University Physics with Modern Physics - Modern Physics by Young and Freedman Problems

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18 Thermal Properties of Matter   17

17 Temperature and Heat

17.1 Guided Practice

17.1.1

(a)

$$\Delta L = \alpha L_0 \Delta T$$
$$\alpha = \frac{\Delta L}{L_0 \Delta T}$$
$$= 2.0 \times 10^{-5}\,\text{K}^{-1}$$

(b)

$$\Delta L = \alpha L_0 \Delta T$$
$$= -0.27\,\text{mm}$$

17.1.2

$$\Delta V_C = \beta V_{C0} \Delta T$$
$$= (5.1 \times 10^{-5})(250)(-70)$$
$$= -0.893\,\text{cm}^3$$
$$\Delta V_E = \beta V_{E0} \Delta T$$
$$= (75 \times 10^{-5})(250)(-70)$$
$$= -13.1\,\text{cm}^3$$
$$\Delta V_C - \Delta V_E = 12.2\,\text{cm}^3$$
$$= 12.2\,\text{mL}$$

17.1.3

$$\frac{\Delta L}{L_0} = \alpha \Delta T$$
$$Y = \frac{F/A}{\Delta L/L_0}$$
$$\frac{\Delta L}{L_0} = \frac{F}{AY}$$
$$\alpha \Delta T + \frac{F}{AY} = 0$$
$$\frac{F}{AY} = -\alpha \Delta T$$
$$F = -\alpha AY \Delta T$$
$$= -(2.0 \times 10^{-5})(\pi 0.005^2)(9.0 \times 10^{10})(-12)$$
$$= 1.70 \times 10^3\,\text{N}$$

Tensile

17.1.4

$$\Delta L = \alpha_A L_A \Delta T + \alpha_B L_B \Delta T$$
$$\frac{\Delta L}{\Delta T} = \alpha_A L_A + \alpha_B (L - L_A)$$
$$= (\alpha_A - \alpha_B) L_A + \alpha_B L$$
$$L_A = \frac{1}{\alpha_A - \alpha_B} \left(\frac{\Delta L}{\Delta T} - \alpha_B L \right)$$

17.1.5

$$0 = m_{Al} c_{Al} \Delta T_{Al} + m_W c_W \Delta T_W$$
$$= m_{Al} c_{Al} (T - T_{Al}) + m_W c_W (T - T_W)$$
$$m_{Al} = -\frac{m_W c_W (T - T_W)}{c_{Al}(T - T_{Al})}$$
$$= 0.20 \, \text{kg}$$

17.1.6

$$0 = m_I L_f + m_C c_C \Delta T$$
$$= m_I L_f - m_C c_C T$$
$$T = \frac{m_I L_f}{m_C c_C}$$
$$= 14.0 \, °\text{C}$$

17.1.7

$$0 = m_I L_F + m_I c_I \Delta T_I + m_E c_E \Delta T_E$$
$$= m_I (L_F + c_I \Delta T_I) + m_E c_E \Delta T_E$$
$$m_I = -\frac{m_E c_E \Delta T_E}{L_F + c_I \Delta T_I}$$
$$= 0.176 \, \text{kg}$$

17.1.8

Cooling the silver to $0 \, °\text{C}$ would take

$$Q = mc\Delta T = 92\,137.5 \, \text{J}$$

whereas melting all of the ice would take

$$Q = mL_f = 83\,500\,\text{J}$$

so all of the ice will melt.

$$
\begin{aligned}
0 &= m_{Ag}c_{Ag}\Delta T_{Ag} + m_I L_f + m_I c_I \Delta T_I + m_I c_W \Delta T_W \\
&= m_{Ag}c_{Ag}(T - T_{Ag}) + m_I L_F - m_I c_I T_I + m_I c_W T \\
&= (m_{Ag}c_{Ag} + m_I c_W)T - m_{Ag}c_{Ag}T_{Ag} + m_I L_F - m_I c_I T_I \\
T &= \frac{m_{Ag}c_{Ag}T_{Ag} + m_I c_I T_I - m_I L_F}{m_{Ag}c_{Ag} + m_I c_W} \\
&= 3.31\,^\circ\text{C}
\end{aligned}
$$

17.1.9

(a)

$$
\begin{aligned}
H &= kA\frac{T_H - T_C}{L} \\
k &= \frac{HL}{A(T_H - T_C)} \\
&= 0.754\,\text{W}/(\text{m K})
\end{aligned}
$$

(b)

$$H = kA\frac{T_H - T_C}{L} = 733\,\text{W}$$

17.1.10

(a)

$$
\begin{aligned}
L &= 0.250\,\text{m} \\
A &= 2.00 \times 10^{-4}\,\text{m}^2 \\
k_B &= 109.0\,\text{W}/(\text{m K}) \\
k_{Pb} &= 34.7\,\text{W}/(\text{m K}) \\
T &= 185\,^\circ\text{C} \\
H &= 6.00\,\text{W}
\end{aligned}
$$

$$
\begin{aligned}
H &= k_B A\frac{T_H - T}{L} \\
T_H &= \frac{HL}{k_B A} + T \\
&= 254\,^\circ\text{C}
\end{aligned}
$$

(b)

$$H = k_{Pb}A\frac{T - T_C}{L}$$

$$T_C = T - \frac{HL}{k_{Pb}A}$$

$$= -31.1\,°\text{C}$$

17.1.11

$$H = 4\pi(kr_E)^2 e\sigma T^4$$

$$(kr_E)^2 = \frac{H}{4\pi e\sigma T^4}$$

$$k = \frac{1}{r_E}\sqrt{\frac{H}{4\pi e\sigma T^4}}$$

$$= 1.70$$

17.1.12

(a)

$$H = Ae\sigma T^4$$

$$= \pi r^2 \sigma T^4$$

$$H = kA\frac{T_H - T_C}{L}$$

$$= k\pi r^2\frac{T_H - T_C}{L}$$

$$\pi r^2 \sigma T^4 = k\pi r^2\frac{T_H - T_C}{L}$$

$$T_H = \frac{L\sigma T^4}{k} + T_C$$

$$= 14.26\,\text{K}$$

(b)

$$H = mL_f$$

$$\pi r^2 \sigma T^4 = mL_f$$

$$m = \frac{\pi r^2 \sigma T^4}{L_f}$$

$$= 1.19 \times 10^{-4}\,\text{kg/s}$$

$$= 0.427\,\text{kg/h}$$

17.2   Exercises and Problems

17.2.15

$$\Delta V = \beta V_0 \Delta T$$
$$\frac{\Delta V}{V_0} = \beta(T - T_0)$$
$$T = T_0 + \frac{\Delta V}{\beta V_0}$$
$$= 49\,^{\circ}\text{C}$$

17.2.25

$$Q = (m_{Al}c_{Al} + m_W c_W)\Delta T$$
$$= 5.55 \times 10^5\,\text{J}$$

17.2.33

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2$$
$$= \frac{1}{2}m(v^2 - v'^2)$$
$$= 3.47\,\text{kJ}$$
$$\Delta K = mc\Delta T$$
$$\Delta T = \frac{\Delta K}{mc}$$
$$= 6.14 \times 10^{-2}\,^{\circ}\text{C}$$

17.2.35

(a)

$$0 = m_m c_m \Delta T_m + m_w c_w \Delta T_w$$
$$c_m = -\frac{m_w c_w \Delta T_w}{m_m \Delta T_m}$$
$$= 215\,\text{J/(kg\,K)}$$

(b) Water because it has a higher specific heat

(c) It would be too small

17.2.45

$$\frac{1}{2}mv^2 = mc\Delta T + mL_F$$

$$v = \sqrt{2(c\Delta T + L_F)}$$

$$= 366\,\text{m/s}$$

17.2.55

$$k_C A \frac{T_H - T}{L} = kA\frac{T}{L}$$

$$k_C T_H - k_C T = kT$$

$$k_C T_H = (k + k_C)T$$

$$T = \frac{k_C}{k + k_C}T_H$$

$$0.71 = \frac{k_C}{k + k_C}$$

$$0.71(k + k_C) = k_C$$

$$0.71k + 0.71k_C = k_C$$

$$0.71k = 0.29k_C$$

$$k = \frac{0.29}{0.71}k_C$$

$$\approx 157\,\text{W/(m\,K)}$$

17.2.57

(a)

$$k_W \frac{T - T_C}{L_W} = k_S \frac{T_H - T}{L_S}$$

$$\left(\frac{k_W}{L_W} + \frac{k_S}{L_S}\right)T = \frac{k_S}{L_S}T_H + \frac{k_W}{L_W}T_C$$

$$T = \frac{\frac{k_S}{L_S}T_H + \frac{k_W}{L_W}T_C}{\frac{k_W}{L_W} + \frac{k_S}{L_S}}$$

$$= -0.86\,^\circ\text{C}$$

(b)

$$H = k_W \frac{T - T_C}{L_W}$$

$$= 24.4\,\text{W/m}^2$$

17.2.65

$$H = Ae\sigma T^4$$
$$A = \frac{H}{e\sigma T^4}$$
$$= 2.1\,\text{cm}^2$$

17.2.69

$$\Delta L = (\alpha_B L_B + \alpha_S L_S)\Delta T$$
$$T = T_0 + \frac{\Delta L}{\alpha_B L_B + \alpha_S L_S}$$
$$= 35.0\,^\circ\text{C}$$

17.2.71

$$Q = mc\Delta T$$
$$= \rho V c \Delta T$$
$$\Delta T = \frac{Q}{\rho V c}$$
$$\Delta V = \beta V \Delta T$$
$$= \frac{\beta Q}{\rho c}$$
$$c = \frac{\beta Q}{\rho \Delta V}$$

17.2.73

(a)

$$0.0\,^\circ\text{M} = -39\,^\circ\text{C}$$
$$100.0\,^\circ\text{M} = 357\,^\circ\text{C}$$
$$T_M = \frac{T_C + 39\,^\circ\text{C}}{3.96}$$
$$\frac{100\,^\circ\text{C} + 39\,^\circ\text{C}}{3.96} = 35.1\,^\circ\text{M}$$

(b)

$$10\,\text{M}^\circ = 10\frac{357\,^\circ\text{C} - (-39\,^\circ\text{C})}{100} = 39.6\,\text{C}^\circ$$

17.2.75

$$Ah + \beta_G Ah(T - T_0) = Ah' + \beta_O Ah'(T - T_0)$$
$$Ah + \beta_G AhT - \beta_G AhT_0 = Ah' + \beta_O Ah'T - \beta_O Ah'T_0$$
$$(\beta_G Ah - \beta_O Ah')T = (Ah' - \beta_O Ah'T_0) - (Ah - \beta_G AhT_0)$$
$$T = \frac{(1 - \beta_O T_0)h' - (1 - \beta_G T_0)h}{\beta_G h - \beta_O h'}$$
$$= 69.4\,°\text{C}$$

17.2.79

(a)

$$Y = \frac{F/A}{\Delta L/L_0}$$
$$\Delta L = \frac{FL_0}{AY}$$
$$\Delta L = \alpha L_0 \Delta T$$
$$\Delta L = \alpha L_0 \Delta T + \frac{FL_0}{AY}$$
$$\frac{F}{A} = Y\left(\frac{\Delta L}{L_0} - \alpha \Delta T\right)$$

(b)

$$\Delta L_B = \alpha_B L_{B0} \Delta T$$
$$\frac{\Delta L_B}{L_{B0}} = \alpha_B \Delta T$$
$$\frac{F}{A} = Y_S(\alpha_B - \alpha_S)\Delta T$$
$$= 1.9 \times 10^8\,\text{Pa}$$

17.2.85

(a)

$$\frac{dQ}{dT} = nk\frac{T^3}{\theta^3}$$
$$Q = \int_a^b nk\frac{T^3}{\theta^3}$$
$$= \frac{nk}{\theta^3}\left[\frac{1}{4}T^4\right]_a^b$$
$$= \frac{nk}{4\theta^3}(b^4 - a^4)$$
$$= 83.6\,\text{J}$$

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(b)

$$Q = nC\Delta T$$
$$C = \frac{Q}{n\Delta T}$$
$$= 1.86\,\text{J}/(\text{mol}\,\text{K})$$

(c)
$$C = 5.60\,\text{J}/(\text{mol}\,\text{K})$$

17.2.95

(a)

$$0 = m_I L_f + m_I c_W \Delta T_I + m_C c_C \Delta T_I - m_S L_v + m_S c_W \Delta T_S$$
$$= m_I L_f + m_I c_W T + m_C c_C T - m_S L_v + m_S c_W (T - T_S)$$
$$T = \frac{m_S(L_v + c_W T_S) - m_I L_f}{m_I c_W + m_C c_C + m_S c_W}$$
$$= 86.1\,°\text{C}$$

(b) No ice, $0.13\,\text{kg}$ water, no steam

17.2.99

(a)

$$H = kA\frac{T_H - T_C}{L}$$
$$= 94\,\text{W}$$

(b)

$$H_{\text{wood}} = 12.4\,\text{W}$$
$$H_{\text{glass}} = 45.0\,\text{W}$$
$$H' = H + (H_{\text{glass}} - H_{\text{wood}})$$
$$= 126.6\,\text{W}$$
$$\frac{H'}{H} = 1.35$$

17.2.105

(b)

$$\frac{dQ}{dt} = k\frac{T_H - T_C}{L}$$

$$\frac{dQ}{dL} = \rho L_f$$

$$\frac{dL}{dt} = \frac{dL}{dQ}\frac{dQ}{dt}$$

$$= \frac{1}{\rho L_f}k\frac{T_H - T_C}{L}$$

$$L\frac{dL}{dt} = \frac{k(T_H - T_C)}{\rho L_f}$$

$$\int_0^t L\frac{dL}{dt}\,dt = \int_0^t \frac{k(T_H - T_C)}{\rho L_f}\,dt$$

$$\int_0^L L'\,dL' = \frac{k(T_H - T_C)}{\rho L_f}t$$

$$\frac{1}{2}L^2 = \frac{k(T_H - T_C)}{\rho L_f}t$$

$$L = \sqrt{\frac{2k(T_H - T_C)}{\rho L_f}t}$$

(c)

$$t = \frac{L^2 \rho L_f}{2k(T_H - T_C)}$$

$$= 7.5\,\text{days}$$

(d) $t \approx 530\,\text{years}$; no

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17.2.107

$$A = 2\pi \left(\frac{d}{2}\right)^2 + 2\pi \left(\frac{d}{2}\right) h$$

$$= 8.34 \times 10^{-2}\,\mathrm{m}^2$$

$$H = Ae\sigma(T^4 - T_s^4)$$

$$= Ae\sigma(T^4 - T_s^4)$$

$$= -3.38 \times 10^{-2}\,\mathrm{W}$$

$$m = \frac{H \times 60 \times 60}{L_v}$$

$$= 5.82 \times 10^{-3}\,\mathrm{kg/h}$$

$$= 5.82\,\mathrm{g/h}$$

17.2.113

$$r(x) = R_2 - (R_2 - R_1)\frac{x}{L}$$

$$A(x) = \pi r(x)^2$$

$$= \pi \left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2$$

$$H = kA(x)\frac{dT}{dx}$$

$$= k\pi \left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2 \frac{dT}{dx}$$

$$\frac{1}{\left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2} H\,dx = k\pi\,dT$$

$$\int_0^L \frac{1}{\left[R_2 - (R_2 - R_1)\frac{x}{L}\right]^2} H\,dx = \int_{T_H}^{T_C} k\pi\,dT$$

$$\frac{HL}{R_2 - R_1} \left[\frac{1}{R_2 - (R_2 - R_1)\frac{x}{L}}\right]_0^L = k\pi(T_C - T_H)$$

$$\frac{HL}{R_2 - R_1} \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = k\pi(T_C - T_H)$$

$$\frac{HL}{R_2 - R_1} \frac{R_2 - R_1}{R_1 R_2} = k\pi(T_C - T_H)$$

$$H = \frac{k\pi R_1 R_2 (T_C - T_H)}{L}$$

17.2.115

(a)

$$H = k(2\pi r L)\frac{dT}{dr}$$

$$\frac{1}{r}H\,dr = 2\pi k L\,dT$$

$$\int_a^b \frac{1}{r}H\,dr = \int_{T_1}^{T_2} 2\pi k L\,dT$$

$$H\ln\frac{b}{a} = 2\pi k L(T_2 - T_1)$$

$$H = \frac{2\pi k L(T_2 - T_1)}{\ln b/a}$$

(b)

$$\frac{2\pi k L(T - T_2)}{\ln r/a} = \frac{2\pi k L(T_2 - T_1)}{\ln b/a}$$

$$\frac{T - T_2}{\ln r/a} = \frac{T_2 - T_1}{\ln b/a}$$

$$T - T_2 = \frac{\ln r/a}{\ln b/a}(T_2 - T_1)$$

$$T = T_2 + \frac{\ln r/a}{\ln b/a}(T_2 - T_1)$$

17.2.117

a

17.2.119

a

18 Thermal Properties of Matter

18.1 Guided Practice

18.1.1

(a)

$$pV = nRT$$

$$\frac{p}{T} = \frac{nR}{V}$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$p_2 = p_1 \frac{T_2}{T_1}$$

$$= 4.67 \times 10^5 \,\text{Pa}$$

(b)

$$pV = nRT$$

$$n = \frac{pV}{RT}$$

$$= 0.280 \,\text{mol}$$

18.1.2

(a)

$$pV = nRT$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$V_2 = \frac{V_1 p_1 T_2}{p_2 T_1}$$

$$= 1.2 \times 10^3 \,\text{m}^3$$

(b)

$$\frac{V_2}{V_1} = \frac{\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi r_1^3}$$

$$= \left(\frac{r_2}{r_1}\right)^3$$

$$\frac{r_2}{r_1} = \sqrt[3]{\frac{V_2}{V_1}}$$

$$= 4.5$$

18.1.3

(a)

$$pV = nRT$$
$$n = \frac{pV}{RT}$$
$$= 2.9 \times 10^{-3} \, \text{mol/m}^3$$

(b)

$$8.0 \times 10^{-5} \, \text{kg/m}^3$$

18.1.4

(a)

$$pV = \frac{m_{\text{total}}}{M} RT$$
$$\frac{p}{\rho T} = \frac{R}{M}$$
$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2}$$
$$= \frac{p_2}{\rho_1 (p_2/p_1)^{3/5} T_2}$$
$$T_2 = \left(\frac{p_2}{p_1} \right)^{2/5} T_1$$

(b)

$$\frac{\rho_2}{\rho_1} = \frac{\rho_1 (p_2/p_1)^{3/5}}{\rho_1}$$
$$= \left(\frac{\frac{1}{2} p_1}{p_1} \right)^{3/5}$$
$$= \left(\frac{1}{2} \right)^{3/5}$$
$$\approx 0.660$$
$$\frac{T_2}{T_1} = \frac{(p_2/p_1)^{2/5} T_1}{T_1}$$
$$= \left(\frac{\frac{1}{2} p_1}{p_1} \right)^{2/5}$$
$$= \left(\frac{1}{2} \right)^{2/5}$$
$$\approx 0.758$$

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(c)

$$\frac{\rho_2}{\rho_1} = 2^{3/5}$$
$$\approx 1.52$$
$$\frac{T_2}{T_1} = 2^{2/5}$$
$$\approx 1.32$$

18.1.5

$$\sqrt{\frac{3RT}{M_{\mathrm{H}}}} = \sqrt{\frac{3RT_{\mathrm{N}}}{M_{\mathrm{N}}}}$$
$$T = \frac{M_{\mathrm{H}}}{M_{\mathrm{N}}} T_{\mathrm{N}}$$
$$= 41.9\,\mathrm{K}$$
$$= -231\,^{\circ}\mathrm{C}$$

18.1.6

(a)

$$K_{\mathrm{tr}} = \frac{3}{2}kT = 6.21 \times 10^{-20}\,\mathrm{J}$$

(b)

$$v_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}} = 8.63 \times 10^{3}\,\mathrm{m/s}$$

18.1.7

(a)

$$pV = \frac{N}{N_A}RT$$
$$N = \frac{N_A pV}{RT}$$
$$= 1.50 \times 10^{27}$$

(b)

$$K_{\mathrm{tr}} = \frac{3}{2}nRT = 9.11 \times 10^{6}\,\mathrm{J}$$

(c)

$$\frac{1}{2}mv^2 = K_{\text{tr}}$$

$$v = \sqrt{\frac{2K_{\text{tr}}}{m}}$$

$$= 110\,\text{m/s}$$

18.1.8

(a) 5.5

(b) 38.5

(c) 6.2

18.1.9

(a)

$$\lambda = \frac{kT}{4\pi\sqrt{2}r^2 p} = 6.8 \times 10^{-6}\,\text{m}$$

(b)

$$\lambda_{\text{Earth}} = 5.54 \times 10^{-8}\,\text{m}$$

$$\frac{\lambda_{\text{Mars}}}{\lambda_{\text{Earth}}} = 1.2 \times 10^2$$

18.1.10

(a)

$$\lambda = \frac{kT}{4\pi\sqrt{2}r^2 p}$$

$$p = \frac{kT}{4\pi\sqrt{2}r^2 \lambda}$$

$$= 5.7 \times 10^{-3}\,\text{Pa}$$

(b)

$$pV = nRT$$

$$n = \frac{pV}{RT}$$

$$= 2.3 \times 10^{-6}\,\text{mol}$$

18.1.11

(a)

$$pV = nRT$$
$$p = \frac{nRT}{V}$$
$$= 2.0 \times 10^7 \, \text{Pa}$$
$$\lambda = \frac{kT}{4\pi\sqrt{2}r^2 p}$$
$$= 1.2 \times 10^{-8} \, \text{m}$$

(b)

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$
$$= 1.4 \times 10^3 \, \text{m/s}$$
$$\lambda = vt_{\text{mean}}$$
$$t_{\text{mean}} = \frac{\lambda}{v}$$
$$= 8.6 \times 10^{-12} \, \text{s}$$

18.1.12

(a)

$$v_{\text{rms}} t_{\text{mean}} = \lambda$$
$$\sqrt{\frac{3kT}{m}} t_{\text{mean}} = \frac{kT}{4\pi\sqrt{2}r^2 p}$$
$$t_{\text{mean}} = \frac{kT}{4\pi\sqrt{2}r^2 p} \sqrt{\frac{m}{3kT}}$$
$$= \frac{1}{4\pi r^2 p} \sqrt{\frac{mkT}{6}}$$

(b) Doubling r.

18.1.13

(a)

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$= 515\,\text{m/s}$$

$$\frac{1}{2}mv_{\text{rms}}^2 = mgh$$

$$h = \frac{v_{\text{rms}}^2}{2g}$$

$$= 102\,\text{km}$$

(b)

$$\int_{2025}^{\infty} 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}\,dv$$

$$= (3.03 \times 10^{-8}) \int_{2025}^{\infty} v^2 e^{-(5.65\times10^{-6})v^2}\,dv$$

$$= 4.8 \times 10^{-10}$$

Yes, some escape.

18.2 Exercises and Problems

18.2.7

$$pV = nRT$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$T_2 = \frac{p_2 V_2 T_1}{p_1 V_1}$$

$$= 776\,\text{K}$$

$$= 503\,°\text{C}$$

18.2.9

$$pV = nRT$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$p_2 = \frac{p_1 V_1 T_2}{T_1 V_2}$$

$$= 1.97 \times 10^4\,\text{Pa}$$

18.2.13

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$V_2 = \frac{p_1 T_2}{T_1 p_2} V_1$$

$$= (5.08 \times 10^{-2}) V_1$$

18.2.17

(a)

$$pV = \frac{m_{\text{total}}}{M} RT$$

$$m_{\text{total}} = \frac{pVM}{RT}$$

$$= 6.91 \times 10^{-16} \, \text{kg}$$

(b)

$$\rho = \frac{m_{\text{total}}}{V} = 2.30 \times 10^{-13} \, \text{kg/m}^3$$

18.2.21

(a)

$$pV = \frac{N}{N_A} RT$$

$$N = \frac{pV N_A}{RT}$$

$$= 2.19 \times 10^6$$

(b)

$$2.44 \times 10^{19}$$

18.2.23

(a)

$$pV = \frac{N}{N_A}RT$$

$$\frac{V}{N} = \frac{RT}{N_A p}$$

$$s = \sqrt[3]{\frac{V}{N}}$$

$$= \sqrt[3]{\frac{RT}{N_A p}}$$

$$= 3.45 \times 10^{-9}\,\text{m}$$

18.2.25

(a)

$$K_{\text{tr}} = \frac{3}{2}nRT$$

$$= \frac{3}{2}pV$$

$$= 5.82 \times 10^7\,\text{J}$$

(b)

$$\frac{1}{2}mv^2 = K_{\text{tr}}$$

$$v = \sqrt{\frac{2K_{\text{tr}}}{m}}$$

$$= 241\,\text{m/s}$$

18.2.27

$$pV = nRT$$

$$p = \frac{nR}{V}T$$

$$\frac{nR}{V} = m$$

$$n = \frac{mV}{R}$$

$$= 1.07\,\text{mol}$$

$$N = nN_A$$

$$= 6.44 \times 10^{23}$$

18.2.29

(a)

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$
$$= 1.93 \times 10^6 \text{ m/s}$$
$$= 0.006c$$

Not a significant fraction of c.

(b)

$$0.10c = \sqrt{\frac{3kT}{m}}$$
$$(0.10c)^2 = \frac{3kT}{m}$$
$$T = \frac{(0.10c)^2 m}{3k}$$
$$= 7.26 \times 10^{10} \text{ K}$$

18.2.31

(a)
$$\frac{3}{2}kT = 6.21 \times 10^{-21} \text{ J}$$

(b)
$$(v^2)_{\text{av}} = \frac{2}{m}\left(\frac{3}{2}kT\right) = 2.34 \times 10^5 \text{ (m/s)}^2$$

(c)
$$v_{\text{rms}} = \sqrt{(v^2)_{\text{av}}} = 484 \text{ m/s}$$

(d)
$$p = mv = \frac{M}{N_A}v = 2.57 \times 10^{-23} \text{ kg m/s}$$

(e)

$$\Delta P = 2P$$
$$= 5.14 \times 10^{-23} \text{ kg m/s}$$
$$\Delta t = \frac{2l}{v}$$
$$= 4.13 \times 10^{-4} \text{ s}$$
$$F_{\text{av}} = \frac{\Delta P}{\Delta t}$$
$$= 1.24 \times 10^{-19} \text{ N}$$

(f)
$$p_{av} = \frac{F_{av}}{A} = 1.24 \times 10^{-17}\,\text{Pa}$$

(g)
$$p = N p_{av}$$
$$N = \frac{p}{p_{av}}$$
$$= 8.15 \times 10^{21}$$

(h)
$$pV = \frac{N}{N_A} RT$$
$$N = \frac{pVN_A}{RT}$$
$$= 2.44 \times 10^{22}$$

18.2.33

$$\sqrt{\frac{3RT}{M_N}} = \sqrt{\frac{3RT_H}{M_H}}$$
$$T = \frac{M_N}{M_H} T_H$$
$$= 4074\,\text{K}$$
$$= 3800\,°\text{C}$$

18.2.35

$$C_V = \frac{5}{2}R$$

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$T = \frac{Mv_{\text{rms}}^2}{3R}$$

$$Q = nC_V\Delta T$$

$$\Delta T = \frac{Q}{nC_V}$$

$$v'_{\text{rms}} = \sqrt{\frac{3R(T+\Delta T)}{M}}$$

$$= \sqrt{\frac{3R\left(\frac{Mv_{\text{rms}}^2}{3R} + \frac{Q}{nC_V}\right)}{M}}$$

$$= \sqrt{v_{\text{rms}}^2 + \frac{3RQ}{Mn\frac{5}{2}R}}$$

$$= \sqrt{v_{\text{rms}}^2 + \frac{6Q}{5Mn}}$$

$$= 1.02 \times 10^3 \,\text{m/s}$$

18.2.39

(a)

$$c_{V,\text{N}} = \frac{5}{2}R$$

$$= 742 \,\text{J/(kg K)}$$

$$c_{V,\text{water}} = 4190 \,\text{J/(kg K)}$$

$$= 5.6 C_{V,\text{N}}$$

(b)

$$Q = mc_{V,\text{water}}\Delta T$$
$$= 4.19 \times 10^4 \,\text{J}$$
$$m = \frac{Q}{c_{V,\text{N}}\Delta T}$$
$$= 5.65 \,\text{kg}$$
$$pV = \frac{m_{\text{total}}}{M}RT$$
$$V = \frac{m_{\text{total}}RT}{Mp}$$
$$= 4.87 \,\text{m}^3$$
$$= 4.87 \times 10^3 \,\text{L}$$

18.2.41

(a)
$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}} = 337 \,\text{m/s}$$

(b)
$$v_{\text{av}} = 380 \,\text{m/s}$$

(c)
$$v_{\text{rms}} = 412 \,\text{m/s}$$

18.2.43

(a)
$$\frac{v_{\text{rms}}}{v} = \sqrt{\frac{3RT}{M}}\sqrt{\frac{M}{\gamma RT}} = \sqrt{\frac{3}{\gamma}} = 1.34$$

(b)
$$\frac{v_{\text{av}}}{v} = \sqrt{\frac{8RT}{\pi M}}\sqrt{\frac{M}{\gamma RT}} = \sqrt{\frac{8}{\pi\gamma}} = 1.23$$

18.2.45

(a) The minimum pressure is $p_1 = 611.657 \,\text{Pa}$. If $p < p_1$ the ice sublimates directly to gas.

(b) The maximum pressure is $p_2 = 2.212 \times 10^7 \,\text{Pa}$. The ice melts to water, then the properties of the water gradually change to those of steam with no phase transition.

18.2.49

(a)

$$p' - p = -\rho g h$$
$$= -1.18 \times 10^4 \, \text{Pa}$$

(b)

$$p_1 V_1 = p_2 V_2$$
$$V_2 = \frac{p_1}{p_2} V_1$$
$$= 0.56 \, \text{L}$$

18.2.51

$$0 = \rho_{\text{cold}} V g - \rho_{\text{hot}} V g - mg$$
$$= \rho_{\text{cold}} V - \rho_{\text{hot}} V - m$$
$$\rho_{\text{hot}} = \rho_{\text{cold}} - \frac{m}{V}$$
$$\frac{Mp}{RT} = \rho_{\text{cold}} - \frac{m}{V}$$
$$T = \frac{Mp}{R(\rho_{\text{cold}} - m/V)}$$
$$= 542 \, \text{K}$$
$$= 269 \, ^\circ\text{C}$$

18.2.53

$$pV = \frac{m_{\text{total}}}{M} RT$$
$$m_{\text{total}} = \frac{pVM}{RT}$$
$$= 0.285 \, \text{kg}$$
$$m'_{\text{total}} = 0.0896 \, \text{kg}$$
$$\Delta m = 0.195 \, \text{kg}$$

18.2.57

(a)

$$0 = \rho V g - (m_{\text{adventurer}} + m_{\text{bell}} + m_{\text{water}})g$$
$$= \rho V - m_{\text{adventurer}} - m_{\text{bell}} - m_{\text{water}}$$
$$m_{\text{water}} = \rho V - m_{\text{adventurer}} - m_{\text{bell}}$$
$$= 98\,\text{kg}$$
$$V_{\text{water}} = \frac{m_{\text{water}}}{\rho_{\text{water}}}$$
$$= 0.0956\,\text{m}^3$$

(b)

$$pV = nRT$$
$$p = \frac{nRT}{V}$$
$$p = \rho g y$$
$$\rho g y = \frac{nRT}{V}$$
$$n = \frac{\rho g V}{RT} y$$
$$\frac{dn}{dt} = \frac{\rho g V}{RT} \frac{dy}{dt}$$
$$= 18.2\,\text{mol/s}$$

(c)

$$pV = nRT$$
$$n = \frac{pV}{RT}$$
$$= 756\,\text{mol}$$
$$\frac{n}{dn/dt} = 41.5\,\text{m}$$

18.2.59

(a)

$$pV = nRT$$

$$n_{\text{balloon}} = \frac{pV}{RT}$$

$$= (9.11 \times 10^6)\frac{1}{T}$$

$$n_{\text{cylinder}} = \frac{pV}{RT}$$

$$= (2.97 \times 10^5)\frac{1}{T}$$

$$\frac{n_{\text{balloon}}}{n_{\text{cylinder}}} = 30.7$$

(b)

$$0 = \rho V g - Mng - mg$$

$$mg = (\rho V - Mn)g$$

$$= 8420\,\text{N}$$

(c)

$$mg = 7810\,\text{N}$$

18.2.67

(c)

$$U(r) = U_0 \left[\left(\frac{R_0}{r} \right)^{12} - 2 \left(\frac{R_0}{r} \right)^6 \right]$$

$$F(r) = 12 \frac{U_0}{R_0} \left[\left(\frac{R_0}{r} \right)^{13} - \left(\frac{R_0}{r} \right)^7 \right]$$

$$0 = U_0 \left[\left(\frac{R_0}{r_1} \right)^{12} - 2 \left(\frac{R_0}{r_1} \right)^6 \right]$$

$$= \left(\frac{R_0}{r_1} \right)^{12} - 2 \left(\frac{R_0}{r_1} \right)^6$$

$$= \left(\frac{R_0}{r_1} \right)^6 - 2$$

$$2 = \left(\frac{R_0}{r_1} \right)^6$$

$$2r_1^6 = R_0^6$$

$$r_1 = \frac{1}{\sqrt[6]{2}} R_0$$

$$\approx 0.89 R_0$$

$$0 = 12 \frac{U_0}{R_0} \left[\left(\frac{R_0}{r_2} \right)^{13} - \left(\frac{R_0}{r_2} \right)^7 \right]$$

$$0 = \left(\frac{R_0}{r_2} \right)^{13} - \left(\frac{R_0}{r_2} \right)^7$$

$$= \left(\frac{R_0}{r_2} \right)^6 - 1$$

$$r_2 = R_0$$

$$\frac{r_1}{r_2} = \frac{1}{\sqrt[6]{2}}$$

(d)

$$W = \int_{r_2}^{\infty} -F \, dr$$

$$= \int_{R_0}^{\infty} -12\frac{U_0}{R_0} \left[\left(\frac{R_0}{r}\right)^{13} - \left(\frac{R_0}{r}\right)^{7} \right] dr$$

$$= -12\frac{U_0}{R_0} \left(-\frac{R_0}{12} \right)$$

$$= U_0$$

18.2.69

(a)
$$C_V = 2R = 16.63 \, \text{J/(mol K)}$$

(b) Less than because vibrational energy will play a smaller role.

18.2.71

(a)
$$\frac{1}{2}mv^2 \geq \frac{GmM}{R_p}$$

$$\geq gmR_p$$

(b)
$$\frac{3}{2}kT \geq mgR_p$$

$$T_{\text{N}} \geq \frac{2mgR_p}{3k}$$

$$\geq 1.40 \times 10^5 \, \text{K}$$

$$T_{\text{H}} \geq 1.02 \times 10^4 \, \text{K}$$

(c)
$$T_{\text{N}} \geq 6.37 \times 10^3 \, \text{K}$$

$$T_{\text{H}} \geq 459 \, \text{K}$$

(d) Because it's very easy to atmospheric particles to escape.

18.2.73

$$\int_0^\infty v^2 f(v)\, dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^4 e^{-mv^2/2kT}\, dv$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{3}{2^3 (m/2kT)^2} \sqrt{\frac{\pi}{(m/2kT)}}$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{3}{8} \left(\frac{2kT}{m}\right)^2 \sqrt{\frac{2\pi kT}{m}}$$

$$= \frac{3kT}{m}$$

18.2.75

(b)

$$v_{\mathrm{mp}} = \sqrt{\frac{2kT}{m}}$$
$$= 395\,\mathrm{m/s}$$
$$f(v_{\mathrm{mp}}) = 2.10 \times 10^{-3}$$
$$\Delta N \approx N f(v_{\mathrm{mp}}) \Delta v$$
$$\approx (4.20 \times 10^{-2}) N$$

(c)

$$7 v_{\mathrm{mp}} = 2765\,\mathrm{m/s}$$
$$f(7 v_{\mathrm{mp}}) = 1.43 \times 10^{-22}$$
$$\Delta N \approx (2.85 \times 10^{-21}) N$$

18.2.77

(a)

$$0 = pA - p_0 A - mg$$
$$p = p_0 + \frac{mg}{A}$$
$$= p_0 + \frac{mg}{\pi r^2}$$

(b)

$$p_1 V_1 = p_2 V_2$$

$$p_2 = \frac{V_1}{V_2} p_1$$

$$= \frac{Ah}{A(h+y)} p_1$$

$$= \frac{h}{h+y} p_1$$

$$\approx \left(1 - \frac{y}{h}\right) p_1$$

$$F = \left(1 - \frac{y}{h}\right) p_1 \pi r^2 - p_0 \pi r^2 - mg$$

$$= \left(1 - \frac{y}{h}\right) \left(p_0 + \frac{mg}{\pi r^2}\right) \pi r^2 - p_0 \pi r^2 - mg$$

$$= \left(1 - \frac{y}{h}\right) \left(p_0 \pi r^2 + mg\right) - p_0 \pi r^2 - mg$$

$$= -\frac{y}{h} \left(p_0 \pi r^2 + mg\right)$$

(c)

$$F = -kx$$

$$k = \frac{1}{h} \left(p_0 \pi r^2 + mg\right)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{1}{h} \left(\frac{p_0 \pi r^2}{m} + g\right)}$$

$$f = \frac{\omega}{2\pi}$$

$$= \frac{1}{2\pi} \sqrt{\frac{g}{h} \left(1 + \frac{p_0 \pi r^2}{gm}\right)}$$

If the displacement is not small the oscillation is not simple harmonic because we can't use the approximation $\frac{h}{h+y} \approx 1 - \frac{y}{h}$.

18.2.81

(a)

$$I = 2mr^2 = 4.1 \times 10^{-46} \, \text{kg m}^2$$

(b)

$$2\left(\frac{1}{2}(2m)v_i^2\right) = 2\left(\frac{1}{2}(2m)v_f^2 + \frac{1}{2}I\omega^2\right)$$

$$2mv_i^2 = 2mv_f^2 + 2mr^2\omega^2$$

$$v_i^2 = v_f^2 + r^2\omega^2$$

$$-2r(2m)v_i = -2I\omega$$

$$2mrv_i = 2mr^2\omega$$

$$v_i = r\omega$$

(c)

$$\omega = \frac{v_i}{r}$$

$$v_i^2 = v_f^2 + r^2\left(\frac{v_i}{r}\right)^2$$

$$v_f = 0$$

(d)

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

$$= 514\,\text{m/s}$$

$$\omega = 5.47 \times 10^{12}\,\text{rad/s}$$

18.2.83

(a)

$$\lambda = \frac{V}{4\pi\sqrt{2}r^2N}$$

$$= 4.50 \times 10^{11}\,\text{m}$$

(b)

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

$$= 704\,\text{m/s}$$

$$t_{\text{mean}} = \frac{\lambda}{v_{\text{rms}}}$$

$$= 6.39 \times 10^8\,\text{s}$$

$$= 20\,\text{years}$$

(c)

$$pV = NkT$$
$$p = \frac{NkT}{V}$$
$$= 1.38 \times 10^{-14}\,\text{Pa}$$

(d)

$$m_\text{total} = \rho V$$
$$= \rho \frac{4}{3}\pi \left(\frac{d}{2}\right)^3$$
$$= 2.96 \times 10^{32}\,\text{kg}$$

$$\frac{1}{2}mv^2 = \frac{Gmm_\text{total}}{r}$$
$$v = \sqrt{\frac{4Gm_\text{total}}{d}}$$
$$= 640\,\text{m/s}$$

It would evaporate.

(f)

$$T_\text{ISM} = \frac{(N/V)_\text{nebula}}{(N/V)_\text{ISM}} T_\text{nebula}$$
$$= 2.0 \times 10^5\,\text{K}$$

34 times hotter than the sun.

18.2.85

a

18.2.87

c

19 The First Law of Thermodynamics

19.1 Guided Practice

19.1.1

(a)

$$\Delta U = Q - W$$
$$Q = \Delta U + W$$
$$= 5.75 \times 10^3 \, \text{J}$$

(b)

$$\Delta U = Q - W$$
$$= -3.2 \times 10^4 \, \text{J}$$

(c)

$$\Delta U = Q - W$$
$$W = Q - \Delta U$$
$$= -1.85 \times 10^3 \, \text{J}$$

19.1.2

(a)
$$W = p(V_2 - V_1) = 155 \, \text{J}$$

(b)
$$W = 0$$

(c)
$$W = p(V_2 - V_1) = -375 \, \text{J}$$

(d)
$$W = \frac{1}{2}(p_1 + p_2)(V_2 - V_1) = 875 \, \text{J}$$

19.1.3

(a)

$$W = p(V_2 - V_1)$$
$$= -240 \, \text{J}$$
$$\Delta U = Q - W$$
$$= 1.80 \times 10^3 \, \text{J}$$

(b)

$$W = p(V_2 - V_1)$$
$$= -720 \, \text{J}$$
$$\Delta U = Q - W$$
$$Q = \Delta U + W$$
$$= 1.08 \times 10^3 \, \text{J}$$

19.1.4

(a)
$$Q = mL = 3.43 \times 10^6 \, \text{J}$$

(b)
$$W = p(V_2 - V_1) = 3.43 \times 10^5 \, \text{J}$$

(c)
$$\Delta U = Q - W = 3.09 \times 10^6 \, \text{J}$$

19.1.5

(a)
$$\Delta U = \Delta Q = nC_V \Delta T = 998 \, \text{J}$$

(b)
$$\Delta U = \Delta Q = nC_V \Delta T = 748 \, \text{J}$$

(c)
$$\Delta U = \Delta Q = nC_V \Delta T = 599 \, \text{J}$$

19.1.6

(a)
$$V = \frac{nRT}{p} = 5.24 \times 10^{-2} \, \text{m}^3$$

(b)   (i)

$$T = 327 \, °\text{C}$$
$$\Delta U = Q$$
$$= nC_V \Delta T$$
$$= 1.31 \times 10^4 \, \text{J}$$

(ii)

$$T = 327\,^\circ\text{C}$$
$$\Delta U = Q$$
$$= nC_V\Delta T$$
$$= 1.31 \times 10^4\,\text{J}$$

(iii)

$$T = 927\,^\circ\text{C}$$
$$\Delta U = 3.92 \times 10^4\,\text{J}$$

19.1.7

(a)

$$pV = nRT$$
$$\frac{pV}{R} = nT$$

$$(2p) = nR(2T)$$
$$\Delta T = T$$

$$\Delta U = Q - W$$
$$= nC_V\Delta T$$
$$= C_V(nT)$$
$$= \frac{3}{2}R\frac{pV}{R}$$
$$= \frac{3}{2}pV$$
$$= 4.50 \times 10^4\,\text{J}$$

(b)

$$pV = nRT$$
$$\frac{pV}{R} = nT$$

$$pV = nRT$$
$$p\left(\frac{1}{2}V\right) = nR\left(\frac{1}{2}T\right)$$
$$\Delta T = -\frac{1}{2}T$$

$$\Delta U = nC_V\Delta T$$
$$= C_V\left(-\frac{1}{2}nT\right)$$
$$= -\frac{3}{4}R\frac{pV}{R}$$
$$= -\frac{3}{4}pV$$
$$= -2.25 \times 10^4 \text{ J}$$

(c)

$$\Delta U = 1.17 \times 10^5 \text{ J}$$

19.1.8

(a)

$$Q = nC_V\Delta T$$
$$= \frac{5}{2}nRT$$

$$W = 0$$

$$\Delta U = Q - W$$
$$= \frac{5}{2}nRT$$

(b)

$$Q = nC_P \Delta T$$
$$= \frac{7}{2}nRT$$

$$W = p(V_2 - V_1)$$

$$\Delta U = \frac{7}{2}nRT - p(V_2 - V_1)$$
$$= \frac{7}{2}nRT - 2nRT + nRT$$
$$= \frac{5}{2}nRT$$

(c)

$$Q = 0$$

$$W = nC_V(T_1 - T_2)$$
$$= -\frac{5}{2}nRT$$

$$\Delta U = Q - W$$
$$= \frac{5}{2}nRT$$

19.1.9

(a)

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

(b)

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$
$$p_2 = \left(\frac{V_1}{V_2}\right)^\gamma p_1$$
$$= 6.41 \times 10^4 \, \text{Pa}$$

(c)

$$W = \frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2)$$
$$= 623 \, \text{J}$$

19.1.10

(a)

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5}$$

(b)

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$
$$V_2^{\gamma-1} = \frac{T_1}{T_2} V_1^{\gamma-1}$$
$$V_2 = \left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} V_1$$
$$= 5.79 \times 10^{-4}\,\text{m}^3$$

(c)

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$
$$p_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} p_1$$
$$= 2.95 \times 10^6\,\text{Pa}$$

(d)

$$W = \frac{1}{\gamma-1}(p_1 V_1 - p_2 V_2)$$
$$= -2.65 \times 10^3\,\text{J}$$

19.1.11

(a)

$$pV = nRT$$
$$p = \frac{nRT}{V}$$
$$= 3.17 \times 10^5\,\text{Pa}$$

(b)

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$
$$p_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} p_1$$
$$= 8.21 \times 10^4\,\text{Pa}$$

(c)

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1$$

$$= 178\,\text{K}$$

(d)

$$W = \frac{1}{\gamma-1}(p_1 V_1 - p_2 V_2)$$

$$= 7.94 \times 10^3\,\text{J}$$

19.1.12

(a)

$$\left[p + \left(\frac{an^2}{V^2}\right)\right](V - nb) = nRT$$

$$p + \left(\frac{an^2}{V^2}\right) = \frac{nRT}{V - nb}$$

$$p = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

$$W = \int_{V_1}^{V_2} p\,dV$$

$$= \int_{V_1}^{V_2}\left(\frac{nRT}{V - nb} - \frac{an^2}{V^2}\right)dV$$

$$= \left[nRT\ln(V - nb) + \frac{an^2}{V}\right]_{V_1}^{V_2}$$

$$= nRT\ln(V_2 - nb) + \frac{an^2}{V_2} - nRT\ln(V_1 - nb) - \frac{an^2}{V_1}$$

$$= nRT\ln\frac{V_2 - nb}{V_1 - nb} + an^2\frac{V_1 - V_2}{V_1 V_2}$$

(b) (i)

$$W = 2.80 \times 10^3\,\text{J}$$

(ii)

$$W = \int_{V_1}^{V_2} p \, dV$$

$$= \int_{V_1}^{V_2} \frac{nRT}{V} \, dV$$

$$= nRT[\ln V]_{V_1}^{V_2}$$

$$= 3.11 \times 10^3 \, \text{J}$$

19.2   Exercises and Problems

19.2.1

(b)

$$W = p(V_2 - V_1)$$

$$= nR(T_2 - T_1)$$

$$= 1.33 \times 10^3 \, \text{J}$$

19.2.3

(b)

$$p_1 V_1 = nRT$$

$$p_2 V_2 = nRT$$

$$3p_1 V_2 = nRT$$

$$V_2 = \frac{1}{3} V_1$$

$$W = \int_{V_1}^{V_2} p \, dV$$

$$= \int_{V_1}^{V_1/3} \frac{nRT}{V} \, dV$$

$$= nRT \ln \frac{1}{3}$$

$$= -6.18 \times 10^3 \, \text{J}$$

19.2.5

(a)

$$pV = nRT$$
$$V = \frac{nRT}{p}$$

$$W = \int_{V_1}^{V_2} p\,dV$$
$$= \int_{nRT/p_1}^{nRT/p_2} \frac{nRT}{V}\,dV$$
$$= nRT \ln \frac{nRT/p_2}{nRT/p_1}$$
$$= nRT \ln \frac{p_1}{p_2}$$
$$\frac{W}{nRT} = \ln \frac{p_1}{p_2}$$
$$p_1 = p_2 e^{W/nRT}$$
$$= 1.05 \times 10^5 \,\text{Pa}$$
$$= 1.04 \,\text{atm}$$

19.2.9

(a)
$$W = p(V_2 - V_1) = 3.47 \times 10^4 \,\text{J}$$

(b)
$$\Delta U = Q - W = 8.03 \times 10^4 \,\text{J}$$

(c) No, because it's an isobaric process.

19.2.11

(a)

$$T_a = \frac{pV}{nR}$$
$$= 278 \,\text{K}$$
$$T_b = 694 \,\text{K}$$
$$T_c = 1250 \,\text{K}$$

The lowest temperature is $278 \,\text{K}$ and it occured at point a.

(b)

$$W_{\mathrm{ab}} = 0$$
$$W_{\mathrm{bc}} = 162\,\mathrm{J}$$

(c)

$$\Delta U = Q - W = 52\,\mathrm{J}$$

19.2.13

(a)

$$T_a = \frac{pV}{nR}$$
$$= 5.35 \times 10^2\,\mathrm{K}$$
$$T_b = 9.36 \times 10^3\,\mathrm{K}$$
$$T_c = 1.50 \times 10^4\,\mathrm{K}$$

(b)

$$W = 2.10 \times 10^4\,\mathrm{J}$$

(c)

$$Q = \Delta U + W = 3.60 \times 10^4\,\mathrm{J}$$

19.2.17

(b)

$$V_1 = \frac{nRT_1}{p_1}$$
$$= 6.18 \times 10^{-3}\,\mathrm{m^3}$$
$$V_2 = 8.23 \times 10^{-3}\,\mathrm{m^3}$$
$$W = p(V_2 - V_1)$$
$$= 207\,\mathrm{J}$$

(c) The piston

(d)

$$C_V = \frac{5}{2}R$$
$$C_P = \frac{7}{2}R$$
$$\Delta U = nC_P\Delta T$$
$$= 727\,\mathrm{J}$$
$$Q = \Delta U + W$$
$$= 934\,\mathrm{J}$$

19.2.19

(a)

$$\Delta U = Q - W$$
$$= Q - 0$$
$$= nC_V \Delta T$$
$$\Delta T = \frac{\Delta U}{nC_V}$$
$$= 168\,\text{K}$$
$$T_2 = T_1 + \Delta T$$
$$= 948\,\text{K}$$

(b)

$$Q = nC_P \Delta T$$
$$\Delta T = \frac{Q}{nC_P}$$
$$= 120\,\text{K}$$
$$T_2 = T_1 + \Delta T$$
$$= 900\,\text{K}$$

19.2.21

$$C_V = \frac{3}{2}R$$
$$C_P = \frac{5}{2}R$$
$$Q = nC_P \Delta T$$
$$= \frac{5}{2}nR(T_2 - T_1)$$
$$W = p(V_2 - V_1)$$
$$= nR(T_2 - T_1)$$
$$\frac{W}{Q} = \frac{2}{5}$$

19.2.23

(a)

$$\Delta U = Q - W$$
$$= 747\,\text{J}$$

(b)

$$Q = nC_P\Delta T$$
$$C_P = \frac{Q}{n\Delta T}$$
$$= 37.0\,\text{J}/(\text{mol k})$$
$$C_V = C_P - R$$
$$= 28.6\,\text{J}/(\text{mol K})$$
$$\gamma = \frac{C_P}{C_V}$$
$$= 1.29$$

19.2.25

(a)

$$V_1 = \frac{nRT}{p_1}$$
$$= 3.46 \times 10^{-3}\,\text{m}^3$$
$$V_2 = 8.64 \times 10^{-4}\,\text{m}^3$$
$$W = \int_{V_1}^{V_2} p\,dV$$
$$= \int_{V_1}^{V_2} \frac{nRT}{V}\,dV$$
$$= nRT\ln\frac{V_2}{V_1}$$
$$= -606\,\text{J}$$

(b)
$$\Delta U = 0\,\text{J}$$

(c) Yes, liberate
$$Q = \Delta U + W = -606\,\text{J}$$

19.2.27

(a)

$$C_V = \frac{3}{2}R$$

$$C_P = \frac{5}{2}R$$

$$\gamma = \frac{C_P}{C_V}$$

$$= \frac{5}{3}$$

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$p_2 = \left(\frac{V_1}{V_2}\right)^\gamma p_1$$

$$= 4.76 \times 10^5 \, \text{Pa}$$

(b)

$$W = \frac{C_V}{R}(p_1 V_1 - p_2 V_2)$$

$$= -1.06 \times 10^4 \, \text{J}$$

(c)

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 1.59$$

Heated

19.2.29

(b)

$$W = nC_V(T_1 - T_2)$$

$$= 314 \, \text{J}$$

(c)

$$\Delta U = Q - W$$

$$= 0 - W$$

$$= -314 \, \text{J}$$

50

19.2.31

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_1 \left(\frac{nRT_1}{p_1} \right)^{\gamma-1} = T_2 \left(\frac{nRT_2}{p_2} \right)^{\gamma-1}$$

$$T_2^\gamma = T_1^\gamma \left(\frac{p_2}{p_1} \right)^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma}$$

$$= 285\,\text{K}$$

$$= 11.6\,^\circ\text{C}$$

19.2.33

$$C_V = \frac{3}{2}R$$

$$C_P = \frac{5}{2}R$$

$$\gamma = \frac{5}{3}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_1 \left(\frac{nRT_1}{p_1} \right)^{\gamma-1} = 2T_1 \left(\frac{2nRT_1}{p_2} \right)^{\gamma-1}$$

$$\frac{1}{p_1^{\gamma-1}} = \frac{2^\gamma}{p_2^{\gamma-1}}$$

$$p_1^{\gamma-1} = \frac{p_2^{\gamma-1}}{2^\gamma}$$

$$p_2 = 2^{\gamma/(\gamma-1)} p_1$$

$$= 2^{5/2} p_1$$

$$= 4\sqrt{2} p_1$$

19.2.35

(a) Increase

(b)

$$W = \frac{1}{2}(p_a + p_b)(V_B - V_A) = 4.8\,\text{kJ}$$

19.2.37

(a)

$$pV = nRT$$
$$n = \frac{pV}{RT}$$
$$= 0.678 \, \text{mol}$$

(b)

$$pV = nRT$$
$$V = \frac{nRT}{p}$$
$$= 3.33 \times 10^{-2} \, \text{m}^3$$

(c)

$$W = \int_{V_1}^{V_2} p \, dV$$
$$= \int_{V_1}^{V_2} \frac{nRT}{V} \, dV$$
$$= nRT \ln \frac{V_2}{V_1}$$
$$= 2.22 \, \text{kJ}$$

(d)
$$\Delta U = 0$$

19.2.39

(a)

$$\Delta U = Q - W$$
$$= 30.0 \, \text{J}$$
$$Q = \Delta U + W$$
$$= 45.0 \, \text{J}$$

(b) Liberate

$$Q = \Delta U + W = -65.0 \, \text{J}$$

(c)

$$\Delta U_{\mathrm{ad}} = 8.0\,\mathrm{J}$$
$$W_{\mathrm{ad}} = 15.0\,\mathrm{J}$$
$$Q_{\mathrm{ad}} = \Delta U_{\mathrm{ad}} + W_{\mathrm{ad}}$$
$$= 23.0\,\mathrm{J}$$
$$Q_{\mathrm{db}} = \Delta U_{\mathrm{ab}} - \Delta U_{\mathrm{ad}}$$
$$= 22.0\,\mathrm{J}$$

19.2.43

(a)

$$p_1 V_1 = p_2 V_2$$
$$V_2 = \frac{p_1}{p_2} V_1$$
$$= 8.0 \times 10^{-4}\,\mathrm{m}^3$$
$$= 0.80\,\mathrm{L}$$

(b)

$$T_a = \frac{pV}{nR}$$
$$= 304\,\mathrm{K}$$
$$T_b = 1.21 \times 10^3\,\mathrm{K}$$
$$T_c = 1.21 \times 10^3\,\mathrm{K}$$

(c)

$$\Delta U_{ab} = Q_{ab} - W_{ab}$$
$$= Q_{ab}$$
$$= nC_V \Delta T$$
$$= 74.0\,\text{J into the gas}$$

$$V_c = \frac{nRT_c}{p_c}$$
$$= 7.97 \times 10^{-4}\,\text{m}^3$$
$$\Delta U_{ca} = Q_{ca} - W_{ca}$$
$$nC_V \Delta T = Q_{ca} - p(V_a - V_c)$$
$$Q_{ca} = nC_V \Delta T + p(V_a - V_c)$$
$$= -104\,\text{J out of the gas}$$

$$\Delta U_{bc} = Q_{bc} - W_{bc}$$
$$Q_{bc} = \Delta U_{bc} + W_{bc}$$
$$= nC_V \Delta T + \int_{V_b}^{V_c} p\,dV$$
$$= nRT \ln \frac{V_c}{V_b}$$
$$= 55.6\,\text{J into the gas}$$

(d)

$$\Delta U_{ab} = 74.0\,\text{J increase}$$

$$\Delta U_{bc} = 0.0\,\text{J no change}$$

$$\Delta U_{ca} = nC_V \Delta T$$
$$= -74.0\,\text{J decrease}$$

19.2.47

(b)

$$V_2 = \frac{p_1}{p_2}V_1 = 6.0\,\text{L}$$

(c)

$$n = \frac{pV}{RT}$$
$$= 6.01 \times 10^{-2} \,\mathrm{mol}$$
$$W_{12} = \int_{V_1}^{V_2} p\,dV$$
$$= nRT_1 \ln \frac{V_2}{V_1}$$
$$= 208\,\mathrm{J}$$

$$W_{23} = p_2(V_3 - V_2)$$
$$= -113\,\mathrm{J}$$

$$W = 95.0\,\mathrm{J}$$

(d) Heat it at constant volume

19.2.49

(a) As the wind descends it experiences greater atmospheric pressure. This compresses the wind, increasing its temperature. It's important that it be moving fast so it's an adiabatic process.

(b)

$$T_2 = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} T_1$$
$$= 287\,\mathrm{K}$$
$$= 13.9\,^\circ\mathrm{C}$$
$$\Delta T = T_2 - T_1$$
$$= 11.9\,\mathrm{C}^\circ$$

19.2.51

(a)

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$
$$p_1(Ah)^\gamma = p_2[A(h - \Delta h)]^\gamma$$
$$\frac{p_1}{p_2} h^\gamma = (h - \Delta h)^\gamma$$
$$\left(\frac{p_1}{p_2}\right)^{1/\gamma} h = h - \Delta h$$
$$\Delta h = h\left[1 - \left(\frac{p_1}{p_2}\right)^{1/\gamma}\right]$$
$$= 16.8\,\text{cm}$$

(b)

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$
$$T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1$$
$$= \left(\frac{Ah}{A(h - \Delta h)}\right)^{\gamma-1} T_1$$
$$= \left(\frac{1}{1 - \Delta h/h}\right)^{\gamma-1} T_1$$
$$= 469\,\text{K}$$
$$= 196\,^\circ\text{C}$$

(c)
$$W = nC_V(T_1 - T_2) = -7.03 \times 10^4\,\text{J}$$

19.2.59

(a) a is abiabatic, b is isochoric, c is isobaric

(b)

$$\Delta U = Q_b - W_b$$
$$= Q_b - 0$$
$$= Q_b$$

$$\Delta U = Q_c - W_c$$
$$= Q_c - p(V_2 - V_1)$$
$$= Q_c - nR(T_2 - T_1)$$

$$Q_b = Q_c - nR(T_2 - T_1)$$
$$T_2 = T_1 + \frac{Q_c - Q_b}{nR}$$
$$= 28.0\,^\circ\mathrm{C}$$

(c)

$$Q_b = nC_V\Delta T$$
$$C_V = \frac{Q_b}{n\Delta T}$$
$$= 12.5\,\mathrm{J/(mol\,K)}$$

$$W_a = nC_V(T_1 - T_2)$$
$$= -30.0\,\mathrm{J}$$

$$W_b = 0$$

$$\Delta U_c = Q_c - W_c$$
$$W_c = Q_c - \Delta U_c$$
$$= Q_c - nC_V\Delta T$$
$$= 20.0\,\mathrm{J}$$

(d)

$$\gamma = \frac{C_P}{C_V}$$
$$= \frac{C_V + R}{C_V}$$
$$= 1.67$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$
$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_1}{T_2}$$
$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)}$$
$$= 0.961$$

$$\Delta V_b = 0$$

$$\frac{V_2}{V_1} = \frac{nRT_2/p}{nRT_1/p}$$
$$= \frac{T_2}{T_1}$$
$$= 1.03$$

a

(e) Decrease, stay the same, increase

19.2.61

(a)

$$r = 1.50\,\text{cm}$$

$$l_{\max} = 30.0\,\text{cm}$$

$$l_{\min} = l_{\max}/v$$

$$p = 101\,\text{kPa}$$

$$T = 30.0\,°\text{C}$$

$$V_1 = \pi r^2 l_{\max}$$

$$= 2.12 \times 10^{-4}\,\text{m}^3$$

$$V_2 = \pi r^2 l_{\min}$$

$$= \pi r^2 \frac{l_{\max}}{v}$$

$$= \frac{V_1}{v}$$

$$n = \frac{pV}{RT}$$

$$= 8.50 \times 10^{-3}\,\text{mol}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= T_1 v^{\gamma-1}$$

$$W_{\text{adiabatic}} = nC_V(T_1 - T_2)$$

$$= nC_V(T_1 - T_1 v^{\gamma-1})$$

$$= nC_V T_1(1 - v^{\gamma-1})$$

$$= 53.5(1 - v^{0.4})$$

$$W_{\text{isothermal}} = \int_{V_1}^{V_2} p\,dV$$

$$= \int_{V_1}^{V_2} \frac{nRT_2}{V}\,dV$$

$$= nRT_2 \ln \frac{V_2}{V_1}$$

$$= nRT_1 v^{\gamma-1} \ln v$$

$$= 21.4 v^{0.4} \ln v$$

$$W = 53.5(1 - v^{0.4}) + 21.4 v^{0.4} \ln v$$

$$= 53.5 + v^{0.40}(21.4 \ln v - 53.5)$$

(b)

$$T_2 \leq T_{\text{max}}$$
$$T_1 v^{\gamma - 1} \leq T_{\text{max}}$$
$$v \leq \left(\frac{T_{\text{max}}}{T_1} \right)^{1/(\gamma - 1)}$$
$$\leq 7.35$$

The largest integer value of v is 7.

(c) 7

(d) 7

(e)

$$T_2 = T_1 v^{\gamma - 1}$$
$$= 660 \, \text{K}$$
$$= 387 \, ^\circ\text{C}$$
$$Q = nC_V \Delta T$$
$$= -63.0 \, \text{J}$$

19.2.63

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$
$$p_2 = \frac{T_2}{T_1} p_1$$
$$= 1.27 \times 10^7 \, \text{Pa}$$
$$= 1.84 \times 10^3 \, \text{psi}$$

c

19.2.65

$$p_1 V_1 = p_2 V_2$$
$$V_1 = \frac{p_2}{p_1} V_2$$
$$= 6.01 \times 10^{-5} \, \text{m}^3$$
$$= 6.01 \times 10^{-2} \, \text{L}$$

d

20 The Second Law of Thermodynamics

20.1 Guided Practice

20.1.1

(a)
$$W = eQ_H \Rightarrow Q_H = \frac{W}{e} = 6.89 \times 10^4 \text{ J}$$

(b)
$$|W| = |Q_H| - |Q_C| \Rightarrow |Q_C| = |Q_H| - |W| = 5.65 \times 10^4 \text{ J}$$

20.1.2
$$e = \frac{W}{Q_H} = \frac{Q_H + Q_C}{Q_H} = 1 + \frac{Q_C}{Q_H} = 0.173 = 17.3\%$$

20.1.3

(a)
$$(1-e)Q_H = Q_C \Rightarrow Q_H = \frac{Q_C}{1-e} = 6.17 \times 10^8 \text{ J}$$

(b)
$$W = eQ_H = 1.21 \times 10^8 \text{ J}$$

20.1.4

(a)
$$W = 3600P = 3.96 \times 10^8 \text{ J}$$

(b)
$$Q_H = mL_c = 1.70 \times 10^9 \text{ J}$$

(c)
$$e = \frac{W}{Q_H} = 0.233 = 23.3\%$$

20.1.5

(a)
$$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$$
$$= 1 + \frac{Q_C}{Q_H}$$
$$= 1 + \frac{W - Q_H}{Q_H}$$
$$= 0.21$$

(b)
$$|Q_C| = |Q_H| - |W| = 6.32 \times 10^4 \, \text{J}$$

(c)
$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H} \Rightarrow T_H = -\frac{Q_H}{Q_C} T_C = 377 \, \text{K} = 104 \, °\text{C}$$

20.1.6

(a)
$$e = 1 - \frac{T_C}{T_H} = 0.6$$

(b)

$$n = 0.200\,\mathrm{mol}$$

$$\gamma = 1.40$$

$$T_H = 227\,^\circ\mathrm{C} = 500\,\mathrm{K}$$

$$T_C = -73\,^\circ\mathrm{C} = 200\,\mathrm{K}$$

$$p_a = 10.0 \times 10^5\,\mathrm{Pa}$$

$$V_a = \frac{nRT_H}{p_a}$$

$$= 8.31 \times 10^{-4}\,\mathrm{m^3}$$

$$V_b = 2V_a$$

$$= 1.66 \times 10^{-3}\,\mathrm{m^3}$$

$$p_b = \frac{nRT_H}{V_b}$$

$$= 5.01 \times 10^5\,\mathrm{Pa}$$

$$W_{ab} = \int_{V_a}^{V_b} p\,dV$$

$$= nRT_H \ln 2$$

$$= 576\,\mathrm{J}$$

$$V_c = \left(\frac{T_H}{T_C}\right)^{1/(\gamma-1)} V_b$$

$$= 1.64 \times 10^{-2}\,\mathrm{m^3}$$

$$p_c = \frac{nrT_C}{V_c}$$

$$= 2.03 \times 10^4\,\mathrm{Pa}$$

$$W_{bc} = \frac{1}{\gamma - 1}(p_b V_b - p_c V_c)$$

$$= 1.25\,\mathrm{kJ}$$

$$V_d = \frac{1}{2}V_c$$

$$= 8.20 \times 10^{-3}\,\mathrm{m^3}$$

$$p_d = 4.06 \times 10^4\,\mathrm{Pa}$$

$$W_{cd} = \int_{V_c}^{V_d} p\,dV$$

$$= nRT_C \ln \frac{1}{2}$$

$$= -231\,\mathrm{J}$$

$$W_{da} = \frac{1}{\gamma - 1}(p_d V_d - p_a V_a)$$

$$= -1.25\,\mathrm{kJ}$$

63

20.1.7

(a)
$$K = \frac{T_C}{T_H - T_C} = 7.52$$

(b)
$$W = \frac{Q_C}{K} = 5.32 \times 10^5 \, \text{J}$$

20.1.8

(a)
$$W = \int_{V_a}^{V_b} p \, dV$$
$$= \int_{V_a}^{2V_a} \frac{nRT_H}{V} \, dV$$
$$= nRT_H \ln 2$$

(b)
$$W = nC_V(T_H - T_C)$$
$$= \frac{3}{2} nR(T_H - T_C)$$

(c)
$$nRT_H \ln 2 = \frac{3}{2} nR(T_H - T_C)$$
$$\ln 2 = \frac{3}{2} \left(1 - \frac{T_C}{T_H} \right)$$
$$\frac{T_C}{T_H} = 1 - \frac{2}{3} \ln 2$$
$$e = 1 - \frac{T_C}{T_H}$$
$$= \frac{2}{3} \ln 2$$
$$= 0.462$$

20.1.9

(a)
$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = 655 \, \text{J/K}$$

(b)

$$\Delta S = mc \int_{159}^{351} \frac{dT}{T} = 1.92 \times 10^3 \, \text{J/K}$$

(c)

$$\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = 2.43 \times 10^3 \, \text{J/K}$$

20.1.10

(a)

$$n = 5.00 \, \text{mol}$$
$$V_1 = 0.120 \, \text{m}^3$$
$$T_1 = 20.0 \, ^\circ\text{C}$$
$$V_2 = 0.360 \, \text{m}^3$$
$$T_2 = 20.0 \, ^\circ\text{C}$$
$$\Delta U = nC_V \Delta T$$
$$= 0$$
$$Q = W$$
$$= \int_{V_1}^{V_2} p \, dV$$
$$= \int_{V_1}^{V_2} \frac{nRT}{V} \, dV$$
$$= nRT \ln \frac{V_2}{V_1}$$
$$= 1.34 \times 10^4 \, \text{J}$$
$$\Delta S = \frac{Q}{T}$$
$$= 45.7 \, \text{J/K}$$

(b) Change in entropy is path independent, so $\Delta S = 45.7 \, \text{J/K}$.

20.1.11

(a)

$$\Delta S = 0$$

(b)

$$\Delta S = \frac{Q}{T} = -150 \, \text{J/K}$$

(c)

$$\Delta S = \frac{Q}{T} = 218 \, \text{J/K}$$

(d)
$$\Delta S = 68 \, \text{J/K}$$

The net entropy increases.

20.1.12

(a)
$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = 1.22 \times 10^3 \, \text{J/K}$$

(b)
$$\Delta S = \int \frac{dQ}{T}$$
$$= mc \int_{368}^{273} \frac{dT}{T}$$
$$= -1.05 \times 10^3 \, \text{J/K}$$

(c)
$$\Delta S = 160 \, \text{J/K}$$

The net entropy increases.

20.1.13

(a)
$$0 = m_i L_f + m_w c(T - T_w) + m_i c(T - T_i)$$
$$T = \frac{(m_w T_w + m_i T_i)c - m_i L_f}{(m_w + m_i)c}$$
$$= 307 \, \text{K}$$
$$= 34.3 \, ^\circ\text{C}$$

(b)
$$\Delta S_i = \frac{Q}{T_1} + \int \frac{dQ}{T}$$
$$= \frac{m_i L_f}{T_1} + m_i c \ln \frac{T_2}{T_1}$$
$$= 101 \, \text{J/K}$$
$$\Delta S_w = m_w c \ln \frac{T_2}{T_1}$$
$$= -86.0 \, \text{J/K}$$
$$\Delta S = 15.0 \, \text{J/K}$$

20.2 Exercises and Problems

20.2.5

(a)

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$p_1 = \left(\frac{V_2}{V_1}\right)^\gamma p_2$$

$$= 12.3 \, \text{atm}$$

(b) It enters during process ca.

$$T_a = \frac{p_a V_a}{nR}$$

$$= 1.20 \times 10^3 \, \text{K}$$

$$T_c = \frac{p_c V_c}{nR}$$

$$= 146 \, \text{K}$$

$$\gamma = \frac{C_P}{C_V}$$

$$= \frac{C_V + R}{C_V}$$

$$C_V \gamma = C_V + R$$

$$C_V (\gamma - 1) = R$$

$$C_V = \frac{R}{\gamma - 1}$$

$$Q = nC_V \Delta T$$

$$= 5.48 \, \text{kJ}$$

(c) It leaves during bc.

$$T_b = \frac{p_b V_b}{nR}$$

$$= 656 \, \text{K}$$

$$\Delta U = Q - W$$

$$Q = \Delta U + W$$

$$= nC_V \Delta T + p_b (V_c - V_b)$$

$$= -3.71 \, \text{kJ}$$

(d)

$$W = W_{ab} + W_{bc} + W_{ca}$$
$$= nC_V(T_a - T_b) + p_b(V_c - V_b) + 0$$
$$= 1.77\,\text{kJ}$$

(e)

$$e = \frac{W}{Q_H} = 0.323 = 32.3\%$$

20.2.7

(a)

$$e = 1 - \frac{1}{r^{\gamma-1}}$$
$$r^{\gamma-1} = \frac{1}{1-e}$$
$$r = \left(\frac{1}{1-e}\right)^{1/(\gamma-1)}$$
$$= 12$$

(b)

$$Q_C = 7.4\,\text{kJ}$$
$$Q_H = 20\,\text{kJ}$$

20.2.9

(a)
$$e = 1 - \frac{1}{r^{\gamma-1}} = 0.581 = 58.1\%$$

(b)

$$e' = 0.595 = 59.5\%$$
$$\Delta e = 0.014$$
$$= 1.4\%$$

20.2.11

$$K = 2.25$$
$$W = 135\,\text{W}$$
$$H = KW$$
$$= 304\,\text{W}$$
$$Q = mc\Delta T$$
$$= 1.30 \times 10^6\,\text{J}$$
$$\frac{Q}{H} = 4.29 \times 10^3\,\text{s}$$
$$= 1.2\,\text{h}$$

20.2.13

$$T_H = T_C + 72.0\,\text{C}^\circ$$
$$e = 1 - \frac{T_C}{T_H}$$
$$= 1 - \frac{T_H - 72.0\,\text{C}^\circ}{T_H}$$
$$= 1 - 1 + \frac{72\,\text{C}^\circ}{T_H}$$
$$T_H = \frac{72\,\text{C}^\circ}{e}$$
$$= 576\,\text{K}$$
$$T_C = 504\,\text{K}$$

20.2.21

$$\frac{T_{CA}}{T_{HA} - T_{CA}} = 1.16\frac{T_{CB}}{T_{HB} - T_{CB}}$$
$$(T_{HB} - T_{CB}) = 1.3(T_{HA} - T_{CA})$$
$$T_{CB} = 180\,\text{K}$$
$$\frac{T_{CA}}{T_{HA} - T_{CA}} = 1.16\frac{T_{CB}}{1.3(T_{HA} - T_{CA})}$$
$$T_{CA} = \frac{1.16}{1.3}T_{CB}$$
$$= 161\,\text{K}$$

20.2.29

$$\Delta S = S' - S$$
$$= k \ln w' - k \ln w$$
$$= k \ln \frac{w'}{w}$$
$$= k \ln \frac{(425/0.0024)^N w}{w}$$
$$= k \ln(1.77 \times 10^5)^{nN_A}$$
$$= 10.0 \, \text{J/K}$$

20.2.31

(a)

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$$
$$Q_C = -\frac{T_C}{T_H} Q_H$$
$$= -121 \, \text{J}$$

(b)

$$n = \frac{U}{W}$$
$$= \frac{mgh}{Q_H + Q_C}$$
$$= 3.80 \times 10^3$$

20.2.33

(a)
$$W = eQ_H = 90.2 \, \text{J}$$

(b)
$$Q_C = Q_H - W = 320 \, \text{J}$$

(c)
$$T_C = T_H(1 - e) = 318 \, \text{K} = 45 \, ^\circ\text{C}$$

(d) 0

(e)
$$m = \frac{W}{gh} = 0.263 \, \text{kg}$$

20.2.37

(a)

$$T_A = \frac{p_A V_A}{nR}$$
$$= 241\,\text{K}$$
$$T_B = 241\,\text{K}$$

(b) Absorbed during bc, rejected during ab and ca.

(c)

$$T_C = 481\,\text{K}$$

(d)

$$W_{AB} = \int_{V_A}^{V_B} p\,dV$$
$$= nRT_A \ln \frac{V_B}{V_A}$$
$$= -1389\,\text{J}$$
$$Q_{AB} = -1389\,\text{J}$$
$$W_{BC} = p_B(V_C - V_B)$$
$$= 2000\,\text{J}$$
$$\Delta U_{BC} = Q_{BC} - W_{BC}$$
$$Q_{BC} = \Delta U_{BC} + W_{BC}$$
$$= nC_V \Delta T + W_{BC}$$
$$= nC_V(T_C - T_B) + W_{BC}$$
$$= \frac{5}{2}nR(T_C - T_B) + W_{BC}$$
$$= 6988\,\text{J}$$
$$W_{CA} = 0$$
$$Q_{CA} = nC_V \Delta T$$
$$= nC_V(T_A - T_C)$$
$$= \frac{5}{2}nR(T_A - T_C)$$
$$= -4988\,\text{J}$$
$$Q_{\text{net}} = 611\,\text{J}$$
$$W_{\text{net}} = 611\,\text{J}$$

(e)

$$e = \frac{W}{Q_H} = 0.0874 = 8.7\%$$

71

20.2.41

(a)

$$e = 1 - \frac{T_C}{T_H} = 7.0\%$$

(b)

$$P = eQ_H$$
$$Q_H = \frac{P}{e}$$
$$= 3.0 \, \text{MW}$$
$$Q_C = Q_H(1 - e)$$
$$= 2.8 \, \text{MW}$$

(c)

$$Q_C = mc\Delta T$$
$$m = \frac{Q_C}{c\Delta T}$$
$$= 167 \, \text{kg/s}$$
$$= 6.0 \times 10^5 \, \text{kg/h}$$
$$= 6.0 \times 10^5 \, \text{L/h}$$

20.2.45

$$e = 1 - \frac{T_C}{T_H}$$

$$e' = 1 - \frac{T'}{T_H}$$

$$W' = Q_H e'$$

$$= Q_H \left(1 - \frac{T'}{T_H}\right)$$

$$e'' = 1 - \frac{T_C}{T'}$$

$$W'' = Q'_H e''$$

$$= (Q_H - W')e''$$

$$= \left[Q_H - Q_H \left(1 - \frac{T'}{T_H}\right)\right]\left(1 - \frac{T_C}{T'}\right)$$

$$= Q_H \left(1 - 1 + \frac{T'}{T_H}\right)\left(1 - \frac{T_C}{T'}\right)$$

$$= Q_H \frac{T'}{T_H}\left(1 - \frac{T_C}{T'}\right)$$

$$e_{\text{Total}} = \frac{W' + W''}{Q_H}$$

$$= 1 - \frac{T'}{T_H} + \frac{T'}{T_H}\left(1 - \frac{T_C}{T'}\right)$$

$$= 1 - \frac{T_C}{T_H}$$

The efficiency is the same.

20.2.49

$$T_A = 300\,\text{K}$$

$$W_{AB} = nRT_A \ln \frac{V_B}{V_A}$$

$$= 2553\,\text{J}$$

$$Q_{AB} = 2553\,\text{J}$$

$$T_B = 300\,\text{K}$$

$$T_C = 1000\,\text{K}$$

$$W_{BC} = 0$$

$$Q_{BC} = nC_V(T_C - T_B)$$

$$= \frac{5}{2}nR(T_C - T_B)$$

$$= 1.24 \times 10^4\,\text{J}$$

$$W_{CA} = p_A(V_A - V_C)$$

$$= -4949\,\text{J}$$

$$\Delta U_{CA} = Q_{CA} - W_{CA}$$

$$Q_{CA} = \Delta U_{CA} + W_{CA}$$

$$= nC_V(T_A - T_C) + W_{CA}$$

$$= \frac{5}{2}nR(T_A - T_C) + W_{CA}$$

$$= -1.73 \times 10^4\,\text{J}$$

$$W = -2396\,\text{J}$$

$$K = \frac{|Q_C|}{|W|}$$

$$= 6.24$$

20.2.51

(a)

$$\Delta S = \int \frac{dQ}{T}$$

$$= m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \frac{T'}{T_2}$$

$$0 = m_1 c_1(T - T_1) + m_2 c_2(T' - T_2)$$

$$m_1 c_1(T - T_1) = m_2 c_2(T_2 - T')$$

(b)

$$T' = T_2 - \frac{m_1 c_1}{m_2 c_2}(T - T_1)$$

$$\Delta S = m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \left(\frac{T_2 - \frac{m_1 c_1}{m_2 c_2}(T - T_1)}{T_2} \right)$$

$$= m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \left(1 - \frac{m_1 c_1}{m_2 c_2} \frac{T - T_1}{T_2} \right)$$

$$\frac{d}{dT} \Delta S = \frac{m_1 c_1}{T} - m_2 c_2 \frac{m_1 c_1}{m_1 c_1 (T_1 - T) + m_2 c_2 T_2}$$

$$0 = \frac{1}{T} - \frac{m_2 c_2}{m_1 c_1 (T_1 - T) + m_2 c_2 T_2}$$

$$T = \frac{m_1 c_1 (T_1 - T) + m_2 c_2 T_2}{m_2 c_2}$$

$$= \frac{m_1 c_1}{m_2 c_2}(T_1 - T) + T_2$$

$$= T'$$

20.2.55

20.2.57

(a)

$$r_b = 8.0\,\mathrm{cm}$$

$$V_b = \frac{4}{3}\pi r_b^3$$

$$= 2.14 \times 10^{-3}\,\mathrm{m}^3$$

$$V_a = 2V_b$$

$$= 4.29 \times 10^{-3}\,\mathrm{m}^3$$

$$r_a = \sqrt[3]{\frac{3V_a}{4\pi}}$$

$$= 10\,\mathrm{cm}$$

$$r_d = 3r_a$$

$$= 30\,\mathrm{cm}$$

$$p_a V_a^{\gamma} = p_b V_b^{\gamma}$$

$$p_a = \left(\frac{V_b}{V_a}\right)^{\gamma} p_b$$

$$= 8.24\,\mathrm{kPa}$$

$$\frac{p_b V_b}{T_b} = \frac{p_a V_a}{T_a}$$

$$T_b = \frac{p_b V_b}{p_a V_a} T_a$$

$$= 152\,\mathrm{K}$$

(b)

$$n = \frac{p_b V_b}{R T_b}$$

$$= 3.44 \times 10^{-2}\,\mathrm{mol}$$

$$V_d = \frac{4}{3}\pi r_d^3$$

$$= 0.113\,\mathrm{m}^3$$

$$p_d = \frac{n R T_d}{V_d}$$

$$= 311\,\mathrm{Pa}$$

(c)

$$e = 1 - \frac{T_C}{T_H}$$
$$= 0.191$$
$$= 19.1\%$$
$$W = eQ_H$$
$$Q_H = \frac{W}{e}$$
$$= 5.24\,\text{kJ}$$

(d)

$$Q_C = (1 - e)Q_H$$
$$= 4.24\,\text{kJ}$$

20.2.59

$$e = 1 - \frac{T_C}{T_H}$$
$$T_C = (1 - e)T_H$$
$$= 281\,\text{K}$$
$$= 7.5\,^{\circ}\text{C}$$

b

20.2.61

d