

Introduction to Electrodynamics by David J.
Griffiths Problems

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Contents

2	Electrostatics	2
2.1	2
2.2	2
2.3	2
2.4	2
2.5	3
2.6	3
2.7	3
2.8	3
2.9	4
2.10	4
2.11	4
2.12	5
2.13	5
2.14	5
2.15	5
2.16	6
2.17	6
2.18	7
2.20	7
2.21	8
2.22	8
2.23	9
2.24	9
2.25	9
2.26	10
2.28	10
2.31	11
2.32	11
2.33	12

2.34	12
2.36	14
2.37	16

2 Electrostatics

2.1

- (a) **0**
- (b) The same as if only the opposite charge were present — all others are cancelled out.

2.2

$$\begin{aligned}\mathbf{E} &= \frac{1}{4\pi\epsilon_0} 2 \frac{q}{z^2} \cos\theta \hat{\mathbf{x}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{dq}{[(d/2)^2 + z^2]^{3/2}} \hat{\mathbf{x}}\end{aligned}$$

2.3

$$\begin{aligned}\mathbf{r} &= z\hat{\mathbf{z}} \\ \mathbf{r}' &= x\hat{\mathbf{x}} \\ \boldsymbol{r} &= z\hat{\mathbf{z}} - x\hat{\mathbf{x}} \\ r &= \sqrt{x^2 + z^2} \\ \hat{\boldsymbol{r}} &= \frac{z\hat{\mathbf{z}} - x\hat{\mathbf{x}}}{\sqrt{x^2 + z^2}} \\ \mathbf{E} &= \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda}{x^2 + z^2} \frac{z\hat{\mathbf{z}} - x\hat{\mathbf{x}}}{\sqrt{x^2 + z^2}} dx \\ &= \frac{1}{4\pi\epsilon_0} \lambda \left(z\hat{\mathbf{z}} \int_0^L \frac{1}{(x^2 + z^2)^{3/2}} dx - \hat{\mathbf{x}} \int_0^L \frac{x}{(x^2 + z^2)} dx \right) \\ &= \frac{1}{4\pi\epsilon_0} \lambda \left[\frac{L}{z\sqrt{L^2 + z^2}} \hat{\mathbf{z}} - \left(\frac{1}{z} - \frac{1}{\sqrt{L^2 + z^2}} \right) \hat{\mathbf{x}} \right] \\ &= \frac{1}{4\pi\epsilon_0} \frac{\lambda}{z} \left[\left(-1 + \frac{z}{\sqrt{L^2 + z^2}} \right) \hat{\mathbf{x}} + \frac{L}{\sqrt{L^2 + z^2}} \hat{\mathbf{z}} \right]\end{aligned}$$

2.4

The electric field a distance z above the midpoint of a line segment of length $2L$ and uniform line charge λ is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z\sqrt{z^2 + L^2}} \hat{\mathbf{z}}.$$

Applying this to the four sides of the square, the horizontal components of opposite sides cancel leaving only the vertical component.

$$\begin{aligned}
\cos \theta &= \frac{z}{\hat{z}} \\
&= \frac{z}{\sqrt{(a/2)^2 + z^2}} \\
\mathbf{E} &= 4 \left(\frac{1}{4\pi\epsilon_0} \frac{\lambda a}{\sqrt{(a/2)^2 + z^2} \sqrt{(a/2)^2 + (a/2)^2 + z^2}} \hat{\mathbf{z}} \right) \cos \theta \\
&= \frac{1}{4\pi\epsilon_0} \frac{4a\lambda z}{[(a/2)^2 + z^2] \sqrt{(a/2)^2 + z^2}} \hat{\mathbf{z}}
\end{aligned}$$

2.5

$$\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda r}{r^2 + z^2} \cos \alpha \, d\theta \, \hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0} \frac{2\pi\lambda r z}{(r^2 + z^2)^{3/2}} \hat{\mathbf{z}}
\end{aligned}$$

2.6

$$\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0} \int \frac{dq}{\hat{z}^2} \cos \theta \hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^R \frac{\sigma}{r^2 + z^2} \frac{z}{\sqrt{r^2 + z^2}} r \, dr \, d\theta \hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0} 2\pi\sigma z \int_0^R \frac{r}{(r^2 + z^2)^{3/2}} \, dr \hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0} 2\pi\sigma z \left(\frac{1}{z} - \frac{1}{\sqrt{R^2 + z^2}} \right) \hat{\mathbf{z}}
\end{aligned}$$

When $R \rightarrow \infty$

$$\mathbf{E} = \frac{\sigma}{2\epsilon_0} \hat{\mathbf{z}}.$$

2.7

$$\mathbf{E} = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{q}{z^2} \hat{\mathbf{z}} & z > R \\ \mathbf{0} & z < R \end{cases}$$

2.8

$$\mathbf{E} = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{q}{z^2} \hat{\mathbf{z}} & z > R \\ \frac{1}{4\pi\epsilon_0} \frac{qz}{R^3} \hat{\mathbf{z}} & z < R \end{cases}$$

2.9

(a)

$$\begin{aligned}\rho &= \epsilon_0 \nabla \cdot \mathbf{E} \\ &= \epsilon_0 \frac{1}{r^2} \frac{\partial}{\partial r} (kr^5) \\ &= 5\epsilon_0 kr^2\end{aligned}$$

(b)

$$\begin{aligned}Q_{\text{enc}} &= \epsilon_0 \oint \mathbf{E} \cdot d\mathbf{a} \\ &= \epsilon_0 \int_0^{2\pi} \int_0^\pi kR^3 R d\theta R \sin \theta d\phi \\ &= 2\pi\epsilon_0 kR^5 [-\cos \theta]_0^\pi \\ &= 4\pi\epsilon_0 kR^5 \\ Q_{\text{enc}} &= \int_V \rho d\tau \\ &= \int_0^{2\pi} \int_0^\pi \int_0^R 5\epsilon_0 kr^2 dr r d\theta r \sin \theta d\phi \\ &= 10\pi\epsilon_0 k \int_0^\pi \int_0^R r^4 \sin \theta dr d\theta \\ &= 2\pi\epsilon_0 kR^5 [-\cos \theta]_0^\pi \\ &= 4\pi\epsilon_0 kR^5\end{aligned}$$

2.10

If the charge was surrounded by 8 such cubes the total flux through all the cubes would be q/ϵ_0 . There are 24 outside faces to the larger cube, so the total flux through the shaded face is $q/(24\epsilon_0)$.

2.11

$$\begin{aligned}\int \mathbf{E}_{\text{inside}} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\ &= 0 \\ \mathbf{E}_{\text{inside}} &= \mathbf{0} \\ \int \mathbf{E}_{\text{outside}} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\ 4\pi r^2 E_{\text{outside}} &= \frac{4\pi R^2 \sigma}{\epsilon_0} \\ \mathbf{E}_{\text{outside}} &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}\end{aligned}$$

2.12

$$\begin{aligned}\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\ 4\pi r^2 E &= \frac{\frac{4}{3}\pi r^3 \rho}{\epsilon_0} \\ \mathbf{E} &= \frac{r\rho}{3\epsilon_0} \hat{\mathbf{r}}\end{aligned}$$

2.13

$$\begin{aligned}\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\ 2\pi s l E &= \frac{l\lambda}{\epsilon_0} \\ \mathbf{E} &= \frac{1}{2\pi\epsilon_0} \frac{\lambda}{s} \hat{\mathbf{s}}\end{aligned}$$

2.14

$$\begin{aligned}Q_{\text{enc}} &= \int_V \rho \, d\tau \\ &= \int_0^{2\pi} \int_0^\pi \int_0^r k r'^3 \sin \theta \, dr' \, d\theta \, d\phi \\ &= 2\pi k \int_0^\pi \left[\frac{1}{4} r'^4 \sin \theta \right]_0^r d\theta \\ &= \frac{1}{2} \pi k r^4 [-\cos \theta]_0^\pi \\ &= \pi k r^4 \\ \int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\ 4\pi r^2 E &= \frac{\pi k r^4}{\epsilon_0} \\ \mathbf{E} &= \frac{k r^2}{4\epsilon_0} \hat{\mathbf{r}}\end{aligned}$$

2.15

(a) $\mathbf{E} = \mathbf{0}$

(b)

$$\begin{aligned}
 Q_{\text{enc}} &= \int_0^{2\pi} \int_0^\pi \int_a^r k \sin \theta \, dr' \, d\theta \, d\phi \\
 &= 4\pi k(r-a) \\
 4\pi r^2 E &= \frac{4\pi k(r-a)}{\epsilon_0} \\
 \mathbf{E} &= \frac{k(r-a)}{\epsilon_0 r^2} \hat{\mathbf{r}}
 \end{aligned}$$

(c) $\mathbf{E} = \frac{k(b-a)}{\epsilon_0 r^2} \hat{\mathbf{r}}$

2.16

(a)

$$\begin{aligned}
 Q_{\text{enc}} &= \pi s^2 l \rho \\
 2\pi s l E &= \frac{\pi s^2 l \rho}{\epsilon_0} \\
 \mathbf{E} &= \frac{s\rho}{2\epsilon_0} \hat{\mathbf{s}}
 \end{aligned}$$

(b)

$$\mathbf{E} = \frac{a^2 \rho}{2\epsilon_0 s} \hat{\mathbf{s}}$$

(c)

$$\mathbf{E} = \mathbf{0}$$

2.17

$$\begin{aligned}
 2AE_{\text{inside}} &= \frac{2Ay\rho}{\epsilon_0} \\
 \mathbf{E}_{\text{inside}} &= \frac{y\rho}{\epsilon_0} \\
 \mathbf{E} &= \begin{cases} \frac{d\rho}{\epsilon_0} & d < y \\ \frac{y\rho}{\epsilon_0} & 0 < y < d \\ -\frac{y\rho}{\epsilon_0} & -d < y < 0 \\ -\frac{d\rho}{\epsilon_0} & y < -d \end{cases}
 \end{aligned}$$

2.18

The electric field inside a uniformly charged solid sphere is

$$\mathbf{E} = \frac{r\rho}{3\epsilon_0} \hat{\mathbf{r}}.$$

$$\begin{aligned}\mathbf{d} &= \mathbf{r}_1 - \mathbf{r}_2 \\ \mathbf{E} &= \frac{r_1\rho}{3\epsilon_0} \hat{\mathbf{r}}_1 - \frac{r_2\rho}{3\epsilon_0} \hat{\mathbf{r}}_2 \\ &= \frac{\rho}{3\epsilon_0} (\mathbf{r}_1 - \mathbf{r}_2) \\ &= \frac{\rho}{3\epsilon_0} \mathbf{d}\end{aligned}$$

2.20

a is impossible because its curl is nonzero.

$$\begin{aligned}V &= - \int_0^y 2kxy' dy' - \int_0^z 2kyz' dz \\ &= -2kx \left[\frac{1}{2}y'^2 \right]_0^y - 2ky \left[\frac{1}{2}z'^2 \right]_0^z \\ &= -k(xy^2 + yz^2) \\ -\nabla V &= k[y^2 \hat{\mathbf{x}} + (2xy + z^2) \hat{\mathbf{y}} + 2yz \hat{\mathbf{z}}] \\ &= \mathbf{E}\end{aligned}$$

2.21

$$\begin{aligned}
\mathbf{E} &= \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} & r > R \\ \frac{1}{4\pi\epsilon_0} \frac{qr}{R^3} & r < R \end{cases} \\
V_{\text{outside}}(r) &= - \int_{\infty}^r \frac{1}{4\pi\epsilon_0} \frac{q}{r'^2} dr' \\
&= - \frac{1}{4\pi\epsilon_0} q \left[-\frac{1}{r'} \right]_{\infty}^r \\
&= \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\
-\nabla V_{\text{outside}} &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}} \\
&= \mathbf{E}_{\text{outside}} \\
V_{\text{inside}}(r) &= - \left(\int_{\infty}^R \frac{1}{4\pi\epsilon_0} \frac{q}{r'^2} dr' + \int_R^r \frac{1}{4\pi\epsilon_0} \frac{qr'}{R^3} dr' \right) \\
&= - \left(-\frac{1}{4\pi\epsilon_0} \frac{q}{R} + \frac{1}{4\pi\epsilon_0} \frac{q}{R^3} \left[\frac{1}{2} r'^2 \right]_R^r \right) \\
&= \frac{1}{4\pi\epsilon_0} \frac{q}{2R} \left[3 - \left(\frac{r}{R} \right)^2 \right] \\
-\nabla V_{\text{inside}} &= \frac{1}{4\pi\epsilon_0} \frac{qr}{R^3} \hat{\mathbf{r}} \\
&= \mathbf{E}_{\text{inside}}
\end{aligned}$$

2.22

$$\begin{aligned}
\mathbf{E} &= \frac{1}{2\pi\epsilon_0} \frac{\lambda}{s} \hat{\mathbf{s}} \\
V &= - \int_O^s \frac{1}{2\pi\epsilon_0} \frac{\lambda}{s'} ds' \\
&= - \frac{1}{2\pi\epsilon_0} \lambda \ln \frac{s}{O} \\
-\nabla V &= \frac{1}{2\pi\epsilon_0} \frac{\lambda}{s} \hat{\mathbf{s}}
\end{aligned}$$

2.23

$$\begin{aligned}
\mathbf{E} &= \begin{cases} \mathbf{0} & r < a \\ \frac{k(r-a)}{\epsilon_0 r^2} \hat{\mathbf{r}} & a < r < b \\ \frac{k(b-a)}{\epsilon_0 r^2} \hat{\mathbf{r}} & b < r \end{cases} \\
V(0) &= - \int_{\infty}^0 E dr \\
&= - \left(\int_{\infty}^b \frac{k(b-a)}{\epsilon_0 r^2} dr + \int_b^a \frac{k(r-a)}{\epsilon_0 r^2} dr \right) \\
&= - \left(\frac{k(b-a)}{\epsilon_0} \left[-\frac{1}{r} \right]_{\infty}^b + \frac{k}{\epsilon_0} \left[\ln r + \frac{a}{r} \right]_b^a \right) \\
&= - \left[-\frac{k(b-a)}{\epsilon_0 b} + \frac{k}{\epsilon_0} \left(\ln a + 1 - \ln b - \frac{a}{b} \right) \right] \\
&= -\frac{k}{\epsilon_0} \left(-1 + \frac{a}{b} + \ln \frac{a}{b} + 1 - \frac{a}{b} \right) \\
&= \frac{k}{\epsilon_0} \ln \frac{b}{a}
\end{aligned}$$

2.24

$$\begin{aligned}
V(b) - V(0) &= - \int_0^b E dr \\
&= - \left(\int_0^a \frac{s\rho}{2\epsilon_0} ds + \int_a^b \frac{a^2\rho}{2\epsilon_0 s} ds \right) \\
&= - \left(\frac{\rho}{2\epsilon_0} \left[\frac{1}{2} s^2 \right]_0^a + \frac{a^2\rho}{2\epsilon_0} \ln \frac{b}{a} \right) \\
&= - \left(\frac{a^2\rho}{4\epsilon_0} + \frac{a^2\rho}{2\epsilon_0} \ln \frac{b}{a} \right) \\
&= -\frac{a^2\rho}{4\epsilon_0} \left(1 + 2 \ln \frac{a}{b} \right)
\end{aligned}$$

2.25

(a)

$$V = \frac{1}{4\pi\epsilon_0} \frac{2q}{\sqrt{(d/2)^2 + z^2}}$$

(b)

$$\begin{aligned} V &= \frac{1}{4\pi\epsilon_0} \int_{-L}^L \frac{\lambda}{\sqrt{x^2 + z^2}} dx \\ &= \frac{1}{4\pi\epsilon_0} \lambda \ln \left(1 + \frac{2L(L + \sqrt{L^2 + z^2})}{z^2} \right) \end{aligned}$$

(c)

$$\begin{aligned} V &= \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^R \frac{\sigma}{\sqrt{r^2 + z^2}} r dr d\theta \\ &= \frac{1}{4\pi\epsilon_0} 2\pi\sigma(\sqrt{R^2 + z^2} - z) \end{aligned}$$

2.26

$$\begin{aligned} V_{\text{bottom}} &= \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^h \frac{\sqrt{2}\sigma z}{\sqrt{2}z} d\phi dz \\ &= \frac{\sigma h}{2\epsilon_0} \\ V_{\text{top}} &= \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^h \frac{\sqrt{2}\sigma z}{\sqrt{z^2 + (h-z)^2}} d\phi dz \\ &= \frac{\sqrt{2}\sigma}{2\epsilon_0} \int_0^h \frac{z}{\sqrt{z^2 + (h-z)^2}} dz \\ &= \frac{\sigma h}{4\epsilon_0} \ln(3 + 2\sqrt{2}) \\ V_{\text{bottom}} - V_{\text{top}} &= \frac{\sigma h}{2\epsilon_0} \left[1 - \frac{1}{2} \ln(3 + 2\sqrt{2}) \right] \end{aligned}$$

2.28

$$\begin{aligned} V(r) &= \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^\pi \int_0^R \frac{\rho r'^2 \sin \theta}{\sqrt{r^2 + r'^2 - 2rr' \cos \theta}} dr' d\theta d\phi \\ &= \frac{\rho}{2\epsilon_0} \int_0^\pi \int_0^R \frac{r'^2 \sin \theta}{\sqrt{r^2 + r'^2 - 2rr' \cos \theta}} dr' d\theta \\ &= \frac{\rho}{2\epsilon_0} \left(R^2 - \frac{r^2}{3} \right) \\ &= \frac{q}{8\pi\epsilon_0 R} \left(3 - \frac{r^2}{R^2} \right) \end{aligned}$$

2.31

(a)

$$W = \frac{q^2}{4\pi\epsilon_0 a} \left(\frac{1}{\sqrt{2}} - 2 \right)$$

(b)

$$\begin{aligned} W &= \frac{1}{4\pi\epsilon_0} \left(-\frac{q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a} \right) \\ &= \frac{q^2}{2\pi\epsilon_0 a} \left(\frac{1}{\sqrt{2}} - 2 \right) \end{aligned}$$

2.32

$$W = \frac{1}{4\pi\epsilon_0} \frac{q_A q_B}{a}$$

$$W = K_1 + K_2$$

$$\frac{1}{4\pi\epsilon_0} \frac{q_A q_B}{a} = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$\frac{1}{2\pi\epsilon_0} \frac{q_A q_B}{a} = m_A v_A^2 + m_B v_B^2$$

$$0 = m_B v_B - m_A v_A$$

$$v_B = \frac{m_A}{m_B} v_A$$

$$\frac{1}{2\pi\epsilon_0} \frac{q_A q_B}{a} = m_A v_A^2 + m_B \left(\frac{m_A}{m_B} v_A \right)^2$$

$$= m_A v_A^2 + \frac{m_A^2}{m_B} v_A^2$$

$$= \frac{m_A(m_A + m_B)}{m_B} v_A^2$$

$$v_A = \sqrt{\frac{1}{2\pi\epsilon_0} \frac{q_A q_B}{(m_A + m_B)a} \frac{m_B}{m_A}}$$

$$v_B = \sqrt{\frac{1}{2\pi\epsilon_0} \frac{q_A q_B}{(m_A + m_B)a} \frac{m_A}{m_B}}$$

2.33

$$\begin{aligned}
 W &= \frac{1}{4\pi\epsilon_0} \left(-\frac{q^2}{a} + \frac{q^2}{2a} - \frac{q^2}{3a} + \dots \right) \\
 &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{a} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \\
 &= -\frac{1}{4\pi\epsilon_0} \frac{q^2}{a} \ln 2
 \end{aligned}$$

2.34

(a)

$$\begin{aligned}
 V &= \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{q}{2R} \left[3 - \left(\frac{r}{R} \right)^2 \right] & r < R \\ \frac{1}{4\pi\epsilon_0} \frac{q}{r} & r > R \end{cases} \\
 W &= \frac{1}{2} \int \rho V d\tau \\
 &= \frac{1}{2} \int_0^{2\pi} \int_0^\pi \int_0^R \rho \frac{1}{4\pi\epsilon_0} \frac{q}{2R} \left[3 - \left(\frac{r}{R} \right)^2 \right] r^2 \sin \theta dr d\theta d\phi \\
 &= \frac{q\rho}{8\epsilon_0 R} \int_0^\pi \int_0^R \left[3 - \left(\frac{r}{R} \right)^2 \right] r^2 \sin \theta dr d\theta \\
 &= \frac{q\rho R^2}{5\epsilon_0} \\
 &= \frac{qR^2}{5\epsilon_0} \frac{q}{\frac{4}{3}\pi R^3} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{3q^2}{5R}
 \end{aligned}$$

(b)

$$\begin{aligned}
\mathbf{E} &= \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}} & r > R \\ \frac{1}{4\pi\epsilon_0} \frac{qr}{R^3} \hat{\mathbf{r}} & r < R \end{cases} \\
E^2 &= \begin{cases} \frac{1}{16\pi^2\epsilon_0^2} \frac{q^2}{r^4} & r > R \\ \frac{1}{16\pi^2\epsilon_0^2} \frac{q^2 r^2}{R^6} & r < R \end{cases} \\
W &= \frac{\epsilon_0}{2} \int E^2 d\tau \\
&= \frac{\epsilon_0}{2} \left(\int_0^{2\pi} \int_0^\pi \int_0^R \frac{1}{16\pi^2\epsilon_0^2} \frac{q^2 r^2}{R^6} r^2 \sin\theta dr d\theta d\phi \right. \\
&\quad \left. + \int_0^{2\pi} \int_0^\pi \int_R^\infty \frac{1}{16\pi^2\epsilon_0^2} \frac{q^2}{r^4} r^2 \sin\theta dr d\theta d\phi \right) \\
&= \frac{\epsilon_0}{2} \frac{1}{16\pi^2\epsilon_0^2} 2\pi q^2 \left(\int_0^\pi \int_0^R \frac{r^4}{R^6} \sin\theta dr d\theta + \int_0^\pi \int_R^\infty \frac{1}{r^2} \sin\theta dr d\theta \right) \\
&= \frac{1}{16\pi\epsilon_0} q^2 \left(\int_0^\pi \int_0^R \frac{r^4}{R^6} \sin\theta dr d\theta + \int_0^\pi \int_R^\infty \frac{1}{r^2} \sin\theta dr d\theta \right) \\
&= \frac{1}{16\pi\epsilon_0} q^2 \left(\frac{2}{5R} + \frac{2}{R} \right) \\
&= \frac{1}{4\pi\epsilon_0} \frac{3q^2}{5R}
\end{aligned}$$

(c)

$$\begin{aligned}
W &= \frac{\epsilon_0}{2} \left(\int_V E^2 d\tau + \oint_S V \mathbf{E} \cdot d\mathbf{a} \right) \\
&= \frac{\epsilon_0}{2} \left(\int_0^{2\pi} \int_0^\pi \int_0^R \frac{1}{(4\pi\epsilon_0)^2} \frac{q^2 r^2}{R^6} r^2 \sin \theta dr d\theta d\phi \right. \\
&\quad + \int_0^{2\pi} \int_0^\pi \int_R^a \frac{1}{(4\pi\epsilon_0)^2} \frac{q^2}{r^4} r^2 \sin \theta dr d\theta d\phi \\
&\quad \left. + \int_0^{2\pi} \int_0^\pi \frac{1}{4\pi\epsilon_0} \frac{q}{a} \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} a^2 \sin \theta d\theta d\phi \right) \\
&= \frac{\epsilon_0}{2} \frac{1}{(4\pi\epsilon_0)^2} 2\pi q^2 \left(\int_0^\pi \int_0^R \frac{r^4}{R^6} \sin \theta dr d\theta \right. \\
&\quad \left. + \int_0^\pi \int_R^a \frac{1}{r^2} \sin \theta dr d\theta + \int_0^\pi \frac{1}{a} \sin \theta d\theta \right) \\
&= \frac{\epsilon_0}{2} \frac{1}{(4\pi\epsilon_0)^2} 2\pi q^2 \left[\frac{2}{5R} + 2 \left(\frac{1}{R} - \frac{1}{a} \right) + \frac{2}{a} \right] \\
&= \frac{1}{8\pi\epsilon_0} q^2 \left[\frac{1}{5R} + \frac{1}{R} \right] \\
&= \frac{1}{4\pi\epsilon_0} \frac{3q^2}{5R}
\end{aligned}$$

2.36

(a)

$$\begin{aligned}
\mathbf{E} &= \begin{cases} \mathbf{0} & r < a \\ \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}} & a < r < b \\ \mathbf{0} & b < r \end{cases} \\
E^2 &= \begin{cases} 0 & r < a \\ \frac{1}{(4\pi\epsilon_0)^2} \frac{q^2}{r^4} & a < r < b \\ 0 & b < r \end{cases} \\
W &= \frac{\epsilon_0}{2} \int E^2 d\tau \\
&= \frac{\epsilon_0}{2} \int_0^{2\pi} \int_0^\pi \int_a^b \frac{1}{(4\pi\epsilon_0)^2} \frac{q^2}{r^4} r^2 \sin \theta dr d\theta d\phi \\
&= \frac{\epsilon_0}{2} \frac{1}{(4\pi\epsilon_0)^2} 2\pi q^2 \int_0^\pi \int_a^b \frac{\sin \theta}{r^2} dr d\theta \\
&= \frac{q^2}{8\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)
\end{aligned}$$

(b)

$$\begin{aligned}
W_{\text{shell}} &= \frac{1}{8\pi\epsilon_0} \frac{q^2}{R} \\
\mathbf{E} &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}} \\
\mathbf{E}_1 \cdot \mathbf{E}_2 &= -\frac{1}{(4\pi\epsilon_0)^2} \frac{q^2}{r^4} \\
W_{\text{total}} &= W_1 + W_2 + \epsilon_0 \int \mathbf{E}_1 \cdot \mathbf{E}_2 d\tau \\
&= \frac{q^2}{8\pi\epsilon_0} \left(\frac{1}{a} + \frac{1}{b} \right) - \epsilon_0 \int_0^{2\pi} \int_0^\pi \int_b^\infty \frac{1}{(4\pi\epsilon_0)^2} \frac{q^2}{r^4} r^2 \sin\theta dr d\theta d\phi \\
&= \frac{q^2}{8\pi\epsilon_0} \left(\frac{1}{a} + \frac{1}{b} \right) - \frac{1}{8\pi\epsilon_0} q^2 \int_0^\pi \int_b^\infty \frac{1}{r^2} \sin\theta dr d\theta \\
&= \frac{q^2}{8\pi\epsilon_0} \left(\frac{1}{a} + \frac{1}{b} \right) - \frac{1}{4\pi\epsilon_0} q^2 \int_b^\infty \frac{1}{r^2} dr \\
&= \frac{q^2}{8\pi\epsilon_0} \left(\frac{1}{a} + \frac{1}{b} \right) - \frac{1}{4\pi\epsilon_0} \frac{q^2}{b} \\
&= \frac{q^2}{8\pi\epsilon_0} \left(\frac{1}{a} + \frac{1}{b} - \frac{2}{b} \right) \\
&= \frac{q^2}{8\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)
\end{aligned}$$

2.37

$$\begin{aligned}
r_1 &= r \\
E_1 &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1^2} \\
&= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \\
r_2 &= \sqrt{a^2 + r^2 - 2ar \cos \theta} \\
E_2 &= \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2^2} \\
&= \frac{1}{4\pi\epsilon_0} \frac{q_2}{a^2 + r^2 - 2ar \cos \theta} \\
\cos \alpha &= \frac{r - a \cos \theta}{\sqrt{a^2 + r^2 - 2ar \cos \theta}} \\
\mathbf{E}_1 \cdot \mathbf{E}_2 &= E_1 E_2 \cos \alpha \\
&= \frac{1}{(4\pi\epsilon_0)^2} \frac{q_1 q_2}{r^2 (a^2 + r^2 - 2ar \cos \theta)} \frac{r - a \cos \theta}{\sqrt{a^2 + r^2 - 2ar \cos \theta}} \\
&= \frac{1}{(4\pi\epsilon_0)^2} \frac{q_1 q_2 (r - a \cos \theta)}{r^2 (a^2 + r^2 - 2ar \cos \theta)^{3/2}} \\
\epsilon_0 \int \mathbf{E}_1 \cdot \mathbf{E}_2 d\tau &= \epsilon_0 \int_0^{2\pi} \int_0^\pi \int_0^\infty \frac{1}{(4\pi\epsilon_0)^2} \frac{q_1 q_2 (r - a \cos \theta)}{r^2 (a^2 + r^2 - 2ar \cos \theta)^{3/2}} r^2 \sin \theta dr d\theta d\phi \\
&= \frac{q_1 q_2}{8\pi\epsilon_0} \int_0^\pi \int_0^\infty \frac{(r - a \cos \theta) \sin \theta}{(a^2 + r^2 - 2ar \cos \theta)^{3/2}} dr d\theta
\end{aligned}$$