University Physics with Modern Physics notes

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Contents

1 Units, Physical Quantities, and Vectors

1.5 Uncertainty and Significant Figures

- If the uncertainty of a number isn't explicitly stated it can be indicated by the number of meaningful digits which is the number of digits excluding leading zeros, e.g. 0.1 and 1 have one, 1.0 has two, and 1.00 has three
- It is assumed that the value has an uncertainty $\pm$ one unit of the least significant digit, e.g. 1 is ± 1, 0.1 and 1.0 are ± 0.1, and 1.00 is ± 0.01
- When you multiply or divide numbers with uncertainties the result can have no more significant figures than the input with the fewest significant figures, e.g. $3.1416 \times 2.34 \times 0.58 = 4.3$
- When you add or subtract numbers with uncertainties the number of significant figures in the result is determined by the term with the largest uncertainty, e.g. $123.62 + 8.9 = 132.5$
- When reducing the number of significant figures in a result round rather than truncate, e.g. 1.688102894 to three significant figures is 1.69 not 1.68
- For the most accurate results, preserve extra significant figures throughout a calculation and only round the result

2 Motion Along a Straight Line

2.4 Motion with Constant Acceleration

- $v_x = v_{0x} + a_x t$
- $x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$
- $v_x^2 = v_{0x}^2 + 2a_x \left(x - x_0\right)$
- $x - x_0 = \frac{1}{2} \left(v_{0x} + v_x\right) t$

3 Motion in Two or Three Dimensions

3.1 Position and Velocity Vectors

- $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$
- $\mathbf{v}_{av} = \frac{\Delta\mathbf{r}}{\Delta t}$
- $\mathbf{v} = \frac{d\mathbf{r}}{dt}$

3.2 The Acceleration Vector

- $\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t}$
- $\mathbf{a} = \frac{d\mathbf{v}}{dt}$
- $\mathbf{a}$ can be decomposed into $\mathbf{a}_{\|}$, the component parallel to $\mathbf{v}$ which affects speed, and $\mathbf{a}_{\perp}$, the component perpendicular to $\mathbf{v}$ which affects direction

3.4 Uniform Circular Motion

- $a_{rad} = \frac{v^2}{R}$
- $T = \frac{2\pi R}{v}$
- $a_{rad} = \frac{4\pi^2 R}{T^2}$
- Because the acceleration in uniform circular motion is always directed towards the centre of the circle, it is sometimes called **centripetal acceleration** meaning "seeking the centre"
- $a_{tan} = \frac{dv}{dt}$

3.5 Relative Velocity

- An observer equipped with a meter stick and a stopwatch forms a **frame of reference** — a coordinate system plus a time scale
- The position of a point or frame of reference A relative to a frame of reference B is given by $\mathbf{r}_{A/B}$ and its velocity is given by $\mathbf{v}_{A/B-x}$
- $\mathbf{r}_{P/A} = \mathbf{r}_{P/B} + \mathbf{r}_{B/A}$
- $\mathbf{v}_{P/A} = \mathbf{v}_{P/B} + \mathbf{v}_{B/A}$

4 Newton's Laws of Motion

4.1 Force and Interactions

- The principle of **superposition of forces** states that any number of forces applied at a point on an object have the same effect as a single force equal to the vector sum of the forces
- The net force acting on an object is $\mathbf{R} = \Sigma\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \cdots$

4.2 Newton's First Law

- **Newton's first law of motion** states: An object acted on by no net external force has constant velocity (which may be zero) and zero acceleration
- Newton's first law only applies in **inertial frames of reference**, i.e. frames of reference that aren't undergoing acceleration

4.3 Newton's Second Law

- **Newton's second law of motion** states: If a net external force acts on an object, the object accelerates. The direction of acceleration is the same as the direction of the net external force. The mass of the object times the acceleration vector of the object equals the net external force vector
- $\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a}$
- Newton's second law only applies in **inertial frames of reference**, i.e. frames of reference that aren't undergoing acceleration

4.4 Newton's Third Law

- **Newton's third law of motion** states: If object A exerts a force on object B (an "action"), then object B exerts a force on object A (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on different objects.

5 Applying Newton's Laws

5.3 Friction Forces

- Friction forces always oppose the direction of motion
- The friction that acts when an object slides over a surface is called **kinetic friction** and is proportional to the normal force

$$f_k = \mu_k n$$

where f_k is the magnitude of the kinetic friction force, μ_k is the **coefficient of kinetic friction**, and n is the magnitude of the normal force

- The friction that acts when an object isn't yet moving is called **static friction** and is proportional to the normal force

$$f_s \leq (f_s)_{\max} = \mu_s n$$

where f_s is the magnitude of the static friction force, $(f_s)_{\max}$ is the maximum magnitude of the static friction force, μ_s is the **coefficient of static friction**, and n is the magnitude of the normal force

- **Rolling resistance** is the force resisting motion when a body such as a ball, tire, or wheel rolls on a surface. It includes factors such as deformation of the surface, friction between bearings, etc.

$$f_r = \mu_r n$$

where f_r is the magnitude of the rolling resistance, μ_r is the **coefficient of rolling friction**, and n is the magnitude of the normal force

- The force that a fluid (a gas or liquid) exerts on an object moving through it is known as **fluid resistance**. It opposes the direction of motion and, for small objects moving at low speeds, is proportional to the object's velocity

$$f = kv$$

where f is the magnitude of the fluid resistance, k is a proportionality constant that depends on the shape and size of the object and the properties of the fluid, and v is the magnitude of the object's velocity

- For large objects moving through air at larger speeds, the resisting force is proportional to v^2

$$f = Dv^2$$

where f is the magnitude of the **air drag** or **air resistance**, D is a proportionality constant similar to k above, and v is the magnitude of the object's velocity

- When an object's speed is large enough that air resistance equals weight, the object's acceleration is 0 and it is said to be at **terminal speed**

- For small objects moving at low speeds

$$v_t = \frac{mg}{k}$$

and for all other objects

$$v_t = \sqrt{\frac{mg}{D}}$$

5.5 The Fundamental Forces of Nature

- All forces are expressions of four fundamental forces: **electromagnetic interactions**, **gravitational interactions**, **the strong interaction**, and **the weak interaction**.

6 Work and Kinetic Energy

6.1 Work

- $W = Fs$ in Joules (Newton meters) for straight line displacement where W is the work done to an object, F is the force applied to the object, and s is the object's resulting displacement

- $W = \mathbf{F} \cdot \mathbf{s}$

- Work can be negative when a force is opposite to the direction of displacement

- The total work done on an object by multiple forces can be calculated as the work done by the net force or by summing the work done by each individual force

6.2 Kinetic Energy and the Work-Energy Theorem

- $K = \frac{1}{2}mv^2$ where K is an object's **kinetic energy**, m is its mass, and v is its speed

- The **work-energy theorem** states that the work done by the net force on a particle equals the change in the particle's kinetic energy

$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$

  where W_{tot} is the total work done on the particle, K_2 is its final kinetic energy, K_1 is its initial kinetic energy, and ΔK is its change in kinetic energy

- The work-energy theorem may only be used in inertial frames of reference, and kinetic energy values may differ between different frames

6.3 Work and Energy with Varying Forces

- $W = \int_{x_1}^{x_2} F dx$

- On a force vs. position graph, the area under the curve is the work done

- **Hooke's law** states that a compressed or stretched spring supplies a restorative force equal to

$$F = kx$$

where k is the **force constant** and x is the displacement from the unstretched position

- The work required to compress or stretch a spring a distance x from its equilibrium position is given by

$$W = \frac{1}{2}kx^2$$

- $W = \int_{P_1}^{P_2} \mathbf{F} \cdot \mathbf{dl}$

6.4 Power

- **Power** is the rate at which work is done
- $P = \frac{dW}{dt}$ in watts (W) where 1 watt is is 1 joule per second
- $P = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v}$

7 Potential Energy and Energy Conservation

7.1 Gravitational Potential Energy

- **Potential energy** is energy associated with an object's position
- **Gravitational potential energy** is energy associated with an object's height above ground and its weight

$$U_{\text{gravity}} = mgh$$

  where m is the object's mass, g is the acceleration due to gravity, and h is its height above ground

- $W_{\text{gravity}} = -\Delta U_{\text{gravity}}$
- The sum of kinetic and potential energies $E = K + U$ is called the **total mechanical energy of the system**
- When only gravity does work the total mechanical energy is conserved
- Gravitational potential energy values differ depending on where you choose to be $y = 0$, but it's the differences between values that matter

7.2 Elastic Potential Energy

- Energy stored in a deformable object that returns to its original shape and size after being deformed is called **elastic potential energy**

$$U_{\text{elastic}} = \frac{1}{2}kx^2$$

  where k is the force constant of the spring and x is its displacement from equilibrium

- $W_{\text{elastic}} = -\Delta U_{\text{elastic}}$ where W_{elastic} is the work done by the elastic force

- When only the elastic force does work the total mechanical energy is conserved

- In general, the relationship between kinetic energy and potential energy can be expressed as

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

  where

$$U = U_{\text{elastic}} + U_{\text{gravity}}$$

7.3 Conservative and Nonconservative Forces

- The work done by a **conservative force** has four properties:

  1. It can be expressed as the difference between the initial and final values of a potential energy function.
  2. It is reversible.
  3. It is independent of the path of the object and depends only on the starting and ending points.
  4. When the starting and ending points are the same the total work is zero.

- When only conservative forces do work the total mechanical energy $E = K + U$ is conserved or constant

- A force that is not conservative is called a **nonconservative force** and the work done by such a force can't be represented by a potential energy function

- Nonconservative forces that cause total mechanical energy to be lost are called **dissipative forces**

- The **law of conservation of energy** states that in a given process, the kinetic energy, potential energy, and internal energy of a system may all change but the sum of those changes is always 0

$$\Delta K + \Delta U + \Delta U_{\text{internal}} = 0$$

7.4 Force and Potential Energy

- $F_x(x) = -\frac{dU(x)}{dx}$ where $F_x(x)$ is the force from potential energy and $U(x)$ is the potential energy
- The physical interpretation of the above is that a conservative force always acts to push the system towards lower potential energy
- $\mathbf{F} = -\left(\frac{\partial U}{\partial x}\hat{\mathbf{x}} + \frac{\partial U}{\partial y}\hat{\mathbf{y}} + \frac{\partial U}{\partial z}\hat{\mathbf{z}}\right) = -\nabla U$

8 Momentum, Impulse, and Collisions

8.1 Momentum and Impulse

- $\mathbf{p} = m\mathbf{v}$ where $\mathbf{p}$ is an object's momentum, m is its mass, and $\mathbf{v}$ is its velocity
- $\mathbf{F} = \frac{d\mathbf{p}}{dt}$
- The **impulse-momentum theorem** states that the impulse of the net external force on a particle during a time interval equals the change in momentum of that particle during that interval

$$\mathbf{J} = \int_{t_1}^{t_2} \Sigma\mathbf{F}dt = \mathbf{F}_{\text{av}}\Delta t = \mathbf{p}_2 - \mathbf{p}_1 = \Delta\mathbf{p}$$

8.2 Conservation of Momentum

- For any system, the forces that the particles of the system exert on each other are called **internal forces**
- Forces exerted on any part of the system by some object outside it are called **external forces**
- When there are no external forces the system is **isolated**
- The **total momentum** of a system is the sum of the momenta of its particles

$$\mathbf{P} = \mathbf{p}_A + \mathbf{p}_B + \cdots = m\mathbf{v}_A + m\mathbf{v}_B + \cdots$$

- The **principle of conservation of momentum** states that if the vector sum of the external forces on a system is zero, the total momentum of the system is constant

8.3 Momentum Conservation and Collisions

- An **elastic collision** is one where the system's total kinetic energy is the same before and after the collision
- An **inelastic collision** is one where the system's total kinetic energy is reduced after the collision
- A **completely inelastic collision** is one where the objects stick together and move as one after colliding

8.4 Elastic Collisions

- In an elastic collision, the relative velocity of the two objects has the same magnitude before and after the collision

8.5 Centre of Mass

- The **centre of mass** is the mass-weighted average position of a collection of particles
$$\mathbf{r}_{\mathrm{cm}} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\Sigma_i m_i \mathbf{r}_i}{\Sigma_i m_i}$$
- Whenever a homogeneous object has a geometric centre, the centre of mass is at the geometric centre
- Whenever an object has an axis of symmetry, the centre of mass lies along that axis
- The total mass of a collection of particles times the velocity of their centre of mass equals the total momentum of the collection of particles
$$M\mathbf{v}_{\mathrm{cm}} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \cdots = \mathbf{P}$$
- This means that if there are no external forces, the velocity of the centre of mass doesn't change
- When an object or a collection of particles is acted on by external forces, the centre of mass moves as though all the mass were concentrated at that point and it were acted on by a net external force equal to the sum of the external forces on the system
$$\Sigma\mathbf{F}_{\mathrm{ext}} = M\mathbf{a}_{\mathrm{cm}} = \frac{d\mathbf{P}}{dt}$$

9 Rotation of Rigid Bodies

9.1 Angular Velocity and Acceleration

- θ is often used to specify the **angular position** of a rigid body
- The **instantaneous angular velocity** of a rigid body is defined as

$$\omega = \frac{d\theta}{dt}$$

- $\boldsymbol{\omega}$ is the angular velocity vector and it points along the axis of rotation as defined by the right hand rule
- The **instantaneous angular acceleration** of a rigid body is defined as

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

- $\boldsymbol{\alpha}$ is the angular acceleration vector

9.2 Rotation with Constant Angular Acceleration

- $\omega_z = \omega_{0z} + \alpha_z t$
- $\theta - \theta_0 = \frac{1}{2}\left(\omega_{0z} + \omega_z\right) t$
- $\theta = \theta_0 + \omega_{0z} t + \frac{1}{2}\alpha_z t^2$
- $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z\left(\theta - \theta_0\right)$

9.3 Relating Linear and Angular Kinematics

- The linear speed of a point on a rotating rigid body is given by $v = r\omega$ where r is the distance between the axis of rotation and the point and ω is the body's angular velocity
- The tangential component of acceleration for a point on a rotating rigid body acts to change the magnitude of its velocity and is given by

$$a_{\text{tan}} = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\alpha$$

- The centripetal (radial) component of acceleration for a point on a rotating rigid body acts to change the direction of its velocity and is given by

$$a_{\text{rad}} = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

9.4 Energy in Rotational Motion

- The **moment of inertia** of a body for a given rotation axis is defined as

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots = \Sigma m_i r_i^2$$

  where m_i is the mass of the i-th particle making up the body and r_i is its perpendicular distance from the axis of rotation

- The **rotational kinetic energy** of a rigid body is given by

$$K = \frac{1}{2} I \omega^2$$

9.5 Parallel-axis Theorem

- The **parallel axis theorem** gives a simple way to calculate moments of inertia

$$I_P = I_{\mathrm{cm}} + Md^2$$

  where I_P is the moment of inertia for a rotational axis passing through point P, I_{cm} is the moment of inertia for a rotational axis parallel to the first but passing through the object's centre of mass, M is the object's mass, and d is the distance between the two parallel axes

9.6 Moment of Inertia Calculations

- For continuous bodies

$$I = \int r^2 dm$$

10 Dynamics of Rotational Motion

10.1 Torque

- **Torque** is to angular acceleration what force is to linear acceleration
- The **line of action** of a force is the line along which the force vector lies
- The **lever arm** (or **moment arm**) of a force about an axis of rotation is the perpendicular distance between the axis of rotation and the line of action of the force
- $\tau = Fl = rFsin\phi = F_{\mathrm{tan}} r$ where τ is the magnitude of the torque, F is the magnitude of the force, l is the lever arm of the force about the axis of rotation, r is the distance between the axis of rotation and the point at which the force is applied, and F_{tan} is the magnitude of the tangential component of the force
- The SI unit of torque is the Newton-meter (not to be confused with the Joule)
- $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

10.2 Torque and Angular Acceleration for a Rigid Body

- $\Sigma\tau_z = I\alpha_z$

10.3 Rigid Body Rotation About a Moving Axis

- For rotation without slipping, $v_{\mathrm{cm}} = R\omega$

10.4 Work and Power in Rotational Motion

- $W = \int_{\theta_0}^{\theta_1} \tau_z d\theta$
- If the torque is constant $W = \tau_z(\theta_1 - \theta_0) = \tau_z\Delta\theta$
- $P = \tau_z\omega_z$

10.5 Angular Momentum

- **Angular momentum** is the rotational equivalent to linear momentum

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$$

  where $\mathbf{L}$ is the angular momentum of a particle about a particular origin O, $\mathbf{r}$ is its displacement vector from 0, and p is its linear momentum

- The rate of change of the angular momentum of a particle equals the torque of the net force acting on it

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \mathbf{F} = \boldsymbol{\tau}$$

- When a rigid body rotates around an axis of symmetry

$$\mathbf{L} = I\boldsymbol{\omega}$$

10.6 Conservation of Angular Momentum

- The **principle of conservation of angular momentum** states that when the net external torque acting on a system is zero, the total angular momentum of the system is constant (conserved)

11 Equilibrium and Elasticity

11.1 Conditions for Equilibrium

- The **first condition for equilibrium** states that in order for the centre of mass of an object at reset to remain at rest, the net external force on the object must be zero
- The **second condition for equilibrium** states that in order for a non-rotating object to remain non-rotating, the net external torque around any point on the object must be zero

11.2 Centre of Gravity

- If g has the same value at all points on a body...
  - its **centre of gravity** is identical to its centre of mass
  - the torque it experiences due to its weight is given by $\boldsymbol{\tau} = \mathbf{r}_{\mathrm{cm}} \times \mathbf{w}$
- If an extended object supported at two or more points is to be in equilibrium, its center of gravity must be somewhere within the area bounded by the supports. If the object is supported at only one point, its center of gravity must be above that point.

11.4 Stress, Strain, and Elastic Moduli

- Hooke's law states
$$\frac{\text{Stress}}{\text{Strain}} = \text{Elastic modulus}$$
where **stress** is a measure of the forces applied to deform an object, **strain** is a measure of how much deformation results from the stress, and **elastic modulus** is a property of the material of the object
- **Elastic** materials return to their original state after stress is removed
- **Plastic** materials remain deformed after stress is removed
- When pulling on either end of an object we say the object is in **tension**
- The **tensile stress** at the cross section is defined as
$$\text{Tensile stress} = \frac{F_{\perp}}{A}$$
where $F_{\perp}$ is the force applied at either end of the object and A is the cross sectional area
- The SI unit of stress is the **pascal** (Pa) which is also used for pressure
- **Tensile strain** is defined as
$$\text{Tensile strain} = \frac{\Delta l}{l_0}$$
where Δl is the object's change in length when under tension and l_0 is its original length
- Tensile strain can be thought of as stretch per unit length
- For sufficiently small tensile stress, stress and strain are proportional and the constant of proportionality is known as **Young's modulus**
$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_{\perp}/A}{\Delta l/l_0} = \frac{F_{\perp}}{A}\frac{l_0}{\Delta l}$$

- When the forces on either side of the object are pushes rather than pulls the object is under **compressive strain**
- For many materials Young's modulus is the same for compressive and tensile strains, but not for all (e.g. concrete)
- Uniform pressure on all sides of an object, e.g. water on a diver, is an example of **bulk stress** (or **volume stress**)
- Bulk stress is equivalent to **pressure**, defined as

$$p = \frac{F_\perp}{A}$$

- Bulk stress tends to compress an object resulting in **bulk strain** (or **volume strain**)
- Bulk strain is defined as

$$\text{Bulk (volume) strain} = \frac{\Delta V}{V_0}$$

  where ΔV is the object's change in volume when under strain and V_0 is its original volume
- When Hooke's law is obeyed bulk strain is proportional to bulk stress and their ratio is called the **bulk modulus**

$$B = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\Delta p}{\Delta V/V_0}$$

- The reciprocal of the bulk modulus is called the **compressability** and is denoted k
- Unlike bulk and tensile stresses which act perpendicular to an object's surface, **shear stress** acts tangent and in opposite directions

$$\text{Shear stress} = \frac{F_\parallel}{A}$$

- **Shear strain** is defined as

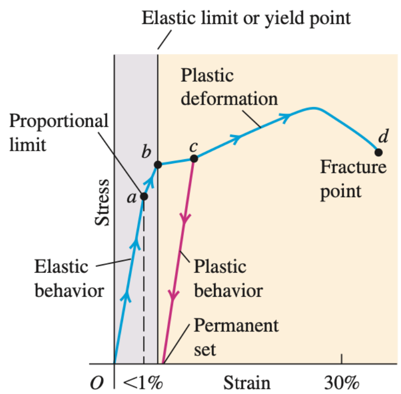
$$\text{Shear strain} = \frac{x}{h}$$

  where x is the displacement of one of the object's faces and h is the height of the transverse dimension
- When Hooke's law is obeyed shear strain is proportional to shear stress and their ratio is called the **shear modulus**

$$S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_\parallel/A}{x/h} = \frac{F_\parallel}{A}\frac{h}{x}$$

11.5 Elasticity and Plasticity

- Hooke's law has a limited range of validity



- Between points O and a Hooke's law applies and the deformation is elastic
- Point a is known as the proportional limit
- Between points a and b Hooke's law doesn't apply but the deformation is still elastic
- Point b is known as the yield point
- After point b the deformation becomes plastic
- At point d the material breaks
- Point d is known as the fracture point

12 Fluid Mechanics

12.1 Gases, Liquids, and Density

- A **fluid** is any substance that can flow and change the shape of the volume it occupies — this includes gases
- The **density** of a material is its mass per unit volume

$$\rho_{\text{avg}} = \frac{m}{V}$$

- The **specific gravity** of a material is the ratio of its density to the density of water at 4.0ºC

12.2 Pressure in a Fluid

- **Pressure** is defined as

$$p = \frac{F_\perp}{A}$$

- Pressure is measured in pascals where

$$1 \text{ pascal} = 1 \text{ Pa} = 1 \text{ N/m}^2$$

- The pressure at a depth h in a fluid of uniform density ρ is given by

$$p = p_0 + \rho g h$$

  where p_0 is the pressure at the surface of the fluid

- **Pascal's law** states that pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel

- Sometimes pressure is measured relative to atmospheric pressure, e.g. a car tire, and is called **gauge pressure** otherwise it's called **absolute pressure**

12.3 Buoyancy

- **Archimedes's Principle** states that when an object is completely or partially immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid displaced by the object.

12.4 Fluid Flow

- An **ideal fluid** is incompressible and has no internal friction (viscosity)

- The path of an individual particle in a moving fluid is called a **flow line**

- In **steady flow** the overall flow pattern doesn't change

- The **volume flow rate** is given by

$$\frac{dV}{dt} = Av$$

- The **continuity equation** states that

$$A_1 v_1 = A_2 v_2$$

  for an incompressible fluid and

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

  for a compressible fluid where ρ is the density the fluid, A is the cross-sectional area, and v is its velocity

12.5 Bernoulli's Equation

- **Bernoulli's equation** relates the pressure, gravitational potential energy, and kinetic energy of a fluid

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

- Bernoulli's equation is only valid for incompressible, steady state fluid flow with no internal friction (viscosity)

12.6 Viscosity and Turbulence

- **Viscosity** is internal friction in a fluid — the opposition of motion of one portion of fluid relative to another

13 Gravitation

13.1 Newton's Law of Gravitation

- **Newton's law of gravitation** states that

$$F_g = \frac{G m_1 m_2}{r^2}$$

  where F_g is the magnitude of the attractive gravitational force between two particles, G is the **gravitational constant**, m_1 and m_2 are the masses of the particles, and r is the distance between them

- The gravitational interaction between any two spherically symmetric mass distributions is the same as if all the mass were located at their centres

13.2 Gravitational Potential Energy

- **Gravitational potential energy** is defined as

$$U = -\frac{G m_1 m_2}{r}$$

13.4 The Motion of Satellites

- In a **closed orbit** the object returns to its starting point
- In an **open orbit** the object doesn't return to its starting point
- The speed of a satellite in a circular orbit around Earth is given by

$$v = \sqrt{\frac{G m_E}{r}}$$

- The period of a satellite in a circular orbit around Earth is given by

$$T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$

- The total energy of a satellite in a circular orbit around Earth is given by

$$E = -\frac{Gm_E m}{2r}$$

13.5 Kepler's Laws and the Motion of Planets

- **Kepler's Laws** are

  1. Each planet moves in an elliptical orbit, with the sun at one focus of the ellipse
  2. A line from the sun to a given planet sweeps out equal areas in equal times
  3. The periods of the planets are proportional to the $\frac{3}{2}$ powers of the major axis lengths of their orbits

- The distance between each focus and the centre of the ellipse is ea where e is a dimensionless number known as the **eccentricity**

- If the eccentricity is 0 the two foci coincide and the ellipse is a circle

- The period of an object in an elliptical orbit is given by

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM}}$$

  where a is the semi-major axis of the orbit and M is the mass of the object that is being orbited (e.g. the sun)

13.6 Spherical Mass Distributions

- The gravitational interaction between two spherically symmetric mass distributions is the same as though all the mass of each were concentrated at the centre

- Points inside a spherically symmetric mass distribution don't experience a gravitational force from mass at a greater radius from the centre

13.7 Apparent Weight and the Earth's Rotation

- Objects that aren't at the Earth's poles undergo circular motion and must experience a net centripetal force. This force means their **apparent weight** is less than their **true weight**

13.8 Black Holes

- The escape speed of an object with mass M and radius R is

$$v = \sqrt{\frac{2GM}{R}}$$

- If the escape speed of an object is equal to c it becomes a black hole
- If the radius of an object of mass M is less than or equal to the **Schwarzchild radius** it becomes a black hole

$$R_s = \frac{2GM}{c^2}$$

14 Periodic Motion

14.1 Describing Oscillation

- The **amplitude** A of the motion is the maximum magnitude of displacement from equilibrium
- A complete vibration or **cycle** is one complete round trip — from one extreme, through equilibrium, to the other extreme, through equilibrium, and back to the starting position
- The **period** T of the motion is the duration of one cycle
- The **frequency** f is the number of cycles per second
- The **angular frequency** ω is 2π times the frequency f
- Frequency and period are reciprocals of each other

$$f = \frac{1}{T}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

14.2 Simple Harmonic Motion

- When the restoring force is directly proportional to the displacement from equilibrium the oscillation is called **simple harmonic motion**
- An object that undergoes simple harmonic oscillation is called a **harmonic oscillator**
- If k is the force constant of the restoring force and m is the mass of the object undergoing oscillation, then

$$\omega = \sqrt{\frac{k}{m}}$$

- This is because simple harmonic motion is analogous to uniform circular motion projected onto an axis — ω is the angular frequency of the object undergoing uniform circular motion

- This also means that
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
and
$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

- The displacement of an object undergoing simple harmonic motion is
$$x = A\cos(\omega t + \phi)$$
where x is the object's displacement, A is its amplitude, ω is its angular frequency, t is time, and ϕ is the phase angle

14.3 Energy in Simple Harmonic Motion

- At maximum displacement from equilibrium $x = A$ and $v = 0$ so
$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

- Only conservative forces act in simple harmonic motion, to total mechanical energy is conserved and $E = \frac{1}{2}kA^2$ always

14.4 Applications of Simple Harmonic Motion

- Vertical SHM is the same as horizontal SHM except the equilibrium position isn't when the spring is unstretched but when its restoring force is equal to the weight of the object

- Under **angular SHM**
$$\omega = \sqrt{\frac{\kappa}{I}}$$
$$f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$$
$$\theta = \Theta\cos(\omega t + \phi)$$
where Ω is the amplitude of the oscillation

14.5 The Simple Pendulum

- A **simple pendulum** is an idealized model consisting of a point mass suspended by a massless, unstretchable string
- The restoring force for a simple pendulum is $-mg\sin\theta$ so the motion isn't SHM, however $\sin\theta \approx \theta$ for small θ in which case the motion is SHM
- For small θ
$$k = \frac{mg}{L}$$
$$\omega = \sqrt{\frac{g}{L}}$$
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

14.6 The Physical Pendulum

- A **physical pendulum** is any real pendulum that uses an extended object, in contrast to the idealized simple pendulum with all of its mass concentrated at a point
- When the object is displaced from equilibrium its weight produces a restoring torque
$$\tau = -mg\sin\theta$$
- Like the simple pendulum, this doesn't describe SHM because it's proportional to $\sin\theta$ rather than θ but it approximates SHM for small θ
- This gives us
$$\begin{aligned} -mgd\theta &= \tau \\ &= I\alpha \\ &= I\frac{d^2\theta}{dt^2} \\ -\frac{mgd}{I}\theta &= \frac{d^2\theta}{dt^2} \end{aligned}$$
so
$$\omega = \sqrt{\frac{mgd}{I}}$$
$$T = 2\pi\sqrt{\frac{I}{mgd}}$$