Advanced Engineering Mathematics Complex Analysis by Dennis G. Zill Problems

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17 Functions of a Complex Variable

17.1 Complex Numbers

17.1.1

$$3 + 3i$$

17.1.3

$$i^8 = (i^2)^4 = (-1)^4 = 1$$

17.1.5

$$7 - 13i$$

17.1.7

$$-7 + 5i$$

17.1.9

$$11 - 10i$$

17.1.11

$$-5 + 12i$$

17.1.13

$$-2i$$

17.1.15

$$\frac{2-4i}{3+5i} = \frac{(2-4i)(3-5i)}{34}$$
$$= \frac{-14-22i}{34}$$
$$= -\frac{7}{17} - \frac{11}{17}i$$

17.1.17

$$\frac{(3-i)(2+3i)}{1+i} = \frac{9+7i}{1+i}$$
$$= \frac{(9+7i)(1-i)}{2}$$
$$= \frac{16-2i}{2}$$
$$= 8-i$$

17.1.27

$$\frac{1}{z} = \frac{\bar{z}}{z\bar{z}}$$
$$= \frac{x-iy}{x^2+y^2}$$
$$\operatorname{Re}\left(\frac{1}{z}\right) = \frac{x}{x^2+y^2}$$

17.1.29

$$2z + 4\bar{z} - 4i = 2(x+iy) + 4(x-iy) - 4i$$
$$= 6x - 2(y+2)i$$
$$\operatorname{Im}(2z + 4\bar{z} - 4i) = -2y - 4$$

17.1.31

$$z - 1 - 3i = x + iy - 1 - 3i$$
$$= (x-1) + (y-3)i$$
$$|z| = \sqrt{(x-1)^2 + (y-3)^2}$$

17.1.33

$$2z = i(2 + 9i)$$
$$= -9 + 2i$$
$$z = -\frac{9}{2} + i$$

17.1.35

$$(x + iy)^2 = x^2 + 2xyi - y^2$$
$$= (x^2 - y^2) + 2xyi$$
$$x^2 = y^2$$
$$x = y$$
$$2xy = 1$$
$$x^2 = \frac{1}{2}$$
$$x = \frac{\sqrt{2}}{2}$$
$$z = \frac{\sqrt{2}}{2}(1 + i)$$

17.1.37

$$z + 2\bar{z} = x + iy + 2x - 2iy$$
$$= 3x - iy$$
$$\frac{2 - i}{1 + 3i} = \frac{(2 - i)(1 - 3i)}{10}$$
$$= \frac{-1 - 7i}{10}$$
$$3x - iy = \frac{-1 - 7i}{10}$$
$$x = -\frac{1}{30}$$
$$y = \frac{7}{10}$$
$$z = -\frac{1}{30} + \frac{7}{10}i$$

17.1.39

$$|10 + 8i| \approx 12.8$$
$$|11 - 6i| \approx 12.5$$

$11 - 6i$ is closer.

17.2 Powers and Roots

17.2.1

$$2(\cos 0 + i \sin 0)$$

17.2.3

$$-3[\cos(-\pi/2) + i \sin(-\pi/2)]$$

17.2.5

$$\sqrt{2}[\cos(\pi/4) + i \sin(\pi/4)]$$

17.2.7

$$2[\cos(5\pi/6) + i \sin(5\pi/6)]$$

17.2.9

$$\begin{aligned}
\frac{3}{-1+i} &= \frac{3(-1-i)}{2} \\
&= \frac{-3 - 3i}{2} \\
&= -\frac{3}{2} - \frac{3}{2}i \\
&= \frac{3\sqrt{2}}{2}[\cos(5\pi/4) + i \sin(5\pi/4)]
\end{aligned}$$

17.2.11

$$-\frac{5\sqrt{3}}{2} - \frac{5}{2}i$$

17.2.13

$$5.54 + 2.30i$$

17.2.15

$$8[\cos(\pi/2) + i \sin(\pi/2)] = 8i$$
$$\frac{1}{2}[\cos(-\pi/4) + i \sin(-\pi/4)] = \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4}i$$

17.2.21

$$(1 + \sqrt{3}i)^9 = \{2[\cos(\pi/3) + i\sin(\pi/3)]\}^9$$
$$= 512(\cos\pi + i\sin\pi)$$
$$= -512$$

17.2.23

$$\left(\frac{1}{2} + \frac{1}{2}i\right)^1 0 = \left\{\frac{\sqrt{2}}{2}[\cos(\pi/4) + i\sin(\pi/4)]\right\}^{10}$$
$$= \frac{1}{32}[\cos(\pi/2) + i\sin(\pi/2)]$$
$$= \frac{1}{32}i$$

17.2.27

$$w_k = 2[\cos(2\pi k/3) + i\sin(2\pi k/3)]$$
$$w_0 = 2$$
$$w_1 = -1 + \sqrt{3}i$$
$$w_2 = -1 - \sqrt{3}i$$

17.2.29

$$w_k = \cos(\pi/4 + k\pi) + i\sin(\pi/4 + k\pi)$$
$$w_0 = \frac{\sqrt{2}}{2}(1 + i)$$
$$w_1 = -\frac{\sqrt{2}}{2}(1 + i)$$

17.2.31

$$w_k = \sqrt{2}[\cos(\pi/3 + k\pi) + i\sin(\pi/3 + k\pi)]$$
$$w_0 = \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$$
$$w_1 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$$

17.2.33

$$z^4 + 1 = 0$$
$$z^4 = -1$$
$$w_k = \cos(\pi/4 + k\pi/2) + \sin(\pi/4 + k\pi/2)$$
$$w_0 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$
$$w_1 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$
$$w_2 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$
$$w_3 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$

17.3   Sets in the Complex Plane

17.3.1

A vertical line at $\mathrm{Re}(z) = 5$.

17.3.3

A horizontal line at $\mathrm{Im}(z) = -3$.

17.3.5

A circle of radius 2 centred at $3i$.

17.3.7

A circle of radius 5 centred at $4 - 3i$.

17.3.9

The region of the plane to the left of (but not including) $\mathrm{Re}(z) = -1$. It is a domain.

17.3.11

The region of the plane above (but not including) $\mathrm{Im}(z) = 3$. It is a domain.

17.3.13

The region of the plane between (but not including) $\mathrm{Re}(z) = 3$ and $\mathrm{Re}(z) = 5$. It is a domain.

17.3.15

$$z^2 = (a + ib)^2$$
$$= a^2 - b^2 + 2iab$$
$$\text{Re}(z^2) = a^2 - b^2$$
$$\text{Re}(z^2) > 0$$
$$a^2 - b^2 > 0$$
$$a^2 > b^2$$

The region between $y = x$ and $y = -x$. Not a domain.

17.3.17

The region between $\theta = 0$ and $\theta = 2\pi/3$. Not a domain.

17.3.19

The region outside a circle of radius 1 centred at i. It is a domain.

17.3.21

The region between the circles of radius 2 and 3 centred at i. It is a domain.

17.3.23

$$y = -x$$

17.3.25

$$z^2 + \bar{z}^2 = (a + ib)^2 + (a - ib)^2$$
$$= a^2 + 2iab - b^2 + a^2 - 2iab - b^2$$
$$= 2(a^2 - b^2)$$
$$2(a^2 - b^2) = 2$$
$$a^2 - b^2 = 1$$
$$a^2 = b^2 + 1$$

The hyperbola $x^2 - y^2 = 1$.

17.4 Functions of a Complex Variable

17.4.1

$$f(z) = z^2$$
$$= (x + iy)^2$$
$$= x^2 - y^2 + 2ixy$$
$$u(x, y) = x^2 - y^2$$
$$= x^2 - 4$$
$$v(x, y) = 2xy$$
$$= 4x$$
$$x = \frac{v}{4}$$
$$u = \left(\frac{v}{4}\right)^2 - 4$$
$$= \frac{1}{16}v^2 - 4$$

17.4.3

$$u = -y^2$$
$$v = 0$$

Line on the left half of the real axis.

17.4.5

$$u = 0$$
$$v = 2x^2$$

Line on the top half of the imaginary axis.

17.4.7

$$f(x) = (6x - 5) + i(6y + 9)$$

17.4.9

$$f(z) = (x^2 - y^2 - 3x) + i(2xy - 3y + 4)$$

17.4.11

$$f(z) = (x^3 - 3xy^2 - 4x) + i(3x^2y - y^3 - 4y)$$

17.4.13

$$f(z) = \left(x + \frac{x}{x^2 + y^2}\right) i \left(y - \frac{y}{x^2 + y^2}\right)$$

17.4.15

(a) $-4 + i$

(b) $3 - 9i$

(c) $1 + 86i$

17.4.17

(a) $14 - 20i$

(b) $-13 + 43i$

(c) $3 - 26i$

17.4.19

$$6 - 5i$$

17.4.21

$$-4i$$

17.4.27

$$f'(z) = 12z^2 - 2(3 + i)z - 5$$

17.4.29

$$f'(z) = 2(z^2 - 4z + 8i) + (2z + 1)(2z - 4)$$
$$= 2z^2 - 8z + 16i + 4z^2 - 8z + 2z - 4$$
$$= 6z^2 - 14z - 4 + 16i$$

17.4.31

$$f'(z) = 6z(z^2 - 4i)^2$$

17.4.33

$$f'(z) = \frac{3(2z + i) - 2(3z - 4 + 8i)}{(2z + i)^2}$$
$$= \frac{6z + 3i - 6z + 8 - 16i}{(2z + i)^2}$$
$$= \frac{8 - 13i}{(2z + i)^2}$$

17.4.35

$3i$

17.4.37

$\pm 2i$

17.4.41

$$\frac{dx}{dt} = 2x$$
$$x = c_1 e^{2t}$$
$$\frac{dy}{dt} = 2y$$
$$y = c_2 e^{2t}$$

17.4.43

$$f(z) = \frac{1}{z}$$
$$= \frac{1}{x - iy}$$
$$= \frac{x + iy}{x^2 + y^2}$$
$$= \frac{x}{x^2 + y^2} + i\frac{y}{x^2 + y^2}$$
$$\frac{dx}{dt} = \frac{x}{x^2 + y^2}$$
$$\frac{dy}{dt} = \frac{y}{x^2 + y^2}$$
$$\frac{dy}{dx} = \frac{y}{x}$$
$$\frac{dy}{y} = \frac{dx}{x}$$
$$\ln y = \ln x + c_1$$
$$y = c_2 x$$

17.5 Cauchy-Riemann Equations

17.5.1

$$f(z) = z^3$$
$$= (x + iy)^3$$
$$= (x^2 + 2ixy - y^2)(x + iy)$$
$$= x^3 + ix^2 y + 2ix^2 y - 2xy^2 - xy^2 - iy^3$$
$$= (x^3 - 3xy^2) + i(3x^2 y - y^3)$$
$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$$
$$= \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} = -6xy$$
$$= -\frac{\partial v}{\partial x}$$

17.5.3

$$f(z) = \text{Re}(z)$$
$$= x$$
$$\frac{\partial u}{\partial x} = 1$$
$$\neq \frac{\partial v}{\partial y}$$

17.5.5

$$f(z) = 4z - 6\overline{z} + 3$$
$$= 4(x + iy) - 6(x - iy) + 3$$
$$= (-2x + 3) + 10iy$$
$$\frac{\partial u}{\partial x} = -2$$
$$\neq \frac{\partial v}{\partial y}$$

17.5.7

$$f(z) = x^2 + y^2$$
$$\frac{\partial u}{\partial x} = 2x$$
$$\neq \frac{\partial v}{\partial y}$$

17.5.9

$$f(z) = e^x \cos y + ie^x \sin y$$
$$u = e^x \cos y$$
$$\frac{\partial u}{\partial x} = e^x \cos y$$
$$\frac{\partial u}{\partial y} = -e^x \sin y$$
$$v = e^x \sin y$$
$$\frac{\partial v}{\partial x} = e^x \sin y$$
$$\frac{\partial v}{\partial y} = e^x \cos y$$

Analytic everywhere.

17.5.11

$$f(z) = x + \sin x \cosh y + i(y + \cos x \sinh y)$$
$$u = x + \sin x \cosh y$$
$$\frac{\partial u}{\partial x} = 1 + \cos x \cosh y$$
$$\frac{\partial u}{\partial y} = \sin x \sinh y$$
$$v = y + \cos x \sinh y$$
$$\frac{\partial v}{\partial x} = -\sin x \sinh y$$
$$\frac{\partial v}{\partial y} = 1 + \cos x \cosh y$$

Analytic everywhere.

17.5.15

$$f(z) = 3x - y + 5 + i(ax + by - 3)$$
$$u = 3x - y + 5$$
$$\frac{\partial u}{\partial x} = 3$$
$$\frac{\partial u}{\partial y} = -1$$
$$v = ax + by - 3$$
$$\frac{\partial v}{\partial x} = a$$
$$\frac{\partial v}{\partial y} = b$$
$$a = 1$$
$$b = 3$$

17.5.17

$$f(z) = x^2 + y^2 + 2ixy$$
$$u = x^2 + y^2$$
$$\frac{\partial u}{\partial x} = 2x$$
$$\frac{\partial u}{\partial y} = 2y$$
$$v = 2xy$$
$$\frac{\partial v}{\partial x} = 2y$$
$$\frac{\partial v}{\partial y} = 2x$$

Only differentiable when $y = 0$.

17.5.19

$$f(z) = x^3 + 3xy^2 - x + i(y^3 + 3x^2y - y)$$
$$u = x^3 + 3xy^2 - x$$
$$\frac{\partial u}{\partial x} = 3x^2 + 3y^2 - 1$$
$$\frac{\partial u}{\partial y} = 6xy$$
$$v = y^3 + 3x^2y - y$$
$$\frac{\partial v}{\partial x} = 6xy$$
$$\frac{\partial v}{\partial y} = 3y^2 + 3x^2 - 1$$

Only differentiable when $x = 0$ or $y = 0$.

17.5.21

$$f(z) = e^x \cos y + ie^x \sin y$$
$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$
$$= e^x \cos y + ie^x \sin y$$

17.5.23

$$u = x$$
$$\frac{\partial^2 u}{\partial x^2} = 0$$
$$\frac{\partial^2 u}{\partial y^2} = 0$$
$$\frac{\partial v}{\partial y} = 1$$
$$v = y + h(x)$$
$$h'(x) = 0$$
$$v = y + c$$
$$f(z) = x + i(y + c)$$

17.5.25

$$u = x^2 - y^2$$
$$\frac{\partial^2 u}{\partial x^2} = 2$$
$$\frac{\partial^2 u}{\partial y^2} = -2$$
$$\frac{\partial v}{\partial y} = 2x$$
$$v = 2xy + h(x)$$
$$2y = 2y + h'(x)$$
$$h'(x) = 0$$
$$h(x) = c$$
$$v = 2xy + c$$
$$f(z) = (x^2 - y^2) + i(2xy + c)$$

17.6   Exponential and Logarithmic Functions

17.6.1

$$\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

17.6.3

$$e^{-1}\frac{\sqrt{2}}{2}(1 + i)$$

17.6.5

$$-e^{\pi}$$

17.6.7

$$e^{1.5}(\cos 2 + i \sin 2) = -1.865 + 4.075i$$

17.6.9

$$\cos 5 + i \sin 5 = 0.2836 - 0.9589i$$

17.6.11

$$e^{1+5\pi i/4}e^{-1-\pi i/3} = e^{11\pi i/12}$$
$$= \cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12}$$
$$= -0.9659 + 0.2588i$$

17.6.13

$$f(z) = e^{-iz}$$
$$= e^{-i(x+iy)}$$
$$= e^{y-ix}$$
$$= e^{y}(\cos x - i \sin x)$$

17.6.15

$$f(z) = e^{z^2}$$
$$= e^{x^2-y^2+2ixy}$$
$$= e^{x^2-y^2}[\cos(2xy) + i \sin(2xy)]$$

17.6.17

$$e^{z} = e^{x+iy}$$
$$= e^{x}(\cos y + i \sin y)$$
$$|e^{z}| = \sqrt{e^{2x}[\cos^2 y + \sin^2 y]}$$
$$= e^{x}$$

17.6.19

$$
\begin{aligned}
e^{z+\pi i} &= e^{x+i(y+\pi)} \\
&= e^x[\cos(y+\pi) + i\sin(y+\pi)] \\
&= e^x[-\cos y - i\sin y] \\
&= -e^x(\cos y + i\sin y) \\
e^{z-\pi i} &= e^{x+i(y-\pi)} \\
&= e^x[\cos(y-\pi) + i\sin(y-\pi)] \\
&= e^x(-\cos y - i\sin y) \\
&= -e^x(\cos y + i\sin y)
\end{aligned}
$$

17.6.21

$$
\begin{aligned}
e^{\overline{z}} &= e^{x-iy} \\
&= e^x(\cos y - i\sin y) \\
u &= e^x\cos y \\
v &= -e^x\sin y \\
\frac{\partial u}{\partial x} &= e^x\cos y \\
&\neq \frac{\partial v}{\partial y}
\end{aligned}
$$

17.6.23

$$
\log_e 5 + i(\pi + 2n\pi) = 1.6094 + i(\pi + 2n\pi)
$$

17.6.25

$$
\log_e(2\sqrt{2}) + i\left(\frac{3}{4}\pi + 2n\pi\right) = 1.0397 + i\left(\frac{3}{4}\pi + 2n\pi\right)
$$

17.6.27

$$
\log_e(2\sqrt{2}) + i\left(\frac{1}{3}\pi + 2n\pi\right) = 1.0397 + i\left(\frac{1}{3}\pi + 2n\pi\right)
$$

17.6.29

$$
\log_e(6\sqrt{2}) - \frac{\pi}{4}i = 2.1383 - \frac{\pi}{4}i
$$

17.6.31

$$
\log_e 13 + 2.7468i = 2.5649 + 2.7468i
$$

17.6.33

$$5\left(\log_e 2 + \frac{\pi}{3}i\right) = 3.4657 - \frac{\pi}{3}i$$

17.6.35

$$z = \log_e 4 + i\left(\frac{\pi}{2} + 2n\pi\right) = 1.3863 + i\left(\frac{\pi}{2} + 2n\pi\right)$$

17.6.37

$$z - 1 = 2 + i\left(-\frac{\pi}{2} + 2n\pi\right)$$
$$z = 3 + i\left(-\frac{\pi}{2} + 2n\pi\right)$$

17.6.39

$$\ln(-i) = i\left(-\frac{\pi}{2} + 2n\pi\right)$$
$$(-i)^{4i} = e^{4i\ln(-i)}$$
$$= e^{4i\times i(-\pi/2 + 2n\pi)}$$
$$= e^{2\pi(1-4n)}$$

17.6.41

$$\ln(1+i) = \log_e \sqrt{2} + i\left(\frac{\pi}{4} + 2n\pi\right)$$
$$(1+i)^{(1+i)} = e^{(1+i)\ln(1+i)}$$
$$= e^{(1+i)[\log_e \sqrt{2} + i(\pi/4 + 2n\pi)]}$$
$$= e^{\log_e \sqrt{2} + i(\pi/4 + 2n\pi) + i\log_e \sqrt{2} - (\pi/4 + 2n\pi)}$$
$$= e^{(\log_e \sqrt{2} - \pi/4 - 2n\pi) + i(\log_e \sqrt{2} + \pi/4 + 2n\pi)}$$
$$= e^{-2n\pi}e^{(\log_e \sqrt{2} - \pi/4) + i(\log_e \sqrt{2} + \pi/4)}$$
$$= e^{-2n\pi}e^{\log_e \sqrt{2} - \pi/4}e^{i(\log_e \sqrt{2} + \pi/4)}$$
$$= e^{-2n\pi}(0.2739 + 0.5837i)$$

17.6.43

$$\text{Ln}(-1) = \pi i$$
$$(-1)^{(-2i/\pi)} = e^{(-2i/\pi)\text{Ln}(-1)}$$
$$= e^{(-2i/\pi)(\pi i)}$$
$$= e^2$$

17.6.47

(a)

$$(-1+i)^2 = -2i$$
$$\mathrm{Ln}(-1+i)^2 = \mathrm{Ln}(-2i)$$
$$= \log_e 2 - \frac{\pi}{2}i$$
$$2\,\mathrm{Ln}(-1+i) = 2\log_e \sqrt{2} + \frac{3\pi}{2}i$$
$$\neq \mathrm{Ln}(-1+i)^2$$

Not true

(b)

$$\mathrm{Ln}\,i^3 = \mathrm{Ln}(-i)$$
$$= -\frac{\pi}{2}i$$
$$3\,\mathrm{Ln}\,i = \frac{3\pi}{2}i$$
$$\neq \mathrm{Ln}\,i^3$$

Not true

(c)

$$\ln i^3 = i\left(-\frac{\pi}{2} + 2n\pi\right)$$
$$3\ln i = 3i\left(\frac{\pi}{2} + 2n\pi\right)$$
$$\neq \ln i^3$$

Not true

17.7 Trigonometric and Hyperbolic Functions

17.7.1

$$\cos(3i) = \cos 0 \cosh 3 - i\sin 0 \sinh 3$$
$$= \cosh 3$$
$$= 10.0677$$

17.7.3

$$\sin(\pi/4 + i) = \sin\frac{\pi}{4}\cosh 1 + i\cos\frac{\pi}{4}\sinh 1$$
$$= 1.0911 + 0.8309i$$

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17.7.5

$$\begin{aligned}
\tan i &= \frac{\sin i}{\cos i} \\
&= \frac{\sin 0 \cosh 1 + i \cos 0 \sinh 1}{\cos 0 \cosh 1 + i \sin 0 \sinh 1} \\
&= \frac{i \sinh 1}{\cosh 1} \\
&= i \tanh 1 \\
&= 0.7615i
\end{aligned}$$

17.7.7

$$\begin{aligned}
\sec(\pi + i) &= \frac{1}{\cos(\pi + i)} \\
&= \frac{1}{\cos \pi \cosh 1 + \sin \pi \sinh 1} \\
&= -\frac{1}{\cosh 1} \\
&= -0.6480
\end{aligned}$$

17.7.9

$$\begin{aligned}
\cosh(\pi i) &= \cosh 0 \cos \pi + i \sinh 0 \sin \pi \\
&= -1
\end{aligned}$$

17.7.11

$$\begin{aligned}
\sinh(1 + \pi i/3) &= \sinh 1 \cos(\pi/3) + i \cosh 1 \sin(\pi/3) \\
&= 0.5876 + 1.3363i
\end{aligned}$$

17.7.15

$$\sin z = 2$$

$$\frac{e^{iz} - e^{-iz}}{2i} = 2$$

$$e^{iz} - e^{-iz} = 4i$$

$$e^{2iz} - 1 = 4ie^{iz}$$

$$e^{2iz} - 4ie^{iz} - 1 = 0$$

$$e^{iz} = \frac{4i \pm \sqrt{-16 + 4}}{2}$$

$$= (2 \pm \sqrt{3})i$$

$$iz = \log_e(2 \pm \sqrt{3}) + i(\pi/2 + 2n\pi)$$

$$z = (\pi/2 + 2n\pi) - i\log_e(2 \pm \sqrt{3})$$

17.7.17

$$\sinh z = -i$$

$$\frac{e^z - e^{-z}}{2} = -i$$

$$e^{2z} + 2ie^z - 1 = 0$$

$$e^z = \frac{-2i \pm \sqrt{-4 + 4}}{2}$$

$$= -i$$

$$z = \ln(-i)$$

$$= i\left(-\frac{\pi}{2} + 2n\pi\right)$$

17.7.19

$$\cos z = \sin z$$

$$\frac{e^{iz} + e^{-iz}}{2} = \frac{e^{iz} - e^{-iz}}{2i}$$

$$e^{iz} + e^{-iz} = \frac{e^{iz} - e^{-iz}}{i}$$

$$= -i(e^{iz} - e^{-iz})$$

$$e^{2iz} + 1 = -i(e^{2iz} - 1)$$

$$e^{2iz}(1 + i) = -1 + i$$

$$e^{2iz} = \frac{-1 + i}{1 + i}$$

$$= \frac{(-1 + i)(1 - i)}{(1 + i)(1 - i)}$$

$$= \frac{-1 + i + i + 1}{1 - i + i + 1}$$

$$= \frac{2i}{2}$$

$$= i$$

$$2iz = \ln i$$

$$= i\left(\frac{\pi}{2} + 2n\pi\right)$$

$$z = \frac{\pi}{4} + n\pi$$

17.7.21

$$\cos z = \cosh 2$$

$$\cos x \cosh y - i \sin x \sinh y = \cosh 2$$

$$y = \pm 2$$

$$x = 2n\pi$$

$$z = 2n\pi \pm 2i$$

17.8 Inverse Trigonometric and Hyperbolic Functions

17.8.1

$$\arcsin z = -i \ln[iz + (1 - z^2)^{1/2}]$$
$$\arcsin(-i) = -i \ln[i(-i) + (1 - (-i)^2)^{1/2}]$$
$$= -i \ln[1 \pm \sqrt{2}]$$
$$\ln(1 + \sqrt{2}) = \log_e(1 + \sqrt{2}) + 2n\pi i$$
$$\ln(1 - \sqrt{2}) = \ln\left(-\frac{1}{1 + \sqrt{2}}\right)$$
$$= -\ln[-(1 + \sqrt{2})]$$
$$= -[\log_e(1 + \sqrt{2}) + i(\pi + 2n\pi)]$$
$$= -\log_e(1 + \sqrt{2}) + i(\pi + 2n\pi)$$
$$\ln(1 \pm \sqrt{2}) = (-1)^n \log_e(1 + \sqrt{2}) + n\pi i$$
$$\arcsin(-i) = -i[(-1)^n \log_e(1 + \sqrt{2}) + n\pi i]$$
$$= n\pi - (-1)^n i \log_e(1 + \sqrt{2})$$
$$= n\pi + (-1)^{n+1} i \log_e(1 + \sqrt{2})$$

17.8.3

$$\arcsin 0 = -i \ln(\pm 1)$$
$$= -i(n\pi i)$$
$$= n\pi$$

17.8.5

$$\arccos 2 = -i \ln[2 + i(1 - 2^2)^{1/2}]$$
$$= -i \ln[2 \pm \sqrt{3}]$$
$$\ln(2 + \sqrt{3}) = \log_e(2 + \sqrt{3}) + 2n\pi i$$
$$\ln(2 - \sqrt{3}) = \log_e(2 - \sqrt{3}) + 2n\pi i$$
$$= -\log_e(2 + \sqrt{3}) + 2n\pi i$$
$$\ln(2 \pm \sqrt{3}) = \pm \log_e(2 + \sqrt{3}) + 2n\pi i$$
$$\arccos 2 = 2n\pi \pm i \log_e(2 + \sqrt{3})$$

17.8.7

$$\arccos \frac{1}{2} = -i \ln \left\{ \frac{1}{2} + i \left[1 - \left(\frac{1}{2} \right)^2 \right]^{1/2} \right\}$$

$$= -i \ln \left(\frac{1}{2} \pm \frac{\sqrt{3}}{2} i \right)$$

$$\ln \left(\frac{1}{2} \pm \frac{\sqrt{3}}{2} i \right) = i \left(\pm \frac{\pi}{3} + 2n\pi \right)$$

$$\arccos \frac{1}{2} = \pm \frac{\pi}{3} + 2n\pi$$

17.8.9

$$\arctan 1 = \frac{i}{2} \ln \frac{1+i}{-1+i}$$

$$\frac{1+i}{-1+i} = \frac{(1+i)(-1-i)}{(-1+i)(-1-i)}$$

$$= -i$$

$$\ln(-i) = i \left(-\frac{\pi}{2} + 2n\pi \right)$$

$$\arctan 1 = \frac{\pi}{4} + n\pi$$

17.8.11

$$\operatorname{arcsinh}\frac{4}{3} = \ln\left\{\frac{4}{3} + \left[\left(\frac{4}{3}\right)^2 + 1\right]^{1/2}\right\}$$

$$= \ln\left(\frac{4}{3} \pm \frac{5}{3}\right)$$

$$\ln\left(\frac{4}{3} + \frac{5}{3}\right) = \ln\frac{9}{3}$$

$$= \ln 3$$

$$= \log_e 3 + 2n\pi i$$

$$\ln\left(\frac{4}{3} - \frac{5}{3}\right) = \ln\left(-\frac{1}{3}\right)$$

$$= \log_e \frac{1}{3} + i(\pi + 2n\pi)$$

$$= -\log_e 3 + i(\pi + 2n\pi)$$

$$\operatorname{arcsinh}\frac{4}{3} = (-1)^n \log_e 3 + n\pi i$$

17.9 Chapter in Review

17.9.1

0, 32

17.9.3

$$\frac{3 + 4i}{3 - 4i} = \frac{(3 + 4i)^2}{(3 - 4i)(3 + 4i)}$$

$$= \frac{-7 + 24i}{25}$$

$$= -\frac{7}{25} + \frac{24}{25}i$$

$$\operatorname{Re}\left(\frac{z}{\bar{z}}\right) = -\frac{7}{25}$$

17.9.5

$$\frac{4i}{-3 - 4i} = \frac{(4i)(-3 + 4i)}{(-3 - 4i)(-3 + 4i)}$$
$$= \frac{-16 - 12i}{25}$$
$$= -\frac{16}{25} - \frac{12}{25}i$$
$$|z| = \sqrt{\left(\frac{16}{25}\right)^2 + \left(\frac{12}{25}\right)^2}$$
$$= \frac{4}{5}$$

17.9.7

False

17.9.9

$$e^z = 2i$$
$$z = \ln(2i)$$
$$= \log_e 2 + i\left(\frac{\pi}{2} + 2n\pi\right)$$

17.9.11

$$(1 + i)^{(2+i)} = e^{(2+i)\ln(1+i)}$$
$$ln(1 + i) = \log_e \sqrt{2} + \frac{\pi}{4}i$$
$$(2 + i)\left(\log_e \sqrt{2} + \frac{\pi}{4}i\right) = 2\log_e \sqrt{2} + \frac{\pi}{2}i + i\log_e \sqrt{2} - \frac{\pi}{4}$$
$$= \left(2\log_e \sqrt{2} - \frac{\pi}{4}\right) + i\left(\log_e \sqrt{2} + \frac{\pi}{2}\right)$$
$$(1 + i)^{(2+i)} = e^{2\log_e \sqrt{2} - \pi/4}\left[\cos\left(\log_e \sqrt{2} + \frac{\pi}{2}\right) + i\sin\left(\log_e \sqrt{2} + \frac{\pi}{2}\right)\right]$$
$$\approx -0.3097 + 0.8576i$$

17.9.13

False

17.9.15

$$\text{Ln}(-ie^3) = 3 - \frac{\pi}{2}i$$

17.9.21

$$z^2 = x^2 - y^2 + 2ixy$$
$$\text{Im}(z^2) \leq 2$$
$$2xy \leq 2$$

17.9.23

$$\frac{1}{\sqrt{x^2 + y^2}} \leq 1$$

17.9.27

$$z^4 = 1 - i$$
$$z_k = 2^{1/8} e^{(-\pi/4 + 2k\pi)i/4}$$
$$= 2^{1/8} e^{i(k\pi/2 - \pi/16)}$$
$$z_0 = 1.0695 - 0.2127i$$
$$z_1 = 0.2127 + 1.0695i$$
$$z_2 = -1.0695 + 0.2127i$$
$$x_3 = -0.2127 - 1.0695i$$