Vibrations and Waves by A. P. French Notes

Chris Doble

May 2022

Contents

1 Periodic motions

- Fouriers theorem states that any repeating signal of period T can be expressed as a sum of sin waves with periods T, $T/2$, etc.

- It's important to define the domain of a SHM equation, e.g. for what values of t is the motion defined?

- SHM can be considered a projection of uniform circular motion

- That uniform circular motion can be represented by a number in the complex plane, with the projection being its real part

- Multiplication by j can be considered a counter-clockwise rotation of $90°$ in the complex plane

- Euler's formula states
$$e^{j\theta} = \cos\theta + j\sin\theta$$

- Multiplication of a complex number z by $e^{j\theta}$ is equivalent to a counter-clockwise rotation of z by an angle of θ

2 The superposition of periodic motions

- The combination of two SHM's of the same period

$$x_1 = A_1 \cos(\omega t + \alpha_1)$$

$$x_2 = A_2 \cos(\omega t + \alpha_2)$$

is given by

$$x = A \cos(\omega t + \alpha)$$

where

$$A^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos(\alpha_2 - \alpha_1),$$
$$A \sin \beta = A_2 \sin(\alpha_2 - \alpha_1),$$

and

$$\alpha = \alpha_1 + \beta.$$

- The combination in complex representation

$$z_1 = A_1 e^{j(\omega t + \alpha_1)}$$
$$z_2 = A_2 e^{j(\omega t + \alpha_2)}$$

is given by

$$z = e^{j(\omega t + \alpha_1)}[A_1 + A_2 e^{j(\alpha_2 - \alpha_1)}]$$

- In the case where $A_1 = A_2$ if we denote $\delta = \alpha_2 - \alpha_1$ then

$$\beta = \frac{\delta}{2}$$

and

$$A = 2A_1 \cos \beta = 2A_1 \cos \frac{\delta}{2}$$

- The superposition of two sinusoids with different periods will itself be periodic if there exist integers n_1 and n_2 such that

$$T = n_1 T_1 = n_2 T_2$$

where T_1 and T_2 are the periods of the two sinusoids

- Periodic motion in two or more dimensions can be represented by extending the "projection of a rotating vector" approach, with one vector for each axis, e.g.

$$x = A_1 \cos \omega t$$
$$y = A_2 \cos \omega t$$

where differing amplitudes, frequencies, and phase differences product different curves called **Lissajous curves**

3 The free vibrations of physical systems

- When a tensile force is applied to a material it elongates. The ratio of the elongation to the original length x/l_0 is known as the **tensile strain**

- The ratio of the tensile force to the cross sectional area of the material F/A is known as the **tensile stress**

- The ratio of stress and strain is a constant known as **Young's modulus** Y

- The force exerted by the stretched material on another object is given by

$$\frac{F/A}{x/l_0} = -Y \Rightarrow F = -\frac{AY}{l_0}x$$

which is in the form of Hooke's law with $k = -\frac{AY}{l_0}$