Advanced Engineering Mathematics Vectors, Matrices, and Vector Calculus by Dennis G. Zill Problems

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Contents

1 Vectors

1.1 Vectors in 2-Space

1.1.1

(a) $3\mathbf{a} = 6\mathbf{i} + 12\mathbf{j}$

(b) $\mathbf{a} + \mathbf{b} = \mathbf{i} + 8\mathbf{j}$

(c) $\mathbf{a} - \mathbf{b} = 3\mathbf{i}$

(d) $||\mathbf{a} + \mathbf{b}|| = \sqrt{1 + 8^2} = \sqrt{65}$

(e) $||\mathbf{a} - \mathbf{b}|| = 3$

1.1.9

(a) $4\mathbf{a} - 2\mathbf{b} = \langle 6, -14 \rangle$

(b) $-3\mathbf{a} - 5\mathbf{b} = \langle 2, 4 \rangle$

1.1.15

$\overrightarrow{P_1P_2} = \langle 2, 5 \rangle$

1.1.19

$(1, 18)$

1.1.21

(a) Yes

(b) Yes

(c) Yes

(d) No

(e) Yes

(f) Yes

1.1.25

(a) $\frac{\mathbf{a}}{||\mathbf{a}||} = \frac{\langle 2,2\rangle}{\sqrt{2^2+2^2}} = \frac{1}{2\sqrt{2}}\langle 2, 2\rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$

(b) $\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\rangle$

1.1.31

$2\frac{\mathbf{a}}{||\mathbf{a}||} = 2\frac{\langle 3,7\rangle}{\sqrt{3^2+7^2}} = \frac{2}{\sqrt{58}}\langle 3, 7\rangle = \langle \frac{6}{\sqrt{58}}, \frac{14}{\sqrt{58}}\rangle$

1.1.37

$\mathbf{x} = -(\mathbf{a} + \mathbf{b})$

1.1.41

$$\begin{aligned}
\mathbf{a} &= 2\mathbf{i} + 3\mathbf{j} \\
\mathbf{b} &= \mathbf{i} + \mathbf{j} \\
\mathbf{c} &= \mathbf{i} - \mathbf{j}
\end{aligned}$$

$$\begin{aligned}
\mathbf{i} &= \frac{1}{2}(\mathbf{b} + \mathbf{c}) \\
\mathbf{j} &= \frac{1}{2}(\mathbf{b} - \mathbf{c}) \\
\mathbf{a} &= 2\left(\frac{1}{2}(\mathbf{b} + \mathbf{c})\right) + 3\left(\frac{1}{2}(\mathbf{b} - \mathbf{c})\right) \\
&= \mathbf{b} + \mathbf{c} + \frac{3}{2}\mathbf{b} - \frac{3}{2}\mathbf{c} \\
&= \frac{5}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}
\end{aligned}$$

1.1.43

$$
\begin{aligned}
y &= \frac{1}{4}x^2 + 1 \\
y(2) &= 2 \\
y' &= \frac{1}{2}x \\
y'(2) &= 1 \\
\mathbf{v} &= \pm\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle
\end{aligned}
$$

1.1.45

(a)

$$
\begin{aligned}
\mathbf{F}_n &= \mathbf{F}\cos\theta \\
\mathbf{F}_g &= \mathbf{F}\sin\theta \\
||\mathbf{F}_f|| &= \mu||\mathbf{F}_n|| \\
||-\mathbf{F}_g|| &= \mu||\mathbf{F}_n|| \\
||-\mathbf{F}\sin\theta|| &= \mu||\mathbf{F}\cos\theta|| \\
||\mathbf{F}||\sin\theta &= \mu||\mathbf{F}||\cos\theta \\
\tan\theta &= \mu
\end{aligned}
$$

(b) $\theta = \arctan\mu \approx 31^\circ$

1.1.47

$$
\begin{aligned}
F_x &= \frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{L\,dy}{2a(L^2+y^2)^{3/2}} \\
&= \frac{LqQ}{8\pi\epsilon_0}\int_{-a}^{a}(L^2+y^2)^{-3/2}\,dy \\
&= \frac{LqQ}{8\pi\epsilon_0}\frac{2a}{L^2\sqrt{a^2+L^2}} \\
&= \frac{aqQ}{4\pi\epsilon_0 L\sqrt{a^2+L^2}} \\
F_y &= -\frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{y\,dy}{2a(L^2+y^2)^{3/2}} \\
&= 0 \\
\mathbf{F} &= \langle \frac{1}{4\pi\epsilon_0}\frac{qQ}{L\sqrt{a^2+L^2}}, 0\rangle
\end{aligned}
$$

1.1.49

Let the three sides of the triangle be vectors **a**, **b**, and **c**. The triangle is closed so it must be the case that

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0.$$

This gives

$$\mathbf{c} = -(\mathbf{a} + \mathbf{b}).$$

The vector from the midpoint of side **a** to the midpoint of side **b** is

$$\left(\mathbf{a} + \frac{1}{2}\mathbf{b}\right) - \frac{1}{2}\mathbf{a} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$$

which is parallel with **c** and half its length.

1.2 Vectors in 3-Space

1.2.7

A plane at $z = 5$ parellel with the x-y plane.

1.2.9

A line parallel to the z axis at $x = 2$ and $y = 3$.

1.2.13

(a) $(0, 5, 4)$, $(-2, 0, 4)$, $(-2, 5, 0)$

(b) $(-2, 5, -2)$

(c) $(3, 5, 4)$

1.2.15

The planes $x = 0$, $y = 0$, and $z = 0$.

1.2.17

$(-1, 2, -3)$

1.2.19

The planes $z = \pm 5$.

1.2.21

$\sqrt{(6-3)^2 + (4+1)^2 + (8-2)^2} = \sqrt{9 + 25 + 36} = \sqrt{70}$

1.2.31

$$\begin{aligned}\sqrt{(2-x)^2+(1-2)^2+(1-3)^2} &= \sqrt{21}\\ (2-x)^2+1+4 &= 21\\ (2-x)^2 &= 16\\ 2-x &= \pm 4\\ x &= 2\pm 4\\ &= -2 \text{ or } 6\end{aligned}$$

1.2.33

$\left(4, \frac{1}{2}, \frac{3}{2}\right)$

1.2.37

$(-3, -6, 1)$

1.3 Dot Product

1.3.1

$\mathbf{a}\cdot\mathbf{b} = 12$

1.3.11

$\left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b} = \frac{12}{30}\mathbf{b} = \langle -\frac{2}{5}, \frac{4}{5}, 2\rangle$

1.3.13

$\mathbf{a}\cdot\mathbf{b} = ||\mathbf{a}||||\mathbf{b}||\cos\theta = 25\sqrt{2}$

1.3.17

$$\begin{aligned} \mathbf{a} \cdot \mathbf{v} &= 0 \\ 3x_1 + y_1 - 1 &= 0 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \cdot \mathbf{v} &= 0 \\ -3x_1 + 2y_2 + 2 &= 0 \end{aligned}$$

$$\begin{aligned} 3y_2 + 1 &= 0 \\ y_2 &= -\frac{1}{3} \end{aligned}$$

$$\begin{aligned} 3x_1 - \frac{1}{3} - 1 &= 0 \\ x_1 &= \frac{4}{9} \end{aligned}$$

$$\mathbf{v} = \langle \frac{4}{9}, -\frac{1}{3}, 1 \rangle$$

1.3.19

$$\begin{aligned} \mathbf{a} \cdot \mathbf{c} &= \mathbf{a} \cdot \left(\mathbf{b} - \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||^2} \mathbf{a} \right) \\ &= \mathbf{a} \cdot \mathbf{b} - \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||^2} \mathbf{a} \cdot \mathbf{a} \\ &= 0 \end{aligned}$$

1.3.21

$$\begin{aligned}
||\mathbf{a}|| &= \sqrt{3^2+1^2}\\
&= \sqrt{10}\\
||\mathbf{b}|| &= \sqrt{2^2+2^2}\\
&= \sqrt{8}\\
&= 2\sqrt{2}\\
\mathbf{a}\cdot\mathbf{b} &= 4\\
\theta &= \arccos\frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{a}||||\mathbf{b}||}\\
&= \arccos\frac{4}{(\sqrt{10})(2\sqrt{2})}\\
&= \arccos\frac{1}{\sqrt{5}}\\
&\approx 63^\circ
\end{aligned}$$

1.3.25

$$\begin{aligned}
||\mathbf{a}|| &= \sqrt{1^2+2^2+3^3}\\
&= \sqrt{14}\\
\cos\alpha &= \frac{1}{\sqrt{14}}\\
\alpha &\approx 75^\circ\\
\cos\beta &= \frac{2}{\sqrt{14}}\\
\beta &\approx 58^\circ\\
\cos\gamma &= \frac{3}{\sqrt{14}}\\
\gamma &\approx 37^\circ
\end{aligned}$$

1.3.29

$$\begin{aligned}
\overrightarrow{AD} &= \langle s, -s, s\rangle \\
||\overrightarrow{AD}|| &= s\sqrt{3} \\
\overrightarrow{AB} &= \langle s, 0, 0\rangle \\
||\overrightarrow{AB}|| &= s \\
\theta &= \arccos \frac{\overrightarrow{AD}\cdot\overrightarrow{AB}}{||\overrightarrow{AD}||||\overrightarrow{AB}||} \\
&= \arccos \frac{s^2}{s^2\sqrt{3}} \\
&= \arccos \frac{1}{\sqrt{3}} \\
&\approx 55^\circ
\end{aligned}$$

1.3.33

$$\begin{aligned}
\text{comp}_{\mathbf{b}}\mathbf{a} &= \frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{b}||} \\
&= \frac{5}{7}
\end{aligned}$$

1.3.37

$$\begin{aligned}
\text{comp}_{\overrightarrow{OP}}\mathbf{a} &= \frac{\mathbf{a}\cdot\overrightarrow{OP}}{||\overrightarrow{OP}||} \\
&= \frac{72}{\sqrt{109}}
\end{aligned}$$

1.3.39

$$\begin{aligned}
\text{proj}_{\mathbf{b}}\mathbf{a} &= \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b} \\
&= \frac{35}{25}\mathbf{b} \\
&= \langle -\frac{21}{5}, \frac{28}{5}\rangle
\end{aligned}$$

1.3.43

$$\begin{aligned}\mathbf{a}+\mathbf{b} &= \langle 3,4\rangle \\ \mathrm{proj}_{\mathbf{a}+\mathbf{b}}\mathbf{a} &= \left(\frac{\mathbf{a}\cdot(\mathbf{a}+\mathbf{b})}{(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})}\right)(\mathbf{a}+\mathbf{b}) \\ &= \frac{24}{25}(\mathbf{a}+\mathbf{b}) \\ &= \langle\frac{72}{25},\frac{96}{25}\rangle\end{aligned}$$

1.3.45

$W = \mathbf{F}\cdot\mathbf{d} = Fd\cos\theta = 1000$

1.3.47

(a) $W = 0$

(b)

$$\begin{aligned}||\mathbf{d}|| &= \sqrt{4^2+3^2} \\ &= 5 \\ \mathbf{F} &= F\hat{\mathbf{d}} \\ &= F\frac{\mathbf{d}}{||\mathbf{d}||} \\ &= F\langle\frac{4}{5},\frac{3}{5}\rangle \\ &= \langle 24, 18\rangle \\ W &= \mathbf{F}\cdot\mathbf{d} \\ &= 150\,\mathrm{J}\end{aligned}$$

1.4 Cross Product

1.4.1

$$\begin{aligned}\mathbf{a}\times\mathbf{b} &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ 0 & 3 & 5\end{vmatrix} \\ &= -5\mathbf{i}-5\mathbf{j}+3\mathbf{k}\end{aligned}$$

1.4.11

$$\begin{aligned}\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 2 & -4 \\ -3 & 1 & 1 \end{vmatrix} \\ &= 6\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}\end{aligned}$$

1.4.17

(a)

$$\begin{aligned}\mathbf{b} \times \mathbf{c} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix} \\ &= \mathbf{j} - \mathbf{k} \\ \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 0 & 1 & -1 \end{vmatrix} \\ &= -\mathbf{i} + \mathbf{j} + \mathbf{k}\end{aligned}$$

1.4.19

$2\mathbf{k}$

1.4.21

$$\begin{aligned}\mathbf{k} \times (2\mathbf{i} - \mathbf{j}) &= (\mathbf{k} \times 2\mathbf{i}) - (\mathbf{k} \times \mathbf{j}) \\ &= \mathbf{i} + 2\mathbf{j}\end{aligned}$$

1.4.23

$$\begin{aligned}[(2\mathbf{k}) \times (3\mathbf{j})] \times (4\mathbf{j}) &= (-6\mathbf{i}) \times (4\mathbf{j}) \\ &= -24\mathbf{k}\end{aligned}$$

1.4.37

$12\mathbf{i} - 9\mathbf{j} + 18\mathbf{k}$

1.4.53

$$\begin{aligned}
\mathbf{b} \times \mathbf{c} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 6 & -6 \\ \frac{5}{2} & 3 & \frac{1}{2} \end{vmatrix} \\
&= 21\mathbf{i} - 14\mathbf{j} - 21\mathbf{k} \\
\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) &= 4 \times 21 + 6 \times (-14) \\
&= 0
\end{aligned}$$

They are coplanar.