Task list 3

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Computer Simulations of Stochastic Processes

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1. Simulate $N = 1000$ trajectories of process $\{X(t)\}$ where $t \in \{0, \Delta t, 2\Delta t, \ldots, 999\Delta t\}$ and $\Delta t = 0.01$ of length $n = 1000$. Consider the following processes:

   - Brownian motion;
   - 1.5-stable symmetric Lévy motion;
   - fractional Brownian motion with $H = 0.3$ and $H = 0.7$;
   - fractional Levy stable motion with $H = 0.3$ and $H = 0.7$ and $\alpha = 1.5$;
   - Ornstein-Uhlenbeck process;

   For these trajectories present the following:

   (a) quantile lines estimator (of order $a = 0.1, 0.3, 0.5, 0.7, 0.9$) and their comparison to appropriate power-law in the Brownian motion and α- stable case;

   (b) self-similarity. It can be done by comparing distributions

   (c) characteristic function $\varphi_{X(t)}(s) = \mathbb{E}(\exp\{\mathrm{i}X(t)s\})$ estimator. For the Brownian motion, Levy stable motion and fractional Brownian motion compare it with the analytical formulae;

   (d) Differrent kinds of MSD's: TAMSD, EAMSD, EATAMSD. Compare them;

   (e) Create confidence intervals for TAMSD and check if EAMSD lies within;

   (f) Check the distribution of your processes for some fixed times or for trajectories (whichever is applicable and has some sense).