University Physics with Modern Physics Electromagnetism Problems

Chris Doble

December 2022

Contents

21 Electric Charge and Electric Field

21.3 Coulomb's Law

21.3.1 Example 21.1

The magnitude of electric repulsion between two α particles is given by

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

and the magnitude of gravitational attraction is given by

$$F_g = \frac{Gm^2}{r^2}$$

. The ratio of the two values is

$$\begin{aligned}\frac{F_e}{F_g} &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}\frac{r^2}{Gm^2} \\ &= \frac{1}{4\pi\epsilon_0}\frac{q^2}{Gm^2} \\ &= 3.1 \times 10^{35}\end{aligned}$$

showing that the electric repulsion is significantly stronger than the gravitational attraction.

21.3.2 Example 21.2

a) The magnitude of the force that q_1 exerts on q_2 is

$$\begin{aligned}F &= \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \\ &= (9.0 \times 10^9)\frac{|(25 \times 10^{-9})(-75 \times 10^{-9})|}{0.030^2} \\ &= 1.9 \times 10^{-2}\,\mathrm{N}.\end{aligned}$$

Since q_1 and q_2 have opposite charge, the force is attractive (from q_2 to q_1).

b) The magnitude of the force that q_2 exerts on q_1 is the same as in part a, but the direction is reversed (from q_1 to q_2).

21.3.3 Example 21.3

By the principle of superposition of forces, the net force exerted on q_3 is equal to the vector sum of the forces exerted on it by q_1 and q_2 separately.

Both q_1 and q_3 have positive charge so they repel each other. q_1 is to the right of q_3 so q_3 experiences a force to the left of magnitude

$$\begin{aligned}F_{1\text{ on }3} &= \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r^2} \\ &= (9.0 \times 10^9)\frac{|(1.0 \times 10^{-9})(5.0 \times 10^{-9})|}{0.020^2} \\ &= 1.1 \times 10^{-4}\,\mathrm{N}.\end{aligned}$$

However q_2 has a negative charge so it attracts q_3. It is also to the right of q_3 so q_3 experiences a force to the right of magnitude

$$
\begin{aligned}
F_{2 \text{ on } 3} &= \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{r^2} \\
&= (9.0 \times 10^9)\frac{|(-3.0 \times 10^{-9})(5.0 \times 10^{-9})}{0.040^2} \\
&= 8.4 \times 10^{-5}\,\text{N}.
\end{aligned}
$$

The net force experienced by q_3 is therefore

$$
\begin{aligned}
F &= -F_{1 \text{ on } 3} + F_{2 \text{ on } 3} \\
&= -1.1 \times 10^{-4} + 8.4 \times 10^{-5} \\
&= -2.6 \times 10^{-5}\,\text{N}.
\end{aligned}
$$

21.3.4 Example 21.4

Since q_1 and q_2 are of equal charge and are symmetric about the x axis on which Q lies, the vertical components of their forces cancel leaving only the horizontal.

The horizontal component of q_1's force on Q is given by

$$
\begin{aligned}
F_{1 \text{ on Q, x}} &= \frac{1}{4\pi\epsilon_0}\frac{q_1 Q}{r_{1,\text{Q}}^2}\cos\alpha \\
&= (9.0 \times 10^9)\frac{(2.0 \times 10^{-6})(4.0 \times 10^{-6})}{\sqrt{0.30^2 + 0.40^2}^2}\frac{0.40}{0.50} \\
&= 0.23\,\text{N}.
\end{aligned}
$$

Again, since q_1 and q_2 are of equal charge and symmetric about the x axis, $F_{1 \text{ on Q, x}} = F_{2 \text{ on Q, x}}$ and the total force experienced by Q is in the positive x direction of magnitude

$$
F = 2F_{1 \text{ on Q, x}} = 0.46\,\text{N}.
$$

21.4 Electric Field and Electric Forces

21.4.1 Example 21.5

The magnitude of the electric field vector is given by

$$
\begin{aligned}
E &= \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \\
&= (9.0 \times 10^9)\frac{|4.0 \times 10^{-9}|}{2.0^2} \\
&= 9.0\,\text{N/C}.
\end{aligned}
$$

21.4.2 Example 21.6

The magnitude of the electric field vector is given by

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \\ &= (9.0\times 10^9)\frac{|-8.0\times 10^{-9}|}{1.2^2+1.6^2} \\ &= 18\,\mathrm{N/C} \end{aligned}$$

and it is directed towards the origin. If θ is the angle between the positive x axis and $\hat{\mathbf{r}}$ then the component form of $\mathbf{E}$ is

$$\begin{aligned} E &= -E\left(\cos\theta\hat{\mathbf{i}}+\sin\theta\hat{\mathbf{j}}\right) \\ &= -E\left(\frac{x}{r}\hat{\mathbf{i}}+\frac{-y}{r}\hat{\mathbf{j}}\right) \\ &= \frac{-18}{\sqrt{1.2^2+1.6^2}}\left(1.2\hat{\mathbf{i}}+1.6\hat{\mathbf{j}}\right) \\ &= (-11\,\mathrm{N/C})\hat{\mathbf{i}}-(14\,\mathrm{N/C})\hat{\mathbf{j}}. \end{aligned}$$

21.4.3 Example 21.7

a) Electrons have a negative charge and the electric field is directed upwards, so the electron will move downwards. The magnitude of its acceleration is

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{eE}{m} \\ &= \frac{(1.60\times 10^{-19})(1.00\times 10^4)}{9.11\times 10^{-31}} \\ &= 1.76\times 10^{15}\,\mathrm{m/s^2}. \end{aligned}$$

b) Its acceleration is constant between the plates, so its final speed is

$$\begin{aligned} v^2 &= v_0^2+2a(x-x_0) \\ &= 2ax \\ v &= \sqrt{2ax} \\ &= \sqrt{2(1.76\times 10^{15})(0.01)} \\ &= 5.9\times 10^6\,\mathrm{m/s^2} \end{aligned}$$

and thus its final kinetic energy is

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(9.11 \times 10^{-31})(5.9 \times 10^6)^2 \\ &= 1.6 \times 10^{-17}\,\text{J}. \end{aligned}$$

c) We can find the time it takes for the electron to travel this distance by rearranging the kinematic equation

$$v = v_0 + at$$

to

$$\begin{aligned} t &= \frac{v - v_0}{a} \\ &= \frac{5.9 \times 10^6}{1.76 \times 10^{15}} \\ &= 3.4 \times 10^{-9}\,\text{s}. \end{aligned}$$

21.5 Electric-Field Calculations

21.5.1 Example 21.8

a) At point a the electric field caused by q_1 points to the right and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} \\ &= (9.0 \times 10^9)\frac{12 \times 10^{-9}}{(0.060)^2} \\ &= 3.0 \times 10^4\,\text{N/C}. \end{aligned}$$

The electric field caused by q_2 also points to the right and it has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} \\ &= (9.0 \times 10^9)\frac{|-12 \times 10^{-9}|}{(0.040)^2} \\ &= 6.8 \times 10^4\,\text{N/C}. \end{aligned}$$

Thus the total field points to the right and has magnitude

$$E = E_1 + E_2 = 9.8 \times 10^4\,\text{N/C}.$$

b) At point b the electric field caused by q_1 points to the left and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} \\ &= (9.0 \times 10^9)\frac{12 \times 10^{-9}}{(0.040)^2} \\ &= 6.8 \times 10^4\,\text{N/C}. \end{aligned}$$

The electric field caused by q_2 points to the right and has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} \\ &= (9.0 \times 10^9)\frac{|-12 \times 10^{-9}|}{(0.140)^2} \\ &= 0.55 \times 10^4\,\text{N/C}. \end{aligned}$$

Thus the total electric field points to the left and has magnitude

$$E = E_1 - E_2 = 6.3 \times 10^4\,\text{N/C}.$$

c) At point c the electric field caused by q_1 points from q_1 to c and has magnitude

$$\begin{aligned} E_1 &= \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} \\ &= (9.0 \times 10^9)\frac{|12 \times 10^{-9}|}{0.130^2} \\ &= 6.4 \times 10^3\,\text{N/C}. \end{aligned}$$

The electric field caused by q_2 points from c to q_2 and has magnitude

$$\begin{aligned} E_2 &= \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} \\ &= (9.0 \times 10^9)\frac{|-12 \times 10^{-9}|}{0.130^2} \\ &= 6.4 \times 10^3\,\text{N/C} \\ &= E_1. \end{aligned}$$

The vertical components of $\mathbf{E_1}$ and $\mathbf{E_2}$ cancel, leaving only a horizontal component pointing to the right of magnitude

$$
\begin{aligned}
E &= 2E_1 \cos\alpha \\
&= 2(6.4 \times 10^3)\frac{0.050}{0.130} \\
&= 4.9 \times 10^3\,\mathrm{N/C}.
\end{aligned}
$$

21.5.2 Example 21.9

By symmetry, each point on the ring has a corresponding point on the opposite side. The components of their electric fields perpendicular to the axis of the ring cancel, leaving only a component parallel to the axis of the ring. Thus the total magnetic field at P is parallel to the axis of the ring and can be calculated as

$$
\begin{aligned}
E &= \int_0^{2\pi} \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r^2}\cos\alpha\, d\theta \\
&= \frac{1}{4\pi\epsilon_0}\frac{Qx}{2\pi(a^2+x^2)^{3/2}}\int_0^{2\pi} d\theta \\
&= \frac{1}{4\pi\epsilon_0}\frac{Qx}{(a^2+x^2)^{3/2}}.
\end{aligned}
$$

21.5.3 Example 21.10

By symmetry, each point on the line has a corresponding point on the opposite side of the x-axis. The y components of their electric fields cancel, leaving only the x components. Thus the total magnetic field at P only has an x component and can be calculated as

$$
\begin{aligned}
E &= \int_{-a}^{a} \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r^2}\cos\alpha\, dy \\
&= \frac{1}{4\pi\epsilon_0}\frac{Qx}{2a}\int_{-a}^{a}\frac{1}{(x^2+y^2)^{3/2}}\,dy \\
&= \frac{1}{4\pi\epsilon_0}\frac{Qx}{2a}\left[\frac{y}{x^2\sqrt{x^2+y^2}}\right]_{-a}^{a} \\
&= \frac{1}{4\pi\epsilon_0}\frac{Q}{2ax}\left(\frac{a}{\sqrt{x^2+a^2}}+\frac{a}{\sqrt{x^2+(-a)^2}}\right) \\
&= \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}.
\end{aligned}
$$

21.5.4 Example 21.11

By symmetry, each point on the disk has a corresponding point 180° rotation around the x-axis. The y and z components of their electric fields cancel, leaving only the x components. Thus the total magnetic field at P only has an x component and can be calculated as

$$\begin{aligned}
E &= \int_0^R \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{\sigma}{r^2} s \cos\alpha \, d\theta \, ds \\
&= \frac{\sigma}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{s}{s^2+x^2} \frac{x}{\sqrt{s^2+x^2}} \, d\theta \, ds \\
&= \frac{\sigma x}{2\epsilon_0} \int_0^R \frac{s}{(s^2+x^2)^{3/2}} \, ds \\
&= \frac{\sigma x}{2\epsilon_0} \left[-\frac{1}{\sqrt{s^2+x^2}} \right]_0^R \\
&= \frac{\sigma x}{2\epsilon_0} \left(-\frac{1}{\sqrt{R^2+x^2}} + \frac{1}{x} \right) \\
&= \frac{\sigma}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{(R/x)^2+1}} \right).
\end{aligned}$$

21.5.5 Example 21.12

From Example 21.11 we know that the electric field produced by an infinite plane sheet of charge is

$$E = \frac{\sigma}{2\epsilon_0}.$$

Therefore the electric field outside the sheets is $\mathbf{0}$ and between the sheets is σ/ϵ_0 towards the negative sheet.