Introduction to Electrodynamics by David J. Griffiths Problems

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Contents

2 Electrostatics

2.1

(a) $\mathbf{0}$

(b) The same as if only the opposite charge were present — all others are cancelled out.

2.2

$$\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0} 2 \frac{q}{\mathscr{r}^2} \cos\theta \hat{\mathbf{x}} \\
&= \frac{1}{4\pi\epsilon_0} \frac{dq}{[(d/2)^2 + z^2]^{3/2}} \hat{\mathbf{x}}
\end{aligned}$$

2.3

$$\begin{aligned}
\mathbf{r} &= z\hat{\mathbf{z}} \\
\mathbf{r}' &= x\hat{\mathbf{x}} \\
\boldsymbol{\mathscr{r}} &= z\hat{\mathbf{z}} - x\hat{\mathbf{x}} \\
\mathscr{r} &= \sqrt{x^2 + z^2} \\
\hat{\boldsymbol{\mathscr{r}}} &= \frac{z\hat{\mathbf{z}} - x\hat{\mathbf{x}}}{\sqrt{x^2 + z^2}} \\
\mathbf{E} &= \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda}{x^2 + z^2} \frac{z\hat{\mathbf{z}} - x\hat{\mathbf{x}}}{\sqrt{x^2 + z^2}} \, dx \\
&= \frac{1}{4\pi\epsilon_0} \lambda \left(z\hat{\mathbf{z}} \int_0^L \frac{1}{(x^2 + z^2)^{3/2}} \, dx - \hat{\mathbf{x}} \int_0^L \frac{x}{(x^2 + z^2)} \, dx \right) \\
&= \frac{1}{4\pi\epsilon_0} \lambda \left[\frac{L}{z\sqrt{L^2 + z^2}} \hat{\mathbf{z}} - \left(\frac{1}{z} - \frac{1}{\sqrt{L^2 + z^2}} \right) \hat{\mathbf{x}} \right] \\
&= \frac{1}{4\pi\epsilon_0} \frac{\lambda}{z} \left[\left(-1 + \frac{z}{\sqrt{L^2 + z^2}} \right) \hat{\mathbf{x}} + \frac{L}{\sqrt{L^2 + z^2}} \hat{\mathbf{z}} \right]
\end{aligned}$$

2.4

The electric field a distance z above the midpoint of a line segment of length $2L$ and uniform line charge λ is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z\sqrt{z^2 + L^2}} \hat{\mathbf{z}}.$$

Applying this to the four sides of the square, the horizontal components of opposite sides cancel leaving only the vertical component.

$$
\begin{aligned}
\cos\theta &= \frac{z}{\mathscr{r}} \\
&= \frac{z}{\sqrt{(a/2)^2 + z^2}} \\
\mathbf{E} &= 4\left(\frac{1}{4\pi\epsilon_0}\frac{\lambda a}{\sqrt{(a/2)^2+z^2}\sqrt{(a/2)^2+(a/2)^2+z^2}}\hat{\mathbf{z}}\right)\cos\theta \\
&= \frac{1}{4\pi\epsilon_0}\frac{4a\lambda z}{[(a/2)^2+z^2]\sqrt{(a^2/2)+z^2}}\hat{\mathbf{z}}
\end{aligned}
$$

2.5

$$
\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\frac{\lambda r}{r^2+z^2}\cos\alpha\, d\theta\,\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}\frac{2\pi\lambda r z}{(r^2+z^2)^{3/2}}\hat{\mathbf{z}}
\end{aligned}
$$

2.6

$$
\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int\frac{dq}{\mathscr{r}^2}\cos\theta\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^R\frac{\sigma}{r^2+z^2}\frac{z}{\sqrt{r^2+z^2}}r\,dr\,d\theta\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}2\pi\sigma z\int_0^R\frac{r}{(r^2+z^2)^{3/2}}\,dr\,\hat{\mathbf{z}} \\
&= \frac{1}{4\pi\epsilon_0}2\pi\sigma z\left(\frac{1}{z}-\frac{1}{\sqrt{R^2+z^2}}\right)\hat{\mathbf{z}}
\end{aligned}
$$

When $R \to \infty$

$$
\mathbf{E} = \frac{\sigma}{2\epsilon_0}\hat{\mathbf{z}}.
$$

2.7

$$
\mathbf{E} = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\hat{\mathbf{z}} & z > R \\ \mathbf{0} & z < R \end{cases}
$$

2.8

$$
\mathbf{E} = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\hat{\mathbf{z}} & z > R \\ \frac{1}{4\pi\epsilon_0}\frac{qz}{R^3}\hat{\mathbf{z}} & z < R \end{cases}
$$

2.9

(a)

$$\begin{aligned}\rho &= \epsilon_0 \nabla \cdot \mathbf{E}\\ &= \epsilon_0 \frac{1}{r^2}\frac{\partial}{\partial r}(kr^5)\\ &= 5\epsilon_0 k r^2\end{aligned}$$

(b)

$$\begin{aligned}Q_{\text{enc}} &= \epsilon_0 \oint \mathbf{E}\cdot d\mathbf{a}\\ &= \epsilon_0 \int_0^{2\pi}\int_0^{\pi} kR^3 R\, d\theta R \sin\theta\, d\phi\\ &= 2\pi\epsilon_0 kR^5[-\cos\theta]_0^{\pi}\\ &= 4\pi\epsilon_0 kR^5\\ Q_{\text{enc}} &= \int_V \rho\, d\tau\\ &= \int_0^{2\pi}\int_0^{\pi}\int_0^{R} 5\epsilon_0 kr^2\, dr r\, d\theta r \sin\theta\, d\phi\\ &= 10\pi\epsilon_0 k \int_0^{\pi}\int_0^{R} r^4 \sin\theta\, dr\, d\theta\\ &= 2\pi\epsilon_0 kR^5[-\cos\theta]_0^{\pi}\\ &= 4\pi\epsilon_0 kR^5\end{aligned}$$

2.10

If the charge was surrounded by 8 such cubes the total flux through all the cubes would be q/ϵ_0. There are 24 outside faces to the larger cube, so the total flux through the shaded face is $q/(24\epsilon_0)$.

2.11

$$\begin{aligned}\int \mathbf{E}_{\text{inside}}\cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0}\\ &= 0\\ \mathbf{E}_{\text{inside}} &= \mathbf{0}\\ \int \mathbf{E}_{\text{outside}}\cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0}\\ 4\pi r^2 E_{\text{outside}} &= \frac{4\pi R^2\sigma}{\epsilon_0}\\ \mathbf{E}_{\text{outside}} &= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}\end{aligned}$$

2.12

$$\begin{aligned}
\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
4\pi r^2 E &= \frac{\frac{4}{3}\pi r^3 \rho}{\epsilon_0} \\
\mathbf{E} &= \frac{r\rho}{3\epsilon_0}\hat{\mathbf{r}}
\end{aligned}$$

2.13

$$\begin{aligned}
\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
2\pi s l E &= \frac{l\lambda}{\epsilon_0} \\
\mathbf{E} &= \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}\hat{\mathbf{s}}
\end{aligned}$$

2.14

$$\begin{aligned}
Q_{\text{enc}} &= \int_V \rho\, d\tau \\
&= \int_0^{2\pi} \int_0^{\pi} \int_0^{r} k r'^3 \sin\theta \, dr' \, d\theta \, d\phi \\
&= 2\pi k \int_0^{\pi} \left[\frac{1}{4} r'^4 \sin\theta \right]_0^r d\theta \\
&= \frac{1}{2}\pi k r^4 [-\cos\theta]_0^{\pi} \\
&= \pi k r^4 \\
\int \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
4\pi r^2 E &= \frac{\pi k r^4}{\epsilon_0} \\
\mathbf{E} &= \frac{k r^2}{4\epsilon_0}\hat{\mathbf{r}}
\end{aligned}$$

2.15

(a) $\mathbf{E} = \mathbf{0}$

(b)

$$\begin{aligned}
Q_{\text{enc}} &= \int_0^{2\pi} \int_0^{\pi} \int_a^r k \sin\theta \, dr' \, d\theta \, d\phi \\
&= 4\pi k(r-a) \\
4\pi r^2 E &= \frac{4\pi k(r-a)}{\epsilon_0} \\
\mathbf{E} &= \frac{k(r-a)}{\epsilon_0 r^2}\hat{\mathbf{r}}
\end{aligned}$$

(c) $\mathbf{E} = \frac{k(b-a)}{\epsilon_0 r^2}\hat{\mathbf{r}}$

2.16

(a)

$$\begin{aligned}
Q_{\text{enc}} &= \pi s^2 l \rho \\
2\pi s l E &= \frac{\pi s^2 l \rho}{\epsilon_0} \\
\mathbf{E} &= \frac{s\rho}{2\epsilon_0}\hat{\mathbf{s}}
\end{aligned}$$

(b)

$$\mathbf{E} = \frac{a^2 \rho}{2\epsilon_0 s}\hat{\mathbf{s}}$$

(c)

$$\mathbf{E} = \mathbf{0}$$

2.17

$$\begin{aligned}
2AE_{\text{inside}} &= \frac{2Ay\rho}{\epsilon_0} \\
\mathbf{E}_{\text{inside}} &= \frac{y\rho}{\epsilon_0} \\
\mathbf{E} &= \begin{cases} \frac{d\rho}{\epsilon_0} & d < y \\ \frac{y\rho}{\epsilon_0} & 0 < y < d \\ -\frac{y\rho}{\epsilon_0} & -d < y < 0 \\ -\frac{d\rho}{\epsilon_0} & y < -d \end{cases}
\end{aligned}$$

2.18

The electric field inside a uniformly charged solid sphere is

$$\mathbf{E} = \frac{r\rho}{3\epsilon_0}\hat{\mathbf{r}}.$$

$$\begin{aligned}
\mathbf{d} &= \mathbf{r}_1 - \mathbf{r}_2 \\
\mathbf{E} &= \frac{r_1\rho}{3\epsilon_0}\hat{\mathbf{r}}_1 - \frac{r_2\rho}{3\epsilon_0}\hat{\mathbf{r}}_2 \\
&= \frac{\rho}{3\epsilon_0}(\mathbf{r}_1 - \mathbf{r}_2) \\
&= \frac{\rho}{3\epsilon_0}\mathbf{d}
\end{aligned}$$

2.20

a is impossible because its curl is nonzero.

$$\begin{aligned}
V &= -\int_0^y 2kxy'\,dy' - \int_0^z 2kyz'\,dz \\
&= -2kx\left[\frac{1}{2}y'^2\right]_0^y - 2ky\left[\frac{1}{2}z'^2\right]_0^z \\
&= -k(xy^2 + yz^2) \\
-\nabla V &= k[y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}] \\
&= \mathbf{E}
\end{aligned}$$

2.21

$$
\begin{aligned}
\mathbf{E} &= \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} & r > R \\ \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3} & r < R \end{cases} \\
V_{\text{outside}}(r) &= -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{q}{r'^2}\,dr' \\
&= -\frac{1}{4\pi\epsilon_0} q \left[-\frac{1}{r'}\right]_{\infty}^{r} \\
&= \frac{1}{4\pi\epsilon_0}\frac{q}{r} \\
-\nabla V_{\text{outside}} &= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \\
&= \mathbf{E}_{\text{outside}} \\
V_{\text{inside}}(r) &= -\left(\int_{\infty}^{R} \frac{1}{4\pi\epsilon_0}\frac{q}{r'^2}\,dr' + \int_{R}^{r} \frac{1}{4\pi\epsilon_0}\frac{qr'}{R^3}\,dr'\right) \\
&= -\left(-\frac{1}{4\pi\epsilon_0}\frac{q}{R} + \frac{1}{4\pi\epsilon_0}\frac{q}{R^3}\left[\frac{1}{2}r'^2\right]_{R}^{r}\right) \\
&= \frac{1}{4\pi\epsilon_0}\frac{q}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right] \\
-\nabla V_{\text{inside}} &= \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3}\hat{\mathbf{r}} \\
&= \mathbf{E}_{\text{inside}}
\end{aligned}
$$

2.22

$$
\begin{aligned}
\mathbf{E} &= \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}\hat{\mathbf{s}} \\
V &= -\int_{O}^{s} \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s'}\,ds' \\
&= -\frac{1}{2\pi\epsilon_0}\lambda \ln\frac{s}{O} \\
-\nabla V &= \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}\hat{\mathbf{s}}
\end{aligned}
$$

2.23

$$
\begin{aligned}
\mathbf{E} &= \begin{cases} \mathbf{0} & r < a \\ \frac{k(r-a)}{\epsilon_0 r^2}\hat{\mathbf{r}} & a < r < b \\ \frac{k(b-a)}{\epsilon_0 r^2}\hat{\mathbf{r}} & b < r \end{cases} \\
V(0) &= -\int_\infty^0 E\,dr \\
&= -\left(\int_\infty^b \frac{k(b-a)}{\epsilon_0 r^2}\,dr + \int_b^a \frac{k(r-a)}{\epsilon_0 r^2}\,dr\right) \\
&= -\left(\frac{k(b-a)}{\epsilon_0}\left[-\frac{1}{r}\right]_\infty^b + \frac{k}{\epsilon_0}\left[\ln r + \frac{a}{r}\right]_b^a\right) \\
&= -\left[-\frac{k(b-a)}{\epsilon_0 b} + \frac{k}{\epsilon_0}\left(\ln a + 1 - \ln b - \frac{a}{b}\right)\right] \\
&= -\frac{k}{\epsilon_0}\left(-1 + \frac{a}{b} + \ln\frac{a}{b} + 1 - \frac{a}{b}\right) \\
&= \frac{k}{\epsilon_0}\ln\frac{b}{a}
\end{aligned}
$$

2.24

$$
\begin{aligned}
V(b) - V(0) &= -\int_0^b E\,dr \\
&= -\left(\int_0^a \frac{s\rho}{2\epsilon_0}\,ds + \int_a^b \frac{a^2\rho}{2\epsilon_0 s}\,ds\right) \\
&= -\left(\frac{\rho}{2\epsilon_0}\left[\frac{1}{2}s^2\right]_0^a + \frac{a^2\rho}{2\epsilon_0}\ln\frac{b}{a}\right) \\
&= -\left(\frac{a^2\rho}{4\epsilon_0} + \frac{a^2\rho}{2\epsilon_0}\ln\frac{b}{a}\right) \\
&= -\frac{a^2\rho}{4\epsilon_0}\left(1 + 2\ln\frac{a}{b}\right)
\end{aligned}
$$

2.25

(a)

$$
V = \frac{1}{4\pi\epsilon_0}\frac{2q}{\sqrt{(d/2)^2 + z^2}}
$$

(b)

$$
\begin{aligned}
V &= \frac{1}{4\pi\epsilon_0}\int_{-L}^{L}\frac{\lambda}{\sqrt{x^2+z^2}}\,dx \\
&= \frac{1}{4\pi\epsilon_0}\lambda\ln\left(1+\frac{2L(L+\sqrt{L^2+z^2})}{z^2}\right)
\end{aligned}
$$

(c)

$$
\begin{aligned}
V &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{R}\frac{\sigma}{\sqrt{r^2+z^2}}r\,dr\,d\theta \\
&= \frac{1}{4\pi\epsilon_0}2\pi\sigma(\sqrt{R^2+z^2}-z)
\end{aligned}
$$

2.26

$$
\begin{aligned}
V_{\text{bottom}} &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{h}\frac{\sqrt{2}\sigma z}{\sqrt{2}z}\,d\phi\,dz \\
&= \frac{\sigma h}{2\epsilon_0} \\
V_{\text{top}} &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{h}\frac{\sqrt{2}\sigma z}{\sqrt{z^2+(h-z)^2}}\,d\phi\,dz \\
&= \frac{\sqrt{2}\sigma}{2\epsilon_0}\int_0^{h}\frac{z}{\sqrt{z^2+(h-z)^2}}\,dz \\
&= \frac{\sigma h}{4\epsilon_0}\ln(3+2\sqrt{2}) \\
V_{\text{bottom}}-V_{\text{top}} &= \frac{\sigma h}{2\epsilon_0}\left[1-\frac{1}{2}\ln(3+2\sqrt{2})\right]
\end{aligned}
$$

2.28

$$
\begin{aligned}
V(r) &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^{R}\frac{\rho r'^2\sin\theta}{\sqrt{r^2+r'^2-2rr'\cos\theta}}\,dr'\,d\theta\,d\phi \\
&= \frac{\rho}{2\epsilon_0}\int_0^{\pi}\int_0^{R}\frac{r'^2\sin\theta}{\sqrt{r^2+r'^2-2rr'\cos\theta}}\,dr'\,d\theta \\
&= \frac{\rho}{2\epsilon_0}\left(R^2-\frac{r^2}{3}\right) \\
&= \frac{q}{8\pi\epsilon_0 R}\left(3-\frac{r^2}{R^2}\right)
\end{aligned}
$$