List 12

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Computer Simulations of Stochastic Processes

06.2022

For a stochastic process $(X_t)_{t\geq 0}$ starting in $x \in \mathbb{R}$, let us define the exit time from interval (a, b) $(a \leq x \leq b)$ as

$$\tau^x_{(a,b)} \stackrel{def}{=} \inf\{t \geq 0 : X_t \notin (a, b)\}. \tag{1}$$

Problem 1. For $x \in (a, b)$, estimate the expected time to exit

$$x \mapsto \mathbb{E}\tau^x_{(a,b)}$$

for the following processes:

a) Brownian motion

b) fractional Brownian motion with $H = 0.3, 0.7$

c) Lévy stable motion with $\alpha \in \{0.5, 1, 1.5, 2\}$ and $\beta \in \{-1, -0.5, 0, 0.5, 1\}$.

Hint First focus on symmetric processes.

Problem 2. Calculate the probability that the exit happened through b, i.e.

$$\mathbb{P}\left(X_{\tau^x_{(a,b)}} \geq b\right),$$

(again, for various $x \in (a, b)$).

Problem 3. (might be hard) Fit the appropriate function to $x \mapsto \mathbb{E}\tau^x_{(a,b)}$ and find the appropriate distribution for $x \mapsto \mathbb{P}\left(X_{\tau^x_{(a,b)}} \geq b\right)$