Task list 1

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Computer Simulations of Stochastic Processes

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1. Write a function simulating α-stable distribution in different parameterizations, i.e. $\mathbf{S}(\alpha, \beta, \gamma, \delta; k)$ for $k \in \{0, 1\}$. [This relates to problem 2 on list 2.]

2. (ECDF) Given a sample, write a function/script that

   (a) calculates and plots its empirical cumulative distribution function

   (b) calculates and plots its tail of empirical cumulative distribution function

   (c) compares its ecdf to a given function or distribution (e.g. exponential tails $F(t) \sim e^{-\lambda|t|}$ or power-law tails $F(t) \sim c|t|^{-\alpha}$ for $t \to -\infty$).

   (d) compares its tail of ecdf to tail of a given function or distribution (e.g. exponential tails $1 - F(t) \sim e^{-\lambda t}$ or power-law tails $1 - F(t) \sim ct^{-\alpha}$ for $t \to \infty$).

   (e) presents all of the plots in a proper scale (x-log, y-log or log-log).

3. (Characteristic function) Given a sample, write a function/script that

   (a) calculates and plots its empirical characteristic function

   (b) compares its empirical characteristic function to a given characteristic function

4. (Divergence of moments) Given a sample from stable distribution, present a running moment plot, i.e. for sample $[x_1, x_2, \ldots, x_N]$ running τ'th moment would be

$$y_n(\tau) \stackrel{def}{=} \frac{1}{n} \sum_{k=1}^{n} x_k^{\tau}, \quad n = 1, \ldots, N$$

5. (Histogram) Given a sample, write a function/script that

   (a) calculates and plots its normalized histogram

   (b) compares it to a given probability density