

University Physics with Modern Physics

Mechanics Problems

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Contents

14 Periodic Motion	1
14.1 VP14.3.1	1
14.2 VP14.3.2	2
14.3 VP14.3.3	3
14.4 VP14.3.4	3
14.5 VP14.4.1	4
14.6 VP14.4.2	4
14.7 VP14.4.3	5
14.8 VP14.4.4	6
14.9 VP14.9.1	6
14.10VP14.9.2	6
14.11VP14.9.3	7
14.12VP14.9.4	7
14.13Bridging Problem	7

14 Periodic Motion

14.1 VP14.3.1

a)

$$T = \frac{1}{f} = \frac{1}{4.15} = 0.241 \text{ s}$$

$$\omega = 2\pi f = 2\pi(4.15) = 26.1 \text{ rad/s}$$

b)

$$\begin{aligned}\omega &= \sqrt{\frac{k}{m}} \\ \omega^2 &= \frac{k}{m} \\ m\omega^2 &= k \\ (0.400)(26.1)^2 &= k \\ 272 \text{ N/m} &= k\end{aligned}$$

c)

$$\begin{aligned}F &= kx \\ &= (272)(0.0200) \\ &= 5.44 \text{ N}\end{aligned}$$

14.2 VP14.3.2

a)

$$\begin{aligned}T &= 2\pi\sqrt{\frac{m}{k}} \\ \frac{T}{2\pi} &= \sqrt{\frac{m}{k}} \\ \frac{T^2}{4\pi^2} &= \frac{m}{k} \\ \frac{kT^2}{4\pi^2} &= m \\ \frac{(4.50)(1.20)^2}{4\pi^2} &= m \\ 0.164 \text{ kg} &= m\end{aligned}$$

b)

$$\begin{aligned}F &= ma_{\text{max}} = kA \\ \frac{ma_{\text{max}}}{k} &= A \\ \frac{(0.164)(1.20)}{(4.50)} &= A \\ 0.0437 \text{ m} &= A\end{aligned}$$

14.3 VP14.3.3

a)

$$\begin{aligned}\frac{1}{2}mv^2 + \frac{1}{2}kx^2 &= \frac{1}{2}kA^2 \\ \sqrt{\frac{m}{k}v^2 + x^2} &= A \\ \sqrt{\frac{1}{(2\pi f)^2}v^2 + x^2} &= A \\ \sqrt{\frac{1}{(2\pi(50.0))^2}(12.4)^2 + (0.0300)^2} &= A \\ 0.0496 \text{ m} &= A\end{aligned}$$

b)

$$\begin{aligned}\frac{1}{2}mv^2 &= \frac{1}{2}kA^2 \\ v &= \sqrt{\frac{k}{m}}A \\ &= 2\pi fA \\ &= 2\pi(50.0)(0.0496) \\ &= 15.6 \text{ m/s}\end{aligned}$$

14.4 VP14.3.4

a)

$$\begin{aligned}f &= \frac{1}{2\pi}\sqrt{\frac{k}{m}} \\ &= \frac{1}{2\pi}\sqrt{\frac{185}{5.00}} \\ &= 0.968 \text{ Hz}\end{aligned}$$

b)

$$\begin{aligned}F &= ma_{\max} = kA \\ \frac{ma_{\max}}{k} &= A \\ \frac{(5.00)(1.52)}{185} &= A \\ 4.11 \times 10^{-2} \text{ m} &= A\end{aligned}$$

14.5 VP14.4.1

a)

$$\begin{aligned}\frac{1}{2}mv^2 &= \frac{1}{2}kA^2 \\ \sqrt{\frac{m}{k}}v &= A \\ \sqrt{\frac{0.150}{8.00}}0.350 &= A \\ 4.79 \times 10^{-2} \text{ m} &= A\end{aligned}$$

b)

$$\frac{1}{2}mv^2 = \frac{1}{2}(0.150)(0.350)^2 = 9.19 \times 10^{-3} \text{ J}$$

c)

$$\begin{aligned}U &= \frac{1}{2}kx^2 = \frac{1}{2}(8.00)(0.0300)^2 = 3.60 \times 10^{-3} \text{ J} \\ K &= E - U = 9.19 \times 10^{-3} - 3.60 \times 10^{-3} = 5.59 \times 10^{-3} \text{ J}\end{aligned}$$

14.6 VP14.4.2

a)

$$\begin{aligned}E &= \frac{1}{2}kA^2 \\ \frac{2E}{A^2} &= k \\ \frac{2(6.00 \times 10^{-2})}{(0.0440)^2} &= k \\ 62.0 \text{ N/m} &= k\end{aligned}$$

b)

$$\begin{aligned}E &= \frac{1}{2}mv^2 + \frac{E}{2} \\ \sqrt{\frac{E}{m}} &= v \\ \sqrt{\frac{6.00 \times 10^{-2}}{0.300}} &= v \\ 0.447 \text{ m/s} &= v\end{aligned}$$

14.7 VP14.4.3

a)

$$\begin{aligned}E &= \frac{1}{2}kA^2 \\ \frac{2E}{A^2} &= k \\ \frac{2(4.00 \times 10^{-3})}{(0.0300)^2} &= k \\ 8.89 \text{ N/m} &= k\end{aligned}$$

$$\begin{aligned}E &= \frac{1}{2}mv^2 \\ \frac{2E}{v^2} &= m \\ \frac{2(4.00 \times 10^{-3})}{(0.125)^2} &= m \\ 0.512 \text{ kg} &= m\end{aligned}$$

b)

$$\begin{aligned}F &= ma = kA \\ a &= \frac{kA}{m} \\ &= \frac{(8.89)(0.0300)}{0.512} \\ &= 0.521 \text{ m/s}^2\end{aligned}$$

c)

$$\begin{aligned}U &= \frac{1}{2}kx^2 \\ \sqrt{\frac{2U}{k}} &= x\end{aligned}$$

$$\begin{aligned}F &= ma = kx \\ a &= \frac{\sqrt{2kU}}{m} \\ &= \frac{\sqrt{2(8.89)(3.00 \times 10^{-3})}}{0.512} \\ &= 0.451 \text{ m/s}^2\end{aligned}$$

14.8 VP14.4.4

- a) At maximum displacement from equilibrium the object's velocity is 0 and all energy is stored in elastic potential energy

$$E = \frac{1}{2}kA^2.$$

The object's kinetic energy will be equal to $\frac{1}{3}$ of its total mechanical energy when its potential energy is equal to $\frac{2}{3}$

$$\begin{aligned}U &= \frac{2}{3}E \\ \frac{1}{2}kx^2 &= \frac{2}{3}\left(\frac{1}{2}kA^2\right) \\ x &= \pm\sqrt{\frac{2}{3}}A.\end{aligned}$$

- b) Similarly, the object's kinetic energy will be equal to $\frac{4}{5}$ of the total mechanical energy when its potential energy is equal to $\frac{1}{5}$

$$\begin{aligned}U &= \frac{1}{5}E \\ \frac{1}{2}kx^2 &= \frac{1}{5}\left(\frac{1}{2}kA^2\right) \\ x &= \pm\frac{A}{\sqrt{5}}.\end{aligned}$$

14.9 VP14.9.1

- a) The period is given by

$$T = \frac{1}{f} = \frac{1}{0.609} = 1.64 \text{ s.}$$

- b) Rearranging the formula for f we find that

$$\begin{aligned}f &= \frac{1}{2\pi}\sqrt{\frac{g}{L}} \\ (2\pi f)^2 L &= g \\ (2\pi(0.609))^2(0.500) &= g \\ 7.32 \text{ m/s}^2 &= g.\end{aligned}$$

14.10 VP14.9.2

The frequency of the air-track glider is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

The frequency of the simple pendulum is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}.$$

If their frequencies are to be equal then

$$\begin{aligned} \frac{1}{2\pi} \sqrt{\frac{k}{m}} &= \frac{1}{2\pi} \sqrt{\frac{g}{L}} \\ \frac{k}{m} &= \frac{g}{L} \\ L &= \frac{gm}{k} \\ &= \frac{(9.80)(0.350)}{8.75} \\ &= 0.392 \text{ m} \end{aligned}$$

14.11 VP14.9.3

Assuming the tire is hung from its rim then it is a physical pendulum and

$$\begin{aligned} f &= \frac{1}{2\pi} \sqrt{\frac{Mgd}{I}} \\ &= \frac{1}{2\pi} \sqrt{\frac{MgR}{2MR^2}} \\ &= \frac{1}{2\pi} \sqrt{\frac{g}{2R}}. \end{aligned}$$

14.12 VP14.9.4

The rod is a physical pendulum. Rearranging the formula for period we find

$$\begin{aligned} T &= 2\pi \sqrt{\frac{I}{mgd}} \\ mgd \left(\frac{T}{2\pi} \right)^2 &= I \\ (0.600)(9.80)(0.500) \left(\frac{1.59}{2\pi} \right)^2 &= I \\ 0.188 \text{ kg/m}^2 &= I \end{aligned}$$

14.13 Bridging Problem

In order for the centre of mass of the cylinders to undergo simple harmonic motion it must experience a net force of the form

$$F = -k'x.$$

The normal and weight forces cancel, leaving only the friction force from the cylinders on the ground f and the restorative force from the spring $-kx$

$$F = f - kx.$$

The cylinders roll without slipping so

$$\begin{aligned}\omega &= -\frac{v}{R} \\ \frac{d}{dt}(\omega) &= \frac{d}{dt}\left(-\frac{v}{R}\right) \\ \alpha &= -\frac{a}{R}.\end{aligned}$$

The friction force generates a torque

$$\begin{aligned}\tau &= I\alpha = fR \\ \left(\frac{1}{2}MR^2\right)\left(-\frac{a}{R}\right) &= fR \\ -\frac{1}{2}Ma &= f.\end{aligned}$$

Substituting this into the force equation we find that

$$\begin{aligned}F = Ma &= -\frac{1}{2}Ma - kx \\ \frac{3}{2}Ma &= -kx \\ Ma &= -\frac{2}{3}kx\end{aligned}$$

which matches the required form of $F = -k'x$ where $k' = \frac{2}{3}k$. This gives a period of

$$\begin{aligned}T &= 2\pi\sqrt{\frac{m}{k'}} \\ &= 2\pi\sqrt{\frac{3m}{2k}}.\end{aligned}$$