

### 21.8.4 VP21.4.4

The magnitude of the electric force exerted by  $q_1$  on  $q_3$  is

$$\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_3|}{r^2} \\
&= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{|(4.00 \times 10^{-9})(-1.50 \times 10^{-9})|}{0.250^2 + 0.200^2} \\
&= 5.26 \times 10^{-7} \text{ N}.
\end{aligned}$$

They have opposite charges so the force is directed from  $q_3$  to  $q_1$ . In component form the force is

$$\begin{aligned}
\mathbf{F}_1 &= -F_1 \cos \alpha \hat{\mathbf{i}} + F_1 \sin \alpha \hat{\mathbf{j}} \\
&= F_1 \left( -\frac{x}{r} \hat{\mathbf{i}} + \frac{y}{r} \hat{\mathbf{j}} \right) \\
&= \frac{5.26 \times 10^{-7}}{\sqrt{0.250^2 + 0.200^2}} \left( -0.250 \hat{\mathbf{i}} + 0.200 \hat{\mathbf{j}} \right) \\
&= (-4.11 \times 10^{-7} \text{ N}) \hat{\mathbf{i}} + (3.29 \times 10^{-7} \text{ N}) \hat{\mathbf{j}}.
\end{aligned}$$

The magnitude of the electric force exerted by  $q_2$  on  $q_3$  is

$$\begin{aligned} F_2 &= \frac{1}{4\pi\epsilon_0} \frac{|q_2 q_3|}{r^2} \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{|(-4.00 \times 10^{-9})(-1.50 \times 10^{-9})|}{0.250^2} \\ &= 8.63 \times 10^{-7} \text{ N}. \end{aligned}$$

The have like charges so the force is directed from  $q_2$  to  $q_3$ , i.e. along the positive  $x$ -axis. In component form the force is

$$\mathbf{F}_2 = (8.64 \times 10^{-7} \text{ N})\hat{\mathbf{i}}.$$

Thus the net force experienced by  $q_3$  is

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= (4.53 \times 10^{-7} \text{ N})\hat{\mathbf{i}} + (3.29 \times 10^{-7} \text{ N})\hat{\mathbf{j}}. \end{aligned}$$

### 21.8.5 VP21.10.1

- a) The source points and field point all lie on the  $y$ -axis, so the source points' electric fields have no  $x$  components.  $q_1$  is positive and the field point is below it, so its contribution is negative.  $q_2$  is negative and the field point is above it, so its contribution is also negative. Thus the  $y$  component of the net electric field is

$$\begin{aligned} E_y &= \frac{1}{4\pi\epsilon_0} \left( -\frac{q_1}{(y_1 - y)^2} + \frac{q_2}{(y_2 - y)^2} \right) \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \left( \frac{4.00 \times 10^{-9}}{(0.200 - 0.100)^2} - \frac{5.00 \times 10^{-9}}{(0 - 0.100)^2} \right) \\ &= -8.09 \times 10^3 \text{ N/C}. \end{aligned}$$

- b) The source points and field point all lie on the  $y$ -axis, so the source points' electric fields have no  $x$  components.  $q_1$  is positive and the field point is above it, so its contribution is positive.  $q_2$  is negative and the field point is above it, so its contribution is also negative. Thus the  $y$  component of the net electric field is

$$\begin{aligned} E_y &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{(y_1 - y)^2} + \frac{q_2}{(y_2 - y)^2} \right) \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \left( \frac{4.00 \times 10^{-9}}{(0.200 - 0.400)^2} - \frac{5.00 \times 10^{-9}}{(0 - 0.400)^2} \right) \\ &= 618 \text{ N/C}. \end{aligned}$$

c) The electric field of  $q_1$  has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{4.00 \times 10^{-9}}{0.200^2 + 0.200^2} \\ &= 449 \text{ N/C}. \end{aligned}$$

It is directed from  $q_1$  to the field point and thus in component form is

$$\begin{aligned} \mathbf{E}_1 &= E_1(\cos \phi \hat{\mathbf{i}} - \sin \phi \hat{\mathbf{j}}) \\ &= \frac{449}{\sqrt{0.200^2 + 0.200^2}}(0.200\hat{\mathbf{i}} - 0.200\hat{\mathbf{j}}) \\ &= (317 \text{ N/C})\hat{\mathbf{i}} - (317 \text{ N/C})\hat{\mathbf{j}}. \end{aligned}$$

$q_2$  and the field point both lie on the  $x$ -axis, and thus its electric field has no  $y$  component. In component form it is

$$\begin{aligned} \mathbf{E}_2 &= \frac{1}{4\pi\epsilon_0} \frac{q_2}{r^2} \hat{\mathbf{i}} \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{-5.00 \times 10^{-9}}{0.200^2} \hat{\mathbf{i}} \\ &= (-1.12 \times 10^3 \text{ N/C})\hat{\mathbf{i}}. \end{aligned}$$

The total electric field is thus

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 \\ &= (-8.03 \times 10^2 \text{ N/C})\hat{\mathbf{i}} + (-3.17 \times 10^2 \text{ N/C})\hat{\mathbf{j}}. \end{aligned}$$

### 21.8.6 VP21.10.2

a) Both source points and the field point are on the  $x$ -axis, so the electric fields at  $P$  have no  $y$  components.

$q_1$  is positive and  $P$  is to the right of  $q_1$ , so its electric field points to the right and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{1.80 \times 10^{-9}}{0.0200^2} \\ &= 4.04 \times 10^4 \text{ N/C}. \end{aligned}$$



- b) The magnitude and direction of the electric field that  $q_2$  causes at  $P$  can be calculated as

$$\begin{aligned} E &= E_1 + E_2 \\ E_2 &= E - E_1 \\ &= 6.75 \times 10^4 - 4.04 \times 10^4 \\ &= 2.71 \times 10^4 \text{ N/C}. \end{aligned}$$

- c)  $E_2$  is positive at  $P$ , so  $q_2$  must be negative. Its value is

$$\begin{aligned} E_2 &= -\frac{1}{4\pi\epsilon_0} \frac{q_2}{r^2} \\ q_2 &= -4\pi\epsilon_0 E_2 r^2 \\ &= -4\pi(8.854 \times 10^{-12})(2.71 \times 10^4)(0.0200)^2 \\ &= -1.21 \times 10^{-9} \text{ C}. \end{aligned}$$

### 21.8.7 VP21.10.3

- a) From Example 21.9 we know that the electric field of a charged ring of radius  $a$  at a distance  $x$  along the ring's axis is directed away from the ring along its axis and has magnitude

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}}.$$

By the principle of superposition of electric fields, the electric field of the hydrogen atom is

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \left( \frac{e}{a^2} - \frac{ea}{(a^2 + a^2)^{3/2}} \right) \\ &= \frac{1}{4\pi\epsilon_0} e \left( \frac{1}{a^2} - \frac{a}{2\sqrt{2}a^3} \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{e}{a^2} \left( 1 - \frac{1}{2\sqrt{2}} \right). \end{aligned}$$

- b)  $1 - 1/(2\sqrt{2}) \approx 0.65$  so the field points away from the proton.

### 21.8.8 VP21.10.4

- a) The charge per unit length is

$$\lambda = \frac{Q}{L}$$

so the charge contained in a segment of length  $dx$  is

$$\lambda dx = \frac{Q}{L} dx.$$

- b) The field and source points both lie on the  $x$ -axis, so the differential electric field has no  $y$  component. The  $x$  component is

$$dE_x = -\frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{x^2} = -\frac{1}{4\pi\epsilon_0} \frac{Q}{Lx^2} dx.$$

- c) The total electric field at the origin is

$$\begin{aligned} E &= \int dE_x \\ &= \int_L^{2L} -\frac{1}{4\pi\epsilon_0} \frac{Q}{Lx^2} dx \\ &= -\frac{1}{4\pi\epsilon_0} \frac{Q}{L} \left[ -\frac{1}{x} \right]_L^{2L} \\ &= -\frac{1}{4\pi\epsilon_0} \frac{Q}{L} \left( -\frac{1}{2L} + \frac{1}{L} \right) \\ &= -\frac{1}{4\pi\epsilon_0} \frac{Q}{2L^2}. \end{aligned}$$

### 21.8.9 VP21.14.1

- a) The magnitude of the torque is given by

$$\begin{aligned} \tau &= pE \sin \theta \\ &= (6.13 \times 10^{-30})(3.00 \times 10^5) \sin 50.0^\circ \\ &= 1.41 \times 10^{-24} \text{ N}\cdot\text{m}. \end{aligned}$$

- b) The potential energy is given by

$$\begin{aligned} U &= -pE \cos \theta \\ &= -(6.13 \times 10^{-30})(3.00 \times 10^5) \cos 50.0^\circ \\ &= -1.18 \times 10^{-24} \text{ J}. \end{aligned}$$

**21.8.10 VP21.14.2**

To find the magnitude of the charges we can rearrange the torque equation

$$\begin{aligned}
 \tau &= pE \sin \theta \\
 &= qdE \sin \theta \\
 q &= \frac{\tau}{dE \sin \theta} \\
 &= \frac{6.60 \times 10^{-26}}{(1.10 \times 10^{-10})(8.50 \times 10^4) \sin 90^\circ} \\
 &= 7.06 \times 10^{-21} \text{ C.}
 \end{aligned}$$

**21.8.11 VP21.14.3**

When the dipole moment is parallel to the field its potential energy is  $-pE$  and when it is antiparallel its potential energy is  $pE$ . Thus, the work required to perform the rotation is  $2pE$  and

$$\begin{aligned}
 W &= 2pE \\
 p &= \frac{W}{2E} \\
 &= \frac{4.60 \times 10^{-25}}{2(1.20 \times 10^5)} \\
 &= 1.92 \times 10^{-30} \text{ C m.}
 \end{aligned}$$

**21.8.12 VP21.14.4**

a) Rearranging the equation for the dipole moment gives

$$\begin{aligned}
 p &= qd \\
 d &= \frac{p}{q} \\
 &= \frac{3.50 \times 10^{-29}}{1.60 \times 10^{-19}} \\
 &= 2.19 \times 10^{-10} \text{ m.}
 \end{aligned}$$

b) From Example 21.14, the electric field of the molecule along its axis is

$$E_y = \frac{1}{4\pi\epsilon_0} \frac{2p}{y^3}.$$

Rearranging for  $y$  and substituting in the desired field strength gives

$$\begin{aligned}
y &= \left( \frac{1}{4\pi\epsilon_0} \frac{2p}{E_y} \right)^{1/3} \\
&= \left( (8.988 \times 10^9) \frac{2(1.60 \times 10^{-19})(2.19 \times 10^{-10})}{8.00 \times 10^4} \right)^{1/3} \\
&= 1.99 \times 10^{-8} \text{ m.}
\end{aligned}$$

### 21.8.13 Bridging Problem

By symmetry, each point on the semicircle has a corresponding point on the opposite side of the  $y$ -axis. The  $x$  components of their electric fields cancel, leaving only the  $y$  components. Thus, the total electric field at  $P$  points in the negative  $y$  direction and has magnitude

$$\begin{aligned}
E &= \int_0^\pi \frac{1}{4\pi\epsilon_0} \frac{\lambda a d\theta}{a^2} \sin \theta \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{\pi a^2} \int_0^\pi \sin \theta d\theta \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{\pi a^2} [-\cos \theta]_0^\pi \\
&= \frac{1}{4\pi\epsilon_0} \frac{2Q}{\pi a^2}.
\end{aligned}$$

## 22 Gauss's Law

### 22.2 Calculating Electric Flux

#### 22.2.1 Example 22.1

a) The electric flux is given by

$$\begin{aligned}
\Phi_E &= AE \cos \theta \\
&= \pi(0.10)^2(2.0 \times 10^3) \cos 30^\circ \\
&= 54 \text{ N m}^2/\text{C}.
\end{aligned}$$

b)  $\Phi_E = 0$

c)  $\Phi_E = AE = 63 \text{ N m}^2/\text{C}$

#### 22.2.2 Example 22.2

a) Four of the six sides are parallel to  $E$  and thus contribute no flux. The remaining two surfaces contribute  $EL^2$  and  $-EL^2$  which add to 0 and also contribute no flux. The total flux is 0.

- b) Two of the six sides are parallel to  $E$  and thus contribute no flux. Two of the remaining four sides contribute  $EL^2 \cos \theta$  and  $-EL^2 \cos \theta$  which add to 0 and also contribute no flux. The final two sides contribute  $EL^2 \sin \theta$  and  $-EL^2 \sin \theta$  which also add to 0 and contribute no flux. The total flux is 0.

### 22.2.3 Example 22.3

At all points the electric field is normal to the sphere and its magnitude is

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = (8.988 \times 10^9) \frac{3.0 \times 10^{-6}}{0.20^2} = 6.74 \times 10^5 \text{ N/C}.$$

The total flux is thus

$$\Phi_E = AE = 4\pi r^2 E = 4\pi (0.20)^2 (6.74 \times 10^5) = 3.4 \times 10^5 \text{ N m}^2/\text{C}.$$

## 22.4 Applications of Gauss's Law

### 22.4.1 Example 22.5

Under electrostatics the electric field inside a conductor is always  $\mathbf{0}$ .

Using a Gaussian sphere of radius  $r > R$  centred on the conducting sphere we can use Gauss's law to calculate the electric field because  $\mathbf{E}$  will be perpendicular to  $d\mathbf{A}$  and have a constant magnitude

$$\begin{aligned} \oint \mathbf{E} \cdot d\mathbf{A} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\ E \oint dA &= \frac{Q_{\text{enc}}}{\epsilon_0} \\ E &= \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{enc}}}{r^2} \\ \mathbf{E} &= \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{enc}}}{r^2} \hat{\mathbf{r}}. \end{aligned}$$

### 22.4.2 Example 22.6

By symmetry the electric field must only have a radial component. Using a Gaussian cylinder of length  $L$  and radius  $r$  centred on the line  $E$  will be perpendicular to  $dA$  and have constant magnitude so we can use Gauss's law to derive the electric field

$$\begin{aligned}
\oint \mathbf{E} \cdot d\mathbf{A} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
E \oint dA &= \frac{L\lambda}{\epsilon_0} \\
E &= \frac{L\lambda}{2\pi r L \epsilon_0} \\
\mathbf{E} &= \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r} \hat{\mathbf{r}}.
\end{aligned}$$

### 22.4.3 Example 22.7

By symmetry, the electric field must be directed away from the sheet. Using a Gaussian cube of side  $L$ ,  $E$  will be perpendicular to  $dA$  on the far sides of the box and have constant magnitude so we can use Gauss's law to derive the electric field

$$\begin{aligned}
\oint \mathbf{E} \cdot d\mathbf{A} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
2EL^2 &= \frac{\sigma L^2}{\epsilon_0} \\
E &= \frac{\sigma}{2\epsilon_0}.
\end{aligned}$$

### 22.4.4 Example 22.8

First we position a Gaussian cylinder such that one end is within the plate and the other is between the plates. The end that is within the plate experiences no flux because there are no electric fields within conductors. We can use Gauss's law to find the electric field at the other end and thus between the plates

$$\begin{aligned}
\oint \mathbf{E} \cdot d\mathbf{A} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
EA &= \frac{\sigma A}{\epsilon_0} \\
E &= \frac{\sigma}{\epsilon_0}.
\end{aligned}$$

### 22.4.5 Example 22.9

From previous examples we know that for  $r > R$  the sphere's electric field is equivalent to that of a point charge

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}.$$

The sphere is insulating so for  $r < R$  its electric field isn't necessarily  $\mathbf{0}$ . However by symmetry its electric field must only have a radial component so we can use Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$E \oint dA = \frac{V\rho}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{4}{3}\pi r^3 \frac{Q}{(4/3)\pi R^3} \frac{1}{\epsilon_0}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{Qr}{R^3} \hat{\mathbf{r}}.$$

#### 22.4.6 Example 22.10

For  $r > R$  the electric field is as if all the charge were concentrated at the centre of the sphere. We can rearrange the equation for the electric field of a point charge to find the magnitude of the charge

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$q = 4\pi\epsilon_0 E r^2$$

$$= -1.80 \times 10^{-9} \text{ C}.$$

## 22.5 Charges on Conductors

### 22.5.1 Example 22.12

a) Rearranging the equation for the electric field at the surface of a conductor gives

$$E_{\perp} = \frac{\sigma}{\epsilon_0}$$

$$\sigma = E_{\perp} \epsilon_0$$

$$= (-150)(8.85 \times 10^{-12})$$

$$= -1.33 \times 10^{-9} \text{ C/m}^2.$$

b) Multiplying  $\sigma$  by the surface area of the Earth gives

$$Q = 4\pi R^2 \sigma$$

$$= 4\pi (6.38 \times 10^6)^2 (-1.33 \times 10^{-9})$$

$$= -6.80 \times 10^5 \text{ C}.$$

## 22.6 Guided Practice

### 22.6.1 VP22.4.4

a)  $q_1$  is within the Gaussian surface, so

$$\Phi_E = \frac{Q_{\text{enc}}}{\epsilon_0} = \frac{3.00 \times 10^{-9}}{8.854 \times 10^{-12}} = 339 \text{ N m}^2/\text{C}.$$

b)  $q_2$  is also within the Gaussian surface, so

$$\Phi_E = \frac{(3.00 \times 10^{-9}) + (-8.00 \times 10^{-9})}{8.854 \times 10^{-12}} = -565 \text{ N m}^2/\text{C}.$$

c)  $q_3$  is outside the Gaussian surface so the flux is unchanged from part b.

### 22.6.2 VP22.10.4

From Examples 21.12 and 22.8 we know that the electric field between the plates of a capacitor is

$$E = \frac{\sigma}{\epsilon_0}.$$

Rearranging the equation for electric force we find

$$\begin{aligned} F &= qE \\ &= q \frac{\sigma}{\epsilon_0} \\ \Rightarrow \sigma &= \frac{\epsilon_0 F}{q} \\ &= \frac{(8.854 \times 10^{-12})(22.0 \times 10^{-6})}{3.60 \times 10^{-9}} \\ &= 5.41 \times 10^{-8} \text{ C/m}^2. \end{aligned}$$

### 22.6.3 VP22.12.1

The inner surface of the block must have a total charge of  $-3.00 \text{ nC}$  to balance the charge within the cavity, leaving  $-5.00 \text{ nC}$  for the outer surface.

### 22.6.4 VP22.12.4

a) The inner surface must have a total charge of  $-4.00 \text{ nC}$  to balance charge within the cavity, leaving  $-2.00 \text{ nC}$  for the outer surface.

b) Found in part a:  $-2.00 \text{ nC}$ .

c)  $E = 2.50 \times 10^3 \text{ N/C}$

d)  $E = 1.65 \times 10^2 \text{ N/C}$



### 22.6.5 Bridging Problem

- a) The sphere will include the proton's charge (+ $Q$ ) plus a portion of the electron's charge.

$$\begin{aligned}
 q &= Q + \int_0^r \rho \, dV \\
 &= Q + \int_0^r -\frac{Q}{\pi a_0^3} e^{-2r'/a_0} 4\pi r'^2 \, dr' \\
 &= Q - \frac{4Q}{a_0^3} \int_0^r e^{-2r'/a_0} r'^2 \, dr'.
 \end{aligned}$$

Using integration by parts

$$\begin{aligned}
 \int e^{-2r/a_0} r^2 \, dr &= -\frac{a_0}{2} e^{-2r/a_0} r^2 - \int -a_0 e^{-2r/a_0} r \, dr \\
 &= -\frac{a_0}{2} e^{-2r/a_0} r^2 - \left( \frac{a_0^2}{2} e^{-2r/a_0} r - \int \frac{a_0^2}{2} e^{-2r/a_0} \, dr \right) \\
 &= -\frac{a_0}{2} e^{-2r/a_0} r^2 - \frac{a_0^2}{2} e^{-2r/a_0} r - \frac{a_0^3}{4} e^{-2r/a_0} + c \\
 &= -\frac{a_0}{4} e^{-2r/a_0} (2r^2 + 2a_0 r + a_0^2) + c \\
 \Rightarrow \int_0^r e^{-2r'/a_0} r'^2 \, dr' &= \frac{1}{4} a_0 \left( a_0^2 - e^{-2r/a_0} (2r^2 + 2a_0 r + a_0^2) \right)
 \end{aligned}$$

Therefore

$$\begin{aligned}
 q &= Q - \frac{4Q}{a_0^3} \frac{1}{4} a_0 \left( a_0^2 - e^{-2r/a_0} (2r^2 + 2a_0 r + a_0^2) \right) \\
 &= Q \left( 1 - 1 + \frac{e^{-2r/a_0}}{a_0^2} (2r^2 + 2a_0 r + a_0^2) \right) \\
 &= Q e^{-2r/a_0} \left( 2 \left( \frac{r}{a_0} \right)^2 + 2 \frac{r}{a_0} + 1 \right).
 \end{aligned}$$

- b) By Gauss's law and symmetry we know that the electric field will only have a radial component and will have magnitude

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}.$$

## 22.7 Exercises

### 22.7.1 22.2

The electric field is constant so we can calculate the electric flux as

$$\begin{aligned}\Phi_E &= \mathbf{A} \cdot \mathbf{E} \\ &= AE \cos \theta \\ &= (0.400)(0.600)(90.0) \cos 70^\circ \\ &= 7.4 \text{ N m}^2/\text{C}.\end{aligned}$$

### 22.7.2 22.9

- a) Assuming the plastic sphere has no charge, the electric field is  $\mathbf{0}$ .
- b) By Gauss's law and symmetry the electric field is as if all charge were concentrated at the centre of the sphere, giving  $E = -1.22 \times 10^8 \text{ N/C}$ .
- c) Using the same logic as in part b,  $E = -3.64 \times 10^7 \text{ N/C}$ .

### 22.7.3 22.13

The electric field of an infinite line charge is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r} \hat{\mathbf{r}}.$$

The force experienced by the section is

$$\begin{aligned}F &= \int dF \\ &= \int E dq \\ &= \int_0^L \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r} \lambda dx \\ &= \frac{1}{4\pi\epsilon_0} \frac{2\lambda^2}{r} \int_0^L dx \\ &= (8.988 \times 10^9) \frac{2(5.20 \times 10^{-6})^2}{0.300} (0.0500) \\ &= 0.0810 \text{ N}.\end{aligned}$$

### 22.7.4 22.23

- a) The electric field of an infinite plane is

$$E = \frac{\sigma}{2\epsilon_0}.$$

The point charge and sheet have opposite charges so are attracted to each other. The work done on the electron by the electric field of the sheet is

$$W = Fd = q \frac{\sigma}{2\epsilon_0} d = (1.60 \times 10^{-19}) \frac{2.90 \times 10^{-12}}{2(8.85 \times 10^{-12})} (0.250) = 6.56 \times 10^{-21} \text{ J.}$$

- b) By the work-energy theorem, we can use the kinetic energy of the electron to calculate its velocity

$$\begin{aligned} W &= \frac{1}{2}mv^2 \\ \Rightarrow v &= \sqrt{\frac{2W}{m}} \\ &= \sqrt{\frac{2(6.56 \times 10^{-21})}{9.11 \times 10^{-31}}} \\ &= 1.20 \times 10^5 \text{ m/s.} \end{aligned}$$

#### 22.7.5 22.34

The electric field is non-uniform so to calculate the electric flux we must perform an integral

$$\begin{aligned} \Phi_E &= \int E dA \\ &= \int_0^L \int_0^L kx dx dy \\ &= kL \left[ \frac{1}{2}x^2 \right]_0^L \\ &= \frac{1}{2}kL^3 \\ &= \frac{1}{2}(964)(0.350)^3 \\ &= 20.7 \text{ N m}^2/\text{C.} \end{aligned}$$

#### 22.7.6 22.45

- a) The charge contained within a sphere of radius  $r$  is given by

$$\begin{aligned}
q &= \int_a^r \rho \, dV \\
&= \int_a^r \frac{\alpha}{r} 4\pi r'^2 \, dr' \\
&= 4\pi\alpha \int_a^r r' \, dr' \\
&= 4\pi\alpha \left[ \frac{1}{2} r'^2 \right]_a^r \\
&= 2\pi\alpha(r^2 - a^2).
\end{aligned}$$

By symmetry and Gauss's law, the electric field at radius  $r$  is the same as if all the charge were concentrated at the centre of the sphere so

$$\begin{aligned}
E &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \\
&= \frac{1}{4\pi\epsilon_0} \frac{2\pi\alpha(r^2 - a^2)}{r^2}.
\end{aligned}$$

b) The net electric field must be equal at  $r = a$  and  $r = b$

$$\begin{aligned}
\frac{1}{4\pi\epsilon_0} \frac{1}{a^2} (2\pi\alpha(a^2 - a^2) + q) &= \frac{1}{4\pi\epsilon_0} \frac{1}{b^2} (2\pi\alpha(b^2 - a^2) + q) \\
q &= \frac{a^2}{b^2} (2\pi\alpha(b^2 - a^2) + q) \\
q \left( 1 - \frac{a^2}{b^2} \right) &= \frac{a^2}{b^2} 2\pi\alpha(b^2 - a^2) \\
q &= 2\pi\alpha a^2.
\end{aligned}$$

c) The net electric field between  $r = a$  and  $r = b$  will be

$$\begin{aligned}
E &= \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} (2\pi\alpha(r^2 - a^2) + 2\pi\alpha a^2) \\
&= \frac{\alpha}{2\epsilon_0}.
\end{aligned}$$

## 22.7.7 22.61

a) By Gauss's law and symmetry, the electric field of the sphere is the same as if the charge were concentrated at its centre. Thus the electric flux through the round side of the cylinder is

$$\begin{aligned}
\Phi_a &= \int \mathbf{E} \cdot d\mathbf{A} \\
&= \int_{-L/2}^{L/2} \frac{1}{4\pi\epsilon_0} \frac{Q}{r'^2} \cos\theta 2\pi R dz \\
&= \frac{QR^2}{2\epsilon_0} \int_{-L/2}^{L/2} (R^2 + z^2)^{-3/2} dz \\
&= \frac{QR^2}{2\epsilon_0} \left[ \frac{z}{R^2\sqrt{R^2 + z^2}} \right]_{-L/2}^{L/2} \\
&= \frac{QR^2}{2\epsilon_0} \left( \frac{L/2}{R^2\sqrt{R^2 + (L/2)^2}} + \frac{L/2}{R^2\sqrt{R^2 + (-L/2)^2}} \right) \\
&= \frac{QL}{2\epsilon_0\sqrt{R^2 + (L/2)^2}}.
\end{aligned}$$

b) The electric flux through the top of the cylinder is

$$\begin{aligned}
\Phi_b &= \int \mathbf{E} \cdot d\mathbf{A} \\
&= \int_0^R \frac{1}{4\pi\epsilon_0} \frac{Q}{r'^2} \cos\theta 2\pi x dx \\
&= \frac{Q}{2\epsilon_0} \int_0^R \frac{(L/2)x}{((L/2)^2 + x^2)^{3/2}} dx \\
&= \frac{QL}{4\epsilon_0} \int_0^R x((L/2)^2 + x^2)^{-3/2} dx
\end{aligned}$$

Let  $u = (L/2)^2 + x^2$  so  $du = 2x dx$  and thus

$$\begin{aligned}
\Phi_b &= \frac{QL}{8\epsilon_0} \int_{(L/2)^2}^{(L/2)^2 + R^2} u^{-3/2} du \\
&= \frac{QL}{8\epsilon_0} \left[ -2u^{-1/2} \right]_{(L/2)^2}^{(L/2)^2 + R^2} \\
&= \frac{QL}{4\epsilon_0} \left( \frac{1}{\sqrt{(L/2)^2}} - \frac{1}{\sqrt{(L/2)^2 + R^2}} \right) \\
&= \frac{QL}{4\epsilon_0} \left( \frac{2}{L} - \frac{1}{\sqrt{(L/2)^2 + R^2}} \right).
\end{aligned}$$

c) The same as in part b.

## 23 Electric Potential

### 23.1 Electric Potential Energy

#### 23.1.1 Example 23.1

- a) The work done on the positron by the  $\alpha$  particle's electric field is equal to the negative of the change in electric potential between them

$$\begin{aligned} W &= -\Delta U \\ &= -(U_b - U_a) \\ &= \frac{1}{4\pi\epsilon_0} qq_0 \left( \frac{1}{r_a} - \frac{1}{r_b} \right) \\ &= 2.30 \times 10^{-18} \text{ J.} \end{aligned}$$

By the work-energy theorem this is equal to the positron's change in kinetic energy, so its final speed is

$$\begin{aligned} K_a + W &= K_b \\ \frac{1}{2}mv_a^2 + W &= \frac{1}{2}mv_b^2 \\ \Rightarrow v_b &= \sqrt{v_a^2 + \frac{2W}{m}} \\ &= 3.75 \times 10^6 \text{ m/s.} \end{aligned}$$

- b) When the positron is very far from the  $\alpha$  particle all of their electric potential energy has been converted into kinetic energy

$$\begin{aligned} K_a + U_a &= K_c \\ \frac{1}{2}mv_a^2 + \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r_a} &= \frac{1}{2}mv_c^2 \\ \Rightarrow v_c &= \sqrt{v_a^2 + \frac{1}{2\pi\epsilon_0} \frac{qq_0}{mr_a}} \\ &= 4.37 \times 10^6 \text{ m/s.} \end{aligned}$$

- c) The electron's speed would decrease until it's stationary, then it would accelerate towards the  $\alpha$  particle.

### 23.1.2 Example 23.2

- a) The work that must be done to bring  $q_3$  in from infinity is equal to its electric potential energy with the system at its final position

$$\begin{aligned} W &= U \\ &= \frac{q_3}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\ &= \frac{e}{4\pi\epsilon_0} \left( \frac{-e}{2a} + \frac{e}{a} \right) \\ &= \frac{e^2}{8\pi\epsilon_0 a} \end{aligned}$$

- b) The total potential energy is equal to

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \sum_{i < j} \frac{q_i q_j}{r_{ij}} \\ &= \frac{1}{4\pi\epsilon_0} \left( \frac{-e^2}{a} - \frac{e^2}{2a} + \frac{e^2}{a} \right) \\ &= -\frac{e^2}{8\pi\epsilon_0 a} \end{aligned}$$

## 23.2 Electric Potential

### 23.2.1 Example 23.3

- a) The force on the proton is

$$F = eE = (1.602 \times 10^{-19})(1.5 \times 10^7) = 2.4 \times 10^{-12} \text{ N}$$

- b) The work done on it by the field is

$$W = dF = (0.50)(2.4 \times 10^{-12}) = 1.2 \times 10^{-12} \text{ J} = 7.5 \text{ MeV}$$

- c) The potential difference between the two points is

$$V_a - V_b = dE = (0.50)(1.5 \times 10^7) = 7.5 \times 10^6 \text{ V}$$

### 23.2.2 Example 23.4

- a) The electric potential at  $a$  is

$$\begin{aligned}
V &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\
&= (8.988 \times 10^9) \left( \frac{12 \times 10^{-9}}{0.060} - \frac{12 \times 10^{-9}}{0.040} \right) \\
&= -9.0 \times 10^2 \text{ V}
\end{aligned}$$

b) The electric potential at  $b$  is

$$\begin{aligned}
V &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\
&= (8.988 \times 10^9) \left( \frac{12 \times 10^{-9}}{0.040} - \frac{12 \times 10^{-9}}{0.140} \right) \\
&= 1.9 \times 10^3 \text{ V}
\end{aligned}$$

c)  $c$  is equidistant from both charges so the electric potential is 0

### 23.2.3 Example 23.5

a)  $U = qV = (4.0 \times 10^{-9})(-9.0 \times 10^2) = -3.6 \times 10^{-6} \text{ J}$

b)  $U = qV = (4.0 \times 10^{-9})(1.9 \times 10^3) = 7.6 \times 10^{-6} \text{ J}$

c)  $U = 0$

### 23.2.4 Example 23.6

$$\begin{aligned}
V_a - V_b &= \int_a^b \mathbf{E} \cdot d\mathbf{l} \\
\Rightarrow V_r - 0 &= \int_r^\infty \frac{1}{4\pi\epsilon_0} \frac{q}{r'^2} dr' \\
V_r &= \frac{1}{4\pi\epsilon_0} q \left[ -\frac{1}{r'} \right]_r^\infty \\
&= \frac{1}{4\pi\epsilon_0} \frac{q}{r}
\end{aligned}$$

### 23.2.5 Example 23.7

The potential at  $a$  is



$$\begin{aligned}
V_a &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\
&= (8.988 \times 10^9) \left( \frac{3.0 \times 10^{-9}}{0.010} - \frac{3.0 \times 10^{-9}}{0.020} \right) \\
&= 1.3 \times 10^3 \text{ V}.
\end{aligned}$$

The potential at  $b$  is

$$\begin{aligned}
V_b &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\
&= (8.988 \times 10^9) \left( \frac{3.0 \times 10^{-9}}{0.020} - \frac{3.0 \times 10^{-9}}{0.010} \right) \\
&= -1.3 \times 10^3 \text{ V}.
\end{aligned}$$

So the potential difference is

$$V_a - V_b = 2.6 \times 10^3 \text{ V}$$

the work done on the dust particle as it moves from  $a$  to  $b$  is

$$q_0(V_a - V_b) = (2.0 \times 10^{-9})(2.6 \times 10^3) = 5.3 \times 10^{-6} \text{ J}$$

and its speed at  $b$  is

$$\begin{aligned}
K &= \frac{1}{2}mv^2 \\
\Rightarrow v &= \sqrt{\frac{2K}{m}} \\
&= \sqrt{\frac{2(5.3 \times 10^{-6})}{5.0 \times 10^{-9}}} \\
&= 46 \text{ m/s}.
\end{aligned}$$

## 23.3 Calculating Electric Potential

### 23.3.1 Example 23.8

By Gauss's law and symmetry, the electric field outside the sphere is the same as if all of its charge were concentrated at its centre. So, for  $r \geq R$

$$V_{\text{out}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}.$$

There is no charge inside the sphere so the electric potential is equal to that of the surface of the sphere

$$V_{\text{in}} = \frac{1}{4\pi\epsilon_0} \frac{q}{R}.$$

### 23.3.2 Example 23.10

The line's electric field points radially away from it with magnitude

$$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}.$$

Setting the electric potential to be 0 at  $r = r_0$  we find that

$$\begin{aligned} V &= \int_r^{r_0} \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r'} dr' \\ &= \frac{\lambda}{2\pi\epsilon_0} [\ln r']_r^{r_0} \\ &= \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r}. \end{aligned}$$

### 23.3.3 Example 23.11

The electric potential is

$$\begin{aligned} V &= \int \frac{1}{4\pi\epsilon_0} \frac{dq}{r} \\ &= \frac{1}{4\pi\epsilon_0} \frac{1}{\sqrt{a^2 + x^2}} \int dq \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{a^2 + x^2}}. \end{aligned}$$

### 23.3.4 Example 23.12

The electric potential is

$$\begin{aligned} V &= \int \frac{1}{4\pi\epsilon_0} \frac{dq}{r} \\ &= \frac{1}{4\pi\epsilon_0} \int_{-a}^a \frac{1}{\sqrt{x^2 + y^2}} \frac{Q}{2a} dy \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \left[ \ln \left( y + \sqrt{x^2 + y^2} \right) \right]_{-a}^a \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \ln \left( \frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a} \right) \end{aligned}$$

## 23.4 Potential Gradient

### 23.4.1 23.13

The electric field is

$$\begin{aligned}\mathbf{E} &= -\frac{\partial V}{\partial r} \hat{\mathbf{r}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}.\end{aligned}$$

### 23.4.2 Example 23.14

The electric field is

$$\begin{aligned}\mathbf{E} &= -\nabla V \\ &= -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right) \\ &= -\frac{\partial}{\partial x}\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+a^2}}\right)\hat{\mathbf{i}} \\ &= -\frac{1}{4\pi\epsilon_0}Q\left(-\frac{1}{2}\frac{2x}{(x^2+a^2)^{3/2}}\right)\hat{\mathbf{i}} \\ &= \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}\hat{\mathbf{i}}.\end{aligned}$$

## 23.5 Guided Practice

### 23.5.1 VP23.2.1

- a) The work required to move the electron is equal to the change in the system's potential energy

$$\begin{aligned}W &= -\frac{1}{4\pi\epsilon_0}(82e^2)\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \\ &= -(8.988 \times 10^9)(82)(-1.60 \times 10^{-19})^2\left(\frac{1}{5.00 \times 10^{-10}} - \frac{1}{1.00 \times 10^{-10}}\right) \\ &= 1.51 \times 10^{-16} \text{ J}\end{aligned}$$

- b) The electron will be attracted to the lead nucleus and some of its electric potential energy will be converted to kinetic energy

$$\begin{aligned}
K_1 + U_1 &= K_2 + U_2 \\
0 - \frac{1}{4\pi\epsilon_0} \frac{82e^2}{r_1} &= K_2 - \frac{1}{4\pi\epsilon_0} \frac{82e^2}{r_2} \\
\Rightarrow K_2 &= \frac{1}{4\pi\epsilon_0} 82e^2 \left( \frac{1}{r_2} - \frac{1}{r_1} \right) \\
&= (8.988 \times 10^9) 82 (-1.60 \times 10^{-19})^2 \left( \frac{1}{8.00 \times 10^{-12}} - \frac{1}{5.00 \times 10^{-10}} \right) \\
&= 2.32 \times 10^{-15} \text{ J}
\end{aligned}$$

### 23.5.2 VP23.2.4

- a) The potential energy of the charges is equal to the work done to bring in the second charge, so

$$\begin{aligned}
W &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \\
\Rightarrow q_2 &= \frac{4\pi\epsilon_0 r W}{q_1} \\
&= \frac{4\pi(8.854 \times 10^{-12})(0.0400)(8.10 \times 10^{-6})}{5.00 \times 10^{-9}} \\
&= 7.21 \times 10^{-9} \text{ C}
\end{aligned}$$

- b) The work required to bring in the third charge is

$$\begin{aligned}
W &= \frac{1}{4\pi\epsilon_0} q_3 \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\
&= (8.988 \times 10^9)(2.00 \times 10^{-9}) \left( \frac{5.00 \times 10^{-9}}{0.0300} + \frac{7.21 \times 10^{-9}}{\sqrt{0.0300^2 + 0.0400^2}} \right) \\
&= 5.59 \times 10^{-6} \text{ J.}
\end{aligned}$$

### 23.5.3 VP23.7.1

- a) The electric field is uniform and both points lie on the  $x$ -axis so

$$V_0 - V_P = dE = (0.0500)(5.00 \times 10^2) = 25.0 \text{ V}$$

### 23.5.4 VP23.7.2

- a) The electric potential is

$$V = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{r} = 2(8.988 \times 10^9) \frac{6.0 \times 10^{-9}}{0.030} = 3.6 \times 10^3 \text{ V}$$

- d) The electric field is 0 at the origin

**23.5.5 VP23.7.4**

- a) Let the electric potential at the surface of the sphere be 0, then the potential difference between the centre of the sphere and the surface is

$$\begin{aligned}
 V_{\text{centre}} - V_{\text{surface}} &= \int \mathbf{E} \cdot d\mathbf{l} \\
 \Rightarrow V_{\text{centre}} &= \int_0^R \frac{1}{4\pi\epsilon_0} \frac{Qr}{R^3} dr \\
 &= \frac{Q}{4\pi\epsilon_0 R^3} \left[ \frac{1}{2} r^2 \right]_0^R \\
 &= \frac{Q}{8\pi\epsilon_0 R}
 \end{aligned}$$

**23.5.6 VP23.12.1**

- a) By Gauss's law and symmetry, the electric field (and thus the electric potential) are the same as if the sphere's charge were concentrated at its centre. Thus the work required to move a test charge from  $5R$  to  $3R$  is

$$\begin{aligned}
 W &= \frac{1}{4\pi\epsilon_0} qq_0 \left( \frac{1}{3R} - \frac{1}{5R} \right) \\
 &= \frac{1}{4\pi\epsilon_0} \frac{2qq_0}{15R} \\
 &= \frac{qq_0}{30\pi\epsilon_0 R}
 \end{aligned}$$

**23.5.7 VP23.12.2**

- a) The electric field is constant, so the electric potential decreases linearly over the distance between the two plates and thus

$$V = \frac{d'}{d} V_{ab} = \frac{3.00}{4.50} 24.0 = 16.0 \text{ V}$$

- b)  $U = qV = (2.00 \times 10^{-9})(16.0) = 3.20 \times 10^{-8} \text{ J}$   
 c) It will move towards the lower plate. When it reaches the plate its speed will be

$$\begin{aligned}
U &= K \\
&= \frac{1}{2}mv^2 \\
\Rightarrow v &= \sqrt{\frac{2U}{m}} \\
&= \sqrt{\frac{2(3.20 \times 10^{-8})}{5.00 \times 10^{-9}}} \\
&= 3.58 \text{ m/s}
\end{aligned}$$

### 23.5.8 VP23.12.4

a)  $dq = \frac{Q}{L} dx$

b) The electric potential at the origin is

$$\begin{aligned}
V &= \int \frac{1}{4\pi\epsilon_0} \frac{dq}{r} \\
&= \frac{1}{4\pi\epsilon_0} \int_L^{2L} \frac{Q}{Lx} dx \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{L} [\ln x]_L^{2L} \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{L} \ln 2
\end{aligned}$$

### 23.5.9 Bridging Problem

The electric potential at  $x = L$  due to the rod is

$$\begin{aligned}
V &= \int \frac{1}{4\pi\epsilon_0} \frac{dq}{r} \\
&= \frac{1}{4\pi\epsilon_0} \int_{-a}^a \frac{Q}{2a} \frac{1}{L-x} dx \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} [-\ln(L-x)]_{-a}^a \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} (-\ln(L-a) + \ln(L+a)) \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \ln\left(\frac{L+a}{L-a}\right).
\end{aligned}$$

The work required to bring in a charge  $q$  from infinity to  $x = L$  is

$$\begin{aligned}
W &= qV \\
&= \frac{1}{4\pi\epsilon_0} \frac{qQ}{2a} \ln \left( \frac{L+a}{L-a} \right).
\end{aligned}$$

## 23.6 Exercises

### 23.6.1 23.1

The electric potential energy at the first position is

$$\begin{aligned}
U &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_1} \\
&= (8.988 \times 10^9) \frac{(2.40 \times 10^{-6})(-4.30 \times 10^{-6})}{0.150} \\
&= -0.618 \text{ J}.
\end{aligned}$$

The electric potential energy at the second position is

$$\begin{aligned}
U &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_2} \\
&= (8.988 \times 10^9) \frac{(2.40 \times 10^{-6})(-4.30 \times 10^{-6})}{\sqrt{2(0.250)^2}} \\
&= -0.262 \text{ J}.
\end{aligned}$$

The work done by the electric force on  $q_2$  is

$$W = U_1 - U_2 = (-0.618) - (-0.262) = -0.356 \text{ J}.$$

### 23.6.2 23.3

The total energy of the system (and thus the work required to assemble it) is

$$\begin{aligned}
E &= \frac{1}{4\pi\epsilon_0} \sum_{i < j} \frac{q_i q_j}{r_{ij}} \\
&= \frac{1}{4\pi\epsilon_0} 3 \frac{e^2}{r} \\
&= (8.988 \times 10^9) 3 \frac{(1.60 \times 10^{-19})^2}{2.00 \times 10^{-15}} \\
&= 3.45 \times 10^{-13} \text{ J}.
\end{aligned}$$

### 23.6.3 23.5

- a) When the spheres are 0.400 m apart some of the second sphere's initial kinetic energy will have been converted into electric potential energy, slowing it down. Using the principle of conservation of energy we find

$$\begin{aligned}
 K_1 + U_1 &= K_2 + U_2 \\
 \frac{1}{2}mv_1^2 + \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r_1} &= \frac{1}{2}mv_2^2 + \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r_2} \\
 v_2 &= \sqrt{v_1^2 + \frac{q_1q_2}{2\pi\epsilon_0 m} \left( \frac{1}{r_1} - \frac{1}{r_2} \right)} \\
 &= 12.5 \text{ m/s}
 \end{aligned}$$

- b) When the sphere's are closest, all of the second sphere's initial kinetic energy will have been converted into electric potential energy. Again, using the principle of conservation of energy we find

$$\begin{aligned}
 K_1 + U_1 &= U_2 \\
 \frac{1}{2}mv_1^2 + \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r_1} &= \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r_2} \\
 mv_1^2 + \frac{q_1q_2}{2\pi\epsilon_0 r_1} &= \frac{q_1q_2}{2\pi\epsilon_0 r_2} \\
 r_2 \left( 2\pi\epsilon_0 mv_1^2 + \frac{q_1q_2}{r_1} \right) &= q_1q_2 \\
 r_2 &= \frac{q_1q_2 r_1}{2\pi\epsilon_0 m r_1 v_1^2 + q_1q_2} \\
 &= 0.323 \text{ m}
 \end{aligned}$$

### 23.6.4 23.7

The protons will exert the maximum electric force on each other when they're closest. This will happen when all of their initial kinetic energy has been converted into electric potential energy. Assuming their electric potential energy is 0 at the start and using the principle of conservation of energy we find that

$$\begin{aligned}
 2K_1 &= U_2 \\
 2 \frac{1}{2}mv_1^2 &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_2} \\
 r_2 &= \frac{e^2}{4\pi\epsilon_0 mv_1^2} \\
 &= 3.44 \times 10^{-12} \text{ m}
 \end{aligned}$$



at which point the electric force between them will be

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_2^2} \\ &= 1.94 \times 10^{-5} \text{ N} \end{aligned}$$

### 23.6.5 23.9

- a) The protons will reach their maximum speed when all of their initial electric potential energy has been converted into kinetic energy. Using the principle of conservation of energy, the maximum speed they will reach is

$$\begin{aligned} U_1 &= 2K_2 \\ \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_1} &= 2 \frac{1}{2} m v_2^2 \\ v_2 &= \sqrt{\frac{e^2}{4\pi\epsilon_0 m r_1}} \\ &= 1.36 \times 10^4 \text{ m/s} \end{aligned}$$

- b) The protons will experience their maximum acceleration when they are closest, i.e. at the start. The maximum acceleration will be

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{m r^2} \\ &= 2.45 \times 10^{17} \text{ m/s}^2 \end{aligned}$$

### 23.6.6 23.11

The potential difference between two points is equal to the work done by the electric force while moving between them, so

$$\begin{aligned}
V_a - V_b &= \int_a^b \mathbf{E} \cdot d\mathbf{l} \\
&= \int_{y_a}^{y_b} \left( \alpha + \frac{\beta}{y^2} \right) dy \\
&= \left[ \alpha y - \frac{\beta}{y} \right]_{y_a}^{y_b} \\
&= \alpha y_b - \frac{\beta}{y_b} - \alpha y_a + \frac{\beta}{y_a} \\
&= 89.3 \text{ V}
\end{aligned}$$

### 23.6.7 23.13

The particle has negative charge and thus moves from areas of low to high electric potential. Point  $B$  has greater electric potential than point  $A$  so the particle has lost electric potential energy. As the electric force is the only force acting on the particle, some of its electric potential energy has been converted into kinetic energy and thus it is moving faster. Its final speed is

$$\begin{aligned}
K_A + U_A &= K_B + U_B \\
\frac{1}{2}mv_A^2 + qV_A &= \frac{1}{2}mv_B^2 + qV_B \\
v_B &= \sqrt{v_A^2 + \frac{2q}{m}(V_A - V_B)} \\
&= 7.42 \text{ m/s}
\end{aligned}$$

### 23.6.8 23.15

- a)  $W = 0 \text{ J}$
- b)  $W = dEq = 7.50 \times 10^{-4} \text{ J}$
- c)  $W = -d \cos \theta Eq = -2.06 \times 10^{-3} \text{ J}$

### 23.6.9 23.17

- a) The electric potential at point  $a$  is

$$\begin{aligned}
V &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r} + \frac{q_2}{r} \right) \\
&= (8.988 \times 10^9) \left( \frac{2.00 \times 10^{-6}}{\sqrt{3^2 + 3^2}/2} - \frac{2.00 \times 10^{-6}}{\sqrt{3^2 + 3^2}/2} \right) \\
&= 0
\end{aligned}$$

b) The electric potential at  $b$  is

$$\begin{aligned} V &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\ &= (8.988 \times 10^9) \left( \frac{2.00 \times 10^{-6}}{\sqrt{0.0300^2 + 0.0300^2}} - \frac{2.00 \times 10^{-6}}{0.0300} \right) \\ &= -176 \text{ kV} \end{aligned}$$

c) The work done on  $q_3$  is

$$W = q_3(V_a - V_b) = -0.88 \text{ J}$$

### 23.6.10 32.19

a) The potential at point  $A$  is

$$V = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r} + \frac{q_2}{r} \right) = -737 \text{ V}$$

b) The potential at point  $B$  is

$$V = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r} + \frac{q_2}{r} \right) = -704 \text{ V}$$

c) The work done by the electric field on the charge is

$$W = q(V_B - V_A) = 8.25 \times 10^{-8} \text{ J}$$

### 23.6.11 23.21

a) Negative

b) The electric field is uniform, so

$$\begin{aligned} V_B - V_A &= (y_B - y_A)E_y \\ \Rightarrow E_y &= \frac{V_B - V_A}{y_B - y_A} \\ &= 171 \text{ N/C} \end{aligned}$$

c) The electric field only does work when moving along the  $y$ -axis, so the  $x$  coordinate can be ignored and

$$V_B - V_C = dE_y = 17.1 \text{ V}$$

**23.6.12 23.23**a)  $b$ 

b)  $E = \frac{V_B - V_A}{x_B - x_A} = 800 \text{ N/C}$

c)  $W = qEd = -4.80 \times 10^{-5} \text{ J}$

**23.6.13 23.25**

a) The work done on the sphere is equal to its change in electric potential energy

$$\begin{aligned}
W &= U_1 - U_2 \\
&= 0 - \frac{1}{4\pi\epsilon_0} Q \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \\
&= -(8.988 \times 10^9)(-0.200 \times 10^{-6}) \left( \frac{6.00 \times 10^{-6}}{0.400} + \frac{6.00 \times 10^{-6}}{0.400} \right) \\
&= 5.39 \times 10^{-2} \text{ J}
\end{aligned}$$

b) By the work-energy theorem, the work done on the sphere equals its change in kinetic energy. It was released from rest, so

$$\begin{aligned}
K &= W \\
\frac{1}{2}mv^2 &= W \\
v &= \sqrt{\frac{2W}{m}} \\
&= 3.00 \text{ m/s}
\end{aligned}$$

**23.6.14 23.27**

a) i)

$$\begin{aligned}
V &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{R_1} + \frac{q_2}{R_2} \right) \\
&= (8.988 \times 10^9) \left( \frac{6.00 \times 10^{-9}}{0.0300} + \frac{-9.00 \times 10^{-9}}{0.0500} \right) \\
&= 180 \text{ V}
\end{aligned}$$

ii)

$$\begin{aligned}
 V &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r} + \frac{q_2}{R_2} \right) \\
 &= (8.988 \times 10^9) \left( \frac{6.00 \times 10^{-9}}{0.0400} + \frac{-9.00 \times 10^{-9}}{0.0500} \right) \\
 &= -270 \text{ V}
 \end{aligned}$$

iii)

$$\begin{aligned}
 V &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r} + \frac{q_2}{R_2} \right) \\
 &= (8.988 \times 10^9) \left( \frac{6.00 \times 10^{-9}}{0.0600} + \frac{-9.00 \times 10^{-9}}{0.0600} \right) \\
 &= -450 \text{ V}
 \end{aligned}$$

b) The potential difference is

$$\begin{aligned}
 V_1 - V_2 &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{R_1} + \frac{q_2}{R_2} - \frac{q_1}{R_2} - \frac{q_2}{R_2} \right) \\
 &= \frac{1}{4\pi\epsilon_0} q_1 \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \\
 &= 719 \text{ V}
 \end{aligned}$$

so the surface of the inner shell has a higher electric potential

### 23.6.15 23.29

- a) The electric force pulls the electron towards the ring, causing it to accelerate. It will speed up, move towards the ring, and eventually pass through its centre. Once on the other side the electric force will change direction and pull the electron back towards the ring. It will oscillate forever.
- b) From Example 23.11 we know that the electric potential along the axis of a charged ring is given by

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + a^2}}.$$

The electric potential at the electron's initial position is

$$\begin{aligned}
 V_1 &= \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + a^2}} \\
 &= (8.988 \times 10^9) \frac{24 \times 10^{-9}}{\sqrt{0.3^2 + 0.15^2}} \\
 &= 643 \text{ V}.
 \end{aligned}$$

The electric potential at the centre of the ring is

$$\begin{aligned} V_2 &= \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \\ &= (8.988 \times 10^9) \frac{24 \times 10^{-9}}{0.15} \\ &= 1440 \text{ V.} \end{aligned}$$

By the work-energy principle, all of the work done by the electric force on the electron will be converted to kinetic energy so its speed at the centre of the ring is

$$\begin{aligned} W &= K \\ e(V_1 - V_2) &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2e(V_1 - V_2)}{m}} \\ &= 1.67 \times 10^7 \text{ m/s.} \end{aligned}$$

### 23.6.16 23.31

By Gauss's law both spheres can be treated as point charges. By the principle of conservation of energy, the small sphere will be closest to the large sphere when all of its kinetic energy has been converted into electric potential energy

$$\begin{aligned} K_1 &= U_2 \\ \frac{1}{2}mv_1^2 &= \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \\ v_1 &= \sqrt{\frac{qQ}{2\pi\epsilon_0 mr}} \\ &= 150 \text{ m/s.} \end{aligned}$$

### 23.6.17 23.33

a) By Gauss's law the cylinder's electric potential is equivalent to that of an infinite line charge with the same linear charge density

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r}.$$

Setting  $r_0 = 0.1000$  the potential difference is

$$\begin{aligned}
 V &= \frac{8.50 \times 10^{-6}}{2\pi(8.854 \times 10^{-12})} \ln \frac{0.10000}{0.0600} \\
 &= 7.80 \times 10^4 \text{ V}
 \end{aligned}$$

- b) There is no charge inside the cylinder so it is an equipotential and the potential difference is 0

### 23.6.18 23.35

The electric potential of the line of charge is

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r}.$$

Setting  $r_0 = 0.0450$  the potential difference between the sphere's initial and final positions is

$$\begin{aligned}
 V_1 - V_2 &= \frac{\lambda}{2\pi\epsilon_0} \left( \ln \frac{r_0}{r_1} - \ln \frac{r_0}{r_2} \right) \\
 &= \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_2}{r_1} \\
 &= 5.92 \times 10^4 \text{ V}
 \end{aligned}$$

By the work-energy theorem, all of the work done by the electric force on the sphere is converted into kinetic energy so

$$\begin{aligned}
 K &= W \\
 &= q(V_1 - V_2) \\
 &= 0.474 \text{ J}
 \end{aligned}$$

### 23.6.19 23.37

- a)  $E = V/d = 8 \times 10^3 \text{ N/C}$   
b)  $F = qE = 1.92 \times 10^{-5} \text{ N}$   
c)  $W = Fd = 8.64 \times 10^{-7} \text{ J}$   
d)  $\Delta U = -q\Delta V = -8.64 \times 10^{-7} \text{ J}$

**23.6.20 23.39**

We can find the total charge of the sphere by rearranging the equation for the magnitude of the electric field at the surface of the sphere

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \\ \Rightarrow q &= 4\pi\epsilon_0 ER^2 \\ &= -1.69 \times 10^{-8} \text{ C.} \end{aligned}$$

The sphere is an equipotential so the electric potential inside is equal to that of the surface

$$\begin{aligned} V &= \frac{1}{4\pi\epsilon_0} \frac{q}{R} \\ &= -760 \text{ V} \end{aligned}$$

**23.6.21 23.41**

a) i)  $V = \frac{1}{4\pi\epsilon_0} q \left( \frac{1}{r_a} - \frac{1}{r_b} \right)$

ii)  $V = \frac{1}{4\pi\epsilon_0} q \left( \frac{1}{r} - \frac{1}{r_b} \right)$

iii)  $V = 0$

b) From above, the potential at the surface of the outer sphere is 0 so the potential difference is equal to the potential at the surface of the inner sphere

$$V = \frac{1}{4\pi\epsilon_0} q \left( \frac{1}{r_a} - \frac{1}{r_b} \right).$$

c) The electric field between the spheres has magnitude

$$\begin{aligned} E &= -\frac{\partial V}{\partial r} \\ &= -\frac{\partial}{\partial r} \left( \frac{1}{4\pi\epsilon_0} q \left( \frac{1}{r} - \frac{1}{r_b} \right) \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \\ &= \frac{V_{ab}}{1/r_a - 1/r_b} \frac{1}{r^2} \end{aligned}$$

d) The electric field outside the larger sphere has magnitude 0



## 23.7 Problems

### 23.7.1 23.47

a) The potential energy of the system is

$$\begin{aligned} U &= \frac{1}{4\pi\epsilon_0} \sum_{i < j} \frac{q_i q_j}{r_{ij}} \\ &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \\ &= -3.60 \times 10^{-7} \text{ J} \end{aligned}$$

b) If the potential energy of the system is 0 then

$$\begin{aligned} 0 &= \frac{1}{4\pi\epsilon_0} \left( \frac{q_1 q_2}{r_2} + \frac{q_1 q_3}{r_3} + \frac{q_2 q_3}{r_2 - r_3} \right) \\ &= (r_2 - r_3) r_3 \frac{q_1 q_2}{r_2} + (r_2 - r_3) q_1 q_3 + r_3 q_2 q_3 \\ &= q_1 q_2 r_3 - \frac{q_1 q_2}{r_2} r_3^2 + q_1 q_3 r_2 - q_1 q_3 r_3 + q_2 q_3 r_3 \\ &= -\frac{q_1 q_2}{r_2} r_3^2 + (q_1 q_2 - q_1 q_3 + q_2 q_3) r_3 + q_1 q_3 r_2. \end{aligned}$$

Let

$$\begin{aligned} a &= -\frac{q_1 q_2}{r_2} \\ &= -\frac{(4.00 \times 10^{-9})(-3.00 \times 10^{-9})}{0.200} \\ &= 6.00 \times 10^{-17} \end{aligned}$$

$$\begin{aligned} b &= q_1 q_2 - q_1 q_3 + q_2 q_3 \\ &= (4.00 \times 10^{-9})(-3.00 \times 10^{-9}) - (4.00 \times 10^{-9})(2.00 \times 10^{-9}) + (-3.00 \times 10^{-9})(2.00 \times 10^{-9}) \\ &= -2.60 \times 10^{-17} \end{aligned}$$

$$\begin{aligned} c &= q_1 q_3 r_2 \\ &= (4.00 \times 10^{-9})(2.00 \times 10^{-9})(0.200) \\ &= 1.60 \times 10^{-18} \end{aligned}$$

then

$$\begin{aligned}
 r_3 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{2.60 \times 10^{-17} \pm \sqrt{(-2.60 \times 10^{-17})^2 - 4(6.00 \times 10^{-17})(1.60 \times 10^{-18})}}{2(6.00 \times 10^{-17})} \\
 &= 0.074 \text{ m or } 0.359 \text{ m}
 \end{aligned}$$

$r_3 = 0.359 \text{ m}$  doesn't lie between  $q_1$  and  $q_2$  so it can be discarded and the answer is  $r_3 = 0.074 \text{ m}$ .

### 23.7.2 23.49

The potential of an infinite line charge is

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r}.$$

Setting  $r_0 = 0.300$ , the potential of the line at point  $A$  is  $V_{A,\text{line}} = 0$ , at point  $B$  is  $V_{B,\text{line}} = 3.64 \times 10^4 \text{ V}$ , and thus the potential difference is  $V_{AB,\text{line}} = V_{A,\text{line}} - V_{B,\text{line}} = -3.64 \times 10^4 \text{ V}$ .

The electric field of an infinite sheet of charge is directed away from the sheet and has magnitude

$$E = \frac{\sigma}{2\epsilon_0}.$$

The potential difference due to the sheet is equal to the work per unit charge done by its electric field between points  $A$  and  $B$  so  $V_{AB,\text{sheet}} = Ed = 2.26 \times 10^5 \text{ V}$ .

Thus the total potential difference is  $V_{AB} = V_{AB,\text{line}} + V_{AB,\text{sheet}} = 1.90 \times 10^5 \text{ V}$  meaning point  $A$  is at higher potential.

### 23.7.3 23.51

In order for the alpha particle to reach a distance  $2.0 \times 10^{-14} \text{ m}$  from the surface of the gold nucleus its potential energy at that point must equal the energy it gained from the potential difference

$$\begin{aligned}
 U &= (2e)V \\
 \Rightarrow V &= \frac{U}{2e} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{79e}{r} \\
 &= (8.988 \times 10^9) \frac{79(1.60 \times 10^{-19})}{2.73 \times 10^{-14}} \\
 &= 4.2 \times 10^6 \text{ V}.
 \end{aligned}$$

### 23.7.4 23.55

a)

$$\begin{aligned} V &= Cx^{4/3} \\ \Rightarrow C &= \frac{V}{x^{4/3}} \\ &= \frac{240}{(0.013)^{4/3}} \\ &= 7.85 \times 10^4 \text{ V/m}^{4/3} \end{aligned}$$

b)

$$\begin{aligned} \mathbf{E} &= -\nabla V \\ &= -\frac{4}{3}Cx^{1/3}\hat{\mathbf{i}} \\ &= -(1.05 \times 10^5)x^{1/3}\hat{\mathbf{i}} \end{aligned}$$

c)

$$\begin{aligned} \mathbf{F} &= e\mathbf{E} \\ &= (1.60 \times 10^{-19})(1.05 \times 10^5)(0.0065)^{1/3}\hat{\mathbf{i}} \\ &= (3.14 \times 10^{-15} \text{ N})\hat{\mathbf{i}} \end{aligned}$$

### 23.7.5 23.59

When the sphere is hanging at an angle  $\theta$  it experiences net forces

$$\begin{aligned} F_x &= qE - T \sin \theta = 0 \\ F_y &= T \cos \theta - mg = 0 \end{aligned}$$

Rearranging  $F_y$

$$T = \frac{mg}{\cos \theta}.$$

Substituting this into  $F_x$

$$E = \frac{mg \tan \theta}{q}.$$

The electric field between the two plates is constant so their potential difference is

$$V = Ed = \frac{dmg \tan \theta}{q}.$$

Thus, in order for the sphere to hang at  $30.0^\circ$  the potential difference between the two plates must be

$$\begin{aligned}
 V &= \frac{(0.0500)(0.00150)(9.81) \tan 30.0^\circ}{8.90 \times 10^{-6}} \\
 &= 47.7 \text{ V}.
 \end{aligned}$$

### 23.7.6 23.67

Consider a sphere of radius  $r$  and charge density

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}.$$

The charge of the sphere is

$$q = \frac{4}{3}\pi r^3 \rho.$$

Adding a shell of charge density  $\rho$  and thickness  $dr$  requires energy

$$\begin{aligned}
 dE &= \int \frac{1}{4\pi\epsilon_0} \frac{q dq}{r} \\
 &= \int_0^{2\pi} \int_0^\pi \frac{1}{4\pi\epsilon_0} \frac{\frac{4}{3}\pi r^3 \rho r d\theta r \sin\theta d\phi dr \rho}{r} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{8}{3} \pi^2 r^4 \rho^2 dr \int_0^\pi \sin\theta d\theta \\
 &= \frac{1}{4\pi\epsilon_0} \frac{16}{3} \pi^2 r^4 \rho^2 dr
 \end{aligned}$$

so assembling a sphere of radius  $R$  requires energy

$$\begin{aligned}
 E &= \int dE \\
 &= \int_0^R \frac{1}{4\pi\epsilon_0} \frac{16}{3} \pi^2 r^4 \rho^2 dr \\
 &= \frac{1}{4\pi\epsilon_0} \frac{16}{3} \pi^2 \left( \frac{3Q}{4\pi R^3} \right)^2 \left[ \frac{1}{5} r^5 \right]_0^R \\
 &= \frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \frac{Q^2}{R} \right).
 \end{aligned}$$

### 23.7.7 23.73

a)  $U = \frac{1}{4\pi\epsilon_0} \frac{e^2}{d} + \frac{1}{2} k d^2$

b)

$$\begin{aligned}\left. \frac{dU}{dd} \right|_{d=d_0} &= 0 \\ -\frac{1}{4\pi\epsilon_0} \frac{e^2}{d_0^2} + kd_0 &= 0 \\ d_0 &= \sqrt[3]{\frac{1}{4\pi\epsilon_0} \frac{e^2}{k}}\end{aligned}$$

c)

$$\begin{aligned}k &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{d_0^3} \\ &= (8.988 \times 10^9) \frac{(1.60 \times 10^{-19})^2}{(1.00 \times 10^{-15})^3} \\ &= 2.30 \times 10^{17} \text{ N/m}\end{aligned}$$

d)

$$\begin{aligned}U &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{d_0} + \frac{1}{2}kd_0^2 \\ &= (8.988 \times 10^9) \frac{(1.60 \times 10^{-19})^2}{1.00 \times 10^{-15}} + \frac{1}{2}(2.30 \times 10^{17})(1.00 \times 10^{-15})^2 \\ &= 3.45 \times 10^{-13} \text{ J} \\ &= 2.16 \times 10^6 \text{ eV}\end{aligned}$$

e)

$$\begin{aligned}2K &= U \\ 2\left(\frac{1}{2}mv^2\right) &= U \\ v &= \sqrt{\frac{U}{m}} \\ &= \sqrt{\frac{3.45 \times 10^{-13}}{1.67 \times 10^{-27}}} \\ &= 1.44 \times 10^7 \text{ m/s}\end{aligned}$$

### 23.7.8 23.79

- a) The oil drop experiences an electric force in the positive  $y$  direction and a weight force in the negative  $y$  direction. If it is at rest then

$$\begin{aligned}
0 &= qE - mg \\
&= q \frac{V_{AB}}{d} - \frac{4}{3} \pi r^3 \rho g \\
\Rightarrow q &= \frac{4\pi}{3} \frac{\rho r^3 g d}{V_{AB}}
\end{aligned}$$

b) At terminal velocity the viscous force equals the drop's weight

$$\begin{aligned}
0 &= 6\pi\eta r v_t - mg \\
&= 6\pi\eta r v_t - \frac{4}{3} \pi r^3 \rho g \\
\Rightarrow r &= \sqrt{\frac{9}{2} \frac{\eta v_t}{\rho g}}
\end{aligned}$$

Substituting this into the equation for  $q$

$$\begin{aligned}
q &= \frac{4\pi}{3} \frac{\rho g d}{V_{AB}} \left( \sqrt{\frac{9}{2} \frac{\eta v_t}{\rho g}} \right)^3 \\
&= 18\pi \frac{d}{V_{AB}} \sqrt{\frac{\eta^3 v_t^3}{2\rho g}}
\end{aligned}$$

c)

$$\begin{aligned}
q_1 &= 4.79 \times 10^{-19} \text{ C} \\
q_2 &= 1.59 \times 10^{-19} \text{ C} \\
q_3 &= 8.09 \times 10^{-19} \text{ C} \\
q_4 &= 3.23 \times 10^{-19} \text{ C}
\end{aligned}$$

d)

$$\begin{aligned}
n_1 &= 3 \\
n_2 &= 1 \\
n_3 &= 5 \\
n_4 &= 2
\end{aligned}$$

e)  $e \approx 1.60 \times 10^{-19} \text{ C}$

## 24 Capacitance and Dielectrics

### 24.1 Capacitors and Capacitance

#### 24.1.1 Example 24.1

$$\begin{aligned}C &= \epsilon_0 \frac{A}{d} \\ \Rightarrow A &= \frac{Cd}{\epsilon_0} \\ &= \frac{(1.0)(0.001)}{8.854 \times 10^{-12}} \\ &= 1.13 \times 10^8 \text{ m}^2\end{aligned}$$

#### 24.1.2 Example 24.2

(a)

$$\begin{aligned}C &= \epsilon_0 \frac{A}{d} \\ &= (8.854 \times 10^{-12} \text{ F/m}) \frac{2.00 \text{ m}^2}{0.00500 \text{ m}} \\ &= 3.54 \times 10^{-9} \text{ F}\end{aligned}$$

(b)

$$\begin{aligned}C &= \frac{Q}{V} \\ \Rightarrow Q &= CV \\ &= (3.54 \times 10^{-9} \text{ F})(10.0 \times 10^3 \text{ V}) \\ &= 3.54 \times 10^{-5} \text{ C}\end{aligned}$$

(c)

$$\begin{aligned}E &= \frac{Q}{\epsilon_0 A} \\ &= \frac{3.54 \times 10^{-5} \text{ C}}{(8.854 \times 10^{-12} \text{ F/m})(2.00 \text{ m}^2)} \\ &= 2.00 \times 10^6 \text{ N/C}\end{aligned}$$

#### 24.1.3 Example 24.3

By Gauss's law and symmetry the electric field between the shells is directed radially outward and has magnitude

$$\begin{aligned}
\oint \mathbf{E} \cdot d\mathbf{A} &= \frac{Q}{\epsilon_0} \\
E 4\pi r^2 &= \frac{Q}{\epsilon_0} \\
E &= \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}.
\end{aligned}$$

The potential difference between the shells is

$$\begin{aligned}
V_a - V_b &= \int_a^b \mathbf{E} \cdot d\mathbf{l} \\
&= \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} dr \\
&= \frac{1}{4\pi\epsilon_0} Q \left[ -\frac{1}{r} \right]_{r_a}^{r_b} \\
&= \frac{1}{4\pi\epsilon_0} Q \left( \frac{1}{r_a} - \frac{1}{r_b} \right) \\
&= \frac{1}{4\pi\epsilon_0} Q \frac{r_b - r_a}{r_a r_b}.
\end{aligned}$$

The capacitance is thus

$$\begin{aligned}
C &= \frac{Q}{V} \\
&= 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}.
\end{aligned}$$

#### 24.1.4 Example 24.4

By Gauss's law and symmetry, the electric field between the cylinders is directed radially outward and has magnitude

$$\begin{aligned}
\oint \mathbf{E} \cdot d\mathbf{A} &= \frac{Q}{\epsilon_0} \\
E 2\pi r L &= \frac{\lambda L}{\epsilon_0} \\
E &= \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}.
\end{aligned}$$

The potential difference between the cylinders is thus



$$\begin{aligned}
V_a - V_b &= \int_a^b \mathbf{E} \cdot d\mathbf{l} \\
&= \int_{r_a}^{r_b} \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} dr \\
&= \frac{1}{2\pi\epsilon_0} \lambda [\ln r]_{r_a}^{r_b} \\
&= \frac{1}{2\pi\epsilon_0} \lambda \ln \frac{r_b}{r_a}
\end{aligned}$$

and the capacitance per unit length is

$$\begin{aligned}
C &= \frac{Q}{V} \\
&= \frac{\lambda}{\frac{1}{2\pi\epsilon_0} \lambda \ln \frac{r_b}{r_a}} \\
&= \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}.
\end{aligned}$$

## 24.2 Capacitors in Series and Parallel

### 24.2.1 Example 24.5

(a)

$$\begin{aligned}
C_{\text{eq}} &= \frac{1}{(1/C_1) + (1/C_2)} = 2.0 \times 10^{-6} \text{ F} \\
Q_1 &= Q_2 = 3.6 \times 10^{-5} \text{ C} \\
V_1 &= 6.0 \text{ V} \\
V_2 &= 12 \text{ V}
\end{aligned}$$

(b)

$$\begin{aligned}
C_{\text{eq}} &= C_1 + C_2 = 9.0 \times 10^{-6} \text{ F} \\
Q_1 &= 1.08 \times 10^{-4} \text{ C} \\
Q_2 &= 5.4 \times 10^{-5} \text{ C} \\
V_1 &= V_2 = 18 \text{ V}
\end{aligned}$$

### 24.2.2 Example 24.6

The two capacitors on the right are equivalent to a single capacitor of capacitance

$$C = \frac{1}{(1/(12 \mu\text{F})) + (1/(6 \mu\text{F}))} = 4 \mu\text{F}.$$

The three capacitors in the middle are then equivalent to a single capacitor of capacitance

$$C = 3\,\mu\text{F} + 11\,\mu\text{F} + 4\,\mu\text{F} = 18\,\mu\text{F}.$$

Finally the two remaining capacitors are equivalent to a single capacitor of capacitance

$$C = \frac{1}{(1/(18\,\mu\text{F})) + (1/(9\,\mu\text{F}))} = 6\,\mu\text{F}.$$

## 24.3 Energy Storage in Capacitors and Electric-Field Energy

### 24.3.1 Example 24.7

(a)

$$Q = CV = (8.0\,\mu\text{F})(120\,\text{V}) = 9.6 \times 10^{-4}\,\text{C}$$

(b)

$$U = \frac{1}{2}QV = \frac{1}{2}(8.0\,\mu\text{F})(120\,\text{V}) = 5.8 \times 10^{-2}\,\text{J}$$

### 24.3.2 Example 24.8

(a)

$$\begin{aligned} U &= uV \\ &= \frac{1}{2}\epsilon_0 E^2 V \\ \Rightarrow E &= \sqrt{\frac{2U}{\epsilon_0 V}} \\ &= 4.75 \times 10^5\,\text{N/C} \end{aligned}$$

(b) 100 times the amount of energy

### 24.3.3 Example 24.9

(a)

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q^2 \frac{1}{4\pi\epsilon_0} \frac{r_b - r_a}{r_a r_b}$$

(b) The electric field energy density is

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \left( \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \right)^2 = \frac{Q^2}{32\pi^2\epsilon_0 r^4}.$$

The electric potential energy stored in the capacitor is thus

$$\begin{aligned}
U &= \int u \, dV \\
&= \int_{r_a}^{r_b} \frac{Q^2}{32\pi^2\epsilon_0 r^4} 4\pi r^2 \, dr \\
&= \frac{Q^2}{8\pi\epsilon_0} \left[ -\frac{1}{r} \right]_{r_a}^{r_b} \\
&= \frac{1}{2} Q^2 \frac{1}{4\pi\epsilon_0} \frac{r_b - r_a}{r_a r_b}
\end{aligned}$$

## 24.4 Dielectrics

### 24.4.1 Example 24.10

- (a)  $C_0 = \epsilon_0 \frac{A}{d} = 1.77 \times 10^{-10} \text{ F}$
- (b)  $C_0 = \frac{Q}{V} \Rightarrow Q = C_0 V = 5.31 \times 10^{-7} \text{ C}$
- (c)  $C = \frac{Q}{V'} = 5.31 \times 10^{-10} \text{ F}$
- (d)  $K = \frac{C}{C_0} = 3.00$
- (e)  $\epsilon = K\epsilon_0 = 2.66 \times 10^{-11} \text{ C}^2/\text{Nm}^2$
- (f)  $Q_i = Q \left(1 - \frac{1}{K}\right) = C_0 V \left(1 - \frac{1}{K}\right) = 3.54 \times 10^{-7} \text{ C}$
- (g)  $E_0 = \frac{\sigma}{\epsilon_0} = \frac{C_0 V}{A\epsilon_0} = 3.00 \times 10^5 \text{ N/C}$
- (h)  $E = \frac{E_0}{K} = 1.00 \times 10^5 \text{ N/C}$

### 24.4.2 Example 24.11

$$\begin{aligned}
U_{\text{before}} &= \frac{1}{2} C V^2 = 7.97 \times 10^{-4} \text{ J} \\
u_{\text{before}} &= \frac{1}{2} \epsilon_0 E^2 = 3.98 \times 10^{-1} \text{ J/m}^2 \\
U_{\text{after}} &= \frac{1}{2} C V^2 = 2.66 \times 10^{-4} \text{ J} \\
u_{\text{after}} &= \frac{1}{2} \epsilon E^2 = 1.33 \times 10^{-1} \text{ J/m}^3
\end{aligned}$$

## 24.5 Gauss's Law in Dielectrics

### 24.5.1 Example 24.12

By Gauss's law and symmetry, the electric field between the shells is

$$\begin{aligned}
\oint K \mathbf{E} \cdot d\mathbf{A} &= \frac{Q}{\epsilon_0} \\
KE4\pi r^2 &= \frac{Q}{\epsilon_0} \\
E &= \frac{Q}{4\pi\epsilon r^2}.
\end{aligned}$$

The potential difference between the shells is

$$\begin{aligned}
V &= \int_a^b \mathbf{E} \cdot d\mathbf{l} \\
&= \int_{r_a}^{r_b} \frac{Q}{4\pi\epsilon r^2} dr \\
&= \frac{Q}{4\pi\epsilon} \left( \frac{1}{r_a} - \frac{1}{r_b} \right) \\
&= \frac{Q(r_b - r_a)}{4\pi\epsilon r_a r_b}
\end{aligned}$$

and thus the capacitance is

$$\begin{aligned}
C &= \frac{Q}{V} \\
&= \frac{4\pi\epsilon r_a r_b}{r_b - r_a} \\
&= KC_0
\end{aligned}$$

## 24.6 Guided Practice

### 24.6.1 VP24.4.1

(a) The capacitance is given by

$$C = \epsilon_0 \frac{A}{d} = 6.58 \times 10^{-9} \text{ F}$$

(b) The potential difference is given by

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C} = \frac{\sigma A}{C} = 5.85 \times 10^3 \text{ V}$$

**24.6.2 VP24.4.4**

(a)

$$\begin{aligned}\frac{C}{L} &= \frac{2\pi\epsilon_0}{\ln(r_b/r_a)} \\ \frac{r_b}{r_a} &= e^{\frac{2\pi\epsilon_0}{C/L}} \\ &= 2.24\end{aligned}$$

(b)

$$\begin{aligned}\frac{C}{L} &= \frac{Q}{LV} \\ V &= \frac{Q/L}{C/L} \\ &= 125 \text{ V}\end{aligned}$$

**24.6.3 VP24.9.1**

(a)

$$C_{\text{eq}} = \frac{1}{(1/C_1) + (1/C_2) + (1/C_3)} = 0.625 \mu\text{F}$$

(b)

$$C_{\text{eq}} = C_1 + C_2 + C_2 = 8.50 \mu\text{F}$$

(c)

$$\begin{aligned}C_{12} &= C_1 + C_2 = 3.50 \mu\text{F} \\ C_{\text{eq}} &= \frac{1}{(1/C_{12}) + (1/C_3)} = 2.06 \mu\text{F}\end{aligned}$$

(d)

$$\begin{aligned}C_{12} &= \frac{1}{(1/C_1) + (1/C_2)} = 0.714 \mu\text{F} \\ C_{\text{eq}} &= C_{12} + C_3 = 5.71 \mu\text{F}\end{aligned}$$

**24.6.4 VP24.9.4**

$$(a) \quad \sigma = Q/A = 5.10 \times 10^{-6} \text{ C/m}^2$$

$$(b) \quad E = \sigma/\epsilon_0 = 5.76 \times 10^5 \text{ N/C}$$

$$(c) \quad u = \frac{1}{2}\epsilon_0 E^2 = 1.47 \text{ J/m}^3$$

$$(d) \quad U = uV = uAd = 5.04 \times 10^{-3} \text{ J}$$

**24.6.5 VP24.11.1**

- (a)  $C = \epsilon_0 A/d = 1.36 \text{ nF}$
- (b)  $C = Q/V \Rightarrow Q = CV = 5.44 \mu\text{C}$
- (c)  $K = V_0/V = 1.60$
- (d)  $C = C_0 K = 2.18 \text{ nF}$
- (e)  $Q_i = Q(1 - 1/K) = 2.04 \times 10^{-6} \text{ C}$

**24.6.6 24.11.4**

- (a)  $U = \frac{1}{2} CV^2 = 6.30 \text{ J}$
- (b)  $U = \frac{1}{2} KCV^2 = 18.0 \text{ J}$

**24.7 Bridging Problem**

- (a) The electric field inside a conductor is 0 so the energy density is also 0
- (b) By Gauss's law and symmetry the electric field at a distance  $r$  from the centre of the sphere points radially outwards and has magnitude

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

so the energy density is

$$u = \frac{1}{2} \epsilon_0 E^2 = \frac{Q^2}{32\pi^2 \epsilon_0 r^4}$$

- (c) The electric field energy of a shell of thickness  $dr$  is

$$dU = 4\pi r^2 u dr = \frac{Q^2}{8\pi\epsilon_0 r^2} dr$$

so the electric field energy of all space is

$$\begin{aligned} U &= \int dU \\ &= \int_R^\infty \frac{Q^2}{8\pi\epsilon_0 r^2} dr \\ &= \frac{Q^2}{8\pi\epsilon_0} \left[ -\frac{1}{r} \right]_R^\infty \\ &= \frac{Q^2}{8\pi\epsilon_0 R} \end{aligned}$$

- (d)  $Q^2/8\pi\epsilon_0 R$
- (e)  $U = Q^2/2C \Rightarrow C = Q^2/2U = 4\pi\epsilon_0 R$

## 24.8 Exercises

### 24.8.1 24.1

(a)  $V = Ed = 10 \text{ kV}$

(b)  $C = Q/V = \epsilon_0 A/d \Rightarrow A = dQ/\epsilon_0 V = 2.26 \times 10^{-3} \text{ m}^2$

(c)  $C = Q/V = 8.00 \text{ pF}$

### 24.8.2 24.11

(a)  $C = Q/V = 15.0 \text{ pF}$

(b)

$$\begin{aligned} C &= 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a} \\ \frac{r_b - r_a}{r_a r_b} &= \frac{4\pi\epsilon_0}{C} \\ \frac{1}{r_a} - \frac{1}{r_b} &= \frac{4\pi\epsilon_0}{C} \\ \frac{1}{r_a} &= \frac{4\pi\epsilon_0}{C} + \frac{1}{r_b} \\ &= \frac{4\pi\epsilon_0 r_b + C}{C r_b} \\ r_a &= \frac{C r_b}{4\pi\epsilon_0 r_b + C} \\ &= 3.08 \text{ cm} \end{aligned}$$

(c)

$$\begin{aligned} E &= \frac{\sigma}{\epsilon_0} \\ &= \frac{Q}{4\pi r_a^2 \epsilon_0} \\ &= 3.13 \times 10^4 \text{ N/C} \end{aligned}$$

### 24.8.3 24.13

$$\begin{aligned} V_2 &= kV \\ &= k(V_1 + V_2) \\ V_2(1 - k) &= kV_1 \\ \frac{Q}{C_2}(1 - k) &= k \frac{Q}{C_1} \\ C_1 &= \frac{C_2 k}{1 - k} \\ &= 5.57 \mu\text{F} \end{aligned}$$

**24.8.4 24.15**

- (a) In series  
 (b) 5000 cells

**24.8.5 24.21**

- (a) The equivalent capacitance for the three capacitors in the middle is

$$C_{\text{middle}} = \frac{1}{(1/18.0 \text{ nF}) + (1/30.0 \text{ nF}) + (1/10.0 \text{ nF})} = 5.29 \text{ nF}.$$

The equivalent capacitance for the remaining three capacitors is

$$C_{\text{equiv}} = 7.5 \text{ nF} + 5.29 \text{ nF} + 6.5 \text{ nF} = 19.3 \text{ nF}.$$

- (b)  $C = Q/V \Rightarrow Q = CV = 483 \text{ nC}$   
 (c)  $C = Q/V \Rightarrow Q = CV = 163 \text{ nC}$   
 (d) 25 V

**24.8.6 24.23**

$$C = \epsilon_0 \frac{A}{d} \Rightarrow A = \frac{Cd}{\epsilon_0} = 3.28 \times 10^3 \text{ m}^2$$

$$u = \frac{\frac{1}{2}CV^2}{Ad} = 2.83 \times 10^{-2} \text{ J/m}^3$$

**24.8.7 24.31**

- (a)

$$U_{\text{before}} = \frac{1}{2}CV^2 = 3.60 \text{ mJ}$$

$$U_{\text{after}} = \frac{1}{2}KCV^2 = 13.5 \text{ mJ}$$

- (b) The energy increased by 9.9 mJ

**24.8.8 24.33**

- (a)

$$E = E_0/K \Rightarrow K = E_0/E = 1.28$$

and

$$\sigma_i = \sigma(1 - 1/K) = \epsilon_0 E_0(1 - 1/K) = 6.20 \times 10^{-7} \text{ C/m}^2$$

- (b)  $K = 1.28$



**24.8.9 24.35**

$$\begin{aligned}
 Q &= C_{\text{equiv}} V \\
 &= \frac{1}{\frac{1}{KC_1} + \frac{1}{C_2}} V \\
 &= \frac{KC_1 C_2}{KC_1 + C_2} V \\
 &= \frac{K \frac{Q_0}{V/2} \frac{Q_0}{V/2}}{K \frac{Q_0}{V/2} + \frac{Q_0}{V/2}} V \\
 &= \frac{K \frac{Q_0}{V/2}}{K + 1} V \\
 &= \frac{2KQ_0}{V(K + 1)} V \\
 &= \frac{2KQ_0}{K + 1}
 \end{aligned}$$

The charge on  $C_1$  increases

**24.8.10 24.37**

At maximum potential difference the charge on the capacitor will be

$$Q = CV = 6.88 \mu\text{C}.$$

The surface charge density will be  $\sigma = Q/A$  and the magnitude of the electric field will be  $E = \sigma/\epsilon$ . If  $E$  is to be less than the dielectric strength of the material then

$$\begin{aligned}
 E_m &\geq E \\
 &\geq \frac{\sigma}{\epsilon} \\
 &\geq \frac{Q}{AK\epsilon_0} \\
 \Rightarrow A &\geq \frac{Q}{E_m K \epsilon_0} \\
 &\geq 1.35 \times 10^{-2} \text{ m}^2
 \end{aligned}$$

**24.8.11 24.39**

(a)

$$\begin{aligned} Q_{\text{after}} - Q_{\text{before}} &= C_{\text{after}}V - C_{\text{before}}V \\ &= (KC_{\text{before}} - C_{\text{before}})V \\ &= (K - 1)C_{\text{before}}V \\ &= 6.3 \mu\text{C} \end{aligned}$$

(b)

$$\begin{aligned} Q_i &= Q(1 - 1/K) \\ &= KCV(1 - 1/K) \\ &= 6.3 \mu\text{C} \end{aligned}$$

(c) No change

**24.8.12 24.41**

(a)  $U = CV^2/2 \Rightarrow V = \sqrt{2U/C} = 10.1 \text{ V}$

(b)  $U = KU_0 \Rightarrow K = U/U_0 = 2.25$

**24.8.13 24.43**

(a)

$$\begin{aligned} \oint K \mathbf{E} \cdot d\mathbf{A} &= \frac{Q_{\text{encl-free}}}{\epsilon_0} \\ KEA &= \frac{\sigma A}{\epsilon_0} \\ E &= \frac{Q}{AK\epsilon_0} \end{aligned}$$

(b)

$$V = Ed = \frac{Qd}{AK\epsilon_0}$$

(c)

$$C = \frac{Q}{V} = K \frac{A\epsilon_0}{d} = KC_0$$

## 24.9 Problems

### 24.9.1 24.45

- (a) At 100% efficiency the energy required is

$$E = Pt = 400 \text{ J}$$

so at 95% efficiency the energy required is

$$E/0.95 = 421 \text{ J}$$

- (b)

$$\begin{aligned} U &= \frac{E}{0.95} \\ \frac{1}{2}CV^2 &= \frac{E}{0.95} \\ C &= \frac{E}{0.95} \frac{2}{V^2} \\ &= 5.39 \times 10^{-2} \text{ F} \end{aligned}$$

### 24.9.2 24.47

- (a)  $C = \epsilon_0 A/d = 0.531 \text{ pF}$

- (b)

$$\begin{aligned} C - C_0 &= \Delta C \\ \epsilon_0 \frac{A}{d + \Delta d} - \epsilon_0 \frac{A}{d} &= \Delta C \\ \frac{A}{d + \Delta d} &= \frac{\Delta C}{\epsilon_0} + \frac{A}{d} \\ &= \frac{\epsilon_0 A + \Delta C d}{\epsilon_0 d} \\ \Delta d &= \frac{\epsilon_0 A d}{\epsilon_0 A + \Delta C d} - d \\ &= -0.224 \text{ mm} \end{aligned}$$

### 24.9.3 24.49

- (a)  $Q = CV = 16.0 \text{ mC}$

- (b) The capacitors are connected in parallel so the initial charge will remain and their potential differences will be equal. The equivalent capacitance is

$$C = C_1 + C_2 = 30 \mu\text{F}$$

so the potential difference across both capacitors is

$$V = Q/C = 533 \text{ V}$$

(c)  $U = CV^2/2 = 4.26 \text{ J}$

(d)  $\Delta U = U_0 - U = 2.14 \text{ J}$

#### 24.9.4 24.55

The potential difference across  $C_1$  is

$$V_1 = Q_1/C_1 = 50 \text{ V}.$$

It is in parallel with  $C_2$  so

$$Q_{12} = Q_1 + Q_2$$

and

$$V_2 = V_1.$$

The equivalent capacitor for  $C_1$  and  $C_2$  is in series with  $C_3$  so

$$Q_{12} = Q_3$$

meaning

$$Q_2 = Q_{12} - Q_1 = Q_3 - Q_1 = 300 \mu\text{C}$$

and thus

$$C_2 = Q_2/V_2 = 6.00 \mu\text{F}.$$

Finally,

$$V = V_1 + V_3 \Rightarrow V_3 = V - V_1$$

so

$$C_3 = Q_3/V_3 = Q_3/(V - V_1) = 4.50 \mu\text{C}.$$

#### 24.9.5 24.57

- (a) Because the capacitors are connected in series they all have the same charge which is equal to that of the equivalent capacitor. Its capacitance is

$$C = \frac{1}{(1/C_1) + (1/C_2) + (1/C_3)} = 2.10 \mu\text{F}$$

so its charge is

$$Q = CV = 76 \mu\text{C}$$

(b)  $U = CV^2/2 = 1.4 \text{ mJ}$

- (c) The capacitance of the equivalent capacitor is

$$C = C_1 + C_2 + C_3 = 21 \mu\text{F}$$

and its charge is

$$Q = Q_1 + Q_2 + Q_3 = 3Q_0 = 0.23 \text{ mJ}$$

so its potential difference (and that of each individual capacitor) is

$$V = Q/C = 11 \text{ V}$$

(d)  $U = CV^2/2 = 1.3 \text{ mJ}$

**24.9.6 24.63**

(a) The electric field of an infinite charged sheet is

$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0}.$$

When the dielectric is present this becomes

$$E = \frac{Q}{2A\epsilon} = \frac{Q}{2AK\epsilon_0}$$

and thus the other sheet experiences an attractive force of

$$F = \frac{Q^2}{2AK\epsilon_0}$$

(b)

$$\begin{aligned} d &= d_0 - \Delta d \\ &= d_0 - \frac{F}{A} \frac{d_0}{Y} \\ &= d_0 - \frac{Q^2}{2A^2K\epsilon_0} \frac{d_0}{Y} \\ &= d_0 - \frac{d_0}{2A^2K\epsilon_0 Y} Q^2 \end{aligned}$$

(c)

$$s = \frac{d_0}{2A^2K\epsilon_0 Y} = 7.53 \times 10^7 \text{ m/C}^2$$

(d)

$$\begin{aligned} V &= \frac{Q}{C} \\ &= \frac{Qd}{A\epsilon} \\ &= \frac{Q(d_0 - sQ^2)}{AK\epsilon_0} \\ &= 95.7 \text{ kV} \end{aligned}$$

(e)

$$\begin{aligned} V' &= \frac{2Q(d_0 - s(2Q)^2)}{AK\epsilon_0} \\ &= 133 \text{ kV} \end{aligned}$$

**24.9.7 24.65**

(a) The capacitance is

$$C = \epsilon_0 \frac{A}{d} = \epsilon_0 \frac{\pi r^2}{d}$$

so the charge on each plate is

$$Q = CV = \epsilon_0 \frac{\pi r^2}{d} V$$

and the tension in the rope is

$$\begin{aligned} T &= \frac{Q^2}{2A\epsilon_0} \\ &= \frac{(\epsilon_0 \frac{A}{d} V)^2}{2A\epsilon_0} \\ &= \frac{\epsilon_0 A V^2}{2d^2} \\ &= \frac{\epsilon_0 \pi r^2 V^2}{2d^2} \end{aligned}$$

(b)

$$\begin{aligned} W &= \int_d^{2d} \frac{\pi \epsilon_0 r^2 V^2}{2z^2} dz \\ &= \frac{\pi \epsilon_0 r^2 V^2}{2} \left[ -\frac{1}{z} \right]_d^{2d} \\ &= \frac{\pi \epsilon_0 r^2 V^2}{4d} \end{aligned}$$

(c) The energy in the electric field is equal to the energy in the capacitor so

$$E_{\text{before}} = \frac{1}{2} C V^2 = \frac{\pi \epsilon_0 r^2 V^2}{2d}$$

(d)

$$E_{\text{after}} = \frac{1}{2} C V^2 = \frac{\pi \epsilon_0 r^2 V^2}{4d}$$

(e)

$$\Delta E = E_{\text{after}} - E_{\text{before}} = -\frac{\pi \epsilon_0 r^2 V^2}{4d}$$

**24.9.8 24.71**

(a) If the dielectric constant at point  $x$  is

$$K(x) = 1 + (K - 1)\frac{x}{d} = \frac{d + (K - 1)x}{d}$$

then the reciprocal of the capacitance is

$$\begin{aligned}\frac{1}{C} &= \int_0^d \frac{dx}{K(x)\epsilon_0 A} \\ &= \int_0^d \frac{d}{(d + (K - 1)x)\epsilon_0 A} dx \\ &= \frac{d}{\epsilon_0 A} \int_0^d \frac{1}{d + (K - 1)x} dx \\ &= \frac{d}{\epsilon_0 A} \left[ \frac{\ln(d + (K - 1)x)}{K - 1} \right]_0^d \\ &= \frac{d}{\epsilon_0 A(K - 1)} (\ln(d + (K - 1)d) - \ln(d)) \\ &= \frac{d \ln K}{\epsilon_0 A(K - 1)}\end{aligned}$$

and the capacitance is

$$C = \frac{K - 1}{\ln K} \epsilon_0 \frac{A}{d}$$

(b)

$$\begin{aligned}\lim_{K \rightarrow 1} C &= \lim_{K \rightarrow 1} \frac{K - 1}{\ln K} \epsilon_0 \frac{A}{d} \\ &= \epsilon_0 \frac{A}{d} \lim_{K \rightarrow 1} \frac{1}{\frac{1}{K}} \\ &= \epsilon_0 \frac{A}{d}\end{aligned}$$