Session 2 Decision Tree

S2.1 Elements of Decision Tree

How to understand decision under **certainty**.

How to understand EVUC, let's start from under risk, the second scenario.

	Interest Rates			
	Decline	Stable	Increase	
Prob. of Scenarios	$p_D = 0.5$	$p_S = 0.35$	$p_I = 0.15$	Expected Value
Office park	0.5	1.7	4.5	1.52
Office building	1.5	1.9	2.4	1.775
Warehouse	1.7	1.4	1.0	1.49
Shopping center	0.7	2.4	3.6	1.73
Apartment	3.2	1.5	0.6	2.215

Suppose we repeat this investment 100 times, among which, in expectation, we will have 50 times decline, 35 times stable, and 15 times increase. Our decision will always be "Apartment", the total payoff is $50\times 3.2 + 35\times 1.5 + 15\times 0.6 = 221.5$.

The payoff for each investment (i.e. EV) = 221.5/100 = 2.215

Our decisions are not **perfect**. We make some "wrong" decisions. For example, when it is stable, we should choose "Shopping center", but actually we always choose "apartment".

If we knew what would happen in the future, our decision would be "perfect":

Decline scenario-Apartment

Stable-shopping center

Increase-office park

The total payoff is $50\times 3.2 + 35\times 2.4 + 15\times 4.5 = 311.5$

The payoff for each investment (i.e. EVUC) = 311.5/100 = 3.115

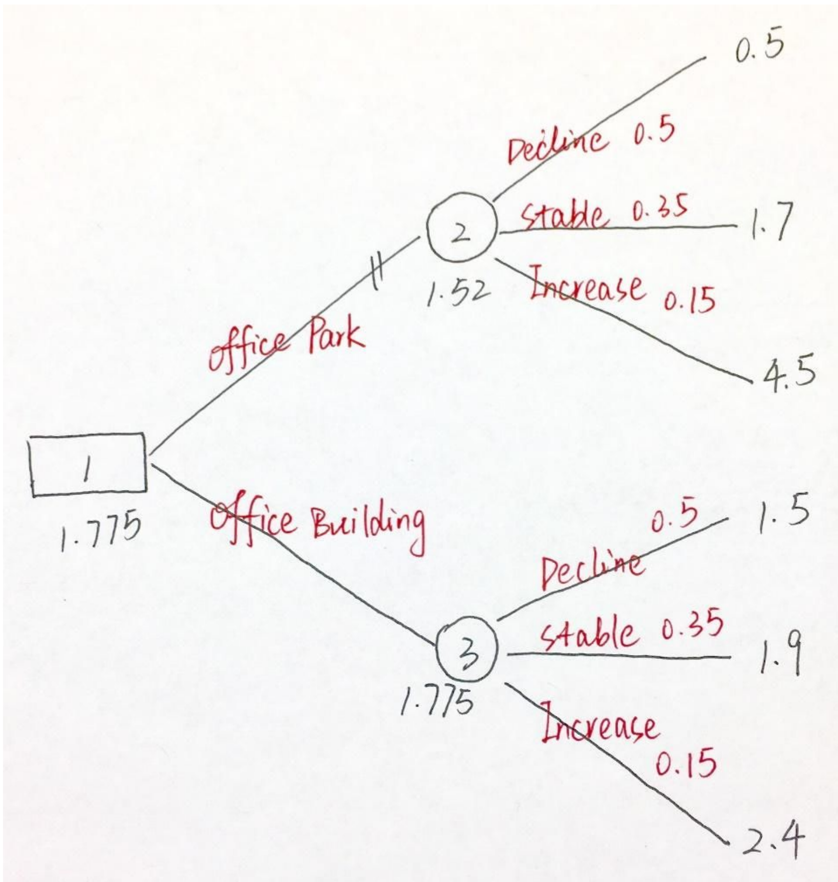
This is exactly the EVUC of the third scenario.

First, is there any **order** for human's decision and nature's decision? Which happens first?

Human beings first take action, the it is the nature's turn.

Example from last class:

	Decline	Stable	Increase	
Prob. of Scenarios	$p_D = 0.5$	$p_S = 0.35$	$p_I = 0.15$	Expected Value
Office park	0.5	1.7	4.5	1.52
Office building	1.5	1.9	2.4	1.775



EV of Node 2: $0.5\times0.5 + 1.7\times0.35 + 4.5\times0.15 = 1.52$

EV of Node 3: $1.5\times0.5 + 1.9\times0.35 + 2.4\times0.15 = 1.775$

EV of Node 1: choose the larger one, i.e. 1.775

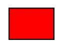
Decision: we should choose the office building.

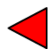
The **overall** expected value is 1.775.

<u>How to draw a decision tree</u>

1. Two types of nodes

State-of-nature node: **circle** 

Alternative node: **rectangle** 

End node: triangle  (not commonly used)

<u>Remark:</u>

There exists probability only for **states of nature**. e.g., how the weather will be, or which football team will win a game. Each state of nature may happen with corresponding probability.

Alternative is an option without likelihood. For example, we choose among alternatives A, B, and C. If we choose A, then the "probability" of choosing A is "1", the probability of choosing B and C are "0". Thus, it is meaningless to use probability when talking about **alternatives**. Because reasonable people always choose the optimal option, so alternatives have no "probability". Never write "probability" on the line out of "rectangles", just choose the optimal one.

2. Each Node contains a single value (write ordinal numbers on each node)

3. End node contains a given value.

4. One possible state of nature will happen out of a circle (state-of-nature node) and we need to take expectation to calculate the EV of a state-of-nature node; Alternative node contains an optimal value (maximum or minimum).

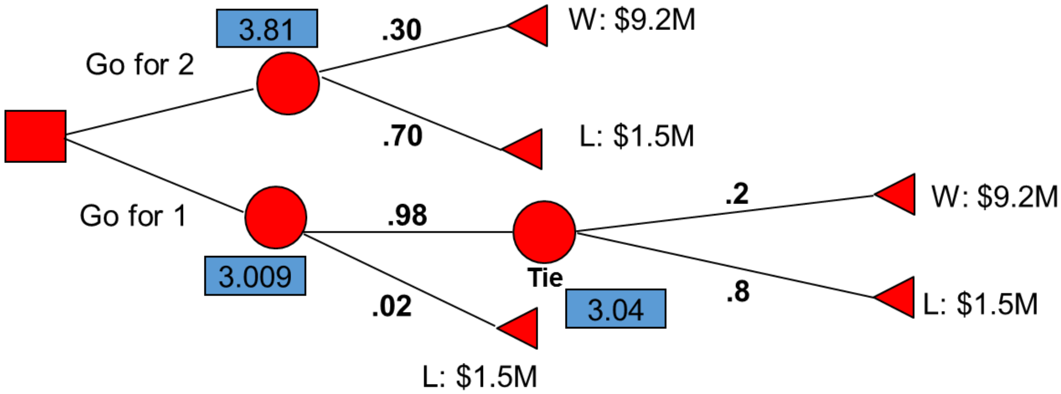
5. The root of the tree (the first rectangle) returns the **overall expected value**.

S2.2 Example

Should the team go for two points (for the win) or one point (for the tie) with no time remaining?

Two-point success rate: 30%; One-point success rate: 98%; Chances in overtime: 20%

<u>Payoffs</u> Win: Sugar Bowl = 9.2M, Lose: Gator Bowl = 1.5M



S2.3 A Sequence of Decisions

Now, let's simplify the first example, and add one more potential decision process.

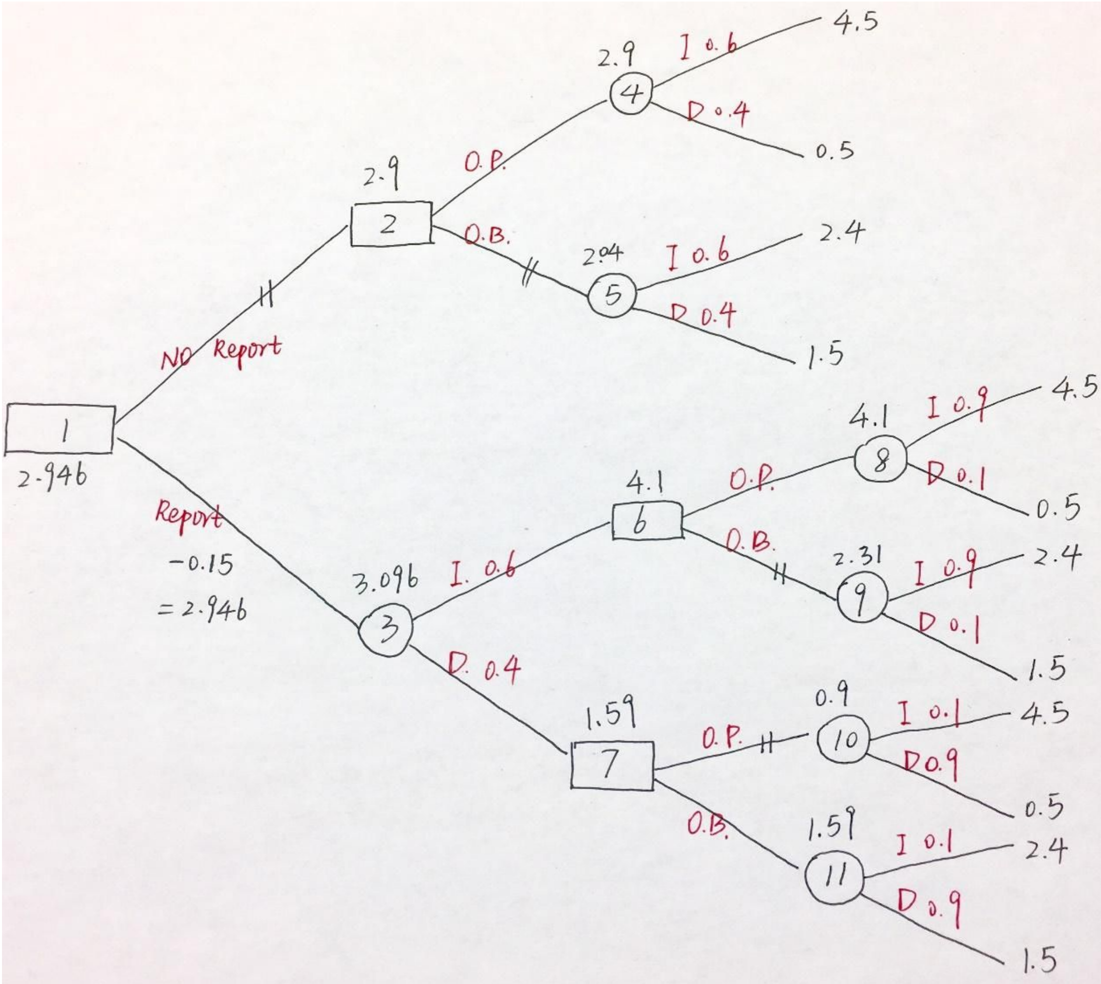
	Decline	Increase
Prob. of Scenarios	$p_D = 0.4$	$p_I = 0.6$
Office park	0.5	4.5
Office building	1.5	2.4

Suppose before choosing from office park and office building, we could either make the decision right away, or pay 0.15% rate of benefits to a consulting firm for a report, and then decide what action to take.

The report is 90% accurate. (If the report says Decline, then the probability of decline will change to 0.9. If the report says Increase, then the probability of Increase will change to 0.9.)

There is a 60% chance of an Increase prediction from the report.

Now this problem becomes much more complicated, and can hardly be solved by a decision table.



<u>When **a sequence of decisions** must be made, decision trees are more powerful.</u>

How to describe decisions? You need to tell **all** the information of a sequence of decisions.

First, we need to ask for the report; then, we choose the Office park.