Advanced Engineering Mathematics Vectors, Matrices, and Vector Calculus by Dennis G. Zill Notes

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June 2023

Contents

1 Vectors

1.1 Vectors in 2-Space

- The zero vector can be assigned any direction
- The vectors $\mathbf{i}$ and $\mathbf{j}$ are known as the **standard basis vectors** for $\mathbb{R}^2$

1.2 Vectors in 3-Space

- In $\mathbb{R}^3$ the octant in which all coordinates are positive is known as the **first octant**. There is no agreement for naming the other seven octants.

1.3 Dot Product

- The **dot product** is also known as the **inner product** or the **scalar product** and is denoted $\mathbf{a} \cdot \mathbf{b}$
- Two non-zero vectors are orthogonal iff their dot product is 0
- The zero vector is considered orthogonal to all vectors
- The angles α, β, and γ between a vector and the unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, respectively are called the **direction angles** of the vector

- The cosines of a vectors direction angles (the **direction cosines**) can be calculated as

$$\begin{aligned}
\cos\alpha &= \frac{\mathbf{a}\cdot\mathbf{i}}{||\mathbf{a}||||\mathbf{i}||} \\
&= \frac{a_1}{||\mathbf{a}||} \\
\cos\beta &= \frac{\mathbf{a}\cdot\mathbf{j}}{||\mathbf{a}||||\mathbf{j}||} \\
&= \frac{a_2}{||\mathbf{a}||} \\
\cos\gamma &= \frac{\mathbf{a}\cdot\mathbf{k}}{||\mathbf{a}||||\mathbf{k}||} \\
&= \frac{a_3}{||\mathbf{a}||}
\end{aligned}$$

  Equivalently, these can be calculated as the components of the unit vector $\mathbf{a}/||\mathbf{a}||$.

- To find the component of a vector **a** in the direction of a vector **b**

$$\text{comp}_{\mathbf{b}}\mathbf{a} = ||\mathbf{a}||\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{b}||}$$

- To project a vector **a** onto a vector **b**

$$\text{proj}_{\mathbf{b}}\mathbf{a} = (\text{comp}_{\mathbf{b}}\mathbf{a})\frac{\mathbf{b}}{||\mathbf{b}||} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b}$$

1.4 Cross Product

- The cross product is only defined in $\mathbb{R}^3$
- The **scalar triple product** of vectors **a**, **b**, and **c** is defined as

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

- The area of a parallelogram with sides **a** and **b** is $||\mathbf{a}\times\mathbf{b}||$
- The area of a triangle with sides **a** and **b** is $\frac{1}{2}||\mathbf{a}\times\mathbf{b}||$
- The volume of a paralleleipied with sides **a**, **b**, and **c** is $|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$
- $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = 0$ iff **a**, **b**, and **c** are coplanar