Advanced Engineering Mathematics Vectors, Matrices, and Vector Calculus by Dennis G. Zill Problems

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Contents

1 Vectors

1.1 Vectors in 2-Space

1.1.1

(a) $3\mathbf{a} = 6\mathbf{i} + 12\mathbf{j}$

(b) $\mathbf{a} + \mathbf{b} = \mathbf{i} + 8\mathbf{j}$

(c) $\mathbf{a} - \mathbf{b} = 3\mathbf{i}$

(d) $||\mathbf{a} + \mathbf{b}|| = \sqrt{1 + 8^2} = \sqrt{65}$

(e) $||\mathbf{a} - \mathbf{b}|| = 3$

1.1.9

(a) $4\mathbf{a} - 2\mathbf{b} = \langle 6, -14 \rangle$

(b) $-3\mathbf{a} - 5\mathbf{b} = \langle 2, 4 \rangle$

1.1.15

$\overrightarrow{P_1P_2} = \langle 2, 5 \rangle$

1.1.19

$(1, 18)$

1.1.21

(a) Yes

(b) Yes

(c) Yes

(d) No

(e) Yes

(f) Yes

1.1.25

(a) $\frac{\mathbf{a}}{||\mathbf{a}||} = \frac{\langle 2,2\rangle}{\sqrt{2^2+2^2}} = \frac{1}{2\sqrt{2}}\langle 2, 2\rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$

(b) $\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\rangle$

1.1.31

$2\frac{\mathbf{a}}{||\mathbf{a}||} = 2\frac{\langle 3,7\rangle}{\sqrt{3^2+7^2}} = \frac{2}{\sqrt{58}}\langle 3, 7\rangle = \langle \frac{6}{\sqrt{58}}, \frac{14}{\sqrt{58}}\rangle$

1.1.37

$\mathbf{x} = -(\mathbf{a} + \mathbf{b})$

1.1.41

$$\begin{aligned}
\mathbf{a} &= 2\mathbf{i} + 3\mathbf{j} \\
\mathbf{b} &= \mathbf{i} + \mathbf{j} \\
\mathbf{c} &= \mathbf{i} - \mathbf{j}
\end{aligned}$$

$$\begin{aligned}
\mathbf{i} &= \frac{1}{2}(\mathbf{b} + \mathbf{c}) \\
\mathbf{j} &= \frac{1}{2}(\mathbf{b} - \mathbf{c}) \\
\mathbf{a} &= 2\left(\frac{1}{2}(\mathbf{b} + \mathbf{c})\right) + 3\left(\frac{1}{2}(\mathbf{b} - \mathbf{c})\right) \\
&= \mathbf{b} + \mathbf{c} + \frac{3}{2}\mathbf{b} - \frac{3}{2}\mathbf{c} \\
&= \frac{5}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}
\end{aligned}$$

1.1.43

$$\begin{aligned}
y &= \frac{1}{4}x^2 + 1 \\
y(2) &= 2 \\
y' &= \frac{1}{2}x \\
y'(2) &= 1 \\
\mathbf{v} &= \pm\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle
\end{aligned}$$

1.1.45

(a)

$$\begin{aligned}
\mathbf{F}_n &= \mathbf{F}\cos\theta \\
\mathbf{F}_g &= \mathbf{F}\sin\theta \\
||\mathbf{F}_f|| &= \mu||\mathbf{F}_n|| \\
||-\mathbf{F}_g|| &= \mu||\mathbf{F}_n|| \\
||-\mathbf{F}\sin\theta|| &= \mu||\mathbf{F}\cos\theta|| \\
||\mathbf{F}||\sin\theta &= \mu||\mathbf{F}||\cos\theta \\
\tan\theta &= \mu
\end{aligned}$$

(b) $\theta = \arctan\mu \approx 31°$

1.1.47

$$\begin{aligned}
F_x &= \frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{L\,dy}{2a(L^2+y^2)^{3/2}} \\
&= \frac{LqQ}{8\pi\epsilon_0}\int_{-a}^{a}(L^2+y^2)^{-3/2}\,dy \\
&= \frac{LqQ}{8\pi\epsilon_0}\frac{2a}{L^2\sqrt{a^2+L^2}} \\
&= \frac{aqQ}{4\pi\epsilon_0 L\sqrt{a^2+L^2}} \\
F_y &= -\frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{y\,dy}{2a(L^2+y^2)^{3/2}} \\
&= 0 \\
\mathbf{F} &= \langle\frac{1}{4\pi\epsilon_0}\frac{qQ}{L\sqrt{a^2+L^2}}, 0\rangle
\end{aligned}$$

1.1.49

Let the three sides of the triangle be vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. The triangle is closed so it must be the case that

$$\mathbf{a}+\mathbf{b}+\mathbf{c} = 0.$$

This gives

$$\mathbf{c} = -(\mathbf{a}+\mathbf{b}).$$

The vector from the midpoint of side $\mathbf{a}$ to the midpoint of side $\mathbf{b}$ is

$$\left(\mathbf{a}+\frac{1}{2}\mathbf{b}\right) - \frac{1}{2}\mathbf{a} = \frac{1}{2}(\mathbf{a}+\mathbf{b})$$

which is parallel with $\mathbf{c}$ and half its length.

1.2 Vectors in 3-Space

1.2.7

A plane at $z = 5$ parellel with the x-y plane.

1.2.9

A line parallel to the z axis at $x = 2$ and $y = 3$.

1.2.13

(a) $(0, 5, 4)$, $(-2, 0, 4)$, $(-2, 5, 0)$

(b) $(-2, 5, -2)$

(c) $(3, 5, 4)$

1.2.15

The planes $x = 0$, $y = 0$, and $z = 0$.

1.2.17

$(-1, 2, -3)$

1.2.19

The planes $z = \pm 5$.

1.2.21

$\sqrt{(6-3)^2 + (4+1)^2 + (8-2)^2} = \sqrt{9 + 25 + 36} = \sqrt{70}$

1.2.31

$$\begin{aligned}
\sqrt{(2-x)^2 + (1-2)^2 + (1-3)^2} &= \sqrt{21} \\
(2-x)^2 + 1 + 4 &= 21 \\
(2-x)^2 &= 16 \\
2 - x &= \pm 4 \\
x &= 2 \pm 4 \\
&= -2 \text{ or } 6
\end{aligned}$$

1.2.33

$\left(4, \frac{1}{2}, \frac{3}{2}\right)$

1.2.37

$(-3, -6, 1)$

1.3 Dot Product

1.3.1

$\mathbf{a} \cdot \mathbf{b} = 12$

1.3.11

$\left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{b} \cdot \mathbf{b}}\right) \mathbf{b} = \frac{12}{30}\mathbf{b} = \langle -\frac{2}{5}, \frac{4}{5}, 2 \rangle$

1.3.13

$\mathbf{a} \cdot \mathbf{b} = ||\mathbf{a}||||\mathbf{b}|| \cos\theta = 25\sqrt{2}$

1.3.17

$$\begin{aligned}
\mathbf{a} \cdot \mathbf{v} &= 0 \\
3x_1 + y_1 - 1 &= 0 \\
\\
\mathbf{b} \cdot \mathbf{v} &= 0 \\
-3x_1 + 2y_2 + 2 &= 0 \\
\\
3y_2 + 1 &= 0 \\
y_2 &= -\frac{1}{3} \\
\\
3x_1 - \frac{1}{3} - 1 &= 0 \\
x_1 &= \frac{4}{9} \\
\\
\mathbf{v} &= \langle \frac{4}{9}, -\frac{1}{3}, 1 \rangle
\end{aligned}$$

1.3.19

$$\begin{aligned}\mathbf{a}\cdot\mathbf{c} &= \mathbf{a}\cdot\left(\mathbf{b}-\frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{a}||^2}\mathbf{a}\right)\\ &= \mathbf{a}\cdot\mathbf{b}-\frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{a}||^2}\mathbf{a}\cdot\mathbf{a}\\ &= 0\end{aligned}$$

1.3.21

$$\begin{aligned}||\mathbf{a}|| &= \sqrt{3^2+1^2}\\ &= \sqrt{10}\\ ||\mathbf{b}|| &= \sqrt{2^2+2^2}\\ &= \sqrt{8}\\ &= 2\sqrt{2}\\ \mathbf{a}\cdot\mathbf{b} &= 4\\ \theta &= \arccos\frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{a}||||\mathbf{b}||}\\ &= \arccos\frac{4}{(\sqrt{10})(2\sqrt{2})}\\ &= \arccos\frac{1}{\sqrt{5}}\\ &\approx 63^\circ\end{aligned}$$

1.3.25

$$\begin{aligned}||\mathbf{a}|| &= \sqrt{1^2+2^2+3^3}\\ &= \sqrt{14}\\ \cos\alpha &= \frac{1}{\sqrt{14}}\\ \alpha &\approx 75^\circ\\ \cos\beta &= \frac{2}{\sqrt{14}}\\ \beta &\approx 58^\circ\\ \cos\gamma &= \frac{3}{\sqrt{14}}\\ \gamma &\approx 37^\circ\end{aligned}$$

1.3.29

$$\begin{aligned}
\overrightarrow{AD} &= \langle s, -s, s\rangle \\
||\overrightarrow{AD}|| &= s\sqrt{3} \\
\overrightarrow{AB} &= \langle s, 0, 0\rangle \\
||\overrightarrow{AB}|| &= s \\
\theta &= \arccos\frac{\overrightarrow{AD}\cdot\overrightarrow{AB}}{||\overrightarrow{AD}||||\overrightarrow{AB}||} \\
&= \arccos\frac{s^2}{s^2\sqrt{3}} \\
&= \arccos\frac{1}{\sqrt{3}} \\
&\approx 55^\circ
\end{aligned}$$

1.3.33

$$\begin{aligned}
\mathrm{comp}_{\mathbf{b}}\mathbf{a} &= \frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{b}||} \\
&= \frac{5}{7}
\end{aligned}$$

1.3.37

$$\begin{aligned}
\mathrm{comp}_{\overrightarrow{OP}}\mathbf{a} &= \frac{\mathbf{a}\cdot\overrightarrow{OP}}{||\overrightarrow{OP}||} \\
&= \frac{72}{\sqrt{109}}
\end{aligned}$$

1.3.39

$$\begin{aligned}
\mathrm{proj}_{\mathbf{b}}\mathbf{a} &= \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b} \\
&= \frac{35}{25}\mathbf{b} \\
&= \langle -\frac{21}{5}, \frac{28}{5}\rangle
\end{aligned}$$

1.3.43

$$\begin{aligned}
\mathbf{a}+\mathbf{b} &= \langle 3,4\rangle \\
\text{proj}_{\mathbf{a}+\mathbf{b}}\mathbf{a} &= \left(\frac{\mathbf{a}\cdot(\mathbf{a}+\mathbf{b})}{(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})}\right)(\mathbf{a}+\mathbf{b}) \\
&= \frac{24}{25}(\mathbf{a}+\mathbf{b}) \\
&= \langle \frac{72}{25}, \frac{96}{25}\rangle
\end{aligned}$$

1.3.45

$W = \mathbf{F}\cdot\mathbf{d} = Fd\cos\theta = 1000$

1.3.47

(a) $W = 0$

(b)

$$\begin{aligned}
||\mathbf{d}|| &= \sqrt{4^2+3^2} \\
&= 5 \\
\mathbf{F} &= F\hat{\mathbf{d}} \\
&= F\frac{\mathbf{d}}{||\mathbf{d}||} \\
&= F\langle \frac{4}{5}, \frac{3}{5}\rangle \\
&= \langle 24, 18\rangle \\
W &= \mathbf{F}\cdot\mathbf{d} \\
&= 150\,\text{J}
\end{aligned}$$

1.4 Cross Product

1.4.1

$$\begin{aligned}
\mathbf{a}\times\mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ 0 & 3 & 5 \end{vmatrix} \\
&= -5\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}
\end{aligned}$$

1.4.11

$$\begin{aligned}\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 2 & -4 \\ -3 & 1 & 1 \end{vmatrix} \\ &= 6\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}\end{aligned}$$

1.4.17

(a)

$$\begin{aligned}\mathbf{b} \times \mathbf{c} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix} \\ &= \mathbf{j} - \mathbf{k} \\ \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 0 & 1 & -1 \end{vmatrix} \\ &= -\mathbf{i} + \mathbf{j} + \mathbf{k}\end{aligned}$$

1.4.19

$2\mathbf{k}$

1.4.21

$$\begin{aligned}\mathbf{k} \times (2\mathbf{i} - \mathbf{j}) &= (\mathbf{k} \times 2\mathbf{i}) - (\mathbf{k} \times \mathbf{j}) \\ &= \mathbf{i} + 2\mathbf{j}\end{aligned}$$

1.4.23

$$\begin{aligned}[(2\mathbf{k}) \times (3\mathbf{j})] \times (4\mathbf{j}) &= (-6\mathbf{i}) \times (4\mathbf{j}) \\ &= -24\mathbf{k}\end{aligned}$$

1.4.37

$12\mathbf{i} - 9\mathbf{j} + 18\mathbf{k}$

1.4.53

$$\begin{aligned}
\mathbf{b}\times\mathbf{c} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 6 & -6 \\ \frac{5}{2} & 3 & \frac{1}{2} \end{vmatrix} \\
&= 21\mathbf{i} - 14\mathbf{j} - 21\mathbf{k} \\
\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) &= 4\times 21 + 6\times(-14) \\
&= 0
\end{aligned}$$

They are coplanar.

1.5 Lines and Planes in 3-Space

1.5.1

$\mathbf{r} = \langle 1, 2, 1\rangle + t\langle 2, 3, -3\rangle$

1.5.7

$$\begin{aligned}
x &= 2 + 4t \\
y &= 3 - 4t \\
z &= 5 + 3t
\end{aligned}$$

1.5.13

$$\begin{aligned}
x &= 1 + 9t \\
y &= 4 + 10t \\
z &= -9 + 7t \\
\frac{x-1}{9} = \frac{y-4}{10} &= \frac{z+9}{7}
\end{aligned}$$

1.5.19

$$\begin{aligned}
x &= 4 + 3t \\
y &= 6 + \frac{1}{2}t \\
z &= -7 - \frac{3}{2}t \\
\frac{x-4}{3} = \frac{y-6}{1/2} &= -\frac{z+7}{3/2}
\end{aligned}$$

1.5.23

$$\begin{aligned}x &= 6 + 2t\\ y &= 4 - 3t\\ z &= -2 + 6t\end{aligned}$$

1.5.25

$$\begin{aligned}x &= 2 + t\\ y &= -2\\ z &= 15\end{aligned}$$

1.5.29

$(0, 5, 15)$, $(5, 0, \frac{15}{2})$, $(10, -5, 0)$

1.5.31

$$\begin{aligned}4 + t_x &= 6 + 2t_x\\ t_x &= -2\\ \\ 5 + t_y &= 11 + 4t_y\\ t_y &= -2\\ \\ -1 + 2t_z &= -3 + t_z\\ t_z &= -2\end{aligned}$$

$(2, 3, -5)$

1.5.35

$$\begin{aligned}\mathbf{a} &= \langle -1, 2, -2\rangle\\ ||\mathbf{a}|| &= 3\\ \mathbf{b} &= \langle 2, 3, -6\rangle\\ ||\mathbf{b}|| &= 7\\ \theta &= \arccos \frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{a}||||\mathbf{b}||}\\ &\approx 40.37^\circ\end{aligned}$$

1.5.37

$$\begin{aligned}
\mathbf{a} &= \langle 1,1,1\rangle \\
\mathbf{b} &= \langle -2,1,-5\rangle \\
\mathbf{a}\times\mathbf{b} &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ -2 & 1 & -5 \end{vmatrix} \\
&= \langle -6,3,3\rangle \\
x &= 4-6t \\
y &= 1+3t \\
z &= 6+3t
\end{aligned}$$

1.5.39

$$\begin{aligned}
\langle 2,-3,4\rangle\cdot(\mathbf{r}-\langle 5,1,3\rangle) &= 0 \\
2(x-5)-3(y-1)+4(z-3) &= 0 \\
2x-3y+4z-19 &= 0
\end{aligned}$$

1.5.45

$$\begin{aligned}
\mathbf{a} &= \langle 3,5,2\rangle \\
\mathbf{b} &= \langle 2,3,1\rangle \\
\mathbf{c} &= \langle -1,-1,4\rangle \\
\mathbf{a}-\mathbf{c} &= \langle 4,6,-2\rangle \\
\mathbf{b}-\mathbf{c} &= \langle 3,4,-3\rangle \\
(\mathbf{a}-\mathbf{c})\times(\mathbf{b}-\mathbf{c}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 6 & -2 \\ 3 & 4 & -3 \end{vmatrix} \\
&= \langle -10,6,-2\rangle \\
\mathbf{n}\cdot(\mathbf{r}-\mathbf{c}) &= 0 \\
\langle -10,6,-2\rangle\cdot(\langle x,y,z\rangle-\langle -1,-1,4\rangle) &= 0 \\
-10(x+1)+6(y+1)-2(z-4) &= 0 \\
-10x+6y-2z+4 &= 0
\end{aligned}$$

1.5.51

$$\begin{aligned}
\langle 1,1,-4\rangle\cdot(\mathbf{r}-\langle 2,3,-5\rangle) &= 0 \\
(x-2)+(y-3)-4(z+5) &= 0 \\
x+y-4z &= 25
\end{aligned}$$

1.5.63

(a) Not perpendicular

(b) Not perpendicular

(c) Perpendicular

(d) Perpendicular

1.5.65

$$\begin{aligned} 5x - 4y - 9t &= 8 \\ x + 4y + 3t &= 4 \end{aligned}$$

$$\begin{aligned} 6x - 6t &= 12 \\ x &= 2 + t \end{aligned}$$

$$y = \frac{1}{2} - t$$

$$z = t$$

1.5.69

$$\begin{aligned} 2(1 + 2t) - 3(2 - t) + 2(-3t) &= -7 \\ t &= -3 \\ x &= -5 \\ y &= 5 \\ z &= 9 \end{aligned}$$

1.5.73

$$
\begin{aligned}
x + y - 4t &= 2 \\
2x - y + t &= 10 \\
\\
3x - 3t &= 12 \\
x &= 4 + t \\
\\
2(4+t) - y + t &= 10 \\
8 + 2t - y + t &= 10 \\
y &= -2 + 3t \\
\\
z &= t \\
\\
x &= 5 + t \\
y &= 6 + 3t \\
z &= -12 + t
\end{aligned}
$$

1.5.75

$$
\begin{aligned}
\mathbf{n} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 5 \\ 1 & 1 & 1 \end{vmatrix} \\
&= \langle -6, 2, 4 \rangle \\
\mathbf{n} \cdot (\mathbf{r} - \langle 4, 0, 1 \rangle) &= 0 \\
-6(x-4) + 2y + 4(z-1) &= 0 \\
-6x + 2y + 4z &= -20 \\
3x - y - 2z &= 10
\end{aligned}
$$

1.6 Vector Spaces

1.6.1

Violates axiom 6

1.6.3

Violates axiom 10

1.6.5

Vector space

1.6.7

Violates axiom 2

1.6.9

Vector space

1.6.11

Subspace

1.6.13

Not a subspace

1.6.15

Subspace

1.6.17

Subspace

1.6.19

Not a subspace

1.6.23

(a)

$$\begin{aligned} k_1\mathbf{u}_1 + k_2\mathbf{u}_2 + k_3\mathbf{u}_3 &= \mathbf{0} \\ k_1\langle 1,0,0\rangle + k_2\langle 1,1,0\rangle + k_3\langle 1,1,1\rangle &= \mathbf{0} \\ \\ k_3 &= 0 \\ \\ k_2 + k_3 &= 0 \\ k_2 &= 0 \\ \\ k_1 + k_2 + k_3 &= 0 \\ k_1 &= 0 \end{aligned}$$

(b)

$$\mathbf{a} = 7\mathbf{u}_1 - 12\mathbf{u}_2 + 8\mathbf{u}_3$$

1.6.25

Dependent

1.6.27

Independent

1.6.29

$f(x)$ is undefined at $x = -3$ and $x = -1$.

1.6.31

$$
\begin{aligned}
||x|| &= \sqrt{(x, x)} \\
&= \sqrt{\int_0^{2\pi} x^2 \, dx} \\
&= \sqrt{\left[\frac{1}{3}x^3\right]_0^{2\pi}} \\
&= \sqrt{\frac{8}{3}\pi^3} \\
||\sin x|| &= \sqrt{(\sin x, \sin x)} \\
&= \sqrt{\int_0^{2\pi} \sin^2 x \, dx} \\
&= \sqrt{\left[\frac{x}{2} - \frac{1}{4}\sin 2x\right]_0^{2\pi}} \\
&= \sqrt{\pi}
\end{aligned}
$$

1.7 Gram–Schmidt Orthogonalization Process

1.7.1

$$
\begin{aligned}
\langle \frac{12}{13}, \frac{5}{13} \rangle \cdot \langle \frac{5}{13}, -\frac{12}{13} \rangle &= 0 \\
\sqrt{\left(\frac{12}{13}\right)^2 + \left(\frac{5}{13}\right)^2} &= 1
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{u} &= \left(\langle 4, 2 \rangle \cdot \langle \frac{12}{13}, \frac{5}{13} \rangle \right) \langle \frac{12}{13}, \frac{5}{13} \rangle \\
&\quad + \left(\langle 4, 2 \rangle \cdot \langle \frac{5}{13}, -\frac{12}{13} \rangle \right) \langle \frac{5}{13}, -\frac{12}{13} \rangle \\
&= \left(\frac{58}{13} \right) \langle \frac{12}{13}, \frac{5}{13} \rangle - \left(\frac{4}{13} \right) \langle \frac{5}{13}, -\frac{12}{13} \rangle
\end{aligned}
$$

1.7.3

$$
\begin{aligned}
\langle 1, 0, 1 \rangle \cdot \langle 0, 1, 0 \rangle &= 0 \\
\langle 1, 0, 1 \rangle \cdot \langle -1, 0, 1 \rangle &= 0 \\
\langle 0, 1, 0 \rangle \cdot \langle -1, 0, 1 \rangle &= 0
\end{aligned}
$$

$$
\begin{aligned}
B' &= \{ \langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle, \langle 0, 1, 0 \rangle, \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle \} \\
\mathbf{u} &= -\frac{3}{\sqrt{2}} \langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle + 7 \langle 0, 1, 0 \rangle - \frac{23}{\sqrt{2}} \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle
\end{aligned}
$$

1.7.5

(a)

$$\begin{aligned}
B &= \{\langle -3, 2\rangle, \langle -1, -1\rangle\} \\
\mathbf{v}_1 &= \mathbf{u}_1 \\
&= \langle -3, 2\rangle \\
\mathbf{v}_2 &= \mathbf{u}_2 - \operatorname{proj}_{\mathbf{v}_1}\mathbf{u}_2 \\
&= \langle -1, -1\rangle - \left(\frac{\langle -1, -1\rangle \cdot \langle -3, 2\rangle}{\langle -3, 2\rangle \cdot \langle -3, 2\rangle}\right)\langle -3, 2\rangle \\
&= \langle -1, -1\rangle - \frac{1}{13}\langle -3, 2\rangle \\
&= \langle -\frac{10}{13}, -\frac{15}{13}\rangle \\
\mathbf{w}_1 &= \langle -\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\rangle \\
\mathbf{w}_2 &= \sqrt{\frac{169}{325}}\langle -\frac{10}{13}, -\frac{15}{13}\rangle \\
&= \frac{\sqrt{13}}{5}\langle -\frac{10}{13}, -\frac{15}{13}\rangle \\
&= \langle -\frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}\rangle
\end{aligned}$$

1.7.9

$$\begin{aligned}
B &= \{\langle 1,1,0\rangle, \langle 1,2,2\rangle, \langle 2,2,1\rangle\} \\
\mathbf{v}_1 &= \langle 1,1,0\rangle \\
\mathbf{v}_2 &= \mathbf{u}_2 - \mathrm{proj}_{\mathbf{v}_1}\mathbf{u}_2 \\
&= \langle 1,2,2\rangle - \left(\frac{\langle 1,2,2\rangle \cdot \langle 1,1,0\rangle}{\langle 1,1,0\rangle \cdot \langle 1,1,0\rangle}\right)\langle 1,1,0\rangle \\
&= \langle 1,2,2\rangle - \frac{3}{2}\langle 1,1,0\rangle \\
&= \langle -\frac{1}{2}, \frac{1}{2}, 2\rangle \\
\mathbf{v}_3 &= \mathbf{u}_3 - \mathrm{proj}_{\mathbf{v}_1}\mathbf{u}_3 - \mathrm{proj}_{\mathbf{v}_2}\mathbf{u}_3 \\
&= \langle 2,2,1\rangle - \left(\frac{\langle 2,2,1\rangle \cdot \langle 1,1,0\rangle}{\langle 1,1,0\rangle \cdot \langle 1,1,0\rangle}\right)\langle 1,1,0\rangle \\
&\qquad - \left(\frac{\langle 2,2,1\rangle \cdot \langle -\frac{1}{2}, \frac{1}{2}, 2\rangle}{\langle -\frac{1}{2}, \frac{1}{2}, 2\rangle \cdot \langle -\frac{1}{2}, \frac{1}{2}, 2\rangle}\right)\langle -\frac{1}{2}, \frac{1}{2}, 2\rangle \\
&= \langle 2,2,1\rangle - 2\langle 1,1,0\rangle - \frac{4}{9}\langle -\frac{1}{2}, \frac{1}{2}, 2\rangle \\
&= \langle \frac{2}{9}, -\frac{2}{9}, \frac{1}{9}\rangle \\
\mathbf{w}_1 &= \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\rangle \\
\mathbf{w}_2 &= \langle -\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}\rangle \\
\mathbf{w}_3 &= 3\langle \frac{2}{9}, -\frac{2}{9}, \frac{1}{9}\rangle \\
&= \langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\rangle
\end{aligned}$$

1.7.17

$$
\begin{aligned}
B &= \{1, x, x^2\} \\
\mathbf{v}_1 &= \mathbf{u}_1 \\
&= 1 \\
\mathbf{v}_2 &= \mathbf{u}_2 - \mathrm{proj}_{\mathbf{v}_1}\mathbf{u}_2 \\
&= \mathbf{u}_2 - \left(\frac{\mathbf{u}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\right)\mathbf{v}_1 \\
&= x - \frac{\int_{-1}^{1} x\,dx}{\int_{-1}^{1} dx} \\
&= x - \frac{\left[\frac{1}{2}x^2\right]_{-1}^{1}}{2} \\
&= x \\
\mathbf{v}_3 &= \mathbf{u}_3 - \mathrm{proj}_{\mathbf{v}_1}\mathbf{u}_3 - \mathrm{proj}_{\mathbf{v}_2}\mathbf{u}_3 \\
&= \mathbf{u}_3 - \left(\frac{\mathbf{u}_3 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\right)\mathbf{v}_1 - \left(\frac{\mathbf{u}_3 \cdot \mathbf{v}_2}{\mathbf{v}_2 \cdot \mathbf{v}_2}\right)\mathbf{v}_2 \\
&= x^2 - \frac{\int_{-1}^{1} x^2\,dx}{\int_{-1}^{1} dx} - \frac{\int_{-1}^{1} x^3\,dx}{\int_{-1}^{1} x^2\,dx}x \\
&= x^2 - \frac{\left[\frac{1}{3}x^3\right]_{-1}^{1}}{2} - \frac{\left[\frac{1}{4}x^4\right]_{-1}^{1}}{\left[\frac{1}{3}x^3\right]_{-1}^{1}}x \\
&= x^2 - \frac{1}{3}
\end{aligned}
$$

1.7.19

$$
\begin{aligned}
||\mathbf{v}_1||^2 &= \int_{-1}^{1} dx \\
&= 2 \\
\mathbf{w}_1 &= \frac{1}{\sqrt{2}} \\
||\mathbf{v}_2||^2 &= \int_{-1}^{1} x^2 \, dx \\
&= \left[\frac{1}{3}x^3\right]_{-1}^{1} \\
&= \frac{2}{3} \\
\mathbf{w}_2 &= \frac{3}{\sqrt{6}}x \\
||\mathbf{v}_3||^2 &= \int_{-1}^{1} \left(x^2 - \frac{1}{3}\right)^2 \, dx \\
&= \int_{-1}^{1} \left(x^4 - \frac{2}{3}x^2 + \frac{1}{9}\right) \, dx \\
&= \left[\frac{1}{5}x^5 - \frac{2}{9}x^3 + \frac{1}{9}x\right]_{-1}^{1} \\
&= \frac{1}{5} - \frac{2}{9} + \frac{1}{9} + \frac{1}{5} - \frac{2}{9} + \frac{1}{9} \\
&= \frac{2}{5} - \frac{2}{9} \\
&= \frac{8}{45} \\
\mathbf{w}_3 &= \sqrt{\frac{45}{8}} \left(x^2 - \frac{1}{3}\right) \\
&= \frac{5}{2\sqrt{10}} \left(3x^2 - 1\right)
\end{aligned}
$$

1.7.21

$$
\begin{aligned}
(\mathbf{p}, \mathbf{w}_1) &= \int_{-1}^{1} \frac{1}{\sqrt{2}}(9x^2 - 6x + 5)\,dx \\
&= \frac{1}{\sqrt{2}}\left[3x^3 - 3x^2 + 5x\right]_{-1}^{1} \\
&= \frac{1}{\sqrt{2}}(3 - 3 + 5 + 3 + 3 + 5) \\
&= \frac{16}{\sqrt{2}} \\
(\mathbf{p}, \mathbf{w}_2) &= \int_{-1}^{1} \frac{3}{\sqrt{6}}x(9x^2 - 6x + 5)\,dx \\
&= \frac{3}{\sqrt{6}}\left[\frac{9}{4}x^4 - 2x^3 + \frac{5}{2}x^2\right]_{-1}^{1} \\
&= \frac{3}{\sqrt{6}}\left(\frac{9}{4} - 2 + \frac{5}{2} - \frac{9}{4} - 2 - \frac{5}{2}\right) \\
&= \frac{3}{\sqrt{6}}\left(\frac{9}{4} - \frac{8}{4} + \frac{10}{4} - \frac{9}{4} - \frac{8}{4} - \frac{10}{4}\right) \\
&= -\frac{12}{\sqrt{6}} \\
(\mathbf{p}, \mathbf{w}_3) &= \int_{-1}^{1} \frac{5}{2\sqrt{10}}(3x^2 - 1)(9x^2 - 6x + 5)\,dx \\
&= \frac{5}{2\sqrt{10}}\int_{-1}^{1}(27x^4 - 18x^3 + 6x^2 + 6x - 5)\,dx \\
&= \frac{5}{2\sqrt{10}}\left[\frac{27}{5}x^5 - \frac{9}{2}x^4 + 2x^3 + 3x^2 - 5x\right]_{-1}^{1} \\
&= \frac{5}{2\sqrt{10}}\left(\frac{27}{5} - \frac{9}{2} + 2 + 3 - 5 + \frac{27}{5} + \frac{9}{2} + 2 - 3 - 5\right) \\
&= \frac{5}{2\sqrt{10}}\left(\frac{54}{10} - \frac{45}{10} + \frac{20}{10} + \frac{30}{10} - \frac{50}{10} + \frac{54}{10} + \frac{45}{10} + \frac{20}{10} - \frac{30}{10} - \frac{50}{10}\right) \\
&= \frac{5}{2\sqrt{10}}\frac{48}{10} \\
&= \frac{12}{\sqrt{10}} \\
\mathbf{p} &= \frac{16}{\sqrt{2}}\mathbf{w}_1 - \frac{12}{\sqrt{6}}\mathbf{w}_2 + \frac{12}{\sqrt{10}}\mathbf{w}_3
\end{aligned}
$$

1.8 Chapter in Review

1.8.1

True

1.8.3

$$\begin{aligned}\mathbf{u} &= \langle 5, -2, 1\rangle \\ \mathbf{v} &= \langle 2, 3, -4\rangle\end{aligned}$$

False

1.8.5

True

1.8.7

True

1.8.9

True

1.8.11

$9\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

1.8.13

$$\begin{aligned}(-\mathbf{k}) \times (5\mathbf{j}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1 \\ 0 & 5 & 0 \end{vmatrix} \\ &= 5\mathbf{i}\end{aligned}$$

1.8.15

$$||-12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}|| = \sqrt{12^2 + 4^2 + 6^2} = 14$$

1.8.17

$\langle -6, 1, -7\rangle$

1.8.19

$$\begin{aligned} x &= 1 + t \\ y &= -2 + 3t \\ z &= -1 + 2t \end{aligned}$$

$$\begin{aligned} x + 2y - z &= 13 \\ (1 + t) + 2(-2 + 3t) - (-1 + 2t) &= 13 \\ 1 + t - 4 + 6t + 1 - 2t &= 13 \\ -2 + 5t &= 13 \\ t &= 3 \end{aligned}$$

$$\begin{aligned} x &= 4 \\ y &= 7 \\ z &= 5 \end{aligned}$$

1.8.21

$$\begin{aligned} \overrightarrow{P_1P_2} &= \overrightarrow{P_2} - \overrightarrow{P_1} \\ \overrightarrow{P_2} &= \overrightarrow{P_1P_2} + \overrightarrow{P_1} \\ &= \langle 3, 5, -4 \rangle + \langle 2, 1, 7 \rangle \\ &= \langle 5, 6, 3 \rangle \end{aligned}$$

1.8.23

$\mathbf{a} \cdot \mathbf{b} = -36\sqrt{2}$

1.8.25

$x = 12,\ y = -8,\ z = 6$

1.8.27

$$\begin{aligned} \frac{1}{2}(\mathbf{a} \times \mathbf{b}) &= \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -1 \\ 2 & -1 & 2 \end{vmatrix} \\ &= \frac{1}{2}\langle 5, -4, -7 \rangle \\ &= \langle \frac{5}{2}, -2, -\frac{7}{2} \rangle \end{aligned}$$

The area is $\sqrt{\left(\frac{5}{2}\right)^2 + (-2)^2 + \left(-\frac{7}{2}\right)^2} = \frac{3}{2}\sqrt{10}$

1.8.29

2

1.8.31

$$\begin{aligned}
\mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 1 & -2 & 1 \end{vmatrix} \\
&= \langle 1, -1, -3 \rangle \\
||\mathbf{a} \times \mathbf{b}|| &= \sqrt{11} \\
\text{norm}(\mathbf{a} \times \mathbf{b}) &= \langle \frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, -\frac{3}{\sqrt{11}} \rangle
\end{aligned}$$

1.8.33

$$\text{comp}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{b}||} = \frac{10}{5} = 2$$

1.8.35

$$\begin{aligned}
\mathbf{a} &= \langle 1, 2, -2 \rangle \\
\mathbf{b} &= \langle 4, 3, 0 \rangle \\
\mathbf{a} + \mathbf{b} &= \langle 5, 5, -2 \rangle \\
\text{proj}_{\mathbf{a}}(\mathbf{a} + \mathbf{b}) &= \left(\frac{(\mathbf{a} + \mathbf{b}) \cdot \mathbf{a}}{\mathbf{a} \cdot \mathbf{a}} \right) \mathbf{a} \\
&= \frac{19}{9} \langle 1, 2, -2 \rangle \\
&= \langle \frac{19}{9}, \frac{38}{9}, -\frac{38}{9} \rangle
\end{aligned}$$

1.8.37

(a)

(b) A plane with normal $\mathbf{a}$

1.8.39

$$\frac{x-7}{4} = \frac{y-3}{-2} = \frac{z+5}{6}$$

1.8.41

$$\begin{aligned} \langle -2,3,1\rangle \cdot \langle 2,1,1\rangle &= -4+3+1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} 1-2t &= 1+2s \\ t &= -s \end{aligned}$$

$$\begin{aligned} 3t &= -4+s \\ t &= -\frac{4}{3}+\frac{s}{3} \\ &= -\frac{4}{3}-\frac{t}{3} \\ \frac{4}{3}t &= -\frac{4}{3} \\ t &= -1 \end{aligned}$$

$$s = 1$$

$$\langle 3,-3,0\rangle$$

1.8.43

$$\begin{aligned} \mathbf{u} &= \langle 1,4,-2\rangle \\ \mathbf{v} &= \langle 1,1,3\rangle \\ \mathbf{n} &= \mathbf{u}\times\mathbf{v} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & -2 \\ 1 & 1 & 3 \end{vmatrix} \\ &= \langle 14,-5,-3\rangle \end{aligned}$$

$$\begin{aligned} \mathbf{n}\cdot(\mathbf{r}-\mathbf{v}) &= 0 \\ \langle 14,-5,-3\rangle \cdot (\langle x,y,z\rangle - \langle 1,1,3\rangle) &= 0 \\ 14(x-1)-5(y-1)-3(z-3) &= 0 \\ 14x-5y-3z &= 0 \end{aligned}$$

1.8.45

$$
\begin{aligned}
\mathbf{F} &= \langle \frac{10}{\sqrt{2}}, \frac{10}{\sqrt{2}}, 0 \rangle \\
\mathbf{d} &= \langle 3, 3, 0 \rangle \\
\mathbf{F} \cdot \mathbf{d} &= 30\sqrt{2}\,\mathrm{J}
\end{aligned}
$$

1.8.47

$$
\begin{aligned}
\mathbf{F}_1 &= \langle 200, 0, 0 \rangle \\
\mathbf{F}_2 &= \langle \frac{200}{\sqrt{2}}, \frac{200}{\sqrt{2}}, 0 \rangle \\
\mathbf{F}_2 &= \mathbf{F}_1 + \mathbf{F}_3 \\
\mathbf{F}_3 &= \mathbf{F}_2 - \mathbf{F}_1 \\
&= \langle \frac{200}{\sqrt{2}}, \frac{200}{\sqrt{2}}, 0 \rangle - \langle 200, 0, 0 \rangle \\
&= \langle \frac{200}{\sqrt{2}} - 200, \frac{200}{\sqrt{2}}, 0 \rangle \\
||\mathbf{F}_3|| &= \sqrt{\left(\frac{200}{\sqrt{2}} - 200\right)^2 + \left(\frac{200}{\sqrt{2}}\right)^2} \\
&= \sqrt{\frac{40000}{2} - \frac{80000}{\sqrt{2}} + 40000 + \frac{40000}{2}} \\
&= 200\sqrt{2\left(1 - \frac{1}{\sqrt{2}}\right)} \\
&\approx 153\,\mathrm{lb}
\end{aligned}
$$

2 Matrices

2.1 Matrix Algebra

2.1.1

2×4

2.1.3

3×3

2.1.5

3×4

2.1.7

No

2.1.9

No

2.1.11

$$\begin{aligned} x &= y - 2 \\ 3x - 2 &= y \end{aligned}$$

$$\begin{aligned} 2x - 2 &= 2 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} 2 &= y - 2 \\ y &= 4 \end{aligned}$$

2.1.13

$$\begin{aligned} c_{23} &= 9 \\ c_{12} &= 12 \end{aligned}$$

2.1.15

(a) $\begin{pmatrix} 2 & 11 \\ 2 & -1 \end{pmatrix}$

(b) $\begin{pmatrix} -6 & 1 \\ 14 & -19 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 28 \\ 12 & -12 \end{pmatrix}$

2.1.17

(a) $\begin{pmatrix} -11 & 6 \\ 17 & -22 \end{pmatrix}$

(b) $\begin{pmatrix} -32 & 27 \\ -4 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 19 & -18 \\ -30 & 31 \end{pmatrix}$

(d) $\begin{pmatrix} 19 & 6 \\ 3 & 22 \end{pmatrix}$

2.1.21

(a) 180

(b) $\begin{pmatrix} 4 & 8 & 10 \\ 8 & 16 & 20 \\ 10 & 20 & 25 \end{pmatrix}$

(c) $\begin{pmatrix} 6 \\ 12 \\ -5 \end{pmatrix}$

2.1.23

(a) $\begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$

(b) $\begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$

2.1.25

$\begin{pmatrix} -14 \\ 1 \end{pmatrix}$

2.1.27

$\begin{pmatrix} -38 \\ -2 \end{pmatrix}$

2.1.29

4×5

2.1.41

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + x_{22}x_2 &= b_2 \end{aligned}$$

2.1.43

$$\begin{aligned} \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a & \frac{1}{2}b \\ \frac{1}{2}b & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} ax + \frac{1}{2}by \\ \frac{1}{2}bx + cy \end{pmatrix} \\ &= ax^2 + \frac{1}{2}bxy + \frac{1}{2}bxy + cy^2 \\ &= ax^2 + bxy + cy^2 \end{aligned}$$

2.1.45

$\langle -1, 1\rangle$

2.1.47

$\langle -2, 0\rangle$

2.1.49

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

2.1.51

(b)

$$\begin{pmatrix} x_S \\ y_S \\ z_S \end{pmatrix} = \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

2.2 Systems of Linear Algebraic Equations

2.2.1

$$\left(\begin{array}{cc|c} 1 & -1 & 11 \\ 4 & 3 & -5 \end{array}\right)$$
$$\left(\begin{array}{cc|c} 1 & -1 & 11 \\ 0 & 7 & -49 \end{array}\right)$$
$$\left(\begin{array}{cc|c} 1 & -1 & 11 \\ 0 & 1 & -7 \end{array}\right)$$
$$\left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -7 \end{array}\right)$$

2.2.5

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 2 & 3 & 5 & 7 \\ 1 & -2 & 3 & -11 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 5 & 7 & 13 \\ 0 & -1 & 4 & -8 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 1 & \frac{7}{5} & \frac{13}{5} \\ 0 & -1 & 4 & -8 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 1 & \frac{7}{5} & \frac{13}{5} \\ 0 & 0 & \frac{27}{5} & -\frac{27}{5} \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 1 & \frac{7}{5} & \frac{13}{5} \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

2.3 Rank of a Matrix

2.3.1

$$\begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -\frac{1}{3} \\ 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -\frac{1}{3} \\ 0 & \frac{10}{3} \end{pmatrix}$$

Rank 2

2.3.3

$$\begin{pmatrix} 2 & 1 & 3 \\ 6 & 3 & 9 \\ -1 & -\frac{1}{2} & -\frac{3}{2} \end{pmatrix}$$
$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 6 & 3 & 9 \\ -1 & -\frac{1}{2} & -\frac{3}{2} \end{pmatrix}$$
$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Rank 1

2.3.5

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 4 \\ 1 & 4 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 3 \\ 0 & 3 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & 3 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 9 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

Rank 3

2.3.7

$$\begin{pmatrix} 1 & -2 \\ 3 & -6 \\ 7 & -1 \\ 4 & 5 \end{pmatrix}$$
$$\begin{pmatrix} 1 & -2 \\ 0 & 13 \\ 0 & 13 \\ 0 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 1 & -2 \\ 0 & 13 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Rank 2

2.3.11

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & -1 & 5 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & -2 \\ 0 & -3 & 2 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -3 & 2 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 5 \end{pmatrix}$$

Linearly independent

2.3.15

5

2.3.17

$\text{rank}(\mathbf{A}) = 2$

2.4 Determinants

2.4.1

9

2.4.3

1

2.4.5

$$\begin{aligned} M_{33} &= \begin{vmatrix} 0 & 2 & 0 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix} \\ &= -2 \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} \\ &= 2 \end{aligned}$$

2.4.7

$$\begin{aligned} C_{34} &= (-1)^{3+4} \begin{vmatrix} 0 & 2 & 4 \\ 1 & 2 & -2 \\ 1 & 1 & 1 \end{vmatrix} \\ &= -\left(-2 \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} \right) \\ &= 10 \end{aligned}$$

2.4.9

-7

2.4.11

17

2.4.13

$$\begin{aligned} (1-\lambda)(2-\lambda) - 6 &= 2 - \lambda - 2\lambda + \lambda^2 - 6 \\ &= \lambda^2 - 3\lambda - 4 \\ &= (\lambda + 1)(\lambda - 4) \end{aligned}$$

2.4.15

-48

2.4.23

$$\begin{aligned}\begin{vmatrix}1 & 1 & 1\\ x & y & z\\ 2 & 3 & 4\end{vmatrix} &= \begin{vmatrix}y & z\\ 3 & 4\end{vmatrix} - \begin{vmatrix}x & z\\ 2 & 4\end{vmatrix} + \begin{vmatrix}x & y\\ 2 & 3\end{vmatrix}\\ &= 4y - 3z - 4x + 2z + 3x - 2y\\ &= -x + 2y - z\end{aligned}$$

2.4.29

$$\begin{aligned}\begin{vmatrix}(-3-\lambda) & 10\\ 2 & (5-\lambda)\end{vmatrix} &= 0\\ (-3-\lambda)(5-\lambda) - 20 &= 0\\ -15 + 3\lambda - 5\lambda + \lambda^2 - 20 &= 0\\ \lambda^2 - 2\lambda - 35 &= 0\\ (\lambda - 7)(\lambda + 5) &= 0\\ \lambda &= -5 \text{ or } 7\end{aligned}$$

2.5 Properties of Determinants

2.5.1

8.5.4

2.5.3

8.5.7

2.5.5

8.5.5

2.5.7

8.5.3

2.5.9

8.5.1

2.5.11

-5

2.5.13

-5

2.5.15

5

2.5.17

80

2.5.19

-105

2.5.25

$$\begin{aligned}
\mathbf{A}\mathbf{A} &= \mathbf{I} \\
\det \mathbf{A} \cdot \det \mathbf{A} &= \det \mathbf{I} \\
(\det \mathbf{A})^2 &= 1 \\
\det \mathbf{A} &= \pm 1
\end{aligned}$$

2.5.27

$$\begin{aligned}
\begin{vmatrix} a & a+1 & a+2 \\ b & b+1 & b+2 \\ c & c+1 & c+2 \end{vmatrix} &= \begin{vmatrix} a & 1 & 2 \\ b & 1 & 2 \\ c & 1 & 2 \end{vmatrix} \\
&= \begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} \\
&= 0
\end{aligned}$$

2.5.29

$$\begin{aligned}
\begin{vmatrix} 1 & 1 & 5 \\ 4 & 3 & 6 \\ 0 & -1 & 1 \end{vmatrix} &= \begin{vmatrix} 1 & 1 & 5 \\ 0 & -1 & -14 \\ 0 & -1 & 1 \end{vmatrix} \\
&= -\begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 14 \\ 0 & 0 & 15 \end{vmatrix} \\
&= -15
\end{aligned}$$

2.5.37

$$\begin{aligned}
\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} &= \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{vmatrix} \\
&= \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & c^2-a^2-(b+a)(c-a) \end{vmatrix} \\
&= (b-a)(c^2-a^2-(b+a)(c-a)) \\
&= (b-a)(c^2-a^2-bc+ab-ac+a^2) \\
&= (b-a)(c^2+ab-ac-bc) \\
&= (b-a)(c-a)(c-b)
\end{aligned}$$

2.5.39

$$\begin{aligned}
a_{21}C_{11}+a_{22}C_{12}+a_{23}C_{13} &= (-1)(4)+(2)(5)+(1)(-6) \\
&= 0 \\
a_{13}C_{12}+a_{23}C_{22}+a_{33}C_{32} &= (2)(5)+(1)(-7)+(1)(-3) \\
&= 0
\end{aligned}$$

2.5.41

$$\begin{aligned}
\mathbf{A}+\mathbf{B} &= \begin{pmatrix} 10 & 0 \\ 0 & -3 \end{pmatrix} \\
\det(\mathbf{A}+\mathbf{B}) &= -30 \\
\det\mathbf{A} &= 10 \\
\det\mathbf{B} &= -31 \\
-30 &\neq 10-31
\end{aligned}$$

2.6 Inverse of a Matrix

2.6.3

$$\begin{aligned}
\det\mathbf{A} &= 9 \\
\mathbf{A}^{-1} &= \frac{1}{9}\begin{pmatrix} 1 & 1 \\ -4 & 5 \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{9} & \frac{1}{9} \\ -\frac{4}{9} & \frac{5}{9} \end{pmatrix}
\end{aligned}$$

2.6.5

$$\begin{aligned}
\det \mathbf{A} &= 12 \\
\mathbf{A}^{-1} &= \frac{1}{12}\begin{pmatrix} 2 & 0 \\ 3 & 6 \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{6} & 0 \\ \frac{1}{4} & \frac{1}{2} \end{pmatrix}
\end{aligned}$$

2.6.7

$$\begin{aligned}
\det \mathbf{A} &= (1)\begin{vmatrix} 4 & 4 \\ -1 & 1 \end{vmatrix} - (3)\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} + (5)\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix} \\
&= 8 + 6 - 30 \\
&= -16 \\
\mathbf{A}^{-1} &= -\frac{1}{16}\begin{pmatrix} 8 & 2 & -6 \\ -8 & -4 & 4 \\ -8 & 6 & -2 \end{pmatrix}^T \\
&= \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{8} & \frac{1}{4} & -\frac{3}{8} \\ \frac{3}{8} & -\frac{1}{4} & \frac{1}{8} \end{pmatrix}
\end{aligned}$$

2.6.15

$$\left(\begin{array}{cc|cc} 6 & -2 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & -\frac{1}{3} & \frac{1}{6} & 0 \\ 0 & 1 & 0 & \frac{1}{4} \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{6} & \frac{1}{12} \\ 0 & 1 & 0 & \frac{1}{4} \end{array}\right)$$

2.6.17

$$\left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right)$$
$$\left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -12 & -5 & 1 \end{array}\right)$$
$$\left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & \frac{5}{12} & -\frac{1}{12} \end{array}\right)$$
$$\left(\begin{array}{cc|cc} 1 & 0 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{5}{12} & -\frac{1}{12} \end{array}\right)$$

2.6.27

$$\begin{aligned}(\mathbf{AB})^{-1} &= \mathbf{B}^{-1}\mathbf{A}^{-1} \\ &= \begin{pmatrix} \frac{2}{3} & \frac{4}{3} \\ -\frac{1}{3} & \frac{5}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{5}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix} \\ &= \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ -\frac{17}{12} & \frac{55}{12} \end{pmatrix}\end{aligned}$$

2.6.29

$$\begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}$$

2.6.31

$$\begin{aligned}\begin{pmatrix} 4 & -3 \\ x & -4 \end{pmatrix} &= \frac{1}{3x-16}\begin{pmatrix} -4 & -x \\ 3 & 4 \end{pmatrix}^T \\ &= \frac{1}{3x-16}\begin{pmatrix} -4 & 3 \\ -x & 4 \end{pmatrix} \\ -1 &= \frac{1}{3x-16} \\ 16-3x &= 1 \\ 3x &= 15 \\ x &= 5\end{aligned}$$

2.6.45

$$\begin{aligned}
\begin{pmatrix}1 & 1\\ 2 & -1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} &= \begin{pmatrix}4\\ 14\end{pmatrix}\\
\begin{pmatrix}x_1\\ x_2\end{pmatrix} &= -\frac{1}{3}\begin{pmatrix}-1 & -1\\ -2 & 1\end{pmatrix}\begin{pmatrix}4\\ 14\end{pmatrix}\\
&= -\frac{1}{3}\begin{pmatrix}-18\\ 6\end{pmatrix}\\
&= \begin{pmatrix}6\\ -2\end{pmatrix}
\end{aligned}$$

2.6.49

$$\begin{aligned}
\begin{pmatrix}1 & 0 & 1\\ 1 & 1 & 1\\ 5 & -1 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} &= \begin{pmatrix}-4\\ 0\\ 6\end{pmatrix}\\
\det \mathbf{A} &= \begin{vmatrix}1 & 1\\ -1 & 0\end{vmatrix} + \begin{vmatrix}1 & 1\\ 5 & -1\end{vmatrix}\\
&= 1 - 6\\
&= -5\\
\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} &= -\frac{1}{5}\begin{pmatrix}1 & 5 & -6\\ -1 & -5 & 1\\ -1 & 0 & 1\end{pmatrix}^T\begin{pmatrix}-4\\ 0\\ 6\end{pmatrix}\\
&= -\frac{1}{5}\begin{pmatrix}1 & -1 & -1\\ 5 & -5 & 0\\ -6 & 1 & 1\end{pmatrix}\begin{pmatrix}-4\\ 0\\ 6\end{pmatrix}\\
&= -\frac{1}{5}\begin{pmatrix}-10\\ -20\\ 30\end{pmatrix}\\
&= \begin{pmatrix}2\\ 4\\ -6\end{pmatrix}
\end{aligned}$$

2.6.55

$$\begin{aligned}
\det\begin{pmatrix}1 & 2 & -1\\ 4 & -1 & 1\\ 5 & 1 & -2\end{pmatrix} &= (1)\begin{vmatrix}-1 & 1\\ 1 & -2\end{vmatrix} - (2)\begin{vmatrix}4 & 1\\ 5 & -2\end{vmatrix} + (-1)\begin{vmatrix}4 & -1\\ 5 & 1\end{vmatrix}\\
&= 1 + 26 - 9\\
&= 18
\end{aligned}$$

Only trivial solution

2.7 Cramer's Rule

2.7.1

$$\begin{aligned}
\mathbf{A} &= \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix} \\
\mathbf{B} &= \begin{pmatrix} 3 \\ -6 \end{pmatrix} \\
\det \mathbf{A} &= 10 \\
\det \mathbf{A}_1 &= -6 \\
\det \mathbf{A}_2 &= 12 \\
x_1 &= -\frac{3}{5} \\
x_2 &= \frac{6}{5}
\end{aligned}$$

2.7.11

$$\begin{aligned}
\mathbf{A} &= \begin{pmatrix} 2-k & k \\ k & 3-k \end{pmatrix} \\
\mathbf{B} &= \begin{pmatrix} 4 \\ 3 \end{pmatrix} \\
\det \mathbf{A} &= (2-k)(3-k) - k^2 \\
&= 6 - 5k \\
\det \mathbf{A}_1 &= 4(3-k) - 3k \\
&= 12 - 7k \\
\det \mathbf{A}_2 &= 3(2-k) - 4k \\
&= 6 - 7k \\
x_1 &= \frac{12-7k}{6-5k} \\
x_2 &= \frac{6-7k}{6-5k}
\end{aligned}$$

The system is inconsistent when $k = \frac{6}{5}$

2.7.13

$$\begin{aligned}
\mathbf{A} &= \begin{pmatrix} \cos 25^\circ & -\cos 15^\circ \\ \sin 25^\circ & \sin 15^\circ \end{pmatrix} \\
\mathbf{B} &= \begin{pmatrix} 0 \\ 300 \end{pmatrix} \\
\det \mathbf{A} &= \cos 25^\circ \sin 15^\circ + \cos 15^\circ \sin 25^\circ \\
&= \sin 40^\circ \\
\det \mathbf{A}_1 &= 300 \cos 15^\circ \\
\det \mathbf{A}_2 &= 300 \cos 25^\circ \\
T_1 &= \frac{300 \cos 15^\circ}{\sin 40^\circ} \\
&\approx 451\,\mathrm{lb} \\
T_2 &= \frac{300 \cos 25^\circ}{\sin 40^\circ} \\
&\approx 423\,\mathrm{lb}
\end{aligned}$$

2.8 The Eigenvalue Problem

2.8.1

$\mathbf{K}_3$ with $\lambda = -1$

2.8.3

$\mathbf{K}_3$ with $\lambda = 0$

2.8.5

$\mathbf{K}_2$ with $\lambda = 3$
$\mathbf{K}_3$ with $\lambda = 1$

2.8.7

$$\begin{aligned}
\det(\mathbf{A} - \lambda \mathbf{I}) &= \begin{vmatrix} -1-\lambda & 2 \\ -7 & 8-\lambda \end{vmatrix} \\
&= (-1-\lambda)(8-\lambda) + 14 \\
&= -8 + \lambda - 8\lambda + \lambda^2 + 14 \\
&= \lambda^2 - 7\lambda + 6 \\
&= (\lambda - 1)(\lambda - 6) \\
\lambda_1 &= 1 \\
\lambda_2 &= 6
\end{aligned}$$

$$\left(\begin{array}{cc|c} -2 & 2 & 0 \\ -7 & 7 & 0 \end{array}\right)$$

$x_1 = x_2$

$$\mathbf{X}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} -7 & 2 & 0 \\ -7 & 2 & 0 \end{array}\right)$$

$x_1 = \frac{2}{7}x_2$

$$\mathbf{X}_2 = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

Nonsingular

2.8.9

$$\begin{aligned} \det(\mathbf{A} - \lambda\mathbf{I}) &= \begin{vmatrix} -8-\lambda & -1 \\ 16 & -\lambda \end{vmatrix} \\ &= -\lambda(-8-\lambda) + 16 \\ &= 8\lambda + \lambda^2 + 16 \\ &= (\lambda+4)^2 \\ \lambda_1 = \lambda_2 &= -4 \end{aligned}$$

$$\left(\begin{array}{cc|c} -4 & -1 & 0 \\ 16 & 4 & 0 \end{array}\right)$$

$x_1 = -\frac{1}{4}x_2$

$$\mathbf{X}_1 = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

Nonsingular

2.8.11

$$\begin{aligned}
\det(\mathbf{A}-\lambda\mathbf{I}) &= \begin{vmatrix} -1-\lambda & 2 \\ -5 & 1-\lambda \end{vmatrix} \\
&= (-1-\lambda)(1-\lambda)+10 \\
&= -1+\lambda-\lambda+\lambda^2+10 \\
&= \lambda^2+9 \\
&= (\lambda-3i)(\lambda+3i) \\
\lambda_1 &= 3i \\
\lambda_2 &= -3i
\end{aligned}$$

$$\left(\begin{array}{cc|c} -1-3i & 2 & 0 \\ -5 & 1-3i & 0 \end{array}\right)$$

$$\mathbf{X}_1 = \begin{pmatrix} 1-3i \\ 5 \end{pmatrix}$$

$$\mathbf{X}_2 = \begin{pmatrix} 1+3i \\ 5 \end{pmatrix}$$

Nonsingular

2.8.23

$$\begin{aligned}
\det(\mathbf{A}-\lambda\mathbf{I}) &= \begin{vmatrix} 5-\lambda & 1 \\ 1 & 5-\lambda \end{vmatrix} \\
&= (5-\lambda)^2-1 \\
&= 25-10\lambda+\lambda^2-1 \\
&= \lambda^2-10\lambda+24 \\
&= (\lambda-4)(\lambda-6) \\
\lambda_1 &= 4 \\
\lambda_2 &= 6
\end{aligned}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\mathbf{X}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\mathbf{X_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{aligned}
\lambda_1' &= \frac{1}{4} \\
\lambda_2' &= \frac{1}{6} \\
\mathbf{X}_1' &= \mathbf{X}_1 \\
\mathbf{X}_2' &= \mathbf{X}_2
\end{aligned}$$

2.9 Powers of Matrices

2.9.1

$$\begin{aligned}
\det(\mathbf{A} - \lambda \mathbf{I}) &= (1-\lambda)(5-\lambda) + 8 \\
&= 5 - \lambda - 5\lambda + \lambda^2 + 8 \\
&= \lambda^2 - 6\lambda + 13 \\
\mathbf{A}^2 &= 6\mathbf{A} - 13\mathbf{I} \\
\begin{pmatrix} 1 & -2 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 4 & 5 \end{pmatrix} &= 6 \begin{pmatrix} 1 & -2 \\ 4 & 5 \end{pmatrix} - 13 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
\begin{pmatrix} -7 & -12 \\ 24 & 17 \end{pmatrix} &= \begin{pmatrix} -7 & -12 \\ 24 & 17 \end{pmatrix}
\end{aligned}$$

2.9.3

$$\begin{aligned}
\mathbf{A} &= \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix} \\
\det(\mathbf{A} - \lambda \mathbf{I}) &= (-1-\lambda)(4-\lambda) - 6 \\
&= -4 + \lambda - 4\lambda + \lambda^2 - 6 \\
&= \lambda^2 - 3\lambda - 10 \\
&= (\lambda - 5)(\lambda + 2)
\end{aligned}$$

$$\begin{aligned}
\lambda^m &= c_0 + c_1\lambda \\
(-2)^m &= c_0 - 2c_1 \\
(5)^m &= c_0 + 5c_1
\end{aligned}$$

$$\begin{aligned}
(-2)^m + \frac{2}{5}(5)^m &= \frac{7}{5}c_0 \\
\frac{5}{7}(-2)^m + \frac{2}{7}(5)^m &= c_0
\end{aligned}$$

$$\begin{aligned}
(5)^m - (-2)^m &= 7c_1 \\
\frac{1}{7}(5)^m - \frac{1}{7}(-2)^m &= c_1
\end{aligned}$$

$$\begin{aligned}
\mathbf{A}^m &= \frac{1}{7}\begin{pmatrix} 5^m + 6(-2)^m & 3(5)^m - 3(-2)^m \\ 2(5)^m - 2(-2)^m & 6(5)^m + (-2)^m \end{pmatrix} \\
\mathbf{A}^3 &= \frac{1}{7}\begin{pmatrix} 5^3 + 6(-2)^3 & 3(5)^3 - 3(-2)^3 \\ 2(5)^3 - 2(-2)^3 & 6(5)^3 + (-2)^3 \end{pmatrix} \\
&= \begin{pmatrix} 11 & 57 \\ 38 & 106 \end{pmatrix}
\end{aligned}$$

2.9.5

$$
\begin{aligned}
\mathbf{A} &= \begin{pmatrix} 8 & 5 \\ 4 & 0 \end{pmatrix} \\
\det(\mathbf{A} - \lambda\mathbf{I}) &= -\lambda(8-\lambda) - 20 \\
&= -8\lambda + \lambda^2 - 20 \\
&= \lambda^2 - 8\lambda - 20 \\
&= (\lambda - 10)(\lambda + 2)
\end{aligned}
$$

$$
\begin{aligned}
\lambda^m &= c_0 + c_1\lambda \\
(-2)^m &= c_0 - 2c_1 \\
10^m &= c_0 + 10c_1
\end{aligned}
$$

$$
\begin{aligned}
10^m + 5(-2)^m &= 6c_0 \\
\frac{1}{6}(10^m + 5(-2)^m) &= c_0 \\
\frac{1}{12}(2 \cdot 10^m + 10(-2)^m) &= c_0
\end{aligned}
$$

$$
\begin{aligned}
10^m - (-2)^m &= 12c_1 \\
\frac{1}{12}(10^m - (-2)^m) &= c_1
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{A}^m &= \frac{1}{12}\begin{pmatrix} 10 \cdot 10^m + 2(-2)^m & 5 \cdot 10^m - 5(-2)^m \\ 4 \cdot 10^m - 4(-2)^m & 2 \cdot 10^m + 10(-2)^m \end{pmatrix} \\
\mathbf{A}^5 &= \begin{pmatrix} 83328 & 41680 \\ 33344 & 16640 \end{pmatrix}
\end{aligned}
$$

2.9.11

$$
\begin{aligned}
\mathbf{A} &= \begin{pmatrix} 7 & 3 \\ -3 & 1 \end{pmatrix} \\
\det(\mathbf{A} - \lambda\mathbf{I}) &= (7-\lambda)(1-\lambda) + 9 \\
&= 7 - 7\lambda - \lambda + \lambda^2 + 9 \\
&= \lambda^2 - 8\lambda + 16 \\
&= (\lambda - 4)^2
\end{aligned}
$$

$$
\begin{aligned}
\lambda^m &= c_0 + c_1\lambda \\
m\lambda^{m-1} &= c_1
\end{aligned}
$$

$$
4^{m-1}m = c_1
$$

$$
\begin{aligned}
4^m &= c_0 + 4^m m \\
4^m(1-m) &= c_0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{A}^m &= \begin{pmatrix} 4^m(1-m) + 7\cdot 4^{m-1}m & 3\cdot 4^{m-1}m \\ -3\cdot 4^{m-1}m & 4^m(1-m) + 4^{m-1}m \end{pmatrix} \\
&= 4^m \begin{pmatrix} 1 - m + \frac{7}{4}m & \frac{3}{4}m \\ -\frac{3}{4}m & 1 - m + \frac{1}{4}m \end{pmatrix} \\
&= 4^m \begin{pmatrix} 1 + \frac{3}{4}m & \frac{3}{4}m \\ -\frac{3}{4}m & 1 - \frac{3}{4}m \end{pmatrix} \\
\mathbf{A}^6 &= 4^6 \begin{pmatrix} 1 + \frac{3}{4}6 & \frac{3}{4}6 \\ -\frac{3}{4}6 & 1 - \frac{3}{4}6 \end{pmatrix} \\
&= 4^5 \begin{pmatrix} 4 + 18 & 18 \\ -18 & 4 - 18 \end{pmatrix} \\
&= 4^5 \begin{pmatrix} 22 & 18 \\ -18 & -14 \end{pmatrix} \\
&= \begin{pmatrix} 22528 & 18432 \\ -18432 & -14336 \end{pmatrix}
\end{aligned}
$$

2.9.13

(a)

$$\begin{aligned}
\mathbf{A} &= \begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix} \\
\det(\mathbf{A} - \lambda\mathbf{I}) &= (1-\lambda)(3-\lambda) - 3 \\
&= 3 - \lambda - 3\lambda + \lambda^2 - 3 \\
&= \lambda^2 - 4\lambda \\
&= \lambda(\lambda - 4)
\end{aligned}$$

$$\begin{aligned}
\lambda^m &= c_1\lambda \\
4^m &= 4c_1 \\
c_1 &= 4^{m-1}
\end{aligned}$$

$$\mathbf{A}^m = 4^{m-1}\mathbf{A}$$

2.9.15

$$\begin{aligned}
\mathbf{A} &= \begin{pmatrix} 2 & -4 \\ 1 & 3 \end{pmatrix} \\
\det(\mathbf{A} - \lambda\mathbf{I}) &= (2-\lambda)(3-\lambda) + 4 \\
&= 6 - 2\lambda - 3\lambda + \lambda^2 + 4 \\
&= \lambda^2 - 5\lambda + 10 \\
10\mathbf{I} &= 5\mathbf{A} - \mathbf{A}^2 \\
\mathbf{I} &= \frac{1}{2}\mathbf{A} - \frac{1}{10}\mathbf{A}^2 \\
\mathbf{A}^{-1} &= \frac{1}{2}\mathbf{I} - \frac{1}{10}\mathbf{A} \\
&= \begin{pmatrix} \frac{3}{10} & \frac{2}{5} \\ -\frac{1}{10} & \frac{1}{5} \end{pmatrix}
\end{aligned}$$

2.10 Orthogonal Matrices

2.10.5

Orthogonal

2.10.7

Orthogonal

2.10.9

Not orthogonal

2.10.11

$$\begin{aligned}
\mathbf{A} &= \begin{pmatrix} 1 & 9 \\ 9 & 1 \end{pmatrix} \\
\det(\mathbf{A} - \lambda \mathbf{I}) &= (1-\lambda)^2 - 81 \\
&= \lambda^2 - 2\lambda + 1 - 81 \\
&= \lambda^2 - 2\lambda - 80 \\
&= (\lambda - 10)(\lambda + 8)
\end{aligned}$$

$$\begin{pmatrix} 9 & 9 \\ 9 & 9 \end{pmatrix}$$

$$\mathbf{X}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -9 & 9 \\ 9 & -9 \end{pmatrix}$$

$$\mathbf{X}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

2.10.13

$$\begin{aligned}
\mathbf{A} &= \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix} \\
\det(\mathbf{A} - \lambda \mathbf{I}) &= (1-\lambda)(9-\lambda) - 9 \\
&= 9 - \lambda - 9\lambda + \lambda^2 - 9 \\
&= \lambda^2 - 10\lambda \\
&= \lambda(\lambda - 10)
\end{aligned}$$

$$\begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}$$

$$\mathbf{X}_1 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -9 & 3 \\ 3 & -1 \end{pmatrix}$$

$$\mathbf{X}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ -\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{pmatrix}$$

2.10.19

$$\begin{aligned} \frac{3}{5}a + \frac{4}{5}b &= 0 \\ 3a &= -4b \\ a &= -\frac{4}{3}b \\ a &= -\frac{4}{5} \\ b &= \frac{3}{5} \end{aligned}$$

2.10.21

(b) $\lambda_1 = -2, \lambda_2 = -2, \lambda_3 = 4$

(c)

$$
\begin{aligned}
\mathbf{W}_1 &= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \\
\mathbf{V}_2 &= \mathbf{K}_2 - (\mathbf{K}_2 \cdot \mathbf{W}_1)\mathbf{W}_1 \\
&= \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -1 \end{pmatrix} \\
|\mathbf{V}_2| &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (-1)^2} \\
&= \sqrt{\frac{3}{2}} \\
\mathbf{W}_2 &= \sqrt{\frac{2}{3}} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -1 \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\sqrt{\frac{2}{3}} \end{pmatrix}
\end{aligned}
$$

$$
\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}
$$