Contents

1 First-order ODEs

- **Form:** IVP

$$\frac{dy}{dx} = f(x, y)$$

$$y(x_0) = y_0$$

Test: $f(x, y)$ and $\partial f / \partial y$ are continuous over I

Property: A unique solution is guaranteed over I

1.1 Separable ODEs

- **Form:**

$$\frac{dy}{dx} = g(x)h(y)$$

Solution: Divide by $h(y)$ then with respect to x.

$$\frac{dy}{dx} = g(x)h(y)$$

$$\frac{1}{h(y)}\frac{dy}{dx} = g(x)$$

$$\int \frac{1}{h(y)}\frac{dy}{dx}\,dx = \int g(x)\,dx$$

$$\int \frac{1}{h(y)}\,dy = \int g(x)\,dx$$

$$H(y) = G(x) + c$$

1.2 Linear Equations

- **Form:**

$$\frac{dy}{dx} + P(x)y = f(x)$$

Solution:

1. Determine the integrating factor $e^{\int P(x)\,dx}$

2. Multiply by the integrating factor

3. Recognise that the left hand side of the equation is the derivative of the product of the integrating factor and y

4. Integrate both sides of the equation

5. Solve for y