

Advanced Engineering Mathematics Ordinary Differential Equations Notes

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1 Introduction to Differential Equations

1.1 Definitions and Terminology

1.1.1 1

2, linear

1.1.2 3

4, linear

1.1.3 5

2, nonlinear

1.1.4 7

3, linear

1.1.5 9

no; yes

1.1.6 15

The domain of the function is $x \in [-2, \infty)$.

$$y' = 1 + \frac{2}{\sqrt{x+2}}$$

The largest interval of definition of the solution is $x \in (-2, \infty)$.

$$\begin{aligned}(y-x)y' &= y-x+8 \\ (x+4\sqrt{x+2}-x)(1+\frac{2}{\sqrt{x+2}}) &= x+4\sqrt{x+2}-x+8 \\ 4\sqrt{x+2}+8 &= 4\sqrt{x+2}+8\end{aligned}$$

1.1.7 17

The domain of the function is $x \in \mathbb{R}, x \neq \pm 2$.

$$y' = \frac{2x}{(4-x^2)^2}$$

The largest intervals of definition of the solution are $(-\infty, -2)$, $(-2, 2)$, and $(2, \infty)$.

$$\begin{aligned}y' &= 2xy^2 \\ \frac{2x}{(4-x^2)^2} &= 2x \left(\frac{1}{4-x^2} \right)^2 \\ &= \frac{2x}{(4-x^2)^2}\end{aligned}$$

1.1.8 19

$$\begin{aligned}\ln \frac{2X-1}{X-1} &= t \\ 2X-1 &= (X-1)e^t \\ (2-e^t)X &= 1-e^t \\ X &= \frac{e^t-1}{e^t-2}\end{aligned}$$

The solutions intervals of validity are $(\infty, \ln 2)$ and $(\ln 2, \infty)$.

$$\begin{aligned}\frac{dX}{dt} &= (X - 1)(1 - 2X) \\ \frac{e^t}{e^t - 2} - \frac{e^t(e^t - 1)}{(e^t - 2)^2} &= \left(\frac{e^t - 1}{e^t - 2} - 1 \right) \left(1 - 2 \frac{e^t - 1}{e^t - 2} \right) \\ \frac{e^t(e^t - 2) - e^t(e^t - 1)}{(e^t - 2)^2} &= \left(\frac{e^t - 1 - e^t + 2}{e^t - 2} \right) \left(\frac{e^t - 2 - 2e^t + 2}{e^t - 2} \right) \\ \frac{e^{2t} - 2e^t - e^{2t} + e^t}{(e^t - 2)^2} &= \left(\frac{1}{e^t - 2} \right) \left(\frac{-e^t}{e^t - 2} \right) \\ \frac{-e^t}{(e^t - 2)^2} &= \frac{-e^t}{(e^t - 2)^2}\end{aligned}$$

1.1.9 31

$$m = -2$$

1.1.10 33

$$m = 2 \text{ or } 3$$

1.1.11 35

$$m = -1 \text{ or } 0$$

1.1.12 37

$$y = 2$$

1.1.13 39

No constant solutions