Introduction to Electrodynamics by David J. Griffiths Notes

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1 Vector Algebra

1.6 The Theory of Vector Fields

1.6.1 The Helmholtz Theorem

- The **Helmholtz theorem** states that a vector field $\mathbf{F}$ is uniquely determined if you're given its divergence $\nabla \cdot \mathbf{F}$, curl $\nabla \times \mathbf{F}$, and sufficient boundary conditions.

1.6.2 Potentials

- If the curl of a vector field vanishes everywhere, then it can be expressed as the gradient of a **scalar potential**

$$\nabla \times \mathbf{F} = \mathbf{0} \Leftrightarrow \mathbf{F} = -\nabla V.$$

- If the divergence of a vector field vanishes everywhere, then it can be expressed as the curl of a **vector potential**

$$\nabla \cdot \mathbf{F} = 0 \Leftrightarrow \mathbf{F} = \nabla \times \mathbf{A}.$$

2 Electrostatics

2.1 The Electric Field

2.1.2 Coulomb's Law

- **Couloumb's law** gives the force between two point charges q and Q

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}}\hat{\boldsymbol{\mathscr{r}}}$$

  where

$$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/(N\,m^2)}$$

  is the **permittivity of free space** and $\boldsymbol{\mathscr{r}}$ is the separation vector between the two charges.

2.1.3 The Electric Field

- The **electric field E** is a vector field that varies from point to point and gives the force per unit charge that would be exerted on a test charge if placed at a particular point.
- For a collection of n source charges q_i at displacements $\boldsymbol{\mathscr{r}}_i$ from a test charge, the electric field is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\boldsymbol{\mathscr{r}}}.$$

2.1.4 Continuous Charge Distributions

- Couloumb's law for a continuous charge distribution is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int\frac{1}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\,dq.$$

2.2 Divergence and Curl of Electrostatic Fields

2.2.1 Field Lines, Flux, and Gauss's Law

- **Gauss's law** states that the electric field flux through a closed surface is proportional to the amount of charge within that surface

$$\oint \mathbf{E}\cdot d\mathbf{a} = \frac{1}{\epsilon_0}Q_{\mathrm{enc}}$$

  or

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho.$$

2.2.4 The Curl of $\mathbf{E}$

- The curl of an electric field is $\mathbf{0}$

$$\nabla \times \mathbf{E} = \mathbf{0}.$$