Advanced Engineering Mathematics Complex Analysis by Dennis G. Zill Problems

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Contents

17 Functions of a Complex Variable

17.1 Complex Numbers

17.1.1

$$3 + 3i$$

17.1.3

$$i^8 = (i^2)^4 = (-1)^4 = 1$$

17.1.5

$$7 - 13i$$

17.1.7

$$-7 + 5i$$

17.1.9

$$11 - 10i$$

17.1.11

$$-5 + 12i$$

17.1.13

$$-2i$$

17.1.15

$$\begin{aligned}\frac{2-4i}{3+5i} &= \frac{(2-4i)(3-5i)}{34} \\ &= \frac{-14-22i}{34} \\ &= -\frac{7}{17} - \frac{11}{17}i\end{aligned}$$

17.1.17

$$\begin{aligned}\frac{(3-i)(2+3i)}{1+i} &= \frac{9+7i}{1+i} \\ &= \frac{(9+7i)(1-i)}{2} \\ &= \frac{16-2i}{2} \\ &= 8-i\end{aligned}$$

17.1.27

$$\begin{aligned}\frac{1}{z} &= \frac{\overline{z}}{z\overline{z}} \\ &= \frac{x-iy}{x^2+y^2} \\ \operatorname{Re}\left(\frac{1}{z}\right) &= \frac{x}{x^2+y^2}\end{aligned}$$

17.1.29

$$\begin{aligned} 2z + 4\overline{z} - 4i &= 2(x + iy) + 4(x - iy) - 4i \\ &= 6x - 2(y + 2)i \\ \operatorname{Im}(2z + 4\overline{z} - 4i) &= -2y - 4 \end{aligned}$$

17.1.31

$$\begin{aligned} z - 1 - 3i &= x + iy - 1 - 3i \\ &= (x - 1) + (y - 3)i \\ |z| &= \sqrt{(x-1)^2 + (y-3)^2} \end{aligned}$$

17.1.33

$$\begin{aligned} 2z &= i(2 + 9i) \\ &= -9 + 2i \\ z &= -\frac{9}{2} + i \end{aligned}$$

17.1.35

$$\begin{aligned} (x + iy)^2 &= x^2 + 2xyi - y^2 \\ &= (x^2 - y^2) + 2xyi \\ x^2 &= y^2 \\ x &= y \\ 2xy &= 1 \\ x^2 &= \frac{1}{2} \\ x &= \frac{\sqrt{2}}{2} \\ z &= \frac{\sqrt{2}}{2}(1 + i) \end{aligned}$$

17.1.37

$$\begin{aligned} z + 2\overline{z} &= x + iy + 2x - 2iy \\ &= 3x - iy \\ \frac{2-i}{1+3i} &= \frac{(2-i)(1-3i)}{10} \\ &= \frac{-1-7i}{10} \\ 3x - iy &= \frac{-1-7i}{10} \\ x &= -\frac{1}{30} \\ y &= \frac{7}{10} \\ z &= -\frac{1}{30} + \frac{7}{10}i \end{aligned}$$

17.1.39

$$\begin{aligned} |10 + 8i| &\approx 12.8 \\ |11 - 6i| &\approx 12.5 \end{aligned}$$

$11 - 6i$ is closer.