University Physics with Modern Physics Electromagnetism Notes

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21 Electric Charge and Electric Field

21.1 Electric Charge

- Electrons have a much smaller mass than neutrons and protons
- Neutrons and protons have a very similar mass
- Electrons and protons have the same magnitude of charge
- The number of protons in an atom determins its **atomic number**
- If an electron is added to a neutral atom it becomes a **negative ion**, if one is removed it becomes a **positive ion** — this is called **ionisation**
- The **principle of conservation of charge** states that the algebraic sum of all the electric charges in any closed system is constant
- The electron or proton's magnitude of charge is a natural unit of charge — every observable amount of electric charge is an integer multiple of this

21.2 Conductors, Insulators, and Incuded Charges

- **Conductors** pemit easy movement of charge, **insulators** do not
- Holding a charged object near an uncharged object causes free electrons in the latter to move away/towards the former, resulting in a net charge on either side — this is called **induced charge**

21.3 Coulomb's Law

- The SI unit of charge is called one **coulomb** (1 C) and is defined such that $1.602176634 \times 10^{-19}$ C is equal to the charge of an electron or proton
- **Coulomb's law** describes the electric force between two point charges

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2}$$

  where the **electric constant** $\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$, q_1 and q_2 are the magnitudes of the charges, and r is the distance between them

- The electric force is always directed along the line between the two charges, attracting opposite charges and repelling like charges
- $\frac{1}{4\pi\epsilon_0}$ can be approximated as $9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$
- The principle of superposition of forces also applies to electric charges

21.4 Electric Field and Electric Forces

- The electric force on a charged object is exerted by the electric field created by other charged objects
- We can determine if there is an electric field at a point by placing a test charge q_0 there and seeing if it experiences an electric force — the electric field at that point (the electric force per unit charge) is then given by

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

- Rearranging, the force experienced by a charge q_0 at a point is given by

$$\mathbf{F} = q_0 \mathbf{E}$$

- When considering an electric field produced by a point charge, the location of the point charge is called the **source point** and the location at which we're trying to determine the field is called the **field point**

- The electric field produced by a point charge is given by

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

where q is the charge of the point charge, r is the distance between the source and field points, and $\hat{\mathbf{r}}$ is the unit vector from the source to the field point

- Unlike Coulomb's law this equation doesn't use the absolute value of q meaning that the electric fields of positive charges point away from the charge, while those of negative charges point towards them

- In electrostatics, the electric field inside the material of a conductor (but not holes within the material) is $\mathbf{0}$

21.5 Electric-Field Calculations

- The **principle of superposition of electric fields** states that the total electric field at a point P is the vector sum of the fields at P due to each point charge in the charge distribution

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots$$

- For a line charge distribution the **linear charge density** is represented by λ (the charge per unit length, measured in C/m)

- For a surface charge distribution the **surface charge density** is represented by σ (the charge per unit area, measured in $\mathrm{C/m^2}$)

- For a volume charge distribution the **volume charge density** is represented by ρ (the charge per unit volume, measured in $\mathrm{C/m^3}$)

- The electric field of an infinitely long line charge along the y-axis is

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

21.6 Electric Field Lines

- An **electric field line** is a line drawn through space such that its tangent at any point is in the direction of the electric field vector at that point

- Fewer lines are drawn in areas where the electric field is weak and more lines are drawn in areas where it's strong

21.7 Electric Dipoles

- An **electric dipole** is a pair of point charges of equal magnitude q and opposite sign separated by a distance d
- The net force on an electric dipole in a uniform electric field is $\mathbf{0}$
- The **electric dipole moment p** of an electric dipole is a vector directed from the negative charge to the positive charge with magnitude qd
- The net torque on an electric dipole in a uniform electric field is $\mathbf{p} \times \mathbf{E}$ or $qEd \sin \phi$ where ϕ is the angle between the electric dipole and the electric field
- The potential energy of an electric dipole in a uniform electric field is

$$U = -\mathbf{p} \cdot \mathbf{E}$$

22 Gauss's Law

22.1 Calculating Electric Flux

- The electric flux of a uniform electric field through a flat surface A is

$$\Phi_E = \mathbf{E} \cdot \mathbf{A}$$

  where $\mathbf{A}$ is normal to A and has a magnitude equal to its area

- The electric flux of a nonuniform electric field through a curved surface A is

$$\Phi_E = \int \mathbf{E} \cdot \mathbf{dA}$$

22.2 Gauss's Law

- Gauss's law states that the total electric flux through a closed surface is equal to the total electric charge enclosed by the surface divided by ϵ_0

$$\Phi_E = \oint \mathbf{E} \cdot \mathbf{dA} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

22.3 Applications of Gauss's Law

- Gauss's law can be used in two ways:
  - If we know the charge distribution and it has enough symmetry to let us evaluate the integral in Gauss's law, we can find the field
  - If we know the field, we can use Gauss's law to find the charge distribution

- Under electrostatics, excess charge always lies of the surface of a conductor
- The electric field of an infinite line charge is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}\hat{\mathbf{r}}$$