University Physics with Modern Physics - Modern Physics by Young and Freedman Problems

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Contents

17 Temperature and Heat

17.1 Guided Practice

17.1.1

(a)

$$\begin{aligned}\Delta L &= \alpha L_0 \Delta T\\ \alpha &= \frac{\Delta L}{L_0 \Delta T}\\ &= 2.0 \times 10^{-5}\,\mathrm{K}^{-1}\end{aligned}$$

(b)

$$\begin{aligned}\Delta L &= \alpha L_0 \Delta T\\ &= -0.27\,\mathrm{mm}\end{aligned}$$

17.1.2

$$
\begin{aligned}
\Delta V_C &= \beta V_{C0} \Delta T \\
&= (5.1 \times 10^{-5})(250)(-70) \\
&= -0.893\,\mathrm{cm}^3 \\
\Delta V_E &= \beta V_{E0} \Delta T \\
&= (75 \times 10^{-5}\ )(250)(-70) \\
&= -13.1\,\mathrm{cm}^3 \\
\Delta V_C - \Delta V_E &= 12.2\,\mathrm{cm}^3 \\
&= 12.2\,\mathrm{mL}
\end{aligned}
$$

17.1.3

$$
\begin{aligned}
\frac{\Delta L}{L_0} &= \alpha \Delta T \\
Y &= \frac{F/A}{\Delta L/L_0} \\
\frac{\Delta L}{L_0} &= \frac{F}{AY} \\
\alpha \Delta T + \frac{F}{AY} &= 0 \\
\frac{F}{AY} &= -\alpha \Delta T \\
F &= -\alpha A Y \Delta T \\
&= -(2.0 \times 10^{-5})(\pi 0.005^2)(9.0 \times 10^{10})(-12) \\
&= 1.70 \times 10^3\,\mathrm{N}
\end{aligned}
$$

Tensile

17.1.4

$$
\begin{aligned}
\Delta L &= \alpha_A L_A \Delta T + \alpha_B L_B \Delta T \\
\frac{\Delta L}{\Delta T} &= \alpha_A L_A + \alpha_B (L - L_A) \\
&= (\alpha_A - \alpha_B) L_A + \alpha_B L \\
L_A &= \frac{1}{\alpha_A - \alpha_B} \left(\frac{\Delta L}{\Delta T} - \alpha_B L \right)
\end{aligned}
$$

17.1.5

$$
\begin{aligned}
0 &= m_{Al}c_{Al}\Delta T_{Al} + m_W c_W \Delta T_W \\
&= m_{Al}c_{Al}(T - T_{Al}) + m_W c_W (T - T_W) \\
m_{Al} &= -\frac{m_W c_W (T - T_W)}{c_{Al}(T - T_{Al})} \\
&= 0.20\,\mathrm{kg}
\end{aligned}
$$

17.1.6

$$
\begin{aligned}
0 &= m_I L_f + m_C c_C \Delta T \\
&= m_I L_f - m_C c_C T \\
T &= \frac{m_I L_f}{m_C c_C} \\
&= 14.0\,^\circ\mathrm{C}
\end{aligned}
$$

17.1.7

$$
\begin{aligned}
0 &= m_I L_F + m_I c_I \Delta T_I + m_E c_E \Delta T_E \\
&= m_I (L_F + c_I \Delta T_I) + m_E c_E \Delta T_E \\
m_I &= -\frac{m_E c_E \Delta T_E}{L_F + c_I \Delta T_I} \\
&= 0.176\,\mathrm{kg}
\end{aligned}
$$

17.1.8

Cooling the silver to $0\,^\circ\mathrm{C}$ would take

$$Q = mc\Delta T = 92\,137.5\,\mathrm{J}$$

whereas melting all of the ice would take

$$Q = mL_f = 83\,500\,\mathrm{J}$$

so all of the ice will melt.

$$
\begin{aligned}
0 &= m_{Ag}c_{Ag}\Delta T_{Ag} + m_I L_f + m_I c_I \Delta T_I + m_I c_W \Delta T_W \\
&= m_{Ag}c_{Ag}(T - T_{Ag}) + m_I L_F - m_I c_I T_I + m_I c_W T \\
&= (m_{Ag}c_{Ag} + m_I c_W)T - m_{Ag}c_{Ag}T_{Ag} + m_I L_F - m_I c_I T_I \\
T &= \frac{m_{Ag}c_{Ag}T_{Ag} + m_I c_I T_I - m_I L_F}{m_{Ag}c_{Ag} + m_I c_W} \\
&= 3.31\,^\circ\mathrm{C}
\end{aligned}
$$

17.1.9

(a)

$$
\begin{aligned}
H &= kA\frac{T_H - T_C}{L} \\
k &= \frac{HL}{A(T_H - T_C)} \\
&= 0.754\,\mathrm{W/(m\,K)}
\end{aligned}
$$

(b)

$$H = kA\frac{T_H - T_C}{L} = 733\,\mathrm{W}$$

17.1.10

(a)

$$
\begin{aligned}
L &= 0.250\,\mathrm{m} \\
A &= 2.00\times 10^{-4}\,\mathrm{m}^2 \\
k_B &= 109.0\,\mathrm{W/(m\,K)} \\
k_{Pb} &= 34.7\,\mathrm{W/(m\,K)} \\
T &= 185\,^\circ\mathrm{C} \\
H &= 6.00\,\mathrm{W}
\end{aligned}
$$

$$
\begin{aligned}
H &= k_B A\frac{T_H - T}{L} \\
T_H &= \frac{HL}{k_B A} + T \\
&= 254\,^\circ\mathrm{C}
\end{aligned}
$$

(b)

$$
\begin{aligned}
H &= k_{Pb} A\frac{T - T_C}{L} \\
T_C &= T - \frac{HL}{k_{Pb} A} \\
&= -31.1\,^\circ\mathrm{C}
\end{aligned}
$$

17.1.11

$$
\begin{aligned}
H &= 4\pi(kr_E)^2 e\sigma T^4 \\
(kr_E)^2 &= \frac{H}{4\pi e\sigma T^4} \\
k &= \frac{1}{r_E}\sqrt{\frac{H}{4\pi e\sigma T^4}} \\
&= 1.70
\end{aligned}
$$

17.1.12

(a)

$$
\begin{aligned}
H &= Ae\sigma T^4 \\
&= \pi r^2 \sigma T^4 \\
H &= kA\frac{T_H - T_C}{L} \\
&= k\pi r^2 \frac{T_H - T_C}{L} \\
\pi r^2 \sigma T^4 &= k\pi r^2 \frac{T_H - T_C}{L} \\
T_H &= \frac{L\sigma T^4}{k} + T_C \\
&= 14.26\,\mathrm{K}
\end{aligned}
$$

(b)

$$
\begin{aligned}
H &= mL_f \\
\pi r^2 \sigma T^4 &= mL_f \\
m &= \frac{\pi r^2 \sigma T^4}{L_f} \\
&= 1.19 \times 10^{-4}\,\mathrm{kg/s} \\
&= 0.427\,\mathrm{kg/h}
\end{aligned}
$$

17.2 Exercises and Problems

17.2.15

$$
\begin{aligned}
\Delta V &= \beta V_0 \Delta T \\
\frac{\Delta V}{V_0} &= \beta(T - T_0) \\
T &= T_0 + \frac{\Delta V}{\beta V_0} \\
&= 49\,^\circ\mathrm{C}
\end{aligned}
$$

17.2.25

$$\begin{aligned} Q &= (m_{Al}c_{Al} + m_W c_W)\Delta T \\ &= 5.55 \times 10^5\,\mathrm{J} \end{aligned}$$

17.2.33

$$\begin{aligned} \Delta K &= \frac{1}{2}mv^2 - \frac{1}{2}mv'^2 \\ &= \frac{1}{2}m(v^2 - v'^2) \\ &= 3.47\,\mathrm{kJ} \\ \Delta K &= mc\Delta T \\ \Delta T &= \frac{\Delta K}{mc} \\ &= 6.14 \times 10^{-2}\,^\circ\mathrm{C} \end{aligned}$$

17.2.35

(a)

$$\begin{aligned} 0 &= m_m c_m \Delta T_m + m_w c_w \Delta T_w \\ c_m &= -\frac{m_w c_w \Delta T_w}{m_m \Delta T_m} \\ &= 215\,\mathrm{J/(kg\,K)} \end{aligned}$$

(b) Water because it has a higher specific heat

(c) It would be too small

17.2.45

$$\begin{aligned} \frac{1}{2}mv^2 &= mc\Delta T + mL_F \\ v &= \sqrt{2(c\Delta T + L_F)} \\ &= 366\,\mathrm{m/s} \end{aligned}$$

17.2.55

$$
\begin{aligned}
k_C A \frac{T_H - T}{L} &= kA\frac{T}{L} \\
k_C T_H - k_C T &= kT \\
k_C T_H &= (k + k_C)T \\
T &= \frac{k_C}{k + k_C} T_H \\
0.71 &= \frac{k_C}{k + k_C} \\
0.71(k + k_C) &= k_C \\
0.71k + 0.71k_C &= k_C \\
0.71k &= 0.29k_C \\
k &= \frac{0.29}{0.71} k_C \\
&\approx 157\,\mathrm{W/(m\,K)}
\end{aligned}
$$

17.2.57

(a)

$$
\begin{aligned}
k_W \frac{T - T_C}{L_W} &= k_S \frac{T_H - T}{L_S} \\
\left(\frac{k_W}{L_W} + \frac{k_S}{L_S}\right) T &= \frac{k_S}{L_S} T_H + \frac{k_W}{L_W} T_C \\
T &= \frac{\frac{k_S}{L_S} T_H + \frac{k_W}{L_W} T_C}{\frac{k_W}{L_W} + \frac{k_S}{L_S}} \\
&= -0.86\,{}^\circ\mathrm{C}
\end{aligned}
$$

(b)

$$
\begin{aligned}
H &= k_W \frac{T - T_C}{L_W} \\
&= 24.4\,\mathrm{W/m^2}
\end{aligned}
$$

17.2.65

$$
\begin{aligned}
H &= Ae\sigma T^4 \\
A &= \frac{H}{e\sigma T^4} \\
&= 2.1\,\mathrm{cm^2}
\end{aligned}
$$

17.2.69

$$
\begin{aligned}
\Delta L &= (\alpha_B L_B + \alpha_S L_S)\Delta T \\
T &= T_0 + \frac{\Delta L}{\alpha_B L_B + \alpha_S L_S} \\
&= 35.0\,^\circ\mathrm{C}
\end{aligned}
$$

17.2.71

$$
\begin{aligned}
Q &= mc\Delta T \\
&= \rho V c\Delta T \\
\Delta T &= \frac{Q}{\rho V c} \\
\Delta V &= \beta V \Delta T \\
&= \frac{\beta Q}{\rho c} \\
c &= \frac{\beta Q}{\rho \Delta V}
\end{aligned}
$$

17.2.73

(a)

$$
\begin{aligned}
0.0\,^\circ\mathrm{M} &= -39\,^\circ\mathrm{C} \\
100.0\,^\circ\mathrm{M} &= 357\,^\circ\mathrm{C} \\
T_M &= \frac{T_C + 39\,^\circ\mathrm{C}}{3.96} \\
\frac{100\,^\circ\mathrm{C} + 39\,^\circ\mathrm{C}}{3.96} &= 35.1\,^\circ\mathrm{M}
\end{aligned}
$$

(b)

$$
10\,\mathrm{M}^\circ = 10\frac{357\,^\circ\mathrm{C} - (-39\,^\circ\mathrm{C})}{100} = 39.6\,\mathrm{C}^\circ
$$

17.2.75

$$
\begin{aligned}
Ah + \beta_G Ah(T - T_0) &= Ah' + \beta_O Ah'(T - T_0) \\
Ah + \beta_G AhT - \beta_G AhT_0 &= Ah' + \beta_O Ah'T - \beta_O Ah'T_0 \\
(\beta_G Ah - \beta_O Ah')T &= (Ah' - \beta_O Ah'T_0) - (Ah - \beta_G AhT_0) \\
T &= \frac{(1 - \beta_O T_0)h' - (1 - \beta_G T_0)h}{\beta_G h - \beta_O h'} \\
&= 69.4\,^\circ\mathrm{C}
\end{aligned}
$$

17.2.79

(a)

$$
\begin{aligned}
Y &= \frac{F/A}{\Delta L/L_0} \\
\Delta L &= \frac{FL_0}{AY} \\
\Delta L &= \alpha L_0 \Delta T \\
\Delta L &= \alpha L_0 \Delta T + \frac{FL_0}{AY} \\
\frac{F}{A} &= Y\left(\frac{\Delta L}{L_0} - \alpha \Delta T\right)
\end{aligned}
$$

(b)

$$
\begin{aligned}
\Delta L_B &= \alpha_B L_{B0} \Delta T \\
\frac{\Delta L_B}{L_{B0}} &= \alpha_B \Delta T \\
\frac{F}{A} &= Y_S(\alpha_B - \alpha_S)\Delta T \\
&= 1.9 \times 10^8\,\mathrm{Pa}
\end{aligned}
$$

17.2.85

(a)

$$
\begin{aligned}
\frac{dQ}{dT} &= nk\frac{T^3}{\theta^3} \\
Q &= \int_a^b nk\frac{T^3}{\theta^3} \\
&= \frac{nk}{\theta^3}\left[\frac{1}{4}T^4\right]_a^b \\
&= \frac{nk}{4\theta^3}(b^4 - a^4) \\
&= 83.6\,\mathrm{J}
\end{aligned}
$$

(b)

$$
\begin{aligned}
Q &= nC\Delta T \\
C &= \frac{Q}{n\Delta T} \\
&= 1.86\,\mathrm{J/(mol\,K)}
\end{aligned}
$$

(c)

$$
C = 5.60\,\mathrm{J/(mol\,K)}
$$

17.2.95

(a)

$$\begin{aligned}
0 &= m_I L_f + m_I c_W \Delta T_I + m_C c_C \Delta T_I - m_S L_v + m_S c_W \Delta T_S \\
&= m_I L_f + m_I c_W T + m_C c_C T - m_S L_v + m_S c_W (T - T_S) \\
T &= \frac{m_S (L_v + c_W T_S) - m_I L_f}{m_I c_W + m_C c_C + m_S c_W} \\
&= 86.1\,^\circ\mathrm{C}
\end{aligned}$$

(b) No ice, 0.13 kg water, no steam

17.2.99

(a)

$$\begin{aligned}
H &= kA \frac{T_H - T_C}{L} \\
&= 94\,\mathrm{W}
\end{aligned}$$

(b)

$$\begin{aligned}
H_{\text{wood}} &= 12.4\,\mathrm{W} \\
H_{\text{glass}} &= 45.0\,\mathrm{W} \\
H' &= H + (H_{\text{glass}} - H_{\text{wood}}) \\
&= 126.6\,\mathrm{W} \\
\frac{H'}{H} &= 1.35
\end{aligned}$$

17.2.105

(b)

$$\begin{aligned}
\frac{dQ}{dt} &= k\frac{T_H - T_C}{L} \\
\frac{dQ}{dL} &= \rho L_f \\
\frac{dL}{dt} &= \frac{dL}{dQ}\frac{dQ}{dt} \\
&= \frac{1}{\rho L_f} k\frac{T_H - T_C}{L} \\
L\frac{dL}{dt} &= \frac{k(T_H - T_C)}{\rho L_f} \\
\int_0^t L\frac{dL}{dt}\,dt &= \int_0^t \frac{k(T_H - T_C)}{\rho L_f}\,dt \\
\int_0^L L'\,dL' &= \frac{k(T_H - T_C)}{\rho L_f}t \\
\frac{1}{2}L^2 &= \frac{k(T_H - T_C)}{\rho L_f}t \\
L &= \sqrt{\frac{2k(T_H - T_C)}{\rho L_f}t}
\end{aligned}$$

(c)

$$\begin{aligned}
t &= \frac{L^2 \rho L_f}{2k(T_H - T_C)} \\
&= 7.5\,\text{days}
\end{aligned}$$

(d) $t \approx 530\,\text{years}$; no

17.2.107

$$
\begin{aligned}
A &= 2\pi\left(\frac{d}{2}\right)^2 + 2\pi\left(\frac{d}{2}\right)h \\
&= 8.34\times 10^{-2}\,\mathrm{m}^2 \\
H &= Ae\sigma(T^4 - T_s^4) \\
&= Ae\sigma(T^4 - T_s^4) \\
&= -3.38\times 10^{-2}\,\mathrm{W} \\
m &= \frac{H\times 60\times 60}{L_v} \\
&= 5.82\times 10^{-3}\,\mathrm{kg/h} \\
&= 5.82\,\mathrm{g/h}
\end{aligned}
$$

17.2.113

$$
\begin{aligned}
r(x) &= R_2 - (R_2 - R_1)\frac{x}{L} \\
A(x) &= \pi r(x)^2 \\
&= \pi\left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2 \\
H &= kA(x)\frac{dT}{dx} \\
&= k\pi\left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2\frac{dT}{dx} \\
\frac{1}{\left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2}H\,dx &= k\pi\,dT \\
\int_0^L \frac{1}{\left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2}H\,dx &= \int_{T_H}^{T_C} k\pi\,dT \\
\frac{HL}{R_2 - R_1}\left[\frac{1}{R_2 - (R_2 - R_1)\frac{x}{L}}\right]_0^L &= k\pi(T_C - T_H) \\
\frac{HL}{R_2 - R_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) &= k\pi(T_C - T_H) \\
\frac{HL}{R_2 - R_1}\frac{R_2 - R_1}{R_1R_2} &= k\pi(T_C - T_H) \\
H &= \frac{k\pi R_1R_2(T_C - T_H)}{L}
\end{aligned}
$$

17.2.115

(a)

$$
\begin{aligned}
H &= k(2\pi r L)\frac{dT}{dr} \\
\frac{1}{r}H\,dr &= 2\pi k L\,dT \\
\int_a^b \frac{1}{r}H\,dr &= \int_{T_1}^{T_2} 2\pi k L\,dT \\
H\ln\frac{b}{a} &= 2\pi k L(T_2 - T_1) \\
H &= \frac{2\pi k L(T_2 - T_1)}{\ln b/a}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{2\pi k L(T - T_2)}{\ln r/a} &= \frac{2\pi k L(T_2 - T_1)}{\ln b/a} \\
\frac{T - T_2}{\ln r/a} &= \frac{T_2 - T_1}{\ln b/a} \\
T - T_2 &= \frac{\ln r/a}{\ln b/a}(T_2 - T_1) \\
T &= T_2 + \frac{\ln r/a}{\ln b/a}(T_2 - T_1)
\end{aligned}
$$

17.2.117

a

17.2.119

a

18 Thermal Properties of Matter

18.1 Guided Practice

18.1.1

(a)

$$\begin{aligned}
pV &= nRT \\
\frac{p}{T} &= \frac{nR}{V} \\
\frac{p_1}{T_1} &= \frac{p_2}{T_2} \\
p_2 &= p_1 \frac{T_2}{T_1} \\
&= 4.67 \times 10^5 \, \mathrm{Pa}
\end{aligned}$$

(b)

$$\begin{aligned}
pV &= nRT \\
n &= \frac{pV}{RT} \\
&= 0.280 \, \mathrm{mol}
\end{aligned}$$

18.1.2

(a)

$$\begin{aligned}
pV &= nRT \\
\frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\
V_2 &= \frac{V_1 p_1 T_2}{p_2 T_1} \\
&= 1.2 \times 10^3 \, \mathrm{m}^3
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{V_2}{V_1} &= \frac{\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi r_1^3} \\
&= \left(\frac{r_2}{r_1}\right)^3 \\
\frac{r_2}{r_1} &= \sqrt[3]{\frac{V_2}{V_1}} \\
&= 4.5
\end{aligned}$$

18.1.3

(a)

$$
\begin{aligned}
pV &= nRT \\
n &= \frac{pV}{RT} \\
&= 2.9 \times 10^{-3}\,\mathrm{mol/m^3}
\end{aligned}
$$

(b)

$$8.0 \times 10^{-5}\,\mathrm{kg/m^3}$$

18.1.4

(a)

$$
\begin{aligned}
pV &= \frac{m_{\text{total}}}{M} RT \\
\frac{p}{\rho T} &= \frac{R}{M} \\
\frac{p_1}{\rho_1 T_1} &= \frac{p_2}{\rho_2 T_2} \\
&= \frac{p_2}{\rho_1 (p_2/p_1)^{3/5} T_2} \\
T_2 &= \left(\frac{p_2}{p_1}\right)^{2/5} T_1
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{\rho_2}{\rho_1} &= \frac{\rho_1 (p_2/p_1)^{3/5}}{\rho_1} \\
&= \left(\frac{\frac{1}{2}p_1}{p_1}\right)^{3/5} \\
&= \left(\frac{1}{2}\right)^{3/5} \\
&\approx 0.660 \\
\frac{T_2}{T_1} &= \frac{(p_2/p_1)^{2/5} T_1}{T_1} \\
&= \left(\frac{\frac{1}{2}p_1}{p_1}\right)^{2/5} \\
&= \left(\frac{1}{2}\right)^{2/5} \\
&\approx 0.758
\end{aligned}
$$

(c)

$$\begin{aligned}\frac{\rho_2}{\rho_1} &= 2^{3/5} \\ &\approx 1.52 \\ \frac{T_2}{T_1} &= 2^{2/5} \\ &\approx 1.32\end{aligned}$$

18.1.5

$$\begin{aligned}\sqrt{\frac{3RT}{M_\mathrm{H}}} &= \sqrt{\frac{3RT_\mathrm{N}}{M_\mathrm{N}}} \\ T &= \frac{M_\mathrm{H}}{M_\mathrm{N}}T_\mathrm{N} \\ &= 41.9\,\mathrm{K} \\ &= -231\,^\circ\mathrm{C}\end{aligned}$$

18.1.6

(a)

$$K_\mathrm{tr} = \frac{3}{2}kT = 6.21 \times 10^{-20}\,\mathrm{J}$$

(b)

$$v_\mathrm{rms} = \sqrt{\frac{3RT}{M}} = 8.63 \times 10^3\,\mathrm{m/s}$$

18.1.7

(a)

$$\begin{aligned}pV &= \frac{N}{N_A}RT \\ N &= \frac{N_A pV}{RT} \\ &= 1.50 \times 10^{27}\end{aligned}$$

(b)

$$K_\mathrm{tr} = \frac{3}{2}nRT = 9.11 \times 10^6\,\mathrm{J}$$

(c)

$$\begin{aligned}\frac{1}{2}mv^2 &= K_{\mathrm{tr}} \\ v &= \sqrt{\frac{2K_{\mathrm{tr}}}{m}} \\ &= 110\,\mathrm{m/s}\end{aligned}$$

18.1.8

(a) 5.5

(b) 38.5

(c) 6.2

18.1.9

(a)

$$\lambda = \frac{kT}{4\pi\sqrt{2}r^2p} = 6.8\times10^{-6}\,\mathrm{m}$$

(b)

$$\begin{aligned}\lambda_{\mathrm{Earth}} &= 5.54\times10^{-8}\,\mathrm{m} \\ \frac{\lambda_{\mathrm{Mars}}}{\lambda_{\mathrm{Earth}}} &= 1.2\times10^{2}\end{aligned}$$

18.1.10

(a)

$$\begin{aligned}\lambda &= \frac{kT}{4\pi\sqrt{2}r^2p} \\ p &= \frac{kT}{4\pi\sqrt{2}r^2\lambda} \\ &= 5.7\times10^{-3}\,\mathrm{Pa}\end{aligned}$$

(b)

$$\begin{aligned}pV &= nRT \\ n &= \frac{pV}{RT} \\ &= 2.3\times10^{-6}\,\mathrm{mol}\end{aligned}$$

18.1.11

(a)

$$\begin{aligned}
pV &= nRT \\
p &= \frac{nRT}{V} \\
&= 2.0 \times 10^{7}\,\text{Pa} \\
\lambda &= \frac{kT}{4\pi\sqrt{2}r^2 p} \\
&= 1.2 \times 10^{-8}\,\text{m}
\end{aligned}$$

(b)

$$\begin{aligned}
v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\
&= 1.4 \times 10^{3}\,\text{m/s} \\
\lambda &= v t_{\text{mean}} \\
t_{\text{mean}} &= \frac{\lambda}{v} \\
&= 8.6 \times 10^{-12}\,\text{s}
\end{aligned}$$

18.1.12

(a)

$$\begin{aligned}
v_{\text{rms}} t_{\text{mean}} &= \lambda \\
\sqrt{\frac{3kT}{m}} t_{\text{mean}} &= \frac{kT}{4\pi\sqrt{2}r^2 p} \\
t_{\text{mean}} &= \frac{kT}{4\pi\sqrt{2}r^2 p}\sqrt{\frac{m}{3kT}} \\
&= \frac{1}{4\pi r^2 p}\sqrt{\frac{mkT}{6}}
\end{aligned}$$

(b) Doubling r.

18.1.13

(a)

$$\begin{aligned} v_{\mathrm{rms}} &= \sqrt{\frac{3RT}{M}} \\ &= 515\,\mathrm{m/s} \\ \frac{1}{2}mv_{\mathrm{rms}}^2 &= mgh \\ h &= \frac{v_{\mathrm{rms}}^2}{2g} \\ &= 102\,\mathrm{km} \end{aligned}$$

(b)

$$\begin{aligned} &\int_{2025}^{\infty} 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}\,dv \\ &= (3.03\times 10^{-8}) \int_{2025}^{\infty} v^2 e^{-(5.65\times10^{-6})v^2}\,dv \\ &= 4.8\times 10^{-10} \end{aligned}$$

Yes, some escape.

18.2 Exercises and Problems

18.2.7

$$\begin{aligned} pV &= nRT \\ \frac{p_1V_1}{T_1} &= \frac{p_2V_2}{T_2} \\ T_2 &= \frac{p_2V_2T_1}{p_1V_1} \\ &= 776\,\mathrm{K} \\ &= 503\,^{\circ}\mathrm{C} \end{aligned}$$

18.2.9

$$\begin{aligned} pV &= nRT \\ \frac{p_1V_1}{T_1} &= \frac{p_2V_2}{T_2} \\ p_2 &= \frac{p_1V_1T_2}{T_1V_2} \\ &= 1.97\times 10^4\,\mathrm{Pa} \end{aligned}$$

18.2.13

$$\begin{aligned}\frac{p_1V_1}{T_1} &= \frac{p_2V_2}{T_2}\\ V_2 &= \frac{p_1T_2}{T_1p_2}V_1\\ &= (5.08\times 10^{-2})V_1\end{aligned}$$

18.2.17

(a)

$$\begin{aligned}pV &= \frac{m_{\text{total}}}{M}RT\\ m_{\text{total}} &= \frac{pVM}{RT}\\ &= 6.91\times 10^{-16}\,\text{kg}\end{aligned}$$

(b)

$$\rho = \frac{m_{\text{total}}}{V} = 2.30\times 10^{-13}\,\text{kg/m}^3$$

18.2.21

(a)

$$\begin{aligned}pV &= \frac{N}{N_A}RT\\ N &= \frac{pVN_A}{RT}\\ &= 2.19\times 10^{6}\end{aligned}$$

(b)

$$2.44\times 10^{19}$$

18.2.23

(a)

$$\begin{aligned}
pV &= \frac{N}{N_A}RT \\
\frac{V}{N} &= \frac{RT}{N_A p} \\
s &= \sqrt[3]{\frac{V}{N}} \\
&= \sqrt[3]{\frac{RT}{N_A p}} \\
&= 3.45 \times 10^{-9}\,\mathrm{m}
\end{aligned}$$

18.2.25

(a)

$$\begin{aligned}
K_{\mathrm{tr}} &= \frac{3}{2}nRT \\
&= \frac{3}{2}pV \\
&= 5.82 \times 10^{7}\,\mathrm{J}
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{1}{2}mv^2 &= K_{\mathrm{tr}} \\
v &= \sqrt{\frac{2K_{\mathrm{tr}}}{m}} \\
&= 241\,\mathrm{m/s}
\end{aligned}$$

18.2.27

$$\begin{aligned}
pV &= nRT \\
p &= \frac{nR}{V}T \\
\frac{nR}{V} &= m \\
n &= \frac{mV}{R} \\
&= 1.07\,\mathrm{mol} \\
N &= nN_A \\
&= 6.44 \times 10^{23}
\end{aligned}$$

18.2.29

(a)

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} \\ &= 1.93 \times 10^6\,\text{m/s} \\ &= 0.006c \end{aligned}$$

Not a significant fraction of c.

(b)

$$\begin{aligned} 0.10c &= \sqrt{\frac{3kT}{m}} \\ (0.10c)^2 &= \frac{3kT}{m} \\ T &= \frac{(0.10c)^2 m}{3k} \\ &= 7.26 \times 10^{10}\,\text{K} \end{aligned}$$

18.2.31

(a)

$$\frac{3}{2}kT = 6.21 \times 10^{-21}\,\text{J}$$

(b)

$$(v^2)_{\text{av}} = \frac{2}{m}\left(\frac{3}{2}kT\right) = 2.34 \times 10^5\,(\text{m/s})^2$$

(c)

$$v_{\text{rms}} = \sqrt{(v^2)_{\text{av}}} = 484\,\text{m/s}$$

(d)

$$p = mv = \frac{M}{N_A}v = 2.57 \times 10^{-23}\,\text{kg m/s}$$

(e)

$$\begin{aligned} \Delta P &= 2P \\ &= 5.14 \times 10^{-23}\,\text{kg m/s} \\ \Delta t &= \frac{2l}{v} \\ &= 4.13 \times 10^{-4}\,\text{s} \\ F_{\text{av}} &= \frac{\Delta P}{\Delta t} \\ &= 1.24 \times 10^{-19}\,\text{N} \end{aligned}$$

(f)

$$p_{\mathrm{av}} = \frac{F_{\mathrm{av}}}{A} = 1.24 \times 10^{-17}\,\mathrm{Pa}$$

(g)

$$\begin{aligned}
p &= N p_{\mathrm{av}} \\
N &= \frac{p}{p_{\mathrm{av}}} \\
&= 8.15 \times 10^{21}
\end{aligned}$$

(h)

$$\begin{aligned}
pV &= \frac{N}{N_A} RT \\
N &= \frac{pVN_A}{RT} \\
&= 2.44 \times 10^{22}
\end{aligned}$$

18.2.33

$$\begin{aligned}
\sqrt{\frac{3RT}{M_{\mathrm{N}}}} &= \sqrt{\frac{3RT_{\mathrm{H}}}{M_{\mathrm{H}}}} \\
T &= \frac{M_{\mathrm{N}}}{M_{\mathrm{H}}} T_{\mathrm{H}} \\
&= 4074\,\mathrm{K} \\
&= 3800\,{}^{\circ}\mathrm{C}
\end{aligned}$$

18.2.35

$$
\begin{aligned}
C_V &= \frac{5}{2}R \\
v_{\mathrm{rms}} &= \sqrt{\frac{3RT}{M}} \\
T &= \frac{Mv_{\mathrm{rms}}^2}{3R} \\
Q &= nC_V\Delta T \\
\Delta T &= \frac{Q}{nC_V} \\
v'_{\mathrm{rms}} &= \sqrt{\frac{3R(T+\Delta T)}{M}} \\
&= \sqrt{\frac{3R\left(\frac{Mv_{\mathrm{rms}}^2}{3R} + \frac{Q}{nC_V}\right)}{M}} \\
&= \sqrt{v_{\mathrm{rms}}^2 + \frac{3RQ}{Mn\frac{5}{2}R}} \\
&= \sqrt{v_{\mathrm{rms}}^2 + \frac{6Q}{5Mn}} \\
&= 1.02\times 10^3\,\mathrm{m/s}
\end{aligned}
$$

18.2.39

(a)

$$
\begin{aligned}
c_{V,\mathrm{N}} &= \frac{5}{2}R \\
&= 742\,\mathrm{J/(kg\,K)} \\
c_{V,\mathrm{water}} &= 4190\,\mathrm{J/(kg\,K)} \\
&= 5.6C_{V,\mathrm{N}}
\end{aligned}
$$

(b)

$$\begin{aligned}
Q &= mc_{V,\text{water}}\Delta T \\
&= 4.19\times 10^4\,\text{J} \\
m &= \frac{Q}{c_{V,\text{N}}\Delta T} \\
&= 5.65\,\text{kg} \\
pV &= \frac{m_{\text{total}}}{M}RT \\
V &= \frac{m_{\text{total}}RT}{Mp} \\
&= 4.87\,\text{m}^3 \\
&= 4.87\times 10^3\,\text{L}
\end{aligned}$$

18.2.41

(a)

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}} = 337\,\text{m/s}$$

(b)

$$v_{\text{av}} = 380\,\text{m/s}$$

(c)

$$v_{\text{rms}} = 412\,\text{m/s}$$

18.2.43

(a)

$$\frac{v_{\text{rms}}}{v} = \sqrt{\frac{3RT}{M}}\sqrt{\frac{M}{\gamma RT}} = \sqrt{\frac{3}{\gamma}} = 1.34$$

(b)

$$\frac{v_{\text{av}}}{v} = \sqrt{\frac{8RT}{\pi M}}\sqrt{\frac{M}{\gamma RT}} = \sqrt{\frac{8}{\pi\gamma}} = 1.23$$

18.2.45

(a) The minimum pressure is $p_1 = 611.657\,\text{Pa}$. If $p < p_1$ the ice sublimates directly to gas.

(b) The maximum pressure is $p_2 = 2.212\times 10^7\,\text{Pa}$. The ice melts to water, then the properties of the water gradually change to those of steam with no phase transition.

18.2.49

(a)

$$\begin{aligned} p' - p &= -\rho g h \\ &= -1.18 \times 10^4\,\mathrm{Pa} \end{aligned}$$

(b)

$$\begin{aligned} p_1 V_1 &= p_2 V_2 \\ V_2 &= \frac{p_1}{p_2} V_1 \\ &= 0.56\,\mathrm{L} \end{aligned}$$

18.2.51

$$\begin{aligned} 0 &= \rho_{\mathrm{cold}} V g - \rho_{\mathrm{hot}} V g - m g \\ &= \rho_{\mathrm{cold}} V - \rho_{\mathrm{hot}} V - m \\ \rho_{\mathrm{hot}} &= \rho_{\mathrm{cold}} - \frac{m}{V} \\ \frac{Mp}{RT} &= \rho_{\mathrm{cold}} - \frac{m}{V} \\ T &= \frac{Mp}{R(\rho_{\mathrm{cold}} - m/V)} \\ &= 542\,\mathrm{K} \\ &= 269\,^{\circ}\mathrm{C} \end{aligned}$$

18.2.53

$$\begin{aligned} pV &= \frac{m_{\mathrm{total}}}{M} RT \\ m_{\mathrm{total}} &= \frac{pVM}{RT} \\ &= 0.285\,\mathrm{kg} \\ m'_{\mathrm{total}} &= 0.0896\,\mathrm{kg} \\ \Delta m &= 0.195\,\mathrm{kg} \end{aligned}$$

18.2.57

(a)

$$\begin{aligned}
0 &= \rho V g - (m_{\text{adventurer}} + m_{\text{bell}} + m_{\text{water}})g \\
&= \rho V - m_{\text{adventurer}} - m_{\text{bell}} - m_{\text{water}} \\
m_{\text{water}} &= \rho V - m_{\text{adventurer}} - m_{\text{bell}} \\
&= 98\,\text{kg} \\
V_{\text{water}} &= \frac{m_{\text{water}}}{\rho_{\text{water}}} \\
&= 0.0956\,\text{m}^3
\end{aligned}$$

(b)

$$\begin{aligned}
pV &= nRT \\
p &= \frac{nRT}{V} \\
p &= \rho g y \\
\rho g y &= \frac{nRT}{V} \\
n &= \frac{\rho g V}{RT} y \\
\frac{dn}{dt} &= \frac{\rho g V}{RT}\frac{dy}{dt} \\
&= 18.2\,\text{mol/s}
\end{aligned}$$

(c)

$$\begin{aligned}
pV &= nRT \\
n &= \frac{pV}{RT} \\
&= 756\,\text{mol} \\
\frac{n}{dn/dt} &= 41.5\,\text{m}
\end{aligned}$$

18.2.59

(a)

$$\begin{aligned}
pV &= nRT \\
n_{\text{balloon}} &= \frac{pV}{RT} \\
&= (9.11 \times 10^6)\frac{1}{T} \\
n_{\text{cylinder}} &= \frac{pV}{RT} \\
&= (2.97 \times 10^5)\frac{1}{T} \\
\frac{n_{\text{balloon}}}{n_{\text{cylinder}}} &= 30.7
\end{aligned}$$

(b)

$$\begin{aligned}
0 &= \rho V g - M n g - m g \\
mg &= (\rho V - M n) g \\
&= 8420\,\mathrm{N}
\end{aligned}$$

(c)

$$mg = 7810\,\mathrm{N}$$

18.2.67

(c)

$$
\begin{aligned}
U(r) &= U_0 \left[\left(\frac{R_0}{r} \right)^{12} - 2 \left(\frac{R_0}{r} \right)^{6} \right] \\
F(r) &= 12 \frac{U_0}{R_0} \left[\left(\frac{R_0}{r} \right)^{13} - \left(\frac{R_0}{r} \right)^{7} \right]
\end{aligned}
$$

$$
\begin{aligned}
0 &= U_0 \left[\left(\frac{R_0}{r_1} \right)^{12} - 2 \left(\frac{R_0}{r_1} \right)^{6} \right] \\
&= \left(\frac{R_0}{r_1} \right)^{12} - 2 \left(\frac{R_0}{r_1} \right)^{6} \\
&= \left(\frac{R_0}{r_1} \right)^{6} - 2 \\
2 &= \left(\frac{R_0}{r_1} \right)^{6} \\
2r_1^6 &= R_0^6 \\
r_1 &= \frac{1}{\sqrt[6]{2}} R_0 \\
&\approx 0.89 R_0
\end{aligned}
$$

$$
\begin{aligned}
0 &= 12 \frac{U_0}{R_0} \left[\left(\frac{R_0}{r_2} \right)^{13} - \left(\frac{R_0}{r_2} \right)^{7} \right] \\
0 &= \left(\frac{R_0}{r_2} \right)^{13} - \left(\frac{R_0}{r_2} \right)^{7} \\
&= \left(\frac{R_0}{r_2} \right)^{6} - 1 \\
r_2 &= R_0
\end{aligned}
$$

$$
\frac{r_1}{r_2} = \frac{1}{\sqrt[6]{2}}
$$

(d)

$$\begin{aligned}
W &= \int_{r_2}^{\infty} -F\,dr \\
&= \int_{R_0}^{\infty} -12\frac{U_0}{R_0}\left[\left(\frac{R_0}{r}\right)^{13} - \left(\frac{R_0}{r}\right)^{7}\right] dr \\
&= -12\frac{U_0}{R_0}\left(-\frac{R_0}{12}\right) \\
&= U_0
\end{aligned}$$

18.2.69

(a)

$$C_V = 2R = 16.63\,\mathrm{J/(mol\,K)}$$

(b) Less than because vibrational energy will play a smaller role.

18.2.71

(a)

$$\begin{aligned}
\frac{1}{2}mv^2 &\geq \frac{GmM}{R_p} \\
&\geq gmR_p
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{3}{2}kT &\geq mgR_p \\
T_\mathrm{N} &\geq \frac{2mgR_p}{3k} \\
&\geq 1.40\times 10^5\,\mathrm{K} \\
T_\mathrm{H} &\geq 1.02\times 10^4\,\mathrm{K}
\end{aligned}$$

(c)

$$\begin{aligned}
T_\mathrm{N} &\geq 6.37\times 10^3\,\mathrm{K} \\
T_\mathrm{H} &\geq 459\,\mathrm{K}
\end{aligned}$$

(d) Because it's very easy to atmospheric particles to escape.

18.2.73

$$\begin{aligned}\int_0^\infty v^2 f(v)\,dv &= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^4 e^{-mv^2/2kT}\,dv\\ &= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\frac{3}{2^3(m/2kT)^2}\sqrt{\frac{\pi}{(m/2kT)}}\\ &= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\frac{3}{8}\left(\frac{2kT}{m}\right)^2\sqrt{\frac{2\pi kT}{m}}\\ &= \frac{3kT}{m}\end{aligned}$$

18.2.75

(b)

$$\begin{aligned}v_{\mathrm{mp}} &= \sqrt{\frac{2kT}{m}}\\ &= 395\,\mathrm{m/s}\\ f(v_{\mathrm{mp}}) &= 2.10\times 10^{-3}\\ \Delta N &\approx Nf(v_{\mathrm{mp}})\Delta v\\ &\approx (4.20\times 10^{-2})N\end{aligned}$$

(c)

$$\begin{aligned}7v_{\mathrm{mp}} &= 2765\,\mathrm{m/s}\\ f(7v_{\mathrm{mp}}) &= 1.43\times 10^{-22}\\ \Delta N &\approx (2.85\times 10^{-21})N\end{aligned}$$

18.2.77

(a)

$$\begin{aligned}0 &= pA - p_0A - mg\\ p &= p_0 + \frac{mg}{A}\\ &= p_0 + \frac{mg}{\pi r^2}\end{aligned}$$

(b)

$$
\begin{aligned}
p_1 V_1 &= p_2 V_2 \\
p_2 &= \frac{V_1}{V_2} p_1 \\
&= \frac{Ah}{A(h+y)} p_1 \\
&= \frac{h}{h+y} p_1 \\
&\approx \left(1 - \frac{y}{h}\right) p_1 \\
F &= \left(1 - \frac{y}{h}\right) p_1 \pi r^2 - p_0 \pi r^2 - mg \\
&= \left(1 - \frac{y}{h}\right) \left(p_0 + \frac{mg}{\pi r^2}\right) \pi r^2 - p_0 \pi r^2 - mg \\
&= \left(1 - \frac{y}{h}\right) (p_0 \pi r^2 + mg) - p_0 \pi r^2 - mg \\
&= -\frac{y}{h}(p_0 \pi r^2 + mg)
\end{aligned}
$$

(c)

$$
\begin{aligned}
F &= -kx \\
k &= \frac{1}{h}(p_0 \pi r^2 + mg) \\
\omega &= \sqrt{\frac{k}{m}} \\
&= \sqrt{\frac{1}{h}\left(\frac{p_0 \pi r^2}{m} + g\right)} \\
f &= \frac{\omega}{2\pi} \\
&= \frac{1}{2\pi}\sqrt{\frac{g}{h}\left(1 + \frac{p_0 \pi r^2}{gm}\right)}
\end{aligned}
$$

If the displacement is not small the oscillation is not simple harmonic because we can't use the approximation $\frac{h}{h+y} \approx 1 - \frac{y}{h}$.

18.2.81

(a)

$$
I = 2mr^2 = 4.1 \times 10^{-46}\,\mathrm{kg\,m^2}
$$

(b)

$$\begin{aligned} 2\left(\frac{1}{2}(2m)v_i^2\right) &= 2\left(\frac{1}{2}(2m)v_f^2 + \frac{1}{2}I\omega^2\right) \\ 2mv_i^2 &= 2mv_f^2 + 2mr^2\omega^2 \\ v_i^2 &= v_f^2 + r^2\omega^2 \end{aligned}$$

$$\begin{aligned} -2r(2m)v_i &= -2I\omega \\ 2mrv_i &= 2mr^2\omega \\ v_i &= r\omega \end{aligned}$$

(c)

$$\omega = \frac{v_i}{r}$$

$$\begin{aligned} v_i^2 &= v_f^2 + r^2\left(\frac{v_i}{r}\right)^2 \\ v_f &= 0 \end{aligned}$$

(d)

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} \\ &= 514\,\text{m/s} \\ \omega &= 5.47 \times 10^{12}\,\text{rad/s} \end{aligned}$$

18.2.83

(a)

$$\begin{aligned} \lambda &= \frac{V}{4\pi\sqrt{2}r^2N} \\ &= 4.50 \times 10^{11}\,\text{m} \end{aligned}$$

(b)

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} \\ &= 704\,\text{m/s} \\ t_{\text{mean}} &= \frac{\lambda}{v_{\text{rms}}} \\ &= 6.39 \times 10^{8}\,\text{s} \\ &= 20\,\text{years} \end{aligned}$$

(c)

$$\begin{aligned} pV &= NkT \\ p &= \frac{NkT}{V} \\ &= 1.38 \times 10^{-14}\,\mathrm{Pa} \end{aligned}$$

(d)

$$\begin{aligned} m_{\text{total}} &= \rho V \\ &= \rho \frac{4}{3}\pi \left(\frac{d}{2}\right)^3 \\ &= 2.96 \times 10^{32}\,\mathrm{kg} \end{aligned}$$

$$\begin{aligned} \frac{1}{2}mv^2 &= \frac{Gmm_{\text{total}}}{r} \\ v &= \sqrt{\frac{4Gm_{\text{total}}}{d}} \\ &= 640\,\mathrm{m/s} \end{aligned}$$

It would evaporate.

(f)

$$\begin{aligned} T_{\text{ISM}} &= \frac{(N/V)_{\text{nebula}}}{(N/V)_{\text{ISM}}} T_{\text{nebula}} \\ &= 2.0 \times 10^5\,\mathrm{K} \end{aligned}$$

34 times hotter than the sun.

18.2.85

a

18.2.87

c

19 The First Law of Thermodynamics

19.1 Guided Practice

19.1.1

(a)

$$\begin{aligned}\Delta U &= Q - W \\ Q &= \Delta U + W \\ &= 5.75 \times 10^3\,\mathrm{J}\end{aligned}$$

(b)

$$\begin{aligned}\Delta U &= Q - W \\ &= -3.2 \times 10^4\,\mathrm{J}\end{aligned}$$

(c)

$$\begin{aligned}\Delta U &= Q - W \\ W &= Q - \Delta U \\ &= -1.85 \times 10^3\,\mathrm{J}\end{aligned}$$

19.1.2

(a)

$$W = p(V_2 - V_1) = 155\,\mathrm{J}$$

(b)

$$W = 0$$

(c)

$$W = p(V_2 - V_1) = -375\,\mathrm{J}$$

(d)

$$W = \frac{1}{2}(p_1 + p_2)(V_2 - V_1) = 875\,\mathrm{J}$$

19.1.3

(a)

$$\begin{aligned}W &= p(V_2 - V_1) \\ &= -240\,\mathrm{J} \\ \Delta U &= Q - W \\ &= 1.80 \times 10^3\,\mathrm{J}\end{aligned}$$

(b)

$$\begin{aligned}
W &= p(V_2 - V_1) \\
&= -720\,\mathrm{J} \\
\Delta U &= Q - W \\
Q &= \Delta U + W \\
&= 1.08 \times 10^3\,\mathrm{J}
\end{aligned}$$

19.1.4

(a)

$$Q = mL = 3.43 \times 10^6\,\mathrm{J}$$

(b)

$$W = p(V_2 - V_1) = 3.43 \times 10^5\,\mathrm{J}$$

(c)

$$\Delta U = Q - W = 3.09 \times 10^6\,\mathrm{J}$$

19.1.5

(a)

$$\Delta U = \Delta Q = nC_V \Delta T = 998\,\mathrm{J}$$

(b)

$$\Delta U = \Delta Q = nC_V \Delta T = 748\,\mathrm{J}$$

(c)

$$\Delta U = \Delta Q = nC_V \Delta T = 599\,\mathrm{J}$$

19.1.6

(a)

$$V = \frac{nRT}{p} = 5.24 \times 10^{-2}\,\mathrm{m}^3$$

(b) (i)

$$\begin{aligned}
T &= 327\,^\circ\mathrm{C} \\
\Delta U &= Q \\
&= nC_V \Delta T \\
&= 1.31 \times 10^4\,\mathrm{J}
\end{aligned}$$

(ii)

$$\begin{aligned}
T &= 327\,^\circ\mathrm{C} \\
\Delta U &= Q \\
&= nC_V \Delta T \\
&= 1.31 \times 10^4\,\mathrm{J}
\end{aligned}$$

(iii)

$$\begin{aligned}
T &= 927\,^\circ\mathrm{C} \\
\Delta U &= 3.92 \times 10^4\,\mathrm{J}
\end{aligned}$$

19.1.7

(a)

$$\begin{aligned}
pV &= nRT \\
\frac{pV}{R} &= nT
\end{aligned}$$

$$\begin{aligned}
(2p) &= nR(2T) \\
\Delta T &= T
\end{aligned}$$

$$\begin{aligned}
\Delta U &= Q - W \\
&= nC_V \Delta T \\
&= C_V(nT) \\
&= \frac{3}{2} R \frac{pV}{R} \\
&= \frac{3}{2} pV \\
&= 4.50 \times 10^4\,\mathrm{J}
\end{aligned}$$

(b)

$$\begin{aligned} pV &= nRT \\ \frac{pV}{R} &= nT \end{aligned}$$

$$\begin{aligned} pV &= nRT \\ p\left(\frac{1}{2}V\right) &= nR\left(\frac{1}{2}T\right) \\ \Delta T &= -\frac{1}{2}T \end{aligned}$$

$$\begin{aligned} \Delta U &= nC_V\Delta T \\ &= C_V\left(-\frac{1}{2}nT\right) \\ &= -\frac{3}{4}R\frac{pV}{R} \\ &= -\frac{3}{4}pV \\ &= -2.25\times 10^4\ \mathrm{J} \end{aligned}$$

(c)

$$\Delta U = 1.17\times 10^5\ \mathrm{J}$$

19.1.8

(a)

$$\begin{aligned} Q &= nC_V\Delta T \\ &= \frac{5}{2}nRT \end{aligned}$$

$$W = 0$$

$$\begin{aligned} \Delta U &= Q - W \\ &= \frac{5}{2}nRT \end{aligned}$$

(b)

$$\begin{aligned} Q &= nC_P\Delta T \\ &= \frac{7}{2}nRT \end{aligned}$$

$$W = p(V_2 - V_1)$$

$$\begin{aligned} \Delta U &= \frac{7}{2}nRT - p(V_2 - V_1) \\ &= \frac{7}{2}nRT - 2nRT + nRT \\ &= \frac{5}{2}nRT \end{aligned}$$

(c)

$$Q = 0$$

$$\begin{aligned} W &= nC_V(T_1 - T_2) \\ &= -\frac{5}{2}nRT \end{aligned}$$

$$\begin{aligned} \Delta U &= Q - W \\ &= \frac{5}{2}nRT \end{aligned}$$

19.1.9

(a)

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

(b)

$$\begin{aligned} p_1V_1^\gamma &= p_2V_2^\gamma \\ p_2 &= \left(\frac{V_1}{V_2}\right)^\gamma p_1 \\ &= 6.41 \times 10^4\,\mathrm{Pa} \end{aligned}$$

(c)

$$\begin{aligned} W &= \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2) \\ &= 623\,\mathrm{J} \end{aligned}$$

19.1.10

(a)

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5}$$

(b)

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
V_2^{\gamma-1} &= \frac{T_1}{T_2} V_1^{\gamma-1} \\
V_2 &= \left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} V_1 \\
&= 5.79 \times 10^{-4}\,\mathrm{m}^3
\end{aligned}$$

(c)

$$\begin{aligned}
p_1 V_1^{\gamma} &= p_2 V_2^{\gamma} \\
p_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma} p_1 \\
&= 2.95 \times 10^{6}\,\mathrm{Pa}
\end{aligned}$$

(d)

$$\begin{aligned}
W &= \frac{1}{\gamma-1}(p_1 V_1 - p_2 V_2) \\
&= -2.65 \times 10^{3}\,\mathrm{J}
\end{aligned}$$

19.1.11

(a)

$$\begin{aligned}
pV &= nRT \\
p &= \frac{nRT}{V} \\
&= 3.17 \times 10^{5}\,\mathrm{Pa}
\end{aligned}$$

(b)

$$\begin{aligned}
p_1 V_1^{\gamma} &= p_2 V_2^{\gamma} \\
p_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma} p_1 \\
&= 8.21 \times 10^{4}\,\mathrm{Pa}
\end{aligned}$$

(c)

$$
\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 \\
&= 178\,\mathrm{K}
\end{aligned}
$$

(d)

$$
\begin{aligned}
W &= \frac{1}{\gamma-1}(p_1 V_1 - p_2 V_2) \\
&= 7.94 \times 10^3\,\mathrm{J}
\end{aligned}
$$

19.1.12

(a)

$$
\begin{aligned}
\left[p + \left(\frac{an^2}{V^2}\right)\right](V - nb) &= nRT \\
p + \left(\frac{an^2}{V^2}\right) &= \frac{nRT}{V - nb} \\
p &= \frac{nRT}{V - nb} - \frac{an^2}{V^2}
\end{aligned}
$$

$$
\begin{aligned}
W &= \int_{V_1}^{V_2} p\,dV \\
&= \int_{V_1}^{V_2} \left(\frac{nRT}{V - nb} - \frac{an^2}{V^2}\right) dV \\
&= \left[nRT\ln(V - nb) + \frac{an^2}{V}\right]_{V_1}^{V_2} \\
&= nRT\ln(V_2 - nb) + \frac{an^2}{V_2} - nRT\ln(V_1 - nb) - \frac{an^2}{V_1} \\
&= nRT\ln\frac{V_2 - nb}{V_1 - nb} + an^2\frac{V_1 - V_2}{V_1 V_2}
\end{aligned}
$$

(b) (i)

$$
W = 2.80 \times 10^3\,\mathrm{J}
$$

(ii)

$$
\begin{aligned}
W &= \int_{V_1}^{V_2} p\,dV \\
&= \int_{V_1}^{V_2} \frac{nRT}{V}\,dV \\
&= nRT[\ln V]_{V_1}^{V_2} \\
&= 3.11 \times 10^3\,\mathrm{J}
\end{aligned}
$$

19.2 Exercises and Problems

19.2.1

(b)

$$
\begin{aligned}
W &= p(V_2 - V_1) \\
&= nR(T_2 - T_1) \\
&= 1.33 \times 10^3\,\mathrm{J}
\end{aligned}
$$

19.2.3

(b)

$$
\begin{aligned}
p_1V_1 &= nRT \\
p_2V_2 &= nRT \\
3p_1V_2 &= nRT \\
V_2 &= \frac{1}{3}V_1 \\
W &= \int_{V_1}^{V_2} p\,dV \\
&= \int_{V_1}^{V_1/3} \frac{nRT}{V}\,dV \\
&= nRT \ln\frac{1}{3} \\
&= -6.18 \times 10^3\,\mathrm{J}
\end{aligned}
$$

19.2.5

(a)

$$\begin{aligned}
pV &= nRT \\
V &= \frac{nRT}{p}
\end{aligned}$$

$$\begin{aligned}
W &= \int_{V_1}^{V_2} p\,dV \\
&= \int_{nRT/p_1}^{nRT/p_2} \frac{nRT}{V}\,dV \\
&= nRT \ln \frac{nRT/p_2}{nRT/p_1} \\
&= nRT \ln \frac{p_1}{p_2} \\
\frac{W}{nRT} &= \ln \frac{p_1}{p_2} \\
p_1 &= p_2 e^{W/nRT} \\
&= 1.05 \times 10^5\,\mathrm{Pa} \\
&= 1.04\,\mathrm{atm}
\end{aligned}$$

19.2.9

(a)

$$W = p(V_2 - V_1) = 3.47 \times 10^4\,\mathrm{J}$$

(b)

$$\Delta U = Q - W = 8.03 \times 10^4\,\mathrm{J}$$

(c) No, because it's an isobaric process.

19.2.11

(a)

$$\begin{aligned}
T_a &= \frac{pV}{nR} \\
&= 278\,\mathrm{K} \\
T_b &= 694\,\mathrm{K} \\
T_c &= 1250\,\mathrm{K}
\end{aligned}$$

The lowest temperature is 278 K and it occured at point a.

(b)

$$\begin{aligned} W_{\mathrm{ab}} &= 0 \\ W_{\mathrm{bc}} &= 162\,\mathrm{J} \end{aligned}$$

(c)

$$\Delta U = Q - W = 52\,\mathrm{J}$$

19.2.13

(a)

$$\begin{aligned} T_a &= \frac{pV}{nR} \\ &= 5.35 \times 10^2\,\mathrm{K} \\ T_b &= 9.36 \times 10^3\,\mathrm{K} \\ T_c &= 1.50 \times 10^4\,\mathrm{K} \end{aligned}$$

(b)

$$W = 2.10 \times 10^4\,\mathrm{J}$$

(c)

$$Q = \Delta U + W = 3.60 \times 10^4\,\mathrm{J}$$

19.2.17

(b)

$$\begin{aligned} V_1 &= \frac{nRT_1}{p_1} \\ &= 6.18 \times 10^{-3}\,\mathrm{m}^3 \\ V_2 &= 8.23 \times 10^{-3}\,\mathrm{m}^3 \\ W &= p(V_2 - V_1) \\ &= 207\,\mathrm{J} \end{aligned}$$

(c) The piston

(d)

$$\begin{aligned} C_V &= \frac{5}{2}R \\ C_P &= \frac{7}{2}R \\ \Delta U &= nC_P\Delta T \\ &= 727\,\mathrm{J} \\ Q &= \Delta U + W \\ &= 934\,\mathrm{J} \end{aligned}$$

19.2.19

(a)

$$\begin{aligned}
\Delta U &= Q - W \\
&= Q - 0 \\
&= nC_V \Delta T \\
\Delta T &= \frac{\Delta U}{nC_V} \\
&= 168\,\mathrm{K} \\
T_2 &= T_1 + \Delta T \\
&= 948\,\mathrm{K}
\end{aligned}$$

(b)

$$\begin{aligned}
Q &= nC_P \Delta T \\
\Delta T &= \frac{Q}{nC_P} \\
&= 120\,\mathrm{K} \\
T_2 &= T_1 + \Delta T \\
&= 900\,\mathrm{K}
\end{aligned}$$

19.2.21

$$\begin{aligned}
C_V &= \frac{3}{2}R \\
C_P &= \frac{5}{2}R \\
Q &= nC_P \Delta T \\
&= \frac{5}{2}nR(T_2 - T_1) \\
W &= p(V_2 - V_1) \\
&= nR(T_2 - T_1) \\
\frac{W}{Q} &= \frac{2}{5}
\end{aligned}$$

19.2.23

(a)

$$\begin{aligned}
\Delta U &= Q - W \\
&= 747\,\mathrm{J}
\end{aligned}$$

(b)

$$\begin{aligned}
Q &= nC_P\Delta T \\
C_P &= \frac{Q}{n\Delta T} \\
&= 37.0\,\mathrm{J/(mol\,k)} \\
C_V &= C_P - R \\
&= 28.6\,\mathrm{J/(mol\,K)} \\
\gamma &= \frac{C_P}{C_V} \\
&= 1.29
\end{aligned}$$

19.2.25

(a)

$$\begin{aligned}
V_1 &= \frac{nRT}{p_1} \\
&= 3.46 \times 10^{-3}\,\mathrm{m}^3 \\
V_2 &= 8.64 \times 10^{-4}\,\mathrm{m}^3 \\
W &= \int_{V_1}^{V_2} p\,dV \\
&= \int_{V_1}^{V_2} \frac{nRT}{V}\,dV \\
&= nRT \ln\frac{V_2}{V_1} \\
&= -606\,\mathrm{J}
\end{aligned}$$

(b)

$$\Delta U = 0\,\mathrm{J}$$

(c) Yes, liberate

$$Q = \Delta U + W = -606\,\mathrm{J}$$

19.2.27

(a)

$$\begin{aligned}
C_V &= \frac{3}{2}R \\
C_P &= \frac{5}{2}R \\
\gamma &= \frac{C_P}{C_V} \\
&= \frac{5}{3} \\
p_1 V_1^{\gamma} &= p_2 V_2^{\gamma} \\
p_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma} p_1 \\
&= 4.76 \times 10^5\,\mathrm{Pa}
\end{aligned}$$

(b)

$$\begin{aligned}
W &= \frac{C_V}{R}(p_1 V_1 - p_2 V_2) \\
&= -1.06 \times 10^4\,\mathrm{J}
\end{aligned}$$

(c)

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
\frac{T_2}{T_1} &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} \\
&= 1.59
\end{aligned}$$

Heated

19.2.29

(b)

$$\begin{aligned}
W &= nC_V(T_1 - T_2) \\
&= 314\,\mathrm{J}
\end{aligned}$$

(c)

$$\begin{aligned}
\Delta U &= Q - W \\
&= 0 - W \\
&= -314\,\mathrm{J}
\end{aligned}$$

19.2.31

$$
\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_1 \left(\frac{nRT_1}{p_1}\right)^{\gamma-1} &= T_2 \left(\frac{nRT_2}{p_2}\right)^{\gamma-1} \\
T_2^{\gamma} &= T_1^{\gamma} \left(\frac{p_2}{p_1}\right)^{\gamma-1} \\
T_2 &= T_1 \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} \\
&= 285\,\mathrm{K} \\
&= 11.6\,^{\circ}\mathrm{C}
\end{aligned}
$$

19.2.33

$$
\begin{aligned}
C_V &= \frac{3}{2}R \\
C_P &= \frac{5}{2}R \\
\gamma &= \frac{5}{3} \\
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_1 \left(\frac{nRT_1}{p_1}\right)^{\gamma-1} &= 2T_1 \left(\frac{2nRT_1}{p_2}\right)^{\gamma-1} \\
\frac{1}{p_1^{\gamma-1}} &= \frac{2^{\gamma}}{p_2^{\gamma-1}} \\
p_1^{\gamma-1} &= \frac{p_2^{\gamma-1}}{2^{\gamma}} \\
p_2 &= 2^{\gamma/(\gamma-1)} p_1 \\
&= 2^{5/2} p_1 \\
&= 4\sqrt{2} p_1
\end{aligned}
$$

19.2.35

(a) Increase

(b)

$$
W = \frac{1}{2}(p_a + p_b)(V_B - V_A) = 4.8\,\mathrm{kJ}
$$

19.2.37

(a)

$$\begin{aligned} pV &= nRT \\ n &= \frac{pV}{RT} \\ &= 0.678\,\text{mol} \end{aligned}$$

(b)

$$\begin{aligned} pV &= nRT \\ V &= \frac{nRT}{p} \\ &= 3.33 \times 10^{-2}\,\text{m}^3 \end{aligned}$$

(c)

$$\begin{aligned} W &= \int_{V_1}^{V_2} p\,dV \\ &= \int_{V_1}^{V_2} \frac{nRT}{V}\,dV \\ &= nRT \ln \frac{V_2}{V_1} \\ &= 2.22\,\text{kJ} \end{aligned}$$

(d)

$$\Delta U = 0$$

19.2.39

(a)

$$\begin{aligned} \Delta U &= Q - W \\ &= 30.0\,\text{J} \\ Q &= \Delta U + W \\ &= 45.0\,\text{J} \end{aligned}$$

(b) Liberate

$$Q = \Delta U + W = -65.0\,\text{J}$$

(c)

$$\begin{aligned}
\Delta U_{\text{ad}} &= 8.0\,\text{J} \\
W_{\text{ad}} &= 15.0\,\text{J} \\
Q_{\text{ad}} &= \Delta U_{\text{ad}} + W_{\text{ad}} \\
&= 23.0\,\text{J} \\
Q_{\text{db}} &= \Delta U_{\text{ab}} - \Delta U_{\text{ad}} \\
&= 22.0\,\text{J}
\end{aligned}$$

19.2.43

(a)

$$\begin{aligned}
p_1 V_1 &= p_2 V_2 \\
V_2 &= \frac{p_1}{p_2} V_1 \\
&= 8.0 \times 10^{-4}\,\text{m}^3 \\
&= 0.80\,\text{L}
\end{aligned}$$

(b)

$$\begin{aligned}
T_a &= \frac{pV}{nR} \\
&= 304\,\text{K} \\
T_b &= 1.21 \times 10^3\,\text{K} \\
T_c &= 1.21 \times 10^3\,\text{K}
\end{aligned}$$

(c)

$$\begin{aligned}\Delta U_{\mathrm{ab}} &= Q_{\mathrm{ab}} - W_{\mathrm{ab}} \\ &= Q_{\mathrm{ab}} \\ &= nC_V\Delta T \\ &= 74.0\,\mathrm{J}\text{ into the gas}\end{aligned}$$

$$\begin{aligned}V_c &= \frac{nRT_c}{p_c} \\ &= 7.97\times10^{-4}\,\mathrm{m}^3 \\ \Delta U_{\mathrm{ca}} &= Q_{\mathrm{ca}} - W_{\mathrm{ca}} \\ nC_V\Delta T &= Q_{\mathrm{ca}} - p(V_a - V_c) \\ Q_{\mathrm{ca}} &= nC_V\Delta T + p(V_a - V_c) \\ &= -104\,\mathrm{J}\text{ out of the gas}\end{aligned}$$

$$\begin{aligned}\Delta U_{\mathrm{bc}} &= Q_{\mathrm{bc}} - W_{\mathrm{bc}} \\ Q_{\mathrm{bc}} &= \Delta U_{\mathrm{bc}} + W_{\mathrm{bc}} \\ &= nC_V\Delta T + \int_{V_b}^{V_c} p\,dV \\ &= nRT\ln\frac{V_c}{V_b} \\ &= 55.6\,\mathrm{J}\text{ into the gas}\end{aligned}$$

(d)

$$\Delta U_{\mathrm{ab}} = 74.0\,\mathrm{J}\text{ increase}$$

$$\Delta U_{\mathrm{bc}} = 0.0\,\mathrm{J}\text{ no change}$$

$$\begin{aligned}\Delta U_{\mathrm{ca}} &= nC_V\Delta T \\ &= -74.0\,\mathrm{J}\text{ decrease}\end{aligned}$$

19.2.47

(b)

$$V_2 = \frac{p_1}{p_2}V_1 = 6.0\,\mathrm{L}$$

(c)

$$\begin{aligned}
n &= \frac{pV}{RT} \\
&= 6.01 \times 10^{-2}\,\text{mol} \\
W_{12} &= \int_{V_1}^{V_2} p\,dV \\
&= nRT_1 \ln\frac{V_2}{V_1} \\
&= 208\,\text{J}
\end{aligned}$$

$$\begin{aligned}
W_{23} &= p_2(V_3 - V_2) \\
&= -113\,\text{J}
\end{aligned}$$

$$W = 95.0\,\text{J}$$

(d) Heat it at constant volume

19.2.49

(a) As the wind descends it experiences greater atmospheric pressure. This compresses the wind, increasing its temperature. It's important that it be moving fast so it's an adiabatic process.

(b)

$$\begin{aligned}
T_2 &= \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} T_1 \\
&= 287\,\text{K} \\
&= 13.9\,^{\circ}\text{C} \\
\Delta T &= T_2 - T_1 \\
&= 11.9\,\text{C}^{\circ}
\end{aligned}$$

19.2.51

(a)

$$\begin{aligned}
p_1 V_1^\gamma &= p_2 V_2^\gamma \\
p_1 (Ah)^\gamma &= p_2 [A(h - \Delta h)]^\gamma \\
\frac{p_1}{p_2} h^\gamma &= (h - \Delta h)^\gamma \\
\left(\frac{p_1}{p_2}\right)^{1/\gamma} h &= h - \Delta h \\
\Delta h &= h \left[1 - \left(\frac{p_1}{p_2}\right)^{1/\gamma}\right] \\
&= 16.8\,\mathrm{cm}
\end{aligned}$$

(b)

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_2 &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 \\
&= \left(\frac{Ah}{A(h - \Delta h)}\right)^{\gamma-1} T_1 \\
&= \left(\frac{1}{1 - \Delta h/h}\right)^{\gamma-1} T_1 \\
&= 469\,\mathrm{K} \\
&= 196\,^\circ\mathrm{C}
\end{aligned}$$

(c)

$$W = nC_V(T_1 - T_2) = -7.03 \times 10^4\,\mathrm{J}$$

19.2.59

(a) a is abiabatic, b is isochoric, c is isobaric

(b)

$$
\begin{aligned}
\Delta U &= Q_b - W_b \\
&= Q_b - 0 \\
&= Q_b
\end{aligned}
$$

$$
\begin{aligned}
\Delta U &= Q_c - W_c \\
&= Q_c - p(V_2 - V_1) \\
&= Q_c - nR(T_2 - T_1)
\end{aligned}
$$

$$
\begin{aligned}
Q_b &= Q_c - nR(T_2 - T_1) \\
T_2 &= T_1 + \frac{Q_c - Q_b}{nR} \\
&= 28.0\,^\circ\mathrm{C}
\end{aligned}
$$

(c)

$$
\begin{aligned}
Q_b &= nC_V \Delta T \\
C_V &= \frac{Q_b}{n\Delta T} \\
&= 12.5\,\mathrm{J/(mol\,K)}
\end{aligned}
$$

$$
\begin{aligned}
W_a &= nC_V(T_1 - T_2) \\
&= -30.0\,\mathrm{J}
\end{aligned}
$$

$$
W_b = 0
$$

$$
\begin{aligned}
\Delta U_c &= Q_c - W_c \\
W_c &= Q_c - \Delta U_c \\
&= Q_c - nC_V \Delta T \\
&= 20.0\,\mathrm{J}
\end{aligned}
$$

(d)

$$\begin{aligned}
\gamma &= \frac{C_P}{C_V} \\
&= \frac{C_V + R}{C_V} \\
&= 1.67
\end{aligned}$$

$$\begin{aligned}
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
\left(\frac{V_2}{V_1}\right)^{\gamma-1} &= \frac{T_1}{T_2} \\
\frac{V_2}{V_1} &= \left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} \\
&= 0.961
\end{aligned}$$

$$\Delta V_b = 0$$

$$\begin{aligned}
\frac{V_2}{V_1} &= \frac{nRT_2/p}{nRT_1/p} \\
&= \frac{T_2}{T_1} \\
&= 1.03
\end{aligned}$$

a

(e) Decrease, stay the same, increase

19.2.61

(a)

$$
\begin{aligned}
r &= 1.50\,\mathrm{cm} \\
l_{\max} &= 30.0\,\mathrm{cm} \\
l_{\min} &= l_{\max}/v \\
p &= 101\,\mathrm{kPa} \\
T &= 30.0\,^{\circ}\mathrm{C} \\
V_1 &= \pi r^2 l_{\max} \\
&= 2.12 \times 10^{-4}\,\mathrm{m}^3 \\
V_2 &= \pi r^2 l_{\min} \\
&= \pi r^2 \frac{l_{\max}}{v} \\
&= \frac{V_1}{v} \\
n &= \frac{pV}{RT} \\
&= 8.50 \times 10^{-3}\,\mathrm{mol} \\
T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\
T_2 &= T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} \\
&= T_1 v^{\gamma-1} \\
W_{\text{adiabatic}} &= nC_V(T_1 - T_2) \\
&= nC_V(T_1 - T_1 v^{\gamma-1}) \\
&= nC_V T_1(1 - v^{\gamma-1}) \\
&= 53.5(1 - v^{0.4}) \\
W_{\text{isothermal}} &= \int_{V_1}^{V_2} p\,dV \\
&= \int_{V_1}^{V_2} \frac{nRT_2}{V}\,dV \\
&= nRT_2 \ln\frac{V_2}{V_1} \\
&= nRT_1 v^{\gamma-1} \ln v \\
&= 21.4 v^{0.4} \ln v \\
W &= 53.5(1 - v^{0.4}) + 21.4 v^{0.4} \ln v \\
&= 53.5 + v^{0.40}(21.4 \ln v - 53.5)
\end{aligned}
$$

(b)

$$\begin{aligned}
T_2 &\le T_{\text{max}} \\
T_1 v^{\gamma-1} &\le T_{\text{max}} \\
v &\le \left(\frac{T_{\text{max}}}{T_1}\right)^{1/(\gamma-1)} \\
&\le 7.35
\end{aligned}$$

The largest integer value of v is 7.

(c) 7

(d) 7

(e)

$$\begin{aligned}
T_2 &= T_1 v^{\gamma-1} \\
&= 660\,\text{K} \\
&= 387\,^{\circ}\text{C} \\
Q &= nC_V \Delta T \\
&= -63.0\,\text{J}
\end{aligned}$$

19.2.63

$$\begin{aligned}
\frac{p_1}{T_1} &= \frac{p_2}{T_2} \\
p_2 &= \frac{T_2}{T_1} p_1 \\
&= 1.27 \times 10^7\,\text{Pa} \\
&= 1.84 \times 10^3\,\text{psi}
\end{aligned}$$

c

19.2.65

$$\begin{aligned}
p_1 V_1 &= p_2 V_2 \\
V_1 &= \frac{p_2}{p_1} V_2 \\
&= 6.01 \times 10^{-5}\,\text{m}^3 \\
&= 6.01 \times 10^{-2}\,\text{L}
\end{aligned}$$

d

20 The Second Law of Thermodynamics

20.1 Guided Practice

20.1.1

(a)

$$W = eQ_H \Rightarrow Q_H = \frac{W}{e} = 6.89 \times 10^4\,\mathrm{J}$$

(b)

$$|W| = |Q_H| - |Q_C| \Rightarrow |Q_C| = |Q_H| - |W| = 5.65 \times 10^4\,\mathrm{J}$$

20.1.2

$$e = \frac{W}{Q_H} = \frac{Q_H + Q_C}{Q_H} = 1 + \frac{Q_C}{Q_H} = 0.173 = 17.3\%$$

20.1.3

(a)

$$(1-e)Q_H = Q_C \Rightarrow Q_H = \frac{Q_C}{1-e} = 6.17 \times 10^8\,\mathrm{J}$$

(b)

$$W = eQ_H = 1.21 \times 10^8\,\mathrm{J}$$

20.1.4

(a)

$$W = 3600P = 3.96 \times 10^8\,\mathrm{J}$$

(b)

$$Q_H = mL_c = 1.70 \times 10^9\,\mathrm{J}$$

(c)

$$e = \frac{W}{Q_H} = 0.233 = 23.3\%$$

20.1.5

(a)

$$\begin{aligned} e_{\text{Carnot}} &= 1 - \frac{T_C}{T_H} \\ &= 1 + \frac{Q_C}{Q_H} \\ &= 1 + \frac{W - Q_H}{Q_H} \\ &= 0.21 \end{aligned}$$

(b)

$$|Q_C| = |Q_H| - |W| = 6.32 \times 10^4\,\mathrm{J}$$

(c)

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H} \Rightarrow T_H = -\frac{Q_H}{Q_C}T_C = 377\,\mathrm{K} = 104\,^\circ\mathrm{C}$$

20.1.6

(a)

$$e = 1 - \frac{T_C}{T_H} = 0.6$$

(b)

$$
\begin{aligned}
n &= 0.200\,\mathrm{mol} \\
\gamma &= 1.40 \\
T_H &= 227\,^\circ\mathrm{C} = 500\,\mathrm{K} \\
T_C &= -73\,^\circ\mathrm{C} = 200\,\mathrm{K} \\
p_a &= 10.0 \times 10^5\,\mathrm{Pa} \\
V_a &= \frac{nRT_H}{p_a} \\
&= 8.31 \times 10^{-4}\,\mathrm{m}^3 \\
V_b &= 2V_a \\
&= 1.66 \times 10^{-3}\,\mathrm{m}^3 \\
p_b &= \frac{nRT_H}{V_b} \\
&= 5.01 \times 10^5\,\mathrm{Pa} \\
W_{\mathrm{ab}} &= \int_{V_a}^{V_b} p\,dV \\
&= nRT_H \ln 2 \\
&= 576\,\mathrm{J} \\
V_c &= \left(\frac{T_H}{T_C}\right)^{1/(\gamma-1)} V_b \\
&= 1.64 \times 10^{-2}\,\mathrm{m}^3 \\
p_c &= \frac{nrT_C}{V_c} \\
&= 2.03 \times 10^4\,\mathrm{Pa} \\
W_{\mathrm{bc}} &= \frac{1}{\gamma-1}(p_bV_b - p_cV_c) \\
&= 1.25\,\mathrm{kJ} \\
V_d &= \frac{1}{2}V_c \\
&= 8.20 \times 10^{-3}\,\mathrm{m}^3 \\
p_d &= 4.06 \times 10^4\,\mathrm{Pa} \\
W_{\mathrm{cd}} &= \int_{V_c}^{V_d} p\,dV \\
&= nRT_C \ln \frac{1}{2} \\
&= -231\,\mathrm{J} \\
W_{\mathrm{da}} &= \frac{1}{\gamma-1}(p_dV_d - p_aV_a) \\
&= -1.25\,\mathrm{kJ}
\end{aligned}
$$

20.1.7

(a)

$$K = \frac{T_C}{T_H - T_C} = 7.52$$

(b)

$$W = \frac{Q_C}{K} = 5.32 \times 10^5\,\mathrm{J}$$

20.1.8

(a)

$$\begin{aligned} W &= \int_{V_a}^{V_b} p\,dV \\ &= \int_{V_a}^{2V_a} \frac{nRT_H}{V}\,dV \\ &= nRT_H \ln 2 \end{aligned}$$

(b)

$$\begin{aligned} W &= nC_V(T_H - T_C) \\ &= \frac{3}{2}nR(T_H - T_C) \end{aligned}$$

(c)

$$\begin{aligned} nRT_H \ln 2 &= \frac{3}{2}nR(T_H - T_C) \\ \ln 2 &= \frac{3}{2}\left(1 - \frac{T_C}{T_H}\right) \\ \frac{T_C}{T_H} &= 1 - \frac{2}{3}\ln 2 \\ e &= 1 - \frac{T_C}{T_H} \\ &= \frac{2}{3}\ln 2 \\ &= 0.462 \end{aligned}$$

20.1.9

(a)

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = 655\,\mathrm{J/K}$$

(b)

$$\Delta S = mc\int_{159}^{351}\frac{dT}{T} = 1.92\times 10^3\,\mathrm{J/K}$$

(c)

$$\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = 2.43\times 10^3\,\mathrm{J/K}$$

20.1.10

(a)

$$\begin{aligned}
n &= 5.00\,\mathrm{mol}\\
V_1 &= 0.120\,\mathrm{m}^3\\
T_1 &= 20.0\,^\circ\mathrm{C}\\
V_2 &= 0.360\,\mathrm{m}^3\\
T_2 &= 20.0\,^\circ\mathrm{C}\\
\Delta U &= nC_V\Delta T\\
&= 0\\
Q &= W\\
&= \int_{V_1}^{V_2} p\,dV\\
&= \int_{V_1}^{V_2}\frac{nRT}{V}\,dV\\
&= nRT\ln\frac{V_2}{V_1}\\
&= 1.34\times 10^4\,\mathrm{J}\\
\Delta S &= \frac{Q}{T}\\
&= 45.7\,\mathrm{J/K}
\end{aligned}$$

(b) Change in entropy is path independent, so $\Delta S = 45.7\,\mathrm{J/K}$.

20.1.11

(a)

$$\Delta S = 0$$

(b)

$$\Delta S = \frac{Q}{T} = -150\,\mathrm{J/K}$$

(c)

$$\Delta S = \frac{Q}{T} = 218\,\mathrm{J/K}$$

(d)

$$\Delta S = 68\,\mathrm{J/K}$$

The net entropy increases.

20.1.12

(a)

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = 1.22 \times 10^3\,\mathrm{J/K}$$

(b)

$$\begin{aligned}\Delta S &= \int \frac{dQ}{T} \\ &= mc\int_{368}^{273} \frac{dT}{T} \\ &= -1.05 \times 10^3\,\mathrm{J/K}\end{aligned}$$

(c)

$$\Delta S = 160\,\mathrm{J/K}$$

The net entropy increases.

20.1.13

(a)

$$\begin{aligned}0 &= m_i L_f + m_w c(T - T_w) + m_i c(T - T_i) \\ T &= \frac{(m_w T_w + m_i T_i)c - m_i L_f}{(m_w + m_i)c} \\ &= 307\,\mathrm{K} \\ &= 34.3\,^\circ\mathrm{C}\end{aligned}$$

(b)

$$\begin{aligned}\Delta S_i &= \frac{Q}{T_1} + \int \frac{dQ}{T} \\ &= \frac{m_i L_f}{T_1} + m_i c \ln \frac{T_2}{T_1} \\ &= 101\,\mathrm{J/K} \\ \Delta S_w &= m_w c \ln \frac{T_2}{T_1} \\ &= -86.0\,\mathrm{J/K} \\ \Delta S &= 15.0\,\mathrm{J/K}\end{aligned}$$

20.2 Exercises and Problems

20.2.5

(a)

$$\begin{aligned} p_1 V_1^\gamma &= p_2 V_2^\gamma \\ p_1 &= \left(\frac{V_2}{V_1}\right)^\gamma p_2 \\ &= 12.3\,\text{atm} \end{aligned}$$

(b) It enters during process ca.

$$\begin{aligned} T_a &= \frac{p_a V_a}{nR} \\ &= 1.20 \times 10^3\,\text{K} \\ T_c &= \frac{p_c V_c}{nR} \\ &= 146\,\text{K} \\ \gamma &= \frac{C_P}{C_V} \\ &= \frac{C_V + R}{C_V} \\ C_V \gamma &= C_V + R \\ C_V(\gamma - 1) &= R \\ C_V &= \frac{R}{\gamma - 1} \\ Q &= n C_V \Delta T \\ &= 5.48\,\text{kJ} \end{aligned}$$

(c) It leaves during bc.

$$\begin{aligned} T_b &= \frac{p_b V_b}{nR} \\ &= 656\,\text{K} \\ \Delta U &= Q - W \\ Q &= \Delta U + W \\ &= n C_V \Delta T + p_b(V_c - V_b) \\ &= -3.71\,\text{kJ} \end{aligned}$$

(d)

$$
\begin{aligned}
W &= W_{\mathrm{ab}} + W_{\mathrm{bc}} + W_{\mathrm{ca}} \\
&= nC_V(T_a - T_b) + p_b(V_c - V_b) + 0 \\
&= 1.77\,\mathrm{kJ}
\end{aligned}
$$

(e)

$$
e = \frac{W}{Q_H} = 0.323 = 32.3\%
$$

20.2.7

(a)

$$
\begin{aligned}
e &= 1 - \frac{1}{r^{\gamma-1}} \\
r^{\gamma-1} &= \frac{1}{1-e} \\
r &= \left(\frac{1}{1-e}\right)^{1/(\gamma-1)} \\
&= 12
\end{aligned}
$$

(b)

$$
\begin{aligned}
Q_C &= 7.4\,\mathrm{kJ} \\
Q_H &= 20\,\mathrm{kJ}
\end{aligned}
$$

20.2.9

(a)

$$
e = 1 - \frac{1}{r^{\gamma-1}} = 0.581 = 58.1\%
$$

(b)

$$
\begin{aligned}
e' &= 0.595 = 59.5\% \\
\Delta e &= 0.014 \\
&= 1.4\%
\end{aligned}
$$

20.2.11

$$
\begin{aligned}
K &= 2.25 \\
W &= 135\,\mathrm{W} \\
H &= KW \\
&= 304\,\mathrm{W} \\
Q &= mc\Delta T \\
&= 1.30 \times 10^{6}\,\mathrm{J} \\
\frac{Q}{H} &= 4.29 \times 10^{3}\,\mathrm{s} \\
&= 1.2\,\mathrm{h}
\end{aligned}
$$

20.2.13

$$
\begin{aligned}
T_H &= T_C + 72.0\,\mathrm{C}^\circ \\
e &= 1 - \frac{T_C}{T_H} \\
&= 1 - \frac{T_H - 72.0\,\mathrm{C}^\circ}{T_H} \\
&= 1 - 1 + \frac{72\,\mathrm{C}^\circ}{T_H} \\
T_H &= \frac{72\,\mathrm{C}^\circ}{e} \\
&= 576\,\mathrm{K} \\
T_C &= 504\,\mathrm{K}
\end{aligned}
$$

20.2.21

$$
\begin{aligned}
\frac{T_{CA}}{T_{HA} - T_{CA}} &= 1.16 \frac{T_{CB}}{T_{HB} - T_{CB}} \\
(T_{HB} - T_{CB}) &= 1.3(T_{HA} - T_{CA}) \\
T_{CB} &= 180\,\mathrm{K} \\
\frac{T_{CA}}{T_{HA} - T_{CA}} &= 1.16 \frac{T_{CB}}{1.3(T_{HA} - T_{CA})} \\
T_{CA} &= \frac{1.16}{1.3} T_{CB} \\
&= 161\,\mathrm{K}
\end{aligned}
$$

20.2.29

$$\begin{aligned}
\Delta S &= S' - S \\
&= k \ln w' - k \ln w \\
&= k \ln \frac{w'}{w} \\
&= k \ln \frac{(425/0.0024)^N w}{w} \\
&= k \ln (1.77 \times 10^5)^{nN_A} \\
&= 10.0\,\mathrm{J/K}
\end{aligned}$$

20.2.31

(a)

$$\begin{aligned}
\frac{Q_C}{Q_H} &= -\frac{T_C}{T_H} \\
Q_C &= -\frac{T_C}{T_H} Q_H \\
&= -121\,\mathrm{J}
\end{aligned}$$

(b)

$$\begin{aligned}
n &= \frac{U}{W} \\
&= \frac{mgh}{Q_H + Q_C} \\
&= 3.80 \times 10^3
\end{aligned}$$

20.2.33

(a)

$$W = eQ_H = 90.2\,\mathrm{J}$$

(b)

$$Q_C = Q_H - W = 320\,\mathrm{J}$$

(c)

$$T_C = T_H(1 - e) = 318\,\mathrm{K} = 45\,^\circ\mathrm{C}$$

(d) 0

(e)

$$m = \frac{W}{gh} = 0.263\,\mathrm{kg}$$

20.2.37

(a)

$$\begin{aligned} T_A &= \frac{p_A V_A}{nR} \\ &= 241\,\mathrm{K} \\ T_B &= 241\,\mathrm{K} \end{aligned}$$

(b) Absorbed during bc, rejected during ab and ca.

(c)

$$T_C = 481\,\mathrm{K}$$

(d)

$$\begin{aligned} W_{AB} &= \int_{V_A}^{V_B} p\,dV \\ &= nRT_A \ln\frac{V_B}{V_A} \\ &= -1389\,\mathrm{J} \\ Q_{AB} &= -1389\,\mathrm{J} \\ W_{BC} &= p_B(V_C - V_B) \\ &= 2000\,\mathrm{J} \\ \Delta U_{BC} &= Q_{BC} - W_{BC} \\ Q_{BC} &= \Delta U_{BC} + W_{BC} \\ &= nC_V\Delta T + W_{BC} \\ &= nC_V(T_C - T_B) + W_{BC} \\ &= \frac{5}{2}nR(T_C - T_B) + W_{BC} \\ &= 6988\,\mathrm{J} \\ W_{CA} &= 0 \\ Q_{CA} &= nC_V\Delta T \\ &= nC_V(T_A - T_C) \\ &= \frac{5}{2}nR(T_A - T_C) \\ &= -4988\,\mathrm{J} \\ Q_{\text{net}} &= 611\,\mathrm{J} \\ W_{\text{net}} &= 611\,\mathrm{J} \end{aligned}$$

(e)

$$e = \frac{W}{Q_H} = 0.0874 = 8.7\%$$

20.2.41

(a)

$$e = 1 - \frac{T_C}{T_H} = 7.0\%$$

(b)

$$\begin{aligned} P &= eQ_H \\ Q_H &= \frac{P}{e} \\ &= 3.0\,\mathrm{MW} \\ Q_C &= Q_H(1-e) \\ &= 2.8\,\mathrm{MW} \end{aligned}$$

(c)

$$\begin{aligned} Q_C &= mc\Delta T \\ m &= \frac{Q_C}{c\Delta T} \\ &= 167\,\mathrm{kg/s} \\ &= 6.0 \times 10^5\,\mathrm{kg/h} \\ &= 6.0 \times 10^5\,\mathrm{L/h} \end{aligned}$$

20.2.45

$$
\begin{aligned}
e &= 1 - \frac{T_C}{T_H} \\
e' &= 1 - \frac{T'}{T_H} \\
W' &= Q_H e' \\
&= Q_H \left(1 - \frac{T'}{T_H}\right) \\
e'' &= 1 - \frac{T_C}{T'} \\
W'' &= Q'_H e'' \\
&= (Q_H - W')e'' \\
&= \left[Q_H - Q_H \left(1 - \frac{T'}{T_H}\right)\right] \left(1 - \frac{T_C}{T'}\right) \\
&= Q_H \left(1 - 1 + \frac{T'}{T_H}\right) \left(1 - \frac{T_C}{T'}\right) \\
&= Q_H \frac{T'}{T_H} \left(1 - \frac{T_C}{T'}\right) \\
e_{\text{Total}} &= \frac{W' + W''}{Q_H} \\
&= 1 - \frac{T'}{T_H} + \frac{T'}{T_H} \left(1 - \frac{T_C}{T'}\right) \\
&= 1 - \frac{T_C}{T_H}
\end{aligned}
$$

The efficiency is the same.

20.2.49

$$
\begin{aligned}
T_A &= 300\,\mathrm{K} \\
W_{AB} &= nRT_A \ln \frac{V_B}{V_A} \\
&= 2553\,\mathrm{J} \\
Q_{AB} &= 2553\,\mathrm{J} \\
T_B &= 300\,\mathrm{K} \\
T_C &= 1000\,\mathrm{K} \\
W_{BC} &= 0 \\
Q_{BC} &= nC_V(T_C - T_B) \\
&= \frac{5}{2}nR(T_C - T_B) \\
&= 1.24 \times 10^4\,\mathrm{J} \\
W_{CA} &= p_A(V_A - V_C) \\
&= -4949\,\mathrm{J} \\
\Delta U_{CA} &= Q_{CA} - W_{CA} \\
Q_{CA} &= \Delta U_{CA} + W_{CA} \\
&= nC_V(T_A - T_C) + W_{CA} \\
&= \frac{5}{2}nR(T_A - T_C) + W_{CA} \\
&= -1.73 \times 10^4\,\mathrm{J} \\
W &= -2396\,\mathrm{J} \\
K &= \frac{|Q_C|}{|W|} \\
&= 6.24
\end{aligned}
$$

20.2.51

(a)

$$
\begin{aligned}
\Delta S &= \int \frac{dQ}{T} \\
&= m_1c_1 \ln \frac{T}{T_1} + m_2c_2 \ln \frac{T'}{T_2} \\
0 &= m_1c_1(T - T_1) + m_2c_2(T' - T_2) \\
m_1c_1(T - T_1) &= m_2c_2(T_2 - T')
\end{aligned}
$$

(b)

$$\begin{aligned}
T' &= T_2 - \frac{m_1c_1}{m_2c_2}(T - T_1) \\
\Delta S &= m_1c_1 \ln \frac{T}{T_1} + m_2c_2 \ln \left(\frac{T_2 - \frac{m_1c_1}{m_2c_2}(T - T_1)}{T_2} \right) \\
&= m_1c_1 \ln \frac{T}{T_1} + m_2c_2 \ln \left(1 - \frac{m_1c_1}{m_2c_2} \frac{T - T_1}{T_2} \right) \\
\frac{d}{dT}\Delta S &= \frac{m_1c_1}{T} - m_2c_2 \frac{m_1c_1}{m_1c_1(T_1 - T) + m_2c_2T_2} \\
0 &= \frac{1}{T} - \frac{m_2c_2}{m_1c_1(T_1 - T) + m_2c_2T_2} \\
T &= \frac{m_1c_1(T_1 - T) + m_2c_2T_2}{m_2c_2} \\
&= \frac{m_1c_1}{m_2c_2}(T_1 - T) + T_2 \\
&= T'
\end{aligned}$$

20.2.55

20.2.57

(a)

$$
\begin{aligned}
r_b &= 8.0\,\mathrm{cm} \\
V_b &= \frac{4}{3}\pi r_b^3 \\
&= 2.14\times 10^{-3}\,\mathrm{m}^3 \\
V_a &= 2V_b \\
&= 4.29\times 10^{-3}\,\mathrm{m}^3 \\
r_a &= \sqrt[3]{\frac{3V_a}{4\pi}} \\
&= 10\,\mathrm{cm} \\
r_d &= 3r_a \\
&= 30\,\mathrm{cm} \\
p_a V_a^\gamma &= p_b V_b^\gamma \\
p_a &= \left(\frac{V_b}{V_a}\right)^\gamma p_b \\
&= 8.24\,\mathrm{kPa} \\
\frac{p_b V_b}{T_b} &= \frac{p_a V_a}{T_a} \\
T_b &= \frac{p_b V_b}{p_a V_a} T_a \\
&= 152\,\mathrm{K}
\end{aligned}
$$

(b)

$$
\begin{aligned}
n &= \frac{p_b V_b}{R T_b} \\
&= 3.44\times 10^{-2}\,\mathrm{mol} \\
V_d &= \frac{4}{3}\pi r_d^3 \\
&= 0.113\,\mathrm{m}^3 \\
p_d &= \frac{nRT_d}{V_d} \\
&= 311\,\mathrm{Pa}
\end{aligned}
$$

(c)

$$\begin{aligned} e &= 1 - \frac{T_C}{T_H} \\ &= 0.191 \\ &= 19.1\% \\ W &= eQ_H \\ Q_H &= \frac{W}{e} \\ &= 5.24\,\mathrm{kJ} \end{aligned}$$

(d)

$$\begin{aligned} Q_C &= (1-e)Q_H \\ &= 4.24\,\mathrm{kJ} \end{aligned}$$

20.2.59

$$\begin{aligned} e &= 1 - \frac{T_C}{T_H} \\ T_C &= (1-e)T_H \\ &= 281\,\mathrm{K} \\ &= 7.5\,^\circ\mathrm{C} \end{aligned}$$

b

20.2.61

d

37 Relativity

37.1 Guided Practice

37.1.1

(a) In the laboratory frame, $v_1 = \alpha c$ and $v_2 = -\alpha c$. In the frame of th first proton, $v_1' = 0$ and $v_2' = -\frac{1}{2}c$. Using the Lorentz velocity transformation:

$$
\begin{aligned}
v_2' &= \frac{v_2 - v_1}{1 - v_1 v_2/c^2} \\
-\frac{1}{2}c &= \frac{-\alpha c - \alpha c}{1 + \alpha^2 c^2/c^2} \\
\frac{1}{2}c &= \frac{2\alpha c}{1 + \alpha^2} \\
\alpha^2 - 4\alpha + 1 &= 0 \\
\alpha &= \frac{4 \pm \sqrt{16 - 4}}{2} \\
&= 2 \pm \sqrt{3}
\end{aligned}
$$

α can't be greater than 1, so $\alpha = 2 - \sqrt{3} \approx 0.268$.

(b)

$$
\begin{aligned}
K &= (\gamma - 1)mc^2 \\
&= \left(\frac{1}{\sqrt{1 - (0.268c)^2/c^2}} - 1 \right) mc^2 \\
&= 5.71 \times 10^{-12}\,\mathrm{J} \\
&= 35.6\,\mathrm{MeV}
\end{aligned}
$$

(c)

$$
\begin{aligned}
K &= \left(\frac{1}{\sqrt{1 - (0.5c)^2/c^2}} - 1 \right) mc^2 \\
&= 2.33 \times 10^{-11}\,\mathrm{J} \\
&= 145\,\mathrm{MeV}
\end{aligned}
$$

37.2 Exercises and Problems

37.2.1

$$
\begin{aligned}
x_1' &= -d \\
t_1' &= 0 \\
x_2' &= d \\
t_2' &= 0 \\
x_1 &= \gamma(x_1' + ut_1') \\
&= -\gamma d \\
t_1 &= \gamma(t_1' + ux_1'/c^2) \\
&= -\gamma ud/c^2 \\
x_2 &= \gamma(x_2' + ut_2') \\
&= \gamma d \\
t_2 &= \gamma(t_2' + ux_2'/c^2) \\
&= \gamma ud/c^2
\end{aligned}
$$

The observer measures the lightning strike at point A to come first.

37.2.3

$$
\begin{aligned}
2 &= \gamma \\
&= \frac{1}{\sqrt{1 - v^2/c^2}} \\
2\sqrt{1 - v^2/c^2} &= 1 \\
1 - \frac{v^2}{c^2} &= \frac{1}{4} \\
\frac{v^2}{c^2} &= \frac{3}{4} \\
v &= \frac{\sqrt{3}}{2}c \\
&\approx 0.866c \\
&\approx 2.60 \times 10^8 \text{ m/s}
\end{aligned}
$$

Present day jet planes don't reach this speed.

37.2.5

(a)

$$\begin{aligned}
\Delta t &= \gamma \Delta t_0 \\
\frac{\Delta t}{\Delta t_0} &= \frac{1}{\sqrt{1 - v^2/c^2}} \\
\frac{\Delta t_0}{\Delta t} &= \sqrt{1 - v^2/c^2} \\
\left(\frac{\Delta t_0}{\Delta t}\right)^2 &= 1 - \frac{v^2}{c^2} \\
\frac{v^2}{c^2} &= 1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2 \\
v &= \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2}\, c \\
&= 0.998c
\end{aligned}$$

(b)

$$d = \Delta t v = 126\,\mathrm{m}$$

37.2.7

$$\begin{aligned}
\Delta t - \Delta t_0 &= \Delta t \left(1 - \frac{1}{\gamma}\right) \\
&= \Delta t (1 - \sqrt{1 - v^2/c^2}) \\
&= 9.15\,\mathrm{h}
\end{aligned}$$

The clock on the spacecraft measured a shorter elapsed time.

37.2.9

$$\begin{aligned}
l &= \frac{l_0}{\gamma} \\
l_0 &= \gamma l \\
&= 103\,\mathrm{m}
\end{aligned}$$

37.2.11

(a)

$$d = \Delta t_0 v = 0.66\,\mathrm{km}$$

(b)

$$
\begin{aligned}
\Delta t &= \gamma \Delta t_0 \\
&= 49\,\mu\mathrm{s} \\
d &= \Delta t v \\
&= 15\,\mathrm{km}
\end{aligned}
$$

(c)

$$l = \frac{l_0}{\gamma} = 0.45\,\mathrm{km}$$

37.2.13

(a)

$$l = \frac{l_0}{\gamma} = 3960\,\mathrm{m}$$

(b)

$$t = \frac{d}{v} = 95.2\,\mu\mathrm{s}$$

(c)

$$t_0 = \frac{t}{\gamma} = 94.3\,\mu\mathrm{s}$$

37.2.15

(a)

$$
\begin{aligned}
v &= \frac{v' + u}{1 + uv'/c^2} \\
&= \frac{0.380c + 0.580c}{1 + (0.580c)(0.380c)/c^2} \\
&= \frac{(0.380 + 0.580)c}{1 + (0.580)(0.380)} \\
&= 0.787c
\end{aligned}
$$

(b)

$$v = 0.949c$$

(c)

$$v = 0.997c$$

37.2.17

(a) Toward

(b)

$$v' = \frac{v - u}{1 - uv/c^2} = \frac{0.650c - 0.830c}{1 - (0.830c)(0.650c)/c^2} = -0.391c$$

37.2.19

$$
\begin{aligned}
v &= \frac{v' + u}{1 + uv'/c^2} \\
&= \frac{0.950c - 0.650c}{1 + (-0.650c)(0.950c)/c^2} \\
&= 0.784c
\end{aligned}
$$

37.2.21

$$
\begin{aligned}
v &= \frac{v' + u}{1 + uv'/c^2} \\
-\alpha c &= \frac{-0.890c + \alpha c}{1 + (\alpha c)(-0.890c)/c^2} \\
&= \frac{(\alpha - 0.890)c}{1 - 0.890\alpha} \\
-\alpha(1 - 0.890\alpha) &= (\alpha - 0.890) \\
0.890\alpha^2 - 2\alpha + 0.890 &= 0 \\
\alpha &= \frac{2 \pm \sqrt{4 - 3.1684}}{1.78} \\
&= 0.611
\end{aligned}
$$

The particles are travelling at $0.611c$.

37.2.23

(a)

$$\begin{aligned}
f_0 &= 4.44 \times 10^{14}\,\mathrm{Hz} \\
f &= 5.22 \times 10^{14}\,\mathrm{Hz} \\
f &= \sqrt{\frac{c+u}{c-u}} f_0 \\
\left(\frac{f}{f_0}\right)^2 &= \frac{c+u}{c-u} \\
(c-u)\left(\frac{f}{f_0}\right)^2 &= c+u \\
c\left[\left(\frac{f}{f_0}\right)^2 - 1\right] &= u\left[\left(\frac{f}{f_0}\right)^2 + 1\right] \\
u &= \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1} c \\
\left(\frac{f}{f_0}\right)^2 &= 1.38 \\
u &= 0.160c
\end{aligned}$$

(b) \$173 million

37.2.25

$$\begin{aligned}
1.20 &= \sqrt{\frac{c+u}{c-u}} \\
1.44 &= \frac{c+u}{c-u} \\
1.44(c-u) &= c+u \\
c(1.44-1) &= (1.44+1)u \\
u &= \frac{0.44}{2.44} c \\
&= 0.18c
\end{aligned}$$

Toward

37.2.27

$$\begin{aligned}
\frac{\gamma_{0.800} 0.800c}{\gamma_{0.400} 0.400c} &= 2\sqrt{\frac{1-(0.400c)^2/c^2}{1-(0.800c)^2/c^2}} \\
&= 3.06
\end{aligned}$$

37.2.29

(a)

$$\begin{aligned} 2 &= \gamma \\ &= \frac{1}{\sqrt{1 - v^2/c^2}} \\ 1 - \frac{v^2}{c^2} &= \frac{1}{4} \\ v &= \frac{\sqrt{3}}{2}c \\ &= 0.866c \end{aligned}$$

(b)

$$\begin{aligned} 2 &= \gamma^3 \\ &= \frac{1}{(1 - v^2/c^2)^{3/2}} \\ 1 - \frac{v^2}{c^2} &= \left(\frac{1}{2}\right)^{2/3} \\ v &= \sqrt{1 - \left(\frac{1}{2}\right)^{2/3}}c \\ &= 0.608c \end{aligned}$$

37.2.31

(a)

$$0.866c$$

(b)

$$\begin{aligned} 6 &= \gamma \\ &= \frac{1}{\sqrt{1 - v^2/c^2}} \\ \frac{1}{36} &= 1 - \frac{v^2}{c^2} \\ v &= \sqrt{1 - \frac{1}{36}}c \\ &= 0.986c \end{aligned}$$

37.2.33

(a)

$$K = 3mc^2 = 4.51 \times 10^{-10}\,\mathrm{J}$$

(b)

$$
\begin{aligned}
E^2 &= (mc^2)^2 + (pc)^2 \\
p &= \frac{\sqrt{E^2 - (mc^2)^2}}{c} \\
&= \frac{\sqrt{(4mc^2)^2 - (mc^2)^2}}{c} \\
&= \frac{\sqrt{15(mc^2)^2}}{c} \\
&= \sqrt{15}mc \\
&= 1.94 \times 10^{-18}\,\mathrm{kg\,m/s}
\end{aligned}
$$

(c)

$$p = \gamma m v \Rightarrow v = \frac{p}{\gamma m} = 2.90 \times 10^8\,\mathrm{m/s} = 0.968c$$

37.2.35

(a)

$$E = mc^2 \Rightarrow m = \frac{E}{c^2} = 1.11 \times 10^3\,\mathrm{kg}$$

(b)

$$
\begin{aligned}
\rho &= 7860\,\mathrm{kg/m^3} \\
\rho V &= m \\
\rho s^3 &= m \\
s &= \sqrt[3]{\frac{m}{\rho}} \\
&= 0.521\,\mathrm{m}
\end{aligned}
$$

37.2.41

(a)

$$
\begin{aligned}
qV &= K \\
V &= \frac{(\gamma - 1)mc^2}{q} \\
&= 2.06 \times 10^6\,\mathrm{V}
\end{aligned}
$$

(b)

$$K = 3.30 \times 10^{-13}\,\mathrm{J} = 2.06 \times 10^6\,\mathrm{eV}$$

37.2.43

(a)

$$\begin{aligned}
l &= v\Delta t \\
&= v\gamma\Delta t_0 \\
&= \frac{v\Delta t_0}{\sqrt{1-v^2/c^2}} \\
l\sqrt{1-v^2/c^2} &= v\Delta t_0 \\
l^2(1-v^2/c^2) &= v^2\Delta t_0^2 \\
v^2 &= \frac{l^2}{\Delta t_0^2}(1-v^2/c^2) \\
v^2\left(1+\frac{l^2}{\Delta t_0^2 c^2}\right) &= \frac{l^2}{\Delta t_0^2} \\
(1-\Delta)c &= \sqrt{\frac{l^2/\Delta t_0^2}{1+l^2/\Delta t_0^2 c^2}} \\
\Delta &= 1-\frac{1}{c}\sqrt{\frac{l^2/\Delta t_0^2}{1+l^2/\Delta t_0^2 c^2}} \\
&= 8.43\times 10^{-6}
\end{aligned}$$

(b)

$$\begin{aligned}
E &= \gamma mc^2 \\
&= 5.45\times 10^{-9}\,\mathrm{J} \\
&= 3.40\times 10^{10}\,\mathrm{eV}
\end{aligned}$$

37.2.45

$$\begin{aligned}
1.4 &= \gamma \\
&= \frac{1}{\sqrt{1-v^2/c^2}} \\
\frac{1}{1.4} &= \sqrt{1-v^2/c^2} \\
\frac{1}{1.96} &= 1-\frac{v^2}{c^2} \\
v &= \sqrt{1-\frac{1}{1.96}}c \\
&= 0.700c
\end{aligned}$$

37.2.47

$$
\begin{aligned}
\Delta t &= 42.5\,\mathrm{y} \\
\Delta t_0 &= \frac{\Delta t}{\gamma} \\
&= \Delta t\sqrt{1 - v^2/c^2} \\
&= 5.02\,\mathrm{y}
\end{aligned}
$$

Her biological age will be $19 + 5 = 24$.

37.2.51

$$
\begin{aligned}
\Delta t - \Delta t_0 &= \Delta t - \frac{\Delta t}{\gamma} \\
&= \Delta t\left(1 - \frac{1}{\gamma}\right) \\
&= \Delta t(1 - \sqrt{1 - v^2/c^2}) \\
&= 8.41\,\mathrm{ns}
\end{aligned}
$$

The clock that was on the airliner will show a shorter elapsed time.

37.2.53

$$
\begin{aligned}
v &\geq \frac{c}{n} \\
&\geq 1.97 \times 10^8\,\mathrm{m/s} \\
K &\geq 2.70 \times 10^{-14}\,\mathrm{J} \\
&\geq 167\,\mathrm{keV}
\end{aligned}
$$

37.2.57

(a)

$$
v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.9995c + 0.7500c}{1 + (0.7500c)(0.9995c)/c^2} = 0.9999c
$$

(b)

$$
v = \frac{v' + u}{1 + uv'/c^2} = \frac{-0.9995c + 0.7500c}{1 + (0.7500c)(-0.9995c)/c^2} = -0.9965c
$$

37.2.61

$$\begin{aligned}
\frac{\Delta f}{f_0} &= \frac{f'' - f_0}{f_0} \\
&= \sqrt{\frac{c+u}{c-u}}\frac{f'}{f_0} - 1 \\
&= \frac{c+u}{c-u} - 1 \\
(c-u)\left(1 + \frac{\Delta f}{f_0}\right) &= c + u \\
c\frac{\Delta f}{f_0} &= u\left(2 + \frac{\Delta f}{f_0}\right) \\
u &= \frac{\Delta f/f_0}{2 + \Delta f/f_0}c \\
&= 43\,\mathrm{m/s} \\
&= 154\,\mathrm{k/m}
\end{aligned}$$

37.2.63

$$\begin{aligned}
F &= \gamma m a \\
&= \frac{1}{\sqrt{1 - v^2/c^2}} m \frac{v^2}{r} \\
&= \frac{mv^2}{r\sqrt{1 - v^2/c^2}} \\
&= 2.04 \times 10^{-13}\,\mathrm{N}
\end{aligned}$$

37.2.65

(a)

$$\begin{aligned}
\Delta t &= \gamma \Delta t_0 \\
&= \frac{\Delta t_0}{\sqrt{1 - u^2/v^2}} \\
\Delta t^2 &= \frac{\Delta t_0^2}{1 - u^2/c^2} \\
\Delta t_0 &\approx 2.59 \times 10^{-8}\,\mathrm{s}
\end{aligned}$$

(b)

$$\begin{aligned}(4\Delta t_0)^2 &= \frac{\Delta t_0^2}{1-u^2/c^2}\\ \frac{1}{16} &= 1-\frac{u^2}{c^2}\\ \frac{u}{c} &= \sqrt{1-\frac{1}{16}}\\ &= 0.968\end{aligned}$$

37.2.71

c

37.2.73

b

38 Photons: Light Waves Behaving as Particles

38.1 Guided Practice

38.1.1

(a)

$$\begin{aligned}\lambda'-\lambda &= \frac{h}{mc}(1-\cos\phi)\\ \lambda &= \lambda' - 2\frac{h}{mc}\\ &= 0.078\,\mathrm{nm}\end{aligned}$$

(b)

$$\begin{aligned}p_p &= p_e - p_p'\\ \frac{h}{\lambda} &= p_e - \frac{h}{\lambda'}\\ p_e &= h\left(\frac{1}{\lambda}+\frac{1}{\lambda'}\right)\\ &= 1.65\times10^{-23}\,\mathrm{kg\,m/s}\\ p_e &= m_e v\\ v &= \frac{p_e}{m_e}\\ &= 1.81\times10^{7}\,\mathrm{m/s}\end{aligned}$$

(c)

$$K = \frac{1}{2}m_e v^2 = 1.49\times10^{-16}\,\mathrm{J} = 933\,\mathrm{eV}$$

38.2 Exercises and Problems

38.2.1

$$
\begin{aligned}
\lambda &= 520\,\mathrm{nm} \\
f &= \frac{c}{\lambda} \\
&= 5.77 \times 10^{14}\,\mathrm{Hz} \\
p &= \frac{h}{\lambda} \\
&= 1.27 \times 10^{-27}\,\mathrm{kg\,m/s} \\
E &= \frac{hc}{\lambda} \\
&= 3.82 \times 10^{-19}\,\mathrm{J} \\
&= 2.39\,\mathrm{eV}
\end{aligned}
$$

38.2.3

(a)

$$f = \frac{c}{\lambda} = 5.36 \times 10^{14}\,\mathrm{Hz}$$

(b)

$$
\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= 3.35 \times 10^{-19}\,\mathrm{J} \\
n &= \frac{P}{E} \\
&= 2.24 \times 10^{20}
\end{aligned}
$$

(c) No

38.2.5

(a)

$$E = pc = 2.40 \times 10^{-19}\,\mathrm{J} = 1.50\,\mathrm{eV}$$

(b)

$$\lambda = \frac{hc}{E} = 828\,\mathrm{nm}$$

Infrared

38.2.7

$$
\begin{aligned}
\lambda &= 206\,\text{nm} \\
f &= \frac{c}{\lambda} \\
&= 1.46 \times 10^{15}\,\text{Hz} \\
\phi &= 5.1\,\text{eV} \\
\frac{1}{2}mv_{\text{max}}^2 &= hf - \phi \\
v_{\text{max}} &= \sqrt{\frac{2(hf-\phi)}{m}} \\
&= 5.77 \times 10^{5}\,\text{m/s}
\end{aligned}
$$

38.2.9

$$
\begin{aligned}
\lambda_1 &= 400.0\,\text{nm} \\
f_1 &= 7.50 \times 10^{14}\,\text{Hz} \\
K_1 &= 1.10\,\text{eV} \\
K_1 &= hf - \phi \\
\phi &= hf - K_1 \\
&= 2.94 \times 10^{-19}\,\text{J} \\
&= 1.83\,\text{eV} \\
\lambda_2 &= 300.0\,\text{nm} \\
f_2 &= 1.00 \times 10^{15}\,\text{Hz} \\
K_2 &= hf - \phi \\
&= 3.67 \times 10^{-19}\,\text{J} \\
&= 2.3\,\text{eV}
\end{aligned}
$$

38.2.11

(a)

$$\begin{aligned}
\lambda &= 254\,\text{nm} \\
f &= 1.18 \times 10^{15}\,\text{Hz} \\
V_0 &= 0.181\,\text{V} \\
eV_0 &= hf - \phi \\
\phi &= hf - eV_0 \\
&= 7.53 \times 10^{-19}\,\text{J} \\
&= 4.71\,\text{eV} \\
E &= \phi \\
\frac{hc}{\lambda} &= \phi \\
\lambda &= \frac{hc}{\phi} \\
&= 264\,\text{nm}
\end{aligned}$$

(b)

$$\phi = 4.71\,\text{eV}$$

Same

38.2.13

$$\begin{aligned}
K &= E \\
eV &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{eV} \\
&= 345\,\text{pm}
\end{aligned}$$

38.2.15

$$\begin{aligned}
K &= E' - E \\
&= hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) \\
&= 3.17 \times 10^{-16}\,\text{J} \\
&= 1.98\,\text{keV}
\end{aligned}$$

38.2.17

$$\begin{aligned}
\lambda' - \lambda &= \frac{h}{mc}(1 - \cos\phi) \\
\lambda' &= \lambda + \frac{h}{mc}(1 - \cos\phi) \\
\lambda'_{\max} &= \lambda + 2\frac{h}{mc} \\
&= 0.665\,\mathrm{nm}
\end{aligned}$$

Occurs at $\phi = \pi$.

38.2.19

(a)

$$\begin{aligned}
\lambda &= 0.0430\,\mathrm{nm} \\
\phi &= 32.0^\circ \\
\Delta\lambda &= \frac{h}{mc}(1 - \cos\phi) \\
&= 3.69 \times 10^{-13}\,\mathrm{m}
\end{aligned}$$

(b)

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\phi) = 0.0434\,\mathrm{nm}$$

(c)

$$\Delta E = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) = -4.26 \times 10^{-17} = -\,266\,\mathrm{eV}$$

(d)

$$-\Delta E$$

38.2.21

$$\begin{aligned}
0.01\lambda &= \frac{h}{mc}(1 - \cos\phi) \\
\phi &= \arccos\left(1 - \frac{0.01\lambda}{h/mc}\right) \\
&= 51.0^\circ
\end{aligned}$$

38.2.23

$$\begin{aligned}
\Delta t &= 7.20\,\mathrm{fs} \\
\lambda &= 522\,\mathrm{nm} \\
p &= \frac{h}{\lambda} \\
&= 1.27 \times 10^{-27}\,\mathrm{kg\,m/s} \\
\Delta x &= c\Delta t \\
&= 2.16\,\mu\mathrm{m} \\
\Delta x \Delta p &\geq \frac{\hbar}{2} \\
\Delta p &\geq \frac{\hbar}{2\Delta x} \\
&\geq 2.4 \times 10^{-29}\,\mathrm{kg\,m/s}
\end{aligned}$$

38.2.25

$$\begin{aligned}
\lambda &= 620\,\mathrm{nm} \\
E &= \frac{hc}{\lambda} \\
&= 3.21 \times 10^{-19}\,\mathrm{J} \\
\Delta E &= 3.21 \times 10^{-21}\,\mathrm{J} \\
\Delta t \Delta E &\geq \frac{\hbar}{2} \\
\Delta t &\geq \frac{\hbar}{2\Delta E} \\
&\geq 1.64 \times 10^{-14}\,\mathrm{s} \\
&\geq 16.4\,\mathrm{fs}
\end{aligned}$$

38.2.27

(a)

$$\begin{aligned}
\lambda &= 585\,\mathrm{nm} \\
\Delta t &= 450\,\mu\mathrm{s} \\
c &= 4190\,\mathrm{J/kg\,K} \\
L_v &= 2.256\times10^{6}\,\mathrm{J/kg} \\
m &= 2.0\,\mu\mathrm{g} \\
&= 2.0\times10^{-9}\,\mathrm{kg} \\
T_0 &= 33\,^{\circ}\mathrm{C} \\
&= 306\,\mathrm{K} \\
E &= mc(373-T_0)+mL_v \\
&= 5.07\,\mathrm{mJ}
\end{aligned}$$

(b)

$$P = E/\Delta t = 11.3\,\mathrm{W}$$

(c)

$$\begin{aligned}
E_p &= \frac{hc}{\lambda} \\
&= 3.40\times10^{-19}\,\mathrm{J} \\
n &= \frac{E}{E_p} \\
&= 1.49\times10^{16}
\end{aligned}$$

38.2.29

(a)

$$\begin{aligned}
\lambda &= 0.0930\,\mathrm{nm} \\
\phi &= 180^{\circ} \\
\lambda' &= \lambda + \frac{h}{mc}(1-\cos\phi) \\
&= 0.0978\,\mathrm{nm} \\
p &= \frac{h}{\lambda} \\
&= 6.77\times10^{-24}\,\mathrm{kg\,m/s}
\end{aligned}$$

(b)

$$\begin{aligned} K &= E - E' \\ &= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) \\ &= 1.05 \times 10^{-16}\,\mathrm{J} \\ &= 656\,\mathrm{eV} \end{aligned}$$

38.2.31

$$
\begin{aligned}
\lambda &= 0.1360\,\mathrm{nm} \\
\phi &= 60.0^\circ \\
\lambda' &= \lambda + \frac{h}{mc}(1-\cos\phi) \\
&= \lambda + \frac{1}{2}\frac{h}{mc} \\
&= 0.1372\,\mathrm{nm} \\
K &= E - E' \\
&= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) \\
&= 1.278 \times 10^{-17}\,\mathrm{J} \\
K &= \frac{1}{2}mv^2 \\
v &= \sqrt{\frac{2K}{m}} \\
&= 5.300 \times 10^{6}\,\mathrm{m/s} \\
p &= mv \\
&= 4.823 \times 10^{-24}\,\mathrm{kg\,m/s} \\
p &= p' + P_x \\
\frac{h}{\lambda} &= \frac{h}{\lambda'}\cos\phi + P_x \\
P_x &= h\left(\frac{1}{\lambda} - \frac{1}{2\lambda'}\right) \\
&= 2.457 \times 10^{-24}\,\mathrm{kg\,m/s} \\
0 &= p' - P_y \\
&= \frac{h}{\lambda'}\sin\phi - P_y \\
P_y &= \frac{\sqrt{3}h}{2\lambda'} \\
&= 4.182 \times 10^{-24}\,\mathrm{kg\,m/s} \\
\theta &= \arctan\frac{P_y}{P_x} \\
&= 59.57^\circ
\end{aligned}
$$

38.2.33

(a)

$$
\begin{aligned}
E_1 &= 1\,\text{MeV} \\
\lambda_1 &= \frac{hc}{E_1} \\
&= 0.001\,24\,\text{nm} \\
\lambda_2 &= 500\,\text{nm} \\
n &= 10^{26} \\
\Delta\lambda &= \frac{\lambda_2 - \lambda_1}{n} \\
&= 5 \times 10^{-33}\,\text{m}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\Delta\lambda &= \frac{h}{mc}(1 - \cos\phi) \\
&= \frac{h}{mc}\frac{\phi^2}{2} \\
\phi &= \sqrt{\frac{2\Delta\lambda}{h/mc}} \\
&= 6.42 \times 10^{-11}\,\text{rad} \\
&= (3.68 \times 10^{-9})^\circ
\end{aligned}
$$

38.2.35

(a)

$$
\begin{aligned}
K &= E - E' \\
&= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) \\
&= 3.11 \times 10^{-17}\,\text{J} \\
&= 195\,\text{eV}
\end{aligned}
$$

(b)

$$
\begin{aligned}
K &= E \\
&= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{K} \\
&= 6.37\,\text{nm}
\end{aligned}
$$

38.2.37

(a)

$$\begin{aligned}
eV_0 &= hf - \phi \\
&= h\frac{c}{\lambda} - \phi \\
V_0 &= \frac{hc}{e}\frac{1}{\lambda} - \frac{\phi}{e} \\
\frac{hc}{e} &= 1.232 \times 10^{-6}\,\mathrm{V\,m} \\
\frac{\phi}{e} &= 4.7651\,\mathrm{V}
\end{aligned}$$

(b)

$$\begin{aligned}
h &= 6.57 \times 10^{-34}\,\mathrm{J\,s} \\
\phi &= 4.7651\,\mathrm{eV}
\end{aligned}$$

(c)

$$\begin{aligned}
\phi &\leq E \\
&\leq \frac{hc}{\lambda} \\
\lambda &\leq \frac{hc}{\phi} \\
&\leq 261\,\mathrm{nm}
\end{aligned}$$

(d)

$$\begin{aligned}
K &= hf - \phi \\
&= h\frac{c}{\lambda} - \phi \\
\lambda &= \frac{hc}{K + \phi} \\
&= 84.1\,\mathrm{nm}
\end{aligned}$$

38.2.39

(a)

$$\begin{aligned}
m &= 2.404\,\mathrm{pm} \\
c &= 5.211\,\mathrm{pm}
\end{aligned}$$

(b)

$$\lambda_C = 2.404\,\mathrm{pm}$$

(c)

$$\lambda = 5.211\,\mathrm{pm}$$

39 Particles Behaving as Waves

39.1 Guided Practice

39.1.1

(a)

$$\lambda_m = \frac{b}{T} = 192\,\text{nm}$$

Ultraviolet

(b)

$$\begin{aligned} E &= E_f - E_i \\ \frac{hc}{\frac{1}{2}\lambda_m} &= -\frac{hcR}{n^2} + hcR \\ \frac{2}{\lambda_m} &= R\left(1 - \frac{1}{n^2}\right) \\ \frac{1}{n^2} &= 1 - \frac{2}{\lambda_m R} \\ n &= \sqrt{\frac{\lambda_m R}{\lambda_m R - 2}} \\ &= 4.45 \end{aligned}$$

4

(c)

$$\begin{aligned} 2\pi r_2 &= 2\lambda_2 \\ \lambda_2 &= \epsilon_0 \frac{n^2 h^2}{me^2} \\ &= 0.667\,\text{nm} \end{aligned}$$

$$\begin{aligned} 2\pi r_3 &= 3\lambda_3 \\ \lambda_3 &= \frac{2}{3}\epsilon_0 \frac{n^2 h^2}{me^2} \\ &= 1.00\,\text{nm} \end{aligned}$$

39.2 Exercises and Problems

39.2.1

(a)

$$\lambda = \frac{h}{mv} = 0.162\,\text{nm}$$

(b)

$$\lambda = 8.82 \times 10^{-14}\,\text{m}$$

39.2.3

(a)

$$p = \frac{h}{\lambda} = 2.37 \times 10^{-24}\,\mathrm{kg\,m/s}$$

(b)

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = 3.10 \times 10^{-18}\,\mathrm{J} = 19.4\,\mathrm{eV}$$

39.2.5

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ &= 9.09 \times 10^{-11}\,\mathrm{m}\end{aligned}$$

$$\begin{aligned}v &= \frac{h}{m\lambda} \\ &= 4.36 \times 10^{3}\,\mathrm{m/s}\end{aligned}$$

39.2.7

(a)

$$\begin{aligned}E &= \frac{hc}{\lambda} \\ \lambda &= \frac{hc}{E} \\ &= 62.1\,\mathrm{nm}\end{aligned}$$

$$\begin{aligned}E &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2E}{m}} \\ &= 2.65 \times 10^{6}\,\mathrm{m/s} \\ \lambda &= \frac{h}{mv} \\ &= 0.275\,\mathrm{nm}\end{aligned}$$

(b)

$$\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= 7.95 \times 10^{-19}\,\mathrm{J} \\
&= 4.97\,\mathrm{eV}
\end{aligned}$$

$$\begin{aligned}
\lambda &= \frac{h}{mv} \\
v &= \frac{h}{m\lambda} \\
&= 2.91 \times 10^{3}\,\mathrm{m/s} \\
E &= \frac{1}{2}mv^2 \\
&= 3.86 \times 10^{-24}\,\mathrm{J} \\
&= 2.41 \times 10^{-5}\,\mathrm{eV}
\end{aligned}$$

39.2.9

$$\lambda = \frac{h}{mv} = 3.90 \times 10^{-34}\,\mathrm{nm}$$

No

39.2.11

(a)

$$\begin{aligned}
\lambda &= \frac{h}{\sqrt{2meV_{\mathrm{ba}}}} \\
\lambda^2 &= \frac{h^2}{2meV_{\mathrm{ba}}} \\
V_{\mathrm{ba}} &= \frac{h^2}{2me\lambda^2} \\
&= 23.6\,\mathrm{mV}
\end{aligned}$$

(b)

$$E = \frac{hc}{\lambda} = 2.48 \times 10^{-17}\,\mathrm{J} = 155\,\mathrm{eV}$$

(c)

$$
\begin{aligned}
\lambda &= \frac{h}{mv} \\
v &= \frac{h}{m\lambda} \\
&= 9.09 \times 10^{4}\,\mathrm{m/s} \\
E &= \frac{1}{2}mv^{2} \\
&= 3.77 \times 10^{-21}\,\mathrm{J}
\end{aligned}
$$

$$
\begin{aligned}
E &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{E} \\
&= 52.7\,\mu\mathrm{m}
\end{aligned}
$$

39.2.13

$$
\begin{aligned}
d &= 0.0910\,\mathrm{nm} \\
m &= 1 \\
\theta &= 29.0^{\circ} \\
d\sin\theta &= \lambda \\
&= \frac{h}{\sqrt{2mK}} \\
(d\sin\theta)^{2} &= \frac{h^{2}}{2mK} \\
K &= \frac{h^{2}}{2m(d\sin\theta)^{2}} \\
&= 6.75 \times 10^{-20}\,\mathrm{J} \\
&= 0.422\,\mathrm{eV}
\end{aligned}
$$

39.2.15

(a)

$$\begin{aligned}
d &= 1.60\,\mu\text{m} \\
v &= 1.26 \times 10^4\,\text{m/s} \\
d\sin\theta &= \lambda \\
&= \frac{h}{mv} \\
\sin\theta &= \frac{h}{dmv} \\
\theta &= \arcsin\frac{h}{dmv} \\
&= 2.07^\circ
\end{aligned}$$

$$\begin{aligned}
\theta &= \arcsin\frac{2h}{dmv} \\
&= 4.14^\circ
\end{aligned}$$

(b) 1.8 cm

39.2.17

(a)

$$\begin{aligned}
U &= \frac{1}{4\pi\epsilon_0}\frac{(2e)(82e)}{r} \\
&= \frac{164e^2}{4\pi\epsilon_0 r} \\
&= 5.68 \times 10^{-13}\,\text{J} \\
&= 3.55\,\text{MeV}
\end{aligned}$$

(b)

$$K = 5.68 \times 10^{-13}\,\text{J} = 3.55\,\text{MeV}$$

(c)

$$K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}} = 1.31 \times 10^7\,\text{m/s}$$

39.2.21

(a)

$$
\begin{aligned}
E_1 &= -\frac{2hcme^4}{\epsilon_0^2 h^3 c} \\
&= -3.46 \times 10^{-17}\,\mathrm{J} \\
&= -217\,\mathrm{eV}
\end{aligned}
$$

16 times greater

(b)

$$
-E_1 = 217\,\mathrm{eV}
$$

16 times greater

(c)

$$
\begin{aligned}
E_2 &= -\frac{hcR}{2^2} \\
&= -\frac{16hcme^4}{32\epsilon_0^2 h^3 c} \\
&= -\frac{me^4}{2\epsilon_0^2 h^2} \\
&= -8.66 \times 10^{-18}\,\mathrm{J} \\
&= -54.1\,\mathrm{eV}
\end{aligned}
$$

$$
\begin{aligned}
E &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{E_2 - E_1} \\
&= 7.63\,\mathrm{nm}
\end{aligned}
$$

(d)

$$
r_n = \epsilon_0 \frac{n^2 h^2}{4\pi m e^2}
$$

$\frac{1}{4}$ the radius

39.2.23

(a)

$$
\begin{aligned}
v_1 &= \frac{1}{\epsilon_0}\frac{e^2}{2nh} \\
&= 2.18 \times 10^6\,\mathrm{m/s} \\
v_2 &= 1.09 \times 10^6\,\mathrm{m/s} \\
v_5 &= 4.36 \times 10^5\,\mathrm{m/s}
\end{aligned}
$$

(b)

$$\begin{aligned} a_0 &= \frac{\epsilon_0 h^2}{\pi m e^2} \\ &= 5.31 \times 10^{-11}\,\mathrm{m} \\ T_1 &= \frac{2\pi r_1}{v_1} \\ &= \frac{2\pi n^2 a_0}{v_1} \\ &= 1.53 \times 10^{-16}\,\mathrm{s} \\ T_2 &= 1.22 \times 10^{-15}\,\mathrm{s} \\ T_5 &= 1.91 \times 10^{-14}\,\mathrm{s} \end{aligned}$$

(c)

$$\frac{\Delta t}{T} = 8.20 \times 10^6$$

39.2.25

(a)

$$E = E_\infty - E_1 = -E_1 = 20\,\mathrm{eV}$$

(b) $3\,\mathrm{eV}$, $5\,\mathrm{eV}$, $8\,\mathrm{eV}$, $10\,\mathrm{eV}$, $15\,\mathrm{eV}$, or $18\,\mathrm{eV}$

(c) Nothing because there's no energy level at $-12\,\mathrm{eV}$.

(d)

$$\begin{aligned} E_{32} &= 5\,\mathrm{eV} \\ E_{31} &= 15\,\mathrm{eV} \\ E_{43} &= 3\,\mathrm{eV} \end{aligned}$$

$$3\,\mathrm{eV} < \phi < 5\,\mathrm{eV}$$

39.2.27

(a)

$$\begin{aligned} E_1 &= -17.50\,\mathrm{eV} \\ E_2 &= E_1 + \frac{hc}{\lambda} \\ &= -4.36\,\mathrm{eV} \\ E_3 &= -1.92\,\mathrm{eV} \\ E_4 &= -1.07\,\mathrm{eV} \\ E_5 &= -0.68\,\mathrm{eV} \end{aligned}$$

(b)

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = 378\,\mathrm{nm}$$

39.2.29

(a)

$$E = -5.08\,\text{eV}$$

(b)

$$E = -5.68\,\text{eV}$$

39.2.31

$$\begin{aligned}\lambda &= 10.6\,\mu\text{m}\\ P &= 0.100\,\text{kW}\\ n &= \frac{P}{hc/\lambda}\\ &= 5.33 \times 10^{21}\end{aligned}$$

39.2.37

(a)

$$\begin{aligned}A &= 2\pi\left(\frac{d}{2}\right)^2 + 2\pi\left(\frac{d}{2}\right)L\\ &= 3.83 \times 10^{-4}\,\text{m}^3\end{aligned}$$

$$\begin{aligned}P &= Ae\sigma T^4\\ T &= \sqrt[4]{\frac{P}{Ae\sigma}}\\ &= 2.0 \times 10^3\,\text{K}\\ &= 1.7 \times 10^3\,{}^\circ\text{C}\end{aligned}$$

(b)

$$\lambda_m = \frac{b}{T} = 1.46\,\mu\text{m}$$

39.2.39

$$\lambda_m = \frac{b}{T} = 1.06 \times 10^{-3}\,\text{m}$$

Microwave

39.2.41

(a)

$$\begin{aligned}
4\pi\left(\frac{\frac{5}{2}d}{2}\right)^2\sigma T^4 &= 4\pi\left(\frac{d}{2}\right)^2\sigma T'^4 \\
\frac{25}{16}d^2T^4 &= \frac{1}{4}d^2T'^4 \\
T' &= \sqrt[4]{\frac{25}{4}}T \\
&\approx 1.58T
\end{aligned}$$

(b)

$$\frac{b/T'}{b/T} = \frac{T}{T'} = \sqrt[4]{\frac{4}{25}} \approx 0.63$$

39.2.45

(a)

$$\begin{aligned}
\Delta y\Delta p_y &\geq \frac{\hbar}{2} \\
\Delta v_y &\geq \frac{\hbar}{2m\Delta y} \\
&\geq 1.58\times 10^4\,\mathrm{m/s}
\end{aligned}$$

39.2.47

$$\begin{aligned}
\Delta x\Delta p_x &\geq \frac{\hbar}{2} \\
4.42\times 10^{-35}\,\mathrm{kg\,m^2/s} &\geq 5.27\times 10^{-35}\,\mathrm{kg\,m^2/s}
\end{aligned}$$

Not possible

39.2.49

$$\begin{aligned}
\Delta E\Delta t &\geq \frac{\hbar}{2} \\
\Delta E &\geq \frac{\hbar}{2\Delta t} \\
&\geq 9.25\times 10^{-33}\,\mathrm{J} \\
&\geq 5.78\times 10^{-14}\,\mathrm{eV}
\end{aligned}$$

39.2.51

(a)

$$m_r = \frac{m_1 m_2}{m_1 + m_2} = 1.69 \times 10^{-28}\,\mathrm{kg}$$

(b)

$$E_1 = -\frac{m_r e^4}{8\epsilon_0^2 h^2} = -4.02 \times 10^{-16}\,\mathrm{J} = -2514\,\mathrm{eV}$$

(c)

$$\begin{aligned} E_2 &= -\frac{m_r e^4}{32\epsilon_0^2 h^2} \\ &= -629\,\mathrm{eV} \\ \lambda &= \frac{hc}{E_2 - E_1} \\ &= 0.659\,\mathrm{nm} \end{aligned}$$

39.2.53

(a)

$$E = 12.1\,\mathrm{eV}$$

39.2.55

(a)

$$\begin{aligned} E_{\text{ionization}} &= -E_1 \\ &= 13.6\,\mathrm{eV} \\ E_{\text{photon}} &= \frac{hc}{\lambda} \\ &= 2.34 \times 10^{-18}\,\mathrm{J} \\ &= 14.6\,\mathrm{eV} \\ K_{\max} &= E_{\text{photon}} - E_{\text{ionization}} \\ &= 1.0\,\mathrm{eV} \end{aligned}$$

39.2.57

(a)

$$\begin{aligned}
A &= 4\pi\left(\frac{600d_{\text{sun}}}{2}\right)^2 \\
&= 2.19\times 10^{24}\,\text{m}^2 \\
P &= A\sigma T^4 \\
&= 1.01\times 10^{31}\,\text{W} \\
\lambda &= \frac{b}{T} \\
&= 966\,\text{nm} \\
n &= \frac{P}{hc/\lambda} \\
&= 4.91\times 10^{49}
\end{aligned}$$

(b)

$$\frac{P_\text{B}}{P_\text{S}} = 2.58\times 10^4$$

39.2.59

$$\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= \frac{hc}{b/T} \\
T &= \frac{bE}{hc} \\
&= 29\,741\,\text{K}
\end{aligned}$$

39.2.61

(a)

$$\begin{aligned}
I(f) &= \frac{2\pi hc^2}{(c/f)^5(e^{hc/(c/f)kT}-1)} \\
&= \frac{2\pi f^5 h}{c^3(e^{hf/kT}-1)}
\end{aligned}$$

(b)

$$
\begin{aligned}
\lambda &= \frac{c}{f} \\
d\lambda &= -\frac{c}{f^2}\,df \\
\int_0^\infty I(\lambda)\,d\lambda &= -\int_\infty^0 I(f)\frac{c}{f^2}\,df \\
&= \int_0^\infty \frac{2\pi f^5 h}{c^3(e^{hf/kT}-1)}\frac{c}{f^2}\,df \\
&= \frac{2\pi h}{c^2}\int_0^\infty \frac{f^3}{e^{hf/kT}-1}\,df \\
&= \frac{2\pi h}{c^2}\frac{1}{240}\left(\frac{2\pi}{h/kT}\right)^4 \\
&= \frac{2\pi h}{240c^2}\left(\frac{2\pi kT}{h}\right)^4 \\
&= \frac{2\pi h}{240c^2}\frac{16\pi^4k^4T^4}{h^4} \\
&= \frac{2\pi^5k^4}{15c^2h^3}T^4
\end{aligned}
$$

(c)

$$\frac{2\pi^5k^4}{15c^2h^3} = 5.65\times10^{-8}$$

39.2.63

(a)

$$E = \frac{hc}{\lambda} = 1.10\times10^{-18}\,\mathrm{J} = 6.90\,\mathrm{eV}$$

(b)

$$
\begin{aligned}
E &= K \\
eV &= \frac{1}{2}mv^2 \\
2emV &= p^2 \\
p &= \sqrt{2emV}
\end{aligned}
$$

$$
\begin{aligned}
\lambda &= \frac{h}{p} \\
&= \frac{h}{\sqrt{2emV}} \\
2emV &= \left(\frac{h}{\lambda}\right)^2 \\
V &= \frac{1}{2em}\left(\frac{h}{\lambda}\right)^2 \\
&= 4.65 \times 10^{-5}\,\mathrm{V}
\end{aligned}
$$

$$
\begin{aligned}
eV &= \frac{1}{2}mv^2 \\
v &= \sqrt{\frac{2eV}{m}} \\
&= 4.0\,\mathrm{km/s}
\end{aligned}
$$

(c)

$$
\begin{aligned}
V &= 2.54 \times 10^{-8}\,\mathrm{V} \\
v &= 2.21\,\mathrm{m/s}
\end{aligned}
$$

39.2.67

(a)

$$
\begin{aligned}
\lambda_e &= \lambda_p \\
\frac{hc}{E} &= \frac{h}{p} \\
&= \frac{h}{\sqrt{2mK}} \\
\frac{E}{c} &= \sqrt{2mK} \\
E &= c\sqrt{2mK}
\end{aligned}
$$

(b) Photon

39.2.69

(a)

$$\begin{aligned}
\lambda &= \frac{h}{p} \\
&= \frac{h}{mv/\sqrt{1-v^2/c^2}} \\
&= \frac{h}{mv}\sqrt{1-v^2/c^2} \\
\lambda^2 &= \left(\frac{h}{mv}\right)^2 (1-v^2/c^2) \\
&= \left(\frac{h}{mv}\right)^2 - \left(\frac{h}{mc}\right)^2 \\
\left(\frac{h}{mv}\right)^2 &= \lambda^2 + \left(\frac{h}{mc}\right)^2 \\
&= \frac{\lambda^2 m^2 c^2 + h^2}{m^2 c^2} \\
\left(\frac{mv}{h}\right)^2 &= \frac{m^2 c^2}{\lambda^2 m^2 c^2 + h^2} \\
v^2 &= \frac{h^2 c^2}{\lambda^2 m^2 c^2 + h^2} \\
&= \frac{c^2}{1 + (mc\lambda/h)^2} \\
v &= \frac{c}{\sqrt{1 + (mc\lambda/h)^2}}
\end{aligned}$$

(b)

$$\begin{aligned}
v &= (1-\Delta)c \\
&= \left[1 - \frac{1}{2}\left(\frac{\lambda}{h/mc}\right)^2\right] c \\
\Delta &= \frac{1}{2}\left(\frac{\lambda}{h/mc}\right)^2
\end{aligned}$$

(c)

$$\begin{aligned}
\Delta &= 8.49 \times 10^{-8} \\
v &= (1 - 8.49 \times 10^{-8})c
\end{aligned}$$

39.2.71

(a)

$$\begin{aligned}
K &= 6mc^2 \\
E - mc^2 &= 6mc^2 \\
E &= 7mc^2
\end{aligned}$$

$$\begin{aligned}
(7mc^2)^2 &= (pc)^2 + (mc^2)^2 \\
49(mc^2)^2 &= (pc)^2 + (mc^2)^2 \\
48(mc^2)^2 &= (pc)^2 \\
pc &= \sqrt{48}mc^2 \\
p &= 4\sqrt{3}mc
\end{aligned}$$

$$\begin{aligned}
\lambda &= \frac{h}{p} \\
&= \frac{h}{4\sqrt{3}mc}
\end{aligned}$$

39.2.73

(a)

$$\begin{aligned}
\Delta x \Delta p_x &\geq \frac{\hbar}{2} \\
\Delta p_x &\geq \frac{\hbar}{2\Delta x} \\
&\geq 1.05 \times 10^{-20}\,\mathrm{kg\,m/s}
\end{aligned}$$

(b)

$$\begin{aligned}
K &= E - mc^2 \\
&= \sqrt{(mc^2)^2 + (pc)^2} - mc^2 \\
&= 3.15 \times 10^{-12}\,\mathrm{J} \\
&= 19.7\,\mathrm{MeV}
\end{aligned}$$

(c)

$$\begin{aligned}
U &= \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} \\
&= 4.60 \times 10^{-14}\,\mathrm{J} \\
&= 288\,\mathrm{keV} \\
\frac{K}{U} &= 68.4
\end{aligned}$$

No

39.2.75

(a)

$$\begin{aligned}\lambda &= \frac{h}{mv}\\ v &= \frac{h}{m\lambda}\\ &= 1.10 \times 10^{-35}\,\text{m/s}\end{aligned}$$

(b)

$$t = \frac{d}{v} = 2.31 \times 10^{27}\,\text{years}$$

No

39.2.77

$$\begin{aligned}d\sin\theta &= \lambda\\ d &= \frac{\lambda}{\sin\theta}\\ &= 5.13 \times 10^{-11}\,\text{m}\end{aligned}$$

$$\begin{aligned}E &= \frac{1}{2}mv^2\\ 2mE &= m^2v^2\\ &= p^2\\ p &= \sqrt{2mE}\end{aligned}$$

$$\begin{aligned}\lambda &= \frac{h}{p}\\ &= \frac{h}{\sqrt{2mE}}\end{aligned}$$

$$\begin{aligned}d\sin\theta &= \lambda\\ \theta &= \arcsin\frac{\lambda}{d}\\ &= \arcsin\frac{h}{d\sqrt{2mE}}\\ &= 20.89^\circ\end{aligned}$$

39.2.79

(a)

$$E = \frac{hc}{\lambda} = 3.98 \times 10^{-17}\,\mathrm{J} = 248\,\mathrm{eV}$$

(b)

$$\begin{aligned}
\lambda &= \frac{h}{mv} \\
v &= \frac{h}{m\lambda} \\
&= 1.45 \times 10^{5}\,\mathrm{m/s}
\end{aligned}$$

$$\begin{aligned}
K &= \frac{1}{2}mv^2 \\
&= 9.64 \times 10^{-21}\,\mathrm{J} \\
&= 0.060\,\mathrm{eV}
\end{aligned}$$

39.2.81

(a)

$$\begin{aligned}
F &= -\frac{dU}{dx} \\
&= -\frac{A|x|}{x}, x \neq 0
\end{aligned}$$

(b)

$$
\begin{aligned}
E &= \frac{p^2}{2m} + A|x| \\
&= \frac{h^2}{2mx^2} + A|x| \\
\frac{dE}{dx} &= -\frac{h^2}{mx^3} + \frac{A|x|}{x}, x \neq 0 \\
0 &= -\frac{h^2}{mx^3} + \frac{A|x|}{x} \\
\frac{h^2}{mx^3} &= \frac{A|x|}{x} \\
\frac{h^2}{m} &= A|x|x^2 \\
|x|^3 &= \frac{h^2}{Am} \\
x &= \pm\sqrt[3]{\frac{h^2}{Am}}
\end{aligned}
$$

$$
\begin{aligned}
E &= \frac{h^2}{2m}\left(\pm\sqrt[3]{\frac{Am}{h^2}}\right)^2 + A\sqrt[3]{\frac{h^2}{Am}} \\
&= \frac{h^2}{2m}\left(\frac{Am}{h^2}\right)^{2/3} + A\sqrt[3]{\frac{h^2}{Am}} \\
&= \frac{h^2}{2m}\sqrt[3]{\frac{A^2m^2}{h^4}} + \sqrt[3]{\frac{A^2h^2}{m}} \\
&= \frac{1}{2}\sqrt[3]{\frac{A^2h^2}{m}} + \sqrt[3]{\frac{A^2h^2}{m}} \\
&= \frac{3}{2}\sqrt[3]{\frac{A^2h^2}{m}}
\end{aligned}
$$

39.2.83

(a)

$$\begin{aligned}
E_2 - E_1 &= \frac{hc}{\lambda} \\
-\frac{mZ^2e^4}{32\epsilon_0^2h^2} + \frac{mZ^2e^4}{8\epsilon_0^2h^2} &= \frac{hc}{\lambda} \\
\frac{3mZ^2e^4}{32\epsilon_0^2h^2} &= \frac{hc}{\lambda} \\
Z &= \sqrt{\frac{32\epsilon_0^2h^3c}{3\lambda me^4}} \\
&= 3
\end{aligned}$$

(b)

$$\begin{aligned}
R' &= 9R \\
&= 9.87 \times 10^7\,\mathrm{m}^{-1} \\
E_1 &= -hcR' \\
&= -1.96 \times 10^{-17}\,\mathrm{J} \\
&= -123\,\mathrm{eV} \\
E_3 &= -\frac{hcR'}{9} \\
&= -13.6\,\mathrm{eV} \\
\lambda &= \frac{hc}{E} \\
&= 11.4\,\mathrm{nm}
\end{aligned}$$

(c)

$$\begin{aligned}
K &= E - E_{\mathrm{ionization}} \\
&= E + E_1 \\
&= \frac{hc}{\lambda} + E_1 \\
&= 60.2\,\mathrm{eV}
\end{aligned}$$

39.2.85

(a)

$$\begin{aligned}
P_{\text{Polaris}} &= A\sigma T^4 \\
&= 4\pi(rR_{\text{sun}})^2T^4 \\
&= 1.67\times10^{37}\,\text{W} \\
P_{\text{Vega}} &= 3.86\times10^{35}\,\text{W} \\
P_{\text{Antares}} &= 6.34\times10^{38}\,\text{W} \\
P_{\alpha\text{ Centauri B}} &= 3.49\times10^{33}\,\text{W}
\end{aligned}$$

Antares

(b)

$$\begin{aligned}
\lambda_{\text{Polaris}} &= \frac{b}{T} \\
&= 482\,\text{nm} \\
\lambda_{\text{Vega}} &= 302\,\text{nm} \\
\lambda_{\text{Antares}} &= 852\,\text{nm} \\
\lambda_{\alpha\text{ Centauri B}} &= 551\,\text{nm}
\end{aligned}$$

Polaris and α Centauri B

39.2.87

(a)

$$\begin{aligned}
f &= \frac{1}{T} \\
&= \frac{v_n}{2\pi r_n} \\
&= \frac{\frac{1}{\epsilon_0}\frac{e^2}{2nh}}{2\pi\epsilon_0\frac{n^2h^2}{\pi me^2}} \\
&= \frac{\pi me^4}{4\pi\epsilon_0^2n^3h^3} \\
&= \frac{me^4}{4\epsilon_0^2n^3h^3}
\end{aligned}$$

(b)

$$\begin{aligned}
hf &= E_i - E_f \\
&= -\frac{me^4}{8\epsilon_0^2 h^2 (n+1)^2} + \frac{me^4}{8\epsilon_0 h^2 n^2} \\
f &= \frac{me^4}{8\epsilon_0^2 h^3}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right) \\
&= \frac{me^4}{8\epsilon_0^2 h^3}\left(\frac{(n+1)^2 - n^2}{n^2(n+1)^2}\right) \\
&= \frac{me^4}{8\epsilon_0^2 h^3}\left(\frac{2n+1}{n^2(n+1)^2}\right) \\
&\approx \frac{me^4}{8\epsilon_0^2 h^3}\left(\frac{2n}{n^4}\right) \\
&\approx \frac{me^4}{4\epsilon_0^2 n^3 h^3}
\end{aligned}$$

39.2.89

$$\begin{aligned}
eV &= \frac{1}{2}mv^2 \\
v &= \sqrt{\frac{2eV}{m}}
\end{aligned}$$

$$\begin{aligned}
\lambda &= \frac{h}{mv} \\
&= \frac{h}{m\sqrt{2eV/m}} \\
\frac{2eV}{m} &= \left(\frac{h}{m\lambda}\right)^2 \\
V &= \frac{m}{2e}\left(\frac{h}{m\lambda}\right)^2 \\
&= 21\,\mathrm{kV}
\end{aligned}$$

a

39.2.91

a

40 Quantum Mechanics I: Wave Functions

40.1 Guided Practice

40.1.1

(a)

$$
\begin{aligned}
E_n &= \frac{n^2\pi^2\hbar^2}{2mL^2} \\
E_2 &= 4E_1 \\
\psi_n(x) &= \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L} \\
\Psi(x,t) &= \frac{1}{\sqrt{2}}\psi_1(x)e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}\psi_2(x)e^{-iE_2t/\hbar} \\
&= \frac{1}{\sqrt{L}}\sin\frac{\pi x}{L}e^{-iE_1t/\hbar} + \frac{1}{\sqrt{L}}\sin\frac{2\pi x}{L}e^{-iE_2t/\hbar} \\
|\Psi(x,t)|^2 &= \Psi(x,t)\Psi^*(x,t) \\
&= \frac{1}{L}\sin^2\frac{\pi x}{L} + \frac{1}{L}\sin^2\frac{2\pi x}{L} \\
&\quad + \frac{1}{L}\sin\frac{\pi x}{L}\sin\frac{2\pi x}{L}e^{i3E_1t/\hbar} \\
&\quad + \frac{1}{L}\sin\frac{\pi x}{L}\sin\frac{2\pi x}{L}e^{-i3E_1t/\hbar} \\
&= \frac{1}{L}\left[\sin^2\frac{\pi x}{L} + \sin^2\frac{2\pi x}{L}\right. \\
&\quad \left. +2\sin\frac{\pi x}{L}\sin\frac{2\pi x}{L}\cos\frac{(E_2-E_1)t}{\hbar}\right]
\end{aligned}
$$

(b) No, because $|\Psi(x,t)|^2$ depends on time.

(c)

$$
\begin{aligned}
&\int_0^L |\Psi(x,t)|^2 \, dx \\
&= \int_0^L \frac{1}{L} \left[\sin^2 \frac{\pi x}{L} + \sin^2 \frac{2\pi x}{L} + 2 \sin \frac{\pi x}{L} \sin \frac{2\pi x}{L} \cos \frac{(E_2 - E_1)t}{\hbar} \right] dx \\
&= \frac{1}{L} \int_0^L \left[\frac{1}{2} - \frac{1}{2} \cos \frac{2\pi x}{L} + \frac{1}{2} - \frac{1}{2} \cos \frac{4\pi x}{L} \right. \\
&\qquad \left. + \left(\cos \frac{\pi x}{L} - \cos \frac{3\pi x}{L} \right) \cos \frac{(E_2 - E_1)t}{\hbar} \right] dx \\
&= \frac{1}{L} \int_0^L \left[1 - \frac{1}{2} \cos \frac{2\pi x}{L} - \frac{1}{2} \cos \frac{4\pi x}{L} \right. \\
&\qquad \left. + \left(\cos \frac{\pi x}{L} - \cos \frac{3\pi x}{L} \right) \cos \frac{(E_2 - E_1)t}{\hbar} \right] dx \\
&= \frac{1}{L} [x]_0^L \\
&= 1
\end{aligned}
$$

(d)

$$
\begin{aligned}
\omega &= \frac{E_2 - E_1}{\hbar} \\
&= \frac{3\pi^2 \hbar}{2mL^2}
\end{aligned}
$$

This is the difference $\omega_2 - \omega_1$ or the beat frequency.

(e)

$$
\begin{aligned}
\omega &= \frac{E_2 - E_1}{\hbar} \\
&= \frac{2.43 E_{1\text{-IDW}} - 0.625 E_{1\text{-IDW}}}{\hbar} \\
&= \frac{1.805 E_{1\text{-IDW}}}{\hbar} \\
&= 0.903 \frac{\pi^2 \hbar}{mL^2}
\end{aligned}
$$

40.2 Exercises and Problems

40.2.1

$$
\begin{aligned}
p &= \hbar k \\
k &= \frac{p}{\hbar} \\
&= 4.27 \times 10^{10}\,\mathrm{rad/m} \\
E &= \hbar\omega \\
\omega &= \frac{E}{\hbar} \\
&= \frac{p^2/2m}{\hbar} \\
&= 1.06 \times 10^{17}\,\mathrm{rad/s} \\
\Psi(x,t) &= Ae^{i(-(4.27\times10^{10})x-(1.06\times10^{17})t)}
\end{aligned}
$$

40.2.3

(a)

$$|\Psi(x,t)|^2 = 2|A|^2\{1+\cos[2kx-(\omega_2-\omega_1)t]\}$$

$$\begin{aligned}\hbar\omega &= \frac{\hbar^2k^2}{2m}\\ \omega &= \frac{\hbar k^2}{2m}\end{aligned}$$

$$\begin{aligned}\omega_2 &= \frac{\hbar(3k)^2}{2m}\\ &= \frac{9\hbar k^2}{2m}\end{aligned}$$

$$\omega_1 = \frac{\hbar k^2}{2m}$$

$$\omega_2-\omega_1 = \frac{4\hbar k^2}{m}$$

$$\begin{aligned}|\Psi(x,t)|^2 &= 2|A|^2\left\{1+\cos\left[2kx-\frac{4\hbar k^2}{m}\frac{4\pi m}{\hbar k^2}\right]\right\}\\ &= 2|A|^2[1+\cos(2kx-16\pi)]\\ 2kx &= 16\pi\\ x &= \frac{16\pi}{2k}\\ &= \frac{8\pi}{k}\end{aligned}$$

(b)

$$\begin{aligned}v &= \frac{x}{t}\\ &= \frac{4}{k}\frac{\hbar k^2}{2m}\\ &= \frac{2\hbar k}{m}\end{aligned}$$

$$\begin{aligned}v_{\text{av}} &= \frac{\omega_2-\omega_1}{2k}\\ &= \frac{2\hbar k}{m}\end{aligned}$$

40.2.5

(a)

$$\begin{aligned}
\psi(x) &= A \sin \frac{2\pi}{\lambda} x \\
|\psi(x)|^2 &= A^2 \sin^2 \frac{2\pi}{\lambda} x \\
&= \frac{1}{2} A^2 \left(1 - \cos \frac{4\pi}{\lambda} x\right)
\end{aligned}$$

$$\begin{aligned}
\cos \frac{4\pi}{\lambda} x &= -1 \\
\frac{4\pi}{\lambda} x &= \pi + 2\pi n, n \in \mathbb{Z} \\
x &= \frac{\lambda}{4\pi}(\pi + 2\pi n), n \in \mathbb{Z} \\
&= \lambda \left(\frac{1}{4} + \frac{n}{2}\right), n \in \mathbb{Z} \\
&= \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots
\end{aligned}$$

(b)

$$\begin{aligned}
\cos \frac{4\pi}{\lambda} x &= 1 \\
\frac{4\pi}{\lambda} x &= 2\pi n, n \in \mathbb{Z} \\
x &= \frac{\lambda}{4\pi} 2\pi n, n \in \mathbb{Z} \\
&= \frac{\lambda n}{2}, n \in \mathbb{Z} \\
&= 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots
\end{aligned}$$

40.2.9

(a)

$$\begin{aligned}
E_1 &= \frac{\pi^2 \hbar^2}{2mL^2} \\
&= 1.62 \times 10^{-67}\,\mathrm{J}
\end{aligned}$$

(b)

$$
\begin{aligned}
K &= \frac{1}{2}mv^2 \\
v &= \sqrt{\frac{2K}{m}} \\
&= 1.27 \times 10^{-33}\,\mathrm{m/s} \\
t &= \frac{d}{v} \\
&= 3.75 \times 10^{25}\,\text{years}
\end{aligned}
$$

(c)

$$
\begin{aligned}
E_2 - E_1 &= \frac{4\pi^2\hbar^2}{2mL} - \frac{\pi^2\hbar^2}{2mL} \\
&= \frac{3\pi^2\hbar^2}{2mL} \\
&= 3E_1 \\
&= 4.87 \times 10^{-67}\,\mathrm{J}
\end{aligned}
$$

(d) No

40.2.11

$$
\begin{aligned}
E &= \frac{\pi^2\hbar^2}{2mL^2} \\
L &= \sqrt{\frac{\pi^2\hbar^2}{2mE}} \\
&= 0.166\,\mathrm{nm}
\end{aligned}
$$

40.2.13

$$
\begin{aligned}
E &= E_2 - E_1 \\
&= \frac{3\pi^2\hbar^2}{2mL^2} \\
L &= \sqrt{\frac{3\pi^2\hbar^2}{2mE}} \\
&= 0.568\,\mathrm{nm}
\end{aligned}
$$

40.2.15

(a)

$$
\int_{-\infty}^{\infty} |\psi(x,t)|^2\,dx = 1
$$

(b)

$$\begin{aligned}\int_{-\infty}^{\infty} |e^{ax}|^2\,dx &= \int_{-\infty}^{\infty} e^{2ax}\,dx\\ &= \left[\frac{1}{2a}e^{2ax}\right]_{-\infty}^{\infty}\\ &= \infty\end{aligned}$$

No, no.

(c)

$$\begin{aligned}\int_{0}^{\infty} |Ae^{-bx}|^2\,dx &= A^2\int_{0}^{\infty} e^{-2bx}\,dx\\ &= -\frac{A^2}{2b}[e^{-2bx}]_0^{\infty}\\ &= \frac{A^2}{2b}\\ A &= \sqrt{2b}\,\mathrm{m}^{-1/2}\end{aligned}$$

40.2.17

(a)

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$$

$$\begin{aligned}\sin\frac{2\pi x}{L} &= 0\\ \frac{2\pi x}{L} &= n\pi, n \in \mathbb{Z}\\ x &= \frac{Ln}{2}, n \in \mathbb{Z}\\ &= 0, \frac{L}{2}, L\end{aligned}$$

(b)

$$\begin{aligned}
|\psi(x)|^2 &= \frac{2}{L}\sin^2\frac{2\pi x}{L} \\
&= \frac{1}{L}\left(1-\cos\frac{4\pi x}{L}\right) \\
\cos\frac{4\pi x}{L} &= -1 \\
\frac{4\pi x}{L} &= \pi + 2\pi n, n \in \mathbb{Z} \\
x &= \frac{L}{4\pi}(\pi + 2\pi n), n \in \mathbb{Z} \\
&= \frac{L}{4}(1+2n), n \in \mathbb{Z} \\
&= \frac{L}{4}, \frac{3L}{4}
\end{aligned}$$

(c) Yes

40.2.19

(a)

$$\begin{aligned}
\lambda_1 &= 2L \\
&= 6.8 \times 10^{-10}\,\mathrm{m} \\
p_1 &= \frac{h}{2L} \\
&= 9.74 \times 10^{-25}\,\mathrm{kg\,m/s}
\end{aligned}$$

(b)

$$\begin{aligned}
\lambda_2 &= 3.4 \times 10^{-10}\,\mathrm{m} \\
p_2 &= 1.95 \times 10^{-24}\,\mathrm{kg\,m/s}
\end{aligned}$$

(c)

$$\begin{aligned}
\lambda_3 &= 2.27 \times 10^{-10}\,\mathrm{m} \\
p_3 &= 2.92 \times 10^{-24}\,\mathrm{kg\,m/s}
\end{aligned}$$

40.2.21

$$\begin{aligned}
E_1 &= 0.625 E_{\text{1-IDW}} \\
&= 0.625 \frac{\pi^2 \hbar^2}{2mL^2} \\
L &= \sqrt{0.625 \frac{\pi^2 \hbar^2}{2mE_1}} \\
&= 0.313\,\mathrm{nm}
\end{aligned}$$

40.2.23

$$\begin{aligned}
E &= hf \\
U_0 - E_1 &= \frac{hc}{\lambda} \\
6E_{\text{1-IDW}} - 0.625 E_{\text{1-IDW}} &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{5.375 E_{\text{1-IDW}}} \\
&= \frac{hc}{5.375 \frac{\pi^2 \hbar^2}{2mL^2}} \\
&= \frac{2mL^2 hc}{5.375 \pi^2 \hbar^2} \\
&= 613\,\mathrm{nm}
\end{aligned}$$

40.2.25

$$\begin{aligned}
E &= hf \\
E_3 - E_1 &= \frac{hc}{\lambda} \\
5.09 E_{\text{1-IDW}} - 0.625 E_{\text{1-IDW}} &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{4.465 E_{\text{1-IDW}}} \\
&= \frac{hc}{4.465 \frac{\pi^2 \hbar^2}{2mL^2}} \\
&= \frac{2mL^2 hc}{4.465 \pi^2 \hbar^2} \\
&= 27\,\mathrm{fm}
\end{aligned}$$

40.2.27

(a)

$$
\begin{aligned}
G &= 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) \\
&= 2.74 \\
\kappa &= \frac{\sqrt{2m(U_0 - E)}}{\hbar} \\
&= 1.53 \times 10^{10} \\
T &= Ge^{-2\kappa L} \\
&= 0.0013
\end{aligned}
$$

(b)

$$
\begin{aligned}
\kappa &= 6.57 \times 10^{11} \\
T &= 1 \times 10^{-143}
\end{aligned}
$$

40.2.29

(a)

$$
\begin{aligned}
G &= 3.99 \\
\kappa &= 1.16 \times 10^{10} \\
T &= 1.1 \times 10^{-8}
\end{aligned}
$$

(b)

$$
T = 3.7 \times 10^{-4}
$$

40.2.31

$$
\begin{aligned}
E_1 &= \frac{p_1^2}{2m} \\
U_0 &= \frac{h^2}{2m\lambda_1^2} \\
\lambda_1 &= \sqrt{\frac{h^2}{2mU_0}}
\end{aligned}
$$

$$
\begin{aligned}
\lambda_2 &= \sqrt{\frac{h^2}{4mU_0}} \\
&= \frac{1}{\sqrt{2}}\lambda_1
\end{aligned}
$$

40.2.33

$$\begin{aligned}
E_n &= \left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{1000}{3}} \\
E_0 &= 9.63\times10^{-34}\,\mathrm{J} \\
&= 6.02\times10^{-15}\,\mathrm{eV} \\
E_{n+1}-E_n &= \hbar\sqrt{\frac{1000}{3}} \\
&= 1.93\times10^{-33}\,\mathrm{J} \\
&= 1.2\times10^{-14}\,\mathrm{eV}
\end{aligned}$$

No

40.2.35

(a)

$$E=\frac{hc}{\lambda}=3.43\times10^{-20}\,\mathrm{J}=0.214\,\mathrm{eV}$$

(b)

$$\begin{aligned}
\Delta E &= \hbar\sqrt{\frac{k}{m}} \\
k &= m\left(\frac{\Delta E}{\hbar}\right)^2 \\
&= 5.9\times10^{3}\,\mathrm{N/m}
\end{aligned}$$

40.2.37

$$\begin{aligned}
E_0 &= \frac{1}{2}\hbar\omega \\
\omega &= \frac{2E_0}{\hbar} \\
&= 1.76\times10^{16}\,\mathrm{rad/s}
\end{aligned}$$

$$\begin{aligned}
E &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{E} \\
&= \frac{hc}{\hbar\omega} \\
&= \frac{2\pi c}{\omega} \\
&= 107\,\mathrm{nm}
\end{aligned}$$

40.2.39

$$E = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\begin{aligned}\frac{1}{2}k'A^2 &= \left(n + \frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}} \\ A &= \sqrt{\frac{2\hbar}{\sqrt{k'm}}\left(n + \frac{1}{2}\right)}\end{aligned}$$

$$\begin{aligned}\frac{1}{2}mv_{\max}^2 &= \frac{1}{2}k'A^2 \\ v_{\max} &= A\sqrt{\frac{k'}{m}} \\ &= \sqrt{\frac{2\hbar k'^{1/2}}{m^{3/2}}\left(n + \frac{1}{2}\right)}\end{aligned}$$

$$p_{\max} = \sqrt{2\hbar\sqrt{k'm}\left(n + \frac{1}{2}\right)}$$

$$\begin{aligned}\Delta x\Delta p &= \frac{A}{\sqrt{2}}\frac{p_{\max}}{\sqrt{2}} \\ &= \frac{1}{2}\sqrt{4\hbar^2\left(n + \frac{1}{2}\right)^2} \\ &= \hbar\left(n + \frac{1}{2}\right)\end{aligned}$$

40.2.41

(a)

$$E_0 = \frac{1}{2}\hbar\omega = 9.44 \times 10^{-22}\,\mathrm{J} = 0.0059\,\mathrm{eV}$$

(b)

$$\begin{aligned}
\lambda &= \frac{hc}{E} \\
&= \frac{hc}{\hbar\omega} \\
&= \frac{2\pi c}{\omega} \\
&= 105\,\mu\mathrm{m}
\end{aligned}$$

(c)

$$\Delta E = \hbar\omega = 1.89 \times 10^{-21}\,\mathrm{J} = 0.0118\,\mathrm{eV}$$

40.2.43

(a)

$$\begin{aligned}
E_n &= \frac{n^2\pi^2\hbar^2}{2mL^2} \\
\psi_n(x) &= \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} \\
\Psi(x,t) &= \frac{1}{\sqrt{L}}\sin\frac{\pi x}{L}e^{-i\pi^2\hbar t/2mL^2} + \frac{1}{\sqrt{L}}\sin\frac{3\pi x}{L}e^{-i9\pi^2\hbar t/2mL^2} \\
\Psi(L/2,t) &= \frac{1}{\sqrt{L}}e^{-i\pi^2\hbar t/2mL^2} - \frac{1}{\sqrt{L}}e^{-i9\pi^2\hbar t/2mL^2} \\
|\Psi(L/2,t)|^2 &= \Psi(L/2,t)\Psi^*(L/2,t) \\
&= \frac{1}{L}\left(1 - e^{i8\pi^2\hbar t/2mL^2} - e^{-i8\pi^2\hbar t/2mL^2} + 1\right) \\
&= \frac{2}{L}\left[1 - \cos\left(\frac{4\pi^2\hbar}{mL^2}t\right)\right]
\end{aligned}$$

(b)

$$\omega = \frac{4\pi^2\hbar}{mL^2}$$

40.2.45

$$
\begin{aligned}
A + B &= C \\
k_1(A - B) &= k_2 C
\end{aligned}
$$

$$
\begin{aligned}
k_1(A - C + A) &= k_2 C \\
2k_1 A - k_1 C &= k_2 C \\
C &= \frac{2k_1}{k_1 + k_2} A
\end{aligned}
$$

$$
\begin{aligned}
A + B &= \frac{2k_1}{k_1 + k_2} A \\
B &= \frac{k_1 - k_2}{k_1 + k_2} A
\end{aligned}
$$

40.2.47

(a)

$$
\begin{aligned}
E &= hf \\
E_2 - E_1 &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{\frac{3\pi^2\hbar^2}{2mL^2}} \\
&= \frac{2mL^2 hc}{3\pi^2\hbar^2} \\
&= 1.94 \times 10^{-5}\,\mathrm{m} \\
&= 19.4\,\mu\mathrm{m}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\lambda &= \frac{2mL^2 hc}{5\pi^2\hbar^2} \\
&= 11.6\,\mu\mathrm{m}
\end{aligned}
$$

40.2.49

(a)

$$\begin{aligned}
\psi_2(x) &= \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L} \\
|\psi_2(x)|^2 &= \frac{2}{L} \sin^2 \frac{2\pi x}{L} \\
&= \frac{1}{L} \left(1 - \cos \frac{4\pi x}{L}\right) \\
|\psi_2(L/4)|^2 \, dx &= \frac{2}{L} \, dx
\end{aligned}$$

(b)

$$|\psi_2(L/2)|^2 \, dx = 0$$

(c)

$$|\psi_2(3L/4)|^2 \, dx = \frac{2}{L} \, dx$$

40.2.51

(a)

$$\begin{aligned}
\psi_1(x) &= \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L} \\
|\psi_1(x)|^2 &= \frac{2}{L} \sin^2 \frac{\pi x}{L} \\
&= \frac{1}{L} \left(1 - \cos \frac{2\pi x}{L}\right) \\
\int_{L/4}^{3L/4} |\psi_1(x)|^2 \, dx &= \int_{L/4}^{3L/4} \frac{1}{L} \left(1 - \cos \frac{2\pi x}{L}\right) \, dx \\
&= \frac{1}{L} \left[x - \frac{L}{2\pi} \sin \frac{2\pi x}{L}\right]_{L/4}^{3L/4} \\
&= \frac{1}{L} \left(\frac{3L}{4} + \frac{L}{2\pi} - \frac{L}{4} + \frac{L}{2\pi}\right) \\
&= \frac{1}{2} + \frac{1}{\pi} \\
&\approx 0.818
\end{aligned}$$

(b)

$$
\begin{aligned}
|\psi_2(x)|^2 &= \frac{1}{L}\left(1 - \cos\frac{4\pi x}{L}\right) \\
\int_{L/4}^{3L/4} |\psi_2(x)|^2\,dx &= \int_{L/4}^{3L/4} \frac{1}{L}\left(1 - \cos\frac{4\pi x}{L}\right)\,dx \\
&= \frac{1}{L}\left[x - \frac{L}{4\pi}\sin\frac{4\pi x}{L}\right]_{L/4}^{3L/4} \\
&= \frac{1}{L}\left(\frac{3L}{4} - \frac{L}{4}\right) \\
&= \frac{1}{2}
\end{aligned}
$$

(c) Yes

40.2.53

(b)

$$
\begin{aligned}
-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} &= E\psi(x) \\
-\frac{\hbar^2\kappa^2}{2m}\psi(x) &= E\psi(x) \\
E &= -\frac{\hbar^2\kappa^2}{2m}
\end{aligned}
$$

(c)

$$
\begin{aligned}
-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} &= E\psi(x) \\
-\frac{\hbar^2\kappa^2}{2m}\psi(x) &= E\psi(x) \\
E &= -\frac{\hbar^2\kappa^2}{2m}
\end{aligned}
$$

(d) Because the derivative is discontinuous at $x = 0$.

40.2.55

(a)

$$\begin{aligned}
\psi_{\text{inside}}(x) &= A\cos\left(\frac{\sqrt{2mE}}{\hbar}x\right) + B\sin\left(\frac{\sqrt{2mE}}{\hbar}x\right)\\
\psi_{\text{outside}}(x) &= Ce^{\kappa x} + De^{-\kappa x}
\end{aligned}$$

$$\begin{aligned}
\psi_{\text{outside}}(0) &= \psi_{\text{inside}}(0)\\
C &= A
\end{aligned}$$

$$\begin{aligned}
\psi_{\text{outside}}(L) &= \psi_{\text{inside}}(L)\\
De^{-\kappa L} &= A\cos\left(\frac{\sqrt{2mE}}{\hbar}L\right) + B\sin\left(\frac{\sqrt{2mE}}{\hbar}L\right)
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{d\psi_{\text{outside}}(x)}{dx} &= \kappa Ce^{\kappa x} - \kappa De^{-\kappa x}\\
\frac{d\psi_{\text{inside}}(x)}{dx} &= -kA\sin kx + kB\cos kx
\end{aligned}$$

$$\begin{aligned}
\frac{d\psi_{\text{outside}}(0)}{dx} &= \frac{d\psi_{\text{inside}}(0)}{dx}\\
\kappa C &= kB
\end{aligned}$$

$$\begin{aligned}
\frac{d\psi_{\text{outside}}(L)}{dx} &= \frac{d\psi_{\text{inside}}(L)}{dx}\\
-\kappa De^{-\kappa L} &= -kA\sin kL + kB\cos kL
\end{aligned}$$

40.2.57

$$
\begin{aligned}
T &= 0.500\,\mathrm{s} \\
E_0 &= \frac{1}{2}\hbar\omega \\
&= \frac{1}{2}\frac{h}{2\pi}\frac{2\pi}{T} \\
&= \frac{h}{2T} \\
&= h \\
&= 6.626 \times 10^{-34}\,\mathrm{J} \\
&= 4.414 \times 10^{-15}\,\mathrm{eV} \\
\Delta E &= \hbar\omega \\
&= \frac{h}{2\pi}\frac{2\pi}{T} \\
&= 2h \\
&= 1.33 \times 10^{-33}\,\mathrm{J} \\
&= 8.28 \times 10^{-15}\,\mathrm{eV}
\end{aligned}
$$

No

40.2.59

(a)

$$
\begin{aligned}
\lambda &= \frac{2L}{n} \\
p &= \frac{h}{\lambda} \\
&= \frac{nh}{2L} \\
E &= K \\
&= \frac{1}{2}mv^2 \\
&= \frac{p^2}{2m} \\
&= \left(\frac{nh}{2L}\right)^2 \frac{1}{2m} \\
&= \frac{n^2h^2}{8mL^2}
\end{aligned}
$$

(b)

$$
\begin{aligned}
E_1 &= \frac{h^2}{8mL^2} \\
&= 2.15 \times 10^{-17}\,\mathrm{J} \\
&= 135\,\mathrm{eV}
\end{aligned}
$$

40.2.61

(a)

$$
\begin{aligned}
E_n - E_1 &= \frac{(n^2-1)\pi^2\hbar^2}{2mL^2} \\
L^2 &= \frac{(n^2-1)\pi^2\hbar^2}{2mhf_n}
\end{aligned}
$$

$$
\begin{aligned}
E_{n+1} - E_1 &= \frac{(n^2+2n)\pi^2\hbar^2}{2mL^2} \\
L^2 &= \frac{(n^2+2n)\pi^2\hbar^2}{2mhf_{n+1}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{(n^2-1)\pi^2\hbar^2}{2mhf_n} &= \frac{(n^2+2n)\pi^2\hbar^2}{2mhf_{n+1}} \\
f_{n+1}(n^2-1) &= f_n(n^2+2n) \\
(f_{n+1}-f_n)n^2 - 2f_n n - f_{n+1} &= 0 \\
n &= \frac{2f_n \pm \sqrt{4f_n^2 + 4(f_{n+1}-f_n)f_{n+1}}}{2(f_{n+1}-f_n)} \\
&= \frac{f_n \pm \sqrt{f_n^2 + f_{n+1}(f_{n+1}-f_n)}}{f_{n+1}-f_n} \\
&= 3
\end{aligned}
$$

The two principle quantum numbers are 3 and 4.

(b)

$$
\begin{aligned}
E_4 - E_3 &= \frac{7\pi^2\hbar^2}{2mL^2} \\
h(f_4 - f_3) &= \frac{7\pi^2\hbar^2}{2mL^2} \\
L &= \sqrt{\frac{7\pi^2\hbar^2}{2mh(f_4-f_3)}} \\
&= 0.9\,\mathrm{nm}
\end{aligned}
$$

(c)

$$
\begin{aligned}
E_2 - E_1 &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{\frac{3\pi^2\hbar^2}{2mL^2}} \\
&= \frac{2mL^2hc}{3\pi^2\hbar^2} \\
&= 890\,\mathrm{nm}
\end{aligned}
$$

40.2.63

$$
\begin{aligned}
E &= \frac{1}{2}mv^2 \\
&= \frac{p^2}{2m} \\
p &= \sqrt{2Em}
\end{aligned}
$$

$$
\begin{aligned}
\lambda &= \frac{h}{p} \\
\frac{2L}{n} &= \frac{h}{\sqrt{2Em}} \\
2Em &= \left(\frac{hn}{2L}\right)^2 \\
E &= \left(\frac{hn}{2L}\right)^2 \frac{1}{2m}
\end{aligned}
$$

$$
\begin{aligned}
E_1 &= 12\,\mathrm{eV} \\
E_2 &= 46\,\mathrm{eV} \\
E_3 &= 105\,\mathrm{eV}
\end{aligned}
$$

Adding $10\,\mathrm{eV}$ to each value gives $22\,\mathrm{eV}$, $56\,\mathrm{eV}$, and $115\,\mathrm{eV}$.

40.2.65

(a)

$$
\begin{aligned}
E &= \frac{1}{2}k'A^2 \\
A &= \pm\sqrt{\frac{2E}{k'}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{nh}{2} &= \int_{-A}^{A} \sqrt{2m[E - \frac{1}{2}k'x^2]}\, dx \\
&= 2\sqrt{k'm} \int_{0}^{A} \sqrt{\frac{2E}{k'} - x^2}\, dx \\
&= \sqrt{k'm} \left[x\sqrt{\frac{2E}{k'} - x^2} + \frac{2E}{k'} \arcsin\left(\frac{x}{\sqrt{2E/k'}} \right) \right]_0^{\sqrt{2E/k'}} \\
&= \sqrt{k'm} \left(\frac{2E}{k'} \frac{\pi}{2} \right) \\
&= \sqrt{\frac{m}{k'}} E\pi \\
E &= n\hbar\omega
\end{aligned}
$$

(c) Underestimate

40.2.67

c

40.2.69

a

41 Quantum Mechanics II: Atomic Structure

41.1 Guided Practice

41.1.1

(a)

$$
\begin{aligned}
l^3 &= \frac{4}{3}\pi r^3 \\
l &= \sqrt[3]{\frac{4}{3}}\pi r \\
&= 2.37 \times 10^{-10}\,\text{m}
\end{aligned}
$$

(b) The values of (n_X, n_Y, n_Z, m_s) for the 22 electrons are:

$$\left(1, 1, 1, +\frac{1}{2}\right)$$

$$\left(1, 1, 1, -\frac{1}{2}\right)$$

$$\left(2, 1, 1, +\frac{1}{2}\right)$$

$$\left(2, 1, 1, -\frac{1}{2}\right)$$

$$\left(1, 2, 1, +\frac{1}{2}\right)$$

$$\left(1, 2, 1, -\frac{1}{2}\right)$$

$$\left(1, 1, 2, +\frac{1}{2}\right)$$

$$\left(1, 1, 2, -\frac{1}{2}\right)$$

$$\left(2, 2, 1, +\frac{1}{2}\right)$$

$$\left(2, 2, 1, -\frac{1}{2}\right)$$

$$\left(2, 1, 2, +\frac{1}{2}\right)$$

$$\left(2, 1, 2, -\frac{1}{2}\right)$$

$$\left(1, 2, 2, +\frac{1}{2}\right)$$

$$\left(1, 2, 2, -\frac{1}{2}\right)$$

$$\left(2, 2, 2, +\frac{1}{2}\right)$$

$$\left(2, 2, 2, -\frac{1}{2}\right)$$

$$\left(3, 1, 1, +\frac{1}{2}\right)$$

$$\left(3, 1, 1, -\frac{1}{2}\right)$$

$$\left(1, 3, 1, +\frac{1}{2}\right)$$

$$\left(1, 3, 1, -\frac{1}{2}\right)$$

$$\left(1, 1, 3, +\frac{1}{2}\right)$$

$$\left(1, 1, 3, -\frac{1}{2}\right)$$

(c)

$$
\begin{aligned}
E_{111} &= 3.218 \times 10^{-18}\,\mathrm{J} = 20.1\,\mathrm{eV} \\
E_{211} &= 6.435 \times 10^{-18}\,\mathrm{J} = 40.2\,\mathrm{eV} \\
E_{221} &= 9.653 \times 10^{-18}\,\mathrm{J} = 60.3\,\mathrm{eV} \\
E_{222} &= 1.287 \times 10^{-17}\,\mathrm{J} = 80.4\,\mathrm{eV} \\
E_{311} &= 1.180 \times 10^{-17}\,\mathrm{J} = 73.8\,\mathrm{eV}
\end{aligned}
$$

(d)

$$
\begin{aligned}
E_{222} - E_{111} &= 60.3\,\mathrm{eV} \\
f &= (2.48 \times 10^{15}\,\mathrm{Hz})(Z-1)^2 \\
&= 1.094 \times 10^{18}\,\mathrm{Hz} \\
E &= hf \\
&= 4.529\,\mathrm{keV}
\end{aligned}
$$

41.2 Exercises and Problems

41.2.1

(a) 1

(b) 3

41.2.3

$$
\begin{aligned}
E_{221} &= \frac{(n_X^2 + n_Y^2 + n_Z^2)\pi^2\hbar^2}{2mL^2} \\
&= 6.69 \times 10^{-17}\,\mathrm{J} \\
E_{111} &= 2.23 \times 10^{-17}\,\mathrm{J} \\
\lambda &= \frac{hc}{E} \\
&= \frac{hc}{E_{221} - E_{111}} \\
&= 4.46\,\mathrm{nm}
\end{aligned}
$$

41.2.5

When $n_X = 2$, $n_Y = 2$, and $n_Z = 1$, the probability distribution function is zero at $x = \frac{L}{2}$ and $y = \frac{L}{2}$.

When $n_X = 2$, $n_Y = 1$, and $n_Z = 1$, the probability distribution function is zero at $x = \frac{L}{2}$.

When $n_X = 1$, $n_Y = 1$, and $n_Z = 1$, the probability distribution function only zero on the walls.

41.2.7

(a) $L_{\text{min}} = 0$

(b) $L_{\text{max}} = \sqrt{l(l+1)}\hbar = \sqrt{12}\hbar = 3.65 \times 10^{-34}\,\text{kg}\,\text{m}^2/\text{s}$

(c) $L_{z,\text{max}} = l\hbar = 3\hbar = 3.16 \times 10^{-34}\,\text{kg}\,\text{m}^2/\text{s}$

(d) $S_z = m_s\hbar = \frac{1}{2}\hbar = 5.27 \times 10^{-35}\,\text{kg}\,\text{m}^2/\text{s}$

(e) $\frac{S_z}{L_z} = \frac{1}{6}$

41.2.9

$$\begin{aligned}
L &= \sqrt{l(l+1)}\hbar \\
\left(\frac{L}{\hbar}\right)^2 &= l(l+1) \\
l^2 + l - \left(\frac{L}{\hbar}\right)^2 &= 0 \\
l &= -5 \text{ or } 4 \\
l &= 4
\end{aligned}$$

41.2.11

If it's the smallest angle then m_l is maximised, i.e. $m_l = l$.

$$\begin{aligned}
\cos\theta &= \frac{L_z}{L} \\
&= \frac{l\hbar}{\sqrt{l(l+1)}\hbar} \\
&= \frac{l}{\sqrt{l(l+1)}} \\
\cos^2\theta &= \frac{l^2}{l(l+1)} \\
&= \frac{l}{l+1} \\
(l+1)\cos^2\theta &= l \\
l(\cos^2\theta - 1) + \cos^2\theta &= 0 \\
l &= \cot^2\theta \\
&= 4
\end{aligned}$$

41.2.13

$$\begin{aligned}
L_9 &= \sqrt{l(l+1)}\hbar \\
&= 1.00 \times 10^{-33}\,\mathrm{kg\,m^2/s} \\
&= 1.054(9\hbar) \\
L_{18} &= 1.95 \times 10^{-33}\,\mathrm{kg\,m^2/s} \\
&= 1.0274(18\hbar) \\
L_{208} &= 2.20 \times 10^{-32}\,\mathrm{kg\,m^2/s} \\
&= 1.0024(208\hbar)
\end{aligned}$$

The Bohr model becomes more accurate as n increases.

41.2.15

(a) 10

(b) $m_l = -2$, $\theta = \arccos \frac{L_z}{L} = \arccos \frac{m_l \hbar}{\sqrt{l(l+1)}\hbar} = 145°$

(c) $m_l = 2$, $\theta = 35°$

41.2.17

$$\begin{aligned}
\Phi(\phi + 2\pi) &= e^{im_l(\phi+2\pi)} \\
&= \cos m_l(\phi + 2\pi) + i \sin m_l(\phi + 2\pi) \\
&= \cos(m_l\phi + 2m_l\pi) + i \sin(m_l\phi + 2m_l\pi)
\end{aligned}$$

This is only equal to $\Phi(\phi)$ if $2m_l\pi$ is a multiple of 2π which only happens if $m_l \in \mathbb{Z}$.

41.2.19

(a)

$$\begin{aligned}
\Delta E &= \mu_B B \\
B &= \frac{\Delta E}{\mu_B} \\
&= \frac{2m\Delta E}{e\hbar} \\
&= 0.506\,\mathrm{T}
\end{aligned}$$

(b) 3

41.2.21

(a) 9

(b) $\Delta E = \mu_B B = 1.45 \times 10^{-5}\,\text{eV}$

(c) $\Delta E = 8\mu_B B = 1.16 \times 10^{-4}\,\text{eV}$

41.2.23

(a)

$$\begin{aligned} L &= I\omega \\ &= \frac{2}{5}mr^2\omega \\ \omega &= \frac{5L}{2mr^2} \\ &= 7.74 \times 10^{29}\,\text{rad/s} \end{aligned}$$

(b)

$$v = r\omega = 1.39 \times 10^{13}\,\text{m/s}$$

This isn't valid because it's greater than c.

41.2.25

$$\begin{aligned} \Delta E &= 2 \times 1.00116\mu_B B_z \\ &= 2.00232\frac{e\hbar}{2m}B_z \\ &= 2.78 \times 10^{-23}\,\text{J} \\ &= 1.74 \times 10^{-4}\,\text{eV} \end{aligned}$$

$m_s = \frac{1}{2}$ has lower energy.

41.2.27

$$\left(1, 0, 0, +\frac{1}{2}\right)$$
$$\left(1, 0, 0, -\frac{1}{2}\right)$$
$$\left(2, 0, 0, +\frac{1}{2}\right)$$
$$\left(2, 0, 0, -\frac{1}{2}\right)$$
$$\left(2, 1, -1, +\frac{1}{2}\right)$$
$$\left(2, 1, -1, -\frac{1}{2}\right)$$
$$\left(2, 1, 0, +\frac{1}{2}\right)$$
$$\left(2, 1, 0, -\frac{1}{2}\right)$$
$$\left(2, 1, 1, -\frac{1}{2}\right)$$
$$\left(2, 1, 1, +\frac{1}{2}\right)$$

41.2.29

(a) $1s^2 2s^2$

(b) $1s^2 2s^2 2p^6 3s^2$, $Z = 12$ Magnesium

(c) $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$, $Z = 20$, Calcium

41.2.31

$$\begin{aligned} E_n &= \frac{Z_{\text{eff}}^2}{n^2}(13.6\,\text{eV}) \\ &= 4.81\,\text{eV} \end{aligned}$$

41.2.33

(a) $1s^2 2s^2 2p$

(b)

$$\begin{aligned} E_n &= -\frac{Z_{\text{eff}}^2}{n^2}(13.6\,\text{eV}) \\ &= -\frac{3^2}{2^2}(13.6\,\text{eV}) \\ &= -30.6\,\text{eV} \end{aligned}$$

(c) $1s^2 2s^2 2p^6 3s^2 3p$

(d)

$$\begin{aligned} E_n &= -\frac{Z_{\text{eff}}^2}{n^2}(13.6\,\text{eV}) \\ &= -\frac{3^2}{3^2}(13.6\,\text{eV}) \\ &= -13.6\,\text{eV} \end{aligned}$$

41.2.35

(a)

$$\begin{aligned} E_n &= -\frac{Z_{\text{eff}}^2}{n^2}(13.6\,\text{eV}) \\ &= -\frac{2^2}{2^2}(13.6\,\text{eV}) \\ &= -13.6\,\text{eV} \end{aligned}$$

(b)

$$E_n = -3.40\,\text{eV}$$

41.2.37

(a)

$$\begin{aligned} f &= (2.48 \times 10^{15}\,\text{Hz})(Z-1)^2 \\ &= 8.95 \times 10^{17}\,\text{Hz} \\ E &= 3.71\,\text{keV} \end{aligned}$$

(b)

$$\begin{aligned} f &= 1.68 \times 10^{18}\,\text{Hz} \\ E &= 6.94\,\text{keV} \end{aligned}$$

(c)

$$\begin{aligned} f &= 5.48 \times 10^{18}\,\text{Hz} \\ E &= 22.7\,\text{keV} \end{aligned}$$

41.2.39

$$E_{221} = 3E_{111}$$

41.2.41

(a)

$$\frac{(L/4)^3}{L^3} = \frac{L^3/64}{L^3} = \frac{1}{64}$$

(b)

$$\begin{aligned}
\int |\psi(x,y,z)|^2\,dV &= |C|^2 \left(\int_0^{L/4} \sin^2\frac{\pi x}{L}\,dx\right)\left(\int_0^{L/4} \sin^2\frac{\pi y}{L}\,dy\right) \\
&\quad \left(\int_0^{L/4} \sin^2\frac{\pi z}{L}\,dz\right) \\
&= \left(\frac{2}{L}\right)^3 \left(\frac{L(\pi-2)}{8\pi}\right)^3 \\
&= \frac{8}{L^3}\frac{L^3(\pi-2)^3}{512\pi^3} \\
&= \frac{(\pi-2)^3}{64\pi^3} \\
&\approx 7.50\times 10^{-4}
\end{aligned}$$

(c)

$$\begin{aligned}
\int |\psi(x,y,z)|^2\,dV &= \left(\frac{2}{L}\right)^3 \frac{L}{8}\left(\frac{L(\pi-2)}{8\pi}\right)^2 \\
&= \frac{8}{L^3}\frac{L}{8}\frac{L^2(\pi-2)^2}{64\pi^2} \\
&= \frac{(\pi-2)^2}{64\pi^2} \\
&\approx 2.06\times 10^{-3}
\end{aligned}$$

41.2.43

(a)

$$
\begin{aligned}
&|C|^2 \left(\int_0^{L/2} \sin^2 \frac{\pi x}{L}\,dx\right)\left(\int_0^{L} \sin^2 \frac{\pi y}{L}\,dy\right)\left(\int_0^{L} \sin^2 \frac{\pi z}{L}\,dz\right)\\
&= \left(\frac{2}{L}\right)^3 \frac{L}{4}\left(\frac{L}{2}\right)^2\\
&= \frac{8}{L^3}\frac{L}{4}\frac{L^2}{4}\\
&= \frac{1}{2}
\end{aligned}
$$

(b)

$$
\begin{aligned}
&|C|^2 \left(\int_{L/4}^{L/2} \sin^2 \frac{\pi x}{L}\,dx\right)\left(\int_0^{L} \sin^2 \frac{\pi y}{L}\,dy\right)\left(\int_0^{L} \sin^2 \frac{\pi z}{L}\,dz\right)\\
&= \left(\frac{2}{L}\right)^3 \frac{L(\pi+2)}{8\pi}\left(\frac{L}{2}\right)^2\\
&= \frac{8}{L^3}\frac{L(\pi+2)}{8\pi}\frac{L^2}{4}\\
&= \frac{\pi+2}{4\pi}\\
&\approx 0.409
\end{aligned}
$$

41.2.47

(a)

$$
\begin{aligned}
\sum_{l=0}^{n-1} 2(2l+1) &= \sum_{l=0}^{n-1}(4l+2)\\
&= 2n + 4\sum_{l=0}^{n-1} l\\
&= 2n + 4\frac{(n-1)n}{2}\\
&= 2n + 2(n^2-n)\\
&= 2n^2
\end{aligned}
$$

(b)

$$50 = 2n^2 \to n = 5$$

The O shell has 50 states.

41.2.49

(a)

$$\begin{aligned}
E &= U \\
\frac{1}{(4\pi\epsilon_0)^2}\frac{m_r e^4}{2\hbar^2} &= \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} \\
\frac{1}{4\pi\epsilon_0}\frac{m_e m_p}{m_e + m_p}\frac{e^2}{2\hbar^2} &= \frac{1}{r} \\
r &= \frac{8\pi\epsilon_0(m_e + m_p)\hbar^2}{m_e m_p e^2} \\
&= 1.06 \times 10^{-10}\,\mathrm{m} \\
&= 2a
\end{aligned}$$

(b)

$$\begin{aligned}
\psi_{1s}(r) &= \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \\
\int_{2a}^{\infty} |\psi_{1s}(r)|^2 \, dr &= \int_{2a}^{\infty} \frac{1}{\pi a^3} e^{-2r/a} \, dV \\
&= \frac{1}{\pi a^3} \int_{2a}^{\infty} e^{-2r/a} 4\pi r^2 \, dr \\
&= \frac{4}{a^3} \int_{2a}^{\infty} r^2 e^{-2r/a} \, dr \\
&= 0.238
\end{aligned}$$

41.2.51

$$\begin{aligned}
P(r) &= (R_{2p})^2 r^2 \\
&= \left(\frac{1}{\sqrt{24a^5}} r e^{-r/2a}\right)^2 r^2 \\
&= \frac{1}{24a^5} r^2 e^{-r/a} r^2 \\
&= \frac{r^4 e^{-r/a}}{24a^5} \\
\frac{dP(r)}{dr} &= \frac{1}{24a^5}\left(4r^3 e^{-r/a} - \frac{1}{a} r^4 e^{-r/a}\right) \\
0 &= \frac{1}{24a^5}\left(4r^3 e^{-r/a} - \frac{1}{a} r^4 e^{-r/a}\right) \\
4r^3 e^{-r/a} &= \frac{1}{a} r^4 e^{-r/a} \\
r &= 4a
\end{aligned}$$

Under the Bohr model, $r_2 = 4a$ so they're equal.

41.2.53

(a)

$$\begin{aligned}
\cos\theta &= \frac{L_z}{L} \\
\theta &= \arccos\frac{l\hbar}{\sqrt{l(l+1)}\hbar} \\
&= \arccos\frac{n-1}{\sqrt{n(n-1)}}
\end{aligned}$$

(b)

$$\begin{aligned}
\theta &= \arccos\frac{-(n-1)}{\sqrt{n(n-1)}} \\
&= \arccos-\sqrt{\frac{(n-1)^2}{n(n-1)}} \\
&= \arccos-\sqrt{1-1/n}
\end{aligned}$$

41.2.55

$$\begin{aligned}
\Delta E &= \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} \\
&= 2.64\times10^{-23}\,\mathrm{J} \\
&= 1.65\times10^{-4}\,\mathrm{eV} \\
\Delta E &= U \\
&= \mu_B B \\
B &= \frac{\Delta E}{\mu_B} \\
&= 2.85\,\mathrm{T}
\end{aligned}$$

41.2.57

(a)

$$\begin{aligned}
E_1 &= -13.60\,\mathrm{eV} \\
E &= E_1 \pm 1.00116\mu_B B \\
\frac{n_{+1/2}}{n_{-1/2}} &= e^{-\Delta E/kT} \\
&= e^{-2\times1.00116\mu_B B/kT} \\
&= 1
\end{aligned}$$

(b) 0.996

(c) 0.969

41.2.61

(a)

$$
\begin{aligned}
\Delta E &= E_i - E_f \\
&= -(13.60\,\mathrm{eV})\left(\frac{1}{2^2} - \frac{1}{1^2}\right) \\
&= 10.2\,\mathrm{eV} \\
\Delta E &= \frac{hc}{\lambda} \\
\lambda &= \frac{hc}{\Delta E} \\
&= 122\,\mathrm{nm}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\Delta E' &= E_i' - E_f' \\
&= E_i + m_l \mu_B B - E_f \\
&= -\frac{13.60\,\mathrm{eV}}{2^2} - \mu_B B + 13.60\,\mathrm{eV} \\
&= \frac{3}{4}(13.60\,\mathrm{eV}) - \mu_B B \\
&= 10.2\,\mathrm{eV} - 1.28 \times 10^{-4}\,\mathrm{eV} \\
\lambda &= \frac{hc}{\Delta E'} \\
&= 121.8\,\mathrm{nm} \\
\Delta\lambda &= hc\left(\frac{1}{\Delta E'} - \frac{1}{\Delta E}\right) \\
&= 1.53\,\mathrm{pm}
\end{aligned}
$$

The magnetic field increases the wavelength.

41.2.65

(a)

$$
\begin{aligned}
E_{\mathrm{Li}} &= 5.40\,\mathrm{eV} \\
E_{\mathrm{Na}} &= 5.15\,\mathrm{eV} \\
E_{\mathrm{K}} &= 4.35\,\mathrm{eV} \\
E_{\mathrm{Rb}} &= 4.18\,\mathrm{eV} \\
E_{\mathrm{Cs}} &= 3.90\,\mathrm{eV} \\
E_{\mathrm{Fr}} &= 3.94\,\mathrm{eV}
\end{aligned}
$$

(b)

$$
\begin{aligned}
Z_{\mathrm{Li}} &= 3 \\
n_{\mathrm{Li}} &= 2 \\
Z_{\mathrm{Na}} &= 11 \\
n_{\mathrm{Na}} &= 3 \\
Z_{\mathrm{K}} &= 19 \\
n_{\mathrm{K}} &= 4 \\
Z_{\mathrm{Rb}} &= 37 \\
n_{\mathrm{Rb}} &= 5 \\
Z_{\mathrm{Cs}} &= 55 \\
n_{\mathrm{Cs}} &= 6 \\
Z_{\mathrm{Fr}} &= 87 \\
n_{\mathrm{Fr}} &= 7
\end{aligned}
$$

(c)

$$
\begin{aligned}
E_n &= -\frac{Z_{\mathrm{eff}}^2}{n^2}(13.6\,\mathrm{eV}) \\
Z_{\mathrm{eff}} &= \sqrt{-\frac{E_n n^2}{13.6\,\mathrm{eV}}} \\
Z_{\mathrm{eff,Li}} &= 1.26 \\
Z_{\mathrm{eff,Na}} &= 1.85 \\
Z_{\mathrm{eff,K}} &= 2.26 \\
Z_{\mathrm{eff,Rb}} &= 2.77 \\
Z_{\mathrm{eff,Cs}} &= 3.21 \\
Z_{\mathrm{eff,Fr}} &= 3.77
\end{aligned}
$$

(d) Increase

41.2.67

(a) $m = 2.842 \times 10^{10}\,\mathrm{Hz/T}$

(b)

$$
\begin{aligned}
\Delta E &= 2\mu_z \Delta B_z \\
\mu_z &= \frac{1}{2}\frac{\Delta E}{\Delta B_z} \\
&= \frac{1}{2}hm \\
&= 9.42 \times 10^{-24}\,\mathrm{J/T}
\end{aligned}
$$

(c)

$$
\begin{aligned}
\gamma &= \left|\frac{\mu_z}{S_z}\right| \\
&= \left|\frac{\mu_z}{m_s\hbar}\right| \\
&= \frac{2\mu_z}{\hbar} \\
&= 1.78 \times 10^{11} \\
\frac{\gamma}{e/2m} &= 2.025
\end{aligned}
$$

41.2.69

(a)

$$
\begin{aligned}
L &= \hbar \\
mvr &= \hbar
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{4\pi\epsilon_0}\frac{2e^2}{r^2} - \frac{1}{4\pi\epsilon_0}\frac{e^2}{(2r)^2} &= \frac{mv^2}{r} \\
\frac{1}{4\pi\epsilon_0}\left(\frac{2e^2}{r^2} - \frac{e^2}{4r^2}\right) &= \frac{mv^2}{r} \\
\frac{1}{4\pi\epsilon_0}\frac{7e^2}{4r^2} &= \frac{mv^2}{r}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{4\pi\epsilon_0}\frac{7e^2}{4r^2} &= \frac{m}{r}\left(\frac{\hbar}{mr}\right)^2 \\
&= \frac{\hbar^2}{mr^3} \\
\frac{1}{4\pi\epsilon_0}\frac{7e^2}{4} &= \frac{\hbar^2}{mr} \\
r &= \frac{16\pi\epsilon_0\hbar^2}{7e^2m} \\
&\approx 3.04 \times 10^{-11}\,\mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
v &= \sqrt{\frac{7e^2r}{16\pi\epsilon_0 mr^2}} \\
&= 3.82 \times 10^6\,\mathrm{m/s}
\end{aligned}
$$

(b)

$$
K = 2\frac{1}{2}mv^2 = 1.33 \times 10^{-17}\,\mathrm{J} = 83.0\,\mathrm{eV}
$$

(c)

$$
\begin{aligned}
U &= -2\frac{1}{4\pi\epsilon_0}\frac{2e^2}{r} + \frac{1}{4\pi\epsilon_0}\frac{e^2}{2r} \\
&= \frac{1}{4\pi\epsilon_0}\left(\frac{e^2}{2r} - \frac{4e^2}{r}\right) \\
&= -\frac{1}{4\pi\epsilon_0}\frac{7e^2}{2r} \\
&= -2.65 \times 10^{-17}\,\mathrm{J} \\
&= -166\,\mathrm{eV}
\end{aligned}
$$

(d) 83.0 eV

41.2.71

b

41.2.73

d

42 Molecules and Condensed Matter

42.1 Guided Practice

42.1.1

(a)

$$
\begin{aligned}
m_r &= \frac{m_1 m_2}{m_1 + m_2} \\
&= 1.586 \times 10^{-27}\,\mathrm{kg} \\
I &= m_r r_0^2 \\
&= 1.342 \times 10^{-47}\,\mathrm{kg\,m^2} \\
E_{nl} &= l(l+1)\frac{\hbar^2}{2I} + \left(n + \frac{1}{2}\right)\hbar\omega \\
E_i &= 1.232 \times 10^{-19}\,\mathrm{J} \\
&= 0.770\,\mathrm{eV} \\
E_f &= 4.191 \times 10^{-20}\,\mathrm{J} \\
&= 0.262\,\mathrm{eV} \\
\Delta E &= 0.508\,\mathrm{eV}
\end{aligned}
$$

The electron is $0.508\,\mathrm{eV} - 0.230\,\mathrm{eV} = 0.278\,\mathrm{eV}$ above the top of the band gap.

(b)

$$
\begin{aligned}
E - E_F &= 0.393\,\mathrm{eV} \\
f(E) &= \frac{1}{e^{(E-E_F)/kT} + 1} \\
&= 1.914 \times 10^{-25}
\end{aligned}
$$

42.2 Exercises and Problems

42.2.1

$$
E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = 2.77 \times 10^{-7}\,\mathrm{m} = 277\,\mathrm{nm}
$$

Ultraviolet

42.2.3

$$
\begin{aligned}
r_0 &= 0.074\,\mathrm{nm} \\
m &= 1.67 \times 10^{-27}\,\mathrm{kg} \\
m_r &= \frac{m^2}{2m} \\
&= \frac{1}{2}m \\
I &= m_r r_0^2 \\
&= \frac{1}{2}m r_0^2 \\
E_l &= l(l+1)\frac{\hbar^2}{2I} \\
&= l(l+1)\frac{\hbar^2}{m r_0^2} \\
\Delta E &= E_3 - E_1 \\
&= 1.216 \times 10^{-20}\,\mathrm{J} \\
&= 0.076\,\mathrm{eV} \\
\lambda &= \frac{hc}{\Delta E} \\
&= 16.3\,\mu\mathrm{m}
\end{aligned}
$$

42.2.5

$$
\begin{aligned}
m_{\mathrm{H}} &= 1.67 \times 10^{-27}\,\mathrm{kg} \\
m_{\mathrm{N}} &= 2.33 \times 10^{-26}\,\mathrm{kg} \\
m_r &= \frac{m_{\mathrm{H}} m_{\mathrm{N}}}{m_{\mathrm{H}} + m_{\mathrm{N}}} \\
&= 1.558 \times 10^{-27}\,\mathrm{kg}
\end{aligned}
$$

$$
\begin{aligned}
\Delta E &= E_i - E_f \\
\frac{hc}{\lambda} &= 10 \frac{\hbar^2}{2I} \\
&= 10 \frac{\hbar^2}{2 m_r r_0^2} \\
r_0 &= \sqrt{5 \frac{\hbar^2 \lambda}{m_r h c}} \\
&= 5.54 \times 10^{-13}\,\mathrm{m} \\
&= 0.554\,\mathrm{pm}
\end{aligned}
$$

42.2.7

$$
\begin{aligned}
f &= \frac{E}{h} \\
&= 2.44 \times 10^{9}\,\mathrm{Hz} \\
\lambda &= 0.123\,\mathrm{m}
\end{aligned}
$$

Yes. The magnetron generates elecromagnetic waves at a frequency that can be absorbed by the water molecules in food, heating it up.

42.2.9

(a)

$$
\begin{aligned}
I &= 1.449 \times 10^{-46}\,\mathrm{kg\,m^2} \\
K &= \frac{1}{2} I \omega^2 \\
\omega &= \sqrt{\frac{2K}{I}} \\
&= 1.03 \times 10^{12}\,\mathrm{rad/s}
\end{aligned}
$$

(b)

$$
\begin{aligned}
r_0 &= 0.1128 \times 10^{-9}\,\mathrm{m} \\
x_{cm} &= \frac{1}{M}\sum m_i x_i \\
&= \frac{m_o r_0}{m_c + m_0} \\
&= 6.44 \times 10^{-11}\,\mathrm{m} \\
v_c &= x_{cm}\omega \\
&= 66.3\,\mathrm{m/s} \\
v_o &= (r_0 - x_{cm})\omega \\
&= 49.9\,\mathrm{m/s}
\end{aligned}
$$

(c)

$$
T = \frac{1}{f} = \frac{2\pi}{\omega} = 6.10 \times 10^{-12}\,\mathrm{s}
$$

42.2.11

(a)

$$
\begin{aligned}
m_{\mathrm{Li}} &= 1.17 \times 10^{-26}\,\mathrm{kg} \\
m_{\mathrm{H}} &= 1.67 \times 10^{-27}\,\mathrm{kg} \\
r_0 &= 0.159\,\mathrm{nm} \\
m_r &= \frac{m_{\mathrm{Li}} m_{\mathrm{H}}}{m_{\mathrm{Li}} + m_{\mathrm{H}}} \\
&= 1.47 \times 10^{-27}\,\mathrm{kg} \\
I &= m_r r_0^2 \\
&= 3.69 \times 10^{-47}\,\mathrm{kg\,m^2} \\
E_l &= l(l+1)\frac{\hbar^2}{2I} \\
E_4 - E_3 &= 8\frac{\hbar^2}{2I} \\
&= 1.20 \times 10^{-21}\,\mathrm{J} \\
&= 7.53\,\mathrm{meV}
\end{aligned}
$$

(b)

$$
\lambda = \frac{hc}{\Delta E} = 165\,\mu\mathrm{m}
$$

42.2.13

By selection criteria the rotational transition is $(n = n, l = 2) \rightarrow (n = n, l = 1)$ and the vibrational transition is $(n = n, l = l) \rightarrow (n = n - 1, l = l - 1)$.

$$
\begin{aligned}
E_{l=2} - E_{l=1} &= 6\frac{\hbar^2}{2I} - 2\frac{\hbar^2}{2I} \\
\Delta E_l &= 2\frac{\hbar^2}{m_r r_0^2} \\
m_r &= \frac{2\hbar^2}{\Delta E_l r_0^2} \\
&= 1.99 \times 10^{-28}\,\mathrm{kg}
\end{aligned}
$$

$$
\begin{aligned}
\Delta E_n &= \hbar\omega \\
&= \hbar\sqrt{\frac{k'}{m_r}} \\
k' &= \frac{m_r \Delta E_n}{\hbar^2} \\
&= 30.8\,\mathrm{N/m}
\end{aligned}
$$

42.2.15

$$
\begin{aligned}
m_{\mathrm{av}} &= \frac{m_{\mathrm{Cl}} + m_{\mathrm{Na}}}{2} \\
&= 4.86 \times 10^{-26}\,\mathrm{kg} \\
\rho &= \frac{m_{\mathrm{av}}}{d^3} \\
&= 2.16 \times 10^3\,\mathrm{kg/m^3}
\end{aligned}
$$

42.2.17

(a)

$$
E = \frac{hc}{\lambda} = 1.12\,\mathrm{eV}
$$

(b) Because it can absorb photos of wavelength up to $1.11\,\mu$m which includes visible light.

42.2.19

$$
\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= 1.33\,\mathrm{MeV} \\
n &= \frac{E}{\Delta E} \\
&= 1.19 \times 10^6
\end{aligned}
$$

42.2.21

$$
\begin{aligned}
g(E) &= \frac{(2m)^{3/2}V}{2\pi^2\hbar^3}E^{1/2} \\
&= 1.185 \times 10^{41} \text{ states/J} \\
&= 1.90 \times 10^{22} \text{ states/eV}
\end{aligned}
$$

42.2.23

(a)

$$
\frac{\pi^2 kT}{2E_F}R = 0.022R
$$

(b) 0.00723 or 0.723%

(c) No, it's primarily due to the silver atoms' vibrational energy

42.2.25

$$
\begin{aligned}
f(E) &= \frac{1}{e^{(E-E_F)/kT}+1} \\
&= 0.089 \\
&= 8.9\%
\end{aligned}
$$

42.2.27

$$
\begin{aligned}
f(E_B) &= \frac{1}{e^{(E_B-E_F)/kT}+1} \\
e^{(E_B-E_F)/kT}+1 &= \frac{1}{f(E_B)} \\
E_B - E_F &= kT\ln\left(\frac{1}{f(E_B)}-1\right) \\
&= 0.2\,\mathrm{eV}
\end{aligned}
$$

42.2.29

(a) (i) $I = 0.02\,\mathrm{mA}$
(ii) $I = -0.019\,\mathrm{mA}$
(iii) $I = 26.7\,\mathrm{mA}$
(iv) $I = -0.49\,\mathrm{mA}$

(b) Yes, between around $-2\,\mathrm{mA}$ to $2\,\mathrm{mA}$.

42.2.31

(a)

$$\begin{aligned} V &= 15.0\,\mathrm{mV} \\ I &= 9.25\,\mathrm{mA} \\ T &= 300\,\mathrm{K} \end{aligned}$$

$$\begin{aligned} I &= I_S(e^{eV/kT} - 1) \\ I_S &= \frac{I}{e^{eV/kT} - 1} \\ &= 11.7\,\mathrm{mA} \end{aligned}$$

$$I = 5.52\,\mathrm{mA}$$

(b)

$$\begin{aligned} I_{-15} &= -5.15\,\mathrm{mA} \\ I_{-10} &= -3.75\,\mathrm{mA} \end{aligned}$$

42.2.33

(a)

$$\begin{aligned} m_r &= 1.57 \times 10^{-27}\,\mathrm{kg} \\ \Delta E &= \frac{hc}{\lambda} \\ \hbar\sqrt{\frac{k'}{m_r}} &= \frac{hc}{\lambda} \\ k' &= m_r \left(\frac{hc}{\hbar\lambda}\right)^2 \\ &= 1103\,\mathrm{N/m} \end{aligned}$$

(b)

$$f = \frac{\omega}{2\pi} = \frac{\sqrt{k'/m_r}}{2\pi} = 1.33 \times 10^{14}\,\mathrm{Hz}$$

42.2.35

(a)

$$p = qd = 3.8 \times 10^{-29}\,\mathrm{C\,m}$$

(b)

$$q = \frac{p}{d} = 1.25 \times 10^{-19}\,\mathrm{C}$$

(c) 0.78

(d)

$$\frac{q}{e} = \frac{p/d}{e} = 0.059$$

42.2.41

(a)

$$\begin{aligned}
\lambda_{2\to1} &= \frac{hc}{\Delta E} \\
&= \frac{hc}{4\hbar^2/2I} \\
&= \frac{m_r r_0^2 hc}{2\hbar^2} \\
&= 1.147\,\text{cm} \\
\lambda_{1\to0} &= \frac{m_r r_0^2 hc}{\hbar^2} \\
&= 2.295\,\text{cm}
\end{aligned}$$

(b)

$$\begin{aligned}
\lambda_{2\to1} &= 1.173\,\text{cm}\,(+0.026\,\text{cm}) \\
\lambda_{1\to0} &= 2.345\,\text{cm}\,(+0.05\,\text{cm})
\end{aligned}$$

42.2.43

$$\begin{aligned}
E_0 &= \frac{1}{2}\hbar\sqrt{\frac{k'}{m_r}} \\
&= 4.38 \times 10^{-20}\,\text{J} \\
&= 0.274\,\text{eV}
\end{aligned}$$

This is 6.1% of the bond energy.

42.2.45

(a)

$$\begin{aligned}
x_{\text{cm}} &= \frac{m_{\text{I}} r_0}{m_{\text{H}} + m_{\text{I}}} \\
&= 0.1587\,\text{nm} \\
I &= m_{\text{H}} x_{\text{cm}}^2 + m_{\text{I}}(r_0 - x_{\text{cm}})^2 \\
&= 4.24 \times 10^{-47}\,\text{kg}\,\text{m}^2
\end{aligned}$$

(i) $4.32\,\mu\text{m}$
(ii) $4.31\,\mu\text{m}$
(iii) $4.35\,\mu\text{m}$

42.2.47

$$
\begin{aligned}
\rho &= 851\,\mathrm{kg/m^3} \\
m &= 6.49 \times 10^{-26}\,\mathrm{kg} \\
n &= \frac{\rho}{m} \\
&= 1.311 \times 10^{28}\,\mathrm{atoms/m^3} \\
n &= \int_0^{E_F} g(E)\,dE \\
&= \int_0^{E_F} \frac{(2m)^{3/2}V}{2\pi^2\hbar^3} E^{1/2}\,dE \\
&= \frac{(2m)^{3/2}V}{2\pi^2\hbar^3} \left[\frac{2}{3}E^{3/2}\right]_0^{E_F} \\
&= \frac{(2m)^{3/2}V}{3\pi^2\hbar^3} E_F^{3/2} \\
E_F &= \left(\frac{3\pi^2\hbar^3 n}{(2m)^{3/2}V}\right)^{2/3} \\
&= 2.03\,\mathrm{eV}
\end{aligned}
$$

42.2.49

(a)

$$
\begin{aligned}
\rho &= 0.534\,\mathrm{g/cm^3} \\
&= 5.34 \times 10^{-4}\,\mathrm{kg/cm^3} \\
&= 534\,\mathrm{kg/m^3} \\
m &= 1.15 \times 10^{-26}\,\mathrm{kg} \\
n &= 4.64 \times 10^{28}\,\mathrm{atoms/m^3} \\
&= 2\,\mathrm{atoms/cell} \\
&= \frac{2}{a^3}\,\mathrm{atoms/m^3}
\end{aligned}
$$

(b)

$$
\begin{aligned}
n &= \int_0^{E_{F0}} g(E)\,dE \\
\frac{2}{a^3} &= \frac{(2m)^{3/2}V}{2\pi^2\hbar^3}\int_0^{E_{F0}} E^{1/2}\,dE \\
&= \frac{(2m)^{3/2}V}{3\pi^2\hbar^3}E_{F0}^{3/2} \\
E_{F0} &= \left(\frac{6\pi^2\hbar^3}{a^3(2m)^{3/2}V}\right)^{2/3} \\
&= 4.74\,\mathrm{eV}
\end{aligned}
$$

42.2.51

(a)

$$
\begin{aligned}
m &= 0.4454\,\mathrm{eV} \\
y_0 &= 1.802\,\mathrm{eV}
\end{aligned}
$$

(b)

$$
\begin{aligned}
E_F &= y_0 \\
&= 1.802\,\mathrm{eV}
\end{aligned}
$$

$$
\begin{aligned}
f(E) &= \frac{1}{e^{(E-E_F)/kT}+1} \\
\frac{1}{f(E)} &= e^{(E-E_F)/kT+1} \\
\ln\left(\frac{1}{f(E)}-1\right) &= \frac{E-E_F}{kT} \\
T &= \frac{E-E_F}{k\ln([1/f(E)]-1)} \\
&= 5136\,\mathrm{K}
\end{aligned}
$$

42.2.53

(a)

$$
\begin{aligned}
E_{\text{tot}} &= \int_0^{E_{F0}} g(E)E\,dE \\
&= \frac{(2m)^{3/2}V}{2\pi^2\hbar^3}\int_0^{E_{F0}} E^{3/2}\,dE \\
&= \frac{(2m)^{3/2}V}{5\pi^2\hbar^3}E_{F0}^{5/2} \\
&= \frac{3^{5/3}\pi^{4/3}\hbar^2 N^{5/3}}{10mV^{2/3}} \\
p &= -\frac{dE_{\text{tot}}}{dV} \\
&= \frac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\left(\frac{N}{V}\right)^{5/3}
\end{aligned}
$$

(b)

$$
\begin{aligned}
p &= 3.806\times 10^{10}\,\text{Pa} \\
&= 3.77\times 10^{5}\,\text{atm}
\end{aligned}
$$

42.2.55

(a)

$$
\begin{aligned}
E_{F0} &= \frac{1}{100}mc^2 \\
\frac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\frac{N}{V}\right)^{2/3} &= \frac{1}{100}mc^2 \\
\frac{N}{V} &= \left(\frac{2m^2c^2}{100\times 3^{2/3}\pi^{4/3}\hbar^2}\right)^{3/2} \\
&= \frac{2^{3/2}m^3c^3}{3000\pi^2\hbar^3} \\
&= 1.66\times 10^{33}\,\text{m}^{-3}
\end{aligned}
$$

(b) Yes

(c)

$$
\begin{aligned}
N &= 6\frac{m_{\text{star}}}{m_{\text{C}}} \\
&= 6.03 \times 10^{56}\,\text{atoms} \\
V &= \frac{4}{3}\pi r^3 \\
&= 9.05 \times 10^{20}\,\text{m}^3 \\
\frac{N}{V} &= 6.66 \times 10^{35}\,\text{m}^{-3}
\end{aligned}
$$

(d) No

42.2.57

b