Advanced Engineering Mathematics Systems of Differential Equations by Dennis G. Zill Problems

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June 2023

Contents

10 Systems of Linear Differential Equations

10.1 Theory of Linear Systems

10.1.1

$$\mathbf{X}' = \begin{pmatrix} 3 & -5 \\ 4 & 8 \end{pmatrix} \mathbf{X}$$

10.1.3

$$\mathbf{X}' = \begin{pmatrix} -3 & 4 & -9 \\ 6 & -1 & 0 \\ 10 & 4 & 3 \end{pmatrix} \mathbf{X}$$

10.1.5

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} t-1 \\ -3t^2 \\ t^2 - t + 2 \end{pmatrix}$$

10.1.7

$$\begin{aligned} \frac{dx}{dt} &= 4x + 2y + e^t \\ \frac{dy}{dt} &= -x + 3y - e^t \end{aligned}$$

10.1.9

$$\begin{aligned} \frac{dx}{dt} &= x - y + 2z + e^{-t} - 3t \\ \frac{dy}{dt} &= 3x - 4y + z + 2e^{-t} + t \\ \frac{dz}{dt} &= -2x + 5y + 6z + 2e^{-t} - t \end{aligned}$$

10.1.11

$$\begin{aligned} 3(e^{-5t}) - 4(2e^{-5t}) &= -5e^{-5t} \\ &= \frac{dx}{dt} \\ 4(e^{-5t}) - 7(2e^{-5t}) &= -10e^{-5t} \\ &= \frac{dy}{dt} \end{aligned}$$

10.1.13

$$\begin{aligned} -(-e^{-3t/2}) + \frac{1}{4}(2e^{-3t/2}) &= \frac{3}{2}e^{-3t/2} \\ &= \frac{dx}{dt} \\ (-e^{-3t/2}) - (2e^{-3t/2}) &= -3e^{-3t/2} \\ &= \frac{dy}{dt} \end{aligned}$$

10.1.17

$$\begin{aligned} W(\mathbf{X}_1, \mathbf{X}_2) &= \begin{vmatrix} e^{-2t} & e^{-6t} \\ e^{-2t} & -e^{-6t} \end{vmatrix} \\ &= -e^{-8t} - e^{-8t} \\ &= -2e^{-8t} \\ &\neq 0 \text{ for } t \in (-\infty, \infty) \end{aligned}$$

Yes, they form a fundamental set.

10.1.19

$$\begin{aligned} W(\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3) &= \begin{vmatrix} 1+t & 1 & 3+2t \\ -2+2t & -2 & -6+4t \\ 4+2t & 4 & 12+4t \end{vmatrix} \\ &= 0 \end{aligned}$$

No, they don't form a fundamental set.

10.1.21

$$\begin{aligned} x &= 2t + 5 \\ y &= -t + 1 \\ \frac{dx}{dt} &= (2t+5) + 4(-t+1) + 2t - 7 \\ &= 2 \\ \frac{dy}{dt} &= 3(2t+5) + 2(-t+1) - 4t - 18 \\ &= -1 \end{aligned}$$

10.1.23

$$\begin{aligned} x &= e^t + te^t \\ x' &= 2e^t + te^t \\ y &= e^t - te^t \\ y' &= -te^t \\ \frac{dx}{dt} &= 2(e^t + te^t) + (e^t - te^t) - e^t \\ &= 2e^t + te^t \\ \frac{dy}{dt} &= 3(e^t + te^t) + 4(e^t - te^t) - 7e^t \\ &= -te^t \end{aligned}$$