

Quantum Computation and Quantum
Information by Michael A. Nielsen and Isaac L.
Chuang Problems

Chris Doble

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Part I

Fundamental concepts

2 Linear algebra

Exercise 2.1

$$(1, -1) + (1, 2) - (2, 1) = (0, 0)$$

Exercise 2.2

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Using the basis $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$ we get

$$|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}$$

$$|1\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}}$$

$$\begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$a_{00} + a_{01} = 1$$

$$a_{10} + a_{11} = -1$$

$$\begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$a_{00} - a_{01} = 1$$

$$a_{10} - a_{11} = 1$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Exercise 2.5

$$\begin{aligned}
\begin{bmatrix} y_1^* & \cdots & y_n^* \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} &= \begin{bmatrix} y_1^* & \cdots & y_n^* \end{bmatrix} \left(\begin{bmatrix} z_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ z_2 \\ \vdots \\ 0 \end{bmatrix} + \cdots + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ z_n \end{bmatrix} \right) \\
&= \begin{bmatrix} y_1^* & \cdots & y_n^* \end{bmatrix} \begin{bmatrix} z_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \cdots + \begin{bmatrix} y_1^* & \cdots & y_n^* \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ z_n \end{bmatrix} \\
&= z_1 \begin{bmatrix} y_1^* & \cdots & y_n^* \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \cdots + z_n \begin{bmatrix} y_1^* & \cdots & y_n^* \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \\
\begin{bmatrix} y_1^* & \cdots & y_n^* \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} &= y_1^* z_1 + y_2^* z_2 + \cdots + y_n^* z_n \\
&= (y_1 z_1^* + y_2 z_2^* + \cdots + y_n z_n^*)^* \\
&= \left(\begin{bmatrix} z_1^* & \cdots & z_n^* \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right)^* \\
\begin{bmatrix} v_1^* & \cdots & v_n^* \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} &= |v_1|^2 + \cdots + |v_n|^2 \\
&\geq 0
\end{aligned}$$

Exercise 2.6

$$\begin{aligned}
\left(\sum_i \lambda_i |w_i\rangle, |v\rangle \right) &= \left(|v\rangle, \sum_i \lambda_i |w_i\rangle \right)^* \\
&= \left(\sum_i \lambda_i (|v\rangle, |w_i\rangle) \right)^* \\
&= \sum_i \lambda_i^* (|v\rangle, |w_i\rangle)^* \\
&= \sum_i \lambda_i^* \langle w_i | v \rangle
\end{aligned}$$

Exercise 2.7

$$\begin{aligned}\langle w|v\rangle &= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ &= (1)(1) + (1)(-1) \\ &= 0\end{aligned}$$

$$\begin{aligned}\frac{|w\rangle}{||w\rangle||} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \frac{|v\rangle}{||v\rangle||} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}\end{aligned}$$

Exercise 2.9

$$\begin{aligned}\sigma_0 &= |0\rangle\langle 0| + |1\rangle\langle 1| \\ \sigma_1 &= |1\rangle\langle 0| + |0\rangle\langle 1| \\ \sigma_2 &= i|1\rangle\langle 0| - i|0\rangle\langle 1| \\ \sigma_3 &= |0\rangle\langle 0| - |1\rangle\langle 1|\end{aligned}$$

Exercise 2.11

$$\begin{aligned}\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} &= \lambda^2 - 1 \\ \lambda &= \pm 1\end{aligned}$$

$$\begin{aligned}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} &= \begin{bmatrix} a \\ b \end{bmatrix} \\ b &= a \\ a &= b\end{aligned}$$

$$\begin{aligned}X_1 &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} &= \begin{bmatrix} -a \\ -b \end{bmatrix} \\ b &= -a \\ a &= -b\end{aligned}$$

$$X_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Exercise 2.12

$$\begin{vmatrix} 1-\lambda & 0 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2$$
$$\lambda_1 = 1$$
$$\lambda_2 = 1$$

The eigenvalue 1 is degenerate. Because the matrix only has one eigenvector it can't be diagonalised.