University Physics with Modern Physics Electromagnetism Problems

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21 Electric Charge and Electric Field

21.3 Coulomb's Law

21.3.1 Example 21.1

The magnitude of electric repulsion between two α particles is given by

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

and the magnitude of gravitational attraction is given by

$$F_g = \frac{Gm^2}{r^2}$$

. The ratio of the two values is

$$\begin{aligned}\frac{F_e}{F_g} &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}\frac{r^2}{Gm^2}\\ &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{Gm^2}\\ &= 3.1 \times 10^{35}\end{aligned}$$

showing that the electric repulsion is significantly stronger than the gravitational attraction.

21.3.2 Example 21.2

a) The magnitude of the force that q_1 exerts on q_2 is

$$\begin{aligned}F &= \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\\ &= (9.0 \times 10^9)\frac{|(25 \times 10^{-9})(-75 \times 10^{-9})|}{0.030^2}\\ &= 1.9 \times 10^{-2}\,\mathrm{N}.\end{aligned}$$

Since q_1 and q_2 have opposite charge, the force is attractive (from q_2 to q_1).

b) The magnitude of the force that q_2 exerts on q_1 is the same as in part a, but the direction is reversed (from q_1 to q_2).

21.3.3 Example 21.3

By the principle of superposition of forces, the net force exerted on q_3 is equal to the vector sum of the forces exerted on it by q_1 and q_2 separately.

Both q_1 and q_3 have positive charge so they repel each other. q_1 is to the right of q_3 so q_3 experiences a force to the left of magnitude

$$
\begin{aligned}
F_{1\text{ on }3} &= \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r^2} \\
&= (9.0\times 10^{9})\frac{|(1.0\times 10^{-9})(5.0\times 10^{-9})|}{0.020^2} \\
&= 1.1\times 10^{-4}\,\mathrm{N}.
\end{aligned}
$$

However q_2 has a negative charge so it attracts q_3. It is also to the right of q_3 so q_3 experiences a force to the right of magnitude

$$
\begin{aligned}
F_{2\text{ on }3} &= \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{r^2} \\
&= (9.0\times 10^{9})\frac{|(-3.0\times 10^{-9})(5.0\times 10^{-9})}{0.040^2} \\
&= 8.4\times 10^{-5}\,\mathrm{N}.
\end{aligned}
$$

The net force experienced by q_3 is therefore

$$
\begin{aligned}
F &= -F_{1\text{ on }3} + F_{2\text{ on }3} \\
&= -1.1\times 10^{-4} + 8.4\times 10^{-5} \\
&= -2.6\times 10^{-5}\,\mathrm{N}.
\end{aligned}
$$

21.3.4 Example 21.4

Since q_1 and q_2 are of equal charge and are symmetric about the x axis on which Q lies, the vertical components of their forces cancel leaving only the horizontal.

The horizontal component of q_1's force on Q is given by

$$
\begin{aligned}
F_{1\text{ on Q, x}} &= \frac{1}{4\pi\epsilon_0}\frac{q_1 Q}{r_{1,\mathrm{Q}}^2}\cos\alpha \\
&= (9.0\times 10^{9})\frac{(2.0\times 10^{-6})(4.0\times 10^{-6})}{\sqrt{0.30^2+0.40^2}^2}\frac{0.40}{0.50} \\
&= 0.23\,\mathrm{N}.
\end{aligned}
$$

Again, since q_1 and q_2 are of equal charge and symmetric about the x axis, $F_{1\text{ on Q, x}} = F_{2\text{ on Q, x}}$ and the total force experienced by Q is in the positive x direction of magnitude

$$
F = 2F_{1\text{ on Q, x}} = 0.46\,\mathrm{N}.
$$

21.4 Electric Field and Electric Forces

21.4.1 Example 21.5

The magnitude of the electric field vector is given by

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \\ &= (9.0 \times 10^9)\frac{|4.0 \times 10^{-9}|}{2.0^2} \\ &= 9.0\,\mathrm{N/C}. \end{aligned}$$

21.4.2 Example 21.6

The magnitude of the electric field vector is given by

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \\ &= (9.0 \times 10^9)\frac{|-8.0 \times 10^{-9}|}{1.2^2 + 1.6^2} \\ &= 18\,\mathrm{N/C} \end{aligned}$$

and it is directed towards the origin. If θ is the angle between the positive x axis and $\hat{\mathbf{r}}$ then the component form of $\mathbf{E}$ is

$$\begin{aligned} E &= -E\left(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}\right) \\ &= -E\left(\frac{x}{r}\hat{\mathbf{i}} + \frac{-y}{r}\hat{\mathbf{j}}\right) \\ &= \frac{-18}{\sqrt{1.2^2 + 1.6^2}}\left(1.2\hat{\mathbf{i}} + 1.6\hat{\mathbf{j}}\right) \\ &= (-11\,\mathrm{N/C})\hat{\mathbf{i}} - (14\,\mathrm{N/C})\hat{\mathbf{j}}. \end{aligned}$$

21.4.3 Example 21.7

a) Electrons have a negative charge and the electric field is directed upwards, so the electron will move downwards. The magnitude of its acceleration is

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{eE}{m} \\ &= \frac{(1.60 \times 10^{-19})(1.00 \times 10^4)}{9.11 \times 10^{-31}} \\ &= 1.76 \times 10^{15}\,\mathrm{m/s^2}. \end{aligned}$$

b) Its acceleration is constant between the plates, so its final speed is

$$\begin{aligned} v^2 &= v_0^2 + 2a(x - x_0) \\ &= 2ax \\ v &= \sqrt{2ax} \\ &= \sqrt{2(1.76 \times 10^{15})(0.01)} \\ &= 5.9 \times 10^6 \text{ m/s}^2 \end{aligned}$$

and thus its final kinetic energy is

$$\begin{aligned} K &= \frac{1}{2} m v^2 \\ &= \frac{1}{2}(9.11 \times 10^{-31})(5.9 \times 10^6)^2 \\ &= 1.6 \times 10^{-17} \text{ J.} \end{aligned}$$

c) We can find the time it takes for the electron to travel this distance by rearranging the kinematic equation

$$v = v_0 + at$$

to

$$\begin{aligned} t &= \frac{v - v_0}{a} \\ &= \frac{5.9 \times 10^6}{1.76 \times 10^{15}} \\ &= 3.4 \times 10^{-9} \text{ s.} \end{aligned}$$

21.5 Electric-Field Calculations

21.5.1 Example 21.8

a) At point a the electric field caused by q_1 points to the right and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r^2} \\ &= (9.0 \times 10^9) \frac{12 \times 10^{-9}}{(0.060)^2} \\ &= 3.0 \times 10^4 \text{ N/C.} \end{aligned}$$

The electric field caused by q_2 also points to the right and it has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} \\ &= (9.0\times 10^9)\frac{|-12\times 10^{-9}|}{(0.040)^2} \\ &= 6.8\times 10^4\,\mathrm{N/C}. \end{aligned}$$

Thus the total field points to the right and has magnitude

$$E = E_1 + E_2 = 9.8\times 10^4\,\mathrm{N/C}.$$

b) At point b the electric field caused by q_1 points to the left and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} \\ &= (9.0\times 10^9)\frac{12\times 10^{-9}}{(0.040)^2} \\ &= 6.8\times 10^4\,\mathrm{N/C}. \end{aligned}$$

The electric field caused by q_2 points to the right and has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} \\ &= (9.0\times 10^9)\frac{|-12\times 10^{-9}|}{(0.140)^2} \\ &= 0.55\times 10^4\,\mathrm{N/C}. \end{aligned}$$

Thus the total electric field points to the left and has magnitude

$$E = E_1 - E_2 = 6.3\times 10^4\,\mathrm{N/C}.$$

c) At point c the electric field caused by q_1 points from q_1 to c and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} \\ &= (9.0\times 10^9)\frac{|12\times 10^{-9}|}{0.130^2} \\ &= 6.4\times 10^3\,\mathrm{N/C}. \end{aligned}$$

The electric field caused by q_2 points from c to q_2 and has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} \\ &= (9.0 \times 10^9)\frac{|-12 \times 10^{-9}|}{0.130^2} \\ &= 6.4 \times 10^3\,\mathrm{N/C} \\ &= E_1. \end{aligned}$$

The vertical components of $\mathbf{E_1}$ and $\mathbf{E_2}$ cancel, leaving only a horizontal component pointing to the right of magnitude

$$\begin{aligned} E &= 2E_1 \cos\alpha \\ &= 2(6.4 \times 10^3)\frac{0.050}{0.130} \\ &= 4.9 \times 10^3\,\mathrm{N/C}. \end{aligned}$$

21.5.2 Example 21.9

By symmetry, each point on the ring has a corresponding point on the opposite side. The components of their electric fields perpendicular to the axis of the ring cancel, leaving only a component parallel to the axis of the ring. Thus the total magnetic field at P is parallel to the axis of the ring and can be calculated as

$$\begin{aligned} E &= \int_0^{2\pi} \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r^2}\cos\alpha\, d\theta \\ &= \frac{1}{4\pi\epsilon_0}\frac{Qx}{2\pi(a^2+x^2)^{3/2}}\int_0^{2\pi} d\theta \\ &= \frac{1}{4\pi\epsilon_0}\frac{Qx}{(a^2+x^2)^{3/2}}. \end{aligned}$$

21.5.3 Example 21.10

By symmetry, each point on the line has a corresponding point on the opposite side of the x-axis. The y components of their electric fields cancel, leaving only the x components. Thus the total magnetic field at P only has an x component and can be calculated as

$$
\begin{aligned}
E &= \int_{-a}^{a} \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r^2} \cos\alpha \, dy \\
&= \frac{1}{4\pi\epsilon_0} \frac{Qx}{2a} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} \, dy \\
&= \frac{1}{4\pi\epsilon_0} \frac{Qx}{2a} \left[\frac{y}{x^2\sqrt{x^2+y^2}} \right]_{-a}^{a} \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{2ax} \left(\frac{a}{\sqrt{x^2+a^2}} + \frac{a}{\sqrt{x^2+(-a)^2}} \right) \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2+a^2}}.
\end{aligned}
$$

21.5.4 Example 21.11

By symmetry, each point on the disk has a corresponding point 180° rotation around the x-axis. The y and z components of their electric fields cancel, leaving only the x components. Thus the total magnetic field at P only has an x component and can be calculated as

$$
\begin{aligned}
E &= \int_0^R \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{\sigma}{r^2} s \cos\alpha \, d\theta \, ds \\
&= \frac{\sigma}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{s}{s^2+x^2} \frac{x}{\sqrt{s^2+x^2}} \, d\theta \, ds \\
&= \frac{\sigma x}{2\epsilon_0} \int_0^R \frac{s}{(s^2+x^2)^{3/2}} \, ds \\
&= \frac{\sigma x}{2\epsilon_0} \left[-\frac{1}{\sqrt{s^2+x^2}} \right]_0^R \\
&= \frac{\sigma x}{2\epsilon_0} \left(-\frac{1}{\sqrt{R^2+x^2}} + \frac{1}{x} \right) \\
&= \frac{\sigma}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{(R/x)^2+1}} \right).
\end{aligned}
$$

21.5.5 Example 21.12

From Example 21.11 we know that the electric field produced by an infinite plane sheet of charge is

$$E = \frac{\sigma}{2\epsilon_0}.$$

Therefore the electric field outside the sheets is $\mathbf{0}$ and between the sheets is σ/ϵ_0 towards the negative sheet.

21.6 Electric Dipoles

21.6.1 Example 21.13

a) The electric field is uniform so the net force exerted on the dipole is **0**

b) The electric dipole moment is directed from the negative charge to the positive charge and has magnitude

$$p = qd = (1.6 \times 10^{-19})(0.125 \times 10^{-9}) = 2.0 \times 10^{-29}\,\mathrm{C \cdot m}$$

c) The torque aligns the electric dipole moment with the electric field so it is directed out of the page and has magnitude

$$\tau = qEd \sin\phi = (1.6\times10^{-19})(5.0\times10^{5})(0.125\times10^{-9}) \sin 35 = 5.7\times10^{-24}\,\mathrm{N{\cdot}m}$$

d) The potential energy of an electric dipole in a uniform electric field is given by

$$U = -qdE\cos\phi = (2.0 \times 10^{-29})(5.0 \times 10^{5}) \cos 35 = 8.2 \times 10^{-24}\,\mathrm{J}$$

21.6.2 Example 21.14

As P is on the y-axis, the electric fields of the electric dipole's point charges have no x component and thus the net electric field is directed along the y-axis.

By the principle of superposition of electric fields, the magnitude of the electric field at P is

$$\begin{aligned}
E &= E_- + E_+ \\
&= \frac{1}{4\pi\epsilon_0}\frac{-q}{(y-(-d/2))^2} + \frac{1}{4\pi\epsilon_0}\frac{q}{(y-d/2)^2} \\
&= \frac{1}{4\pi\epsilon_0} q \left(\frac{1}{(y-d/2)^2} - \frac{1}{(y+d/2)^2} \right) \\
&= \frac{1}{4\pi\epsilon_0}\frac{q}{y^2}\left(\left(1-\frac{d}{2y}\right)^{-2} - \left(1+\frac{d}{2y}\right)^{-2} \right) \\
&\approx \frac{1}{4\pi\epsilon_0}\frac{q}{y^2}\left(1+\frac{d}{y}-1+\frac{d}{y}\right) \\
&= \frac{qd}{2\pi\epsilon_0 y^3} \\
&= \frac{p}{2\pi\epsilon_0 y^3}.
\end{aligned}$$

21.7 Guided Practice

21.7.1 VP21.4.1

q_1 attracts q_3 to the left with magnitude

$$\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r^2} \\
&= (9.0\times 10^9)\frac{|(4.00\times 10^{-9})(-2.00\times 10^{-9})}{0.0400^2} \\
&= 4.5\times 10^{-5}\,\mathrm{N}.
\end{aligned}$$

q_2 repels q_3 to the left with magnitude

$$\begin{aligned}
F_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{r^2} \\
&= (9.0\times 10^9)\frac{|(-1.20\times 10^{-9})(-2.00\times 10^{-9}|}{(0.0600-0.0400)^2} \\
&= 5.4\times 10^{-5}\,\mathrm{N}.
\end{aligned}$$

By the principle of superposition of forces, the net force on q_3 is

$$\mathbf{F} = (-F_1 - F_2)\hat{\mathbf{i}} = (-9.9\times 10^{-5}\,\mathrm{N})\hat{\mathbf{i}}.$$

21.7.2 VP21.4.2

a) q_1 repels q_2 in the positive x direction with magnitude

$$\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} \\
&= \frac{1}{4\pi(8.854\times 10^{-12})}\frac{|(3.60\times 10^{-9})(2.00\times 10^{-9})}{0.0400^2} \\
&= 40.4\,\mu\mathrm{N}.
\end{aligned}$$

b) By the superposition of forces

$$\begin{aligned}
F &= F_1 + F_2 \\
F_2 &= F - F_1 \\
&= 54.0 - 40.4 \\
&= 13.6\,\mu\mathrm{N}
\end{aligned}$$

in the positive x direction.

c) q_2 repels q_3 so it must also have a positive charge of magnitude

$$
\begin{aligned}
F &= \frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{r^2} \\
q_2 &= \frac{4\pi\epsilon_0 F r^2}{q_3} \\
&= \frac{4\pi(8.854\times10^{-12})(1.36\times10^{-5})(0.0800)^2}{2.00\times10^{-9}} \\
&= 4.84\times10^{-9}\,\text{C}.
\end{aligned}
$$

21.7.3 VP21.4.3

By symmetry the x components of q_1 and q_2's electric fields cancel leaving only their y components which are directed in the negative y direction and equal. q_3 is negative and thus experiences a net force in the positive y direction of magnitude

$$
\begin{aligned}
F &= 2\frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r^2}\sin\alpha \\
&= 2\frac{1}{4\pi(8.854\times10^{-12})}\frac{(6.00\times10^{-9})(2.50\times10^{-9})}{0.150^2+0.200^2}\frac{0.200}{\sqrt{0.150^2+0.200^2}} \\
&= 3.45\times10^{-6}\,\text{N}.
\end{aligned}
$$

21.8 VP21.4.4

The magnitude of the electric force exerted by q_1 on q_3 is

$$
\begin{aligned}
F_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r^2} \\
&= \frac{1}{4\pi(8.854\times10^{-12})}\frac{|(4.00\times10^{-9})(-1.50\times10^{-9})}{0.250^2+0.200^2} \\
&= 5.26\times10^{-7}\,\text{N}.
\end{aligned}
$$

They have opposite charges so the force is directed from q_3 to q_1. In component form the force is

$$
\begin{aligned}
\mathbf{F_1} &= -F_1\cos\alpha\hat{\mathbf{i}} + F_1\sin\alpha\hat{\mathbf{j}} \\
&= F_1\left(-\frac{x}{r}\hat{\mathbf{i}} + \frac{y}{r}\hat{\mathbf{j}}\right) \\
&= \frac{5.26\times10^{-7}}{\sqrt{0.250^2+0.200^2}}\left(-0.250\hat{\mathbf{i}} + 0.200\hat{\mathbf{j}}\right) \\
&= (-4.11\times10^{-7}\,\text{N})\hat{\mathbf{i}} + (3.29\times10^{-7}\,\text{N})\hat{\mathbf{j}}.
\end{aligned}
$$

The magnitude of the electric force exerted by q_2 on q_3 is

$$
\begin{aligned}
F_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{r^2} \\
&= \frac{1}{4\pi(8.854\times10^{-12})}\frac{|(-4.00\times10^{-9})(-1.50\times10^{-9})|}{0.250^2} \\
&= 8.63\times10^{-7}\,\mathrm{N}.
\end{aligned}
$$

The have like charges so the force is directed from q_2 to q_3, i.e. along the positive x-axis. In component form the force is

$$\mathbf{F_2} = (8.64\times10^{-7}\,\mathrm{N})\hat{\mathbf{i}}.$$

Thus the net force experienced by q_3 is

$$
\begin{aligned}
\mathbf{F} &= \mathbf{F_1} + \mathbf{F_2} \\
&= (4.53\times10^{-7}\,\mathrm{N})\hat{\mathbf{i}} + (3.29\times10^{-7}\,\mathrm{N})\hat{\mathbf{j}}.
\end{aligned}
$$