University Physics with Modern Physics Electromagnetism Problems

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Contents

21 Electric Charge and Electric Field

21.3 Coulomb's Law

21.3.1 Example 21.1

The magnitude of electric repulsion between two α particles is given by

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

and the magnitude of gravitational attraction is given by

$$F_g = \frac{Gm^2}{r^2}$$

. The ratio of the two values is

$$\begin{aligned}\frac{F_e}{F_g} &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}\frac{r^2}{Gm^2} \\ &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{Gm^2} \\ &= 3.1 \times 10^{35}\end{aligned}$$

showing that the electric repulsion is significantly stronger than the gravitational attraction.

21.3.2 Example 21.2

a) The magnitude of the force that q_1 exerts on q_2 is

$$\begin{aligned}F &= \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \\ &= (9.0 \times 10^9)\frac{|(25 \times 10^{-9})(-75 \times 10^{-9})|}{0.030^2} \\ &= 1.9 \times 10^{-2}\,\text{N}.\end{aligned}$$

Since q_1 and q_2 have opposite charge, the force is attractive (from q_2 to q_1).

b) The magnitude of the force that q_2 exerts on q_1 is the same as in part a, but the direction is reversed (from q_1 to q_2).

21.3.3 Example 21.3

By the principle of superposition of forces, the net force exerted on q_3 is equal to the vector sum of the forces exerted on it by q_1 and q_2 separately.

Both q_1 and q_3 have positive charge so they repel each other. q_1 is to the right of q_3 so q_3 experiences a force to the left of magnitude

$$\begin{aligned}F_{1\text{ on }3} &= \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r^2} \\ &= (9.0\times10^9)\frac{|(1.0\times10^{-9})(5.0\times10^{-9})|}{0.020^2} \\ &= 1.1\times10^{-4}\,\text{N}.\end{aligned}$$

However q_2 has a negative charge so it attracts q_3. It is also to the right of q_3 so q_3 experiences a force to the right of magnitude

$$\begin{aligned}F_{2\text{ on }3} &= \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{r^2} \\ &= (9.0\times10^9)\frac{|(-3.0\times10^{-9})(5.0\times10^{-9})}{0.040^2} \\ &= 8.4\times10^{-5}\,\text{N}.\end{aligned}$$

The net force experienced by q_3 is therefore

$$\begin{aligned}F &= -F_{1\text{ on }3} + F_{2\text{ on }3} \\ &= -1.1\times10^{-4} + 8.4\times10^{-5} \\ &= -2.6\times10^{-5}\,\text{N}.\end{aligned}$$

21.3.4 Example 21.4

Since q_1 and q_2 are of equal charge and are symmetric about the x axis on which Q lies, the vertical components of their forces cancel leaving only the horizontal.

The horizontal component of q_1's force on Q is given by

$$\begin{aligned}F_{1\text{ on Q, x}} &= \frac{1}{4\pi\epsilon_0}\frac{q_1 Q}{r_{1,\text{Q}}^2}\cos\alpha \\ &= (9.0\times10^9)\frac{(2.0\times10^{-6})(4.0\times10^{-6})}{\sqrt{0.30^2+0.40^2}^2}\frac{0.40}{0.50} \\ &= 0.23\,\text{N}.\end{aligned}$$

Again, since q_1 and q_2 are of equal charge and symmetric about the x axis, $F_{1\text{ on Q, x}} = F_{2\text{ on Q, x}}$ and the total force experienced by Q is in the positive x direction of magnitude

$$F = 2F_{1\text{ on Q, x}} = 0.46\,\text{N}.$$

21.4 Electric Field and Electric Forces

21.4.1 Example 21.5

The magnitude of the electric field vector is given by

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \\ &= (9.0\times 10^9)\frac{|4.0\times 10^{-9}|}{2.0^2} \\ &= 9.0\,\mathrm{N/C}. \end{aligned}$$

21.4.2 Example 21.6

The magnitude of the electric field vector is given by

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \\ &= (9.0\times 10^9)\frac{|-8.0\times 10^{-9}|}{1.2^2+1.6^2} \\ &= 18\,\mathrm{N/C} \end{aligned}$$

and it is directed towards the origin. If θ is the angle between the positive x axis and $\hat{\mathbf{r}}$ then the component form of $\mathbf{E}$ is

$$\begin{aligned} E &= -E\left(\cos\theta\hat{\mathbf{i}}+\sin\theta\hat{\mathbf{j}}\right) \\ &= -E\left(\frac{x}{r}\hat{\mathbf{i}}+\frac{-y}{r}\hat{\mathbf{j}}\right) \\ &= \frac{-18}{\sqrt{1.2^2+1.6^2}}\left(1.2\hat{\mathbf{i}}+1.6\hat{\mathbf{j}}\right) \\ &= (-11\,\mathrm{N/C})\hat{\mathbf{i}}-(14\,\mathrm{N/C})\hat{\mathbf{j}}. \end{aligned}$$

21.4.3 Example 21.7

a) Electrons have a negative charge and the electric field is directed upwards, so the electron will move downwards. The magnitude of its acceleration is

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{eE}{m} \\ &= \frac{(1.60\times 10^{-19})(1.00\times 10^4)}{9.11\times 10^{-31}} \\ &= 1.76\times 10^{15}\,\mathrm{m/s^2}. \end{aligned}$$

b) Its acceleration is constant between the plates, so its final speed is

$$
\begin{aligned}
v^2 &= v_0^2 + 2a(x - x_0) \\
&= 2ax \\
v &= \sqrt{2ax} \\
&= \sqrt{2(1.76 \times 10^{15})(0.01)} \\
&= 5.9 \times 10^6 \text{ m/s}^2
\end{aligned}
$$

and thus its final kinetic energy is

$$
\begin{aligned}
K &= \frac{1}{2}mv^2 \\
&= \frac{1}{2}(9.11 \times 10^{-31})(5.9 \times 10^6)^2 \\
&= 1.6 \times 10^{-17} \text{ J.}
\end{aligned}
$$

c) We can find the time it takes for the electron to travel this distance by rearranging the kinematic equation

$$v = v_0 + at$$

to

$$
\begin{aligned}
t &= \frac{v - v_0}{a} \\
&= \frac{5.9 \times 10^6}{1.76 \times 10^{15}} \\
&= 3.4 \times 10^{-9} \text{ s.}
\end{aligned}
$$

21.5 Electric-Field Calculations

21.5.1 Example 21.8

a) At point a the electric field caused by q_1 points to the right and has magnitude

$$
\begin{aligned}
E_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} \\
&= (9.0 \times 10^9)\frac{12 \times 10^{-9}}{(0.060)^2} \\
&= 3.0 \times 10^4 \text{ N/C.}
\end{aligned}
$$

The electric field caused by q_2 also points to the right and it has magnitude

$$\begin{aligned}E_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} \\ &= (9.0\times 10^9)\frac{|-12\times 10^{-9}|}{(0.040)^2} \\ &= 6.8\times 10^4\,\mathrm{N/C}.\end{aligned}$$

Thus the total field points to the right and has magnitude

$$E = E_1 + E_2 = 9.8\times 10^4\,\mathrm{N/C}.$$

b) At point b the electric field caused by q_1 points to the left and has magnitude

$$\begin{aligned}E_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} \\ &= (9.0\times 10^9)\frac{12\times 10^{-9}}{(0.040)^2} \\ &= 6.8\times 10^4\,\mathrm{N/C}.\end{aligned}$$

The electric field caused by q_2 points to the right and has magnitude

$$\begin{aligned}E_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} \\ &= (9.0\times 10^9)\frac{|-12\times 10^{-9}|}{(0.140)^2} \\ &= 0.55\times 10^4\,\mathrm{N/C}.\end{aligned}$$

Thus the total electric field points to the left and has magnitude

$$E = E_1 - E_2 = 6.3\times 10^4\,\mathrm{N/C}.$$

c) At point c the electric field caused by q_1 points from q_1 to c and has magnitude

$$\begin{aligned}E_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} \\ &= (9.0\times 10^9)\frac{|12\times 10^{-9}|}{0.130^2} \\ &= 6.4\times 10^3\,\mathrm{N/C}.\end{aligned}$$

The electric field caused by q_2 points from c to q_2 and has magnitude

$$\begin{aligned}
E_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} \\
&= (9.0 \times 10^9)\frac{|-12 \times 10^{-9}|}{0.130^2} \\
&= 6.4 \times 10^3 \, \text{N/C} \\
&= E_1.
\end{aligned}$$

The vertical components of $\mathbf{E_1}$ and $\mathbf{E_2}$ cancel, leaving only a horizontal component pointing to the right of magnitude

$$\begin{aligned}
E &= 2E_1 \cos\alpha \\
&= 2(6.4 \times 10^3)\frac{0.050}{0.130} \\
&= 4.9 \times 10^3 \, \text{N/C}.
\end{aligned}$$

21.5.2 Example 21.9

By symmetry, each point on the ring has a corresponding point on the opposite side. The components of their electric fields perpendicular to the axis of the ring cancel, leaving only a component parallel to the axis of the ring. Thus the total magnetic field at P is parallel to the axis of the ring and can be calculated as

$$\begin{aligned}
E &= \int_0^{2\pi} \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r^2}\cos\alpha \, d\theta \\
&= \frac{1}{4\pi\epsilon_0}\frac{Qx}{2\pi(a^2+x^2)^{3/2}}\int_0^{2\pi} d\theta \\
&= \frac{1}{4\pi\epsilon_0}\frac{Qx}{(a^2+x^2)^{3/2}}.
\end{aligned}$$

21.5.3 Example 21.10

By symmetry, each point on the line has a corresponding point on the opposite side of the x-axis. The y components of their electric fields cancel, leaving only the x components. Thus the total magnetic field at P only has an x component and can be calculated as

$$
\begin{aligned}
E &= \int_{-a}^{a} \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r^2} \cos\alpha \, dy \\
&= \frac{1}{4\pi\epsilon_0} \frac{Qx}{2a} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} \, dy \\
&= \frac{1}{4\pi\epsilon_0} \frac{Qx}{2a} \left[\frac{y}{x^2\sqrt{x^2+y^2}} \right]_{-a}^{a} \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{2ax} \left(\frac{a}{\sqrt{x^2+a^2}} + \frac{a}{\sqrt{x^2+(-a)^2}} \right) \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2+a^2}}.
\end{aligned}
$$

21.5.4 Example 21.11

By symmetry, each point on the disk has a corresponding point 180° rotation around the x-axis. The y and z components of their electric fields cancel, leaving only the x components. Thus the total magnetic field at P only has an x component and can be calculated as

$$
\begin{aligned}
E &= \int_0^R \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{\sigma}{r^2} s \cos\alpha \, d\theta \, ds \\
&= \frac{\sigma}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{s}{s^2+x^2} \frac{x}{\sqrt{s^2+x^2}} \, d\theta \, ds \\
&= \frac{\sigma x}{2\epsilon_0} \int_0^R \frac{s}{(s^2+x^2)^{3/2}} \, ds \\
&= \frac{\sigma x}{2\epsilon_0} \left[-\frac{1}{\sqrt{s^2+x^2}} \right]_0^R \\
&= \frac{\sigma x}{2\epsilon_0} \left(-\frac{1}{\sqrt{R^2+x^2}} + \frac{1}{x} \right) \\
&= \frac{\sigma}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{(R/x)^2+1}} \right).
\end{aligned}
$$

21.5.5 Example 21.12

From Example 21.11 we know that the electric field produced by an infinite plane sheet of charge is

$$E = \frac{\sigma}{2\epsilon_0}.$$

Therefore the electric field outside the sheets is $\mathbf{0}$ and between the sheets is σ/ϵ_0 towards the negative sheet.

21.6 Electric Dipoles

21.6.1 Example 21.13

a) The electric field is uniform so the net force exerted on the dipole is **0**

b) The electric dipole moment is directed from the negative charge to the positive charge and has magnitude

$$p = qd = (1.6 \times 10^{-19})(0.125 \times 10^{-9}) = 2.0 \times 10^{-29}\,\mathrm{C \cdot m}$$

c) The torque aligns the electric dipole moment with the electric field so it is directed out of the page and has magnitude

$$\tau = qEd \sin\phi = (1.6{\times}10^{-19})(5.0{\times}10^{5})(0.125{\times}10^{-9}) \sin 35 = 5.7{\times}10^{-24}\,\mathrm{N{\cdot}m}$$

d) The potential energy of an electric dipole in a uniform electric field is given by

$$U = -qdE\cos\phi = (2.0 \times 10^{-29})(5.0 \times 10^{5}) \cos 35 = 8.2 \times 10^{-24}\,\mathrm{J}$$

21.6.2 Example 21.14

As P is on the y-axis, the electric fields of the electric dipole's point charges have no x component and thus the net electric field is directed along the y-axis.

By the principle of superposition of electric fields, the magnitude of the electric field at P is

$$\begin{aligned}
E &= E_- + E_+ \\
&= \frac{1}{4\pi\epsilon_0}\frac{-q}{(y-(-d/2))^2} + \frac{1}{4\pi\epsilon_0}\frac{q}{(y-d/2)^2} \\
&= \frac{1}{4\pi\epsilon_0} q \left(\frac{1}{(y-d/2)^2} - \frac{1}{(y+d/2)^2}\right) \\
&= \frac{1}{4\pi\epsilon_0}\frac{q}{y^2}\left(\left(1-\frac{d}{2y}\right)^{-2} - \left(1+\frac{d}{2y}\right)^{-2}\right) \\
&\approx \frac{1}{4\pi\epsilon_0}\frac{q}{y^2}\left(1+\frac{d}{y} - 1 + \frac{d}{y}\right) \\
&= \frac{qd}{2\pi\epsilon_0 y^3} \\
&= \frac{p}{2\pi\epsilon_0 y^3}.
\end{aligned}$$

21.7 Guided Practice

21.7.1 VP21.4.1

q_1 attracts q_3 to the left with magnitude

$$\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1q_3|}{r^2} \\
&= (9.0\times 10^9)\frac{|(4.00\times 10^{-9})(-2.00\times 10^{-9})}{0.0400^2} \\
&= 4.5\times 10^{-5}\,\text{N}.
\end{aligned}$$

q_2 repels q_3 to the left with magnitude

$$\begin{aligned}
F_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2q_3|}{r^2} \\
&= (9.0\times 10^9)\frac{|(-1.20\times 10^{-9})(-2.00\times 10^{-9}|}{(0.0600-0.0400)^2} \\
&= 5.4\times 10^{-5}\,\text{N}.
\end{aligned}$$

By the principle of superposition of forces, the net force on q_3 is

$$\mathbf{F} = (-F_1 - F_2)\hat{\mathbf{i}} = (-9.9\times 10^{-5}\,\text{N})\hat{\mathbf{i}}.$$

21.7.2 VP21.4.2

a) q_1 repels q_2 in the positive x direction with magnitude

$$\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} \\
&= \frac{1}{4\pi(8.854\times 10^{-12})}\frac{|(3.60\times 10^{-9})(2.00\times 10^{-9})}{0.0400^2} \\
&= 40.4\,\mu\text{N}.
\end{aligned}$$

b) By the superposition of forces

$$\begin{aligned}
F &= F_1 + F_2 \\
F_2 &= F - F_1 \\
&= 54.0 - 40.4 \\
&= 13.6\,\mu\text{N}
\end{aligned}$$

in the positive x direction.

c) q_2 repels q_3 so it must also have a positive charge of magnitude

$$
\begin{aligned}
F &= \frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{r^2} \\
q_2 &= \frac{4\pi\epsilon_0 F r^2}{q_3} \\
&= \frac{4\pi(8.854\times10^{-12})(1.36\times10^{-5})(0.0800)^2}{2.00\times10^{-9}} \\
&= 4.84\times10^{-9}\,\mathrm{C}.
\end{aligned}
$$

21.7.3 VP21.4.3

By symmetry the x components of q_1 and q_2's electric fields cancel leaving only their y components which are directed in the negative y direction and equal. q_3 is negative and thus experiences a net force in the positive y direction of magnitude

$$
\begin{aligned}
F &= 2\frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r^2}\sin\alpha \\
&= 2\frac{1}{4\pi(8.854\times10^{-12})}\frac{(6.00\times10^{-9})(2.50\times10^{-9})}{0.150^2+0.200^2}\frac{0.200}{\sqrt{0.150^2+0.200^2}} \\
&= 3.45\times10^{-6}\,\mathrm{N}.
\end{aligned}
$$

21.7.4 VP21.4.4

The magnitude of the electric force exerted by q_1 on q_3 is

$$
\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r^2} \\
&= \frac{1}{4\pi(8.854\times10^{-12})}\frac{|(4.00\times10^{-9})(-1.50\times10^{-9})}{0.250^2+0.200^2} \\
&= 5.26\times10^{-7}\,\mathrm{N}.
\end{aligned}
$$

They have opposite charges so the force is directed from q_3 to q_1. In component form the force is

$$
\begin{aligned}
\mathbf{F_1} &= -F_1\cos\alpha\hat{\mathbf{i}} + F_1\sin\alpha\hat{\mathbf{j}} \\
&= F_1\left(-\frac{x}{r}\hat{\mathbf{i}} + \frac{y}{r}\hat{\mathbf{j}}\right) \\
&= \frac{5.26\times10^{-7}}{\sqrt{0.250^2+0.200^2}}\left(-0.250\hat{\mathbf{i}} + 0.200\hat{\mathbf{j}}\right) \\
&= (-4.11\times10^{-7}\,\mathrm{N})\hat{\mathbf{i}} + (3.29\times10^{-7}\,\mathrm{N})\hat{\mathbf{j}}.
\end{aligned}
$$

The magnitude of the electric force exerted by q_2 on q_3 is

$$\begin{aligned}
F_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{r^2} \\
&= \frac{1}{4\pi(8.854\times10^{-12})}\frac{|(-4.00\times10^{-9})(-1.50\times10^{-9})|}{0.250^2} \\
&= 8.63\times10^{-7}\,\mathrm{N}.
\end{aligned}$$

The have like charges so the force is directed from q_2 to q_3, i.e. along the positive x-axis. In component form the force is

$$\mathbf{F_2} = (8.64\times10^{-7}\,\mathrm{N})\hat{\mathbf{i}}.$$

Thus the net force experienced by q_3 is

$$\begin{aligned}
\mathbf{F} &= \mathbf{F_1} + \mathbf{F_2} \\
&= (4.53\times10^{-7}\,\mathrm{N})\hat{\mathbf{i}} + (3.29\times10^{-7}\,\mathrm{N})\hat{\mathbf{j}}.
\end{aligned}$$

21.7.5 VP21.10.1

a) The source points and field point all lie on the y-axis, so the source points' electric fields have no x components. q_1 is positive and the field point is below it, so its contribution is negative. q_2 is negative and the field point is above it, so its contribution is also negative. Thus the y component of the net electric field is

$$\begin{aligned}
E_y &= \frac{1}{4\pi\epsilon_0}\left(-\frac{q_1}{(y_1-y)^2} + \frac{q_2}{(y_2-y)^2}\right) \\
&= \frac{1}{4\pi(8.854\times10^{-12})}\left(\frac{4.00\times10^{-9}}{(0.200-0.100)^2} - \frac{5.00\times10^{-9}}{(0-0.100)^2}\right) \\
&= -8.09\times10^{3}\,\mathrm{N/C}.
\end{aligned}$$

b) The source points and field point all lie on the y-axis, so the source points' electric fields have no x components. q_1 is positive and the field point is above it, so its contribution is positive. q_2 is negative and the field point is above it, so its contribution is also negative. Thus the y component of the net electric field is

$$\begin{aligned}
E_y &= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{(y_1-y)^2} + \frac{q_2}{(y_2-y)^2}\right) \\
&= \frac{1}{4\pi(8.854\times10^{-12})}\left(\frac{4.00\times10^{-9}}{(0.200-0.400)^2} - \frac{5.00\times10^{-9}}{(0-0.400)^2}\right) \\
&= 618\,\mathrm{N/C}.
\end{aligned}$$

c) The electric field of q_1 has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2} \\ &= \frac{1}{4\pi(8.854\times10^{-12})}\frac{4.00\times10^{-9}}{0.200^2+0.200^2} \\ &= 449\,\mathrm{N/C}. \end{aligned}$$

It is directed from q_1 to the field point and thus in component form is

$$\begin{aligned} \mathbf{E_1} &= E_1(\cos\phi\hat{\mathbf{i}} - \sin\phi\hat{\mathbf{j}}) \\ &= \frac{449}{\sqrt{0.200^2+0.200^2}}(0.200\hat{\mathbf{i}} - 0.200\hat{\mathbf{j}}) \\ &= (317\,\mathrm{N/C})\hat{\mathbf{i}} - (317\,\mathrm{N/C})\hat{\mathbf{j}}. \end{aligned}$$

q_2 and the field point both lie on the x-axis, and thus its electric field has no y component. In component form it is

$$\begin{aligned} \mathbf{E_2} &= \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}\hat{\mathbf{i}} \\ &= \frac{1}{4\pi(8.854\times10^{-12})}\frac{-5.00\times10^{-9}}{0.200^2}\hat{\mathbf{i}} \\ &= (-1.12\times10^3\,\mathrm{N/C})\hat{\mathbf{i}}. \end{aligned}$$

The total electric field is thus

$$\begin{aligned} \mathbf{E} &= \mathbf{E_1} + \mathbf{E_2} \\ &= (-8.03\times10^2\,\mathrm{N/C})\hat{\mathbf{i}} + (-3.17\times10^2\,\mathrm{N/C})\hat{\mathbf{j}}. \end{aligned}$$

21.7.6 VP21.10.2

a) Both source points and the field point are on the x-axis, so the electric fields at P have no y components.

   q_1 is positive and P is to the right of q_1, so its electric field points to the right and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2} \\ &= \frac{1}{4\pi(8.854\times10^{-12})}\frac{1.80\times10^{-9}}{0.0200^2} \\ &= 4.04\times10^4\,\mathrm{N/C}. \end{aligned}$$

b) The magnitude and direction of the electric field that q_2 causes at P can be calculated as

$$\begin{aligned} E &= E_1 + E_2 \\ E_2 &= E - E_1 \\ &= 6.75 \times 10^4 - 4.04 \times 10^4 \\ &= 2.71 \times 10^4 \text{ N/C}. \end{aligned}$$

c) E_2 is positive at P, so q_2 must be negative. Its value is

$$\begin{aligned} E_2 &= -\frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2} \\ q_2 &= -4\pi\epsilon_0 E_2 r^2 \\ &= -4\pi(8.854 \times 10^{-12})(2.71 \times 10^4)(0.0200)^2 \\ &= -1.21 \times 10^{-9} \text{ C}. \end{aligned}$$

21.7.7 21.10.3

a) From Example 21.9 we know that the electric field of a charged ring of radius a at a distance x along the ring's axis is directed away from the ring along its axis and has magnitude

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + a^2)^{3/2}}.$$

By the principle of superposition of electric fields, the electric field of the hydrogen atom is

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0}\left(\frac{e}{a^2} - \frac{ea}{(a^2 + a^2)^{3/2}}\right) \\ &= \frac{1}{4\pi\epsilon_0}e\left(\frac{1}{a^2} - \frac{a}{2\sqrt{2}a^3}\right) \\ &= \frac{1}{4\pi\epsilon_0}\frac{e}{a^2}\left(1 - \frac{1}{2\sqrt{2}}\right). \end{aligned}$$

b) $1 - 1/(2\sqrt{2}) \approx 0.65$ so the field points away from the proton.

21.7.8 21.10.4

a) The charge per unit length is

$$\lambda = \frac{Q}{L}$$

so the charge contained in a segment of length dx is

$$\lambda\, dx = \frac{Q}{L}\, dx.$$

b) The field and source points both lie on the x-axis, so the differential electric field has no y component. The x component is

$$dE_x = -\frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{x^2} = -\frac{1}{4\pi\epsilon_0}\frac{Q}{Lx^2}\, dx.$$

c) The total electric field at the origin is

$$\begin{aligned} E &= \int dE_x \\ &= \int_L^{2L} -\frac{1}{4\pi\epsilon_0}\frac{Q}{Lx^2}\, dx \\ &= -\frac{1}{4\pi\epsilon_0}\frac{Q}{L}\left[-\frac{1}{x}\right]_L^{2L} \\ &= -\frac{1}{4\pi\epsilon_0}\frac{Q}{L}\left(-\frac{1}{2L}+\frac{1}{L}\right) \\ &= -\frac{1}{4\pi\epsilon_0}\frac{Q}{2L^2}. \end{aligned}$$

21.7.9 VP21.14.1

a) The magnitude of the torque is given by

$$\begin{aligned} \tau &= pE\sin\theta \\ &= (6.13\times 10^{-30})(3.00\times 10^{5})\sin 50.0^\circ \\ &= 1.41\times 10^{-24}\,\mathrm{N\,m}. \end{aligned}$$

b) The potential energy is given by

$$\begin{aligned} U &= -pE\cos\theta \\ &= -(6.13\times 10^{-30})(3.00\times 10^{5})\cos 50.0^\circ \\ &= -1.18\times 10^{-24}\,\mathrm{J}. \end{aligned}$$

21.7.10 VP21.14.2

To find the magnitude of the charges we can rearrange the torque equation

$$\begin{aligned}
\tau &= pE\sin\theta \\
&= qdE\sin\theta \\
q &= \frac{\tau}{dE\sin\theta} \\
&= \frac{6.60\times10^{-26}}{(1.10\times10^{-10})(8.50\times10^{4})\sin 90^\circ} \\
&= 7.06\times10^{-21}\,\mathrm{C}.
\end{aligned}$$

21.7.11 VP21.14.3

When the dipole moment is parallel to the field its potential energy is $-pE$ and when it is antiparallel its potential energy is pE. Thus, the work required to perform the rotation is $2pE$ and

$$\begin{aligned}
W &= 2pE \\
p &= \frac{W}{2E} \\
&= \frac{4.60\times10^{-25}}{2(1.20\times10^{5})} \\
&= 1.92\times10^{-30}\,\mathrm{C\,m}.
\end{aligned}$$

21.7.12 VP21.14.4

a) Rearranging the equation for the dipole moment gives

$$\begin{aligned}
p &= qd \\
d &= \frac{p}{q} \\
&= \frac{3.50\times10^{-29}}{1.60\times10^{-19}} \\
&= 2.19\times10^{-10}\,\mathrm{m}.
\end{aligned}$$

b) From Example 21.14, the electric field of the molecule along its axis is

$$E_y = \frac{1}{4\pi\epsilon_0}\frac{2p}{y^3}.$$

Rearranging for y and substituting in the desired field strength gives

$$
\begin{aligned}
y &= \left(\frac{1}{4\pi\epsilon_0}\frac{2p}{E_y}\right)^{1/3} \\
&= \left((8.988 \times 10^9)\frac{2(1.60 \times 10^{-19})(2.19 \times 10^{-10})}{8.00 \times 10^4}\right)^{1/3} \\
&= 1.99 \times 10^{-8}\,\text{m}.
\end{aligned}
$$

21.7.13 Bridging Problem

By symmetry, each point on the semicircle has a corresponding point on the opposite side of the y-axis. The x components of their electric fields cancel, leaving only the y components. Thus, the total electric field at P points in the negative y direction and has magnitude

$$
\begin{aligned}
E &= \int_0^\pi \frac{1}{4\pi\epsilon_0}\frac{\lambda a\,d\theta}{a^2}\sin\theta \\
&= \frac{1}{4\pi\epsilon_0}\frac{Q}{\pi a^2}\int_0^\pi \sin\theta\,d\theta \\
&= \frac{1}{4\pi\epsilon_0}\frac{Q}{\pi a^2}\left[-\cos\theta\right]_0^\pi \\
&= \frac{1}{4\pi\epsilon_0}\frac{2Q}{\pi a^2}.
\end{aligned}
$$