List 1

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Computer Simulations of Stochastic Processes

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Theorem 1 (Central Limit Theorem)**.** *What are the assumptions, what is the thesis?*

For the following exercises, to calculate cumulative distribution function (CDF), density function (PDF) and characteristic function use Monte Carlo simulations.

Exercise 1 (Rule of 12)**.** *Compare CDF's, PDF's and characteristic functions of*

$$Z = \frac{\sum_{k=1}^{12} U_k - 12\mathbb{E}(U_1)}{\sqrt{12}\sqrt{\mathrm{Var}U_1}},$$

where $\{U_k\}_{k=1}^{12}$ *have standard uniform distribution, and standard normal variables.*

Exercise 2 (CLT for exponential distribution)**.** *Compare CDF's, PDF's and characteristic functions of*

$$Z = \frac{\sum_{k=1}^{n} X_k - n\mathbb{E}(X_1)}{\sqrt{n}\sqrt{\mathrm{Var}X_1}},$$

where $\{X_k\}_{k=1}^{n}$ *are i.i.d exponential distribution (with* λ *of your choosing), and standard normal variables. Use different* n*'s.*

Exercise 3 (CLT (or lack thereof) for Pareto distribution)**.** *Pareto distribution has CDF:*

$$1 - F(t) = \left(\frac{\lambda}{\lambda + t}\right)^{\alpha}, \quad t \geq 0,$$

for $\lambda > 0, \alpha > 0$.
Compare CDFs, PDFs and characteristic functions of

$$Z = \frac{\sum_{k=1}^{n} X_k - n\mathbb{E}(X_1)}{\sqrt{n}\sqrt{\mathrm{Var}X_1}},$$

where $\{X_k\}_{k=1}^{n}$ *have Pareto distribution (with* $\lambda = 1$ *and* $\alpha > 2$ *of your choosing), and standard normal variables. Use different* n*'s.*
Furthermore, check the behavior of the distribution of

$$Z = \frac{\sum_{k=1}^{n} X_k - n\mathbb{E}(X_1)}{\sqrt{n}},$$

where $\{X_k\}_{k=1}^n$ *have Pareto distribution (with* $\lambda = 1$ *and* $1 < \alpha < 2$ *of your choosing).*
Hint: *To simulate Pareto (or any other distribution with easily inversible CDF) you can use inverse transformation method: to simulate random variable* X *with CDF* F_X *you can simulate standard uniform random variable and plug it in the inverse of the* F_X*.*

$$F_X^{-1}(U) \sim X \quad \text{for } U \sim \mathcal{U}(0,1).$$

Homework 1 (Stable program)**.** *Investigate "stable.exe" on Nolan's webpage.*