

# University Physics with Modern Physics - Modern Physics by Young and Freedman Problems

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## Contents

<b>17 Temperature and Heat</b>	<b>6</b>
17.1 Guided Practice . . . . .	6
17.1.1 . . . . .	6
17.1.2 . . . . .	6
17.1.3 . . . . .	7
17.1.4 . . . . .	7
17.1.5 . . . . .	7
17.1.6 . . . . .	7
17.1.7 . . . . .	8
17.1.8 . . . . .	8
17.1.9 . . . . .	8
17.1.10 . . . . .	9
17.1.11 . . . . .	9
17.1.12 . . . . .	10
17.2 Exercises and Problems . . . . .	10
17.2.15 . . . . .	10
17.2.25 . . . . .	10
17.2.33 . . . . .	11
17.2.35 . . . . .	11
17.2.45 . . . . .	11
17.2.55 . . . . .	12
17.2.57 . . . . .	12
17.2.65 . . . . .	12
17.2.69 . . . . .	13
17.2.71 . . . . .	13
17.2.73 . . . . .	13
17.2.75 . . . . .	13
17.2.79 . . . . .	14
17.2.85 . . . . .	14
17.2.95 . . . . .	15

17.2.99 . . . . .	15
17.2.105 . . . . .	16
17.2.107 . . . . .	17
17.2.113 . . . . .	17
17.2.115 . . . . .	18
17.2.117 . . . . .	18
17.2.119 . . . . .	18
<b>18 Thermal Properties of Matter</b>	<b>19</b>
18.1 Guided Practice . . . . .	19
18.1.1 . . . . .	19
18.1.2 . . . . .	19
18.1.3 . . . . .	20
18.1.4 . . . . .	20
18.1.5 . . . . .	21
18.1.6 . . . . .	21
18.1.7 . . . . .	21
18.1.8 . . . . .	22
18.1.9 . . . . .	22
18.1.10 . . . . .	22
18.1.11 . . . . .	23
18.1.12 . . . . .	23
18.1.13 . . . . .	24
18.2 Exercises and Problems . . . . .	24
18.2.7 . . . . .	24
18.2.9 . . . . .	24
18.2.13 . . . . .	25
18.2.17 . . . . .	25
18.2.21 . . . . .	25
18.2.23 . . . . .	26
18.2.25 . . . . .	26
18.2.27 . . . . .	26
18.2.29 . . . . .	27
18.2.31 . . . . .	27
18.2.33 . . . . .	28
18.2.35 . . . . .	29
18.2.39 . . . . .	29
18.2.41 . . . . .	30
18.2.43 . . . . .	30
18.2.45 . . . . .	30
18.2.49 . . . . .	31
18.2.51 . . . . .	31
18.2.53 . . . . .	31
18.2.57 . . . . .	32
18.2.59 . . . . .	33
18.2.67 . . . . .	34

18.2.69	35
18.2.71	35
18.2.73	36
18.2.75	36
18.2.77	36
18.2.81	37
18.2.83	38
18.2.85	39
18.2.87	39
<b>19 The First Law of Thermodynamics</b>	<b>40</b>
19.1 Guided Practice	40
19.1.1	40
19.1.2	40
19.1.3	40
19.1.4	41
19.1.5	41
19.1.6	41
19.1.7	42
19.1.8	43
19.1.9	44
19.1.10	45
19.1.11	45
19.1.12	46
19.2 Exercises and Problems	47
19.2.1	47
19.2.3	47
19.2.5	48
19.2.9	48
19.2.11	48
19.2.13	49
19.2.17	49
19.2.19	50
19.2.21	50
19.2.23	50
19.2.25	51
19.2.27	52
19.2.29	52
19.2.31	53
19.2.33	53
19.2.35	53
19.2.37	54
19.2.39	54
19.2.43	55
19.2.47	56
19.2.49	57

19.2.51	58
19.2.59	58
19.2.61	61
19.2.63	62
19.2.65	62
<b>20 The Second Law of Thermodynamics</b>	<b>63</b>
20.1 Guided Practice	63
20.1.1	63
20.1.2	63
20.1.3	63
20.1.4	63
20.1.5	63
20.1.6	64
20.1.7	66
20.1.8	66
20.1.9	66
20.1.10	67
20.1.11	67
20.1.12	68
20.1.13	68
20.2 Exercises and Problems	69
20.2.5	69
20.2.7	70
20.2.9	70
20.2.11	71
20.2.13	71
20.2.21	71
20.2.29	72
20.2.31	72
20.2.33	72
20.2.37	73
20.2.41	74
20.2.45	75
20.2.49	76
20.2.51	76
20.2.55	78
20.2.57	78
20.2.59	79
20.2.61	79
<b>37 Relativity</b>	<b>79</b>
37.1 Guided Practice	79
37.1.1	79
37.2 Exercises and Problems	81
37.2.1	81

37.2.3	81
37.2.5	82
37.2.7	82
37.2.9	82
37.2.11	82
37.2.13	83
37.2.15	83
37.2.17	83
37.2.19	84
37.2.21	84
37.2.23	85
37.2.25	85
37.2.27	85
37.2.29	86
37.2.31	86
37.2.33	86
37.2.35	87
37.2.41	87
37.2.43	88
37.2.45	88
37.2.47	89
37.2.51	89
37.2.53	89
37.2.57	89
37.2.61	90
37.2.63	90
37.2.65	90
37.2.71	91
37.2.73	91
<b>38 Photons: Light Waves Behaving as Particles</b>	<b>91</b>
38.1 Guided Practice	91
38.1.1	91
38.2 Exercises and Problems	92
38.2.1	92
38.2.3	92
38.2.5	92
38.2.7	93
38.2.9	93
38.2.11	94
38.2.13	94
38.2.15	94
38.2.17	95
38.2.19	95
38.2.21	95
38.2.23	96

38.2.25	96
38.2.27	97
38.2.29	97
38.2.31	99
38.2.33	100
38.2.35	100
38.2.37	101
38.2.39	101

## 17 Temperature and Heat

### 17.1 Guided Practice

#### 17.1.1

(a)

$$\begin{aligned}
 \Delta L &= \alpha L_0 \Delta T \\
 \alpha &= \frac{\Delta L}{L_0 \Delta T} \\
 &= 2.0 \times 10^{-5} \text{ K}^{-1}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \Delta L &= \alpha L_0 \Delta T \\
 &= -0.27 \text{ mm}
 \end{aligned}$$

#### 17.1.2

$$\begin{aligned}
 \Delta V_C &= \beta V_{C0} \Delta T \\
 &= (5.1 \times 10^{-5})(250)(-70) \\
 &= -0.893 \text{ cm}^3 \\
 \Delta V_E &= \beta V_{E0} \Delta T \\
 &= (75 \times 10^{-5})(250)(-70) \\
 &= -13.1 \text{ cm}^3 \\
 \Delta V_C - \Delta V_E &= 12.2 \text{ cm}^3 \\
 &= 12.2 \text{ mL}
 \end{aligned}$$

### 17.1.3

$$\begin{aligned}
 \frac{\Delta L}{L_0} &= \alpha \Delta T \\
 Y &= \frac{F/A}{\Delta L/L_0} \\
 \frac{\Delta L}{L_0} &= \frac{F}{AY} \\
 \alpha \Delta T + \frac{F}{AY} &= 0 \\
 \frac{F}{AY} &= -\alpha \Delta T \\
 F &= -\alpha AY \Delta T \\
 &= -(2.0 \times 10^{-5})(\pi 0.005^2)(9.0 \times 10^{10})(-12) \\
 &= 1.70 \times 10^3 \text{ N}
 \end{aligned}$$

Tensile

### 17.1.4

$$\begin{aligned}
 \Delta L &= \alpha_A L_A \Delta T + \alpha_B L_B \Delta T \\
 \frac{\Delta L}{\Delta T} &= \alpha_A L_A + \alpha_B (L - L_A) \\
 &= (\alpha_A - \alpha_B) L_A + \alpha_B L \\
 L_A &= \frac{1}{\alpha_A - \alpha_B} \left( \frac{\Delta L}{\Delta T} - \alpha_B L \right)
 \end{aligned}$$

### 17.1.5

$$\begin{aligned}
 0 &= m_{Al} c_{Al} \Delta T_{Al} + m_W c_W \Delta T_W \\
 &= m_{Al} c_{Al} (T - T_{Al}) + m_W c_W (T - T_W) \\
 m_{Al} &= -\frac{m_W c_W (T - T_W)}{c_{Al} (T - T_{Al})} \\
 &= 0.20 \text{ kg}
 \end{aligned}$$

### 17.1.6

$$\begin{aligned}
 0 &= m_I L_f + m_C c_C \Delta T \\
 &= m_I L_f - m_C c_C T \\
 T &= \frac{m_I L_f}{m_C c_C} \\
 &= 14.0^\circ \text{C}
 \end{aligned}$$

### 17.1.7

$$\begin{aligned}
0 &= m_I L_F + m_I c_I \Delta T_I + m_E c_E \Delta T_E \\
&= m_I (L_F + c_I \Delta T_I) + m_E c_E \Delta T_E \\
m_I &= -\frac{m_E c_E \Delta T_E}{L_F + c_I \Delta T_I} \\
&= 0.176 \text{ kg}
\end{aligned}$$

### 17.1.8

Cooling the silver to 0 °C would take

$$Q = mc\Delta T = 92\,137.5 \text{ J}$$

whereas melting all of the ice would take

$$Q = mL_f = 83\,500 \text{ J}$$

so all of the ice will melt.

$$\begin{aligned}
0 &= m_{Ag} c_{Ag} \Delta T_{Ag} + m_I L_f + m_I c_I \Delta T_I + m_I c_W \Delta T_W \\
&= m_{Ag} c_{Ag} (T - T_{Ag}) + m_I L_F - m_I c_I T_I + m_I c_W T \\
&= (m_{Ag} c_{Ag} + m_I c_W) T - m_{Ag} c_{Ag} T_{Ag} + m_I L_F - m_I c_I T_I \\
T &= \frac{m_{Ag} c_{Ag} T_{Ag} + m_I c_I T_I - m_I L_F}{m_{Ag} c_{Ag} + m_I c_W} \\
&= 3.31 \text{ °C}
\end{aligned}$$

### 17.1.9

(a)

$$\begin{aligned}
H &= kA \frac{T_H - T_C}{L} \\
k &= \frac{HL}{A(T_H - T_C)} \\
&= 0.754 \text{ W/(m K)}
\end{aligned}$$

(b)

$$H = kA \frac{T_H - T_C}{L} = 733 \text{ W}$$



**17.1.10**

(a)

$$\begin{aligned}L &= 0.250 \text{ m} \\A &= 2.00 \times 10^{-4} \text{ m}^2 \\k_B &= 109.0 \text{ W}/(\text{m K}) \\k_{Pb} &= 34.7 \text{ W}/(\text{m K}) \\T &= 185^\circ \text{C} \\H &= 6.00 \text{ W}\end{aligned}$$

$$\begin{aligned}H &= k_B A \frac{T_H - T}{L} \\T_H &= \frac{HL}{k_B A} + T \\&= 254^\circ \text{C}\end{aligned}$$

(b)

$$\begin{aligned}H &= k_{Pb} A \frac{T - T_C}{L} \\T_C &= T - \frac{HL}{k_{Pb} A} \\&= -31.1^\circ \text{C}\end{aligned}$$

**17.1.11**

$$\begin{aligned}H &= 4\pi(kr_E)^2 e\sigma T^4 \\(kr_E)^2 &= \frac{H}{4\pi e\sigma T^4} \\k &= \frac{1}{r_E} \sqrt{\frac{H}{4\pi e\sigma T^4}} \\&= 1.70\end{aligned}$$

**17.1.12**

(a)

$$\begin{aligned}
H &= Ae\sigma T^4 \\
&= \pi r^2 \sigma T^4 \\
H &= kA \frac{T_H - T_C}{L} \\
&= k\pi r^2 \frac{T_H - T_C}{L} \\
\pi r^2 \sigma T^4 &= k\pi r^2 \frac{T_H - T_C}{L} \\
T_H &= \frac{L\sigma T^4}{k} + T_C \\
&= 14.26 \text{ K}
\end{aligned}$$

(b)

$$\begin{aligned}
H &= mL_f \\
\pi r^2 \sigma T^4 &= mL_f \\
m &= \frac{\pi r^2 \sigma T^4}{L_f} \\
&= 1.19 \times 10^{-4} \text{ kg/s} \\
&= 0.427 \text{ kg/h}
\end{aligned}$$

**17.2 Exercises and Problems****17.2.15**

$$\begin{aligned}
\Delta V &= \beta V_0 \Delta T \\
\frac{\Delta V}{V_0} &= \beta(T - T_0) \\
T &= T_0 + \frac{\Delta V}{\beta V_0} \\
&= 49^\circ \text{C}
\end{aligned}$$

**17.2.25**

$$\begin{aligned}
Q &= (m_{Al}c_{Al} + m_Wc_W)\Delta T \\
&= 5.55 \times 10^5 \text{ J}
\end{aligned}$$

**17.2.33**

$$\begin{aligned}
\Delta K &= \frac{1}{2}mv^2 - \frac{1}{2}mv'^2 \\
&= \frac{1}{2}m(v^2 - v'^2) \\
&= 3.47 \text{ kJ} \\
\Delta K &= mc\Delta T \\
\Delta T &= \frac{\Delta K}{mc} \\
&= 6.14 \times 10^{-2} \text{ }^\circ\text{C}
\end{aligned}$$

**17.2.35**

(a)

$$\begin{aligned}
0 &= m_m c_m \Delta T_m + m_w c_w \Delta T_w \\
c_m &= -\frac{m_w c_w \Delta T_w}{m_m \Delta T_m} \\
&= 215 \text{ J/(kg K)}
\end{aligned}$$

(b) Water because it has a higher specific heat

(c) It would be too small

**17.2.45**

$$\begin{aligned}
\frac{1}{2}mv^2 &= mc\Delta T + mL_F \\
v &= \sqrt{2(c\Delta T + L_F)} \\
&= 366 \text{ m/s}
\end{aligned}$$

17.2.55

$$\begin{aligned}
k_C A \frac{T_H - T}{L} &= k A \frac{T}{L} \\
k_C T_H - k_C T &= k T \\
k_C T_H &= (k + k_C) T \\
T &= \frac{k_C}{k + k_C} T_H \\
0.71 &= \frac{k_C}{k + k_C} \\
0.71(k + k_C) &= k_C \\
0.71k + 0.71k_C &= k_C \\
0.71k &= 0.29k_C \\
k &= \frac{0.29}{0.71} k_C \\
&\approx 157 \text{ W/(m K)}
\end{aligned}$$

17.2.57

(a)

$$\begin{aligned}
k_W \frac{T - T_C}{L_W} &= k_S \frac{T_H - T}{L_S} \\
\left( \frac{k_W}{L_W} + \frac{k_S}{L_S} \right) T &= \frac{k_S}{L_S} T_H + \frac{k_W}{L_W} T_C \\
T &= \frac{\frac{k_S}{L_S} T_H + \frac{k_W}{L_W} T_C}{\frac{k_W}{L_W} + \frac{k_S}{L_S}} \\
&= -0.86^\circ \text{C}
\end{aligned}$$

(b)

$$\begin{aligned}
H &= k_W \frac{T - T_C}{L_W} \\
&= 24.4 \text{ W/m}^2
\end{aligned}$$

17.2.65

$$\begin{aligned}
H &= Ae\sigma T^4 \\
A &= \frac{H}{e\sigma T^4} \\
&= 2.1 \text{ cm}^2
\end{aligned}$$

17.2.69

$$\begin{aligned}\Delta L &= (\alpha_B L_B + \alpha_S L_S) \Delta T \\ T &= T_0 + \frac{\Delta L}{\alpha_B L_B + \alpha_S L_S} \\ &= 35.0^\circ\text{C}\end{aligned}$$

17.2.71

$$\begin{aligned}Q &= mc\Delta T \\ &= \rho V c \Delta T \\ \Delta T &= \frac{Q}{\rho V c} \\ \Delta V &= \beta V \Delta T \\ &= \frac{\beta Q}{\rho c} \\ c &= \frac{\beta Q}{\rho \Delta V}\end{aligned}$$

17.2.73

(a)

$$\begin{aligned}0.0^\circ\text{M} &= -39^\circ\text{C} \\ 100.0^\circ\text{M} &= 357^\circ\text{C} \\ T_M &= \frac{T_C + 39^\circ\text{C}}{3.96} \\ \frac{100^\circ\text{C} + 39^\circ\text{C}}{3.96} &= 35.1^\circ\text{M}\end{aligned}$$

(b)

$$10\text{M}^\circ = 10 \frac{357^\circ\text{C} - (-39^\circ\text{C})}{100} = 39.6\text{C}^\circ$$

17.2.75

$$\begin{aligned}Ah + \beta_G Ah(T - T_0) &= Ah' + \beta_O Ah'(T - T_0) \\ Ah + \beta_G AhT - \beta_G AhT_0 &= Ah' + \beta_O Ah'T - \beta_O Ah'T_0 \\ (\beta_G Ah - \beta_O Ah')T &= (Ah' - \beta_O Ah'T_0) - (Ah - \beta_G AhT_0) \\ T &= \frac{(1 - \beta_O T_0)h' - (1 - \beta_G T_0)h}{\beta_G h - \beta_O h'} \\ &= 69.4^\circ\text{C}\end{aligned}$$

**17.2.79**

(a)

$$\begin{aligned}
 Y &= \frac{F/A}{\Delta L/L_0} \\
 \Delta L &= \frac{FL_0}{AY} \\
 \Delta L &= \alpha L_0 \Delta T \\
 \Delta L &= \alpha L_0 \Delta T + \frac{FL_0}{AY} \\
 \frac{F}{A} &= Y \left( \frac{\Delta L}{L_0} - \alpha \Delta T \right)
 \end{aligned}$$

(b)

$$\begin{aligned}
 \Delta L_B &= \alpha_B L_{B0} \Delta T \\
 \frac{\Delta L_B}{L_{B0}} &= \alpha_B \Delta T \\
 \frac{F}{A} &= Y_S (\alpha_B - \alpha_S) \Delta T \\
 &= 1.9 \times 10^8 \text{ Pa}
 \end{aligned}$$

**17.2.85**

(a)

$$\begin{aligned}
 \frac{dQ}{dT} &= nk \frac{T^3}{\theta^3} \\
 Q &= \int_a^b nk \frac{T^3}{\theta^3} \\
 &= \frac{nk}{\theta^3} \left[ \frac{1}{4} T^4 \right]_a^b \\
 &= \frac{nk}{4\theta^3} (b^4 - a^4) \\
 &= 83.6 \text{ J}
 \end{aligned}$$

(b)

$$\begin{aligned}
 Q &= nC \Delta T \\
 C &= \frac{Q}{n \Delta T} \\
 &= 1.86 \text{ J}/(\text{mol K})
 \end{aligned}$$

(c)

$$C = 5.60 \text{ J}/(\text{mol K})$$

**17.2.95**

(a)

$$\begin{aligned}
0 &= m_I L_f + m_I c_W \Delta T_I + m_C c_C \Delta T_I - m_S L_v + m_S c_W \Delta T_S \\
&= m_I L_f + m_I c_W T + m_C c_C T - m_S L_v + m_S c_W (T - T_S) \\
T &= \frac{m_S (L_v + c_W T_S) - m_I L_f}{m_I c_W + m_C c_C + m_S c_W} \\
&= 86.1^\circ \text{C}
\end{aligned}$$

(b) No ice, 0.13 kg water, no steam

**17.2.99**

(a)

$$\begin{aligned}
H &= kA \frac{T_H - T_C}{L} \\
&= 94 \text{ W}
\end{aligned}$$

(b)

$$\begin{aligned}
H_{\text{wood}} &= 12.4 \text{ W} \\
H_{\text{glass}} &= 45.0 \text{ W} \\
H' &= H + (H_{\text{glass}} - H_{\text{wood}}) \\
&= 126.6 \text{ W} \\
\frac{H'}{H} &= 1.35
\end{aligned}$$

17.2.105

(b)

$$\begin{aligned}
\frac{dQ}{dt} &= k \frac{T_H - T_C}{L} \\
\frac{dQ}{dL} &= \rho L_f \\
\frac{dL}{dt} &= \frac{dL}{dQ} \frac{dQ}{dt} \\
&= \frac{1}{\rho L_f} k \frac{T_H - T_C}{L} \\
L \frac{dL}{dt} &= \frac{k(T_H - T_C)}{\rho L_f} \\
\int_0^t L \frac{dL}{dt} dt &= \int_0^t \frac{k(T_H - T_C)}{\rho L_f} dt \\
\int_0^L L' dL' &= \frac{k(T_H - T_C)}{\rho L_f} t \\
\frac{1}{2} L^2 &= \frac{k(T_H - T_C)}{\rho L_f} t \\
L &= \sqrt{\frac{2k(T_H - T_C)}{\rho L_f} t}
\end{aligned}$$

(c)

$$\begin{aligned}
t &= \frac{L^2 \rho L_f}{2k(T_H - T_C)} \\
&= 7.5 \text{ days}
\end{aligned}$$

(d)  $t \approx 530$  years; no



17.2.107

$$\begin{aligned}
A &= 2\pi \left(\frac{d}{2}\right)^2 + 2\pi \left(\frac{d}{2}\right) h \\
&= 8.34 \times 10^{-2} \text{ m}^2 \\
H &= Ae\sigma(T^4 - T_s^4) \\
&= Ae\sigma(T^4 - T_s^4) \\
&= -3.38 \times 10^{-2} \text{ W} \\
m &= \frac{H \times 60 \times 60}{L_v} \\
&= 5.82 \times 10^{-3} \text{ kg/h} \\
&= 5.82 \text{ g/h}
\end{aligned}$$

17.2.113

$$\begin{aligned}
r(x) &= R_2 - (R_2 - R_1) \frac{x}{L} \\
A(x) &= \pi r(x)^2 \\
&= \pi \left[ R_2 - (R_2 - R_1) \frac{x}{L} \right]^2 \\
H &= kA(x) \frac{dT}{dx} \\
&= k\pi \left[ R_2 - (R_2 - R_1) \frac{x}{L} \right]^2 \frac{dT}{dx} \\
\frac{1}{\left[ R_2 - (R_2 - R_1) \frac{x}{L} \right]^2} H dx &= k\pi dT \\
\int_0^L \frac{1}{\left[ R_2 - (R_2 - R_1) \frac{x}{L} \right]^2} H dx &= \int_{T_H}^{T_C} k\pi dT \\
\frac{HL}{R_2 - R_1} \left[ \frac{1}{R_2 - (R_2 - R_1) \frac{x}{L}} \right]_0^L &= k\pi(T_C - T_H) \\
\frac{HL}{R_2 - R_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) &= k\pi(T_C - T_H) \\
\frac{HL}{R_2 - R_1} \frac{R_2 - R_1}{R_1 R_2} &= k\pi(T_C - T_H) \\
H &= \frac{k\pi R_1 R_2 (T_C - T_H)}{L}
\end{aligned}$$

**17.2.115**

(a)

$$\begin{aligned}
 H &= k(2\pi rL) \frac{dT}{dr} \\
 \frac{1}{r} H dr &= 2\pi kL dT \\
 \int_a^b \frac{1}{r} H dr &= \int_{T_1}^{T_2} 2\pi kL dT \\
 H \ln \frac{b}{a} &= 2\pi kL(T_2 - T_1) \\
 H &= \frac{2\pi kL(T_2 - T_1)}{\ln b/a}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \frac{2\pi kL(T - T_2)}{\ln r/a} &= \frac{2\pi kL(T_2 - T_1)}{\ln b/a} \\
 \frac{T - T_2}{\ln r/a} &= \frac{T_2 - T_1}{\ln b/a} \\
 T - T_2 &= \frac{\ln r/a}{\ln b/a} (T_2 - T_1) \\
 T &= T_2 + \frac{\ln r/a}{\ln b/a} (T_2 - T_1)
 \end{aligned}$$

**17.2.117**

a

**17.2.119**

a

## 18 Thermal Properties of Matter

### 18.1 Guided Practice

#### 18.1.1

(a)

$$\begin{aligned}pV &= nRT \\ \frac{p}{T} &= \frac{nR}{V} \\ \frac{p_1}{T_1} &= \frac{p_2}{T_2} \\ p_2 &= p_1 \frac{T_2}{T_1} \\ &= 4.67 \times 10^5 \text{ Pa}\end{aligned}$$

(b)

$$\begin{aligned}pV &= nRT \\ n &= \frac{pV}{RT} \\ &= 0.280 \text{ mol}\end{aligned}$$

#### 18.1.2

(a)

$$\begin{aligned}pV &= nRT \\ \frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\ V_2 &= \frac{V_1 p_1 T_2}{p_2 T_1} \\ &= 1.2 \times 10^3 \text{ m}^3\end{aligned}$$

(b)

$$\begin{aligned}\frac{V_2}{V_1} &= \frac{\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi r_1^3} \\ &= \left(\frac{r_2}{r_1}\right)^3 \\ \frac{r_2}{r_1} &= \sqrt[3]{\frac{V_2}{V_1}} \\ &= 4.5\end{aligned}$$

**18.1.3**

(a)

$$\begin{aligned}
 pV &= nRT \\
 n &= \frac{pV}{RT} \\
 &= 2.9 \times 10^{-3} \text{ mol/m}^3
 \end{aligned}$$

(b)

$$8.0 \times 10^{-5} \text{ kg/m}^3$$

**18.1.4**

(a)

$$\begin{aligned}
 pV &= \frac{m_{\text{total}}}{M} RT \\
 \frac{p}{\rho T} &= \frac{R}{M} \\
 \frac{p_1}{\rho_1 T_1} &= \frac{p_2}{\rho_2 T_2} \\
 &= \frac{p_2}{\rho_1 (p_2/p_1)^{3/5} T_2} \\
 T_2 &= \left( \frac{p_2}{p_1} \right)^{2/5} T_1
 \end{aligned}$$

(b)

$$\begin{aligned}
 \frac{\rho_2}{\rho_1} &= \frac{\rho_1 (p_2/p_1)^{3/5}}{\rho_1} \\
 &= \left( \frac{\frac{1}{2} p_1}{p_1} \right)^{3/5} \\
 &= \left( \frac{1}{2} \right)^{3/5} \\
 &\approx 0.660 \\
 \frac{T_2}{T_1} &= \frac{(p_2/p_1)^{2/5} T_1}{T_1} \\
 &= \left( \frac{\frac{1}{2} p_1}{p_1} \right)^{2/5} \\
 &= \left( \frac{1}{2} \right)^{2/5} \\
 &\approx 0.758
 \end{aligned}$$

(c)

$$\frac{\rho_2}{\rho_1} = 2^{3/5}$$

$$\approx 1.52$$

$$\frac{T_2}{T_1} = 2^{2/5}$$

$$\approx 1.32$$

### 18.1.5

$$\sqrt{\frac{3RT}{M_{\text{H}}}} = \sqrt{\frac{3RT_{\text{N}}}{M_{\text{N}}}}$$

$$T = \frac{M_{\text{H}}}{M_{\text{N}}} T_{\text{N}}$$

$$= 41.9 \text{ K}$$

$$= -231 \text{ }^{\circ}\text{C}$$

### 18.1.6

(a)

$$K_{\text{tr}} = \frac{3}{2} kT = 6.21 \times 10^{-20} \text{ J}$$

(b)

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = 8.63 \times 10^3 \text{ m/s}$$

### 18.1.7

(a)

$$pV = \frac{N}{N_A} RT$$

$$N = \frac{N_A pV}{RT}$$

$$= 1.50 \times 10^{27}$$

(b)

$$K_{\text{tr}} = \frac{3}{2} nRT = 9.11 \times 10^6 \text{ J}$$

(c)

$$\begin{aligned}\frac{1}{2}mv^2 &= K_{\text{tr}} \\ v &= \sqrt{\frac{2K_{\text{tr}}}{m}} \\ &= 110 \text{ m/s}\end{aligned}$$

### 18.1.8

(a) 5.5

(b) 38.5

(c) 6.2

### 18.1.9

(a)

$$\lambda = \frac{kT}{4\pi\sqrt{2}r^2p} = 6.8 \times 10^{-6} \text{ m}$$

(b)

$$\lambda_{\text{Earth}} = 5.54 \times 10^{-8} \text{ m}$$

$$\frac{\lambda_{\text{Mars}}}{\lambda_{\text{Earth}}} = 1.2 \times 10^2$$

### 18.1.10

(a)

$$\begin{aligned}\lambda &= \frac{kT}{4\pi\sqrt{2}r^2p} \\ p &= \frac{kT}{4\pi\sqrt{2}r^2\lambda} \\ &= 5.7 \times 10^{-3} \text{ Pa}\end{aligned}$$

(b)

$$\begin{aligned}pV &= nRT \\ n &= \frac{pV}{RT} \\ &= 2.3 \times 10^{-6} \text{ mol}\end{aligned}$$

### 18.1.11

(a)

$$\begin{aligned}
 pV &= nRT \\
 p &= \frac{nRT}{V} \\
 &= 2.0 \times 10^7 \text{ Pa} \\
 \lambda &= \frac{kT}{4\pi\sqrt{2}r^2p} \\
 &= 1.2 \times 10^{-8} \text{ m}
 \end{aligned}$$

(b)

$$\begin{aligned}
 v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\
 &= 1.4 \times 10^3 \text{ m/s} \\
 \lambda &= vt_{\text{mean}} \\
 t_{\text{mean}} &= \frac{\lambda}{v} \\
 &= 8.6 \times 10^{-12} \text{ s}
 \end{aligned}$$

### 18.1.12

(a)

$$\begin{aligned}
 v_{\text{rms}}t_{\text{mean}} &= \lambda \\
 \sqrt{\frac{3kT}{m}}t_{\text{mean}} &= \frac{kT}{4\pi\sqrt{2}r^2p} \\
 t_{\text{mean}} &= \frac{kT}{4\pi\sqrt{2}r^2p} \sqrt{\frac{m}{3kT}} \\
 &= \frac{1}{4\pi r^2p} \sqrt{\frac{mkT}{6}}
 \end{aligned}$$

(b) Doubling  $r$ .

### 18.1.13

(a)

$$\begin{aligned}v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\&= 515 \text{ m/s} \\ \frac{1}{2}mv_{\text{rms}}^2 &= mgh \\ h &= \frac{v_{\text{rms}}^2}{2g} \\&= 102 \text{ km}\end{aligned}$$

(b)

$$\begin{aligned}&\int_{2025}^{\infty} 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} dv \\&= (3.03 \times 10^{-8}) \int_{2025}^{\infty} v^2 e^{-(5.65 \times 10^{-6})v^2} dv \\&= 4.8 \times 10^{-10}\end{aligned}$$

Yes, some escape.

## 18.2 Exercises and Problems

### 18.2.7

$$\begin{aligned}pV &= nRT \\ \frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\ T_2 &= \frac{p_2 V_2 T_1}{p_1 V_1} \\&= 776 \text{ K} \\&= 503^\circ\text{C}\end{aligned}$$

### 18.2.9

$$\begin{aligned}pV &= nRT \\ \frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\ p_2 &= \frac{p_1 V_1 T_2}{T_1 V_2} \\&= 1.97 \times 10^4 \text{ Pa}\end{aligned}$$



**18.2.13**

$$\begin{aligned}\frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\ V_2 &= \frac{p_1 T_2}{T_1 p_2} V_1 \\ &= (5.08 \times 10^{-2}) V_1\end{aligned}$$

**18.2.17**

(a)

$$\begin{aligned}pV &= \frac{m_{\text{total}}}{M} RT \\ m_{\text{total}} &= \frac{pVM}{RT} \\ &= 6.91 \times 10^{-16} \text{ kg}\end{aligned}$$

(b)

$$\rho = \frac{m_{\text{total}}}{V} = 2.30 \times 10^{-13} \text{ kg/m}^3$$

**18.2.21**

(a)

$$\begin{aligned}pV &= \frac{N}{N_A} RT \\ N &= \frac{pV N_A}{RT} \\ &= 2.19 \times 10^6\end{aligned}$$

(b)

$$2.44 \times 10^{19}$$

**18.2.23**

(a)

$$\begin{aligned}
 pV &= \frac{N}{N_A} RT \\
 \frac{V}{N} &= \frac{RT}{N_A p} \\
 s &= \sqrt[3]{\frac{V}{N}} \\
 &= \sqrt[3]{\frac{RT}{N_A p}} \\
 &= 3.45 \times 10^{-9} \text{ m}
 \end{aligned}$$

**18.2.25**

(a)

$$\begin{aligned}
 K_{\text{tr}} &= \frac{3}{2} nRT \\
 &= \frac{3}{2} pV \\
 &= 5.82 \times 10^7 \text{ J}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \frac{1}{2} m v^2 &= K_{\text{tr}} \\
 v &= \sqrt{\frac{2K_{\text{tr}}}{m}} \\
 &= 241 \text{ m/s}
 \end{aligned}$$

**18.2.27**

$$\begin{aligned}
 pV &= nRT \\
 p &= \frac{nR}{V} T \\
 \frac{nR}{V} &= m \\
 n &= \frac{mV}{R} \\
 &= 1.07 \text{ mol} \\
 N &= nN_A \\
 &= 6.44 \times 10^{23}
 \end{aligned}$$

**18.2.29**

(a)

$$\begin{aligned}
 v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} \\
 &= 1.93 \times 10^6 \text{ m/s} \\
 &= 0.006c
 \end{aligned}$$

Not a significant fraction of  $c$ .

(b)

$$\begin{aligned}
 0.10c &= \sqrt{\frac{3kT}{m}} \\
 (0.10c)^2 &= \frac{3kT}{m} \\
 T &= \frac{(0.10c)^2 m}{3k} \\
 &= 7.26 \times 10^{10} \text{ K}
 \end{aligned}$$

**18.2.31**

(a)

$$\frac{3}{2}kT = 6.21 \times 10^{-21} \text{ J}$$

(b)

$$(v^2)_{\text{av}} = \frac{2}{m} \left( \frac{3}{2}kT \right) = 2.34 \times 10^5 \text{ (m/s)}^2$$

(c)

$$v_{\text{rms}} = \sqrt{(v^2)_{\text{av}}} = 484 \text{ m/s}$$

(d)

$$p = mv = \frac{M}{N_A} v = 2.57 \times 10^{-23} \text{ kg m/s}$$

(e)

$$\begin{aligned}
 \Delta P &= 2P \\
 &= 5.14 \times 10^{-23} \text{ kg m/s} \\
 \Delta t &= \frac{2l}{v} \\
 &= 4.13 \times 10^{-4} \text{ s} \\
 F_{\text{av}} &= \frac{\Delta P}{\Delta t} \\
 &= 1.24 \times 10^{-19} \text{ N}
 \end{aligned}$$

(f)

$$p_{\text{av}} = \frac{F_{\text{av}}}{A} = 1.24 \times 10^{-17} \text{ Pa}$$

(g)

$$\begin{aligned} p &= N p_{\text{av}} \\ N &= \frac{p}{p_{\text{av}}} \\ &= 8.15 \times 10^{21} \end{aligned}$$

(h)

$$\begin{aligned} pV &= \frac{N}{N_A} RT \\ N &= \frac{pV N_A}{RT} \\ &= 2.44 \times 10^{22} \end{aligned}$$

### 18.2.33

$$\begin{aligned} \sqrt{\frac{3RT}{M_{\text{N}}}} &= \sqrt{\frac{3RT_{\text{H}}}{M_{\text{H}}}} \\ T &= \frac{M_{\text{N}}}{M_{\text{H}}} T_{\text{H}} \\ &= 4074 \text{ K} \\ &= 3800 ^\circ\text{C} \end{aligned}$$

**18.2.35**

$$\begin{aligned}
 C_V &= \frac{5}{2}R \\
 v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\
 T &= \frac{Mv_{\text{rms}}^2}{3R} \\
 Q &= nC_V\Delta T \\
 \Delta T &= \frac{Q}{nC_V} \\
 v'_{\text{rms}} &= \sqrt{\frac{3R(T + \Delta T)}{M}} \\
 &= \sqrt{\frac{3R\left(\frac{Mv_{\text{rms}}^2}{3R} + \frac{Q}{nC_V}\right)}{M}} \\
 &= \sqrt{v_{\text{rms}}^2 + \frac{3RQ}{Mn\frac{5}{2}R}} \\
 &= \sqrt{v_{\text{rms}}^2 + \frac{6Q}{5Mn}} \\
 &= 1.02 \times 10^3 \text{ m/s}
 \end{aligned}$$

**18.2.39**

(a)

$$\begin{aligned}
 c_{V,\text{N}} &= \frac{5}{2}R \\
 &= 742 \text{ J/(kg K)} \\
 c_{V,\text{water}} &= 4190 \text{ J/(kg K)} \\
 &= 5.6c_{V,\text{N}}
 \end{aligned}$$

(b)

$$\begin{aligned}Q &= mc_{V,\text{water}}\Delta T \\&= 4.19 \times 10^4 \text{ J} \\m &= \frac{Q}{c_{V,\text{N}}\Delta T} \\&= 5.65 \text{ kg} \\pV &= \frac{m_{\text{total}}}{M}RT \\V &= \frac{m_{\text{total}}RT}{Mp} \\&= 4.87 \text{ m}^3 \\&= 4.87 \times 10^3 \text{ L}\end{aligned}$$

#### 18.2.41

(a)

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}} = 337 \text{ m/s}$$

(b)

$$v_{\text{av}} = 380 \text{ m/s}$$

(c)

$$v_{\text{rms}} = 412 \text{ m/s}$$

#### 18.2.43

(a)

$$\frac{v_{\text{rms}}}{v} = \sqrt{\frac{3RT}{M}} \sqrt{\frac{M}{\gamma RT}} = \sqrt{\frac{3}{\gamma}} = 1.34$$

(b)

$$\frac{v_{\text{av}}}{v} = \sqrt{\frac{8RT}{\pi M}} \sqrt{\frac{M}{\gamma RT}} = \sqrt{\frac{8}{\pi\gamma}} = 1.23$$

#### 18.2.45

- (a) The minimum pressure is  $p_1 = 611.657 \text{ Pa}$ . If  $p < p_1$  the ice sublimates directly to gas.
- (b) The maximum pressure is  $p_2 = 2.212 \times 10^7 \text{ Pa}$ . The ice melts to water, then the properties of the water gradually change to those of steam with no phase transition.

**18.2.49**

(a)

$$\begin{aligned}p' - p &= -\rho gh \\ &= -1.18 \times 10^4 \text{ Pa}\end{aligned}$$

(b)

$$\begin{aligned}p_1 V_1 &= p_2 V_2 \\ V_2 &= \frac{p_1}{p_2} V_1 \\ &= 0.56 \text{ L}\end{aligned}$$

**18.2.51**

$$\begin{aligned}0 &= \rho_{\text{cold}} V g - \rho_{\text{hot}} V g - m g \\ &= \rho_{\text{cold}} V - \rho_{\text{hot}} V - m \\ \rho_{\text{hot}} &= \rho_{\text{cold}} - \frac{m}{V} \\ \frac{Mp}{RT} &= \rho_{\text{cold}} - \frac{m}{V} \\ T &= \frac{Mp}{R(\rho_{\text{cold}} - m/V)} \\ &= 542 \text{ K} \\ &= 269^\circ \text{C}\end{aligned}$$

**18.2.53**

$$\begin{aligned}pV &= \frac{m_{\text{total}}}{M} RT \\ m_{\text{total}} &= \frac{pVM}{RT} \\ &= 0.285 \text{ kg} \\ m'_{\text{total}} &= 0.0896 \text{ kg} \\ \Delta m &= 0.195 \text{ kg}\end{aligned}$$

18.2.57

(a)

$$\begin{aligned}
 0 &= \rho V g - (m_{\text{adventurer}} + m_{\text{bell}} + m_{\text{water}})g \\
 &= \rho V - m_{\text{adventurer}} - m_{\text{bell}} - m_{\text{water}} \\
 m_{\text{water}} &= \rho V - m_{\text{adventurer}} - m_{\text{bell}} \\
 &= 98 \text{ kg} \\
 V_{\text{water}} &= \frac{m_{\text{water}}}{\rho_{\text{water}}} \\
 &= 0.0956 \text{ m}^3
 \end{aligned}$$

(b)

$$\begin{aligned}
 pV &= nRT \\
 p &= \frac{nRT}{V} \\
 p &= \rho g y \\
 \rho g y &= \frac{nRT}{V} \\
 n &= \frac{\rho g V}{RT} y \\
 \frac{dn}{dt} &= \frac{\rho g V}{RT} \frac{dy}{dt} \\
 &= 18.2 \text{ mol/s}
 \end{aligned}$$

(c)

$$\begin{aligned}
 pV &= nRT \\
 n &= \frac{pV}{RT} \\
 &= 756 \text{ mol} \\
 \frac{n}{dn/dt} &= 41.5 \text{ m}
 \end{aligned}$$



**18.2.59**

(a)

$$\begin{aligned}pV &= nRT \\n_{\text{balloon}} &= \frac{pV}{RT} \\&= (9.11 \times 10^6) \frac{1}{T} \\n_{\text{cylinder}} &= \frac{pV}{RT} \\&= (2.97 \times 10^5) \frac{1}{T} \\\frac{n_{\text{balloon}}}{n_{\text{cylinder}}} &= 30.7\end{aligned}$$

(b)

$$\begin{aligned}0 &= \rho Vg - Mng - mg \\mg &= (\rho V - Mn)g \\&= 8420 \text{ N}\end{aligned}$$

(c)

$$mg = 7810 \text{ N}$$

18.2.67

(c)

$$U(r) = U_0 \left[ \left( \frac{R_0}{r} \right)^{12} - 2 \left( \frac{R_0}{r} \right)^6 \right]$$

$$F(r) = 12 \frac{U_0}{R_0} \left[ \left( \frac{R_0}{r} \right)^{13} - \left( \frac{R_0}{r} \right)^7 \right]$$

$$\begin{aligned} 0 &= U_0 \left[ \left( \frac{R_0}{r_1} \right)^{12} - 2 \left( \frac{R_0}{r_1} \right)^6 \right] \\ &= \left( \frac{R_0}{r_1} \right)^{12} - 2 \left( \frac{R_0}{r_1} \right)^6 \\ &= \left( \frac{R_0}{r_1} \right)^6 - 2 \\ 2 &= \left( \frac{R_0}{r_1} \right)^6 \\ 2r_1^6 &= R_0^6 \\ r_1 &= \frac{1}{\sqrt[6]{2}} R_0 \\ &\approx 0.89 R_0 \end{aligned}$$

$$\begin{aligned} 0 &= 12 \frac{U_0}{R_0} \left[ \left( \frac{R_0}{r_2} \right)^{13} - \left( \frac{R_0}{r_2} \right)^7 \right] \\ 0 &= \left( \frac{R_0}{r_2} \right)^{13} - \left( \frac{R_0}{r_2} \right)^7 \\ &= \left( \frac{R_0}{r_2} \right)^6 - 1 \\ r_2 &= R_0 \end{aligned}$$

$$\frac{r_1}{r_2} = \frac{1}{\sqrt[6]{2}}$$

(d)

$$\begin{aligned} W &= \int_{r_2}^{\infty} -F dr \\ &= \int_{R_0}^{\infty} -12 \frac{U_0}{R_0} \left[ \left( \frac{R_0}{r} \right)^{13} - \left( \frac{R_0}{r} \right)^7 \right] dr \\ &= -12 \frac{U_0}{R_0} \left( -\frac{R_0}{12} \right) \\ &= U_0 \end{aligned}$$

**18.2.69**

(a)

$$C_V = 2R = 16.63 \text{ J/(mol K)}$$

(b) Less than because vibrational energy will play a smaller role.

**18.2.71**

(a)

$$\begin{aligned} \frac{1}{2}mv^2 &\geq \frac{GmM}{R_p} \\ &\geq gmR_p \end{aligned}$$

(b)

$$\begin{aligned} \frac{3}{2}kT &\geq mgR_p \\ T_N &\geq \frac{2mgR_p}{3k} \\ &\geq 1.40 \times 10^5 \text{ K} \\ T_H &\geq 1.02 \times 10^4 \text{ K} \end{aligned}$$

(c)

$$\begin{aligned} T_N &\geq 6.37 \times 10^3 \text{ K} \\ T_H &\geq 459 \text{ K} \end{aligned}$$

(d) Because it's very easy to atmospheric particles to escape.

**18.2.73**

$$\begin{aligned}
 \int_0^\infty v^2 f(v) dv &= 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} \int_0^\infty v^4 e^{-mv^2/2kT} dv \\
 &= 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} \frac{3}{2^3(m/2kT)^2} \sqrt{\frac{\pi}{(m/2kT)}} \\
 &= 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} \frac{3}{8} \left( \frac{2kT}{m} \right)^2 \sqrt{\frac{2\pi kT}{m}} \\
 &= \frac{3kT}{m}
 \end{aligned}$$

**18.2.75**

(b)

$$\begin{aligned}
 v_{\text{mp}} &= \sqrt{\frac{2kT}{m}} \\
 &= 395 \text{ m/s} \\
 f(v_{\text{mp}}) &= 2.10 \times 10^{-3} \\
 \Delta N &\approx N f(v_{\text{mp}}) \Delta v \\
 &\approx (4.20 \times 10^{-2}) N
 \end{aligned}$$

(c)

$$\begin{aligned}
 7v_{\text{mp}} &= 2765 \text{ m/s} \\
 f(7v_{\text{mp}}) &= 1.43 \times 10^{-22} \\
 \Delta N &\approx (2.85 \times 10^{-21}) N
 \end{aligned}$$

**18.2.77**

(a)

$$\begin{aligned}
 0 &= pA - p_0A - mg \\
 p &= p_0 + \frac{mg}{A} \\
 &= p_0 + \frac{mg}{\pi r^2}
 \end{aligned}$$

(b)

$$\begin{aligned} p_1 V_1 &= p_2 V_2 \\ p_2 &= \frac{V_1}{V_2} p_1 \\ &= \frac{Ah}{A(h+y)} p_1 \\ &= \frac{h}{h+y} p_1 \\ &\approx \left(1 - \frac{y}{h}\right) p_1 \\ F &= \left(1 - \frac{y}{h}\right) p_1 \pi r^2 - p_0 \pi r^2 - mg \\ &= \left(1 - \frac{y}{h}\right) \left(p_0 + \frac{mg}{\pi r^2}\right) \pi r^2 - p_0 \pi r^2 - mg \\ &= \left(1 - \frac{y}{h}\right) (p_0 \pi r^2 + mg) - p_0 \pi r^2 - mg \\ &= -\frac{y}{h} (p_0 \pi r^2 + mg) \end{aligned}$$

(c)

$$\begin{aligned} F &= -kx \\ k &= \frac{1}{h} (p_0 \pi r^2 + mg) \\ \omega &= \sqrt{\frac{k}{m}} \\ &= \sqrt{\frac{1}{h} \left( \frac{p_0 \pi r^2}{m} + g \right)} \\ f &= \frac{\omega}{2\pi} \\ &= \frac{1}{2\pi} \sqrt{\frac{g}{h} \left( 1 + \frac{p_0 \pi r^2}{gm} \right)} \end{aligned}$$

If the displacement is not small the oscillation is not simple harmonic because we can't use the approximation  $\frac{h}{h+y} \approx 1 - \frac{y}{h}$ .

### 18.2.81

(a)

$$I = 2mr^2 = 4.1 \times 10^{-46} \text{ kg m}^2$$

(b)

$$2 \left( \frac{1}{2} (2m) v_i^2 \right) = 2 \left( \frac{1}{2} (2m) v_f^2 + \frac{1}{2} I \omega^2 \right)$$

$$2m v_i^2 = 2m v_f^2 + 2m r^2 \omega^2$$

$$v_i^2 = v_f^2 + r^2 \omega^2$$

$$-2r(2m)v_i = -2I\omega$$

$$2mr v_i = 2mr^2 \omega$$

$$v_i = r\omega$$

(c)

$$\omega = \frac{v_i}{r}$$

$$v_i^2 = v_f^2 + r^2 \left( \frac{v_i}{r} \right)^2$$

$$v_f = 0$$

(d)

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

$$= 514 \text{ m/s}$$

$$\omega = 5.47 \times 10^{12} \text{ rad/s}$$

### 18.2.83

(a)

$$\begin{aligned} \lambda &= \frac{V}{4\pi\sqrt{2}r^2N} \\ &= 4.50 \times 10^{11} \text{ m} \end{aligned}$$

(b)

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} \\ &= 704 \text{ m/s} \end{aligned}$$

$$\begin{aligned} t_{\text{mean}} &= \frac{\lambda}{v_{\text{rms}}} \\ &= 6.39 \times 10^8 \text{ s} \\ &= 20 \text{ years} \end{aligned}$$

(c)

$$\begin{aligned}pV &= NkT \\p &= \frac{NkT}{V} \\&= 1.38 \times 10^{-14} \text{ Pa}\end{aligned}$$

(d)

$$\begin{aligned}m_{\text{total}} &= \rho V \\&= \rho \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 \\&= 2.96 \times 10^{32} \text{ kg}\end{aligned}$$

$$\begin{aligned}\frac{1}{2}mv^2 &= \frac{Gmm_{\text{total}}}{r} \\v &= \sqrt{\frac{4Gm_{\text{total}}}{d}} \\&= 640 \text{ m/s}\end{aligned}$$

It would evaporate.

(f)

$$\begin{aligned}T_{\text{ISM}} &= \frac{(N/V)_{\text{nebula}}}{(N/V)_{\text{ISM}}} T_{\text{nebula}} \\&= 2.0 \times 10^5 \text{ K}\end{aligned}$$

34 times hotter than the sun.

**18.2.85**

a

**18.2.87**

c

## 19 The First Law of Thermodynamics

### 19.1 Guided Practice

#### 19.1.1

(a)

$$\begin{aligned}\Delta U &= Q - W \\ Q &= \Delta U + W \\ &= 5.75 \times 10^3 \text{ J}\end{aligned}$$

(b)

$$\begin{aligned}\Delta U &= Q - W \\ &= -3.2 \times 10^4 \text{ J}\end{aligned}$$

(c)

$$\begin{aligned}\Delta U &= Q - W \\ W &= Q - \Delta U \\ &= -1.85 \times 10^3 \text{ J}\end{aligned}$$

#### 19.1.2

(a)

$$W = p(V_2 - V_1) = 155 \text{ J}$$

(b)

$$W = 0$$

(c)

$$W = p(V_2 - V_1) = -375 \text{ J}$$

(d)

$$W = \frac{1}{2}(p_1 + p_2)(V_2 - V_1) = 875 \text{ J}$$

#### 19.1.3

(a)

$$\begin{aligned}W &= p(V_2 - V_1) \\ &= -240 \text{ J} \\ \Delta U &= Q - W \\ &= 1.80 \times 10^3 \text{ J}\end{aligned}$$



(b)

$$\begin{aligned}W &= p(V_2 - V_1) \\&= -720 \text{ J} \\ \Delta U &= Q - W \\ Q &= \Delta U + W \\&= 1.08 \times 10^3 \text{ J}\end{aligned}$$

#### 19.1.4

(a)

$$Q = mL = 3.43 \times 10^6 \text{ J}$$

(b)

$$W = p(V_2 - V_1) = 3.43 \times 10^5 \text{ J}$$

(c)

$$\Delta U = Q - W = 3.09 \times 10^6 \text{ J}$$

#### 19.1.5

(a)

$$\Delta U = \Delta Q = nC_V \Delta T = 998 \text{ J}$$

(b)

$$\Delta U = \Delta Q = nC_V \Delta T = 748 \text{ J}$$

(c)

$$\Delta U = \Delta Q = nC_V \Delta T = 599 \text{ J}$$

#### 19.1.6

(a)

$$V = \frac{nRT}{p} = 5.24 \times 10^{-2} \text{ m}^3$$

(b) (i)

$$\begin{aligned}T &= 327^\circ \text{C} \\ \Delta U &= Q \\&= nC_V \Delta T \\&= 1.31 \times 10^4 \text{ J}\end{aligned}$$

(ii)

$$\begin{aligned}T &= 327^\circ\text{C} \\ \Delta U &= Q \\ &= nC_V\Delta T \\ &= 1.31 \times 10^4 \text{ J}\end{aligned}$$

(iii)

$$\begin{aligned}T &= 927^\circ\text{C} \\ \Delta U &= 3.92 \times 10^4 \text{ J}\end{aligned}$$

### 19.1.7

(a)

$$\begin{aligned}pV &= nRT \\ \frac{pV}{R} &= nT \\ (2p) &= nR(2T) \\ \Delta T &= T \\ \Delta U &= Q - W \\ &= nC_V\Delta T \\ &= C_V(nT) \\ &= \frac{3}{2}R\frac{pV}{R} \\ &= \frac{3}{2}pV \\ &= 4.50 \times 10^4 \text{ J}\end{aligned}$$

(b)

$$pV = nRT$$

$$\frac{pV}{R} = nT$$

$$pV = nRT$$

$$p\left(\frac{1}{2}V\right) = nR\left(\frac{1}{2}T\right)$$

$$\Delta T = -\frac{1}{2}T$$

$$\Delta U = nC_V\Delta T$$

$$= C_V\left(-\frac{1}{2}nT\right)$$

$$= -\frac{3}{4}R\frac{pV}{R}$$

$$= -\frac{3}{4}pV$$

$$= -2.25 \times 10^4 \text{ J}$$

(c)

$$\Delta U = 1.17 \times 10^5 \text{ J}$$

### 19.1.8

(a)

$$Q = nC_V\Delta T$$

$$= \frac{5}{2}nRT$$

$$W = 0$$

$$\Delta U = Q - W$$

$$= \frac{5}{2}nRT$$

(b)

$$\begin{aligned}Q &= nC_P\Delta T \\&= \frac{7}{2}nRT\end{aligned}$$

$$W = p(V_2 - V_1)$$

$$\begin{aligned}\Delta U &= \frac{7}{2}nRT - p(V_2 - V_1) \\&= \frac{7}{2}nRT - 2nRT + nRT \\&= \frac{5}{2}nRT\end{aligned}$$

(c)

$$Q = 0$$

$$\begin{aligned}W &= nC_V(T_1 - T_2) \\&= -\frac{5}{2}nRT\end{aligned}$$

$$\begin{aligned}\Delta U &= Q - W \\&= \frac{5}{2}nRT\end{aligned}$$

### 19.1.9

(a)

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

(b)

$$\begin{aligned}p_1 V_1^\gamma &= p_2 V_2^\gamma \\p_2 &= \left(\frac{V_1}{V_2}\right)^\gamma p_1 \\&= 6.41 \times 10^4 \text{ Pa}\end{aligned}$$

(c)

$$\begin{aligned}W &= \frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2) \\&= 623 \text{ J}\end{aligned}$$

**19.1.10**

(a)

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5}$$

(b)

$$\begin{aligned} T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ V_2^{\gamma-1} &= \frac{T_1}{T_2} V_1^{\gamma-1} \\ V_2 &= \left( \frac{T_1}{T_2} \right)^{1/(\gamma-1)} V_1 \\ &= 5.79 \times 10^{-4} \text{ m}^3 \end{aligned}$$

(c)

$$\begin{aligned} p_1 V_1^\gamma &= p_2 V_2^\gamma \\ p_2 &= \left( \frac{V_1}{V_2} \right)^\gamma p_1 \\ &= 2.95 \times 10^6 \text{ Pa} \end{aligned}$$

(d)

$$\begin{aligned} W &= \frac{1}{\gamma-1} (p_1 V_1 - p_2 V_2) \\ &= -2.65 \times 10^3 \text{ J} \end{aligned}$$

**19.1.11**

(a)

$$\begin{aligned} pV &= nRT \\ p &= \frac{nRT}{V} \\ &= 3.17 \times 10^5 \text{ Pa} \end{aligned}$$

(b)

$$\begin{aligned} p_1 V_1^\gamma &= p_2 V_2^\gamma \\ p_2 &= \left( \frac{V_1}{V_2} \right)^\gamma p_1 \\ &= 8.21 \times 10^4 \text{ Pa} \end{aligned}$$

(c)

$$\begin{aligned}T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\T_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 \\&= 178 \text{ K}\end{aligned}$$

(d)

$$\begin{aligned}W &= \frac{1}{\gamma-1}(p_1 V_1 - p_2 V_2) \\&= 7.94 \times 10^3 \text{ J}\end{aligned}$$

### 19.1.12

(a)

$$\begin{aligned}\left[p + \left(\frac{an^2}{V^2}\right)\right](V - nb) &= nRT \\p + \left(\frac{an^2}{V^2}\right) &= \frac{nRT}{V - nb} \\p &= \frac{nRT}{V - nb} - \frac{an^2}{V^2}\end{aligned}$$

$$\begin{aligned}W &= \int_{V_1}^{V_2} p \, dV \\&= \int_{V_1}^{V_2} \left(\frac{nRT}{V - nb} - \frac{an^2}{V^2}\right) dV \\&= \left[nRT \ln(V - nb) + \frac{an^2}{V}\right]_{V_1}^{V_2} \\&= nRT \ln(V_2 - nb) + \frac{an^2}{V_2} - nRT \ln(V_1 - nb) - \frac{an^2}{V_1} \\&= nRT \ln \frac{V_2 - nb}{V_1 - nb} + an^2 \frac{V_1 - V_2}{V_1 V_2}\end{aligned}$$

(b) (i)

$$W = 2.80 \times 10^3 \text{ J}$$

(ii)

$$\begin{aligned} W &= \int_{V_1}^{V_2} p \, dV \\ &= \int_{V_1}^{V_2} \frac{nRT}{V} \, dV \\ &= nRT [\ln V]_{V_1}^{V_2} \\ &= 3.11 \times 10^3 \text{ J} \end{aligned}$$

## 19.2 Exercises and Problems

### 19.2.1

(b)

$$\begin{aligned} W &= p(V_2 - V_1) \\ &= nR(T_2 - T_1) \\ &= 1.33 \times 10^3 \text{ J} \end{aligned}$$

### 19.2.3

(b)

$$\begin{aligned} p_1 V_1 &= nRT \\ p_2 V_2 &= nRT \\ 3p_1 V_2 &= nRT \\ V_2 &= \frac{1}{3} V_1 \\ W &= \int_{V_1}^{V_2} p \, dV \\ &= \int_{V_1}^{V_1/3} \frac{nRT}{V} \, dV \\ &= nRT \ln \frac{1}{3} \\ &= -6.18 \times 10^3 \text{ J} \end{aligned}$$

**19.2.5**

(a)

$$pV = nRT$$

$$V = \frac{nRT}{p}$$

$$\begin{aligned} W &= \int_{V_1}^{V_2} p \, dV \\ &= \int_{nRT/p_1}^{nRT/p_2} \frac{nRT}{V} \, dV \\ &= nRT \ln \frac{nRT/p_2}{nRT/p_1} \\ &= nRT \ln \frac{p_1}{p_2} \end{aligned}$$

$$\frac{W}{nRT} = \ln \frac{p_1}{p_2}$$

$$\begin{aligned} p_1 &= p_2 e^{W/nRT} \\ &= 1.05 \times 10^5 \text{ Pa} \\ &= 1.04 \text{ atm} \end{aligned}$$

**19.2.9**

(a)

$$W = p(V_2 - V_1) = 3.47 \times 10^4 \text{ J}$$

(b)

$$\Delta U = Q - W = 8.03 \times 10^4 \text{ J}$$

(c) No, because it's an isobaric process.

**19.2.11**

(a)

$$\begin{aligned} T_a &= \frac{pV}{nR} \\ &= 278 \text{ K} \\ T_b &= 694 \text{ K} \\ T_c &= 1250 \text{ K} \end{aligned}$$

The lowest temperature is 278 K and it occurred at point  $a$ .



(b)

$$W_{ab} = 0$$

$$W_{bc} = 162 \text{ J}$$

(c)

$$\Delta U = Q - W = 52 \text{ J}$$

### 19.2.13

(a)

$$\begin{aligned} T_a &= \frac{pV}{nR} \\ &= 5.35 \times 10^2 \text{ K} \end{aligned}$$

$$T_b = 9.36 \times 10^3 \text{ K}$$

$$T_c = 1.50 \times 10^4 \text{ K}$$

(b)

$$W = 2.10 \times 10^4 \text{ J}$$

(c)

$$Q = \Delta U + W = 3.60 \times 10^4 \text{ J}$$

### 19.2.17

(b)

$$\begin{aligned} V_1 &= \frac{nRT_1}{p_1} \\ &= 6.18 \times 10^{-3} \text{ m}^3 \end{aligned}$$

$$V_2 = 8.23 \times 10^{-3} \text{ m}^3$$

$$W = p(V_2 - V_1)$$

$$= 207 \text{ J}$$

(c) The piston

(d)

$$C_V = \frac{5}{2}R$$

$$C_P = \frac{7}{2}R$$

$$\Delta U = nC_P\Delta T$$

$$= 727 \text{ J}$$

$$Q = \Delta U + W$$

$$= 934 \text{ J}$$

**19.2.19**

(a)

$$\begin{aligned}
 \Delta U &= Q - W \\
 &= Q - 0 \\
 &= nC_V \Delta T \\
 \Delta T &= \frac{\Delta U}{nC_V} \\
 &= 168 \text{ K} \\
 T_2 &= T_1 + \Delta T \\
 &= 948 \text{ K}
 \end{aligned}$$

(b)

$$\begin{aligned}
 Q &= nC_P \Delta T \\
 \Delta T &= \frac{Q}{nC_P} \\
 &= 120 \text{ K} \\
 T_2 &= T_1 + \Delta T \\
 &= 900 \text{ K}
 \end{aligned}$$

**19.2.21**

$$\begin{aligned}
 C_V &= \frac{3}{2}R \\
 C_P &= \frac{5}{2}R \\
 Q &= nC_P \Delta T \\
 &= \frac{5}{2}nR(T_2 - T_1) \\
 W &= p(V_2 - V_1) \\
 &= nR(T_2 - T_1) \\
 \frac{W}{Q} &= \frac{2}{5}
 \end{aligned}$$

**19.2.23**

(a)

$$\begin{aligned}
 \Delta U &= Q - W \\
 &= 747 \text{ J}
 \end{aligned}$$

(b)

$$\begin{aligned}Q &= nC_P\Delta T \\C_P &= \frac{Q}{n\Delta T} \\&= 37.0 \text{ J}/(\text{mol K}) \\C_V &= C_P - R \\&= 28.6 \text{ J}/(\text{mol K}) \\\gamma &= \frac{C_P}{C_V} \\&= 1.29\end{aligned}$$

**19.2.25**

(a)

$$\begin{aligned}V_1 &= \frac{nRT}{p_1} \\&= 3.46 \times 10^{-3} \text{ m}^3 \\V_2 &= 8.64 \times 10^{-4} \text{ m}^3 \\W &= \int_{V_1}^{V_2} p dV \\&= \int_{V_1}^{V_2} \frac{nRT}{V} dV \\&= nRT \ln \frac{V_2}{V_1} \\&= -606 \text{ J}\end{aligned}$$

(b)

$$\Delta U = 0 \text{ J}$$

(c) Yes, liberate

$$Q = \Delta U + W = -606 \text{ J}$$

**19.2.27**

(a)

$$\begin{aligned}
C_V &= \frac{3}{2}R \\
C_P &= \frac{5}{2}R \\
\gamma &= \frac{C_P}{C_V} \\
&= \frac{5}{3} \\
p_1 V_1^\gamma &= p_2 V_2^\gamma \\
p_2 &= \left( \frac{V_1}{V_2} \right)^\gamma p_1 \\
&= 4.76 \times 10^5 \text{ Pa}
\end{aligned}$$

(b)

$$\begin{aligned}
W &= \frac{C_V}{R} (p_1 V_1 - p_2 V_2) \\
&= -1.06 \times 10^4 \text{ J}
\end{aligned}$$

(c)

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
\frac{T_2}{T_1} &= \left( \frac{V_1}{V_2} \right)^{\gamma-1} \\
&= 1.59
\end{aligned}$$

Heated

**19.2.29**

(b)

$$\begin{aligned}
W &= nC_V(T_1 - T_2) \\
&= 314 \text{ J}
\end{aligned}$$

(c)

$$\begin{aligned}
\Delta U &= Q - W \\
&= 0 - W \\
&= -314 \text{ J}
\end{aligned}$$

**19.2.31**

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_1 \left( \frac{nRT_1}{p_1} \right)^{\gamma-1} &= T_2 \left( \frac{nRT_2}{p_2} \right)^{\gamma-1} \\
T_2^\gamma &= T_1^\gamma \left( \frac{p_2}{p_1} \right)^{\gamma-1} \\
T_2 &= T_1 \left( \frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} \\
&= 285 \text{ K} \\
&= 11.6^\circ \text{C}
\end{aligned}$$

**19.2.33**

$$\begin{aligned}
C_V &= \frac{3}{2}R \\
C_P &= \frac{5}{2}R \\
\gamma &= \frac{5}{3} \\
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_1 \left( \frac{nRT_1}{p_1} \right)^{\gamma-1} &= 2T_1 \left( \frac{2nRT_1}{p_2} \right)^{\gamma-1} \\
\frac{1}{p_1^{\gamma-1}} &= \frac{2^\gamma}{p_2^{\gamma-1}} \\
p_1^{\gamma-1} &= \frac{p_2^{\gamma-1}}{2^\gamma} \\
p_2 &= 2^{\gamma/(\gamma-1)} p_1 \\
&= 2^{5/2} p_1 \\
&= 4\sqrt{2} p_1
\end{aligned}$$

**19.2.35**

(a) Increase

(b)

$$W = \frac{1}{2}(p_a + p_b)(V_B - V_A) = 4.8 \text{ kJ}$$

**19.2.37**

(a)

$$\begin{aligned}
 pV &= nRT \\
 n &= \frac{pV}{RT} \\
 &= 0.678 \text{ mol}
 \end{aligned}$$

(b)

$$\begin{aligned}
 pV &= nRT \\
 V &= \frac{nRT}{p} \\
 &= 3.33 \times 10^{-2} \text{ m}^3
 \end{aligned}$$

(c)

$$\begin{aligned}
 W &= \int_{V_1}^{V_2} p dV \\
 &= \int_{V_1}^{V_2} \frac{nRT}{V} dV \\
 &= nRT \ln \frac{V_2}{V_1} \\
 &= 2.22 \text{ kJ}
 \end{aligned}$$

(d)

$$\Delta U = 0$$

**19.2.39**

(a)

$$\begin{aligned}
 \Delta U &= Q - W \\
 &= 30.0 \text{ J} \\
 Q &= \Delta U + W \\
 &= 45.0 \text{ J}
 \end{aligned}$$

(b) Liberate

$$Q = \Delta U + W = -65.0 \text{ J}$$

(c)

$$\Delta U_{\text{ad}} = 8.0 \text{ J}$$

$$W_{\text{ad}} = 15.0 \text{ J}$$

$$\begin{aligned} Q_{\text{ad}} &= \Delta U_{\text{ad}} + W_{\text{ad}} \\ &= 23.0 \text{ J} \end{aligned}$$

$$\begin{aligned} Q_{\text{db}} &= \Delta U_{\text{ab}} - \Delta U_{\text{ad}} \\ &= 22.0 \text{ J} \end{aligned}$$

**19.2.43**

(a)

$$p_1 V_1 = p_2 V_2$$

$$\begin{aligned} V_2 &= \frac{p_1}{p_2} V_1 \\ &= 8.0 \times 10^{-4} \text{ m}^3 \\ &= 0.80 \text{ L} \end{aligned}$$

(b)

$$\begin{aligned} T_a &= \frac{pV}{nR} \\ &= 304 \text{ K} \end{aligned}$$

$$T_b = 1.21 \times 10^3 \text{ K}$$

$$T_c = 1.21 \times 10^3 \text{ K}$$

(c)

$$\begin{aligned}\Delta U_{ab} &= Q_{ab} - W_{ab} \\ &= Q_{ab} \\ &= nC_V\Delta T \\ &= 74.0 \text{ J into the gas}\end{aligned}$$

$$\begin{aligned}V_c &= \frac{nRT_c}{p_c} \\ &= 7.97 \times 10^{-4} \text{ m}^3 \\ \Delta U_{ca} &= Q_{ca} - W_{ca} \\ nC_V\Delta T &= Q_{ca} - p(V_a - V_c) \\ Q_{ca} &= nC_V\Delta T + p(V_a - V_c) \\ &= -104 \text{ J out of the gas}\end{aligned}$$

$$\begin{aligned}\Delta U_{bc} &= Q_{bc} - W_{bc} \\ Q_{bc} &= \Delta U_{bc} + W_{bc} \\ &= nC_V\Delta T + \int_{V_b}^{V_c} p dV \\ &= nRT \ln \frac{V_c}{V_b} \\ &= 55.6 \text{ J into the gas}\end{aligned}$$

(d)

$$\Delta U_{ab} = 74.0 \text{ J increase}$$

$$\Delta U_{bc} = 0.0 \text{ J no change}$$

$$\begin{aligned}\Delta U_{ca} &= nC_V\Delta T \\ &= -74.0 \text{ J decrease}\end{aligned}$$

**19.2.47**

(b)

$$V_2 = \frac{p_1}{p_2} V_1 = 6.0 \text{ L}$$



(c)

$$\begin{aligned}n &= \frac{pV}{RT} \\&= 6.01 \times 10^{-2} \text{ mol} \\W_{12} &= \int_{V_1}^{V_2} p dV \\&= nRT_1 \ln \frac{V_2}{V_1} \\&= 208 \text{ J}\end{aligned}$$

$$\begin{aligned}W_{23} &= p_2(V_3 - V_2) \\&= -113 \text{ J}\end{aligned}$$

$$W = 95.0 \text{ J}$$

(d) Heat it at constant volume

**19.2.49**

(a) As the wind descends it experiences greater atmospheric pressure. This compresses the wind, increasing its temperature. It's important that it be moving fast so it's an adiabatic process.

(b)

$$\begin{aligned}T_2 &= \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} T_1 \\&= 287 \text{ K} \\&= 13.9^\circ\text{C} \\\Delta T &= T_2 - T_1 \\&= 11.9^\circ\text{C}\end{aligned}$$

**19.2.51**

(a)

$$\begin{aligned}
 p_1 V_1^\gamma &= p_2 V_2^\gamma \\
 p_1 (Ah)^\gamma &= p_2 [A(h - \Delta h)]^\gamma \\
 \frac{p_1}{p_2} h^\gamma &= (h - \Delta h)^\gamma \\
 \left(\frac{p_1}{p_2}\right)^{1/\gamma} h &= h - \Delta h \\
 \Delta h &= h \left[ 1 - \left(\frac{p_1}{p_2}\right)^{1/\gamma} \right] \\
 &= 16.8 \text{ cm}
 \end{aligned}$$

(b)

$$\begin{aligned}
 T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
 T_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 \\
 &= \left(\frac{Ah}{A(h - \Delta h)}\right)^{\gamma-1} T_1 \\
 &= \left(\frac{1}{1 - \Delta h/h}\right)^{\gamma-1} T_1 \\
 &= 469 \text{ K} \\
 &= 196^\circ \text{C}
 \end{aligned}$$

(c)

$$W = nC_V(T_1 - T_2) = -7.03 \times 10^4 \text{ J}$$

**19.2.59**

(a) a is adiabatic, b is isochoric, c is isobaric

(b)

$$\begin{aligned}\Delta U &= Q_b - W_b \\ &= Q_b - 0 \\ &= Q_b\end{aligned}$$

$$\begin{aligned}\Delta U &= Q_c - W_c \\ &= Q_c - p(V_2 - V_1) \\ &= Q_c - nR(T_2 - T_1)\end{aligned}$$

$$\begin{aligned}Q_b &= Q_c - nR(T_2 - T_1) \\ T_2 &= T_1 + \frac{Q_c - Q_b}{nR} \\ &= 28.0^\circ\text{C}\end{aligned}$$

(c)

$$\begin{aligned}Q_b &= nC_V\Delta T \\ C_V &= \frac{Q_b}{n\Delta T} \\ &= 12.5\text{ J}/(\text{mol K})\end{aligned}$$

$$\begin{aligned}W_a &= nC_V(T_1 - T_2) \\ &= -30.0\text{ J}\end{aligned}$$

$$W_b = 0$$

$$\begin{aligned}\Delta U_c &= Q_c - W_c \\ W_c &= Q_c - \Delta U_c \\ &= Q_c - nC_V\Delta T \\ &= 20.0\text{ J}\end{aligned}$$

(d)

$$\begin{aligned}\gamma &= \frac{C_P}{C_V} \\ &= \frac{C_V + R}{C_V} \\ &= 1.67\end{aligned}$$

$$\begin{aligned}T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ \left(\frac{V_2}{V_1}\right)^{\gamma-1} &= \frac{T_1}{T_2} \\ \frac{V_2}{V_1} &= \left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} \\ &= 0.961\end{aligned}$$

$$\Delta V_b = 0$$

$$\begin{aligned}\frac{V_2}{V_1} &= \frac{nRT_2/p}{nRT_1/p} \\ &= \frac{T_2}{T_1} \\ &= 1.03\end{aligned}$$

a

(e) Decrease, stay the same, increase

**19.2.61**

(a)

$$\begin{aligned}
 r &= 1.50 \text{ cm} \\
 l_{\max} &= 30.0 \text{ cm} \\
 l_{\min} &= l_{\max}/v \\
 p &= 101 \text{ kPa} \\
 T &= 30.0^\circ\text{C} \\
 V_1 &= \pi r^2 l_{\max} \\
 &= 2.12 \times 10^{-4} \text{ m}^3 \\
 V_2 &= \pi r^2 l_{\min} \\
 &= \pi r^2 \frac{l_{\max}}{v} \\
 &= \frac{V_1}{v} \\
 n &= \frac{pV}{RT} \\
 &= 8.50 \times 10^{-3} \text{ mol} \\
 T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
 T_2 &= T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} \\
 &= T_1 v^{\gamma-1} \\
 W_{\text{adiabatic}} &= nC_V(T_1 - T_2) \\
 &= nC_V(T_1 - T_1 v^{\gamma-1}) \\
 &= nC_V T_1 (1 - v^{\gamma-1}) \\
 &= 53.5(1 - v^{0.4}) \\
 W_{\text{isothermal}} &= \int_{V_1}^{V_2} p dV \\
 &= \int_{V_1}^{V_2} \frac{nRT_2}{V} dV \\
 &= nRT_2 \ln \frac{V_2}{V_1} \\
 &= nRT_1 v^{\gamma-1} \ln v \\
 &= 21.4 v^{0.4} \ln v \\
 W &= 53.5(1 - v^{0.4}) + 21.4 v^{0.4} \ln v \\
 &= 53.5 + v^{0.40}(21.4 \ln v - 53.5)
 \end{aligned}$$

(b)

$$\begin{aligned}T_2 &\leq T_{\max} \\T_1 v^{\gamma-1} &\leq T_{\max} \\v &\leq \left(\frac{T_{\max}}{T_1}\right)^{1/(\gamma-1)} \\&\leq 7.35\end{aligned}$$

The largest integer value of  $v$  is 7.

(c) 7

(d) 7

(e)

$$\begin{aligned}T_2 &= T_1 v^{\gamma-1} \\&= 660 \text{ K} \\&= 387^\circ \text{C} \\Q &= nC_V \Delta T \\&= -63.0 \text{ J}\end{aligned}$$

**19.2.63**

$$\begin{aligned}\frac{p_1}{T_1} &= \frac{p_2}{T_2} \\p_2 &= \frac{T_2}{T_1} p_1 \\&= 1.27 \times 10^7 \text{ Pa} \\&= 1.84 \times 10^3 \text{ psi}\end{aligned}$$

c

**19.2.65**

$$\begin{aligned}p_1 V_1 &= p_2 V_2 \\V_1 &= \frac{p_2}{p_1} V_2 \\&= 6.01 \times 10^{-5} \text{ m}^3 \\&= 6.01 \times 10^{-2} \text{ L}\end{aligned}$$

d

## 20 The Second Law of Thermodynamics

### 20.1 Guided Practice

#### 20.1.1

(a)

$$W = eQ_H \Rightarrow Q_H = \frac{W}{e} = 6.89 \times 10^4 \text{ J}$$

(b)

$$|W| = |Q_H| - |Q_C| \Rightarrow |Q_C| = |Q_H| - |W| = 5.65 \times 10^4 \text{ J}$$

#### 20.1.2

$$e = \frac{W}{Q_H} = \frac{Q_H + Q_C}{Q_H} = 1 + \frac{Q_C}{Q_H} = 0.173 = 17.3\%$$

#### 20.1.3

(a)

$$(1 - e)Q_H = Q_C \Rightarrow Q_H = \frac{Q_C}{1 - e} = 6.17 \times 10^8 \text{ J}$$

(b)

$$W = eQ_H = 1.21 \times 10^8 \text{ J}$$

#### 20.1.4

(a)

$$W = 3600P = 3.96 \times 10^8 \text{ J}$$

(b)

$$Q_H = mL_c = 1.70 \times 10^9 \text{ J}$$

(c)

$$e = \frac{W}{Q_H} = 0.233 = 23.3\%$$

#### 20.1.5

(a)

$$\begin{aligned} e_{\text{Carnot}} &= 1 - \frac{T_C}{T_H} \\ &= 1 + \frac{Q_C}{Q_H} \\ &= 1 + \frac{W - Q_H}{Q_H} \\ &= 0.21 \end{aligned}$$

(b)

$$|Q_C| = |Q_H| - |W| = 6.32 \times 10^4 \text{ J}$$

(c)

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H} \Rightarrow T_H = -\frac{Q_H}{Q_C}T_C = 377 \text{ K} = 104^\circ\text{C}$$

### 20.1.6

(a)

$$e = 1 - \frac{T_C}{T_H} = 0.6$$



(b)

$$n = 0.200 \text{ mol}$$

$$\gamma = 1.40$$

$$T_H = 227^\circ\text{C} = 500 \text{ K}$$

$$T_C = -73^\circ\text{C} = 200 \text{ K}$$

$$p_a = 10.0 \times 10^5 \text{ Pa}$$

$$V_a = \frac{nRT_H}{p_a}$$
$$= 8.31 \times 10^{-4} \text{ m}^3$$

$$V_b = 2V_a$$
$$= 1.66 \times 10^{-3} \text{ m}^3$$

$$p_b = \frac{nRT_H}{V_b}$$
$$= 5.01 \times 10^5 \text{ Pa}$$

$$W_{ab} = \int_{V_a}^{V_b} p dV$$
$$= nRT_H \ln 2$$
$$= 576 \text{ J}$$

$$V_c = \left(\frac{T_H}{T_C}\right)^{1/(\gamma-1)} V_b$$
$$= 1.64 \times 10^{-2} \text{ m}^3$$

$$p_c = \frac{nrT_C}{V_c}$$
$$= 2.03 \times 10^4 \text{ Pa}$$

$$W_{bc} = \frac{1}{\gamma - 1}(p_b V_b - p_c V_c)$$
$$= 1.25 \text{ kJ}$$

$$V_d = \frac{1}{2} V_c$$
$$= 8.20 \times 10^{-3} \text{ m}^3$$

$$p_d = 4.06 \times 10^4 \text{ Pa}$$

$$W_{cd} = \int_{V_c}^{V_d} p dV$$
$$= nRT_C \ln \frac{1}{2}$$
$$= -231 \text{ J}$$

$$W_{da} = \frac{1}{\gamma - 1}(p_d V_d - p_a V_a)$$
$$= -1.25 \text{ kJ}$$

**20.1.7**

(a)

$$K = \frac{T_C}{T_H - T_C} = 7.52$$

(b)

$$W = \frac{Q_C}{K} = 5.32 \times 10^5 \text{ J}$$

**20.1.8**

(a)

$$\begin{aligned} W &= \int_{V_a}^{V_b} p dV \\ &= \int_{V_a}^{2V_a} \frac{nRT_H}{V} dV \\ &= nRT_H \ln 2 \end{aligned}$$

(b)

$$\begin{aligned} W &= nC_V(T_H - T_C) \\ &= \frac{3}{2}nR(T_H - T_C) \end{aligned}$$

(c)

$$\begin{aligned} nRT_H \ln 2 &= \frac{3}{2}nR(T_H - T_C) \\ \ln 2 &= \frac{3}{2} \left( 1 - \frac{T_C}{T_H} \right) \\ \frac{T_C}{T_H} &= 1 - \frac{2}{3} \ln 2 \\ e &= 1 - \frac{T_C}{T_H} \\ &= \frac{2}{3} \ln 2 \\ &= 0.462 \end{aligned}$$

**20.1.9**

(a)

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = 655 \text{ J/K}$$

(b)

$$\Delta S = mc \int_{159}^{351} \frac{dT}{T} = 1.92 \times 10^3 \text{ J/K}$$

(c)

$$\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = 2.43 \times 10^3 \text{ J/K}$$

### 20.1.10

(a)

$$\begin{aligned} n &= 5.00 \text{ mol} \\ V_1 &= 0.120 \text{ m}^3 \\ T_1 &= 20.0^\circ\text{C} \\ V_2 &= 0.360 \text{ m}^3 \\ T_2 &= 20.0^\circ\text{C} \\ \Delta U &= nC_V \Delta T \\ &= 0 \\ Q &= W \\ &= \int_{V_1}^{V_2} p dV \\ &= \int_{V_1}^{V_2} \frac{nRT}{V} dV \\ &= nRT \ln \frac{V_2}{V_1} \\ &= 1.34 \times 10^4 \text{ J} \\ \Delta S &= \frac{Q}{T} \\ &= 45.7 \text{ J/K} \end{aligned}$$

(b) Change in entropy is path independent, so  $\Delta S = 45.7 \text{ J/K}$ .

### 20.1.11

(a)

$$\Delta S = 0$$

(b)

$$\Delta S = \frac{Q}{T} = -150 \text{ J/K}$$

(c)

$$\Delta S = \frac{Q}{T} = 218 \text{ J/K}$$

(d)

$$\Delta S = 68 \text{ J/K}$$

The net entropy increases.

### 20.1.12

(a)

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = 1.22 \times 10^3 \text{ J/K}$$

(b)

$$\begin{aligned}\Delta S &= \int \frac{dQ}{T} \\ &= mc \int_{368}^{273} \frac{dT}{T} \\ &= -1.05 \times 10^3 \text{ J/K}\end{aligned}$$

(c)

$$\Delta S = 160 \text{ J/K}$$

The net entropy increases.

### 20.1.13

(a)

$$\begin{aligned}0 &= m_i L_f + m_w c(T - T_w) + m_i c(T - T_i) \\ T &= \frac{(m_w T_w + m_i T_i)c - m_i L_f}{(m_w + m_i)c} \\ &= 307 \text{ K} \\ &= 34.3^\circ \text{C}\end{aligned}$$

(b)

$$\begin{aligned}\Delta S_i &= \frac{Q}{T_1} + \int \frac{dQ}{T} \\ &= \frac{m_i L_f}{T_1} + m_i c \ln \frac{T_2}{T_1} \\ &= 101 \text{ J/K} \\ \Delta S_w &= m_w c \ln \frac{T_2}{T_1} \\ &= -86.0 \text{ J/K} \\ \Delta S &= 15.0 \text{ J/K}\end{aligned}$$

## 20.2 Exercises and Problems

### 20.2.5

(a)

$$\begin{aligned}p_1 V_1^\gamma &= p_2 V_2^\gamma \\p_1 &= \left(\frac{V_2}{V_1}\right)^\gamma p_2 \\&= 12.3 \text{ atm}\end{aligned}$$

(b) It enters during process  $ca$ .

$$\begin{aligned}T_a &= \frac{p_a V_a}{nR} \\&= 1.20 \times 10^3 \text{ K} \\T_c &= \frac{p_c V_c}{nR} \\&= 146 \text{ K} \\\gamma &= \frac{C_P}{C_V} \\&= \frac{C_V + R}{C_V} \\C_V \gamma &= C_V + R \\C_V(\gamma - 1) &= R \\C_V &= \frac{R}{\gamma - 1} \\Q &= nC_V \Delta T \\&= 5.48 \text{ kJ}\end{aligned}$$

(c) It leaves during  $bc$ .

$$\begin{aligned}T_b &= \frac{p_b V_b}{nR} \\&= 656 \text{ K} \\\Delta U &= Q - W \\Q &= \Delta U + W \\&= nC_V \Delta T + p_b(V_c - V_b) \\&= -3.71 \text{ kJ}\end{aligned}$$

(d)

$$\begin{aligned} W &= W_{ab} + W_{bc} + W_{ca} \\ &= nC_V(T_a - T_b) + p_b(V_c - V_b) + 0 \\ &= 1.77 \text{ kJ} \end{aligned}$$

(e)

$$e = \frac{W}{Q_H} = 0.323 = 32.3\%$$

### 20.2.7

(a)

$$\begin{aligned} e &= 1 - \frac{1}{r^{\gamma-1}} \\ r^{\gamma-1} &= \frac{1}{1-e} \\ r &= \left( \frac{1}{1-e} \right)^{1/(\gamma-1)} \\ &= 12 \end{aligned}$$

(b)

$$\begin{aligned} Q_C &= 7.4 \text{ kJ} \\ Q_H &= 20 \text{ kJ} \end{aligned}$$

### 20.2.9

(a)

$$e = 1 - \frac{1}{r^{\gamma-1}} = 0.581 = 58.1\%$$

(b)

$$\begin{aligned} e' &= 0.595 = 59.5\% \\ \Delta e &= 0.014 \\ &= 1.4\% \end{aligned}$$

**20.2.11**

$$\begin{aligned}
K &= 2.25 \\
W &= 135 \text{ W} \\
H &= KW \\
&= 304 \text{ W} \\
Q &= mc\Delta T \\
&= 1.30 \times 10^6 \text{ J} \\
\frac{Q}{H} &= 4.29 \times 10^3 \text{ s} \\
&= 1.2 \text{ h}
\end{aligned}$$

**20.2.13**

$$\begin{aligned}
T_H &= T_C + 72.0 \text{ C}^\circ \\
e &= 1 - \frac{T_C}{T_H} \\
&= 1 - \frac{T_H - 72.0 \text{ C}^\circ}{T_H} \\
&= 1 - 1 + \frac{72 \text{ C}^\circ}{T_H} \\
T_H &= \frac{72 \text{ C}^\circ}{e} \\
&= 576 \text{ K} \\
T_C &= 504 \text{ K}
\end{aligned}$$

**20.2.21**

$$\begin{aligned}
\frac{T_{CA}}{T_{HA} - T_{CA}} &= 1.16 \frac{T_{CB}}{T_{HB} - T_{CB}} \\
(T_{HB} - T_{CB}) &= 1.3(T_{HA} - T_{CA}) \\
T_{CB} &= 180 \text{ K} \\
\frac{T_{CA}}{T_{HA} - T_{CA}} &= 1.16 \frac{T_{CB}}{1.3(T_{HA} - T_{CA})} \\
T_{CA} &= \frac{1.16}{1.3} T_{CB} \\
&= 161 \text{ K}
\end{aligned}$$

**20.2.29**

$$\begin{aligned}
\Delta S &= S' - S \\
&= k \ln w' - k \ln w \\
&= k \ln \frac{w'}{w} \\
&= k \ln \frac{(425/0.0024)^N w}{w} \\
&= k \ln (1.77 \times 10^5)^{n N_A} \\
&= 10.0 \text{ J/K}
\end{aligned}$$

**20.2.31**

(a)

$$\begin{aligned}
\frac{Q_C}{Q_H} &= -\frac{T_C}{T_H} \\
Q_C &= -\frac{T_C}{T_H} Q_H \\
&= -121 \text{ J}
\end{aligned}$$

(b)

$$\begin{aligned}
n &= \frac{U}{W} \\
&= \frac{mgh}{Q_H + Q_C} \\
&= 3.80 \times 10^3
\end{aligned}$$

**20.2.33**

(a)

$$W = eQ_H = 90.2 \text{ J}$$

(b)

$$Q_C = Q_H - W = 320 \text{ J}$$

(c)

$$T_C = T_H(1 - e) = 318 \text{ K} = 45^\circ \text{C}$$

(d) 0

(e)

$$m = \frac{W}{gh} = 0.263 \text{ kg}$$



**20.2.37**

(a)

$$\begin{aligned}T_A &= \frac{p_A V_A}{nR} \\&= 241 \text{ K} \\T_B &= 241 \text{ K}\end{aligned}$$

(b) Absorbed during  $bc$ , rejected during  $ab$  and  $ca$ .

(c)

$$T_C = 481 \text{ K}$$

(d)

$$\begin{aligned}W_{AB} &= \int_{V_A}^{V_B} p dV \\&= nRT_A \ln \frac{V_B}{V_A} \\&= -1389 \text{ J} \\Q_{AB} &= -1389 \text{ J} \\W_{BC} &= p_B(V_C - V_B) \\&= 2000 \text{ J} \\\Delta U_{BC} &= Q_{BC} - W_{BC} \\Q_{BC} &= \Delta U_{BC} + W_{BC} \\&= nC_V \Delta T + W_{BC} \\&= nC_V(T_C - T_B) + W_{BC} \\&= \frac{5}{2}nR(T_C - T_B) + W_{BC} \\&= 6988 \text{ J} \\W_{CA} &= 0 \\Q_{CA} &= nC_V \Delta T \\&= nC_V(T_A - T_C) \\&= \frac{5}{2}nR(T_A - T_C) \\&= -4988 \text{ J} \\Q_{\text{net}} &= 611 \text{ J} \\W_{\text{net}} &= 611 \text{ J}\end{aligned}$$

(e)

$$e = \frac{W}{Q_H} = 0.0874 = 8.7\%$$

**20.2.41**

(a)

$$e = 1 - \frac{T_C}{T_H} = 7.0\%$$

(b)

$$\begin{aligned}P &= eQ_H \\Q_H &= \frac{P}{e} \\&= 3.0 \text{ MW} \\Q_C &= Q_H(1 - e) \\&= 2.8 \text{ MW}\end{aligned}$$

(c)

$$\begin{aligned}Q_C &= mc\Delta T \\m &= \frac{Q_C}{c\Delta T} \\&= 167 \text{ kg/s} \\&= 6.0 \times 10^5 \text{ kg/h} \\&= 6.0 \times 10^5 \text{ L/h}\end{aligned}$$

20.2.45

$$\begin{aligned}
e &= 1 - \frac{T_C}{T_H} \\
e' &= 1 - \frac{T'}{T_H} \\
W' &= Q_H e' \\
&= Q_H \left( 1 - \frac{T'}{T_H} \right) \\
e'' &= 1 - \frac{T_C}{T'} \\
W'' &= Q'_H e'' \\
&= (Q_H - W') e'' \\
&= \left[ Q_H - Q_H \left( 1 - \frac{T'}{T_H} \right) \right] \left( 1 - \frac{T_C}{T'} \right) \\
&= Q_H \left( 1 - 1 + \frac{T'}{T_H} \right) \left( 1 - \frac{T_C}{T'} \right) \\
&= Q_H \frac{T'}{T_H} \left( 1 - \frac{T_C}{T'} \right) \\
e_{\text{Total}} &= \frac{W' + W''}{Q_H} \\
&= 1 - \frac{T'}{T_H} + \frac{T'}{T_H} \left( 1 - \frac{T_C}{T'} \right) \\
&= 1 - \frac{T_C}{T_H}
\end{aligned}$$

The efficiency is the same.

**20.2.49**

$$\begin{aligned}
 T_A &= 300 \text{ K} \\
 W_{AB} &= nRT_A \ln \frac{V_B}{V_A} \\
 &= 2553 \text{ J} \\
 Q_{AB} &= 2553 \text{ J} \\
 T_B &= 300 \text{ K} \\
 T_C &= 1000 \text{ K} \\
 W_{BC} &= 0 \\
 Q_{BC} &= nC_V(T_C - T_B) \\
 &= \frac{5}{2}nR(T_C - T_B) \\
 &= 1.24 \times 10^4 \text{ J} \\
 W_{CA} &= p_A(V_A - V_C) \\
 &= -4949 \text{ J} \\
 \Delta U_{CA} &= Q_{CA} - W_{CA} \\
 Q_{CA} &= \Delta U_{CA} + W_{CA} \\
 &= nC_V(T_A - T_C) + W_{CA} \\
 &= \frac{5}{2}nR(T_A - T_C) + W_{CA} \\
 &= -1.73 \times 10^4 \text{ J} \\
 W &= -2396 \text{ J} \\
 K &= \frac{|Q_C|}{|W|} \\
 &= 6.24
 \end{aligned}$$

**20.2.51**

(a)

$$\begin{aligned}
 \Delta S &= \int \frac{dQ}{T} \\
 &= m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \frac{T'}{T_2} \\
 0 &= m_1 c_1 (T - T_1) + m_2 c_2 (T' - T_2) \\
 m_1 c_1 (T - T_1) &= m_2 c_2 (T_2 - T')
 \end{aligned}$$

(b)

$$\begin{aligned}T' &= T_2 - \frac{m_1 c_1}{m_2 c_2} (T - T_1) \\ \Delta S &= m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \left( \frac{T_2 - \frac{m_1 c_1}{m_2 c_2} (T - T_1)}{T_2} \right) \\ &= m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \left( 1 - \frac{m_1 c_1}{m_2 c_2} \frac{T - T_1}{T_2} \right) \\ \frac{d}{dT} \Delta S &= \frac{m_1 c_1}{T} - m_2 c_2 \frac{m_1 c_1}{m_1 c_1 (T_1 - T) + m_2 c_2 T_2} \\ 0 &= \frac{1}{T} - \frac{m_2 c_2}{m_1 c_1 (T_1 - T) + m_2 c_2 T_2} \\ T &= \frac{m_1 c_1 (T_1 - T) + m_2 c_2 T_2}{m_2 c_2} \\ &= \frac{m_1 c_1}{m_2 c_2} (T_1 - T) + T_2 \\ &= T'\end{aligned}$$

**20.2.55**

**20.2.57**

(a)

$$r_b = 8.0 \text{ cm}$$

$$V_b = \frac{4}{3}\pi r_b^3$$
$$= 2.14 \times 10^{-3} \text{ m}^3$$

$$V_a = 2V_b$$
$$= 4.29 \times 10^{-3} \text{ m}^3$$

$$r_a = \sqrt[3]{\frac{3V_a}{4\pi}}$$
$$= 10 \text{ cm}$$

$$r_d = 3r_a$$
$$= 30 \text{ cm}$$

$$p_a V_a^\gamma = p_b V_b^\gamma$$
$$p_a = \left(\frac{V_b}{V_a}\right)^\gamma p_b$$
$$= 8.24 \text{ kPa}$$

$$\frac{p_b V_b}{T_b} = \frac{p_a V_a}{T_a}$$
$$T_b = \frac{p_b V_b}{p_a V_a} T_a$$
$$= 152 \text{ K}$$

(b)

$$n = \frac{p_b V_b}{RT_b}$$
$$= 3.44 \times 10^{-2} \text{ mol}$$

$$V_d = \frac{4}{3}\pi r_d^3$$
$$= 0.113 \text{ m}^3$$

$$p_d = \frac{nRT_d}{V_d}$$
$$= 311 \text{ Pa}$$

(c)

$$\begin{aligned}e &= 1 - \frac{T_C}{T_H} \\&= 0.191 \\&= 19.1\% \\W &= eQ_H \\Q_H &= \frac{W}{e} \\&= 5.24 \text{ kJ}\end{aligned}$$

(d)

$$\begin{aligned}Q_C &= (1 - e)Q_H \\&= 4.24 \text{ kJ}\end{aligned}$$

**20.2.59**

$$\begin{aligned}e &= 1 - \frac{T_C}{T_H} \\T_C &= (1 - e)T_H \\&= 281 \text{ K} \\&= 7.5^\circ \text{C}\end{aligned}$$

b

**20.2.61**

d

## 37 Relativity

### 37.1 Guided Practice

#### 37.1.1

- (a) In the laboratory frame,  $v_1 = \alpha c$  and  $v_2 = -\alpha c$ . In the frame of the first proton,  $v'_1 = 0$  and  $v'_2 = -\frac{1}{2}c$ . Using the Lorentz velocity transformation:

$$\begin{aligned}
v_2' &= \frac{v_2 - v_1}{1 - v_1 v_2 / c^2} \\
-\frac{1}{2}c &= \frac{-\alpha c - \alpha c}{1 + \alpha^2 c^2 / c^2} \\
\frac{1}{2}c &= \frac{2\alpha c}{1 + \alpha^2} \\
\alpha^2 - 4\alpha + 1 &= 0 \\
\alpha &= \frac{4 \pm \sqrt{16 - 4}}{2} \\
&= 2 \pm \sqrt{3}
\end{aligned}$$

$\alpha$  can't be greater than 1, so  $\alpha = 2 - \sqrt{3} \approx 0.268$ .

(b)

$$\begin{aligned}
K &= (\gamma - 1)mc^2 \\
&= \left( \frac{1}{\sqrt{1 - (0.268c)^2 / c^2}} - 1 \right) mc^2 \\
&= 5.71 \times 10^{-12} \text{ J} \\
&= 35.6 \text{ MeV}
\end{aligned}$$

(c)

$$\begin{aligned}
K &= \left( \frac{1}{\sqrt{1 - (0.5c)^2 / c^2}} - 1 \right) mc^2 \\
&= 2.33 \times 10^{-11} \text{ J} \\
&= 145 \text{ MeV}
\end{aligned}$$



## 37.2 Exercises and Problems

### 37.2.1

$$\begin{aligned}x'_1 &= -d \\t'_1 &= 0 \\x'_2 &= d \\t'_2 &= 0 \\x_1 &= \gamma(x'_1 + ut'_1) \\&= -\gamma d \\t_1 &= \gamma(t'_1 + ux'_1/c^2) \\&= -\gamma ud/c^2 \\x_2 &= \gamma(x'_2 + ut'_2) \\&= \gamma d \\t_2 &= \gamma(t'_2 + ux'_2/c^2) \\&= \gamma ud/c^2\end{aligned}$$

The observer measures the lightning strike at point  $A$  to come first.

### 37.2.3

$$\begin{aligned}2 &= \gamma \\&= \frac{1}{\sqrt{1 - v^2/c^2}} \\2\sqrt{1 - v^2/c^2} &= 1 \\1 - \frac{v^2}{c^2} &= \frac{1}{4} \\\frac{v^2}{c^2} &= \frac{3}{4} \\v &= \frac{\sqrt{3}}{2}c \\&\approx 0.866c \\&\approx 2.60 \times 10^8 \text{ m/s}\end{aligned}$$

Present day jet planes don't reach this speed.

**37.2.5**

(a)

$$\begin{aligned}
\Delta t &= \gamma \Delta t_0 \\
\frac{\Delta t}{\Delta t_0} &= \frac{1}{\sqrt{1 - v^2/c^2}} \\
\frac{\Delta t_0}{\Delta t} &= \sqrt{1 - v^2/c^2} \\
\left(\frac{\Delta t_0}{\Delta t}\right)^2 &= 1 - \frac{v^2}{c^2} \\
\frac{v^2}{c^2} &= 1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2 \\
v &= \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} c \\
&= 0.998c
\end{aligned}$$

(b)

$$d = \Delta t v = 126 \text{ m}$$

**37.2.7**

$$\begin{aligned}
\Delta t - \Delta t_0 &= \Delta t \left(1 - \frac{1}{\gamma}\right) \\
&= \Delta t (1 - \sqrt{1 - v^2/c^2}) \\
&= 9.15 \text{ h}
\end{aligned}$$

The clock on the spacecraft measured a shorter elapsed time.

**37.2.9**

$$\begin{aligned}
l &= \frac{l_0}{\gamma} \\
l_0 &= \gamma l \\
&= 103 \text{ m}
\end{aligned}$$

**37.2.11**

(a)

$$d = \Delta t_0 v = 0.66 \text{ km}$$

(b)

$$\begin{aligned}\Delta t &= \gamma \Delta t_0 \\ &= 49 \mu\text{s} \\ d &= \Delta t v \\ &= 15 \text{ km}\end{aligned}$$

(c)

$$l = \frac{l_0}{\gamma} = 0.45 \text{ km}$$

### 37.2.13

(a)

$$l = \frac{l_0}{\gamma} = 3960 \text{ m}$$

(b)

$$t = \frac{d}{v} = 95.2 \mu\text{s}$$

(c)

$$t_0 = \frac{t}{\gamma} = 94.3 \mu\text{s}$$

### 37.2.15

(a)

$$\begin{aligned}v &= \frac{v' + u}{1 + uv'/c^2} \\ &= \frac{0.380c + 0.580c}{1 + (0.580c)(0.380c)/c^2} \\ &= \frac{(0.380 + 0.580)c}{1 + (0.580)(0.380)} \\ &= 0.787c\end{aligned}$$

(b)

$$v = 0.949c$$

(c)

$$v = 0.997c$$

### 37.2.17

(a) Toward

(b)

$$v' = \frac{v - u}{1 - uv/c^2} = \frac{0.650c - 0.830c}{1 - (0.830c)(0.650c)/c^2} = -0.391c$$

**37.2.19**

$$\begin{aligned}
v &= \frac{v' + u}{1 + uv'/c^2} \\
&= \frac{0.950c - 0.650c}{1 + (-0.650c)(0.950c)/c^2} \\
&= 0.784c
\end{aligned}$$

**37.2.21**

$$\begin{aligned}
v &= \frac{v' + u}{1 + uv'/c^2} \\
-\alpha c &= \frac{-0.890c + \alpha c}{1 + (\alpha c)(-0.890c)/c^2} \\
&= \frac{(\alpha - 0.890)c}{1 - 0.890\alpha} \\
-\alpha(1 - 0.890\alpha) &= (\alpha - 0.890) \\
0.890\alpha^2 - 2\alpha + 0.890 &= 0 \\
\alpha &= \frac{2 \pm \sqrt{4 - 3.1684}}{1.78} \\
&= 0.611
\end{aligned}$$

The particles are travelling at  $0.611c$ .

**37.2.23**

(a)

$$f_0 = 4.44 \times 10^{14} \text{ Hz}$$

$$f = 5.22 \times 10^{14} \text{ Hz}$$

$$f = \sqrt{\frac{c+u}{c-u}} f_0$$

$$\left(\frac{f}{f_0}\right)^2 = \frac{c+u}{c-u}$$

$$(c-u) \left(\frac{f}{f_0}\right)^2 = c+u$$

$$c \left[ \left(\frac{f}{f_0}\right)^2 - 1 \right] = u \left[ \left(\frac{f}{f_0}\right)^2 + 1 \right]$$

$$u = \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1} c$$

$$\left(\frac{f}{f_0}\right)^2 = 1.38$$

$$u = 0.160c$$

(b) \$173 million

**37.2.25**

$$1.20 = \sqrt{\frac{c+u}{c-u}}$$

$$1.44 = \frac{c+u}{c-u}$$

$$1.44(c-u) = c+u$$

$$c(1.44-1) = (1.44+1)u$$

$$u = \frac{0.44}{2.44} c$$

$$= 0.18c$$

Toward

**37.2.27**

$$\begin{aligned} \frac{\gamma_{0.800} 0.800c}{\gamma_{0.400} 0.400c} &= 2 \sqrt{\frac{1 - (0.400c)^2/c^2}{1 - (0.800c)^2/c^2}} \\ &= 3.06 \end{aligned}$$

**37.2.29**

(a)

$$\begin{aligned}
2 &= \gamma \\
&= \frac{1}{\sqrt{1 - v^2/c^2}} \\
1 - \frac{v^2}{c^2} &= \frac{1}{4} \\
v &= \frac{\sqrt{3}}{2}c \\
&= 0.866c
\end{aligned}$$

(b)

$$\begin{aligned}
2 &= \gamma^3 \\
&= \frac{1}{(1 - v^2/c^2)^{3/2}} \\
1 - \frac{v^2}{c^2} &= \left(\frac{1}{2}\right)^{2/3} \\
v &= \sqrt{1 - \left(\frac{1}{2}\right)^{2/3}} c \\
&= 0.608c
\end{aligned}$$

**37.2.31**

(a)

$$0.866c$$

(b)

$$\begin{aligned}
6 &= \gamma \\
&= \frac{1}{\sqrt{1 - v^2/c^2}} \\
\frac{1}{36} &= 1 - \frac{v^2}{c^2} \\
v &= \sqrt{1 - \frac{1}{36}} c \\
&= 0.986c
\end{aligned}$$

**37.2.33**

(a)

$$K = 3mc^2 = 4.51 \times 10^{-10} \text{ J}$$

(b)

$$\begin{aligned}E^2 &= (mc^2)^2 + (pc)^2 \\p &= \frac{\sqrt{E^2 - (mc^2)^2}}{c} \\&= \frac{\sqrt{(4mc^2)^2 - (mc^2)^2}}{c} \\&= \frac{\sqrt{15(mc^2)^2}}{c} \\&= \sqrt{15}mc \\&= 1.94 \times 10^{-18} \text{ kg m/s}\end{aligned}$$

(c)

$$p = \gamma mv \Rightarrow v = \frac{p}{\gamma m} = 2.90 \times 10^8 \text{ m/s} = 0.968c$$

### 37.2.35

(a)

$$E = mc^2 \Rightarrow m = \frac{E}{c^2} = 1.11 \times 10^3 \text{ kg}$$

(b)

$$\begin{aligned}\rho &= 7860 \text{ kg/m}^3 \\ \rho V &= m \\ \rho s^3 &= m \\ s &= \sqrt[3]{\frac{m}{\rho}} \\ &= 0.521 \text{ m}\end{aligned}$$

### 37.2.41

(a)

$$\begin{aligned}qV &= K \\ V &= \frac{(\gamma - 1)mc^2}{q} \\ &= 2.06 \times 10^6 \text{ V}\end{aligned}$$

(b)

$$K = 3.30 \times 10^{-13} \text{ J} = 2.06 \times 10^6 \text{ eV}$$

**37.2.43**

(a)

$$\begin{aligned}
 l &= v\Delta t \\
 &= v\gamma\Delta t_0 \\
 &= \frac{v\Delta t_0}{\sqrt{1-v^2/c^2}} \\
 l\sqrt{1-v^2/c^2} &= v\Delta t_0 \\
 l^2(1-v^2/c^2) &= v^2\Delta t_0^2 \\
 v^2 &= \frac{l^2}{\Delta t_0^2}(1-v^2/c^2) \\
 v^2\left(1 + \frac{l^2}{\Delta t_0^2 c^2}\right) &= \frac{l^2}{\Delta t_0^2} \\
 (1-\Delta)c &= \sqrt{\frac{l^2/\Delta t_0^2}{1+l^2/\Delta t_0^2 c^2}} \\
 \Delta &= 1 - \frac{1}{c}\sqrt{\frac{l^2/\Delta t_0^2}{1+l^2/\Delta t_0^2 c^2}} \\
 &= 8.43 \times 10^{-6}
 \end{aligned}$$

(b)

$$\begin{aligned}
 E &= \gamma mc^2 \\
 &= 5.45 \times 10^{-9} \text{ J} \\
 &= 3.40 \times 10^{10} \text{ eV}
 \end{aligned}$$

**37.2.45**

$$\begin{aligned}
 1.4 &= \gamma \\
 &= \frac{1}{\sqrt{1-v^2/c^2}} \\
 \frac{1}{1.4} &= \sqrt{1-v^2/c^2} \\
 \frac{1}{1.96} &= 1 - \frac{v^2}{c^2} \\
 v &= \sqrt{1 - \frac{1}{1.96}}c \\
 &= 0.700c
 \end{aligned}$$



**37.2.47**

$$\begin{aligned}
\Delta t &= 42.5 \text{ y} \\
\Delta t_0 &= \frac{\Delta t}{\gamma} \\
&= \Delta t \sqrt{1 - v^2/c^2} \\
&= 5.02 \text{ y}
\end{aligned}$$

Her biological age will be  $19 + 5 = 24$ .

**37.2.51**

$$\begin{aligned}
\Delta t - \Delta t_0 &= \Delta t - \frac{\Delta t}{\gamma} \\
&= \Delta t \left( 1 - \frac{1}{\gamma} \right) \\
&= \Delta t (1 - \sqrt{1 - v^2/c^2}) \\
&= 8.41 \text{ ns}
\end{aligned}$$

The clock that was on the airliner will show a shorter elapsed time.

**37.2.53**

$$\begin{aligned}
v &\geq \frac{c}{n} \\
&\geq 1.97 \times 10^8 \text{ m/s} \\
K &\geq 2.70 \times 10^{-14} \text{ J} \\
&\geq 167 \text{ keV}
\end{aligned}$$

**37.2.57**

(a)

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.9995c + 0.7500c}{1 + (0.7500c)(0.9995c)/c^2} = 0.9999c$$

(b)

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{-0.9995c + 0.7500c}{1 + (0.7500c)(-0.9995c)/c^2} = -0.9965c$$

**37.2.61**

$$\begin{aligned}
 \frac{\Delta f}{f_0} &= \frac{f'' - f_0}{f_0} \\
 &= \sqrt{\frac{c+u}{c-u}} \frac{f'}{f_0} - 1 \\
 &= \frac{c+u}{c-u} - 1 \\
 (c-u) \left( 1 + \frac{\Delta f}{f_0} \right) &= c+u \\
 c \frac{\Delta f}{f_0} &= u \left( 2 + \frac{\Delta f}{f_0} \right) \\
 u &= \frac{\Delta f / f_0}{2 + \Delta f / f_0} c \\
 &= 43 \text{ m/s} \\
 &= 154 \text{ k/m}
 \end{aligned}$$

**37.2.63**

$$\begin{aligned}
 F &= \gamma m a \\
 &= \frac{1}{\sqrt{1 - v^2/c^2}} m \frac{v^2}{r} \\
 &= \frac{mv^2}{r \sqrt{1 - v^2/c^2}} \\
 &= 2.04 \times 10^{-13} \text{ N}
 \end{aligned}$$

**37.2.65**

(a)

$$\begin{aligned}
 \Delta t &= \gamma \Delta t_0 \\
 &= \frac{\Delta t_0}{\sqrt{1 - u^2/v^2}} \\
 \Delta t^2 &= \frac{\Delta t_0^2}{1 - u^2/c^2} \\
 \Delta t_0 &\approx 2.59 \times 10^{-8} \text{ s}
 \end{aligned}$$

(b)

$$\begin{aligned}(4\Delta t_0)^2 &= \frac{\Delta t_0^2}{1 - u^2/c^2} \\ \frac{1}{16} &= 1 - \frac{u^2}{c^2} \\ \frac{u}{c} &= \sqrt{1 - \frac{1}{16}} \\ &= 0.968\end{aligned}$$

**37.2.71**

c

**37.2.73**

b

## 38 Photons: Light Waves Behaving as Particles

### 38.1 Guided Practice

#### 38.1.1

(a)

$$\begin{aligned}\lambda' - \lambda &= \frac{h}{mc}(1 - \cos \phi) \\ \lambda &= \lambda' - 2\frac{h}{mc} \\ &= 0.078 \text{ nm}\end{aligned}$$

(b)

$$\begin{aligned}p_p &= p_e - p'_p \\ \frac{h}{\lambda} &= p_e - \frac{h}{\lambda'} \\ p_e &= h \left( \frac{1}{\lambda} + \frac{1}{\lambda'} \right) \\ &= 1.65 \times 10^{-23} \text{ kg m/s} \\ p_e &= m_e v \\ v &= \frac{p_e}{m_e} \\ &= 1.81 \times 10^7 \text{ m/s}\end{aligned}$$

(c)

$$K = \frac{1}{2} m_e v^2 = 1.49 \times 10^{-16} \text{ J} = 933 \text{ eV}$$

## 38.2 Exercises and Problems

### 38.2.1

$$\begin{aligned}\lambda &= 520 \text{ nm} \\ f &= \frac{c}{\lambda} \\ &= 5.77 \times 10^{14} \text{ Hz} \\ p &= \frac{h}{\lambda} \\ &= 1.27 \times 10^{-27} \text{ kg m/s} \\ E &= \frac{hc}{\lambda} \\ &= 3.82 \times 10^{-19} \text{ J} \\ &= 2.39 \text{ eV}\end{aligned}$$

### 38.2.3

(a)

$$f = \frac{c}{\lambda} = 5.36 \times 10^{14} \text{ Hz}$$

(b)

$$\begin{aligned}E &= \frac{hc}{\lambda} \\ &= 3.35 \times 10^{-19} \text{ J} \\ n &= \frac{P}{E} \\ &= 2.24 \times 10^{20}\end{aligned}$$

(c) No

### 38.2.5

(a)

$$E = pc = 2.40 \times 10^{-19} \text{ J} = 1.50 \text{ eV}$$

(b)

$$\lambda = \frac{hc}{E} = 828 \text{ nm}$$

Infrared

**38.2.7**

$$\lambda = 206 \text{ nm}$$

$$f = \frac{c}{\lambda}$$

$$= 1.46 \times 10^{15} \text{ Hz}$$

$$\phi = 5.1 \text{ eV}$$

$$\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$$

$$v_{\text{max}} = \sqrt{\frac{2(hf - \phi)}{m}}$$

$$= 5.77 \times 10^5 \text{ m/s}$$

**38.2.9**

$$\lambda_1 = 400.0 \text{ nm}$$

$$f_1 = 7.50 \times 10^{14} \text{ Hz}$$

$$K_1 = 1.10 \text{ eV}$$

$$K_1 = hf - \phi$$

$$\phi = hf - K_1$$

$$= 2.94 \times 10^{-19} \text{ J}$$

$$= 1.83 \text{ eV}$$

$$\lambda_2 = 300.0 \text{ nm}$$

$$f_2 = 1.00 \times 10^{15} \text{ Hz}$$

$$K_2 = hf - \phi$$

$$= 3.67 \times 10^{-19} \text{ J}$$

$$= 2.3 \text{ eV}$$

**38.2.11**

(a)

$$\lambda = 254 \text{ nm}$$

$$f = 1.18 \times 10^{15} \text{ Hz}$$

$$V_0 = 0.181 \text{ V}$$

$$eV_0 = hf - \phi$$

$$\phi = hf - eV_0$$

$$= 7.53 \times 10^{-19} \text{ J}$$

$$= 4.71 \text{ eV}$$

$$E = \phi$$

$$\frac{hc}{\lambda} = \phi$$

$$\lambda = \frac{hc}{\phi}$$

$$= 264 \text{ nm}$$

(b)

$$\phi = 4.71 \text{ eV}$$

Same

**38.2.13**

$$K = E$$

$$eV = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{eV}$$

$$= 345 \text{ pm}$$

**38.2.15**

$$K = E' - E$$

$$= hc \left( \frac{1}{\lambda'} - \frac{1}{\lambda} \right)$$

$$= 3.17 \times 10^{-16} \text{ J}$$

$$= 1.98 \text{ keV}$$

**38.2.17**

$$\begin{aligned}\lambda' - \lambda &= \frac{h}{mc}(1 - \cos \phi) \\ \lambda' &= \lambda + \frac{h}{mc}(1 - \cos \phi) \\ \lambda'_{\max} &= \lambda + 2\frac{h}{mc} \\ &= 0.665 \text{ nm}\end{aligned}$$

Occurs at  $\phi = \pi$ .

**38.2.19**

(a)

$$\begin{aligned}\lambda &= 0.0430 \text{ nm} \\ \phi &= 32.0^\circ \\ \Delta\lambda &= \frac{h}{mc}(1 - \cos \phi) \\ &= 3.69 \times 10^{-13} \text{ m}\end{aligned}$$

(b)

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos \phi) = 0.0434 \text{ nm}$$

(c)

$$\Delta E = hc \left( \frac{1}{\lambda'} - \frac{1}{\lambda} \right) = -4.26 \times 10^{-17} = -266 \text{ eV}$$

(d)

$$-\Delta E$$

**38.2.21**

$$\begin{aligned}0.01\lambda &= \frac{h}{mc}(1 - \cos \phi) \\ \phi &= \arccos \left( 1 - \frac{0.01\lambda}{h/mc} \right) \\ &= 51.0^\circ\end{aligned}$$

**38.2.23**

$$\Delta t = 7.20 \text{ fs}$$

$$\lambda = 522 \text{ nm}$$

$$p = \frac{h}{\lambda}$$

$$= 1.27 \times 10^{-27} \text{ kg m/s}$$

$$\Delta x = c\Delta t$$

$$= 2.16 \mu\text{m}$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{\hbar}{2\Delta x}$$

$$\geq 2.4 \times 10^{-29} \text{ kg m/s}$$

**38.2.25**

$$\lambda = 620 \text{ nm}$$

$$E = \frac{hc}{\lambda}$$

$$= 3.21 \times 10^{-19} \text{ J}$$

$$\Delta E = 3.21 \times 10^{-21} \text{ J}$$

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

$$\Delta t \geq \frac{\hbar}{2\Delta E}$$

$$\geq 1.64 \times 10^{-14} \text{ s}$$

$$\geq 16.4 \text{ fs}$$



**38.2.27**

(a)

$$\begin{aligned}
\lambda &= 585 \text{ nm} \\
\Delta t &= 450 \text{ } \mu\text{s} \\
c &= 4190 \text{ J/kg K} \\
L_v &= 2.256 \times 10^6 \text{ J/kg} \\
m &= 2.0 \text{ } \mu\text{g} \\
&= 2.0 \times 10^{-9} \text{ kg} \\
T_0 &= 33 \text{ }^\circ\text{C} \\
&= 306 \text{ K} \\
E &= mc(373 - T_0) + mL_v \\
&= 5.07 \text{ mJ}
\end{aligned}$$

(b)

$$P = E/\Delta t = 11.3 \text{ W}$$

(c)

$$\begin{aligned}
E_p &= \frac{hc}{\lambda} \\
&= 3.40 \times 10^{-19} \text{ J} \\
n &= \frac{E}{E_p} \\
&= 1.49 \times 10^{16}
\end{aligned}$$

**38.2.29**

(a)

$$\begin{aligned}
\lambda &= 0.0930 \text{ nm} \\
\phi &= 180^\circ \\
\lambda' &= \lambda + \frac{h}{mc}(1 - \cos \phi) \\
&= 0.0978 \text{ nm} \\
p &= \frac{h}{\lambda} \\
&= 6.77 \times 10^{-24} \text{ kg m/s}
\end{aligned}$$

(b)

$$\begin{aligned}K &= E - E' \\&= hc \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) \\&= 1.05 \times 10^{-16} \text{ J} \\&= 656 \text{ eV}\end{aligned}$$

**38.2.31**

$$\lambda = 0.1360 \text{ nm}$$

$$\phi = 60.0^\circ$$

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos \phi)$$

$$= \lambda + \frac{1}{2} \frac{h}{mc}$$

$$= 0.1372 \text{ nm}$$

$$K = E - E'$$

$$= hc \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right)$$

$$= 1.278 \times 10^{-17} \text{ J}$$

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}}$$

$$= 5.300 \times 10^6 \text{ m/s}$$

$$p = mv$$

$$= 4.823 \times 10^{-24} \text{ kg m/s}$$

$$p = p' + P_x$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'} \cos \phi + P_x$$

$$P_x = h \left( \frac{1}{\lambda} - \frac{1}{2\lambda'} \right)$$

$$= 2.457 \times 10^{-24} \text{ kg m/s}$$

$$0 = p' - P_y$$

$$= \frac{h}{\lambda'} \sin \phi - P_y$$

$$P_y = \frac{\sqrt{3}h}{2\lambda'}$$

$$= 4.182 \times 10^{-24} \text{ kg m/s}$$

$$\theta = \arctan \frac{P_y}{P_x}$$

$$= 59.57^\circ$$

**38.2.33**

(a)

$$\begin{aligned}
 E_1 &= 1 \text{ MeV} \\
 \lambda_1 &= \frac{hc}{E_1} \\
 &= 0.00124 \text{ nm} \\
 \lambda_2 &= 500 \text{ nm} \\
 n &= 10^{26} \\
 \Delta\lambda &= \frac{\lambda_2 - \lambda_1}{n} \\
 &= 5 \times 10^{-33} \text{ m}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \Delta\lambda &= \frac{h}{mc}(1 - \cos\phi) \\
 &= \frac{h}{mc} \frac{\phi^2}{2} \\
 \phi &= \sqrt{\frac{2\Delta\lambda}{h/mc}} \\
 &= 6.42 \times 10^{-11} \text{ rad} \\
 &= (3.68 \times 10^{-9})^\circ
 \end{aligned}$$

**38.2.35**

(a)

$$\begin{aligned}
 K &= E - E' \\
 &= hc \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) \\
 &= 3.11 \times 10^{-17} \text{ J} \\
 &= 195 \text{ eV}
 \end{aligned}$$

(b)

$$\begin{aligned}
 K &= E \\
 &= \frac{hc}{\lambda} \\
 \lambda &= \frac{hc}{K} \\
 &= 6.37 \text{ nm}
 \end{aligned}$$

**38.2.37**

(a)

$$\begin{aligned}
 eV_0 &= hf - \phi \\
 &= h \frac{c}{\lambda} - \phi \\
 V_0 &= \frac{hc}{e} \frac{1}{\lambda} - \frac{\phi}{e} \\
 \frac{hc}{e} &= 1.232 \times 10^{-6} \text{ V m} \\
 \frac{\phi}{e} &= 4.7651 \text{ V}
 \end{aligned}$$

(b)

$$\begin{aligned}
 h &= 6.57 \times 10^{-34} \text{ J s} \\
 \phi &= 4.7651 \text{ eV}
 \end{aligned}$$

(c)

$$\begin{aligned}
 \phi &\leq E \\
 &\leq \frac{hc}{\lambda} \\
 \lambda &\leq \frac{hc}{\phi} \\
 &\leq 261 \text{ nm}
 \end{aligned}$$

(d)

$$\begin{aligned}
 K &= hf - \phi \\
 &= h \frac{c}{\lambda} - \phi \\
 \lambda &= \frac{hc}{K + \phi} \\
 &= 84.1 \text{ nm}
 \end{aligned}$$

**38.2.39**

(a)

$$\begin{aligned}
 m &= 2.404 \text{ pm} \\
 c &= 5.211 \text{ pm}
 \end{aligned}$$

(b)

$$\lambda_C = 2.404 \text{ pm}$$

(c)

$$\lambda = 5.211 \text{ pm}$$