20200210【Data Analyst Nanodegree】P04M01L04

> Part 04 : Practical Statistics
> Learn how to apply inferential statistics and probability to important, real-world scenarios, such as analyzing A/B tests and building supervised learning models.

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Module 01: Practical Stats

Lesson 04: Probability

> Gain the basics of probability using coins and die.

01. Introduction to Probability

Statistics and **probability** are different but strongly related fields of mathematics.

In **probability**, we **make predictions about future events based on models or causes** that we assume whereas in **statistics** we **analyze the data from the past events to infer what those models or causes** could be.

There's almost an opposite relation between these two.In one, you're predicting data and in the other, you're using data to predict.

Although not all topics and both fields require an understanding of the other, you'll need a good understanding in probability for the foundation you'll be building in statistics. With that in mind, let's get started with the basics of probability.

02. Flipping Coins

I have here a U.S. dollar coin. It has two sides, one showing a head and one showing what's called tails. In probability, I'm giving a description of this coin, and I'm making data. We just make data.

$DATA = HEADS, HEADS, HEADS$

So let me ask a statistical question to test your intuition. Do you think if I twist this coin more frequently will it always come up heads?

I think the best answer is no. This is what's called a **Fair Coin(公平硬币)**, and that means it really has a 50% chance of coming up tails. So probability is a method of describing the anticipated outcome of these coin flips.

03. Fair Coin

Fair Coin

$P(HEADS) = 0.5$

$P(TAILS) = 0.5$

04. Loaded Coin 1

Loaded Coin

A **Loaded Coin(不公正硬币)** is one that comes up with one of the two much more frequently than the other. So, for example, suppose I have a coin that always comes up heads.

$P(HEADS) = 1$

$P(TAILS) = 0$

04. Loaded Coin 2

Loaded Coin

A loaded coin is one that comes up with one of the two much more frequently than the other. So, for example, suppose I have a coin that always comes up heads.

$P(HEADS) = 1$

$P(TAILS) = 0$

$P(HEADS) + P(TAILS) = 1$

06. Loaded Coin 3

Loaded Coin

A loaded coin is one that comes up with one of the two much more frequently than the other. So, for example, suppose I have a coin that always comes up heads.

$P(HEADS) = 0.75$

$P(TAILS) = 0.25$

$P(HEADS) + P(TAILS) = 1$

07. Complementary Outcomes

Basic Law of Probability

$P(A) = 1 - P(\neg A)$

08. Two Flips 1

So now I want to ask you a really tricky question:

What's the probability of observing heads and heads if you flip the same unbiased coin twice? This means in each flip we assume the probability of heads is 0.5.

$P(H) = 0.5$

$\overbrace{P(H,H)}^{\text{HEADS,HEADS}} = 0.25$

Truth Table

Flip-1	Flip-2	
H	H	0.25
H	T	0.25
T	H	0.25
T	T	0.25

The productProbability of Heads

$\overbrace{P(H,H)}^{\text{HEADS,HEADS}} = P(H) * P(H) = 0.5 * 0.5 = 0.25$

09. Two Flips 2

$P(H) = 0.6$

$P(T) = 0.4$

10. Two Flips 3

$P(H) = 0.6$

$P(T) = 0.4$

Truth Table

Flip-1	Flip-2	
H	H	0.36
H	T	0.24
T	H	0.24
T	T	0.16

11. Two Flips 4

$P(H) = 0.6$

$P(T) = 0.4$

Truth Table

Flip-1	Flip-2	1
H	H	0.36
H	T	0.24
T	H	0.24
T	T	0.16

$\overbrace{P(H,H)}^{\text{HEADS,HEADS}} = 0.36$

12. Two Flips 5

$P(H) = 1$

$P(T) = 0$

Truth Table

Flip-1	Flip-2	1
H	H	1
H	T	0
T	H	0
T	T	0

$\overbrace{P(H,H)}^{\text{HEADS,HEADS}} = 1$

13. One Head 1

$P(H) = 0.5$

Flip it twice we would see heads exactly once.

Truth Table

Flip-1	Flip-2	1
H	H	0.25
H	T	0.25
T	H	0.25
T	T	0.25

$P(\text{Exactly one H}) = ?$

14. One Head 2

$P(H) = 0.5$

Flip it twice we would see heads exactly once.

Truth Table

Flip-1	Flip-2	1
H	H	0.25
H	T	0.25
T	H	0.25
T	T	0.25

$P(\text{Exactly one H}) = 0.25 + 0.25 = 0.5$

15. One Of Three 1

$P(H) = 0.5$

Flip it twice we would see heads exactly once.

$P(\text{Exactly one H in 3 Flips}) = 3 * \frac{1}{8} = 0.375$

Truth Table

Flip-1	Flip-2	Flip-2	1
H	H	H	$\frac{1}{8}$
H	H	T	$\frac{1}{8}$
H	T	H	$\frac{1}{8}$
H	T	T	$\frac{1}{8}$
T	T	T	$\frac{1}{8}$
T	T	H	$\frac{1}{8}$
T	H	T	$\frac{1}{8}$
T	H	H	$\frac{1}{8}$

16. One Of Three 2

$P(H) = 0.6$

Flip it twice we would see heads exactly once.

$P(\text{Exactly one H in 3 Flips}) = 3 * \frac{36}{125} = 0.288$

Truth Table

Flip-1	Flip-2	Flip-2	1
H	H	H	$\frac{27}{125}$
H	H	T	$\frac{18}{125}$
H	T	H	$\frac{18}{125}$
H	T	T	$\frac{12}{125}$
T	T	T	$\frac{8}{125}$
T	T	H	$\frac{12}{125}$
T	H	T	$\frac{12}{125}$
T	H	H	$\frac{18}{125}$

17. Even Roll

Now I am throwing dice. The difference between dice and coins is that there are now 6 possible outcomes. Let me just draw them, and say it's a fair die, which means each of the different sides comes up with a probability over 6 for any of the numbers you can plug in over here.

Fair Die: $P() = \frac{1}{6}$

What do you think the probability is the die comes up with an even number?

Truth Table

Die	1
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	$\frac{1}{6}$
4	$\frac{1}{6}$
5	$\frac{1}{6}$
6	$\frac{1}{6}$

$P(DIE = EVEN) = 3 * \frac{1}{6} = \frac{1}{2}$

18. Doubles

Throw a fair die twice!

Truth Table

Throw-1	Throw-2	1
1	1	$\frac{1}{36}$
2	2	$\frac{1}{36}$
3	3	$\frac{1}{36}$
4	4	$\frac{1}{36}$
5	5	$\frac{1}{36}$
6	6	$\frac{1}{36}$

$P(DOUBLE) = 6 * \frac{1}{36} = \frac{1}{6}$

19. Probability Conclusion

Summary

- Probablity of event: P
- Probablity of opposite event: P
- Probablity of composite event(dependent events: $P * P * P ... * P$

20. Text: Recap + Next Steps

Probability

Here you learned some fundamental rules of probability. Using notation, we could say that the outcome of a coin flip could either be **T** or **H** for the event that the coin flips tails or heads, respectively.

Then the following rules are true:

1. $P(H) = 0.5$
2. $1 - P(H) = P() = 0.5$
   where not H is the event of anything other than heads. Since, there are **only two possible outcomes**, we have that $P(\text{not H}) = P(T) = 0.5$. In later concepts, you will see this with the following notation: $\neg H$.
3. Across **multiple coin flips**, we have the probability of seeing n heads as $P(H)^n$. This is because these events are **independent**.

We can get two **generic rules** from this:

1. **The probability of any event** must be between 0 and 1, **inclusive**.
2. The **probability of the compliment event** is 1 minus the probability of an event. That is the **probability of all other possible events** is 1 minus the probability an event itself. Therefore, the **sum of all possible events** is equal to 1.
3. If our events are **independent**, then the probability of the **string of possible events** is the **product** of those events. That is the **probability of one event AND the next AND the next event**, is the **product** of those events.

21. Appendix: Glossary

- Fair Coin(公平硬币)
- Loaded Coin(不公正硬币)