Vibrations and Waves by A. P. French Problems

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Contents

1 Periodic motions

1.4

(a)

$$
\begin{aligned}
z &= Ae^{j\theta} \\
dz &= jAe^{j\theta}\,d\theta \\
&= jz\,d\theta
\end{aligned}
$$

The motion of the point is always perpendicular to its position.

(b)

$$
\begin{aligned}
|2 + j\sqrt{3}| &= \sqrt{2^2 + \sqrt{3}^2} \\
&= \sqrt{7} \\
\arg(2 + j\sqrt{3}) &= \arctan \frac{\sqrt{3}}{2} \\
&= 41^\circ
\end{aligned}
$$

$$
\begin{aligned}
(2 - j\sqrt{3})^2 &= 4 - j4\sqrt{3} - 3 \\
&= 1 - j4\sqrt{3} \\
|1 - j4\sqrt{3}| &= \sqrt{1^2 + (4\sqrt{3})^2} \\
&= 7 \\
\arg(1 - j4\sqrt{3}) &= -\arctan 4\sqrt{3}
\end{aligned}
$$

1.9

$$
\begin{aligned}
\cos\theta + j\sin\theta &= e^{j\theta} \\
\cos\frac{\pi}{2} + j\sin\frac{\pi}{2} &= e^{j\frac{\pi}{2}} \\
j &= e^{j\frac{\pi}{2}} \\
j^j &= (e^{j\frac{\pi}{2}})^j \\
&= e^{-\frac{\pi}{2}} \\
&\approx 0.208
\end{aligned}
$$

Yes, I would be willing to pay 20 cents because I could sell it to the mathematician and gain 0.8 cents.

1.10

$$
\begin{aligned}
y &= A\cos kx + B\sin kx \\
\frac{dy}{dx} &= -Ak\sin kx + Bk\cos kx \\
\frac{d^2y}{dx^2} &= -Ak^2\cos kx - Bk^2\sin kx \\
&= -k^2 y
\end{aligned}
$$

$$
\begin{aligned}
C &= \sqrt{A^2+B^2} \\
\alpha &= \arctan\left(-\frac{B}{A}\right) \\
y &= C\cos(kx+\alpha) \\
&= C\,\mathrm{Re}[e^{j(kx+\alpha)}] \\
&= Re[(Ce^{j\alpha})e^{jkx}]
\end{aligned}
$$

1.11

(a)

$$
\begin{aligned}
x &= A\cos(\omega t+\alpha) \\
A &= 5\,\mathrm{cm} \\
f &= 1\,\mathrm{Hz} \\
\omega &= 2\pi f \\
&= 2\pi\,\mathrm{rad/s} \\
\alpha &= \pm\frac{\pi}{2}
\end{aligned}
$$

(b)

$$
\begin{aligned}
x\left(\frac{8}{3}\right) &= 5\cos\left(2\pi\frac{8}{3}+\alpha\right) \\
&= \pm 4.33\,\mathrm{cm} \\
\frac{dx}{dt} &= -A\omega\sin(\omega t+\alpha) \\
\frac{dx}{dt}\left(\frac{8}{3}\right) &= \pm 15.7\,\mathrm{cm/s} \\
\frac{d^2x}{dt^2} &= -A\omega^2\cos(\omega t+\alpha) \\
\frac{d^2x}{dt^2}\left(\frac{8}{3}\right) &= \mp 171\,\mathrm{cm/s^2}
\end{aligned}
$$

1.12

(a)

$$\begin{aligned}
v &= 50\,\mathrm{cm/s} \\
T &= 6\,\mathrm{s} \\
\theta_0 &= 30^\circ \\
c &= 300\,\mathrm{cm} \\
A &= \frac{c}{2\pi} \\
&= \frac{150}{\pi}\,\mathrm{cm} \\
\omega &= \frac{2\pi}{T} \\
&= \frac{\pi}{3}\,\mathrm{rad/s} \\
\alpha &= \frac{\pi}{6}\,\mathrm{rad} \\
x &= \frac{150}{\pi}\cos\left(\frac{\pi}{3}t + \frac{\pi}{6}\right)
\end{aligned}$$

(b)

$$\begin{aligned}
x(2\,\mathrm{s}) &= -41.3\,\mathrm{cm} \\
\frac{dx}{dt} &= -50\sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) \\
\frac{dx}{dt}(2\,\mathrm{s}) &= -25\,\mathrm{cm/s} \\
\frac{d^2x}{dt^2} &= -\frac{50\pi}{3}\cos\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) \\
\frac{d^2x}{dt^2}(2\,\mathrm{s}) &= 45\,\mathrm{cm/s^2}
\end{aligned}$$