Advanced Engineering Mathematics Vectors, Matrices, and Vector Calculus by Dennis G. Zill Problems

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Contents

4

1   Vectors

1.1   Vectors in 2-Space

1.1.1

(a) $3\mathbf{a} = 6\mathbf{i} + 12\mathbf{j}$

(b) $\mathbf{a} + \mathbf{b} = \mathbf{i} + 8\mathbf{j}$

(c) $\mathbf{a} - \mathbf{b} = 3\mathbf{i}$

(d) $\|\mathbf{a} + \mathbf{b}\| = \sqrt{1 + 8^2} = \sqrt{65}$

(e) $\|\mathbf{a} - \mathbf{b}\| = 3$

1.1.9

(a) $4\mathbf{a} - 2\mathbf{b} = \langle 6, -14 \rangle$

(b) $-3\mathbf{a} - 5\mathbf{b} = \langle 2, 4 \rangle$

1.1.15

$\overrightarrow{P_1 P_2} = \langle 2, 5 \rangle$

1.1.19

$(1, 18)$

1.1.21

  (a) Yes

  (b) Yes

  (c) Yes

  (d) No

  (e) Yes

  (f) Yes

1.1.25

  (a) $\frac{\mathbf{a}}{||\mathbf{a}||} = \frac{\langle 2,2 \rangle}{\sqrt{2^2+2^2}} = \frac{1}{2\sqrt{2}}\langle 2,2 \rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

  (b) $\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$

1.1.31

$2\frac{\mathbf{a}}{||\mathbf{a}||} = 2\frac{\langle 3,7 \rangle}{\sqrt{3^2+7^2}} = \frac{2}{\sqrt{58}}\langle 3,7 \rangle = \langle \frac{6}{\sqrt{58}}, \frac{14}{\sqrt{58}} \rangle$

1.1.37

$\mathbf{x} = -(\mathbf{a} + \mathbf{b})$

1.1.41

$$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$$
$$\mathbf{b} = \mathbf{i} + \mathbf{j}$$
$$\mathbf{c} = \mathbf{i} - \mathbf{j}$$

$$\mathbf{i} = \frac{1}{2}(\mathbf{b} + \mathbf{c})$$
$$\mathbf{j} = \frac{1}{2}(\mathbf{b} - \mathbf{c})$$
$$\mathbf{a} = 2\left(\frac{1}{2}(\mathbf{b} + \mathbf{c})\right) + 3\left(\frac{1}{2}(\mathbf{b} - \mathbf{c})\right)$$
$$= \mathbf{b} + \mathbf{c} + \frac{3}{2}\mathbf{b} - \frac{3}{2}\mathbf{c}$$
$$= \frac{5}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}$$

1.1.43

$$y = \frac{1}{4}x^2 + 1$$
$$y(2) = 2$$
$$y' = \frac{1}{2}x$$
$$y'(2) = 1$$
$$\mathbf{v} = \pm\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$$

1.1.45

(a)

$$\mathbf{F}_n = \mathbf{F}\cos\theta$$
$$\mathbf{F}_g = \mathbf{F}\sin\theta$$
$$||\mathbf{F}_f|| = \mu||\mathbf{F}_n||$$
$$|| - \mathbf{F}_g|| = \mu||\mathbf{F}_n||$$
$$|| - \mathbf{F}\sin\theta|| = \mu||\mathbf{F}\cos\theta||$$
$$||\mathbf{F}||\sin\theta = \mu||\mathbf{F}||\cos\theta$$
$$\tan\theta = \mu$$

(b) $\theta = \arctan\mu \approx 31°$

1.1.47

$$F_x = \frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{L\,dy}{2a(L^2+y^2)^{3/2}}$$
$$= \frac{LqQ}{8\pi\epsilon_0}\int_{-a}^{a}(L^2+y^2)^{-3/2}\,dy$$
$$= \frac{LqQ}{8\pi\epsilon_0}\frac{2a}{L^2\sqrt{a^2+L^2}}$$
$$= \frac{aqQ}{4\pi\epsilon_0 L\sqrt{a^2+L^2}}$$
$$F_y = -\frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{y\,dy}{2a(L^2+y^2)^{3/2}}$$
$$= 0$$
$$\mathbf{F} = \langle \frac{1}{4\pi\epsilon_0}\frac{qQ}{L\sqrt{a^2+L^2}}, 0 \rangle$$

1.1.49

Let the three sides of the triangle be vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. The triangle is closed so it must be the case that

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0.$$

This gives

$$\mathbf{c} = -(\mathbf{a} + \mathbf{b}).$$

The vector from the midpoint of side $\mathbf{a}$ to the midpoint of side $\mathbf{b}$ is

$$\left(\mathbf{a} + \frac{1}{2}\mathbf{b}\right) - \frac{1}{2}\mathbf{a} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$$

which is parallel with $\mathbf{c}$ and half its length.

1.2   Vectors in 3-Space

1.2.7

A plane at $z = 5$ parellel with the x-y plane.

1.2.9

A line parallel to the z axis at $x = 2$ and $y = 3$.

1.2.13

(a) $(0, 5, 4)$, $(-2, 0, 4)$, $(-2, 5, 0)$

(b) $(-2, 5, -2)$

(c) $(3, 5, 4)$

1.2.15

The planes $x = 0$, $y = 0$, and $z = 0$.

1.2.17

$(-1, 2, -3)$

1.2.19

The planes $z = \pm 5$.

1.2.21

$$\sqrt{(6-3)^2 + (4+1)^2 + (8-2)^2} = \sqrt{9 + 25 + 36} = \sqrt{70}$$

1.2.31

$$\sqrt{(2-x)^2 + (1-2)^2 + (1-3)^2} = \sqrt{21}$$
$$(2-x)^2 + 1 + 4 = 21$$
$$(2-x)^2 = 16$$
$$2 - x = \pm 4$$
$$x = 2 \pm 4$$
$$= -2 \text{ or } 6$$

1.2.33

$\left(4, \frac{1}{2}, \frac{3}{2}\right)$

1.2.37

$(-3, -6, 1)$

1.3 Dot Product

1.3.1

$\mathbf{a} \cdot \mathbf{b} = 12$

1.3.11

$\left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{b} \cdot \mathbf{b}}\right) \mathbf{b} = \frac{12}{30} \mathbf{b} = \langle -\frac{2}{5}, \frac{4}{5}, 2 \rangle$

1.3.13

$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta = 25\sqrt{2}$

1.3.17

$$\mathbf{a} \cdot \mathbf{v} = 0$$
$$3x_1 + y_1 - 1 = 0$$

$$\mathbf{b} \cdot \mathbf{v} = 0$$
$$-3x_1 + 2y_2 + 2 = 0$$

$$3y_2 + 1 = 0$$
$$y_2 = -\frac{1}{3}$$

$$3x_1 - \frac{1}{3} - 1 = 0$$
$$x_1 = \frac{4}{9}$$

$$\mathbf{v} = \langle \frac{4}{9}, -\frac{1}{3}, 1 \rangle$$

1.3.19

$$\mathbf{a} \cdot \mathbf{c} = \mathbf{a} \cdot \left(\mathbf{b} - \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||^2} \mathbf{a} \right)$$
$$= \mathbf{a} \cdot \mathbf{b} - \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||^2} \mathbf{a} \cdot \mathbf{a}$$
$$= 0$$

1.3.21

$$||\mathbf{a}|| = \sqrt{3^2 + 1^2}$$
$$= \sqrt{10}$$
$$||\mathbf{b}|| = \sqrt{2^2 + 2^2}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$
$$\mathbf{a} \cdot \mathbf{b} = 4$$
$$\theta = \arccos \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||||\mathbf{b}||}$$
$$= \arccos \frac{4}{(\sqrt{10})(2\sqrt{2})}$$
$$= \arccos \frac{1}{\sqrt{5}}$$
$$\approx 63°$$

1.3.25

$$||\mathbf{a}|| = \sqrt{1^2 + 2^2 + 3^3}$$
$$= \sqrt{14}$$
$$\cos \alpha = \frac{1}{\sqrt{14}}$$
$$\alpha \approx 75°$$
$$\cos \beta = \frac{2}{\sqrt{14}}$$
$$\beta \approx 58°$$
$$\cos \gamma = \frac{3}{\sqrt{14}}$$
$$\gamma \approx 37°$$

1.3.29

$$\overrightarrow{AD} = \langle s, -s, s \rangle$$
$$||\overrightarrow{AD}|| = s\sqrt{3}$$
$$\overrightarrow{AB} = \langle s, 0, 0 \rangle$$
$$||\overrightarrow{AB}|| = s$$
$$\theta = \arccos \frac{\overrightarrow{AD} \cdot \overrightarrow{AB}}{||\overrightarrow{AD}||||\overrightarrow{AB}||}$$
$$= \arccos \frac{s^2}{s^2\sqrt{3}}$$
$$= \arccos \frac{1}{\sqrt{3}}$$
$$\approx 55°$$

1.3.33

$$\text{comp}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{b}||}$$
$$= \frac{5}{7}$$

1.3.37

$$\text{comp}_{\overrightarrow{OP}}\mathbf{a} = \frac{\mathbf{a} \cdot \overrightarrow{OP}}{||\overrightarrow{OP}||}$$
$$= \frac{72}{\sqrt{109}}$$

1.3.39

$$\text{proj}_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{b} \cdot \mathbf{b}} \right) \mathbf{b}$$
$$= \frac{35}{25}\mathbf{b}$$
$$= \langle -\frac{21}{5}, \frac{28}{5} \rangle$$

1.3.43

$$\mathbf{a} + \mathbf{b} = \langle 3, 4 \rangle$$

$$\text{proj}_{\mathbf{a}+\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a} \cdot (\mathbf{a}+\mathbf{b})}{(\mathbf{a}+\mathbf{b}) \cdot (\mathbf{a}+\mathbf{b})} \right) (\mathbf{a}+\mathbf{b})$$

$$= \frac{24}{25}(\mathbf{a}+\mathbf{b})$$

$$= \langle \frac{72}{25}, \frac{96}{25} \rangle$$

1.3.45

$W = \mathbf{F} \cdot \mathbf{d} = Fd\cos\theta = 1000$

1.3.47

(a) $W = 0$

(b)

$$\|\mathbf{d}\| = \sqrt{4^2 + 3^2}$$

$$= 5$$

$$\mathbf{F} = F\hat{\mathbf{d}}$$

$$= F\frac{\mathbf{d}}{\|\mathbf{d}\|}$$

$$= F\langle \frac{4}{5}, \frac{3}{5} \rangle$$

$$= \langle 24, 18 \rangle$$

$$W = \mathbf{F} \cdot \mathbf{d}$$

$$= 150\,\text{J}$$

1.4   Cross Product

1.4.1

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ 0 & 3 & 5 \end{vmatrix}$$

$$= -5\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$$

1.4.11

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 2 & -4 \\ -3 & 1 & 1 \end{vmatrix}$$
$$= 6\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$$

1.4.17

(a)

$$\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix}$$
$$= \mathbf{j} - \mathbf{k}$$
$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 0 & 1 & -1 \end{vmatrix}$$
$$= -\mathbf{i} + \mathbf{j} + \mathbf{k}$$

1.4.19

$2\mathbf{k}$

1.4.21

$$\mathbf{k} \times (2\mathbf{i} - \mathbf{j}) = (\mathbf{k} \times 2\mathbf{i}) - (\mathbf{k} \times \mathbf{j})$$
$$= \mathbf{i} + 2\mathbf{j}$$

1.4.23

$$[(2\mathbf{k}) \times (3\mathbf{j})] \times (4\mathbf{j}) = (-6\mathbf{i}) \times (4\mathbf{j})$$
$$= -24\mathbf{k}$$

1.4.37

$12\mathbf{i} - 9\mathbf{j} + 18\mathbf{k}$

1.4.53

$$\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 6 & -6 \\ \frac{5}{2} & 3 & \frac{1}{2} \end{vmatrix}$$
$$= 21\mathbf{i} - 14\mathbf{j} - 21\mathbf{k}$$
$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 4 \times 21 + 6 \times (-14)$$
$$= 0$$

They are coplanar.

1.5 Lines and Planes in 3-Space

1.5.1

$\mathbf{r} = \langle 1, 2, 1 \rangle + t\langle 2, 3, -3 \rangle$

1.5.7

$$x = 2 + 4t$$
$$y = 3 - 4t$$
$$z = 5 + 3t$$

1.5.13

$$x = 1 + 9t$$
$$y = 4 + 10t$$
$$z = -9 + 7t$$
$$\frac{x-1}{9} = \frac{y-4}{10} = \frac{z+9}{7}$$

1.5.19

$$x = 4 + 3t$$
$$y = 6 + \frac{1}{2}t$$
$$z = -7 - \frac{3}{2}t$$
$$\frac{x-4}{3} = \frac{y-6}{1/2} = -\frac{z+7}{3/2}$$

1.5.23

$$x = 6 + 2t$$
$$y = 4 - 3t$$
$$z = -2 + 6t$$

1.5.25

$$x = 2 + t$$
$$y = -2$$
$$z = 15$$

1.5.29

$(0, 5, 15)$, $(5, 0, \frac{15}{2})$, $(10, -5, 0)$

1.5.31

$$4 + t_x = 6 + 2t_x$$
$$t_x = -2$$

$$5 + t_y = 11 + 4t_y$$
$$t_y = -2$$

$$-1 + 2t_z = -3 + t_z$$
$$t_z = -2$$

$(2, 3, -5)$

1.5.35

$$\mathbf{a} = \langle -1, 2, -2 \rangle$$
$$\|\mathbf{a}\| = 3$$
$$\mathbf{b} = \langle 2, 3, -6 \rangle$$
$$\|\mathbf{b}\| = 7$$
$$\theta = \arccos \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}$$
$$\approx 40.37°$$

1.5.37

$$\mathbf{a} = \langle 1, 1, 1 \rangle$$
$$\mathbf{b} = \langle -2, 1, -5 \rangle$$
$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ -2 & 1 & -5 \end{vmatrix}$$
$$= \langle -6, 3, 3 \rangle$$
$$x = 4 - 6t$$
$$y = 1 + 3t$$
$$z = 6 + 3t$$

1.5.39

$$\langle 2, -3, 4 \rangle \cdot (\mathbf{r} - \langle 5, 1, 3 \rangle) = 0$$
$$2(x - 5) - 3(y - 1) + 4(z - 3) = 0$$
$$2x - 3y + 4z - 19 = 0$$

1.5.45

$$\mathbf{a} = \langle 3, 5, 2 \rangle$$
$$\mathbf{b} = \langle 2, 3, 1 \rangle$$
$$\mathbf{c} = \langle -1, -1, 4 \rangle$$
$$\mathbf{a} - \mathbf{c} = \langle 4, 6, -2 \rangle$$
$$\mathbf{b} - \mathbf{c} = \langle 3, 4, -3 \rangle$$
$$(\mathbf{a} - \mathbf{c}) \times (\mathbf{b} - \mathbf{c}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 6 & -2 \\ 3 & 4 & -3 \end{vmatrix}$$
$$= \langle -10, 6, -2 \rangle$$
$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{c}) = 0$$
$$\langle -10, 6, -2 \rangle \cdot (\langle x, y, z \rangle - \langle -1, -1, 4 \rangle) = 0$$
$$-10(x + 1) + 6(y + 1) - 2(z - 4) = 0$$
$$-10x + 6y - 2z + 4 = 0$$

1.5.51

$$\langle 1, 1, -4 \rangle \cdot (\mathbf{r} - \langle 2, 3, -5 \rangle) = 0$$
$$(x - 2) + (y - 3) - 4(z + 5) = 0$$
$$x + y - 4z = 25$$

1.5.63

  (a) Not perpendicular

  (b) Not perpendicular

  (c) Perpendicular

  (d) Perpendicular

1.5.65

$$5x - 4y - 9t = 8$$
$$x + 4y + 3t = 4$$

$$6x - 6t = 12$$
$$x = 2 + t$$

$$y = \frac{1}{2} - t$$

$$z = t$$

1.5.69

$$2(1 + 2t) - 3(2 - t) + 2(-3t) = -7$$
$$t = -3$$
$$x = -5$$
$$y = 5$$
$$z = 9$$

1.5.73

$$x + y - 4t = 2$$
$$2x - y + t = 10$$

$$3x - 3t = 12$$
$$x = 4 + t$$

$$2(4 + t) - y + t = 10$$
$$8 + 2t - y + t = 10$$
$$y = -2 + 3t$$

$$z = t$$

$$x = 5 + t$$
$$y = 6 + 3t$$
$$z = -12 + t$$

1.5.75

$$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 5 \\ 1 & 1 & 1 \end{vmatrix}$$
$$= \langle -6, 2, 4 \rangle$$
$$\mathbf{n} \cdot (\mathbf{r} - \langle 4, 0, 1 \rangle) = 0$$
$$-6(x - 4) + 2y + 4(z - 1) = 0$$
$$-6x + 2y + 4z = -20$$
$$3x - y - 2z = 10$$

1.6   Vector Spaces

1.6.1

Violates axiom 6

1.6.3

Violates axiom 10

1.6.5

Vector space

1.6.7

Violates axiom 2

1.6.9

Vector space

1.6.11

Subspace

1.6.13

Not a subspace

1.6.15

Subspace

1.6.17

Subspace

1.6.19

Not a subspace

1.6.23

(a)

$$k_1\mathbf{u}_1 + k_2\mathbf{u}_2 + k_3\mathbf{u}_3 = \mathbf{0}$$
$$k_1\langle 1,0,0 \rangle + k_2\langle 1,1,0 \rangle + k_3\langle 1,1,1 \rangle = \mathbf{0}$$

$$k_3 = 0$$

$$k_2 + k_3 = 0$$
$$k_2 = 0$$

$$k_1 + k_2 + k_3 = 0$$
$$k_1 = 0$$

(b)

$$\mathbf{a} = 7\mathbf{u}_1 - 12\mathbf{u}_2 + 8\mathbf{u}_3$$

1.6.25

Dependent

1.6.27

Independent

1.6.29

$f(x)$ is undefined at $x = -3$ and $x = -1$.

1.6.31

$$
\begin{aligned}
||x|| &= \sqrt{(x, x)} \\
&= \sqrt{\int_0^{2\pi} x^2 \, dx} \\
&= \sqrt{\left[\frac{1}{3}x^3\right]_0^{2\pi}} \\
&= \sqrt{\frac{8}{3}\pi^3} \\
||\sin x|| &= \sqrt{(\sin x, \sin x)} \\
&= \sqrt{\int_0^{2\pi} \sin^2 x \, dx} \\
&= \sqrt{\left[\frac{x}{2} - \frac{1}{4}\sin 2x\right]_0^{2\pi}} \\
&= \sqrt{\pi}
\end{aligned}
$$

1.7 Gram–Schmidt Orthogonalization Process

1.7.1

$$\frac{\langle \frac{12}{13}, \frac{5}{13} \rangle \cdot \langle \frac{5}{13}, -\frac{12}{13} \rangle = 0}{\sqrt{\left(\frac{12}{13}\right)^2 + \left(\frac{5}{13}\right)^2} = 1}$$

$$\mathbf{u} = \left(\langle 4, 2 \rangle \cdot \langle \frac{12}{13}, \frac{5}{13} \rangle \right) \langle \frac{12}{13}, \frac{5}{13} \rangle$$

$$+ \left(\langle 4, 2 \rangle \cdot \langle \frac{5}{13}, -\frac{12}{13} \rangle \right) \langle \frac{5}{13}, -\frac{12}{13} \rangle$$

$$= \left(\frac{58}{13} \right) \langle \frac{12}{13}, \frac{5}{13} \rangle - \left(\frac{4}{13} \right) \langle \frac{5}{13}, -\frac{12}{13} \rangle$$

1.7.3

$$\langle 1, 0, 1 \rangle \cdot \langle 0, 1, 0 \rangle = 0$$
$$\langle 1, 0, 1 \rangle \cdot \langle -1, 0, 1 \rangle = 0$$
$$\langle 0, 1, 0 \rangle \cdot \langle -1, 0, 1 \rangle = 0$$

$$B' = \{ \langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle, \langle 0, 1, 0 \rangle, \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle \}$$

$$\mathbf{u} = -\frac{3}{\sqrt{2}} \langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle + 7 \langle 0, 1, 0 \rangle - \frac{23}{\sqrt{2}} \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$$

1.7.5

(a)

$$B = \{\langle -3, 2\rangle, \langle -1, -1\rangle\}$$

$$\mathbf{v}_1 = \mathbf{u}_1$$

$$= \langle -3, 2\rangle$$

$$\mathbf{v}_2 = \mathbf{u}_2 - \text{proj}_{\mathbf{v}_1}\mathbf{u}_2$$

$$= \langle -1, -1\rangle - \left(\frac{\langle -1, -1\rangle \cdot \langle -3, 2\rangle}{\langle -3, 2\rangle \cdot \langle -3, 2\rangle}\right)\langle -3, 2\rangle$$

$$= \langle -1, -1\rangle - \frac{1}{13}\langle -3, 2\rangle$$

$$= \langle -\frac{10}{13}, -\frac{15}{13}\rangle$$

$$\mathbf{w}_1 = \langle -\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\rangle$$

$$\mathbf{w}_2 = \sqrt{\frac{169}{325}}\langle -\frac{10}{13}, -\frac{15}{13}\rangle$$

$$= \frac{\sqrt{13}}{5}\langle -\frac{10}{13}, -\frac{15}{13}\rangle$$

$$= \langle -\frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}\rangle$$

1.7.9

$$B = \{\langle 1, 1, 0 \rangle, \langle 1, 2, 2 \rangle, \langle 2, 2, 1 \rangle\}$$

$$\mathbf{v}_1 = \langle 1, 1, 0 \rangle$$

$$\mathbf{v}_2 = \mathbf{u}_2 - \text{proj}_{\mathbf{v}_1} \mathbf{u}_2$$

$$= \langle 1, 2, 2 \rangle - \left(\frac{\langle 1, 2, 2 \rangle \cdot \langle 1, 1, 0 \rangle}{\langle 1, 1, 0 \rangle \cdot \langle 1, 1, 0 \rangle} \right) \langle 1, 1, 0 \rangle$$

$$= \langle 1, 2, 2 \rangle - \frac{3}{2} \langle 1, 1, 0 \rangle$$

$$= \langle -\frac{1}{2}, \frac{1}{2}, 2 \rangle$$

$$\mathbf{v}_3 = \mathbf{u}_3 - \text{proj}_{\mathbf{v}_1} \mathbf{u}_3 - \text{proj}_{\mathbf{v}_2} \mathbf{u}_3$$

$$= \langle 2, 2, 1 \rangle - \left(\frac{\langle 2, 2, 1 \rangle \cdot \langle 1, 1, 0 \rangle}{\langle 1, 1, 0 \rangle \cdot \langle 1, 1, 0 \rangle} \right) \langle 1, 1, 0 \rangle$$

$$- \left(\frac{\langle 2, 2, 1 \rangle \cdot \langle -\frac{1}{2}, \frac{1}{2}, 2 \rangle}{\langle -\frac{1}{2}, \frac{1}{2}, 2 \rangle \cdot \langle -\frac{1}{2}, \frac{1}{2}, 2 \rangle} \right) \langle -\frac{1}{2}, \frac{1}{2}, 2 \rangle$$

$$= \langle 2, 2, 1 \rangle - 2 \langle 1, 1, 0 \rangle - \frac{4}{9} \langle -\frac{1}{2}, \frac{1}{2}, 2 \rangle$$

$$= \langle \frac{2}{9}, -\frac{2}{9}, \frac{1}{9} \rangle$$

$$\mathbf{w}_1 = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \rangle$$

$$\mathbf{w}_2 = \langle -\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}} \rangle$$

$$\mathbf{w}_3 = 3 \langle \frac{2}{9}, -\frac{2}{9}, \frac{1}{9} \rangle$$

$$= \langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \rangle$$

1.7.17

$$B = \{1, x, x^2\}$$

$$\mathbf{v}_1 = \mathbf{u}_1$$

$$= 1$$

$$\mathbf{v}_2 = \mathbf{u}_2 - \text{proj}_{\mathbf{v}_1} \mathbf{u}_2$$

$$= \mathbf{u}_2 - \left(\frac{\mathbf{u}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1} \right) \mathbf{v}_1$$

$$= x - \frac{\int_{-1}^{1} x \, dx}{\int_{-1}^{1} dx}$$

$$= x - \frac{\left[\frac{1}{2} x^2 \right]_{-1}^{1}}{2}$$

$$= x$$

$$\mathbf{v}_3 = \mathbf{u}_3 - \text{proj}_{\mathbf{v}_1} \mathbf{u}_3 - \text{proj}_{\mathbf{v}_2} \mathbf{u}_3$$

$$= \mathbf{u}_3 - \left(\frac{\mathbf{u}_3 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1} \right) \mathbf{v}_1 - \left(\frac{\mathbf{u}_3 \cdot \mathbf{v}_2}{\mathbf{v}_2 \cdot \mathbf{v}_2} \right) \mathbf{v}_2$$

$$= x^2 - \frac{\int_{-1}^{1} x^2 \, dx}{\int_{-1}^{1} dx} - \frac{\int_{-1}^{1} x^3 \, dx}{\int_{-1}^{1} x^2 \, dx} x$$

$$= x^2 - \frac{\left[\frac{1}{3} x^3 \right]_{-1}^{1}}{2} - \frac{\left[\frac{1}{4} x^4 \right]_{-1}^{1}}{\left[\frac{1}{3} x^3 \right]_{-1}^{1}} x$$

$$= x^2 - \frac{1}{3}$$

1.7.19

$$||\mathbf{v}_1||^2 = \int_{-1}^{1} dx$$

$$= 2$$

$$\mathbf{w}_1 = \frac{1}{\sqrt{2}}$$

$$||\mathbf{v}_2||^2 = \int_{-1}^{1} x^2 \, dx$$

$$= \left[\frac{1}{3}x^3 \right]_{-1}^{1}$$

$$= \frac{2}{3}$$

$$\mathbf{w}_2 = \frac{3}{\sqrt{6}}x$$

$$||\mathbf{v}_3||^2 = \int_{-1}^{1} \left(x^2 - \frac{1}{3} \right)^2 dx$$

$$= \int_{-1}^{1} \left(x^4 - \frac{2}{3}x^2 + \frac{1}{9} \right) dx$$

$$= \left[\frac{1}{5}x^5 - \frac{2}{9}x^3 + \frac{1}{9}x \right]_{-1}^{1}$$

$$= \frac{1}{5} - \frac{2}{9} + \frac{1}{9} + \frac{1}{5} - \frac{2}{9} + \frac{1}{9}$$

$$= \frac{2}{5} - \frac{2}{9}$$

$$= \frac{8}{45}$$

$$\mathbf{w}_3 = \sqrt{\frac{45}{8}} \left(x^2 - \frac{1}{3} \right)$$

$$= \frac{5}{2\sqrt{10}} \left(3x^2 - 1 \right)$$

1.7.21

$$(\mathbf{p}, \mathbf{w}_1) = \int_{-1}^{1} \frac{1}{\sqrt{2}}(9x^2 - 6x + 5)\, dx$$

$$= \frac{1}{\sqrt{2}}\left[3x^3 - 3x^2 + 5x\right]_{-1}^{1}$$

$$= \frac{1}{\sqrt{2}}(3 - 3 + 5 + 3 + 3 + 5)$$

$$= \frac{16}{\sqrt{2}}$$

$$(\mathbf{p}, \mathbf{w}_2) = \int_{-1}^{1} \frac{3}{\sqrt{6}}x(9x^2 - 6x + 5)\, dx$$

$$= \frac{3}{\sqrt{6}}\left[\frac{9}{4}x^4 - 2x^3 + \frac{5}{2}x^2\right]_{-1}^{1}$$

$$= \frac{3}{\sqrt{6}}\left(\frac{9}{4} - 2 + \frac{5}{2} - \frac{9}{4} - 2 - \frac{5}{2}\right)$$

$$= \frac{3}{\sqrt{6}}\left(\frac{9}{4} - \frac{8}{4} + \frac{10}{4} - \frac{9}{4} - \frac{8}{4} - \frac{10}{4}\right)$$

$$= -\frac{12}{\sqrt{6}}$$

$$(\mathbf{p}, \mathbf{w}_3) = \int_{-1}^{1} \frac{5}{2\sqrt{10}}(3x^2 - 1)(9x^2 - 6x + 5)\, dx$$

$$= \frac{5}{2\sqrt{10}}\int_{-1}^{1}(27x^4 - 18x^3 + 6x^2 + 6x - 5)\, dx$$

$$= \frac{5}{2\sqrt{10}}\left[\frac{27}{5}x^5 - \frac{9}{2}x^4 + 2x^3 + 3x^2 - 5x\right]_{-1}^{1}$$

$$= \frac{5}{2\sqrt{10}}\left(\frac{27}{5} - \frac{9}{2} + 2 + 3 - 5 + \frac{27}{5} + \frac{9}{2} + 2 - 3 - 5\right)$$

$$= \frac{5}{2\sqrt{10}}\left(\frac{54}{10} - \frac{45}{10} + \frac{20}{10} + \frac{30}{10} - \frac{50}{10} + \frac{54}{10} + \frac{45}{10} + \frac{20}{10} - \frac{30}{10} - \frac{50}{10}\right)$$

$$= \frac{5}{2\sqrt{10}}\frac{48}{10}$$

$$= \frac{12}{\sqrt{10}}$$

$$\mathbf{p} = \frac{16}{\sqrt{2}}\mathbf{w}_1 - \frac{12}{\sqrt{6}}\mathbf{w}_2 + \frac{12}{\sqrt{10}}\mathbf{w}_3$$

1.8   Chapter in Review

1.8.1

True

1.8.3

$$\mathbf{u} = \langle 5, -2, 1 \rangle$$
$$\mathbf{v} = \langle 2, 3, -4 \rangle$$

False

1.8.5

True

1.8.7

True

1.8.9

True

1.8.11

$9\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

1.8.13

$$(-\mathbf{k}) \times (5\mathbf{j}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1 \\ 0 & 5 & 0 \end{vmatrix}$$
$$= 5\mathbf{i}$$

1.8.15

$$|| - 12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}|| = \sqrt{12^2 + 4^2 + 6^2} = 14$$

1.8.17

$\langle -6, 1, -7 \rangle$

1.8.19

$$x = 1 + t$$
$$y = -2 + 3t$$
$$z = -1 + 2t$$

$$x + 2y - z = 13$$
$$(1 + t) + 2(-2 + 3t) - (-1 + 2t) = 13$$
$$1 + t - 4 + 6t + 1 - 2t = 13$$
$$-2 + 5t = 13$$
$$t = 3$$

$$x = 4$$
$$y = 7$$
$$z = 5$$

1.8.21

$$\overrightarrow{P_1P_2} = \overrightarrow{P_2} - \overrightarrow{P_1}$$
$$\overrightarrow{P_2} = \overrightarrow{P_1P_2} + \overrightarrow{P_1}$$
$$= \langle 3, 5, -4 \rangle + \langle 2, 1, 7 \rangle$$
$$= \langle 5, 6, 3 \rangle$$

1.8.23

$\mathbf{a} \cdot \mathbf{b} = -36\sqrt{2}$

1.8.25

$x = 12$, $y = -8$, $z = 6$

1.8.27

$$\frac{1}{2}(\mathbf{a} \times \mathbf{b}) = \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -1 \\ 2 & -1 & 2 \end{vmatrix}$$
$$= \frac{1}{2}\langle 5, -4, -7 \rangle$$
$$= \langle \frac{5}{2}, -2, -\frac{7}{2} \rangle$$

The area is $\sqrt{\left(\frac{5}{2}\right)^2 + (-2)^2 + \left(-\frac{7}{2}\right)^2} = \frac{3}{2}\sqrt{10}$

1.8.29

2

1.8.31

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 1 & -2 & 1 \end{vmatrix}$$
$$= \langle 1, -1, -3 \rangle$$
$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{11}$$
$$\text{norm}(\mathbf{a} \times \mathbf{b}) = \langle \frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, -\frac{3}{\sqrt{11}} \rangle$$

1.8.33

$$\text{comp}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|} = \frac{10}{5} = 2$$

1.8.35

$$\mathbf{a} = \langle 1, 2, -2 \rangle$$
$$\mathbf{b} = \langle 4, 3, 0 \rangle$$
$$\mathbf{a} + \mathbf{b} = \langle 5, 5, -2 \rangle$$
$$\text{proj}_{\mathbf{a}}(\mathbf{a} + \mathbf{b}) = \left(\frac{(\mathbf{a} + \mathbf{b}) \cdot \mathbf{a}}{\mathbf{a} \cdot \mathbf{a}} \right) \mathbf{a}$$
$$= \frac{19}{9} \langle 1, 2, -2 \rangle$$
$$= \langle \frac{19}{9}, \frac{38}{9}, -\frac{38}{9} \rangle$$

1.8.37

(a)

(b) A plane with normal $\mathbf{a}$

1.8.39

$$\frac{x - 7}{4} = \frac{y - 3}{-2} = \frac{z + 5}{6}$$

1.8.41

$$\langle -2, 3, 1 \rangle \cdot \langle 2, 1, 1 \rangle = -4 + 3 + 1$$
$$= 0$$

$$1 - 2t = 1 + 2s$$
$$t = -s$$

$$3t = -4 + s$$
$$t = -\frac{4}{3} + \frac{s}{3}$$
$$= -\frac{4}{3} - \frac{t}{3}$$
$$\frac{4}{3}t = -\frac{4}{3}$$
$$t = -1$$

$$s = 1$$

$$\langle 3, -3, 0 \rangle$$

1.8.43

$$\mathbf{u} = \langle 1, 4, -2 \rangle$$
$$\mathbf{v} = \langle 1, 1, 3 \rangle$$
$$\mathbf{n} = \mathbf{u} \times \mathbf{v}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & -2 \\ 1 & 1 & 3 \end{vmatrix}$$
$$= \langle 14, -5, -3 \rangle$$

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{v}) = 0$$
$$\langle 14, -5, -3 \rangle \cdot (\langle x, y, z \rangle - \langle 1, 1, 3 \rangle) = 0$$
$$14(x - 1) - 5(y - 1) - 3(z - 3) = 0$$
$$14x - 5y - 3z = 0$$

1.8.45

$$\mathbf{F} = \langle \frac{10}{\sqrt{2}}, \frac{10}{\sqrt{2}}, 0 \rangle$$
$$\mathbf{d} = \langle 3, 3, 0 \rangle$$
$$\mathbf{F} \cdot \mathbf{d} = 30\sqrt{2}\,\text{J}$$

1.8.47

$$\mathbf{F}_1 = \langle 200, 0, 0 \rangle$$
$$\mathbf{F}_2 = \langle \frac{200}{\sqrt{2}}, \frac{200}{\sqrt{2}}, 0 \rangle$$
$$\mathbf{F}_2 = \mathbf{F}_1 + \mathbf{F}_3$$
$$\mathbf{F}_3 = \mathbf{F}_2 - \mathbf{F}_1$$
$$= \langle \frac{200}{\sqrt{2}}, \frac{200}{\sqrt{2}}, 0 \rangle - \langle 200, 0, 0 \rangle$$
$$= \langle \frac{200}{\sqrt{2}} - 200, \frac{200}{\sqrt{2}}, 0 \rangle$$
$$||\mathbf{F}_3|| = \sqrt{\left(\frac{200}{\sqrt{2}} - 200\right)^2 + \left(\frac{200}{\sqrt{2}}\right)^2}$$
$$= \sqrt{\frac{40000}{2} - \frac{80000}{\sqrt{2}} + 40000 + \frac{40000}{2}}$$
$$= 200\sqrt{2\left(1 - \frac{1}{\sqrt{2}}\right)}$$
$$\approx 153\,\text{lb}$$

2 Matrices

2.1 Matrix Algebra

2.1.1

2×4

2.1.3

3×3

2.1.5

3×4

2.1.7

No

2.1.9

No

2.1.11

$$x = y - 2$$
$$3x - 2 = y$$

$$2x - 2 = 2$$
$$2x = 4$$
$$x = 2$$

$$2 = y - 2$$
$$y = 4$$

2.1.13

$$c_{23} = 9$$
$$c_{12} = 12$$

2.1.15

(a) $\begin{pmatrix} 2 & 11 \\ 2 & -1 \end{pmatrix}$

(b) $\begin{pmatrix} -6 & 1 \\ 14 & -19 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 28 \\ 12 & -12 \end{pmatrix}$

2.1.17

(a) $\begin{pmatrix} -11 & 6 \\ 17 & -22 \end{pmatrix}$

(b) $\begin{pmatrix} -32 & 27 \\ -4 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 19 & -18 \\ -30 & 31 \end{pmatrix}$

(d) $\begin{pmatrix} 19 & 6 \\ 3 & 22 \end{pmatrix}$

2.1.21

(a) 180

(b) $\begin{pmatrix} 4 & 8 & 10 \\ 8 & 16 & 20 \\ 10 & 20 & 25 \end{pmatrix}$

(c) $\begin{pmatrix} 6 \\ 12 \\ -5 \end{pmatrix}$

2.1.23

(a) $\begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$

(b) $\begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$

2.1.25

$\begin{pmatrix} -14 \\ 1 \end{pmatrix}$

2.1.27

$\begin{pmatrix} -38 \\ -2 \end{pmatrix}$

2.1.29

4×5

2.1.41

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + x_{22}x_2 = b_2$$

2.1.43

$$(x \quad y) \begin{pmatrix} a & \frac{1}{2}b \\ \frac{1}{2}b & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = (x \quad y) \begin{pmatrix} ax + \frac{1}{2}by \\ \frac{1}{2}bx + cy \end{pmatrix}$$
$$= ax^2 + \frac{1}{2}bxy + \frac{1}{2}bxy + cy^2$$
$$= ax^2 + bxy + cy^2$$

2.1.45

$\langle -1, 1 \rangle$

2.1.47

$\langle -2, 0 \rangle$

2.1.49

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

2.1.51

(b)

$$\begin{pmatrix} x_S \\ y_S \\ z_S \end{pmatrix} = \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

2.2 Systems of Linear Algebraic Equations

2.2.1

$$\begin{pmatrix} 1 & -1 & | & 11 \\ 4 & 3 & | & -5 \end{pmatrix}$$
$$\begin{pmatrix} 1 & -1 & | & 11 \\ 0 & 7 & | & -49 \end{pmatrix}$$
$$\begin{pmatrix} 1 & -1 & | & 11 \\ 0 & 1 & | & -7 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & | & 4 \\ 0 & 1 & | & -7 \end{pmatrix}$$

2.2.5

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 2 & 3 & 5 & 7 \\ 1 & -2 & 3 & -11 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 5 & 7 & 13 \\ 0 & -1 & 4 & -8 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 1 & \frac{7}{5} & \frac{13}{5} \\ 0 & -1 & 4 & -8 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 1 & \frac{7}{5} & \frac{13}{5} \\ 0 & 0 & \frac{27}{5} & -\frac{27}{5} \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 1 & \frac{7}{5} & \frac{13}{5} \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

2.3 Rank of a Matrix

2.3.1

$$\begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -\frac{1}{3} \\ 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -\frac{1}{3} \\ 0 & \frac{10}{3} \end{pmatrix}$$

Rank 2

2.3.3

$$\begin{pmatrix} 2 & 1 & 3 \\ 6 & 3 & 9 \\ -1 & -\frac{1}{2} & -\frac{3}{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 6 & 3 & 9 \\ -1 & -\frac{1}{2} & -\frac{3}{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Rank 1

2.3.5

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 4 \\ 1 & 4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 3 \\ 0 & 3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & 3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

Rank 3

2.3.7

$$\begin{pmatrix} 1 & -2 \\ 3 & -6 \\ 7 & -1 \\ 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 \\ 0 & 13 \\ 0 & 13 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 \\ 0 & 13 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Rank 2

2.3.11

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & -1 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & -2 \\ 0 & -3 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -3 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 5 \end{pmatrix}$$

Linearly independent

2.3.15

5

2.3.17

$\mathrm{rank}(\mathbf{A}) = 2$

2.4 Determinants

2.4.1

9

2.4.3

1

2.4.5

$$M_{33} = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}$$

$$= 2$$

2.4.7

$$C_{34} = (-1)^{3+4} \begin{vmatrix} 0 & 2 & 4 \\ 1 & 2 & -2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= -\left(-2 \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} \right)$$

$$= 10$$

2.4.9

-7

2.4.11

17

2.4.13

$$(1 - \lambda)(2 - \lambda) - 6 = 2 - \lambda - 2\lambda + \lambda^2 - 6$$

$$= \lambda^2 - 3\lambda - 4$$

$$= (\lambda + 1)(\lambda - 4)$$

2.4.15

-48

2.4.23

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ 2 & 3 & 4 \end{vmatrix} = \begin{vmatrix} y & z \\ 3 & 4 \end{vmatrix} - \begin{vmatrix} x & z \\ 2 & 4 \end{vmatrix} + \begin{vmatrix} x & y \\ 2 & 3 \end{vmatrix}$$

$$= 4y - 3z - 4x + 2z + 3x - 2y$$
$$= -x + 2y - z$$

2.4.29

$$\begin{vmatrix} (-3 - \lambda) & 10 \\ 2 & (5 - \lambda) \end{vmatrix} = 0$$

$$(-3 - \lambda)(5 - \lambda) - 20 = 0$$
$$-15 + 3\lambda - 5\lambda + \lambda^2 - 20 = 0$$
$$\lambda^2 - 2\lambda - 35 = 0$$
$$(\lambda - 7)(\lambda + 5) = 0$$
$$\lambda = -5 \text{ or } 7$$

2.5 Properties of Determinants

2.5.1

8.5.4

2.5.3

8.5.7

2.5.5

8.5.5

2.5.7

8.5.3

2.5.9

8.5.1

2.5.11

-5

2.5.13

-5

2.5.15

5

2.5.17

80

2.5.19

-105

2.5.25

$$\mathbf{A}\mathbf{A} = \mathbf{I}$$
$$\det \mathbf{A} \cdot \det \mathbf{A} = \det \mathbf{I}$$
$$(\det \mathbf{A})^2 = 1$$
$$\det \mathbf{A} = \pm 1$$

2.5.27

$$\begin{vmatrix} a & a+1 & a+2 \\ b & b+1 & b+2 \\ c & c+1 & c+2 \end{vmatrix} = \begin{vmatrix} a & 1 & 2 \\ b & 1 & 2 \\ c & 1 & 2 \end{vmatrix}$$
$$= \begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix}$$
$$= 0$$

2.5.29

$$\begin{vmatrix} 1 & 1 & 5 \\ 4 & 3 & 6 \\ 0 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 5 \\ 0 & -1 & -14 \\ 0 & -1 & 1 \end{vmatrix}$$
$$= - \begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 14 \\ 0 & 0 & 15 \end{vmatrix}$$
$$= -15$$

2.5.37

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & c^2-a^2-(b+a)(c-a) \end{vmatrix}$$

$$= (b-a)(c^2-a^2-(b+a)(c-a))$$
$$= (b-a)(c^2-a^2-bc+ab-ac+a^2)$$
$$= (b-a)(c^2+ab-ac-bc)$$
$$= (b-a)(c-a)(c-b)$$

2.5.39

$$a_{21}C_{11}+a_{22}C_{12}+a_{23}C_{13} = (-1)(4)+(2)(5)+(1)(-6)$$
$$= 0$$
$$a_{13}C_{12}+a_{23}C_{22}+a_{33}C_{32} = (2)(5)+(1)(-7)+(1)(-3)$$
$$= 0$$

2.5.41

$$\mathbf{A}+\mathbf{B} = \begin{pmatrix} 10 & 0 \\ 0 & -3 \end{pmatrix}$$
$$\det(\mathbf{A}+\mathbf{B}) = -30$$
$$\det\mathbf{A} = 10$$
$$\det\mathbf{B} = -31$$
$$-30 \neq 10-31$$