Advanced Engineering Mathematics Complex Analysis by Dennis G. Zill Problems

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17 Functions of a Complex Variable

17.1 Complex Numbers

17.1.1

$$3 + 3i$$

17.1.3

$$i^8 = (i^2)^4 = (-1)^4 = 1$$

17.1.5

$$7 - 13i$$

17.1.7

$$-7 + 5i$$

17.1.9

$$11 - 10i$$

17.1.11

$$-5 + 12i$$

17.1.13

$$-2i$$

17.1.15

$$\begin{aligned}
\frac{2-4i}{3+5i} &= \frac{(2-4i)(3-5i)}{34} \\
&= \frac{-14-22i}{34} \\
&= -\frac{7}{17} - \frac{11}{17}i
\end{aligned}$$

17.1.17

$$\begin{aligned}
\frac{(3-i)(2+3i)}{1+i} &= \frac{9+7i}{1+i} \\
&= \frac{(9+7i)(1-i)}{2} \\
&= \frac{16-2i}{2} \\
&= 8-i
\end{aligned}$$

17.1.27

$$\begin{aligned}
\frac{1}{z} &= \frac{\overline{z}}{z\overline{z}} \\
&= \frac{x-iy}{x^2+y^2} \\
\operatorname{Re}\left(\frac{1}{z}\right) &= \frac{x}{x^2+y^2}
\end{aligned}$$

17.1.29

$$\begin{aligned}
2z+4\overline{z}-4i &= 2(x+iy)+4(x-iy)-4i \\
&= 6x-2(y+2)i \\
\operatorname{Im}(2z+4\overline{z}-4i) &= -2y-4
\end{aligned}$$

17.1.31

$$\begin{aligned}
z-1-3i &= x+iy-1-3i \\
&= (x-1)+(y-3)i \\
|z| &= \sqrt{(x-1)^2+(y-3)^2}
\end{aligned}$$

17.1.33

$$\begin{aligned}2z &= i(2+9i)\\ &= -9+2i\\ z &= -\frac{9}{2}+i\end{aligned}$$

17.1.35

$$\begin{aligned}(x+iy)^2 &= x^2+2xyi-y^2\\ &= (x^2-y^2)+2xyi\\ x^2 &= y^2\\ x &= y\\ 2xy &= 1\\ x^2 &= \frac{1}{2}\\ x &= \frac{\sqrt{2}}{2}\\ z &= \frac{\sqrt{2}}{2}(1+i)\end{aligned}$$

17.1.37

$$\begin{aligned}z+2\overline{z} &= x+iy+2x-2iy\\ &= 3x-iy\\ \frac{2-i}{1+3i} &= \frac{(2-i)(1-3i)}{10}\\ &= \frac{-1-7i}{10}\\ 3x-iy &= \frac{-1-7i}{10}\\ x &= -\frac{1}{30}\\ y &= \frac{7}{10}\\ z &= -\frac{1}{30}+\frac{7}{10}i\end{aligned}$$

17.1.39

$$\begin{aligned}|10+8i| &\approx 12.8\\ |11-6i| &\approx 12.5\end{aligned}$$

$11-6i$ is closer.

17.2 Powers and Roots

17.2.1

$$2(\cos 0 + i \sin 0)$$

17.2.3

$$-3[\cos(-\pi/2) + i \sin(-\pi/2)]$$

17.2.5

$$\sqrt{2}[\cos(\pi/4) + i \sin(\pi/4)]$$

17.2.7

$$2[\cos(5\pi/6) + i \sin(5\pi/6)]$$

17.2.9

$$\begin{aligned}
\frac{3}{-1+i} &= \frac{3(-1-i)}{2} \\
&= \frac{-3-3i}{2} \\
&= -\frac{3}{2} - \frac{3}{2}i \\
&= \frac{3\sqrt{2}}{2}[\cos(5\pi/4) + i \sin(5\pi/4)]
\end{aligned}$$

17.2.11

$$-\frac{5\sqrt{3}}{2} - \frac{5}{2}i$$

17.2.13

$$5.54 + 2.30i$$

17.2.15

$$\begin{aligned}
8[\cos(\pi/2) + i \sin(\pi/2)] &= 8i \\
\frac{1}{2}[\cos(-\pi/4) + i \sin(-\pi/4)] &= \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4}i
\end{aligned}$$

17.2.21

$$\begin{aligned}(1+\sqrt{3}i)^9 &= \{2[\cos(\pi/3)+i\sin(\pi/3)]\}^9 \\ &= 512(\cos\pi + i\sin\pi) \\ &= -512\end{aligned}$$

17.2.23

$$\begin{aligned}\left(\frac{1}{2}+\frac{1}{2}i\right)^1 0 &= \left\{\frac{\sqrt{2}}{2}[\cos(\pi/4)+i\sin(\pi/4)]\right\}^{10} \\ &= \frac{1}{32}[\cos(\pi/2)+i\sin(\pi/2)] \\ &= \frac{1}{32}i\end{aligned}$$

17.2.27

$$\begin{aligned}w_k &= 2[\cos(2\pi k/3)+i\sin(2\pi k/3)] \\ w_0 &= 2 \\ w_1 &= -1+\sqrt{3}i \\ w_2 &= -1-\sqrt{3}i\end{aligned}$$

17.2.29

$$\begin{aligned}w_k &= \cos(\pi/4+k\pi)+i\sin(\pi/4+k\pi) \\ w_0 &= \frac{\sqrt{2}}{2}(1+i) \\ w_1 &= -\frac{\sqrt{2}}{2}(1+i)\end{aligned}$$

17.2.31

$$\begin{aligned}w_k &= \sqrt{2}[\cos(\pi/3+k\pi)+i\sin(\pi/3+k\pi)] \\ w_0 &= \frac{\sqrt{2}}{2}+\frac{\sqrt{6}}{2}i \\ w_1 &= -\frac{\sqrt{2}}{2}-\frac{\sqrt{6}}{2}i\end{aligned}$$

17.2.33

$$
\begin{aligned}
z^4 + 1 &= 0 \\
z^4 &= -1 \\
w_k &= \cos(\pi/4 + k\pi/2) + \sin(\pi/4 + k\pi/2) \\
w_0 &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \\
w_1 &= -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \\
w_2 &= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \\
w_3 &= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i
\end{aligned}
$$

17.3 Sets in the Complex Plane

17.3.1

A vertical line at $\text{Re}(z) = 5$.

17.3.3

A horizontal line at $\text{Im}(z) = -3$.

17.3.5

A circle of radius 2 centred at $3i$.

17.3.7

A circle of radius 5 centred at $4 - 3i$.

17.3.9

The region of the plane to the left of (but not including) $\text{Re}(z) = -1$. It is a domain.

17.3.11

The region of the plane above (but not including) $\text{Im}(z) = 3$. It is a domain.

17.3.13

The region of the plane between (but not including) $\text{Re}(z) = 3$ and $\text{Re}(z) = 5$. It is a domain.

17.3.15

$$\begin{aligned}
z^2 &= (a+ib)^2 \\
&= a^2 - b^2 + 2iab \\
\mathrm{Re}(z^2) &= a^2 - b^2 \\
\mathrm{Re}(z^2) &> 0 \\
a^2 - b^2 &> 0 \\
a^2 &> b^2
\end{aligned}$$

The region between $y = x$ and $y = -x$. Not a domain.

17.3.17

The region between $\theta = 0$ and $\theta = 2\pi/3$. Not a domain.

17.3.19

The region outside a circle of radius 1 centred at i. It is a domain.

17.3.21

The region between the circles of radius 2 and 3 centred at i. It is a domain.

17.3.23

$$y = -x$$

17.3.25

$$\begin{aligned}
z^2 + \overline{z}^2 &= (a+ib)^2 + (a-ib)^2 \\
&= a^2 + 2iab - b^2 + a^2 - 2iab - b^2 \\
&= 2(a^2 - b^2) \\
2(a^2 - b^2) &= 2 \\
a^2 - b^2 &= 1 \\
a^2 &= b^2 + 1
\end{aligned}$$

The hyperbola $x^2 - y^2 = 1$