Quantum Computation and Quantum Information by Michael A. Nielsen and Isaac L. Chuang

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Part I
Fundamental concepts

1 Introduction and overview

1.2 Quantum bits

- The special states $|0\rangle$ and $|1\rangle$ form an orthonormal basis and are known as **computational basis states**.
- A quantum bit (**qubit**) is a linear combination of the computational basis states
$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$
where α and β are complex numbers.
- When we measure a qubit we either get $|0\rangle$ with probability $|\alpha|^2$ or $|1\rangle$ with probability $|\beta|^2$. Thus, $|\alpha|^2 + |\beta|^2 = 1$ and a qubit can be thought of as a unit vector in a two-dimensional complex vector space.
- If a qubit is in the state
$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

there's a 50/50 chance of measuring $|0\rangle$ or $|1\rangle$.

- If we let

$$\alpha = e^{i\gamma} \cos \frac{\theta}{2}$$

and

$$\beta = e^{i\gamma} e^{i\varphi} \sin \frac{\theta}{2}$$

then

$$\begin{aligned} |\alpha|^2 + |\beta|^2 &= \alpha^* \alpha + \beta^* \beta \\ &= \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \\ &= 1 \end{aligned}$$

so the qubit is still normalised and it can be written

$$|\psi\rangle = e^{i\gamma} \left(\cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle \right).$$

It turns out that $e^{i\gamma}$ has no observable effects and we can effectively write

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle .$$

This defines a point on a three-dimensional sphere known as the **Bloch sphere** where θ and φ take on their usual roles in a spherical coordinate system.

- Before measurement a qubit is in a linear combination of $|0\rangle$ and $|1\rangle$ but when measured you get one or the other and the state of the system changes to match the measured result.