List 7

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Computer Simulations of Stochastic Processes

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Here we denote $(X_t)_{t\geq 0}$ as a stochastic process.

Problem 1 (Simulating special cases of Lévy process)**.** *Using properties of Lévy processes (especially, stationarity and independence of increments) simulate:*

- *Brownian motion (aka Wiener process);*
- *α-stable process with different sets of parameters.*

See how the change of parameters changes the outcome of your simulations.

Problem 2 (Tools for analysing processes)**.** *Implement such tools:*

- *quantile function estimator;*
- *characteristic function estimator.*

Then answer the following questions. For α-stable processes:

- *how should the characteristic function behave as a function of time t?*

  $$\varphi_{X_t}(s) = \mathbb{E}(\exp\{iX_t s\}) = \dots$$

  Is there any theorem connected with such processes (or maybe the class is even wider, such as Lévy processes?)

- *how should the quantile lines behave?*
- *what is self-similarity and how can we check it?*
- *how should behave the quantile lines of increments of such process (or, in general, of Lévy process?)*

Problem 3 (Bonus problem)**.** *We can treat finite-dimensional distributions $[X_{t_1}, X_{t_2}, \dots, X_{t_n}]$ of the process $(X_t)_{t\geq 0}$ as a vector, and thus, we can simulate such processes' values in finite number of points using multivariate methods (for vectors).*
Compare speed and accuracy of both methods.