Introduction to Electrodynamics by David J. Griffiths Problems

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Contents

2 Electrostatics

2.1

(a) $\mathbf{0}$

(b) The same as if only the opposite charge were present — all others are cancelled out.

2.2

$$\begin{aligned}\mathbf{E} &= \frac{1}{4\pi\epsilon_0} 2 \frac{q}{\mathscr{r}^2} \cos\theta \hat{\mathbf{x}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{dq}{[(d/2)^2 + z^2]^{3/2}} \hat{\mathbf{x}}\end{aligned}$$

2.3

$$
\begin{aligned}
\mathbf{r} &= z\hat{\mathbf{z}} \\
\mathbf{r}' &= x\hat{\mathbf{x}} \\
\boldsymbol{\mathscr{r}} &= z\hat{\mathbf{z}} - x\hat{\mathbf{x}} \\
\mathscr{r} &= \sqrt{x^2 + z^2} \\
\hat{\boldsymbol{\mathscr{r}}} &= \frac{z\hat{\mathbf{z}} - x\hat{\mathbf{x}}}{\sqrt{x^2 + z^2}} \\
\mathbf{E} &= \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda}{x^2 + z^2} \frac{z\hat{\mathbf{z}} - x\hat{\mathbf{x}}}{\sqrt{x^2 + z^2}}\, dx \\
&= \frac{1}{4\pi\epsilon_0}\lambda \left(z\hat{\mathbf{z}} \int_0^L \frac{1}{(x^2 + z^2)^{3/2}}\, dx - \hat{\mathbf{x}} \int_0^L \frac{x}{(x^2 + z^2)}\, dx \right) \\
&= \frac{1}{4\pi\epsilon_0}\lambda \left[\frac{L}{z\sqrt{L^2 + z^2}}\hat{\mathbf{z}} - \left(\frac{1}{z} - \frac{1}{\sqrt{L^2 + z^2}} \right) \hat{\mathbf{x}} \right] \\
&= \frac{1}{4\pi\epsilon_0} \frac{\lambda}{z} \left[\left(-1 + \frac{z}{\sqrt{L^2 + z^2}} \right) \hat{\mathbf{x}} + \frac{L}{\sqrt{L^2 + z^2}}\hat{\mathbf{z}} \right]
\end{aligned}
$$

2.4

The electric field a distance z above the midpoint of a line segment of length $2L$ and uniform line charge λ is

$$
\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z\sqrt{z^2 + L^2}}\hat{\mathbf{z}}.
$$

Applying this to the four sides of the square, the horizontal components of opposite sides cancel leaving only the vertical component.

$$
\begin{aligned}
\cos\theta &= \frac{z}{\mathscr{r}} \\
&= \frac{z}{\sqrt{(a/2)^2 + z^2}} \\
\mathbf{E} &= 4 \left(\frac{1}{4\pi\epsilon_0} \frac{\lambda a}{\sqrt{(a/2)^2 + z^2}\sqrt{(a/2)^2 + (a/2)^2 + z^2}}\hat{\mathbf{z}} \right) \cos\theta \\
&= \frac{1}{4\pi\epsilon_0} \frac{4a\lambda z}{[(a/2)^2 + z^2]\sqrt{(a^2/2) + z^2}}\hat{\mathbf{z}}
\end{aligned}
$$

2.5

$$
\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\frac{\lambda r}{r^2+z^2}\cos\alpha\, d\theta\,\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}\frac{2\pi\lambda r z}{(r^2+z^2)^{3/2}}\hat{\mathbf{z}}
\end{aligned}
$$

2.6

$$
\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int\frac{dq}{\imath^2}\cos\theta\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{R}\frac{\sigma}{r^2+z^2}\frac{z}{\sqrt{r^2+z^2}}r\,dr\,d\theta\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}2\pi\sigma z\int_0^{R}\frac{r}{(r^2+z^2)^{3/2}}\,dr\,\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}2\pi\sigma z\left(\frac{1}{z}-\frac{1}{\sqrt{R^2+z^2}}\right)\hat{\mathbf{z}}
\end{aligned}
$$

When $R \to \infty$

$$
\mathbf{E} = \frac{\sigma}{2\epsilon_0}\hat{\mathbf{z}}.
$$

2.7

$$
\mathbf{E} = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\hat{\mathbf{z}} & z > R \\ \mathbf{0} & z < R \end{cases}
$$

2.8

$$
\mathbf{E} = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\hat{\mathbf{z}} & z > R \\ \frac{1}{4\pi\epsilon_0}\frac{qz}{R^3}\hat{\mathbf{z}} & z < R \end{cases}
$$