List 6

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Computer Simulations of Stochastic Processes

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1 Codifference as a measure in infinite variance case

Let us define the codiffence between random variables X and Y in the following way:

$$\tau_{X,Y} = \log\left(\mathbb{E}\exp(i(X-Y))\right) - \log\left(\mathbb{E}\exp(iX)\right) - \log\left(\mathbb{E}\exp(-iY)\right). \tag{1}$$

Formula (1) can be considered as a linear combination that includes log values of $X-Y$, X and $-Y$'s characteristic functions.

Problem 1. *Implement the code for a codifference measure given by formula (1). Use the standard estimator for characteristic function.*

Problem 2. *Implement Cholesky method of simulation of multivariate normal distribution for given expectation vector and covariance matrix.*

Problem 3. *Let us consider a normal random vector* $\mathbf{Z} = [X, Y]'$ *with mean and covariance matrix*

$$\mu_{\mathbf{Z}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \textit{and} \qquad Cov_{\mathbf{Z}} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \tag{2}$$

Calculate the codifference for marginals of vector $\mathbf{Z}$*. Compare its value with the covariance between the marginals of* $\mathbf{Z}$*.*

Problem 4. *Verify that the codifference is symmetrical in case of model in Problem 3, that is* $\tau_{X,Y} = \tau_{Y,X}$*.*

Problem 5. *Calculate the codifference and covariance for marginals of two-dimensional* α*-stable vector for different* $\alpha \in (0, 2]$ *and the spectral measure as follows:*

a) indepedent symmetric marginals:

- $\gamma_1 = 0.25$ *at* $\mathbf{s}_1 = (1, 0)$,
- $\gamma_2 = 0.25$ *at* $\mathbf{s}_2 = (0, 1)$,
- $\gamma_3 = 0.25$ *at* $\mathbf{s}_3 = (-1, 0)$,

- $\gamma_4 = 0.25$ *at* $\mathbf{s}_4 = (0, -1)$.

b) symmetric spectral measure:

- $\gamma_1 = 0.25$ *at* $\mathbf{s}_1 = (\sqrt{2}/2, \sqrt{2}/2)$,
- $\gamma_2 = 0.25$ *at* $\mathbf{s}_2 = (-\sqrt{2}/2, \sqrt{2}/2)$,
- $\gamma_3 = 0.25$ *at* $\mathbf{s}_3 = (-\sqrt{2}/2, -\sqrt{2}/2)$,
- $\gamma_4 = 0.25$ *at* $\mathbf{s}_4 = (\sqrt{2}/2, -\sqrt{2}/2)$.