Vibrations and Waves by George C. King Notes

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Contents

1 Simple Harmonic Motion

- The equation of motion for a simple harmonic oscillator is

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

  where

$$\omega^2 = \frac{k}{m}$$

- The general solution of the equation of motion for a simple harmonic oscillator is

$$x = A\cos(\omega t + \phi)$$

  or equivalently

$$x = a\cos\omega t + b\sin\omega t$$

- The angular frequency ω is determined entirely by properties of the oscillator, e.g. its mass and spring coefficient

- The total energy of a harmonic oscillator is

$$E = \frac{1}{2}kA^2$$

- Nearly all potential wells have a shape that is parabolic when sufficiently close to the equilibrium position, so most oscillating systems will oscillate with SHM when the amplitude of oscillation is small

- The vibrations of nuclei in a molecule can be modeled by SHM, but only a discrete set of vibrational energies is possible, namely

$$\frac{1}{2}\hbar\omega,\ \frac{3}{2}\hbar\omega,\ \frac{5}{2}\hbar\omega,\ \ldots$$

  where $\hbar$ is Planck's constant divided by 2π

- The total energy of a system undergoing SHM is always given by an expression of the form
$$E = \frac{1}{2}\alpha v^2 + \frac{1}{2}\beta x^2$$
where α and β are physical constants — if we obtain this equation during the analysis of a system we know we have SHM

- The equation of motion for a system described by the energy equation above is
$$\frac{d^2x}{dt^2} = -\frac{\beta}{\alpha}x$$