University Physics with Modern Physics
Mechanics Problems

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December 2022

Contents

14 Periodic Motion

14.1 VP14.3.1

a)

$$T = \frac{1}{f} = \frac{1}{4.15} = 0.241\,\mathrm{s}$$

$$\omega = 2\pi f = 2\pi(4.15) = 26.1\,\mathrm{rad/s}$$

b)

$$\begin{aligned}
\omega &= \sqrt{\frac{k}{m}} \\
\omega^2 &= \frac{k}{m} \\
m\omega^2 &= k \\
(0.400)(26.1)^2 &= k \\
272\,\mathrm{N/m} &= k
\end{aligned}$$

c)

$$\begin{aligned}
F &= kx \\
&= (272)(0.0200) \\
&= 5.44\,\mathrm{N}
\end{aligned}$$

14.2 VP14.3.2

a)

$$\begin{aligned}
T &= 2\pi\sqrt{\frac{m}{k}} \\
\frac{T}{2\pi} &= \sqrt{\frac{m}{k}} \\
\frac{T^2}{4\pi^2} &= \frac{m}{k} \\
\frac{kT^2}{4\pi^2} &= m \\
\frac{(4.50)(1.20)^2}{4\pi^2} &= m \\
0.164\,\mathrm{kg} &= m
\end{aligned}$$

b)

$$\begin{aligned}
F = ma_{\mathrm{max}} &= kA \\
\frac{ma_{\mathrm{max}}}{k} &= A \\
\frac{(0.164)(1.20)}{(4.50)} &= A \\
0.0437\,\mathrm{m} &= A
\end{aligned}$$

14.3 VP14.3.3

a)

$$\begin{aligned}
\frac{1}{2}mv^2 + \frac{1}{2}kx^2 &= \frac{1}{2}kA^2 \\
\sqrt{\frac{m}{k}v^2 + x^2} &= A \\
\sqrt{\frac{1}{(2\pi f)^2}v^2 + x^2} &= A \\
\sqrt{\frac{1}{(2\pi(50.0))^2}(12.4)^2 + (0.0300)^2} &= A \\
0.0496\,\mathrm{m} &= A
\end{aligned}$$

b)

$$\begin{aligned}
\frac{1}{2}mv^2 &= \frac{1}{2}kA^2 \\
v &= \sqrt{\frac{k}{m}}A \\
&= 2\pi f A \\
&= 2\pi(50.0)(0.0496) \\
&= 15.6\,\mathrm{m/s}
\end{aligned}$$

14.4 VP14.3.4

a)

$$\begin{aligned}
f &= \frac{1}{2\pi}\sqrt{\frac{k}{m}} \\
&= \frac{1}{2\pi}\sqrt{\frac{185}{5.00}} \\
&= 0.968\,\mathrm{Hz}
\end{aligned}$$

b)

$$\begin{aligned}
F = ma_{\max} &= kA \\
\frac{ma_{\max}}{k} &= A \\
\frac{(5.00)(1.52)}{185} &= A \\
4.11 \times 10^{-2}\,\mathrm{m} &= A
\end{aligned}$$

14.5 VP14.4.1

a)

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2$$
$$\sqrt{\frac{m}{k}}v = A$$
$$\sqrt{\frac{0.150}{8.00}}0.350 = A$$
$$4.79 \times 10^{-2}\,\mathrm{m} = A$$

b)

$$\frac{1}{2}mv^2 = \frac{1}{2}(0.150)(0.350)^2 = 9.19 \times 10^{-3}\,\mathrm{J}$$

c)

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(8.00)(0.0300)^2 = 3.60 \times 10^{-3}\,\mathrm{J}$$
$$K = E - U = 9.19 \times 10^{-3} - 3.60 \times 10^{-3} = 5.59 \times 10^{-3}\,\mathrm{J}$$

14.6 VP14.4.2

a)

$$E = \frac{1}{2}kA^2$$
$$\frac{2E}{A^2} = k$$
$$\frac{2(6.00 \times 10^{-2})}{(0.0440)^2} = k$$
$$62.0\,\mathrm{N/m} = k$$

b)

$$E = \frac{1}{2}mv^2 + \frac{E}{2}$$
$$\sqrt{\frac{E}{m}} = v$$
$$\sqrt{\frac{6.00 \times 10^{-2}}{0.300}} = v$$
$$0.447\,\mathrm{m/s} = v$$

14.7 VP14.4.3

a)

$$\begin{aligned}
E &= \frac{1}{2}kA^2 \\
\frac{2E}{A^2} &= k \\
\frac{2(4.00 \times 10^{-3})}{(0.0300)^2} &= k \\
8.89\,\mathrm{N/m} &= k
\end{aligned}$$

$$\begin{aligned}
E &= \frac{1}{2}mv^2 \\
\frac{2E}{v^2} &= m \\
\frac{2(4.00 \times 10^{-3})}{(0.125)^2} &= m \\
0.512\,\mathrm{kg} &= m
\end{aligned}$$

b)

$$\begin{aligned}
F = ma &= kA \\
a &= \frac{kA}{m} \\
&= \frac{(8.89)(0.0300)}{0.512} \\
&= 0.521\,\mathrm{m/s^2}
\end{aligned}$$

c)

$$\begin{aligned}
U &= \frac{1}{2}kx^2 \\
\sqrt{\frac{2U}{k}} &= x
\end{aligned}$$

$$\begin{aligned}
F = ma &= kx \\
a &= \frac{\sqrt{2kU}}{m} \\
&= \frac{\sqrt{2(8.89)(3.00 \times 10^{-3})}}{0.512} \\
&= 0.451\,\mathrm{m/s^2}
\end{aligned}$$

14.8 VP14.4.4

a) At maximum displacement from equilibrium the object's velocity is 0 and all energy is stored in elastic potential energy

$$E = \frac{1}{2}kA^2.$$

The object's kinetic energy will be equal to $\frac{1}{3}$ of its total mechanical energy when its potential energy is equal to $\frac{2}{3}$

$$\begin{aligned} U &= \frac{2}{3}E \\ \frac{1}{2}kx^2 &= \frac{2}{3}\left(\frac{1}{2}kA^2\right) \\ x &= \pm\sqrt{\frac{2}{3}}A. \end{aligned}$$

b) Similarly, the object's kinertic energy will be equal to $\frac{4}{5}$ of the total mechanical energy when its potential energy is equal to $\frac{1}{5}$

$$\begin{aligned} U &= \frac{1}{5}E \\ \frac{1}{2}kx^2 &= \frac{1}{5}\left(\frac{1}{2}kA^2\right) \\ x &= \pm\frac{A}{\sqrt{5}}. \end{aligned}$$

14.9 VP14.9.1

a) The period is given by

$$T = \frac{1}{f} = \frac{1}{0.609} = 1.64\,\text{s}.$$

b) Rearranging the formula for f we find that

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{g}{L}} \\ (2\pi f)^2 L &= g \\ (2\pi(0.609))^2(0.500) &= g \\ 7.32\,\text{m/s}^2 &= g. \end{aligned}$$

14.10 VP14.9.2

The frequency of the air-track glider is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

The frequency of the simple pendulum is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}.$$

If their frequencies are to be equal then

$$\begin{aligned}
\frac{1}{2\pi}\sqrt{\frac{k}{m}} &= \frac{1}{2\pi}\sqrt{\frac{g}{L}} \\
\frac{k}{m} &= \frac{g}{L} \\
L &= \frac{gm}{k} \\
&= \frac{(9.80)(0.350)}{8.75} \\
&= 0.392\,\mathrm{m}
\end{aligned}$$

14.11 VP14.9.3

Assuming the tire is hung from its rim then it is a physical pendulum and

$$\begin{aligned}
f &= \frac{1}{2\pi}\sqrt{\frac{Mgd}{I}} \\
&= \frac{1}{2\pi}\sqrt{\frac{MgR}{2MR^2}} \\
&= \frac{1}{2\pi}\sqrt{\frac{g}{2R}}.
\end{aligned}$$

14.12 VP14.9.4

The rod is a physical pendulum. Rearranging the formula for period we find

$$\begin{aligned}
T &= 2\pi\sqrt{\frac{I}{mgd}} \\
mgd\left(\frac{T}{2\pi}\right)^2 &= I \\
(0.600)(9.80)(0.500)\left(\frac{1.59}{2\pi}\right)^2 &= I \\
0.188\,\mathrm{kg/m}^2 &= I
\end{aligned}$$

14.13 Bridging Problem

In order for the centre of mass of the cylinders to undergo simple harmonic motion it must experience a net force of the form

$$F = -k'x.$$

The normal and weight forces cancel, leaving only the friction force from the cylinders on the ground f and the restorative force from the spring $-kx$

$$F = f - kx.$$

The cylinders roll without slipping so

$$\begin{aligned}\omega &= -\frac{v}{R} \\ \frac{d}{dt}(\omega) &= \frac{d}{dt}(-\frac{v}{R}) \\ \alpha &= -\frac{a}{R}.\end{aligned}$$

The friction force generates a torque

$$\begin{aligned}\tau = I\alpha &= fR \\ \left(\frac{1}{2}MR^2\right)\left(-\frac{a}{R}\right) &= fR \\ -\frac{1}{2}Ma &= f.\end{aligned}$$

Substituting this into the force equation we find that

$$\begin{aligned}F = Ma &= -\frac{1}{2}Ma - kx \\ \frac{3}{2}Ma &= -kx \\ Ma &= -\frac{2}{3}kx\end{aligned}$$

which matches the required form of $F = -k'x$ where $k' = \frac{2}{3}k$.
This gives a period of

$$\begin{aligned}T &= 2\pi\sqrt{\frac{m}{k'}} \\ &= 2\pi\sqrt{\frac{3m}{2k}}.\end{aligned}$$