Advanced Engineering Mathematics Vectors, Matrices, and Vector Calculus by Dennis G. Zill Problems

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Contents

1 Vectors

1.1 Vectors in 2-Space

1.1.1

(a) $3\mathbf{a} = 6\mathbf{i} + 12\mathbf{j}$

(b) $\mathbf{a} + \mathbf{b} = \mathbf{i} + 8\mathbf{j}$

(c) $\mathbf{a} - \mathbf{b} = 3\mathbf{i}$

(d) $||\mathbf{a} + \mathbf{b}|| = \sqrt{1 + 8^2} = \sqrt{65}$

(e) $||\mathbf{a} - \mathbf{b}|| = 3$

1.1.9

(a) $4\mathbf{a} - 2\mathbf{b} = \langle 6, -14\rangle$

(b) $-3\mathbf{a} - 5\mathbf{b} = \langle 2, 4\rangle$

1.1.15

$\overrightarrow{P_1P_2} = \langle 2, 5\rangle$

1.1.19

$(1, 18)$

1.1.21

(a) Yes

(b) Yes

(c) Yes

(d) No

(e) Yes

(f) Yes

1.1.25

(a) $\frac{\mathbf{a}}{||\mathbf{a}||} = \frac{\langle 2,2\rangle}{\sqrt{2^2+2^2}} = \frac{1}{2\sqrt{2}}\langle 2, 2\rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$

(b) $\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\rangle$

1.1.31

$2\frac{\mathbf{a}}{||\mathbf{a}||} = 2\frac{\langle 3,7\rangle}{\sqrt{3^2+7^2}} = \frac{2}{\sqrt{58}}\langle 3, 7\rangle = \langle \frac{6}{\sqrt{58}}, \frac{14}{\sqrt{58}}\rangle$

1.1.37

$\mathbf{x} = -(\mathbf{a} + \mathbf{b})$

1.1.41

$$\begin{aligned}
\mathbf{a} &= 2\mathbf{i} + 3\mathbf{j} \\
\mathbf{b} &= \mathbf{i} + \mathbf{j} \\
\mathbf{c} &= \mathbf{i} - \mathbf{j}
\end{aligned}$$

$$\begin{aligned}
\mathbf{i} &= \frac{1}{2}(\mathbf{b} + \mathbf{c}) \\
\mathbf{j} &= \frac{1}{2}(\mathbf{b} - \mathbf{c}) \\
\mathbf{a} &= 2\left(\frac{1}{2}(\mathbf{b} + \mathbf{c})\right) + 3\left(\frac{1}{2}(\mathbf{b} - \mathbf{c})\right) \\
&= \mathbf{b} + \mathbf{c} + \frac{3}{2}\mathbf{b} - \frac{3}{2}\mathbf{c} \\
&= \frac{5}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}
\end{aligned}$$

1.1.43

$$\begin{aligned} y &= \frac{1}{4}x^2 + 1 \\ y(2) &= 2 \\ y' &= \frac{1}{2}x \\ y'(2) &= 1 \\ \mathbf{v} &= \pm\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle \end{aligned}$$

1.1.45

(a)

$$\begin{aligned} \mathbf{F}_n &= \mathbf{F}\cos\theta \\ \mathbf{F}_g &= \mathbf{F}\sin\theta \\ ||\mathbf{F}_f|| &= \mu||\mathbf{F}_n|| \\ ||-\mathbf{F}_g|| &= \mu||\mathbf{F}_n|| \\ ||-\mathbf{F}\sin\theta|| &= \mu||\mathbf{F}\cos\theta|| \\ ||\mathbf{F}||\sin\theta &= \mu||\mathbf{F}||\cos\theta \\ \tan\theta &= \mu \end{aligned}$$

(b) $\theta = \arctan\mu \approx 31^\circ$

1.1.47

$$\begin{aligned} F_x &= \frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a} \frac{L\,dy}{2a(L^2+y^2)^{3/2}} \\ &= \frac{LqQ}{8\pi\epsilon_0}\int_{-a}^{a}(L^2+y^2)^{-3/2}\,dy \\ &= \frac{LqQ}{8\pi\epsilon_0}\frac{2a}{L^2\sqrt{a^2+L^2}} \\ &= \frac{aqQ}{4\pi\epsilon_0 L\sqrt{a^2+L^2}} \\ F_y &= -\frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{y\,dy}{2a(L^2+y^2)^{3/2}} \\ &= 0 \\ \mathbf{F} &= \langle \frac{1}{4\pi\epsilon_0}\frac{qQ}{L\sqrt{a^2+L^2}}, 0\rangle \end{aligned}$$

1.1.49

Let the three sides of the triangle be vectors **a**, **b**, and **c**. The triangle is closed so it must be the case that

$$\mathbf{a}+\mathbf{b}+\mathbf{c}=0.$$

This gives

$$\mathbf{c}=-(\mathbf{a}+\mathbf{b}).$$

The vector from the midpoint of side **a** to the midpoint of side **b** is

$$\left(\mathbf{a}+\frac{1}{2}\mathbf{b}\right)-\frac{1}{2}\mathbf{a}=\frac{1}{2}(\mathbf{a}+\mathbf{b})$$

which is parallel with **c** and half its length.

1.2 Vectors in 3-Space

1.2.7

A plane at $z=5$ parellel with the x-y plane.

1.2.9

A line parallel to the z axis at $x=2$ and $y=3$.

1.2.13

(a) $(0,5,4)$, $(-2,0,4)$, $(-2,5,0)$

(b) $(-2,5,-2)$

(c) $(3,5,4)$

1.2.15

The planes $x=0$, $y=0$, and $z=0$.

1.2.17

$(-1,2,-3)$

1.2.19

The planes $z=\pm 5$.

1.2.21

$\sqrt{(6-3)^2+(4+1)^2+(8-2)^2}=\sqrt{9+25+36}=\sqrt{70}$

1.2.31

$$
\begin{aligned}
\sqrt{(2-x)^2+(1-2)^2+(1-3)^2} &= \sqrt{21} \\
(2-x)^2+1+4 &= 21 \\
(2-x)^2 &= 16 \\
2-x &= \pm 4 \\
x &= 2 \pm 4 \\
&= -2 \text{ or } 6
\end{aligned}
$$

1.2.33

$\left(4, \frac{1}{2}, \frac{3}{2}\right)$

1.2.37

$(-3, -6, 1)$

1.3 Dot Product

1.3.1

$\mathbf{a} \cdot \mathbf{b} = 12$

1.3.11

$\left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b} = \frac{12}{30}\mathbf{b} = \langle -\frac{2}{5}, \frac{4}{5}, 2\rangle$

1.3.13

$\mathbf{a} \cdot \mathbf{b} = ||\mathbf{a}||||\mathbf{b}|| \cos\theta = 25\sqrt{2}$

1.3.17

$$
\begin{aligned}
\mathbf{a} \cdot \mathbf{v} &= 0 \\
3x_1 + y_1 - 1 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{b} \cdot \mathbf{v} &= 0 \\
-3x_1 + 2y_2 + 2 &= 0
\end{aligned}
$$

$$
\begin{aligned}
3y_2 + 1 &= 0 \\
y_2 &= -\frac{1}{3}
\end{aligned}
$$

$$
\begin{aligned}
3x_1 - \frac{1}{3} - 1 &= 0 \\
x_1 &= \frac{4}{9}
\end{aligned}
$$

$$
\mathbf{v} = \langle \frac{4}{9}, -\frac{1}{3}, 1 \rangle
$$

1.3.19

$$
\begin{aligned}
\mathbf{a} \cdot \mathbf{c} &= \mathbf{a} \cdot \left(\mathbf{b} - \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||^2} \mathbf{a} \right) \\
&= \mathbf{a} \cdot \mathbf{b} - \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||^2} \mathbf{a} \cdot \mathbf{a} \\
&= 0
\end{aligned}
$$

1.3.21

$$\begin{aligned}
||\mathbf{a}|| &= \sqrt{3^2 + 1^2} \\
&= \sqrt{10} \\
||\mathbf{b}|| &= \sqrt{2^2 + 2^2} \\
&= \sqrt{8} \\
&= 2\sqrt{2} \\
\mathbf{a} \cdot \mathbf{b} &= 4 \\
\theta &= \arccos \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||||\mathbf{b}||} \\
&= \arccos \frac{4}{(\sqrt{10})(2\sqrt{2})} \\
&= \arccos \frac{1}{\sqrt{5}} \\
&\approx 63^\circ
\end{aligned}$$

1.3.25

$$\begin{aligned}
||\mathbf{a}|| &= \sqrt{1^2 + 2^2 + 3^3} \\
&= \sqrt{14} \\
\cos \alpha &= \frac{1}{\sqrt{14}} \\
\alpha &\approx 75^\circ \\
\cos \beta &= \frac{2}{\sqrt{14}} \\
\beta &\approx 58^\circ \\
\cos \gamma &= \frac{3}{\sqrt{14}} \\
\gamma &\approx 37^\circ
\end{aligned}$$

1.3.29

$$\begin{aligned}
\overrightarrow{AD} &= \langle s, -s, s\rangle \\
||\overrightarrow{AD}|| &= s\sqrt{3} \\
\overrightarrow{AB} &= \langle s, 0, 0\rangle \\
||\overrightarrow{AB}|| &= s \\
\theta &= \arccos \frac{\overrightarrow{AD}\cdot\overrightarrow{AB}}{||\overrightarrow{AD}||||\overrightarrow{AB}||} \\
&= \arccos \frac{s^2}{s^2\sqrt{3}} \\
&= \arccos \frac{1}{\sqrt{3}} \\
&\approx 55^\circ
\end{aligned}$$

1.3.33

$$\begin{aligned}
\mathrm{comp}_{\mathbf{b}}\mathbf{a} &= \frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{b}||} \\
&= \frac{5}{7}
\end{aligned}$$

1.3.37

$$\begin{aligned}
\mathrm{comp}_{\overrightarrow{OP}}\mathbf{a} &= \frac{\mathbf{a}\cdot\overrightarrow{OP}}{||\overrightarrow{OP}||} \\
&= \frac{72}{\sqrt{109}}
\end{aligned}$$

1.3.39

$$\begin{aligned}
\mathrm{proj}_{\mathbf{b}}\mathbf{a} &= \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b} \\
&= \frac{35}{25}\mathbf{b} \\
&= \langle -\frac{21}{5}, \frac{28}{5}\rangle
\end{aligned}$$

1.3.43

$$
\begin{aligned}
\mathbf{a}+\mathbf{b} &= \langle 3,4\rangle \\
\mathrm{proj}_{\mathbf{a}+\mathbf{b}}\mathbf{a} &= \left(\frac{\mathbf{a}\cdot(\mathbf{a}+\mathbf{b})}{(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})}\right)(\mathbf{a}+\mathbf{b}) \\
&= \frac{24}{25}(\mathbf{a}+\mathbf{b}) \\
&= \langle \frac{72}{25}, \frac{96}{25}\rangle
\end{aligned}
$$

1.3.45

$W = \mathbf{F}\cdot\mathbf{d} = Fd\cos\theta = 1000$

1.3.47

(a) $W = 0$

(b)

$$
\begin{aligned}
||\mathbf{d}|| &= \sqrt{4^2+3^2} \\
&= 5 \\
\mathbf{F} &= F\hat{\mathbf{d}} \\
&= F\frac{\mathbf{d}}{||\mathbf{d}||} \\
&= F\langle \frac{4}{5}, \frac{3}{5}\rangle \\
&= \langle 24, 18\rangle \\
W &= \mathbf{F}\cdot\mathbf{d} \\
&= 150\,\mathrm{J}
\end{aligned}
$$

1.4 Cross Product

1.4.1

$$
\begin{aligned}
\mathbf{a}\times\mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ 0 & 3 & 5 \end{vmatrix} \\
&= -5\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}
\end{aligned}
$$

1.4.11

$$\begin{aligned}\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 2 & -4 \\ -3 & 1 & 1 \end{vmatrix} \\ &= 6\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}\end{aligned}$$

1.4.17

(a)

$$\begin{aligned}\mathbf{b} \times \mathbf{c} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix} \\ &= \mathbf{j} - \mathbf{k} \\ \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 0 & 1 & -1 \end{vmatrix} \\ &= -\mathbf{i} + \mathbf{j} + \mathbf{k}\end{aligned}$$

1.4.19

$2\mathbf{k}$

1.4.21

$$\begin{aligned}\mathbf{k} \times (2\mathbf{i} - \mathbf{j}) &= (\mathbf{k} \times 2\mathbf{i}) - (\mathbf{k} \times \mathbf{j}) \\ &= \mathbf{i} + 2\mathbf{j}\end{aligned}$$

1.4.23

$$\begin{aligned}[(2\mathbf{k}) \times (3\mathbf{j})] \times (4\mathbf{j}) &= (-6\mathbf{i}) \times (4\mathbf{j}) \\ &= -24\mathbf{k}\end{aligned}$$

1.4.37

$12\mathbf{i} - 9\mathbf{j} + 18\mathbf{k}$

1.4.53

$$\begin{aligned}
\mathbf{b} \times \mathbf{c} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 6 & -6 \\ \frac{5}{2} & 3 & \frac{1}{2} \end{vmatrix} \\
&= 21\mathbf{i} - 14\mathbf{j} - 21\mathbf{k} \\
\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) &= 4 \times 21 + 6 \times (-14) \\
&= 0
\end{aligned}$$

They are coplanar.

1.5 Lines and Planes in 3-Space

1.5.1

$\mathbf{r} = \langle 1, 2, 1 \rangle + t\langle 2, 3, -3 \rangle$

1.5.7

$$\begin{aligned}
x &= 2 + 4t \\
y &= 3 - 4t \\
z &= 5 + 3t
\end{aligned}$$

1.5.13

$$\begin{aligned}
x &= 1 + 9t \\
y &= 4 + 10t \\
z &= -9 + 7t \\
\frac{x-1}{9} = \frac{y-4}{10} &= \frac{z+9}{7}
\end{aligned}$$

1.5.19

$$\begin{aligned}
x &= 4 + 3t \\
y &= 6 + \frac{1}{2}t \\
z &= -7 - \frac{3}{2}t \\
\frac{x-4}{3} = \frac{y-6}{1/2} &= -\frac{z+7}{3/2}
\end{aligned}$$

1.5.23

$$\begin{aligned} x &= 6 + 2t \\ y &= 4 - 3t \\ z &= -2 + 6t \end{aligned}$$

1.5.25

$$\begin{aligned} x &= 2 + t \\ y &= -2 \\ z &= 15 \end{aligned}$$

1.5.29

$(0, 5, 15)$, $(5, 0, \frac{15}{2})$, $(10, -5, 0)$

1.5.31

$$\begin{aligned} 4 + t_x &= 6 + 2t_x \\ t_x &= -2 \\ \\ 5 + t_y &= 11 + 4t_y \\ t_y &= -2 \\ \\ -1 + 2t_z &= -3 + t_z \\ t_z &= -2 \end{aligned}$$

$(2, 3, -5)$

1.5.35

$$\begin{aligned} \mathbf{a} &= \langle -1, 2, -2 \rangle \\ ||\mathbf{a}|| &= 3 \\ \mathbf{b} &= \langle 2, 3, -6 \rangle \\ ||\mathbf{b}|| &= 7 \\ \theta &= \arccos \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}|| ||\mathbf{b}||} \\ &\approx 40.37^\circ \end{aligned}$$

1.5.37

$$\begin{aligned}
\mathbf{a} &= \langle 1, 1, 1\rangle \\
\mathbf{b} &= \langle -2, 1, -5\rangle \\
\mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ -2 & 1 & -5 \end{vmatrix} \\
&= \langle -6, 3, 3\rangle \\
x &= 4 - 6t \\
y &= 1 + 3t \\
z &= 6 + 3t
\end{aligned}$$

1.5.39

$$\begin{aligned}
\langle 2, -3, 4\rangle \cdot (\mathbf{r} - \langle 5, 1, 3\rangle) &= 0 \\
2(x-5) - 3(y-1) + 4(z-3) &= 0 \\
2x - 3y + 4z - 19 &= 0
\end{aligned}$$

1.5.45

$$\begin{aligned}
\mathbf{a} &= \langle 3, 5, 2\rangle \\
\mathbf{b} &= \langle 2, 3, 1\rangle \\
\mathbf{c} &= \langle -1, -1, 4\rangle \\
\mathbf{a} - \mathbf{c} &= \langle 4, 6, -2\rangle \\
\mathbf{b} - \mathbf{c} &= \langle 3, 4, -3\rangle \\
(\mathbf{a} - \mathbf{c}) \times (\mathbf{b} - \mathbf{c}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 6 & -2 \\ 3 & 4 & -3 \end{vmatrix} \\
&= \langle -10, 6, -2\rangle \\
\mathbf{n} \cdot (\mathbf{r} - \mathbf{c}) &= 0 \\
\langle -10, 6, -2\rangle \cdot (\langle x, y, z\rangle - \langle -1, -1, 4\rangle) &= 0 \\
-10(x+1) + 6(y+1) - 2(z-4) &= 0 \\
-10x + 6y - 2z + 4 &= 0
\end{aligned}$$

1.5.51

$$\begin{aligned}
\langle 1, 1, -4\rangle \cdot (\mathbf{r} - \langle 2, 3, -5\rangle) &= 0 \\
(x-2) + (y-3) - 4(z+5) &= 0 \\
x + y - 4z &= 25
\end{aligned}$$

1.5.63

(a) Not perpendicular

(b) Not perpendicular

(c) Perpendicular

(d) Perpendicular

1.5.65

$$\begin{aligned} 5x - 4y - 9t &= 8 \\ x + 4y + 3t &= 4 \end{aligned}$$

$$\begin{aligned} 6x - 6t &= 12 \\ x &= 2 + t \end{aligned}$$

$$y = \frac{1}{2} - t$$

$$z = t$$

1.5.69

$$\begin{aligned} 2(1 + 2t) - 3(2 - t) + 2(-3t) &= -7 \\ t &= -3 \\ x &= -5 \\ y &= 5 \\ z &= 9 \end{aligned}$$

1.5.73

$$\begin{aligned}
x + y - 4t &= 2\\
2x - y + t &= 10\\
\\
3x - 3t &= 12\\
x &= 4 + t\\
\\
2(4+t) - y + t &= 10\\
8 + 2t - y + t &= 10\\
y &= -2 + 3t\\
\\
z &= t\\
\\
x &= 5 + t\\
y &= 6 + 3t\\
z &= -12 + t
\end{aligned}$$

1.5.75

$$\begin{aligned}
\mathbf{n} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 5 \\ 1 & 1 & 1 \end{vmatrix}\\
&= \langle -6, 2, 4 \rangle\\
\mathbf{n} \cdot (\mathbf{r} - \langle 4, 0, 1 \rangle) &= 0\\
-6(x-4) + 2y + 4(z-1) &= 0\\
-6x + 2y + 4z &= -20\\
3x - y - 2z &= 10
\end{aligned}$$

1.6 Vector Spaces

1.6.1

Violates axiom 6

1.6.3

Violates axiom 10

1.6.5

Vector space

1.6.7

Violates axiom 2

1.6.9

Vector space

1.6.11

Subspace

1.6.13

Not a subspace

1.6.15

Subspace

1.6.17

Subspace

1.6.19

Not a subspace

1.6.23

(a)

$$\begin{aligned} k_1\mathbf{u}_1 + k_2\mathbf{u}_2 + k_3\mathbf{u}_3 &= \mathbf{0} \\ k_1\langle 1,0,0\rangle + k_2\langle 1,1,0\rangle + k_3\langle 1,1,1\rangle &= \mathbf{0} \\ \\ k_3 &= 0 \\ \\ k_2 + k_3 &= 0 \\ k_2 &= 0 \\ \\ k_1 + k_2 + k_3 &= 0 \\ k_1 &= 0 \end{aligned}$$

(b)

$$\mathbf{a} = 7\mathbf{u}_1 - 12\mathbf{u}_2 + 8\mathbf{u}_3$$

1.6.25

Dependent

1.6.27

Independent

1.6.29

$f(x)$ is undefined at $x = -3$ and $x = -1$.

1.6.31

$$\begin{aligned}
||x|| &= \sqrt{(x,x)} \\
&= \sqrt{\int_0^{2\pi} x^2\,dx} \\
&= \sqrt{\left[\frac{1}{3}x^3\right]_0^{2\pi}} \\
&= \sqrt{\frac{8}{3}\pi^3} \\
||\sin x|| &= \sqrt{(\sin x, \sin x)} \\
&= \sqrt{\int_0^{2\pi} \sin^2 x\,dx} \\
&= \sqrt{\left[\frac{x}{2} - \frac{1}{4}\sin 2x\right]_0^{2\pi}} \\
&= \sqrt{\pi}
\end{aligned}$$

1.7 Gram–Schmidt Orthogonalization Process

1.7.1

$$
\begin{aligned}
\langle \frac{12}{13}, \frac{5}{13} \rangle \cdot \langle \frac{5}{13}, -\frac{12}{13} \rangle &= 0 \\
\sqrt{\left(\frac{12}{13}\right)^2 + \left(\frac{5}{13}\right)^2} &= 1
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{u} &= \left(\langle 4, 2 \rangle \cdot \langle \frac{12}{13}, \frac{5}{13} \rangle \right) \langle \frac{12}{13}, \frac{5}{13} \rangle \\
&\quad + \left(\langle 4, 2 \rangle \cdot \langle \frac{5}{13}, -\frac{12}{13} \rangle \right) \langle \frac{5}{13}, -\frac{12}{13} \rangle \\
&= \left(\frac{58}{13} \right) \langle \frac{12}{13}, \frac{5}{13} \rangle - \left(\frac{4}{13} \right) \langle \frac{5}{13}, -\frac{12}{13} \rangle
\end{aligned}
$$

1.7.3

$$
\begin{aligned}
\langle 1, 0, 1 \rangle \cdot \langle 0, 1, 0 \rangle &= 0 \\
\langle 1, 0, 1 \rangle \cdot \langle -1, 0, 1 \rangle &= 0 \\
\langle 0, 1, 0 \rangle \cdot \langle -1, 0, 1 \rangle &= 0
\end{aligned}
$$

$$
\begin{aligned}
B' &= \{ \langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle, \langle 0, 1, 0 \rangle, \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle \} \\
\mathbf{u} &= -\frac{3}{\sqrt{2}} \langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle + 7 \langle 0, 1, 0 \rangle - \frac{23}{\sqrt{2}} \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle
\end{aligned}
$$

1.7.5

(a)

$$
\begin{aligned}
B &= \{\langle -3, 2\rangle, \langle -1, -1\rangle\} \\
\mathbf{v}_1 &= \mathbf{u}_1 \\
&= \langle -3, 2\rangle \\
\mathbf{v}_2 &= \mathbf{u}_2 - \operatorname{proj}_{\mathbf{v}_1} \mathbf{u}_2 \\
&= \langle -1, -1\rangle - \left(\frac{\langle -1, -1\rangle \cdot \langle -3, 2\rangle}{\langle -3, 2\rangle \cdot \langle -3, 2\rangle}\right) \langle -3, 2\rangle \\
&= \langle -1, -1\rangle - \frac{1}{13}\langle -3, 2\rangle \\
&= \langle -\frac{10}{13}, -\frac{15}{13}\rangle \\
\mathbf{w}_1 &= \langle -\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\rangle \\
\mathbf{w}_2 &= \sqrt{\frac{169}{325}}\langle -\frac{10}{13}, -\frac{15}{13}\rangle \\
&= \frac{\sqrt{13}}{5}\langle -\frac{10}{13}, -\frac{15}{13}\rangle \\
&= \langle -\frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}\rangle
\end{aligned}
$$

1.7.9

$$\begin{aligned}
B &= \{\langle 1,1,0\rangle, \langle 1,2,2\rangle, \langle 2,2,1\rangle\} \\
\mathbf{v}_1 &= \langle 1,1,0\rangle \\
\mathbf{v}_2 &= \mathbf{u}_2 - \mathrm{proj}_{\mathbf{v}_1}\mathbf{u}_2 \\
&= \langle 1,2,2\rangle - \left(\frac{\langle 1,2,2\rangle \cdot \langle 1,1,0\rangle}{\langle 1,1,0\rangle \cdot \langle 1,1,0\rangle}\right)\langle 1,1,0\rangle \\
&= \langle 1,2,2\rangle - \frac{3}{2}\langle 1,1,0\rangle \\
&= \langle -\frac{1}{2}, \frac{1}{2}, 2\rangle \\
\mathbf{v}_3 &= \mathbf{u}_3 - \mathrm{proj}_{\mathbf{v}_1}\mathbf{u}_3 - \mathrm{proj}_{\mathbf{v}_2}\mathbf{u}_3 \\
&= \langle 2,2,1\rangle - \left(\frac{\langle 2,2,1\rangle \cdot \langle 1,1,0\rangle}{\langle 1,1,0\rangle \cdot \langle 1,1,0\rangle}\right)\langle 1,1,0\rangle \\
&\qquad - \left(\frac{\langle 2,2,1\rangle \cdot \langle -\frac{1}{2}, \frac{1}{2}, 2\rangle}{\langle -\frac{1}{2}, \frac{1}{2}, 2\rangle \cdot \langle -\frac{1}{2}, \frac{1}{2}, 2\rangle}\right)\langle -\frac{1}{2}, \frac{1}{2}, 2\rangle \\
&= \langle 2,2,1\rangle - 2\langle 1,1,0\rangle - \frac{4}{9}\langle -\frac{1}{2}, \frac{1}{2}, 2\rangle \\
&= \langle \frac{2}{9}, -\frac{2}{9}, \frac{1}{9}\rangle \\
\mathbf{w}_1 &= \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\rangle \\
\mathbf{w}_2 &= \langle -\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}\rangle \\
\mathbf{w}_3 &= 3\langle \frac{2}{9}, -\frac{2}{9}, \frac{1}{9}\rangle \\
&= \langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\rangle
\end{aligned}$$

1.7.17

$$\begin{aligned}
B &= \{1, x, x^2\} \\
\mathbf{v}_1 &= \mathbf{u}_1 \\
&= 1 \\
\mathbf{v}_2 &= \mathbf{u}_2 - \mathrm{proj}_{\mathbf{v}_1}\mathbf{u}_2 \\
&= \mathbf{u}_2 - \left(\frac{\mathbf{u}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\right)\mathbf{v}_1 \\
&= x - \frac{\int_{-1}^{1} x\,dx}{\int_{-1}^{1} dx} \\
&= x - \frac{\left[\frac{1}{2}x^2\right]_{-1}^{1}}{2} \\
&= x \\
\mathbf{v}_3 &= \mathbf{u}_3 - \mathrm{proj}_{\mathbf{v}_1}\mathbf{u}_3 - \mathrm{proj}_{\mathbf{v}_2}\mathbf{u}_3 \\
&= \mathbf{u}_3 - \left(\frac{\mathbf{u}_3 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\right)\mathbf{v}_1 - \left(\frac{\mathbf{u}_3 \cdot \mathbf{v}_2}{\mathbf{v}_2 \cdot \mathbf{v}_2}\right)\mathbf{v}_2 \\
&= x^2 - \frac{\int_{-1}^{1} x^2\,dx}{\int_{-1}^{1} dx} - \frac{\int_{-1}^{1} x^3\,dx}{\int_{-1}^{1} x^2\,dx}x \\
&= x^2 - \frac{\left[\frac{1}{3}x^3\right]_{-1}^{1}}{2} - \frac{\left[\frac{1}{4}x^4\right]_{-1}^{1}}{\left[\frac{1}{3}x^3\right]_{-1}^{1}}x \\
&= x^2 - \frac{1}{3}
\end{aligned}$$

1.7.19

$$
\begin{aligned}
||\mathbf{v}_1||^2 &= \int_{-1}^{1} dx \\
&= 2 \\
\mathbf{w}_1 &= \frac{1}{\sqrt{2}} \\
||\mathbf{v}_2||^2 &= \int_{-1}^{1} x^2\, dx \\
&= \left[\frac{1}{3}x^3\right]_{-1}^{1} \\
&= \frac{2}{3} \\
\mathbf{w}_2 &= \frac{3}{\sqrt{6}}x \\
||\mathbf{v}_3||^2 &= \int_{-1}^{1} \left(x^2 - \frac{1}{3}\right)^2 dx \\
&= \int_{-1}^{1} \left(x^4 - \frac{2}{3}x^2 + \frac{1}{9}\right) dx \\
&= \left[\frac{1}{5}x^5 - \frac{2}{9}x^3 + \frac{1}{9}x\right]_{-1}^{1} \\
&= \frac{1}{5} - \frac{2}{9} + \frac{1}{9} + \frac{1}{5} - \frac{2}{9} + \frac{1}{9} \\
&= \frac{2}{5} - \frac{2}{9} \\
&= \frac{8}{45} \\
\mathbf{w}_3 &= \sqrt{\frac{45}{8}} \left(x^2 - \frac{1}{3}\right) \\
&= \frac{5}{2\sqrt{10}} \left(3x^2 - 1\right)
\end{aligned}
$$

1.7.21

$$
\begin{aligned}
(\mathbf{p}, \mathbf{w}_1) &= \int_{-1}^{1} \frac{1}{\sqrt{2}}(9x^2 - 6x + 5)\, dx \\
&= \frac{1}{\sqrt{2}} \left[3x^3 - 3x^2 + 5x\right]_{-1}^{1} \\
&= \frac{1}{\sqrt{2}}(3 - 3 + 5 + 3 + 3 + 5) \\
&= \frac{16}{\sqrt{2}} \\
(\mathbf{p}, \mathbf{w}_2) &= \int_{-1}^{1} \frac{3}{\sqrt{6}} x(9x^2 - 6x + 5)\, dx \\
&= \frac{3}{\sqrt{6}} \left[\frac{9}{4}x^4 - 2x^3 + \frac{5}{2}x^2\right]_{-1}^{1} \\
&= \frac{3}{\sqrt{6}} \left(\frac{9}{4} - 2 + \frac{5}{2} - \frac{9}{4} - 2 - \frac{5}{2}\right) \\
&= \frac{3}{\sqrt{6}} \left(\frac{9}{4} - \frac{8}{4} + \frac{10}{4} - \frac{9}{4} - \frac{8}{4} - \frac{10}{4}\right) \\
&= -\frac{12}{\sqrt{6}} \\
(\mathbf{p}, \mathbf{w}_3) &= \int_{-1}^{1} \frac{5}{2\sqrt{10}}(3x^2 - 1)(9x^2 - 6x + 5)\, dx \\
&= \frac{5}{2\sqrt{10}} \int_{-1}^{1} (27x^4 - 18x^3 + 6x^2 + 6x - 5)\, dx \\
&= \frac{5}{2\sqrt{10}} \left[\frac{27}{5}x^5 - \frac{9}{2}x^4 + 2x^3 + 3x^2 - 5x\right]_{-1}^{1} \\
&= \frac{5}{2\sqrt{10}} \left(\frac{27}{5} - \frac{9}{2} + 2 + 3 - 5 + \frac{27}{5} + \frac{9}{2} + 2 - 3 - 5\right) \\
&= \frac{5}{2\sqrt{10}} \left(\frac{54}{10} - \frac{45}{10} + \frac{20}{10} + \frac{30}{10} - \frac{50}{10} + \frac{54}{10} + \frac{45}{10} + \frac{20}{10} - \frac{30}{10} - \frac{50}{10}\right) \\
&= \frac{5}{2\sqrt{10}} \frac{48}{10} \\
&= \frac{12}{\sqrt{10}} \\
\mathbf{p} &= \frac{16}{\sqrt{2}}\mathbf{w}_1 - \frac{12}{\sqrt{6}}\mathbf{w}_2 + \frac{12}{\sqrt{10}}\mathbf{w}_3
\end{aligned}
$$

1.8 Chapter in Review

1.8.1

True

1.8.3

$$\begin{aligned}\mathbf{u} &= \langle 5, -2, 1\rangle \\ \mathbf{v} &= \langle 2, 3, -4\rangle\end{aligned}$$

False

1.8.5

True

1.8.7

True

1.8.9

True

1.8.11

$9\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

1.8.13

$$\begin{aligned}(-\mathbf{k}) \times (5\mathbf{j}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1 \\ 0 & 5 & 0 \end{vmatrix} \\ &= 5\mathbf{i}\end{aligned}$$

1.8.15

$$||-12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}|| = \sqrt{12^2 + 4^2 + 6^2} = 14$$

1.8.17

$\langle -6, 1, -7\rangle$

1.8.19

$$\begin{aligned} x &= 1+t \\ y &= -2+3t \\ z &= -1+2t \end{aligned}$$

$$\begin{aligned} x+2y-z &= 13 \\ (1+t)+2(-2+3t)-(-1+2t) &= 13 \\ 1+t-4+6t+1-2t &= 13 \\ -2+5t &= 13 \\ t &= 3 \end{aligned}$$

$$\begin{aligned} x &= 4 \\ y &= 7 \\ z &= 5 \end{aligned}$$

1.8.21

$$\begin{aligned} \overrightarrow{P_1P_2} &= \overrightarrow{P_2} - \overrightarrow{P_1} \\ \overrightarrow{P_2} &= \overrightarrow{P_1P_2} + \overrightarrow{P_1} \\ &= \langle 3,5,-4\rangle + \langle 2,1,7\rangle \\ &= \langle 5,6,3\rangle \end{aligned}$$

1.8.23

$\mathbf{a}\cdot\mathbf{b} = -36\sqrt{2}$

1.8.25

$x = 12,\ y = -8,\ z = 6$

1.8.27

$$\begin{aligned} \frac{1}{2}(\mathbf{a}\times\mathbf{b}) &= \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -1 \\ 2 & -1 & 2 \end{vmatrix} \\ &= \frac{1}{2}\langle 5,-4,-7\rangle \\ &= \langle \frac{5}{2},-2,-\frac{7}{2}\rangle \end{aligned}$$

The area is $\sqrt{\left(\frac{5}{2}\right)^2 + (-2)^2 + \left(-\frac{7}{2}\right)^2} = \frac{3}{2}\sqrt{10}$

1.8.29

2

1.8.31

$$\begin{aligned}
\mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 1 & -2 & 1 \end{vmatrix} \\
&= \langle 1, -1, -3 \rangle \\
||\mathbf{a} \times \mathbf{b}|| &= \sqrt{11} \\
\text{norm}(\mathbf{a} \times \mathbf{b}) &= \langle \frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, -\frac{3}{\sqrt{11}} \rangle
\end{aligned}$$

1.8.33

$$\text{comp}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{b}||} = \frac{10}{5} = 2$$

1.8.35

$$\begin{aligned}
\mathbf{a} &= \langle 1, 2, -2 \rangle \\
\mathbf{b} &= \langle 4, 3, 0 \rangle \\
\mathbf{a} + \mathbf{b} &= \langle 5, 5, -2 \rangle \\
\text{proj}_{\mathbf{a}}(\mathbf{a} + \mathbf{b}) &= \left(\frac{(\mathbf{a} + \mathbf{b}) \cdot \mathbf{a}}{\mathbf{a} \cdot \mathbf{a}} \right) \mathbf{a} \\
&= \frac{19}{9} \langle 1, 2, -2 \rangle \\
&= \langle \frac{19}{9}, \frac{38}{9}, -\frac{38}{9} \rangle
\end{aligned}$$

1.8.37

(a)

(b) A plane with normal $\mathbf{a}$

1.8.39

$$\frac{x-7}{4} = \frac{y-3}{-2} = \frac{z+5}{6}$$

1.8.41

$$\begin{aligned}\langle -2, 3, 1\rangle \cdot \langle 2, 1, 1\rangle &= -4 + 3 + 1 \\ &= 0\end{aligned}$$

$$\begin{aligned}1 - 2t &= 1 + 2s \\ t &= -s\end{aligned}$$

$$\begin{aligned}3t &= -4 + s \\ t &= -\frac{4}{3} + \frac{s}{3} \\ &= -\frac{4}{3} - \frac{t}{3} \\ \frac{4}{3}t &= -\frac{4}{3} \\ t &= -1\end{aligned}$$

$$s = 1$$

$$\langle 3, -3, 0\rangle$$

1.8.43

$$\begin{aligned}\mathbf{u} &= \langle 1, 4, -2\rangle \\ \mathbf{v} &= \langle 1, 1, 3\rangle \\ \mathbf{n} &= \mathbf{u} \times \mathbf{v} \\ &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & -2 \\ 1 & 1 & 3\end{vmatrix} \\ &= \langle 14, -5, -3\rangle\end{aligned}$$

$$\begin{aligned}\mathbf{n} \cdot (\mathbf{r} - \mathbf{v}) &= 0 \\ \langle 14, -5, -3\rangle \cdot (\langle x, y, z\rangle - \langle 1, 1, 3\rangle) &= 0 \\ 14(x - 1) - 5(y - 1) - 3(z - 3) &= 0 \\ 14x - 5y - 3z &= 0\end{aligned}$$

1.8.45

$$\begin{aligned}\mathbf{F} &= \langle \frac{10}{\sqrt{2}}, \frac{10}{\sqrt{2}}, 0\rangle \\ \mathbf{d} &= \langle 3, 3, 0\rangle \\ \mathbf{F}\cdot\mathbf{d} &= 30\sqrt{2}\,\mathrm{J}\end{aligned}$$

1.8.47

$$\begin{aligned}\mathbf{F}_1 &= \langle 200, 0, 0\rangle \\ \mathbf{F}_2 &= \langle \frac{200}{\sqrt{2}}, \frac{200}{\sqrt{2}}, 0\rangle \\ \mathbf{F}_2 &= \mathbf{F}_1 + \mathbf{F}_3 \\ \mathbf{F}_3 &= \mathbf{F}_2 - \mathbf{F}_1 \\ &= \langle \frac{200}{\sqrt{2}}, \frac{200}{\sqrt{2}}, 0\rangle - \langle 200, 0, 0\rangle \\ &= \langle \frac{200}{\sqrt{2}} - 200, \frac{200}{\sqrt{2}}, 0\rangle \\ ||\mathbf{F}_3|| &= \sqrt{\left(\frac{200}{\sqrt{2}} - 200\right)^2 + \left(\frac{200}{\sqrt{2}}\right)^2} \\ &= \sqrt{\frac{40000}{2} - \frac{80000}{\sqrt{2}} + 40000 + \frac{40000}{2}} \\ &= 200\sqrt{2\left(1 - \frac{1}{\sqrt{2}}\right)} \\ &\approx 153\,\mathrm{lb}\end{aligned}$$

2 Matrices

2.1 Matrix Algebra

2.1.1

2×4

2.1.3

3×3

2.1.5

3×4

2.1.7

No

2.1.9

No

2.1.11

$$
\begin{aligned}
x &= y - 2 \\
3x - 2 &= y \\
\\
2x - 2 &= 2 \\
2x &= 4 \\
x &= 2 \\
\\
2 &= y - 2 \\
y &= 4
\end{aligned}
$$

2.1.13

$$
\begin{aligned}
c_{23} &= 9 \\
c_{12} &= 12
\end{aligned}
$$

2.1.15

(a) $\begin{pmatrix} 2 & 11 \\ 2 & -1 \end{pmatrix}$

(b) $\begin{pmatrix} -6 & 1 \\ 14 & -19 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 28 \\ 12 & -12 \end{pmatrix}$

2.1.17

(a) $\begin{pmatrix} -11 & 6 \\ 17 & -22 \end{pmatrix}$

(b) $\begin{pmatrix} -32 & 27 \\ -4 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 19 & -18 \\ -30 & 31 \end{pmatrix}$

(d) $\begin{pmatrix} 19 & 6 \\ 3 & 22 \end{pmatrix}$

2.1.21

(a) 180

(b) $\begin{pmatrix} 4 & 8 & 10 \\ 8 & 16 & 20 \\ 10 & 20 & 25 \end{pmatrix}$

(c) $\begin{pmatrix} 6 \\ 12 \\ -5 \end{pmatrix}$

2.1.23

(a) $\begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$

(b) $\begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$

2.1.25

$\begin{pmatrix} -14 \\ 1 \end{pmatrix}$

2.1.27

$\begin{pmatrix} -38 \\ -2 \end{pmatrix}$

2.1.29

4×5

2.1.41

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + x_{22}x_2 &= b_2 \end{aligned}$$

2.1.43

$$\begin{aligned} \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a & \frac{1}{2}b \\ \frac{1}{2}b & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} ax + \frac{1}{2}by \\ \frac{1}{2}bx + cy \end{pmatrix} \\ &= ax^2 + \frac{1}{2}bxy + \frac{1}{2}bxy + cy^2 \\ &= ax^2 + bxy + cy^2 \end{aligned}$$

2.1.45

$\langle -1, 1\rangle$

2.1.47

$\langle -2, 0\rangle$

2.1.49

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

2.1.51

(b)

$$\begin{pmatrix} x_S \\ y_S \\ z_S \end{pmatrix} = \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$