

University Physics with Modern Physics

Electromagnetism Problems

Chris Doble

December 2022

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21 Electric Charge and Electric Field

21.3 Coulomb's Law

21.3.1 Example 21.1

The magnitude of electric repulsion between two α particles is given by

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$$

and the magnitude of gravitational attraction is given by

$$F_g = \frac{Gm^2}{r^2}$$

. The ratio of the two values is

$$\begin{aligned} \frac{F_e}{F_g} &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \frac{r^2}{Gm^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{Gm^2} \\ &= 3.1 \times 10^{35} \end{aligned}$$

showing that the electric repulsion is significantly stronger than the gravitational attraction.

21.3.2 Example 21.2

a) The magnitude of the force that q_1 exerts on q_2 is

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \\ &= (9.0 \times 10^9) \frac{|(25 \times 10^{-9})(-75 \times 10^{-9})|}{0.030^2} \\ &= 1.9 \times 10^{-2} \text{ N.} \end{aligned}$$

Since q_1 and q_2 have opposite charge, the force is attractive (from q_2 to q_1).

b) The magnitude of the force that q_2 exerts on q_1 is the same as in part a, but the direction is reversed (from q_1 to q_2).

21.3.3 Example 21.3

By the principle of superposition of forces, the net force exerted on q_3 is equal to the vector sum of the forces exerted on it by q_1 and q_2 separately.

Both q_1 and q_3 have positive charge so they repel each other. q_1 is to the right of q_3 so q_3 experiences a force to the left of magnitude

$$\begin{aligned}
F_{1 \text{ on } 3} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_3|}{r^2} \\
&= (9.0 \times 10^9) \frac{|(1.0 \times 10^{-9})(5.0 \times 10^{-9})|}{0.020^2} \\
&= 1.1 \times 10^{-4} \text{ N.}
\end{aligned}$$

However q_2 has a negative charge so it attracts q_3 . It is also to the right of q_3 so q_3 experiences a force to the right of magnitude

$$\begin{aligned}
F_{2 \text{ on } 3} &= \frac{1}{4\pi\epsilon_0} \frac{|q_2 q_3|}{r^2} \\
&= (9.0 \times 10^9) \frac{|(-3.0 \times 10^{-9})(5.0 \times 10^{-9})|}{0.040^2} \\
&= 8.4 \times 10^{-5} \text{ N.}
\end{aligned}$$

The net force experienced by q_3 is therefore

$$\begin{aligned}
F &= -F_{1 \text{ on } 3} + F_{2 \text{ on } 3} \\
&= -1.1 \times 10^{-4} + 8.4 \times 10^{-5} \\
&= -2.6 \times 10^{-5} \text{ N.}
\end{aligned}$$

21.3.4 Example 21.4

Since q_1 and q_2 are of equal charge and are symmetric about the x axis on which Q lies, the vertical components of their forces cancel leaving only the horizontal.

The horizontal component of q_1 's force on Q is given by

$$\begin{aligned}
F_{1 \text{ on } Q, x} &= \frac{1}{4\pi\epsilon_0} \frac{q_1 Q}{r_{1,Q}^2} \cos \alpha \\
&= (9.0 \times 10^9) \frac{(2.0 \times 10^{-6})(4.0 \times 10^{-6})}{\sqrt{0.30^2 + 0.40^2}^2} \frac{0.40}{0.50} \\
&= 0.23 \text{ N.}
\end{aligned}$$

Again, since q_1 and q_2 are of equal charge and symmetric about the x axis, $F_{1 \text{ on } Q, x} = F_{2 \text{ on } Q, x}$ and the total force experienced by Q is in the positive x direction of magnitude

$$F = 2F_{1 \text{ on } Q, x} = 0.46 \text{ N.}$$

21.4 Electric Field and Electric Forces

21.4.1 Example 21.5

The magnitude of the electric field vector is given by

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \\ &= (9.0 \times 10^9) \frac{|4.0 \times 10^{-9}|}{2.0^2} \\ &= 9.0 \text{ N/C}. \end{aligned}$$

21.4.2 Example 21.6

The magnitude of the electric field vector is given by

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \\ &= (9.0 \times 10^9) \frac{|-8.0 \times 10^{-9}|}{1.2^2 + 1.6^2} \\ &= 18 \text{ N/C} \end{aligned}$$

and it is directed towards the origin. If θ is the angle between the positive x axis and $\hat{\mathbf{r}}$ then the component form of \mathbf{E} is

$$\begin{aligned} E &= -E (\cos \theta \hat{\mathbf{i}} + \sin \theta \hat{\mathbf{j}}) \\ &= -E \left(\frac{x}{r} \hat{\mathbf{i}} + \frac{-y}{r} \hat{\mathbf{j}} \right) \\ &= \frac{-18}{\sqrt{1.2^2 + 1.6^2}} (1.2\hat{\mathbf{i}} + 1.6\hat{\mathbf{j}}) \\ &= (-11 \text{ N/C})\hat{\mathbf{i}} - (14 \text{ N/C})\hat{\mathbf{j}}. \end{aligned}$$

21.4.3 Example 21.7

- a) Electrons have a negative charge and the electric field is directed upwards, so the electron will move downwards. The magnitude of its acceleration is

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{eE}{m} \\ &= \frac{(1.60 \times 10^{-19})(1.00 \times 10^4)}{9.11 \times 10^{-31}} \\ &= 1.76 \times 10^{15} \text{ m/s}^2. \end{aligned}$$

b) Its acceleration is constant between the plates, so its final speed is

$$\begin{aligned}
 v^2 &= v_0^2 + 2a(x - x_0) \\
 &= 2ax \\
 v &= \sqrt{2ax} \\
 &= \sqrt{2(1.76 \times 10^{15})(0.01)} \\
 &= 5.9 \times 10^6 \text{ m/s}
 \end{aligned}$$

and thus its final kinetic energy is

$$\begin{aligned}
 K &= \frac{1}{2}mv^2 \\
 &= \frac{1}{2}(9.11 \times 10^{-31})(5.9 \times 10^6)^2 \\
 &= 1.6 \times 10^{-17} \text{ J.}
 \end{aligned}$$

c) We can find the time it takes for the electron to travel this distance by rearranging the kinematic equation

$$v = v_0 + at$$

to

$$\begin{aligned}
 t &= \frac{v - v_0}{a} \\
 &= \frac{5.9 \times 10^6}{1.76 \times 10^{15}} \\
 &= 3.4 \times 10^{-9} \text{ s.}
 \end{aligned}$$

21.5 Electric-Field Calculations

21.5.1 Example 21.8

a) At point a the electric field caused by q_1 points to the right and has magnitude

$$\begin{aligned}
 E_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r^2} \\
 &= (9.0 \times 10^9) \frac{12 \times 10^{-9}}{(0.060)^2} \\
 &= 3.0 \times 10^4 \text{ N/C.}
 \end{aligned}$$

The electric field caused by q_2 also points to the right and it has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r^2} \\ &= (9.0 \times 10^9) \frac{|-12 \times 10^{-9}|}{(0.040)^2} \\ &= 6.8 \times 10^4 \text{ N/C}. \end{aligned}$$

Thus the total field points to the right and has magnitude

$$E = E_1 + E_2 = 9.8 \times 10^4 \text{ N/C}.$$

b) At point b the electric field caused by q_1 points to the left and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r^2} \\ &= (9.0 \times 10^9) \frac{12 \times 10^{-9}}{(0.040)^2} \\ &= 6.8 \times 10^4 \text{ N/C}. \end{aligned}$$

The electric field caused by q_2 points to the right and has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r^2} \\ &= (9.0 \times 10^9) \frac{|-12 \times 10^{-9}|}{(0.140)^2} \\ &= 0.55 \times 10^4 \text{ N/C}. \end{aligned}$$

Thus the total electric field points to the left and has magnitude

$$E = E_1 - E_2 = 6.3 \times 10^4 \text{ N/C}.$$

c) At point c the electric field caused by q_1 points from q_1 to c and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r^2} \\ &= (9.0 \times 10^9) \frac{12 \times 10^{-9}}{0.130^2} \\ &= 6.4 \times 10^3 \text{ N/C}. \end{aligned}$$

The electric field caused by q_2 points from c to q_2 and has magnitude

$$\begin{aligned}
 E_2 &= \frac{1}{4\pi\epsilon_0} \frac{|q_2|}{r^2} \\
 &= (9.0 \times 10^9) \frac{|-12 \times 10^{-9}|}{0.130^2} \\
 &= 6.4 \times 10^3 \text{ N/C} \\
 &= E_1.
 \end{aligned}$$

The vertical components of \mathbf{E}_1 and \mathbf{E}_2 cancel, leaving only a horizontal component pointing to the right of magnitude

$$\begin{aligned}
 E &= 2E_1 \cos \alpha \\
 &= 2(6.4 \times 10^3) \frac{0.050}{0.130} \\
 &= 4.9 \times 10^3 \text{ N/C}.
 \end{aligned}$$

21.5.2 Example 21.9

By symmetry, each point on the ring has a corresponding point on the opposite side. The components of their electric fields perpendicular to the axis of the ring cancel, leaving only a component parallel to the axis of the ring. Thus the total electric field at P is parallel to the axis of the ring and can be calculated as

$$\begin{aligned}
 E &= \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r^2} \cos \alpha \, d\theta \\
 &= \frac{1}{4\pi\epsilon_0} \frac{Qx}{2\pi(a^2 + x^2)^{3/2}} \int_0^{2\pi} d\theta \\
 &= \frac{1}{4\pi\epsilon_0} \frac{Qx}{(a^2 + x^2)^{3/2}}.
 \end{aligned}$$

21.5.3 Example 21.10

By symmetry, each point on the line has a corresponding point on the opposite side of the x -axis. The y components of their electric fields cancel, leaving only the x components. Thus the total electric field at P only has an x component and can be calculated as

$$\begin{aligned}
E &= \int_{-a}^a \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r^2} \cos \alpha \, dy \\
&= \frac{1}{4\pi\epsilon_0} \frac{Qx}{2a} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} \, dy \\
&= \frac{1}{4\pi\epsilon_0} \frac{Qx}{2a} \left[\frac{y}{x^2 \sqrt{x^2 + y^2}} \right]_{-a}^a \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{2ax} \left(\frac{a}{\sqrt{x^2 + a^2}} + \frac{a}{\sqrt{x^2 + (-a)^2}} \right) \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2 + a^2}}.
\end{aligned}$$

21.5.4 Example 21.11

By symmetry, each point on the disk has a corresponding point 180° rotation around the x -axis. The y and z components of their electric fields cancel, leaving only the x components. Thus the total magnetic field at P only has an x component and can be calculated as

$$\begin{aligned}
E &= \int_0^R \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{\sigma}{r^2} s \cos \alpha \, d\theta \, ds \\
&= \frac{\sigma}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{s}{s^2 + x^2} \frac{x}{\sqrt{s^2 + x^2}} \, d\theta \, ds \\
&= \frac{\sigma x}{2\epsilon_0} \int_0^R \frac{s}{(s^2 + x^2)^{3/2}} \, ds \\
&= \frac{\sigma x}{2\epsilon_0} \left[-\frac{1}{\sqrt{s^2 + x^2}} \right]_0^R \\
&= \frac{\sigma x}{2\epsilon_0} \left(-\frac{1}{\sqrt{R^2 + x^2}} + \frac{1}{x} \right) \\
&= \frac{\sigma}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{(R/x)^2 + 1}} \right).
\end{aligned}$$

21.5.5 Example 21.12

From Example 21.11 we know that the electric field produced by an infinite plane sheet of charge is

$$E = \frac{\sigma}{2\epsilon_0}.$$

Therefore the electric field outside the sheets is $\mathbf{0}$ and between the sheets is σ/ϵ_0 towards the negative sheet.

21.6 Electric Dipoles

21.6.1 Example 21.13

- a) The electric field is uniform so the net force exerted on the dipole is $\mathbf{0}$
- b) The electric dipole moment is directed from the negative charge to the positive charge and has magnitude

$$p = qd = (1.6 \times 10^{-19})(0.125 \times 10^{-9}) = 2.0 \times 10^{-29} \text{ C} \cdot \text{m}$$

- c) The torque aligns the electric dipole moment with the electric field so it is directed out of the page and has magnitude

$$\tau = qEd \sin \phi = (1.6 \times 10^{-19})(5.0 \times 10^5)(0.125 \times 10^{-9}) \sin 35 = 5.7 \times 10^{-24} \text{ N} \cdot \text{m}$$

- d) The potential energy of an electric dipole in a uniform electric field is given by

$$U = -qEd \cos \phi = (2.0 \times 10^{-29})(5.0 \times 10^5) \cos 35 = 8.2 \times 10^{-24} \text{ J}$$

21.6.2 Example 21.14

As P is on the y -axis, the electric fields of the electric dipole's point charges have no x component and thus the net electric field is directed along the y -axis.

By the principle of superposition of electric fields, the magnitude of the electric field at P is

$$\begin{aligned} E &= E_- + E_+ \\ &= \frac{1}{4\pi\epsilon_0} \frac{-q}{(y - (-d/2))^2} + \frac{1}{4\pi\epsilon_0} \frac{q}{(y - d/2)^2} \\ &= \frac{1}{4\pi\epsilon_0} q \left(\frac{1}{(y - d/2)^2} - \frac{1}{(y + d/2)^2} \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{y^2} \left(\left(1 - \frac{d}{2y}\right)^{-2} - \left(1 + \frac{d}{2y}\right)^{-2} \right) \\ &\approx \frac{1}{4\pi\epsilon_0} \frac{q}{y^2} \left(1 + \frac{d}{y} - 1 + \frac{d}{y} \right) \\ &= \frac{qd}{2\pi\epsilon_0 y^3} \\ &= \frac{p}{2\pi\epsilon_0 y^3}. \end{aligned}$$

21.7 Guided Practice

21.7.1 VP21.4.1

q_1 attracts q_3 to the left with magnitude

$$\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_3|}{r^2} \\
&= (9.0 \times 10^9) \frac{|(4.00 \times 10^{-9})(-2.00 \times 10^{-9})|}{0.0400^2} \\
&= 4.5 \times 10^{-5} \text{ N}.
\end{aligned}$$

q_2 repels q_3 to the left with magnitude

$$\begin{aligned}
F_2 &= \frac{1}{4\pi\epsilon_0} \frac{|q_2 q_3|}{r^2} \\
&= (9.0 \times 10^9) \frac{|(-1.20 \times 10^{-9})(-2.00 \times 10^{-9})|}{(0.0600 - 0.0400)^2} \\
&= 5.4 \times 10^{-5} \text{ N}.
\end{aligned}$$

By the principle of superposition of forces, the net force on q_3 is

$$\mathbf{F} = (-F_1 - F_2)\hat{\mathbf{i}} = (-9.9 \times 10^{-5} \text{ N})\hat{\mathbf{i}}.$$

21.7.2 VP21.4.2

a) q_1 repels q_2 in the positive x direction with magnitude

$$\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} \\
&= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{|(3.60 \times 10^{-9})(2.00 \times 10^{-9})|}{0.0400^2} \\
&= 40.4 \mu\text{N}.
\end{aligned}$$

b) By the superposition of forces

$$\begin{aligned}
F &= F_1 + F_2 \\
F_2 &= F - F_1 \\
&= 54.0 - 40.4 \\
&= 13.6 \mu\text{N}
\end{aligned}$$

in the positive x direction.

c) q_2 repels q_3 so it must also have a positive charge of magnitude

$$\begin{aligned}
F &= \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r^2} \\
q_2 &= \frac{4\pi\epsilon_0 F r^2}{q_3} \\
&= \frac{4\pi(8.854 \times 10^{-12})(1.36 \times 10^{-5})(0.0800)^2}{2.00 \times 10^{-9}} \\
&= 4.84 \times 10^{-9} \text{ C}.
\end{aligned}$$

21.7.3 VP21.4.3

By symmetry the x components of q_1 and q_2 's electric fields cancel leaving only their y components which are directed in the negative y direction and equal. q_3 is negative and thus experiences a net force in the positive y direction of magnitude

$$\begin{aligned}
F &= 2 \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r^2} \sin \alpha \\
&= 2 \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{(6.00 \times 10^{-9})(2.50 \times 10^{-9})}{0.150^2 + 0.200^2} \frac{0.200}{\sqrt{0.150^2 + 0.200^2}} \\
&= 3.45 \times 10^{-6} \text{ N}.
\end{aligned}$$

21.7.4 VP21.4.4

The magnitude of the electric force exerted by q_1 on q_3 is

$$\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_3|}{r^2} \\
&= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{|(4.00 \times 10^{-9})(-1.50 \times 10^{-9})|}{0.250^2 + 0.200^2} \\
&= 5.26 \times 10^{-7} \text{ N}.
\end{aligned}$$

They have opposite charges so the force is directed from q_3 to q_1 . In component form the force is

$$\begin{aligned}
\mathbf{F}_1 &= -F_1 \cos \alpha \hat{\mathbf{i}} + F_1 \sin \alpha \hat{\mathbf{j}} \\
&= F_1 \left(-\frac{x}{r} \hat{\mathbf{i}} + \frac{y}{r} \hat{\mathbf{j}} \right) \\
&= \frac{5.26 \times 10^{-7}}{\sqrt{0.250^2 + 0.200^2}} \left(-0.250 \hat{\mathbf{i}} + 0.200 \hat{\mathbf{j}} \right) \\
&= (-4.11 \times 10^{-7} \text{ N}) \hat{\mathbf{i}} + (3.29 \times 10^{-7} \text{ N}) \hat{\mathbf{j}}.
\end{aligned}$$

The magnitude of the electric force exerted by q_2 on q_3 is

$$\begin{aligned} F_2 &= \frac{1}{4\pi\epsilon_0} \frac{|q_2 q_3|}{r^2} \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{|(-4.00 \times 10^{-9})(-1.50 \times 10^{-9})|}{0.250^2} \\ &= 8.63 \times 10^{-7} \text{ N}. \end{aligned}$$

The have like charges so the force is directed from q_2 to q_3 , i.e. along the positive x -axis. In component form the force is

$$\mathbf{F}_2 = (8.64 \times 10^{-7} \text{ N})\hat{\mathbf{i}}.$$

Thus the net force experienced by q_3 is

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= (4.53 \times 10^{-7} \text{ N})\hat{\mathbf{i}} + (3.29 \times 10^{-7} \text{ N})\hat{\mathbf{j}}. \end{aligned}$$

21.7.5 VP21.10.1

- a) The source points and field point all lie on the y -axis, so the source points' electric fields have no x components. q_1 is positive and the field point is below it, so its contribution is negative. q_2 is negative and the field point is above it, so its contribution is also negative. Thus the y component of the net electric field is

$$\begin{aligned} E_y &= \frac{1}{4\pi\epsilon_0} \left(-\frac{q_1}{(y_1 - y)^2} + \frac{q_2}{(y_2 - y)^2} \right) \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \left(\frac{4.00 \times 10^{-9}}{(0.200 - 0.100)^2} - \frac{5.00 \times 10^{-9}}{(0 - 0.100)^2} \right) \\ &= -8.09 \times 10^3 \text{ N/C}. \end{aligned}$$

- b) The source points and field point all lie on the y -axis, so the source points' electric fields have no x components. q_1 is positive and the field point is above it, so its contribution is positive. q_2 is negative and the field point is above it, so its contribution is also negative. Thus the y component of the net electric field is

$$\begin{aligned} E_y &= \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{(y_1 - y)^2} + \frac{q_2}{(y_2 - y)^2} \right) \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \left(\frac{4.00 \times 10^{-9}}{(0.200 - 0.400)^2} - \frac{5.00 \times 10^{-9}}{(0 - 0.400)^2} \right) \\ &= 618 \text{ N/C}. \end{aligned}$$

c) The electric field of q_1 has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{4.00 \times 10^{-9}}{0.200^2 + 0.200^2} \\ &= 449 \text{ N/C}. \end{aligned}$$

It is directed from q_1 to the field point and thus in component form is

$$\begin{aligned} \mathbf{E}_1 &= E_1(\cos \phi \hat{\mathbf{i}} - \sin \phi \hat{\mathbf{j}}) \\ &= \frac{449}{\sqrt{0.200^2 + 0.200^2}}(0.200\hat{\mathbf{i}} - 0.200\hat{\mathbf{j}}) \\ &= (317 \text{ N/C})\hat{\mathbf{i}} - (317 \text{ N/C})\hat{\mathbf{j}}. \end{aligned}$$

q_2 and the field point both lie on the x -axis, and thus its electric field has no y component. In component form it is

$$\begin{aligned} \mathbf{E}_2 &= \frac{1}{4\pi\epsilon_0} \frac{q_2}{r^2} \hat{\mathbf{i}} \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{-5.00 \times 10^{-9}}{0.200^2} \hat{\mathbf{i}} \\ &= (-1.12 \times 10^3 \text{ N/C})\hat{\mathbf{i}}. \end{aligned}$$

The total electric field is thus

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 \\ &= (-8.03 \times 10^2 \text{ N/C})\hat{\mathbf{i}} + (-3.17 \times 10^2 \text{ N/C})\hat{\mathbf{j}}. \end{aligned}$$

21.7.6 VP21.10.2

a) Both source points and the field point are on the x -axis, so the electric fields at P have no y components.

q_1 is positive and P is to the right of q_1 , so its electric field points to the right and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \\ &= \frac{1}{4\pi(8.854 \times 10^{-12})} \frac{1.80 \times 10^{-9}}{0.0200^2} \\ &= 4.04 \times 10^4 \text{ N/C}. \end{aligned}$$

- b) The magnitude and direction of the electric field that q_2 causes at P can be calculated as

$$\begin{aligned} E &= E_1 + E_2 \\ E_2 &= E - E_1 \\ &= 6.75 \times 10^4 - 4.04 \times 10^4 \\ &= 2.71 \times 10^4 \text{ N/C.} \end{aligned}$$

- c) E_2 is positive at P , so q_2 must be negative. Its value is

$$\begin{aligned} E_2 &= -\frac{1}{4\pi\epsilon_0} \frac{q_2}{r^2} \\ q_2 &= -4\pi\epsilon_0 E_2 r^2 \\ &= -4\pi(8.854 \times 10^{-12})(2.71 \times 10^4)(0.0200)^2 \\ &= -1.21 \times 10^{-9} \text{ C.} \end{aligned}$$

21.7.7 21.10.3

- a) From Example 21.9 we know that the electric field of a charged ring of radius a at a distance x along the ring's axis is directed away from the ring along its axis and has magnitude

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}}.$$

By the principle of superposition of electric fields, the electric field of the hydrogen atom is

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \left(\frac{e}{a^2} - \frac{ea}{(a^2 + a^2)^{3/2}} \right) \\ &= \frac{1}{4\pi\epsilon_0} e \left(\frac{1}{a^2} - \frac{a}{2\sqrt{2}a^3} \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{e}{a^2} \left(1 - \frac{1}{2\sqrt{2}} \right). \end{aligned}$$

- b) $1 - 1/(2\sqrt{2}) \approx 0.65$ so the field points away from the proton.

21.7.8 21.10.4

- a) The charge per unit length is

$$\lambda = \frac{Q}{L}$$

so the charge contained in a segment of length dx is

$$\lambda dx = \frac{Q}{L} dx.$$

- b) The field and source points both lie on the x -axis, so the differential electric field has no y component. The x component is

$$dE_x = -\frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{x^2} = -\frac{1}{4\pi\epsilon_0} \frac{Q}{Lx^2} dx.$$

- c) The total electric field at the origin is

$$\begin{aligned} E &= \int dE_x \\ &= \int_L^{2L} -\frac{1}{4\pi\epsilon_0} \frac{Q}{Lx^2} dx \\ &= -\frac{1}{4\pi\epsilon_0} \frac{Q}{L} \left[-\frac{1}{x} \right]_L^{2L} \\ &= -\frac{1}{4\pi\epsilon_0} \frac{Q}{L} \left(-\frac{1}{2L} + \frac{1}{L} \right) \\ &= -\frac{1}{4\pi\epsilon_0} \frac{Q}{2L^2}. \end{aligned}$$