

# University Physics with Modern Physics

## Electromagnetism Notes

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## 21 Electric Charge and Electric Field

### 21.1 Electric Charge

- Electrons have a much smaller mass than neutrons and protons
- Neutrons and protons have a very similar mass
- Electrons and protons have the same magnitude of charge
- The number of protons in an atom determines its **atomic number**
- If an electron is added to a neutral atom it becomes a **negative ion**, if one is removed it becomes a **positive ion** — this is called **ionisation**
- The **principle of conservation of charge** states that the algebraic sum of all the electric charges in any closed system is constant
- The electron or proton's magnitude of charge is a natural unit of charge — every observable amount of electric charge is an integer multiple of this

### 21.2 Conductors, Insulators, and Induced Charges

- **Conductors** permit easy movement of charge, **insulators** do not
- Holding a charged object near an uncharged object causes free electrons in the latter to move away/towards the former, resulting in a net charge on either side — this is called **induced charge**

### 21.3 Coulomb's Law

- The SI unit of charge is called one **coulomb** (1 C) and is defined such that  $1.602176634 \times 10^{-19}$  C is equal to the charge of an electron or proton
- **Coulomb's law** describes the electric force between two point charges

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2}$$

where the **electric constant**  $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$ ,  $q_1$  and  $q_2$  are the magnitudes of the charges, and  $r$  is the distance between them

- The electric force is always directed along the line between the two charges, attracting opposite charges and repelling like charges
- $\frac{1}{4\pi\epsilon_0}$  can be approximated as  $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
- The principle of superposition of forces also applies to electric charges

## 21.4 Electric Field and Electric Forces

- The electric force on a charged object is exerted by the electric field created by other charged objects
- We can determine if there is an electric field at a point by placing a test charge  $q_0$  there and seeing if it experiences an electric force — the electric field at that point (the electric force per unit charge) is then given by

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

- Rearranging, the force experienced by a charge  $q_0$  at a point is given by

$$\mathbf{F} = q_0 \mathbf{E}$$

- When considering an electric field produced by a point charge, the location of the point charge is called the **source point** and the location at which we're trying to determine the field is called the **field point**
- The electric field produced by a point charge is given by

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

where  $q$  is the charge of the point charge,  $r$  is the distance between the source and field points, and  $\hat{\mathbf{r}}$  is the unit vector from the source to the field point

- Unlike Coulomb's law this equation doesn't use the absolute value of  $q$  meaning that the electric fields of positive charges point away from the charge, while those of negative charges point towards them
- In electrostatics, the electric field inside the material of a conductor (but not holes within the material) is  $\mathbf{0}$

## 21.5 Electric-Field Calculations

- The **principle of superposition of electric fields** states that the total electric field at a point  $P$  is the vector sum of the fields at  $P$  due to each point charge in the charge distribution

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots$$

- For a line charge distribution the **linear charge density** is represented by  $\lambda$  (the charge per unit length, measured in C/m)
- For a surface charge distribution the **surface charge density** is represented by  $\sigma$  (the charge per unit area, measured in C/m<sup>2</sup>)

- For a volume charge distribution the **volume charge density** is represented by  $\rho$  (the charge per unit volume, measured in C/m<sup>3</sup>)
- The electric field of an infinitely long line charge along the  $y$ -axis is

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

## 21.6 Electric Field Lines

- An **electric field line** is a line drawn through space such that its tangent at any point is in the direction of the electric field vector at that point
- Fewer lines are drawn in areas where the electric field is weak and more lines are drawn in areas where it's strong

## 21.7 Electric Dipoles

- An **electric dipole** is a pair of point charges of equal magnitude  $q$  and opposite sign separated by a distance  $d$
- The net force on an electric dipole in a uniform electric field is  $\mathbf{0}$
- The **electric dipole moment**  $\mathbf{p}$  of an electric dipole is a vector directed from the negative charge to the positive charge with magnitude  $qd$
- The net torque on an electric dipole in a uniform electric field is  $\mathbf{p} \times \mathbf{E}$  or  $qEd \sin \phi$  where  $\phi$  is the angle between the electric dipole and the electric field
- The potential energy of an electric dipole in a uniform electric field is

$$U = -\mathbf{p} \cdot \mathbf{E}$$

## 22 Gauss's Law

### 22.1 Calculating Electric Flux

- The electric flux of a uniform electric field through a flat surface  $A$  is

$$\Phi_E = \mathbf{E} \cdot \mathbf{A}$$

where  $\mathbf{A}$  is normal to  $A$  and has a magnitude equal to its area

- The electric flux of a nonuniform electric field through a curved surface  $A$  is

$$\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$$

## 22.2 Gauss's Law

- Gauss's law states that the total electric flux through a closed surface is equal to the total electric charge enclosed by the surface divided by  $\epsilon_0$

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

## 22.3 Applications of Gauss's Law

- Gauss's law can be used in two ways:
  - If we know the charge distribution and it has enough symmetry to let us evaluate the integral in Gauss's law, we can find the field
  - If we know the field, we can use Gauss's law to find the charge distribution
- Under electrostatics, excess charge always lies on the surface of a conductor
- The electric field of an infinite line charge is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r} \hat{\mathbf{r}}$$

## 22.4 Charges on Conductors

- If there is excess charge at rest on a conductor, all of that charge must lie on the surface of the conductor and the electric field inside the conductor must be zero. If there is a cavity inside the conductor, the net charge on the cavity walls equals the amount of charge enclosed by the cavity
- Charges outside a conductor have no effect on the interior of the conductor, even if it has a cavity inside — this is why Faraday cages work
- At the surface of a conductor, the component of the electric field that is perpendicular to the surface is

$$E_{\perp} = \frac{\sigma}{\epsilon_0}$$

# 23 Electric Potential

## 23.1 Electric Potential Energy

- The electric potential energy of two point charges is

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

- The electric potential energy of a point charge  $q_0$  and a collection of charges  $q_1, q_2$ , etc. is

$$U = \frac{q_0}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} + \dots \right) = \frac{q_0}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

- For every electric field due to a static charge distribution, the force exerted by that field is conservative
- The total electric potential energy of a collection of charges  $q_1, q_2$ , etc. is

$$U = \frac{1}{4\pi\epsilon_0} \sum_{i < j} \frac{q_i q_j}{r_{ij}}$$

where  $r_{ij}$  is the distance between  $q_i$  and  $q_j$

## 23.2 Electric Potential

- **Potential** is potential energy per unit charge
- The unit of potential is the **volt**, equal to 1 joule per coulomb
- The potential difference between two points  $V_{ab} = V_a - V_b$  is called the potential of  $a$  with respect to  $b$  and equals the amount of work done by the electric force when a unit (1 C) of charge moves from  $a$  to  $b$

- The electric potential due to a point charge is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

- The electric potential due to a collection of point charges is

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

- The electric potential due to a continuous charge distribution is

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

- The electric potential difference between two points is given by

$$V_a - V_b = \int_a^b \mathbf{E} \cdot d\mathbf{l} = \int_a^b E \cos \phi \, dl$$

- Positive charges tend to “fall” from high- to low-potential regions while negative charges do the opposite
- When a particle with charge  $e = 1.602 \times 10^{-19} \text{ C}$  moves between two points with a potential difference of  $1 \text{ V} = 1 \text{ J/C}$  the change in energy is  $U_a - U_b = qV_{ab} = (1.602 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.602 \times 10^{-19} \text{ J}$  which is called 1 **electron volt**

## 23.4 Equipotential Surfaces

- An **equipotential surface** is a three-dimensional surface on which the electric potential is the same at every point
- Because electric potential energy doesn't change as a test charge moves over an equipotential surface, the electric field can do no work and thus **field lines and equipotential surfaces are always perpendicular**
- When all charges are at rest, the surface of a conductor is an equipotential surface
- When all charges are at rest, the entire solid volume of a conductor is at the same potential

## 23.5 Potential Gradient

- The relationship between  $\mathbf{E}$  and  $V$  is given by

$$\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$$

- If  $E$  has a radial component  $E_r$  with respect to an axis or a point and  $r$  is the distance from that axis or point, then

$$E_r = -\frac{\partial V}{\partial r}$$

## 24 Capacitance and Dielectrics

### 24.1 Capacitors and Capacitance

- Any two conductors separated by an insulator (or a vacuum) form a **capacitor**
- The **capacitance** of a capacitor measures its ability to store charge

$$C = \frac{Q}{V_{AB}}$$

- Capacitance is measured in **farads** where

$$1 \text{ F} = 1 \text{ C/V}$$

- The capacitance of a parallel plate capacitor in a vacuum is

$$C = \epsilon_0 \frac{A}{d}$$

## 24.2 Capacitors in Series and Parallel

- In a series connection, the magnitude of charge on all plates is the same
- The **equivalent capacitance** of a combination of capacitors is the capacitance of a single capacitor that would have equivalent behaviour
- In a series connection, the reciprocal of the equivalent capacitance equals the sum of the reciprocals of the individual capacitances

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

meaning the equivalent capacitance is always less than any individual capacitance

- In a parallel connection, the potential difference is the same for all capacitors
- In a parallel connection, the equivalent capacitance equals the sum of the individual capacitances

$$C_{\text{eq}} = C_1 + C_2 + \cdots$$

meaning the equivalent capacitance is always greater than any individual capacitance

## 24.3 Energy Storage in Capacitors and Electric-Field Energy

- The potential energy stored in a capacitor is

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- The **energy density** of a parallel plate capacitor is its energy per unit volume

$$u = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\epsilon_0 E^2$$

## 24.4 Dielectrics

- **Dielectrics** are nonconducting materials
- Most capacitors have a dielectric material between their plates because
  1. It preserves the distance between the plates
  2. It increases the maximum potential difference between the plates by avoiding **dielectric breakdown** when the material between the plates becomes ionized and becomes conductive — this happens more easily for air



3. It increases the capacitance by decreasing the potential difference for a given charge

- The **dielectric constant** of a material is defined as

$$K = \frac{C}{C_0}$$

where  $C_0$  is the capacitance of a capacitor with vacuum between the plates and  $C$  is the capacitance of the same capacitor with the material between the plates

- If  $E_0$  is the magnitude of the electric field between the plates of a parallel plate capacitor when separated by a vacuum and  $E$  is the magnitude when separated by a dielectric then

$$E = \frac{E_0}{K}$$

- The electric field (and electric potential) are reduced because the dielectric becomes **polarized** and an induced surface charge appears of magnitude

$$\sigma_i = \sigma \left( 1 - \frac{1}{K} \right)$$

- The **permittivity** of a dielectric is defined as

$$\epsilon = K\epsilon_0$$

- The capacitance of a parallel plate capacitor with dielectric between the plates is thus

$$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

and the electric energy density is

$$u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2$$