# University Physics with Modern Physics
# Electromagnetism Notes

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## Contents

# 21 Electric Charge and Electric Field

## 21.1 Electric Charge

- Electrons have a much smaller mass than neutrons and protons
- Neutrons and protons have a very similar mass
- Electrons and protons have the same magnitude of charge
- The number of protons in an atom determins its **atomic number**
- If an electron is added to a neutral atom it becomes a **negative ion**, if one is removed it becomes a **positive ion** — this is called **ionisation**
- The **principle of conservation of charge** states that the algebraic sum of all the electric charges in any closed system is constant
- The electron or proton's magnitude of charge is a natural unit of charge — every observable amount of electric charge is an integer multiple of this

## 21.2 Conductors, Insulators, and Incuded Charges

- **Conductors** pemit easy movement of charge, **insulators** do not
- Holding a charged object near an uncharged object causes free electrons in the latter to move away/towards the former, resulting in a net charge on either side — this is called **induced charge**

## 21.3 Coulomb's Law

- The SI unit of charge is called one **coulomb** (1 C) and is defined such that $1.602176634 \times 10^{-19}$ C is equal to the charge of an electron or proton
- **Coulomb's law** describes the electric force between two point charges

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

  where the **electric constant** $\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2$, $q_1$ and $q_2$ are the magnitudes of the charges, and $r$ is the distance between them

- The electric force is always directed along the line between the two charges, attracting opposite charges and repelling like charges
- $\frac{1}{4\pi\epsilon_0}$ can be approximated as $9.0 \times 10^9\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2$
- The principle of superposition of forces also applies to electric charges

## 21.4 Electric Field and Electric Forces

- The electric force on a charged object is exerted by the electric field created by other charged objects
- We can determine if there is an electric field at a point by placing a test charge $q_0$ there and seeing if it experiences an electric force — the electric field at that point (the electric force per unit charge) is then given by

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

- Rearranging, the force experienced by a charge $q_0$ at a point is given by

$$\mathbf{F} = q_0\mathbf{E}$$

- When considering an electric field produced by a point charge, the location of the point charge is called the **source point** and the location at which we're trying to determine the field is called the **field point**

- The electric field produced by a point charge is given by

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

where $q$ is the charge of the point charge, $r$ is the distance between the source and field points, and $\hat{\mathbf{r}}$ is the unit vector from the source to the field point

- Unlike Coulomb's law this equation doesn't use the absolute value of $q$ meaning that the electric fields of positive charges point away from the charge, while those of negative charges point towards them

- In electrostatics, the electric field inside the material of a conductor (but not holes within the material) is $\mathbf{0}$

## 21.5 Electric-Field Calculations

- The **principle of superposition of electric fields** states that the total electric field at a point $P$ is the vector sum of the fields at $P$ due to each point charge in the charge distribution

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots$$

- For a line charge distribution the **linear charge density** is represented by $\lambda$ (the charge per unit length, measured in C/m)

- For a surface charge distribution the **surface charge density** is represented by $\sigma$ (the charge per unit area, measured in $\mathrm{C/m^2}$)

- For a volume charge distribution the **volume charge density** is represented by $\rho$ (the charge per unit volume, measured in $\mathrm{C/m^3}$)

- The electric field of an infinitely long line charge along the $y$-axis is

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

## 21.6 Electric Field Lines

- An **electric field line** is a line drawn through space such that its tangent at any point is in the direction of the electric field vector at that point

- Fewer lines are drawn in areas where the electric field is weak and more lines are drawn in areas where it's strong

### 21.7 Electric Dipoles

- An **electric dipole** is a pair of point charges of equal magnitude $q$ and opposite sign separated by a distance $d$
- The net force on an electric dipole in a uniform electric field is $\mathbf{0}$
- The **electric dipole moment p** of an electric dipole is a vector directed from the negative charge to the positive charge with magnitude $qd$
- The net torque on an electric dipole in a uniform electric field is $\mathbf{p} \times \mathbf{E}$ or $qEd \sin \phi$ where $\phi$ is the angle between the electric dipole and the electric field
- The potential energy of an electric dipole in a uniform electric field is

$$U = -\mathbf{p} \cdot \mathbf{E}$$

## 22 Gauss's Law

### 22.1 Calculating Electric Flux

- The electric flux of a uniform electric field through a flat surface $A$ is

$$\Phi_E = \mathbf{E} \cdot \mathbf{A}$$

  where $\mathbf{A}$ is normal to $A$ and has a magnitude equal to its area

- The electric flux of a nonuniform electric field through a curved surface $A$ is

$$\Phi_E = \int \mathbf{E} \cdot \mathbf{dA}$$

### 22.2 Gauss's Law

- Gauss's law states that the total electric flux through a closed surface is equal to the total electric charge enclosed by the surface divided by $\epsilon_0$

$$\Phi_E = \oint \mathbf{E} \cdot \mathbf{dA} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

### 22.3 Applications of Gauss's Law

- Gauss's law can be used in two ways:
  - If we know the charge distribution and it has enough symmetry to let us evaluate the integral in Gauss's law, we can find the field
  - If we know the field, we can use Gauss's law to find the charge distribution

- Under electrostatics, excess charge always lies of the surface of a conductor
- The electric field of an infinite line charge is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}\hat{\mathbf{r}}$$

### 22.4 Charges on Conductors

- If there is excess charge at rest on a conductor, all of that charge must lie on the surface of the conductor and the electric field inside the conductor must be zero. If there is a cavity inside the conductor, the net charge on the cavity walls equals the amount of charge enclosed by the cavity
- Charges outside a conductor have no effect on the interior of the conductor, even if it has a cavity inside — this is why Faraday cages work
- At the surface of a conductor, the component of the electric field that is perpendicular to the surface is

$$E_\perp = \frac{\sigma}{\epsilon_0}$$

## 23 Electric Potential

### 23.1 Electric Potential Energy

- The electric potential energy of two point charges is

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

- The electric potential energy of a point charge $q_0$ and a collection of charges $q_1$, $q_2$, etc. is

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- For every electric field due to a static charge distribution, the force extered by that field is conservative
- The total electric potential energy of a collection of charges $q_1$, $q_2$, etc. is

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

  where $r_{ij}$ is the distance between $q_i$ and $q_j$

## 23.2 Electric Potential

- **Potential** is potential energy per unit charge
- The unit of potential is the **volt**, equal to 1 joule per coulomb
- The potential difference between two points $V_{ab} = V_a - V_b$ is called the potential of $a$ with respect to $b$ and equals the amount of work done by the electric force when a unit (1 C) of charge moves from $a$ to $b$
- The electric potential due to a point charge is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

- The electric potential due to a collection of point charges is

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- The electric potential due to a continuous charge distribution is

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

- The electric potential difference between two points is given by

$$V_a - V_b = \int_a^b \mathbf{E}\cdot d\mathbf{l} = \int_a^b E\cos\phi\, dl$$

- Positive charges tend to "fall" from high- to low-potential regions while negative charges do the opposite
- When a particle with charge $e = 1.602 \times 10^{-19}\,\mathrm{C}$ moves between two points with a potential difference of $1\,\mathrm{V} = 1\,\mathrm{J/C}$ the change in energy is $U_a - U_b = qV_{ab} = (1.602 \times 10^{-19}\,\mathrm{C})(1\,\mathrm{J/C}) = 1.602 \times 10^{-19}\,\mathrm{J}$ which is called 1 **electron volt**

## 23.4 Equipotential Surfaces

- An **equipotential surface** is a three-dimensional surface on which the electric potential is the same at every point
- Because electric potential energy doesn't change as a test charge moves over an equipotential surface, the electric field can do no work and thus **field lines and equipotential surfaces are always perpendicular**
- When all charges are at rest, the surface of a conductor is an equipotential surface
- When all charges are at rest, the entire solid volume of a conductor is at the same potential

## 23.5 Potential Gradient

- The relationship between $\mathbf{E}$ and $V$ is given by

$$\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$$

- If $E$ has a radial component $E_r$ with respect to an axis or a point and $r$ is the distance from that axis or point, then

$$E_r = -\frac{\partial V}{\partial r}$$

# 24 Capacitance and Dielectrics

## 24.1 Capacitors and Capacitance

- Any two conductors separated by an insulator (or a vacuum) form a **capacitor**

- The **capacitance** of a capacitor measures its ability to store charge

$$C = \frac{Q}{V_{AB}}$$

- Capacitance is measured in **farads** where

$$1\,\mathrm{F} = 1\,\mathrm{C/V}$$

- The capacitance of a parallel plate capacitor in a vacuum is

$$C = \epsilon_0 \frac{A}{d}$$

## 24.2 Capacitors in Series and Parallel

- In a series connection, the magnitude of charge on all plates is the same

- The **equivalent capacitance** of a combination of capacitors is the capacitance of a single capacitor that would have equivalent behaviour

- In a series connection, the reciprocal of the equivalent capacitance equals the sum of the reciprocals of the individual capacitances

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

  meaning the equivalent capacitance is always less than any individual capacitance

- In a parallel connection, the potential difference is the same for all capacitors
- In a parallel connection, the equivalent capacitance equals the sum of the individual capacitances

  $$C_{\text{eq}} = C_1 + C_2 + \cdots$$

  meaning the equivalent capacitance is always greater than any individual capacitance

## 24.3 Energy Storage in Capacitors and Electric-Field Energy

- The potential energy stored in a capacitor is

  $$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- The **energy density** of a parallel plate capacitor is its energy per unit volume

  $$u = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\epsilon_0 E^2$$

## 24.4 Dielectrics

- **Dielectrics** are nonconducting materials
- Most capacitors have a dielectric material between their plates because
  1. It preserves the distance between the plates
  2. It increases the maximum potential difference between the plates by avoiding **dielectric breakdown** when the material between the plates becomes ionized and becomes conductive — this happens more easily for air
  3. It increases the capacitance by decreasing the potential difference for a given charge
- The **dielectric constant** of a material is defined as

  $$K = \frac{C}{C_0}$$

  where $C_0$ is the capacitance of a capacitor with vacuum between the plates and $C$ is the capacitance of the same capacitor with the material between the plates

- If $E_0$ is the magnitude of the electric field between the plates of a parallel plate capacitor when separated by a vacuum and $E$ is the magnitude when separated by a dielectric then

$$E = \frac{E_0}{K}$$

- The electric field (and electric potential) are reduced because the dielectric becomes **polarized** and an induced surface charge appears of magnitude

$$\sigma_i = \sigma \left(1 - \frac{1}{K}\right)$$

- The **permittivity** of a dielectric is defined as

$$\epsilon = K\epsilon_0$$

- The capacitance of a parallel plate capacitor with dielectric between the plates is thus

$$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

  and the electric energy density is

$$u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2$$

- The maximum electric-field magnitude that a material can withstand without the occurence of breakdown is called its **dielectric strength** and is denoted $E_m$

## 24.5 Molecular Model of Induced Charge

- If a material is comprised of polar molecules where the net charge of the molecule is 0 but the charge isn't distributed equally, electric fields cause the molecules to rotate which induces a charge
- Even if a material isn't comprised of polar molecules, electric fields cause molecules' positive and negative charges to separate slightly resulting in a dipole which again experiences a torque
- The charges in conductors are free to move so they're known as **free charges** while the charges in dielectrics aren't so they're known as **bound charges**

### 24.6 Gauss's Law in Dielectrics

- Gauss's Law in a dielectric material relates the flux of $K\mathbf{E}$ through the surface to the amount of free (not bound) charge enclosed by the surface

$$\oint K\mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0}$$

- This shows that filling a volume with a dielectric with relative permittivity $K$ reduces the magnitude of the electric field by a factor of $1/K$

## 25 Current, Resistance, and Electromotive Force

### 25.1 Current

- A **current** is any motion of charge from one region to another
- The **drift velocity** $\mathbf{v_d}$ of a current is the velocity of its particles
- While a current may come about through the movement of negative and/or positive charges, **conventional current** dictates that by convention we describe currents as if they were carried by positive charges
- The unit of current is the **ampere** which is defined to be one coulomb per second

$$1\,\mathrm{A} = 1\,\mathrm{C/s}$$

- The **charge concentration** $n$ is the number of moving charged particles per unit volume
- The current through an area is given by

$$I = \frac{dQ}{dt} = n|q|v_d A$$

- The **current density** is the current per unit cross-sectional area

$$\mathbf{J} = nq\mathbf{v_d}$$

### 25.2 Resistivity

- The **resistivity** $\rho$ of a material is defined by **Ohm's law**

$$\rho = \frac{E}{J}$$

- The unit of resistivity is ohm-meters ($\Omega$m)

- The reciprocal of resistivity is **conductivity**
- Materials that obey Ohm's law are called **ohmic** or **linear** conductors
- Materials that don't obey Ohm's law are called **nonohmic** or **nonlinear** conductors
- The resistivity of a metallic conductor nearly always increases with increasing temperature

  $$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

  where $\rho_0$ is the resistivity at reference temperature $T_0$ and $\alpha$ is the **temperature coefficient of resistivity**
- The resistivity of semiconductors decreases with increasing temperature
- Some materials exhibit **superconductivity** where their resistivity drops to 0 below a critical temperature

## 25.3 Resistance

- The ratio of the voltage and current in a conductor is called its **resistance**

  $$R = \frac{V}{I} = \frac{\rho L}{A}$$

  where $\rho$ is the resistivity of the conductor, $L$ is its length, and $A$ is its cross-sectional area
- If $\rho$ is constant (as in ohmic materials), then $R$ is also constant
- The unit of resistance is the ohm

  $$1\,\Omega = 1\,\mathrm{V/A}$$

- Because the resistivity of a material varies with temperature, so too does the resistance of a specific conductor

  $$R(T) = R_0[1 + \alpha(T - T_0)]$$

- A device made to have a specific resistance is called a **resistor**

## 25.4 Electromotive Force and Circuits

- When a charge goes around a complete circuit and returns to its starting position its electric potential energy must be the same, but it experienced losses due to resistance along the way

- **Electromotive force** or **emf** $\mathcal{E}$ is the influence that makes current flow from lower to higher potential in a circuit and restores its original potential energy
- A device that provides emf is called a **source of emf**
- The SI unit of emf is the volt
- In an **ideal source of emf**
  - The potential difference between its terminals is constant regardless of the current passing through it
  - $\mathcal{E} = V = IR$
- Real sources of emf have **internal resistance** $r$ that reduce the **terminal voltage**
$$V_{ab} = \mathcal{E} - Ir$$
- Real sources of emf can be modelled as an ideal source of emf $\mathcal{E}$ in series with a resistor $r$

## 25.5 Energy and Power in Electric Circuits

- **Power** is the time rate change of energy transfer
$$P = VI$$
where $V$ is the voltage across a circuit element and $I$ is the current in it
- The SI unit of power is the watt
$$1\,\mathrm{W} = 1\,\mathrm{J/s}$$
- If the circuit element is a resistor then $V = IR$ and
$$P = VI = I^2R = \frac{V^2}{R}$$
- If the circuit element is a source of emf outputting power then
$$P = VI = (\mathcal{E} - Ir)I = \mathcal{E}I - I^2r$$
where the $\mathcal{E}I$ term is the power generated by the element and the $I^2r$ term is the power dissipated by its internal resistance
- If the circuit element is a source of emf consuming power (charging) then
$$P = VI = (\mathcal{E} + Ir)I = \mathcal{E}I + I^2r$$
where the terms are the same as above

### 25.6 Theory of Metallic Conduction

- The average time between collisions of an electron and positive ions is called the **mean free time** $\tau$
- The resistivity of a metal can be approximated as

$$\rho = \frac{m}{ne^2\tau}$$

  where $m$ is the mass of an electron, $n$ is the number of free electrons per unit volume, $e$ is the charge of an electron, and $\tau$ is the mean free time

## 26 Direct-Current Circuits

### 26.1 Resistors in Series and Parallel

- Circuit elements connected one after another with a single current path between them are said to be connected in **series**
- The current is the same for all circuit elements connected in series
- Circuit elements connected such there is an alternate current path for each element are said to be connected in **parallel**
- The potential difference / voltage is the same for all circuit elements connected in parallel
- For any combination of resistors we can always find a single resistor that could replace the combination and result in the same current and potential difference — the resistance of this resistor is called the **equivalent resistance**
- The equivalent resistance of a series combination of resistors equals the sum of the individual resistances

$$R_{\text{eq}} = R_1 + R_2 + \cdots$$

- The reciprocal of the equivalent resistance of a parallel combination of resistors equals the sum of the reciprocals of the individual resistances

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

### 26.2 Kirchhoff's Rules

- A **junction** in a circuit is a point where three or more conductors meet
- A **loop** is any closed conducting path

- **Kirchhoff's junction rule** states that the sum of the currents into any junction equals zero

$$\sum I = 0$$

- **Kirchhoff's loop rule** states that the sum of the potential differences around any loop equals zero

$$\sum V = 0$$

- Kirchhoff's rules can be used to analyse circuits by following these steps

**PROBLEM-SOLVING STRATEGY 26.2 Kirchhoff's Rules**

**IDENTIFY** *the relevant concepts:* Kirchhoff's rules are useful for analyzing any electric circuit.

**SET UP** *the problem* using the following steps:

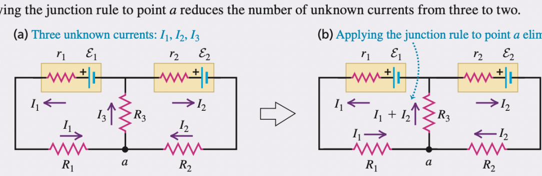
1. Draw a circuit diagram, leaving room to label all quantities, known and unknown. Indicate an assumed direction for each unknown current and emf. (Kirchhoff's rules will yield the magnitudes *and directions* of unknown currents and emfs. If the actual direction of a quantity is opposite to your assumption, the resulting quantity will have a negative sign.)
2. As you label currents, it's helpful to use Kirchhoff's junction rule, as in **Fig. 26.9**, so as to express the currents in terms of as few quantities as possible.
3. Identify the target variables.

**EXECUTE** *the solution* as follows:

1. Choose any loop in the network and choose a direction (clockwise or counterclockwise) to travel around the loop as you apply Kirchhoff's loop rule. The direction need not be the same as any assumed current direction.
2. Travel around the loop in the chosen direction, adding potential differences algebraically as you cross them. Use the sign conventions of Fig. 26.8.
3. Equate the sum obtained in step 2 to zero in accordance with the loop rule.
4. If you need more independent equations, choose another loop and repeat steps 1–3; continue until you have as many independent equations as unknowns or until every circuit element has been included in at least one loop.
5. Solve the equations simultaneously to determine the unknowns.
6. You can use the loop-rule bookkeeping system to find the potential $V_{ab}$ of any point $a$ with respect to any other point $b$. Start at $b$ and add the potential changes you encounter in going from $b$ to $a$; use the same sign rules as in step 2. The algebraic sum of these changes is $V_{ab} = V_a - V_b$.

**EVALUATE** *your answer:* Check all the steps in your algebra. Apply steps 1 and 2 to a loop you have not yet considered; if the sum of potential drops isn't zero, you've made an error somewhere.

Figure **26.9** Applying the junction rule to point $a$ reduces the number of unknown currents from three to two.




## 26.3 Electrical Measuring Instruments

- Many common devices measure current or potential difference with a **d'Arsonval galvanometer** which reports values via the deflection of a pointer
- The maximum deflection is called **full-scale deflection** and occurs at a particular current $I_{\text{fs}}$
- The device also has a resistance $R_c$
- A device that measures the current passing through it is called an **ammeter**
- An ideal ammeter would have 0 resistance so it doesn't affect the rest of the circuit, but real ammeters have a small, finite resistance

- An ammeter can be adjusted to measure currents larger than its full-scale reading by connecting a resistor in parallel so some current passes through the resistor — this is called a **shunt resistor** and obeys the relation

  $$I_{\mathrm{fs}} R_{\mathrm{c}} = (I_{\mathrm{a}} - I_{\mathrm{fs}}) R_{\mathrm{sh}}$$

  where $I_{\mathrm{fs}}$ is the device's full-scale current, $R_{\mathrm{c}}$ is its coil resistance, $I_{\mathrm{a}}$ is the desired full-scale current, and $R_{\mathrm{sh}}$ is the resistance of the shunt resistor

- A device that measures the potential difference between two probes / terminals is called a **voltmeter**

- An ideal voltmeter would have infinite resistance so it doesn't affect the rest of the circuit, but real voltmeters have large, finite resistance

- A voltmeter can be adjusted to measure potential differences larger than its full-scale reading by connecting a resistor in series with it so the voltage drops before reaching the voltmeter — this obeys the relation

  $$V_{\mathrm{V}} = I_{\mathrm{fs}}(R_{\mathrm{c}} + R_{\mathrm{s}})$$

  where $V_{\mathrm{V}}$ is the desired full-scale voltage, $I_{\mathrm{fs}}$ is the device's full-scale current, $R_{\mathrm{c}}$ is its coil resistance, and $R_{\mathrm{s}}$ is the resistance of the resistor

- If you measure the current and voltage across an element simulateneously either the ammeter will be measuring the current across both the element and the voltmeter, or the voltmeter will be measuring the potential difference across both the element and the ammeter — either way you need to correct one of the measurements

- A device that measures resistance is called an **ohmmeter**

- A **potentiometer** is a device that can be used to measure an unknown emf via a known emf and a sliding contact attached to a resistor

## 26.4 R-C Circuits

- A circuit that has a resistor and a capacitor in series is called an **R-C circuit**

- When charging the capacitor in an R-C circuit, the charge on the capacitor is given by

  $$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_{\mathrm{f}}(1 - e^{-t/RC})$$

  and the current in the circuit is given by

  $$i = \frac{\mathcal{E}}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

- At $t = RC$ the capacitor has reached $1 - 1/e$ of its final value and the current has decreased to $1/e$ of its original value — this product $\tau = RC$ is a measure of how quickly the capacitor charges and is called the **time constant** or the **relaxation time**

- When discharging the capacitor in an R-C circuit, the charge on the capacitor is given by
$$q = Q_0 e^{-t/RC}$$
and the current in the circuit is given by
$$i = -\frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$

# 27 Magnetic Field and Magnetic Forces

- Electric forces act on electric charges whether they are moving or not but magnetic forces acts only on moving charges

- Electric forces arise in two stages: (1) a charge produces an electric field in the space around it, and (2) a second charge responds to this field

- Magnetic forces also arise in two stages: (1) a moving charge or a collection of moving charges (i.e. a current) produces a magnetic field, and (2) a second moving charge or current responds to this field

## 27.1 Magnetism

- There is no experimental evidence that **magnetic monopoles** exist

## 27.2 Magnetic Field

- Magnetic fields are vector fields represented by the symbol **B**

- The direction of the field is the direction the North pole of a compass would point at that position

- For any magnet, **B** points out of its North pole towards its South pole

- The magnetic force on a moving charged particle is given by
$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

- The unit of magnetic fields is the **tesla** where
$$1 \text{ tesla} = 1\,\mathrm{T} = 1\,\mathrm{N/A\,m}$$
or the **gauss** where
$$1\,\mathrm{G} = 1 \times 10^{-4}\,\mathrm{T}$$

- When a charged particle moves through a region where both electric and magnetic fields are present the total force is the vector sum of the electric and magnetic forces
$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{E})$$
this is called the **Lorentz force**

## 27.3 Magnetic Field Lines and Magnetic Flux

- **Magnetic field lines** are to magnetic fields what electric field lines are to electric fields, however they don't show the force that would be exerted on a moving charge because that depends on the charge's velocity
- The magnetic flux through a surface is given by
$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$
- Magnetic flux uses the unit **weber** where
$$1\,\mathrm{Wb} = 1\,\mathrm{T\,m^2} = 1\,\mathrm{N\,m/A}$$
- **Gauss's law for magnetism** states that the total magnetic flux through a closed surface is 0
$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$
- Sometimes we need to calculate the magnetic flux through an open surface in which case the direction of $d\mathbf{A}$ is ambiguous — in these scenarios we choose one direction to be positive and use that consistently

## 27.4 Motion of Charge Particles in a Magnetic Field

- Magnetic forces do no work on point charges because they are always perpendicular to the charges' velocity
- The motion of a charge particle affected only by a magnetic force is always motion with constant speed
- The radius of a circular orbit in a magentic field is given by
$$R = \frac{mv}{|q|B},$$
the angular speed is given by
$$\omega = \frac{|q|B}{m},$$
and the frequency is given by
$$f = \frac{\omega}{2\pi} = \frac{|q|B}{2\pi m}$$

## 27.5 Applications of Motion of Charged Particles

- The speed for which there is no deflection in a velocity selector is

$$v = \frac{E}{B}$$

## 27.6 Magnetic Force on a Current-Carrying Conductor

- The magnetic force on a straight wire segment is given by

$$\mathbf{F} = I\mathbf{l} \times \mathbf{B}$$

  where $\mathbf{l}$ is the vector length of the segment (points in the current direction)

- The magnetic force on a non-straight wire segment is given by

$$\mathbf{F} = I \int d\mathbf{l} \times \mathbf{B}$$

## 27.7 Force and Torque on a Current Loop

- The net force on a current loop in a uniform magnetic field is zero, however the net torque is not in general equal to zero
- The **magnetic dipole moment** or **magnetic moment** of a current loop is a vector perpendicular to the plane of the loop with direction determined by the current around the loop and right hand rule with magnitude

$$\mu = IA$$

- A current loop or any any other object that experiences a magnetic torque in a magnetic field is called a **magnetic dipole**
- The magnetic torque experienced by a current loop is given by

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

- The potential energy of a magnetic dipole in a magnetic field is

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

- A coil consisting of $N$ planar loops close together is called a **solenoid** and its magnetic moment, potential energy, and torque are all multiplied by a factor of $N$

## 27.9 The Hall Effect

- The Hall effect is described by the equation

$$nq = \frac{-J_x B_y}{E_z}$$

  where $E_z$ is the induced electrostatic field in the conductor