Advanced Engineering Mathematics Vectors, Matrices, and Vector Calculus by Dennis G. Zill Problems

Chris Doble

June 2023

Contents

1 Vectors

1.1 Vectors in 2-Space

1.1.1

(a) $3\mathbf{a} = 6\mathbf{i} + 12\mathbf{j}$

(b) $\mathbf{a} + \mathbf{b} = \mathbf{i} + 8\mathbf{j}$

(c) $\mathbf{a} - \mathbf{b} = 3\mathbf{i}$

(d) $||\mathbf{a} + \mathbf{b}|| = \sqrt{1 + 8^2} = \sqrt{65}$

(e) $||\mathbf{a} - \mathbf{b}|| = 3$

1.1.9

(a) $4\mathbf{a} - 2\mathbf{b} = \langle 6, -14 \rangle$

(b) $-3\mathbf{a} - 5\mathbf{b} = \langle 2, 4 \rangle$

1.1.15

$\overrightarrow{P_1P_2} = \langle 2, 5 \rangle$

1.1.19

$(1, 18)$

1.1.21

(a) Yes

(b) Yes

(c) Yes

(d) No

(e) Yes

(f) Yes

1.1.25

(a) $\frac{\mathbf{a}}{||\mathbf{a}||} = \frac{\langle 2,2 \rangle}{\sqrt{2^2+2^2}} = \frac{1}{2\sqrt{2}}\langle 2, 2 \rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

(b) $\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$

1.1.31

$2\frac{\mathbf{a}}{||\mathbf{a}||} = 2\frac{\langle 3,7 \rangle}{\sqrt{3^2+7^2}} = \frac{2}{\sqrt{58}}\langle 3, 7 \rangle = \langle \frac{6}{\sqrt{58}}, \frac{14}{\sqrt{58}} \rangle$

1.1.37

$\mathbf{x} = -(\mathbf{a} + \mathbf{b})$

1.1.41

$$\begin{aligned}
\mathbf{a} &= 2\mathbf{i} + 3\mathbf{j} \\
\mathbf{b} &= \mathbf{i} + \mathbf{j} \\
\mathbf{c} &= \mathbf{i} - \mathbf{j}
\end{aligned}$$

$$\begin{aligned}
\mathbf{i} &= \frac{1}{2}(\mathbf{b} + \mathbf{c}) \\
\mathbf{j} &= \frac{1}{2}(\mathbf{b} - \mathbf{c}) \\
\mathbf{a} &= 2\left(\frac{1}{2}(\mathbf{b} + \mathbf{c})\right) + 3\left(\frac{1}{2}(\mathbf{b} - \mathbf{c})\right) \\
&= \mathbf{b} + \mathbf{c} + \frac{3}{2}\mathbf{b} - \frac{3}{2}\mathbf{c} \\
&= \frac{5}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}
\end{aligned}$$

1.1.43

$$\begin{aligned}
y &= \frac{1}{4}x^2 + 1 \\
y(2) &= 2 \\
y' &= \frac{1}{2}x \\
y'(2) &= 1 \\
\mathbf{v} &= \pm\langle\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle
\end{aligned}$$

1.1.45

(a)

$$\begin{aligned}
\mathbf{F}_n &= \mathbf{F}\cos\theta \\
\mathbf{F}_g &= \mathbf{F}\sin\theta \\
||\mathbf{F}_f|| &= \mu||\mathbf{F}_n|| \\
||-\mathbf{F}_g|| &= \mu||\mathbf{F}_n|| \\
||-\mathbf{F}\sin\theta|| &= \mu||\mathbf{F}\cos\theta|| \\
||\mathbf{F}||\sin\theta &= \mu||\mathbf{F}||\cos\theta \\
\tan\theta &= \mu
\end{aligned}$$

(b) $\theta = \arctan\mu \approx 31^\circ$

1.1.47

$$
\begin{aligned}
F_x &= \frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{L\,dy}{2a(L^2+y^2)^{3/2}} \\
&= \frac{LqQ}{8\pi\epsilon_0}\int_{-a}^{a}(L^2+y^2)^{-3/2}\,dy \\
&= \frac{LqQ}{8\pi\epsilon_0}\frac{2a}{L^2\sqrt{a^2+L^2}} \\
&= \frac{aqQ}{4\pi\epsilon_0 L\sqrt{a^2+L^2}} \\
F_y &= -\frac{qQ}{4\pi\epsilon_0}\int_{-a}^{a}\frac{y\,dy}{2a(L^2+y^2)^{3/2}} \\
&= 0 \\
\mathbf{F} &= \langle\frac{1}{4\pi\epsilon_0}\frac{qQ}{L\sqrt{a^2+L^2}}, 0\rangle
\end{aligned}
$$

1.1.49

Let the three sides of the triangle be vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. The triangle is closed so it must be the case that

$$\mathbf{a}+\mathbf{b}+\mathbf{c}=0.$$

This gives

$$\mathbf{c}=-(\mathbf{a}+\mathbf{b}).$$

The vector from the midpoint of side $\mathbf{a}$ to the midpoint of side $\mathbf{b}$ is

$$\left(\mathbf{a}+\frac{1}{2}\mathbf{b}\right)-\frac{1}{2}\mathbf{a}=\frac{1}{2}(\mathbf{a}+\mathbf{b})$$

which is parallel with $\mathbf{c}$ and half its length.