On a Formal Model of Safe and Scalable Self-driving Cars

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Abstract

In recent years, car makers and tech companies have been racing towards self driving cars. It seems that the main parameter in this race is who will have the first car on the road. The goal of this paper is to add to the equation two additional crucial parameters. The first is standardization of safety assurance — what are the minimal requirements that every self-driving car must satisfy, and how can we verify these requirements. The second parameter is scalability — engineering solutions that lead to unleashed costs will not scale to millions of cars, which will push interest in this field into a niche academic corner, and drive the entire field into a "winter of autonomous driving". In the first part of the paper we propose a white-box, interpretable, mathematical model for safety assurance. In the second part we describe a design of a system that adheres to our safety assurance requirements and is scalable to millions of cars.

1 Introduction

The "Winter of AI" is commonly known as the decades long period of inactivity following the collapse of Artificial Intelligence research that over-reached its goals and hyped its promise until the inevitable fall during the early 80s. We believe that the development of Autonomous Vehicles (AV) is dangerously moving along a similar path that might end in great disappointment after which further progress will come to a halt for many years to come.

The challenges posed by most current approaches are centered around lack of safety guarantees, and lack of scalability. Consider the issue of guaranteeing a multi-agent safe driving ("Safety"). Given that society will unlikely tolerate road accident fatalities caused by machines, guarantee of Safety is paramount to the acceptance of AV. Ultimately, our desire is to guarantee zero accidents, but this is impossible since multiple agents are typically involved in an accident and one can easily envision situations where an accident occurs solely due to the blame of other agents (see Fig. 1 for illustration). In light of this, the typical response of practitioners of AV is to resort to a statistical data-driven approach where Safety validation becomes tighter as more mileage is collected.

To appreciate the problematic nature of a data-driven approach to Safety, consider first that the probability of a fatality caused by an accident per one hour of (human) driving is known to be 10^{-6}. It is reasonable to assume that for society to accept machines to replace humans in the task of driving, the fatality rate should be reduced by three orders of magnitude, namely a probability of 10^{-9} per hour[1]. In this regard, attempts to guarantee Safety using a data-driven statistical approach, claiming increasing superiority as more mileage is driven, are naive at best. The amount of data required to guarantee a probability of 10^{-9} fatality per hour of driving is proportional to its inverse, 10^9 hours of data (see details in the sequel), which is roughly in the order of thirty billion miles. Moreover, a multi-agent system interacts with its environment and thus cannot be validated offline[2], thus any change to the software of planning and control will require a new data collection of the same magnitude — clearly unwieldy. Finally, developing a system through data invariably suffers from lack of interpretability and explainability of the actions being taken — if an AV kills someone, we need to know the reason. Consequently, a model-based approach to Safety is required but the existing "functional safety" and ASIL requirements in the automotive industry are not designed to cope with multi-agent environments. Hence the need for a formal model of Safety which is one of the goals of this paper.

[1] This estimate is inspired from the fatality rate of air bags and from aviation standards. In particular, 10^{-9} is the probability that a wing will spontaneously detach from the aircraft in mid air.

[2] unless a realistic simulator emulating real human driving with all its richness and complexities such as reckless driving is available, but the problem of validating the simulator is even harder than creating a Safe AV agent — see Appendix C.

The second area of risk lies with lack of scalability. The difference between AV and other great science and technology achievements of the past is that as a "science project" the effort is not sustainable and will eventually lose steam. The premise underlying AV goes beyond "building a better world" and instead is based on the premise that mobility without a driver can be sustained at a lower cost than with a driver. This premise is invariably coupled with the notion of scalability — in the sense of supporting mass production of AVs (in the millions) and more importantly of supporting a negligible incremental cost to enable driving in a new city. Therefore the cost of computing and sensing does matter, if AV is to be mass manufactured, the cost of validation and the ability to drive "everywhere" rather than in a select few cities is also a necessary requirement to sustain a business.

The issue with most current approaches is centered around a "brute force" state of mind along three axes: (i) the required "computing density", (ii) the way high-definition maps are defined and created, and (iii) the required specification from sensors. A brute-force approach goes against scalability and shifts the weight towards a future in which unlimited on-board computing is ubiquitous, somehow the cost of building and maintaining HD-maps becomes negligible and scalable, and that exotic super advanced sensors would be developed, productized to automotive grade, and at a negligible cost. A future for which any of the above holds is indeed plausible but having all of the above hold becomes a low-probability event. The combined issues of Safety and Scalability contain the risk of "Winter of AV". The goal of this paper is to provide a formal model of how Safety and Scalability are pieced together into an AV program that society can accept and is scalable in the sense of supporting millions of cars driving anywhere in the developed countries.

The contribution of this paper is twofold. On the Safety front we introduce a model called "Responsibility Sensitive Safety" (RSS) which formalizes the notion of "accident blame", is interpretable and explainable, and incorporates a sense of "responsibility" into the actions of a robotic agent. The definition of RSS is agnostic to the manner in which it is implemented — which is a key feature to facilitate our goal of creating a convincing global safety model. RSS is motivated by the observation (as highlighted in Fig. 1) that agents play a non-symmetrical role in an accident where typically only one of the agents is responsible for the accident and therefore is to be blamed for it. The RSS model also includes a formal treatment of "cautious driving" under limited sensing conditions where not all agents are always visible (due to occlusions of kids behind a parking vehicle, for example). Our ultimate goal is to guarantee that an agent will never make an accident of its "blame". Clearly, a model is useful only if it comes with an efficient Policy[3] that complies with RSS — in particular an action that looks innocent at the current moment might lead to a catastrophic event in the far future ("butterfly effect"). We prove that our definition of RSS is useful by constructing a set of local constraints on the short-term future that guarantees Safety for the entire future.

Our second contribution evolves around the introduction of a "semantic" language that consists of units, measurements, and action space, and specification as to how they are incorporated into Planning, Sensing and Actuation of the AV. To get a sense of what we mean by Semantics, consider how a human taking driving lessons is instructed to think about "driving policy". These instructions are not geometric — they do not take the form "drive 13.7 meters at the current speed and then accelerate at a rate of 0.8 m/s^2". Instead, the instructions are of a semantic nature — "follow the car in front of you" or "overtake that car on your left". The language of human driving policy is about longitudinal and lateral goals rather than through geometric units of acceleration vectors. We develop a formal Semantic language and show that the Semantic model is crucial on multiple fronts connected to the computational complexity of Planning that do not scale up exponentially with time and number of agents, to the manner in which Safety and Comfort interact, to the way the computation of sensing is defined and the specification of sensor modalities and how they interact in a fusion methodology. We show how the resulting fusion methodology (based on the semantic langauge) guarantees the RSS model to the required 10^{-9} probability of fatality, per one hour of driving, while performing only offline validation over a dataset of the order of 10^5 hours of driving data.

Specifically, we show that in a reinforcement learning setting we can define the Q function[4] over actions defined over a semantic space in which the number of trajectories to be inspected at any given time is bounded by 10^4 regardless of the time horizon used for Planning. Moreover, the signal to noise ratio in this space is high, allowing for effective machine learning approaches to succeed in modeling the Q function. In the case of computation of sensing, Semantics allow to distinguish between mistakes that affect Safety versus those mistakes that affect the Comfort of driving. We

[3] a function that maps the "sensing state" to an action.

[4] A function evaluating the long term quality of performing an action $a \in A$ when the agent is at state $s \in S$. Given such a Q-function, the natural choice of an action is to pick the one with highest quality, $\pi(s) = \mathrm{argmax}_a \, Q(s, a)$.

define a PAC model[5] for sensing which is tied to the Q-function and show how measurement mistakes are incorporated into Planning in a manner that complies with RSS yet allows to optimize the comfort of driving. The language of semantics is shown to be crucial for the success of this model as other standard measures of error, such as error with respect to a global coordinate system, do not comply with the PAC sensing model. In addition, the semantic language is also a critical enabler for defining HD-maps that can be constructed using low-bandwidth sensing data and thus be constructed through crowd-sourcing and support scalability.

To summarize, we propose a formal model that covers all the important ingredients of an AV: sense, plan and act. The model guarantees that from a Planning perspective there will be no accident of the AV's blame, and also through a PAC-sensing model guarantees that, with sensing errors, a fusion methodology we present will require only offline data collection of a very reasonable magnitude to comply with our Safety model. Furthermore, the model ties together Safety and Scalability through the language of semantics, thereby providing a complete methodology for a safe and scalable AV. Finally, it is worth noting that developing an accepted safety model that would be adopted by the industry and regulatory bodies is a necessary condition for the success of AV — and it is better to do it earlier rather than later. An early adoption of a safety model will enable the industry to focus resources along a path that will lead to acceptance of AV. Our RSS model contains parameters whose values need to be determined through discussion with regulatory bodies and it would serve everyone if this discussion happens early in the process of developing AV solutions.

1.1 Outline

We follow the classic sense-plan-act robotic control methodology. The sensing system is responsible for understanding the present state of the environment. The planning part, which we call "driving policy", is responsible for figuring out what is the best next move (a "what will happen if" type of reasoning). The acting part is responsible for implementing the plan. The focus of the paper is on the sensing and planning parts (since the acting part is by and large well understood by control theory).

Mistakes that might lead to accidents can stem from sensing errors or planning errors. Planning is a multi-agent game, as there are other road users (humans and machines) that react to our actions. Section 2 deals with safety guarantees for the planning part, which we call multi-agent safety. We formally show that statistical estimation of the probability of planning errors must be done "online", namely, after every update of the software we must drive billions of miles with the new version. This is clearly infeasible. As an alternative, we propose a formal mathematical model for multi-agent safety which we call Responsibility Sensitive Safety (RSS). This model gives a 100% guarantee that the planning module will not make mistakes of the AV's blame (the notion of "blame" is formally defined). Such a model is useless without an efficient way to validate that a certain driving policy adheres to it. In Section 3 we accompany the RSS definitions with computationally efficient methods to validate them.

Mistakes of the sensing system are easier to validate, since sensing can be independent[6] of the vehicle actions, and therefore we can validate the probability of a severe sensing error using "offline" data. But, even collecting offline data of more than 10^9 hours of driving is challenging. In Section 5.2, as part of a description of our sensing system, we present a fusion approach that can be validated using a significantly smaller amount of data.

The rest of the sections deal with Scalability. We outline a complete system that is safe and can scale to millions of cars. In Section 4 we describe our driving policy, starting from an explanation of why existing methods are so computationally demanding, and then showing how our semantic-based approach leads to a computationally efficient driving policy. In Section 5 we connect our semantic driving policy to semantic requirements from the sensing system, showing how it leads to sensing and mapping requirements that can scale to millions of cars in today's technology.

2 Multi-agent Safety

We start off by formalizing our arguments with regard to the necessity of a thorough safety definition, a minimal standard to which AV systems must abide. In the following technical lemma, we formally show why a statistical approach to validation of an AV system is infeasible, even for validating a simple claim such as "the system makes N

[5] Probably Approximate Correct (PAC), borrowing Valiant's PAC-learning terminology.

[6] Strictly speaking, the vehicle actions might change the distribution over the way we view the environment. However, this dependency can be rather easily circumvented by data augmentation techniques.

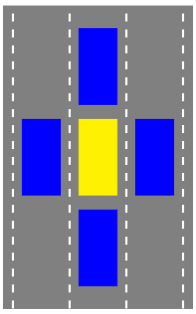


Figure 1: The central car can do nothing to ensure absolute safety.

accidents per hour". This implies that a model-based safety definition is the only feasible tool for validating an AV system.

Lemma 1 *Let X be a probability space, and A be an event for which $\Pr(A) = p_1 < 0.1$. Assume we sample $m = \frac{1}{p_1}$ i.i.d. samples from X, and let $Z = \sum_{i=1}^{m} \mathbf{1}_{[x \in A]}$. Then*

$$\Pr(Z = 0) \geq e^{-2}.$$

Proof We use the inequality $1 - x \geq e^{-2x}$ (proven for completeness in Appendix A.1), to get

$$\Pr(Z = 0) = (1 - p_1)^m \geq e^{-2p_1 m} = e^{-2}.$$

■

Corollary 1 *Assume an AV system AV_1 makes an accident with small yet insufficient probability p_1. Any deterministic validation procedure which is given $1/p_1$ samples, will, with constant probability, not distinguish between AV_1 and a different AV system AV_0 which never makes accidents.*

In order to gain perspective over the typical values for such probabilities, assume we desire an accident probability of 10^{-9} per hour, and a certain AV system provides only 10^{-8} probability. Even if we obtain 10^8 hours of driving, there is constant probability that our validation process will not be able to tell us that the system is dangerous.

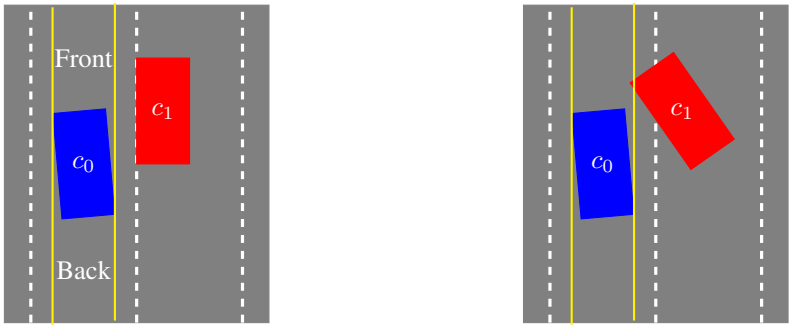
Finally, note that this difficulty is for invalidating a single, specific, dangerous AV system. A full *solution* cannot be a viewed as a single system, as new versions, bug fixes, and updates will be necessary. Each change, even of a single line of code, generates a *new system* from a validator's perspective. Thus, a solution which is validated statistically, must do so online, over new samples after every small fix or change, to account for the shift in the distribution of states observed and arrived-at by the new system. Repeatedly and systematically obtaining such a huge number of samples (and even then, with constant probability, failing to validate the system), is infeasible.

We further add to the discussion the fact that any statistical claim must be formalized in order to be measured. Claiming a statistical property over the number of accidents a system makes is significantly weaker than claiming "it drives in a safe manner". In order to say that, one must formally define what is safety.

Lastly, we stress that the "implementation free" approach which we are adopting in this section, not concerning ourselves with *how* to ensure the given safety definitions, is key to achieving our goal of creating a convincing, global safety model. The problem of implementing a scalable safe system is dealt with in the next sections.

2.1 Absolute Safety is Impossible

We begin with a naive attempt at defining a safe action-taking by a car, and immediately rule it out as infeasible. We say an action a taken by a car c is *absolutely safe* if no accident can follow it at some future time. It is easy to see that



(a) (b)

Figure 2: 2a: $t = 0$. The front and back corridors of c_0. 2b: $t = 1$. c_1 has cut-in to c_0's front corridor.

it is impossible to achieve absolute safety, by observing simple driving scenarios, for example, as depicted in Figure 1: from the central car's perspective, no action can ensure that none of the surrounding cars will crash into it, and no action can help it escape this potentially dangerous situation. We emphasize that solving this problem by forbidding the autonomous car from being in such situations is completely impossible — every highway with more than 2 lanes will lead to it and forbidding this scenario amounts to staying in the garage.

The implications might seem, at first glance, disappointing. Nothing is absolutely safe. However, we claim that this requirement is too harsh, as evident by the fact that humans do not get even close to following absolute safety. Instead, humans follow a safety notion that depends on responsibility.

2.2 Responsibility-Sensitive Safety

A crucial aspect missing from the absolute safety concept is the non-symmetry of most accidents - it is usually one of the drivers who is responsible for a crash, and is to be blamed. Clearly, in the example we consider in Figure 1, the central car is not to be blamed if the left car, for example, suddenly drives into it. We'd like to formalize the fact that considering its lack of responsibility, a behaviour of staying in its own lane can be considered safe. In order to do that, we develop a formal concept of "accident blame", which, we argue, captures the common sense behind safe driving.

Let us gain some intuition with an informal examination of one of the most basic responsibility concepts we will later formally develop. Consider the simple case of two cars c_f, c_r, driving at the same speed, one behind the other, along a straight road. Assume c_f, the car at the front, suddenly brakes because of an obstacle appearing on the road, and manages to avoid it. Unfortunately, c_r did not keep enough of a distance from c_f, is not able to respond in time, and crashes into c_f's rear side. It is clear that the blame is on c_r; it is the responsibility of the rear car to keep safe distance from the front car, and to be ready for unexpected, yet reasonable, braking.

We now move on to formally consider a much broader family of scenarios: driving in a multi-lane road, where cars can freely change lanes, cut into other cars' paths, drive at different speeds, and so on. To simplify the following discussion, we assume a straight road on a planar surface, where the lateral, longitudinal axes are the x, y axes, respectively. This can be achieved, under mild conditions, by defining a homomorphism between the actual curved road and a straight road. See Appendix D. We also consider a discrete time space. We start off with definitions aiding in distinction between two intuitively different sets of cases: simple ones, where no significant lateral manoeuvre is performed, and more complex ones, involving lateral movement.

Definition 1 (Car Corridor) *The* Corridor *of a car c is the range $[c_{x,left}, c_{x,right}] \times [\pm\infty]$, where $c_{x,left}, c_{x,right}$ are the positions of the leftmost, rightmost corners of c.*

Definition 2 (Cut-in) *A car c_1* Cuts-in *to car c_0's corridor at time t if it did not intersect c_0's corridor at time $t-1$, and does intersect it at time t.*

We may make the further distinction between front/back parts of the corridor. We will also use the term "the direction of a cut-in" to describe movement in the direction of the relevant corridor boundary. See Figure 2 for an illustration. We will use these definitions to define cases with lateral movement. For the simple case where there is no such occurence, such as the simple case of a car following another, we define the safe longitudinal distance:

Definition 3 (Safe longitudinal distance) *A* longitudinal distance *between a car c_r and another car c_f that is in c_r's frontal corridor is* safe *w.r.t. a response time ρ if for any braking command a, $|a| < a_{\max,\text{brake}}$, performed by c_f, if c_r will apply its maximal brake from time ρ until a full stop then it won't collide with c_f.*

Lemma 2 below calculates d as a function of the velocities of c_r, c_f, the response time ρ, and the maximal acceleration $a_{\max,\text{brake}}$. Both ρ and $a_{\max,\text{brake}}$ are constants, which should be determined to some reasonable values by regulation.

Lemma 2 *Let c_r be a vehicle which is behind c_f on the longitudinal axis. Let $a_{\max,\text{brake}}, a_{\max,\text{accel}}$ be the maximal braking and acceleration commands, and let ρ be c_r's response time. Let v_r, v_f be the longitudinal velocities of the cars, and let l_f, l_r be their lengths. Define $v_{\rho,\max} = v_r + \rho \cdot a_{\max,\text{accel}}$, and define $T_r = \rho + \frac{v_{\rho,\max}}{a_{\max,\text{brake}}}$ and $T_f = \frac{v_f}{a_{\max,\text{brake}}}$. Let $L = (l_r + l_f)/2$. Then, the minimal safe longitudinal distance for c_r is:*

$$d_{\min} = \begin{cases} L & \text{if } T_r \le T_f \\ L + T_f\left[(v_{\rho,\max} - v_f) + \rho a_{\max,\text{brake}}\right] - \frac{\rho^2 a_{\max,\text{brake}}}{2} + \frac{(T_r - T_f)(v_{\rho,\max} - (T_f - \rho)a_{\max,\text{brake}})}{2} & \text{otherwise} \end{cases}$$

Proof Let d_t be the distance at time t. To prevent an accident, we must have that $d_t > L$ for every t. To construct $d_{\min}$ we need to find the tightest needed lower bound on d_0. Clearly, d_0 must be at least L. As long as the two cars didn't stop after $T \ge \rho$ seconds, the velocity of the preceding car will be $v_f - T\, a_{\max,\text{brake}}$ while c_r's velocity will be upper bounded by $v_{\rho,\max} - (T - \rho)\, a_{\max,\text{accel}}$. So, the distance between the cars after T seconds will be lower bounded by:

$$\begin{aligned} d_T &:= d_0 + \frac{T}{2}\left(2v_f - T\, a_{\max,\text{brake}}\right) - \left[\rho\, v_{\rho,\max} + \frac{T-\rho}{2}\left(2v_{\rho,\max} - (T-\rho)a_{\max,\text{brake}}\right)\right] \\ &= d_0 + T\left[(v_f - v_{\rho,\max}) - \rho a_{\max,\text{brake}}\right] + \frac{\rho^2 a_{\max,\text{brake}}}{2}\,. \end{aligned}$$

Note that T_r is the time on which c_r arrives to a full stop (a velocity of 0) and T_f is the time on which the other vehicle arrives to a full stop. Note that $a_{\max,\text{brake}}(T_r - T_f) = v_{\rho,\max} - v_f + \rho a_{\max,\text{brake}}$, so if $T_r \le T_f$ it suffices to require that $d_0 > L$. If $T_r > T_f$ then

$$d_{T_r} = d_0 + T_f\left[(v_f - v_{\rho,\max}) - \rho a_{\max,\text{brake}}\right] + \frac{\rho^2 a_{\max,\text{brake}}}{2} - \frac{(T_r - T_f)(v_{\rho,\max} - (T_f - \rho)a_{\max,\text{brake}})}{2}\,.$$

Requiring $d_{T_r} > L$ and rearranging terms concludes the proof. ■

Finally, we define a comparison operator which allows us to discuss comparisons with some notion of "margin": when comparing lengths, velocities and so on, it is necessary to accept very similar quantities as "equal".

Definition 4 (μ-comparison) *The μ-comparison of two numbers a, b is $a >_\mu b$ if $a > b + \mu$, $a <_\mu b$ if $a < b - \mu$ and $a =_\mu b$ if $|a - b| \le \mu$.*

All comparisons (argmin, argmax, etc.) below are μ-comparisons for some suitable μs. Assume an accident occurred between cars c_1, c_2. In order to consider who is to blame for the accident, we define the relevant moment which needs to be examined. This is some point in time which preceded the accident, and intuitively, was the "point of no return"; after it, nothing could be done to prevent the accident.

Definition 5 (Blame Time) *The* Blame Time *of an accident is the earliest time preceding the accident in which:*

- *there was an intersection between one of the cars and the other's corridor, and*

- *the longitudinal distance was not safe.*

Clearly there is such a time, since at the moment of accident, both conditions hold. We note that we can split Blame Times into two separate categories:

- Ones in which a cut-in also occurs, namely, they are the first moment of intersection of one car and the other's corridor, and it's in a non safe distance.
- Ones in which a cut-in does not occur, namely, there was intersection with the corridor already, in a safe longitudinal distance, and the distance had changed to unsafe at the Blame Time.

Definition 6 (μ-Losing by Lateral Velocity) *Assume a cut-in occurs between cars c_1, c_2. We say that c_1* μ-Loses by Lateral Velocity *in case its lateral velocity w.r.t. the direction of the cut-in is higher by μ than that of c_2.*

It should be noted that the direction of the velocity is important: For example, velocities of $-1, 1$ (both cars crashing into each other) is a tie, however if the velocities are $1, 1 + \mu/2$, the one with positive direction towards the other car is to be blamed. Intuitively, this definition will allow us to blame a car which drives laterally very fast into another.

Definition 7 ((μ_1, μ_2)-Winning by Lateral Position) *Assume a cut-in occurs between cars c_1, c_2. We say that c_1* (μ_1, μ_2)-Wins by Lateral Position *in case its lateral position w.r.t. the cut-in lane's center (the center closest to the cut-in relevant corridor) is smaller than μ_1 (in absolute value), and smaller by μ_2 than that of c_2.*

Intuitively, we will not blame a car if it's very close to the lane center (μ_1), and much closer than the other car (by μ_2).

Definition 8 (Blame) *The* Blame *of an accident between cars c_1, c_2, is a function of the state at the* Blame Time*, and is defined as follows:*

- *If the Blame Time is not a cut-in time, the blame is on the rear car.*
- *If the Blame Time is also a cut-in time, the blame is on both cars, unless for one of the cars, w.l.o.g. c_1, the two following conditions hold, for some predefined μs:*
  - *It doesn't lose by Lateral Velocity,*
  - *It wins by Lateral Position.*

  In that case, c_1 is spared.

In words, if an unsafe cut-in occurs, both cars are to blame, unless one of the cars is not (significantly) laterally faster, and is (significantly) closer to the lane center. By this, we cleanly capture the desired behaviour: if following a car, keep a safe distance, and if cutting into a corridor of a car which simply drives in its own lane, do it only at a safe distance.

Remark: One may wonder if following the safety guidelines described above will lead to an extremely defensive driving. This is not the case, as empirically evident by managing to drive in complex scenario in a natural way while adhering to the responsibility-sensitive safety constraints. See for example the "double-merge" scenario as described in [5].

2.3 Dealing with Limited Sensing

After considering the highway example, let us move on to a second example from which arises a general concept which cannot be overlooked - the problem of limited sensing. A very common human response, when blamed for an accident, falls into the "but I couldn't see him" category. It is, many times, true. Human sensing capabilities are limited, sometimes because of an unaware decision to focus on a different part of the road, sometimes because of carelessness, and sometimes because of physical limitations - it is impossible to see a little kid hidden behind a parked car. Of those human limitations, advanced automatic sensing systems are only subject to the latter: 360° view of the

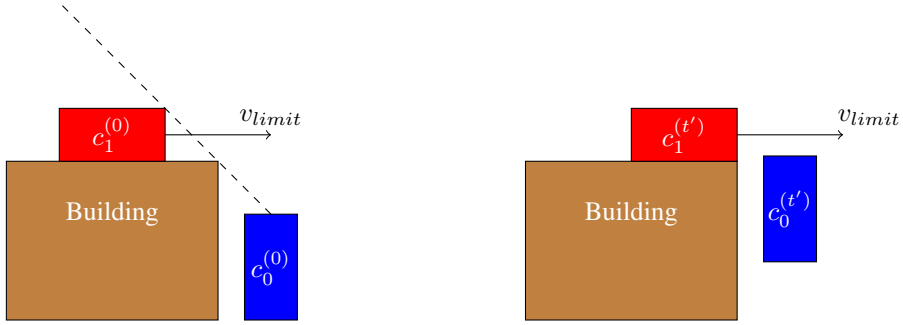


Figure 3: Worst case for a car occluded by a building. c_0 should make sure that at the cut-in time t', there's a safe distance to the "virtual" occluded car c_1.

road, along with the fact that computers are never careless, puts them above human sensing capabilities. Returning to our "but I couldn't see him" example, a fitting answer is "well, you should've been more careful". Let us formalize what is being careful w.r.t. limited sensing, a crucial component of responsibility-sensitive safety (RSS).

Consider a simple scenario, depicted in Figure 3. c_0 is trying to exit a parking lot, merging into a (possibly) busy road, but cannot see whether there are cars in the street. Let us assume that this is an urban, narrow street, with a speed limit of 30 km/h. A human driver's behaviour is to slowly merge onto the road, obtaining more and more field of view, until sensing limitations are eliminated. What assumptions does the human driver make, which allows this manoeuvre? Another significant moment in time should be defined - the first time the occluded object is exposed to us; after its exposure, we can deal with it just like any other object we can sense.

Definition 9 (Exposure Time) *The* Exposure Time *of an object is the first time in which we see it.*

Definition 10 (Blame due to Unreasonable Speed) *Assume that at the exposure time or after it, car c_1 was driving at speed $v > v_{limit}$, and c_0 wasn't doing so. Then, the blame is only on c_1. We say that c_1 is* blamed due to unreasonable speed.

This extension allows c_0 to exit the parking lot safely, in the same manner a human driver does. Using our previous responsibility-sensitive safety definitions, along with a dynamic v_{limit} definition (which uses the road conditions and speed limit, plus reasonable margins), we have that the only necessity is to check whether in the worst case, as illustrated in the figure, the cut-in is in a safe longitudinal distance, while assuming that c_1 will not exceed v_{limit}. Intuitively, this encourages c_0 to drive slower and further from the occluder, thus slowly increasing its field of view and later allowing for safe merging into the street.

Having extended the blame definition to this basic case of limited sensing, we offer a family of extensions for similar cases. Simple assumptions as to **what** can be occluded (a potentially fast car cannot be occluded between two closely parked cars, but a kid can), and what is the **worst case** manoeuvre it can perform (a kid's v_{limit} is much smaller than that of a car), imply restrictions on driving - we must be prepared for the worse, and have the ability to respond if suddenly, the exposure time comes. A more elaborate example, in an urban scenario, can be taken from the scenario of a pedestrian which is possibly occluded by a parked car. Let us define accident blame for accidents with a pedestrian:

Definition 11 (Accident-with-Pedestrian Blame) *The* Accident-with-Pedestrian Blame *is always on the car, unless one of the following three holds:*

- *the car hits the pedestrian with the car's side, and the lateral velocity of the car is smaller than μ, w.r.t. the direction of the hit.*
- *the pedestrian's velocity at the exposure time or later was larger than v_{limit}.*
- *the car is at complete stop.*

Informally, the car is not to blame only if a pedestrian runs into its side, while the car does not ride faster than μ into the pedestrian, or if the car is at stop, or if the pedestrian was running super-humanly fast, in some direction, not necessarily the hitting direction.

2.4 Utopia is Possible

To wrap up the discussion, consider a utopic future, where all cars (and other road users) somehow are able to successfully verify that they will never be blamed for an accident. Since by definition, for every accident there is at least one car to blame, the meaning is that *there will never be any accidents*, leading us to a utopic future of absolute safety.

2.5 A complete responsibility-sensitive safety definition

Not all roads are of a simple structure. Some, like junctions and roundabouts, contain more complex situations, along with various right of way rules. Not all occluded objects are cars or pedestrians, with bicycles and motorcycles all legitimate road users to be considered. The principles introduced in this section can be extended to these additional cases. We leave the formal extensions to a longer version of this manuscript.

3 Efficiently Validated Conditions for Responsibility-Sensitive Safety

In Section 2, we have completely ignored the implementation aspect of RSS. Of course, those definitions will be useless if there's no efficient way of successfully verifying that we will never be blamed for an accident. To appreciate the difficulty, an action that is performed now may have a butterfly effect, that will lead to a chain of events with an accident after 10 minutes of driving. A "brute-force" approach that will check all possible future outcomes is, of course, impossible. It is therefore crucial to accompany the responsibility-sensitive safety definitions with computationally efficient methods to validate them.

3.1 Computationally Feasible Safety Verification

The main mathematical tool for computationally feasible verification is "induction". To prove a claim by induction, one begins with proving the claim for simple cases, and then, each induction step extends the proof to more and more involved cases. To illustrate how this induction tool can be helpful for safety verification, consider again the simple example of a car c_r following another, c_f. We will apply the following constraint on the policy of c_r. At each time step t, the policy can pick any acceleration command such that even if c_f will apply a deceleration of $-a_{\max}$, the resulting distance between c_r and c_f at the next time step will be at least the safe longitudinal distance (defined in Definition 3 and Lemma 2). If no such action exists, c_r must apply the deceleration $-a_{\max}$. The following lemma uses induction to prove that any policy that adheres to the above constraints will never make an accident with c_f.

Lemma 3 *Under the assumptions given in Definition 3, if the policy of c_r adheres to the constraints given above it will never make an accident with c_f.*

Proof The proof is by induction. For the induction base we start with an initial state in which the distance between the two cars is safe (according to Lemma 2). The induction step is as follows. Consider the distance between c_r and c_f at some time t. If there is an action that results in a safe distance (even with c_f making maximal deceleration), we are fine. If all actions cannot guarantee safe distance, let $t' < t$ be the maximal time in which we took an action which was not maximal deceleration. By the induction hypothesis, at time $t' + 1$, we were in a safe distance, and from there on we performed maximal deceleration. Hence, by the definition of safe distance, we did not crash from time t' till now, which concludes the proof. ■

The above example demonstrates a more general idea: there's some emergency manoeuvre which can be performed by c_r in an extreme case, and lead it back to a "safe state". It should be noted that the constraints on the policy we have described above depend on just one future time step, hence it can be verified in a computationally efficient manner.

In order to generalize those ideas of sufficient local properties for RSS, we firstly define a Default Emergency Policy, and use it as a building block for defining a local property of action-taking which we call "cautious". We then show that taking only cautious commands is sufficient for RSS.

Definition 12 (Default Emergency Policy) *The* Default Emergency Policy *(DEP) is to apply maximum braking power, and maximum heading change towards* 0 *heading w.r.t. the lane.*

Remark: The maximum braking power and heading change are derived from physical parameters of the car (and maybe also from weather and road conditions).

Definition 13 (Safe state) *A state* s *is safe if performing DEP starting from it will not lead to an accident of our blame.*

As in the simple case of a car following another, we define a command to be cautious if it leads to a safe state.

Definition 14 (Cautious command) *Suppose we are currently at state* s_0*. A command* a *is cautious if the next state,* s_1*, will be safe with respect to a set* A *of possible commands that other vehicles might perform now.*

Remark: The above definition depends on the worst-case commands, in the set A, other vehicles might perform. We will construct the set A based on reasonable upper bounds on maximum braking/acceleration and lateral movements.

The following theorem proves, by induction again, that if we only issue cautious commands then there will be no accidents of our blame.

Theorem 1 *Assume that in time* 0*,* c *is in a safe state, and for every time step,* c *only issues cautious commands, where if no cautious command exists at some time step,* c *applies DEP. Then,* c *will never make accidents of its blame.*

Proof By induction. The base of the induction follows from the definition of a safe state and step from the defintion of a cautious command. ■

The main benefit of this approach is that there is no need to check infinite future, as we can quickly return to a safe state, and continue safely from there. Moreover, given the fact we will plan again at $t + 1$, and hence be able to perform DEP then if necessary, we should only check the command we are giving at time t, and not a possible longer plan we might have in mind - we can change that plan at $t + 1$. We note that now, incorporating a learning component in a system, when it is verified at run time by this transparent model, is made possible. Finally, this local verification implies full future RSS, which is our desired goal. An implementation obstacle is that the cautiousness definition involves all trajectories another agent can perform until t_{brake}, which, even for moderate t_{brake}, is a huge space. To tackle this, we next turn to develop an efficiently computable way to verify cautiousness, and hence RSS, in a scalable manner.

3.1.1 Efficient cautiousness verification

A first trivial observation is that a state is not safe if and only if there exists a specific vehicle, $\tilde{c}$, which can perform commands from the set A which lead to an accident of our blame while we execute the DEP. Therefore, from now on, we focus on a scene with a single target car, denoted $\tilde{c}$, and in the general case, we will execute the procedure sequentially, for each of the other vehicles in the scene.

When considering a single target car, an action a is not cautious if and only if there is a sequence of commands for $\tilde{c}$, denoted $\tilde{a}_1, \ldots, \tilde{a}_{t_{\text{brake}}}$, all in the set A, that results in an accident of c's blame. As we have already proved, if at time 0, it holds that $\tilde{c}$ is in the frontal corridor of c, there is a simple way to check the cautiousness of a — we just need to verify that even if $\tilde{c}$ will apply maximal brake for one time step (and we'll perform a), the resulting longitudinal distance will remain safe. The lemma below gives a sufficient condition for cautiousness in the more involved cases, where lateral manoeuvres are to be considered too.

Lemma 4 *Assume that at time* $T = 0$*,* $\tilde{c}$ *is not in the frontal corridor of* c*. Then, if at every* $T \in (0, t_{\text{brake}}]$*, there is no non-safe cut in of* c*'s blame, then* a *is cautious.*

Proof Suppose that a is not cautious, namely, there exists $\tilde{a}_1, \ldots, \tilde{a}_{t_{\text{brake}}}$ that leads to an accident of c's blame. Before the accident there must be a cut-in time, T. Assume first that $T > 0$. If this cut-in was at a safe longitudinal distance, then there cannot be an accident of our blame due to the fact that DEP is executed by deceleration of $-a_{\max}$ and based on the definition of safe longitudinal distance (and we assume here that the response time ρ is larger than the time resolution of steps). If the cut-in was non-safe, by the assumption of the lemma it was not of c's blame, hence the accident is also not of c's blame.

Finally, if $T \leq 0$, by the assumption of the lemma, at the moment of cutting, $\tilde{c}$ was at the back corridor of c. By induction, c performed only safe cut-ins in the past, and hence either the cut-in was safe or it was $\tilde{c}$'s blame. In both cases, the current accident is not of c's blame.

■

In light of Lemma 4, we are left with the problem of checking whether there can be a non-safe cut in of c's blame. Below we present an efficient algorithm for checking the possibility of a non-safe cut-in at time t. To validate the entire trajectory we discretize the time interval $[0, t_{\text{brake}}]$ and apply the algorithm on all time intervals (with a slightly larger value of ρ in the definition of safe distance to ensure that the discretization doesn't hurt). Let $\tilde{c}_{\text{diag}}$ be the length of a diagonal of a minimal rectangle bounding $\tilde{c}$. For each time $t \in [0, t_{\text{brake}}]$, define $c_{\text{length}}(t)$ to be the longitudinal "span" of c in time t, and let $L(t) = \frac{\tilde{c}_{\text{diag}} + c_{\text{length}}[t]}{2}$. Define $c_{\text{width}}[t]$ in similar fashion and let $W(t) = \frac{\tilde{c}_{\text{diag}} + c_{\text{width}}[t]}{2}$.

Algorithm 1: Check the possibility of a non-safe cut-in at time t

input:
$\tilde{y}[0], \tilde{v_y}[0], \tilde{x}[0], \tilde{v_x}[0]$ are longitudinal/lateral position/velocity of $\tilde{c}$ at time 0
$y[t], v_y[t], x[t], v_x[t]$ are longitudinal/lateral position/velocity of c at time t
$a_{y,\min}, a_{y,\max}$ are longitudinal acceleration boundaries
$a_{x,\max}$ is a lateral acceleration absolute value boundary
$L = L(t), W = W(t)$

check longitudinal feasibility:
let $\tilde{y}_{\min} = \tilde{y}[0] + \tilde{v_y}[0]t + \frac{1}{2}a_{y,\min}t^2$
let $\tilde{y}_{\max} = \tilde{y}[0] + \tilde{v_y}[0]t + \frac{1}{2}a_{y,\max}t^2$
if $[\tilde{y}_{\min}, \tilde{y}_{\max}] \cap [y[t] - L, y[t] + L] \neq \emptyset$
continue to **check lateral feasibility**
else if $\tilde{y}_{\min} > y[t] + L$ and $(\tilde{y}_{\min}, \tilde{v_y}[0] + a_{y,\min}t)$ is not longitudinally safe w.r.t. $(y[t], v_y[t])$
continue to **check lateral feasibility**
else if $\tilde{y}_{\max} < y[t] - L$ and $(\tilde{y}_{\max}, \tilde{v_y}[0] + a_{y,\max}t)$ is not longitudinally safe w.r.t. $(y[t], v_y[t])$
continue to **check lateral feasibility**
return "non-feasible"

check lateral feasibility:
w.l.o.g. assume $x[t] = 0$ and $\tilde{x}[0] \leq 0$, and $\tilde{v_x}[0] \geq 0$.
if a position $x[t]$ is **not** considered (μ_1, μ_2)-Winning by Lateral Position w.r.t. a position $x[t] - W$
w.l.o.g. assume $v_x[t] = -\mu$, as in Definition 6.
else
w.l.o.g. assume $v_x[t] = -2\mu$, as in Definition 6.
let $t_{\text{top}} = 0.5(t - \tilde{v_x}[0]/a_{x,\max})$
if $t_{\text{top}} < 0$
return "non-feasible"
let $x_{\max} = \tilde{x}[0] + 2(\tilde{v}[0]t_{\text{top}} + 0.5a_{x,\max}t_{\text{top}}^2) + \tilde{v_x}[0]^2/(2a_{x,\max})$
if $x_{\max} < -W$
return "non-feasible"
return "feasible"

The following theorem proves the correctness of the above algorithm.

Theorem 2 *If Algorithm 1 returns "non-feasible" then there cannot be a non-safe cut-in of the ego vehicle's blame at time t.*

To prove the theorem, we rely on the following key lemmas, which prove the correctness of the two building blocks of Algorithm 1. We start off with the longitudinal feasibility:

Lemma 5 *Under the notation of Algorithm 1, if the **check longitudinal feasibility** procedure is concluded by returning "non-feasible", then there cannot be a non-safe cut-in of the ego vehicle's blame at time t.*

Proof Ignoring the lateral aspect of a cut-in manoeuvre, we examine the mere possibility that the longitudinal distance between c and $\tilde{c}$ will be unsafe. It is clear that the positions $(\tilde{y}_{\min}, \tilde{y}_{\max})$ are bounding the position which can be attained by $\tilde{c}$ at time t. By the fact $[\tilde{y}_{\min}, \tilde{y}_{\max}] \cap [y[t] - L, y[t] + L] = \emptyset$, we obtain that any longitudinally non-safe distance which is attainable, is $\geq L$. Assume $\tilde{y}_{\min} > y[t] + L$, and assume by contradiction that a non-safe longitudinal position and velocity, denoted $\tilde{y}_{\text{bad}}[t], \tilde{v_y}_{\text{bad}}[t]$, are attainable using acceleration commands bounded by $a_{y,\min}, a_{y,\max}$. By definition of $\tilde{y}_{\min}$, we have $\tilde{y}_{\text{bad}}[t] > \tilde{y}_{\min}$, and hence the distance between the cars is larger, namely $\tilde{y}_{\text{bad}}[t] - (y[t]+L) > \tilde{y}_{\min} - (y[t]+L)$. Since $(\tilde{y}_{\min}, \tilde{v_y}[0] + a_{y,\min}t)$ is longitudinally safe w.r.t. $(y[t], v_y[t])$, by

definition of longitudinal non-safety, it follows that the attained velocity $\tilde{v_y}_{\text{bad}}[t]$ must be smaller than $\tilde{v_y}[0] + a_{y,\min}t$. However, it is clear that in order to achieve lesser speed, $\tilde{c}$ must use average acceleration which is lesser than $a_{y,\min}$ throughout the time window $[0,t]$, thus contradicting the fact that the longitudinal non-safety was attained using commands bounded by $a_{y,\min}, a_{y,\max}$. By considering a symmetrical argument for the case $\tilde{y}_{\max} < y[t] - L$, the proof is completed. ■

We now move on to the lateral feasibility.

Lemma 6 *Under the notation of Algorithm 1, if the **check lateral feasibility** procedure is concluded by returning "non-feasible", then there cannot be a non-safe cut-in of the ego vehicle's blame at time t.*

Proof Firstly, it is clear that there is no loss of generality by the assumptions $x[t] = 0$, $\tilde{x}[0] \leq 0$ and the ones regarding $v_x[t]$, by a simple change of coordinates and consideration of relative velocity. Moreover, by similar arguments it is simple to extend to the case when $\tilde{v_x}[0] \leq 0$.

Note that the positions of the cars involved in the cut-in, which in our case, are $(x[t], x[t] - W)$, imply some (μ_1, μ_2)-Winning by Lateral Position property, affecting the blame. By our assumptions over $v_x[t]$, we obtain that the maximal lateral velocity $\tilde{c}$ may use at time t, under assumption that c will be blamed, is 0: either in order to μ-"tie" lateral velocity (in the case c does not (μ_1, μ_2)-Win by Lateral Position, this can be enough in order to put the blame on it), or to μ-Win lateral velocity (in the case c does (μ_1, μ_2)-Win by Lateral Position, this is necessary in order to put the blame on it). It is left to check whether exists a manoeuvre starting at $\tilde{v_x}[0]$, ending at $\tilde{v_x}[t] = 0$, using lateral accelerations bounded by $a_{x,\max}$, with final position $\tilde{x}[t] \geq x[t] - W$. In words, a cut-in which ends at the desired lateral velocity, 0.

Recall the definition $t_{\text{top}} = 0.5(t - \tilde{v_x}[0]/a_{x,\max})$ from the algorithm. Assume $t_{\text{top}} < 0$. This implies that the time needed by $\tilde{c}$ in order to reach lateral velocity 0 when using maximal lateral acceleration, namely, $\tilde{v_x}[0]/a_{x,\max}$, is lesser than t. This implies that there is no manoeuvre which it can perform in order to reach the desired velocity in time, and hence no problematic manoeuvre exists. Therefore, if the procedure returned "non-feasible" due to $t_{\text{top}} < 0$, indeed, there is no feasibility of a non-safe cut-in of c's blame.

Consider the case $t_{\text{top}} \geq 0$. Then, the procedure returned "non-feasible" due to $x_{\max} < -W$. Let us consider a family of lateral velocity profiles for $\tilde{c}$ in the time range $[0,t]$, denoted $U = \{u_a \ : \ a > \tilde{v_x}[0]/t\}$ and parameterized by a. We define, for each a, in similar fashion to the one used in the algorithm, $t_{\text{top}}(a) := 0.5(t - \tilde{v_x}[0]/a)$. Note that $t_{\text{top}}(a) > 0$ for all $a > \tilde{v_x}[0]/t$. We now define the velocity profile u_a for all times $t' \in [0,t]$ as follows:

$$u_a(t') = \begin{cases} \tilde{v_x}[0] + a \cdot t' & t' < t_{\text{top}}(a) \\ \tilde{v_x}[0] + a \cdot (2t_{\text{top}}(a) - t') & t' \geq t_{\text{top}}(a) \end{cases}$$

First, It is very easy to see that u_a satisfies the constraints $u_a(0) = \tilde{v_x}[0]$, $u_a(t) = \tilde{v_x}[t]$. Second, it is easy to calculate the distance travelled while using u_a, as this amounts to integration of a piecewise linear function. Let us define the arrived-at position as $\tilde{x}_{u_a}$, and note that $x_{\max}$ defined in the algorithm is precisely $\tilde{x}_{u_{a_{x,\max}}}$. Third, it is easy to see that the travelled distance is monotonically increasing with a, and is unbounded. Hence, for any desired final position $x > \tilde{x}_{u_{\tilde{v_x}[0]/t}}$, there exists a value of a for which $x = \tilde{x}_{u_a}$. In particular, for $x = x[t] - W$, such a value exists, and we denote it by a_{cut}.

Observe that since $x_{\max}$, defined in the algorithm, is $< x[t] - W$, we have that $a_{\text{cut}} > a_{x,\max}$. This in not sufficient in order to show no valid manoeuvre can lead to position $\geq x[t] - W$; this is implied only for members of the family U. We now prove that even outside of U, all velocity profiles which attain a final position of $x[t] - W$ must use an acceleration value of at least a_{cut} on the way, making them invalid, and hence completing the proof.

Assume some velocity profile u satisfies the boundary constraints $u(0) = \tilde{v_x}[0]$, $u(t) = \tilde{v_x}[t]$. Moreover, assume it attains some final position $\tilde{x}_u \geq x[t] - W$. We thus obtain:

$$\int_0^t u(\tau)d\tau \geq \int_0^t u_{a_{\text{cut}}}(\tau)d\tau.$$

Assume $u \geq u_{a_{\text{cut}}}$ for all τ. In particular, $u(t_{\text{top}}(a_{\text{cut}})) \geq u_{a_{\text{cut}}}(t_{\text{top}}(a_{\text{cut}}))$. From the mean value theorem, there

exists $\zeta \in [0, t_{\text{top}}(a_{\text{cut}})]$ s.t.

$$u'(\zeta) = \frac{u(t_{\text{top}}(a_{\text{cut}})) - u(0)}{t_{\text{top}}(a_{\text{cut}})} = \frac{u(t_{\text{top}}(a_{\text{cut}})) - u_{a_{\text{cut}}}(0)}{t_{\text{top}}(a_{\text{cut}})} \geq \frac{u_{a_{\text{cut}}}(t_{\text{top}}(a_{\text{cut}})) - u_{a_{\text{cut}}}(0)}{t_{\text{top}}(a_{\text{cut}})} = a_{\text{cut}} > a_{x,\max},$$

implying infeasibility of u, as it uses acceleration (that is, u', the derivative of the velocity) which exceeds $a_{x,\max}$.

Now, assume $u \geq u_{a_{\text{cut}}}$ does not hold for all τ. Then, due to the fact $\int_0^t u(\tau)d\tau \geq \int_0^t u_{a_{\text{cut}}}(\tau)d\tau$, there must be a point where $u > u_{a_{\text{cut}}}$. If such point τ_{large} exists in $[0, t_{\text{top}}(a_{\text{cut}})]$, then we can easily use the mean value theorem in the same manner as above, to obtain $\zeta \in [0, \tau_{\text{large}}]$ where too large an acceleration is used. If such point only exists in $[t_{\text{top}}(a_{\text{cut}}), t]$, similar argument will give us a point $\zeta \in [\tau_{\text{large}}, t]$ in which an acceleration value lesser than $-a_{x,\max}$ was used, concluding the proof. ■

Equipped with the above lemmas, Theorem 2's proof is immediate.

3.2 Safety Verification - Occlusions

In very similar fashion to dealing with observed objects, we define the extension for cautiousness w.r.t. occluded objects, with a similar theorem to Theorem 1, proving that cautiousness implies there are never accidents of our blame.

Definition 15 (Cautiousness w.r.t. Occluded Objects) *A command given at time t is* Cautious w.r.t. Occluded Objects *if in the case that the exposure time of the object is $t+1$, and we command a Default Emergency Policy (DEP) at $t+1$, there will not be an accident of our blame.*

Lemma 7 *If we only give cautious, w.r.t. occluded objects and non occluded objects, commands, there will never be an accident of our blame.*

Proof Assume that an accident of our blame occurred at time t, with the exposure time being $t' \leq t$. By cautiousness assumption, the command given at $t'-1$ allowed us to command a DEP at t' without being blamed for an accident. As there was an accident of our blame, we apparently did not command a DEP at time t'. But from t' on, we were safe w.r.t. non occluded objects, hence the command we gave was safe, and there was no accident of our blame. ■

Here too, in Appendix B, we provide efficient ways to checking cautiousness w.r.t. a worst-case assumptions over occluded objects, allowing for feasible, scalable, RSS.

4 Driving Policy

A driving policy is a mapping from a sensing state (a description of the world around us) into a driving command (e.g., the command is lateral and longitudinal accelerations for the coming second, which determines where and at what speed should the car be in one second from now). The driving command is passed to a controller, that aims at actually moving the car to the desired position/speed.

In the previous sections we described a formal safety model and proposed constraints on the commands issued by the driving policy that guarantee safety. The constraints on safety are designed for extreme cases. Typically, we do not want to even need these constraints, and would like to construct a driving policy that leads to a comfortable ride. The focus of this section is on how to build an efficient driving policy, in particular, one that requires computational resources that can scale to millions of cars. For now, we ignore the issue of how to obtain the sensing state and assume an utopic sensing state, that faithfully represents the world around us without any limitations. In later sections we will discuss the effect of inaccuracies in the sensing state on the driving policy.

We can cast the problem of defining a driving policy in the language of Reinforcement Learning (RL). At each iteration of RL, an agent observes a state describing the world, denoted s_t, and should pick an action, denoted a_t, based on a policy function, π, that maps states into actions. As a result of its action and other factors out of its control

(such as the actions of other agents), the state of the world is changed to s_{t+1}. We denote a (state,action) sequence by $\bar{s} = ((s_1, a_1), (s_2, a_2), \ldots, (s_{\text{len}(\bar{s})}, a_{\text{len}(\bar{s})}))$. Every policy induces a probability function over (state,action) sequences. This probability function is affected by the actions taken by the agent, but also depends on the environment (and in particular, on how other agents behave). We denote by P_π the probability over (state,action) sequences induced by π. The quality of a policy is defined to be $\mathbb{E}_{\bar{s}\sim P_\pi}[\rho(\bar{s})]$, where $\rho(\bar{s})$ is a reward function that measures how good the sequence $\bar{s}$ is. In most case, $\rho(\bar{s})$ takes the form $\rho(\bar{s}) = \sum_{t=1}^{\text{len}(\bar{s})} \rho(s_t, a_t)$, where $\rho(s, a)$ is an instantaneous reward function, that measures the immediate quality of being at state s and performing action a. For simplicity, we stick to this simpler case.

To cast the driving policy problem in the above RL language, let s_t be some representation of the road, and the positions, velocities, and accelerations, of the ego vehicle as well as other road users. Let a_t be a lateral and longitudinal acceleration command. The next state, s_{t+1}, depends on a_t as well as on how the other agents will behave. The instantaneous reward, $\rho(s_t, a_t)$, may depend on the relative position/velocities/acceleration to other cars, the difference between our speed and the desired speed, whether we follow the desired route, whether our acceleration is comfortable etc.

The main difficulty of deciding what action should the policy take at time t stems from the fact that one needs to estimate the long term effect of this action on the reward. For example, in the context of driving policy, an action that is taken at time t may seem a good action for the present (that is, the reward value $\rho(s_t, a_t)$ is good), but might lead to an accident after 5 seconds (that is, the reward value in 5 seconds would be catastrophic). We therefore need to estimate the long term quality of performing an action a when the agent is at state s. This is often called the Q-function, namely, $Q(s, a)$ should reflect the long term quality of performing action a at time s. Given such a Q-function, the natural choice of an action is to pick the one with highest quality, $\pi(s) = \text{argmax}_a\, Q(s, a)$.

The immediate questions are how to define Q and how to evaluate Q efficiently. Let us first make the (completely non-realistic) simplifying assumption that s_{t+1} is some deterministic function of (s_t, a_t), namely, $s_{t+1} = f(s_t, a_t)$. The reader familiar with Markov Decision Processes (MDPs), will quickly notice that this assumption is even stronger than the Markovian assumption of MDPs (i.e., that s_{t+1} is conditionally independent of the past given (s_t, a_t)). As noted in [5], even the Markovian assumption is not adequate for multi-agent scenarios, such as driving, and we will therefore later relax the assumption.

Under this simplifying assumption, given s_t, for every sequence of decisions for T steps, $(a_t, \ldots, a_{t+T})$, we can calculate exactly the future states $(s_{t+1}, \ldots, s_{t+T+1})$ as well as the reward values for times $t, \ldots, T$. Summarizing all these reward values into a single number, e.g. by taking their sum $\sum_{\tau=t}^{T} \rho(s_\tau, a_\tau)$, we can define $Q(s, a)$ as follows:

$$Q(s,a) = \max_{(a_t,\ldots,a_{t+T})} \sum_{\tau=t}^{T} \rho(s_\tau, a_\tau) \quad \text{s.t.} \quad s_t = s,\ a_t = a,\ \forall \tau,\ s_{\tau+1} = f(s_\tau, a_\tau)$$

That is, $Q(s, a)$ is the best future we can hope for, if we are currently at state s and immediately perform action a.

Let us discuss how to calculate Q. The first idea is to discretize the set of possible actions, A, into a finite set $\hat{A}$, and simply traverse all action sequences in the discretized set. Then, the runtime is dominated by the number of discrete action sequences, $|\hat{A}|^T$. If $\hat{A}$ represents 10 lateral accelerations and 10 longitudinal accelerations, we obtain 100^T possibilities, which becomes infeasible even for small values of T. While there are heuristics for speeding up the search (e.g. coarse-to-fine search), this brute-force approach requires tremendous computational power.

The parameter T is often called the "time horizon of planning", and it controls a natural tradeoff between computation time and quality of evaluation — the larger T is, the better our evaluation of the current action (since we explicitly examine its effect deeper into the future), but on the other hand, a larger T increases the computation time exponentially. To understand why we may need a large value of T, consider a scenario in which we are 200 meters before a highway exit and we should take it. When the time horizon is long enough, the cumulative reward will indicate if at some time τ between t and $t + T$ we have arrived to the exit lane. On the other hand, for a short time horizon, even if we perform the right immediate action we will not know if it will lead us eventually to the exit lane.

A different approach attempts to perform offline calculations in order to construct an approximation of Q, denoted $\hat{Q}$, and then during the online run of the policy, use $\hat{Q}$ as an approximation to Q, without explicitly rolling out the future. One way to construct such an approximation is to discretize both the action domain and the state domain. Denote by $\hat{A}, \hat{S}$ these discretized sets. We can perform an offline calculation for evaluating the value of $Q(s, a)$

for every $(s,a) \in \hat{S} \times \hat{A}$. Then, for every $a \in \hat{A}$ we define $\hat{Q}(s_t,a)$ to be $Q(s,a)$ for $s = \text{argmin}_{s \in \hat{S}} \|s - s_t\|$. Furthermore, based on the pioneering work of Bellman [2, 3], we can calculate $Q(s,a)$ for every $(s,a) \in \hat{S} \times \hat{A}$, based on dynamic programming procedures (such as the Value Iteration algorithm), and under our assumptions, the total runtime is order of $T\,|\hat{A}|\,|\hat{S}|$. The main problem with this approach is that in any reasonable approximation, $\hat{S}$ is extremely large (due to the curse of dimensionality). Indeed, the sensing state should represent 6 parameters for every other relevant vehicle in the sense — the longitudinal and lateral position, velocity, and acceleration. Even if we discretize each dimension to only 10 values (a very crude discretization), since we have 6 dimensions, to describe a single car we need 10^6 states, and to describe k cars we need 10^{6k} states. This leads to unrealistic memory requirements for storing the values of Q for every (s,a) in $\hat{S} \times \hat{A}$.

A popular approach to deal with this curse of dimensionality is to restrict Q to come from a restricted class of functions (often called a hypothesis class), such as linear functions over manually determined features or deep neural networks. For example, [4] learned a deep neural network that approximates Q in the context of playing Atari games. This leads to a resource-efficient solution, provided that the class of functions that approximate Q can be evaluated efficiently. However, there are several disadvantages of this approach. First, it is not known if the chosen class of functions contain a good approximation to the desired Q function. Second, even if such function exists, it is not known if existing algorithms will manage to learn it efficiently. So far, there are not many success stories for learning a Q function for complicated multi-agent problems, such as the ones we are facing in driving. There are several theoretical reasons why this task is difficult. We have already mentioned that the Markovian assumption, underlying existing methods, is problematic. But, a more severe problem is that we are facing a very small signal-to-noise ratio due to the time resolution of decision making, as we explain below.

Consider a simple scenario in which we need to change lane in order to take a highway exit in 200 meters and the road is currently empty. The best decision is to start making the lane change. We are making decisions every 0.1 second, so at the current time t, the best value of $Q(s_t,a)$ should be for the action a corresponding to a small lateral acceleration to the right. Consider the action a' that corresponds to zero lateral acceleration. Since there is a very little difference between starting the change lane now, or in 0.1 seconds, the values of $Q(s_t,a)$ and $Q(s_t,a')$ are almost the same. In other words, there is very little *advantage* for picking a over a'. On the other hand, since we are using a function approximation for Q, and since there is noise in measuring the state s_t, it is likely that our approximation to the Q value is noisy. This yields a very small signal-to-noise ratio, which leads to an extremely slow learning, especially for stochastic learning algorithms which are heavily used for the neural networks approximation class. However, as noted in [1], this problem is not a property of any particular function approximation class, but rather, it is inherent in the definition of the Q function.

In summary, existing approaches can be roughly divided into two camps. The first one is the brute-force approach which includes searching over many sequences of actions or discretizing the sensing state domain and maintaining a huge table in memory. This approach can lead to a very accurate approximation of Q but requires unleashed resources, either in terms of computation time or in terms of memory. The second one is a resource efficient approach in which we either search for short sequences of actions or we apply a function approximation to Q. In both cases, we pay by having a less accurate approximation of Q, that might lead to poor decisions.

Our approach to constructing a Q function that is both resource-efficient and accurate is to depart from geometrical actions and to adapt a semantic action space, as described in the next subsection.

4.1 Semantics to the rescue

To motivate our semantic approach, consider a teenager that just got his driving license. His father seats next to him and gives him "driving policy" instructions. These instructions are not geometric — they do not take the form "drive 13.7 meters at the current speed and then accelerate at a rate of 0.8 m/s^2". Instead, the instructions are of semantic nature — "follow the car in front of you" or "quickly overtake that car on your left". We formalize a semantic language for such instructions, and use them as a semantic action space. We then define the Q function over the semantic action space. We show that a semantic action can have a very long time horizon, which allows us to estimate $Q(s,a)$ without planning for many future semantic actions. Yes, the total number of semantic actions is still small. This allows us to obtain an accurate estimation of the Q function while still being resource efficient. Furthermore, as we show later, we combine learning techniques for further improving the quality function, while not suffering from a small

signal-to-noise ratio due to a significant difference between different semantic actions.

We now define our semantic action space. The main idea is to define lateral and longitudinal goals, as well as the aggressiveness level of achieving them. Lateral goals are desired positions in lane coordinate system (e.g., "my goal is to be in the center of lane number 2"). Longitudinal goals are of three types. The first is relative position and speed w.r.t. other vehicles (e.g., "my goal is to be behind car number 3, at its same speed, and at a distance of 2 seconds from it"). The second is a speed target (e.g., "drive at the allowed speed for this road times 110%"). The third is a speed constraint at a certain position (e.g., when approaching a junction, "speed of 0 at the stop line", or when passing a sharp curve, "speed of at most 60kmh at a certain position on the curve"). For the third option we can instead apply a "speed profile" (few discrete points on the route and the desired speed at each of them). A reasonable number of lateral goals is bounded by $16 = 4 \times 4$ (4 positions in at most 4 relevant lanes). A reasonable number of longitudinal goals of the first type is bounded by $8 \times 2 \times 3 = 48$ (8 relevant cars, whether to be in front or behind them, and 3 relevant distances). A reasonable number of absolute speed targets are 10, and a reasonable upper bound on the number of speed constraints is 2. To implement a given lateral or longitudinal goal, we need to apply acceleration and then deceleration (or the other way around). The aggressiveness of achieving the goal is a maximal (in absolute value) acceleration/deceleration to achieve the goal. With the goal and aggressivness defined, we have a closed form formula to implement the goal, using kinematic calculations. The only remaining part is to determine the combination between the lateral and longitudinal goals (e.g., "start with the lateral goal, and exactly at the middle of it, start to apply also the longitudinal goal"). A set of 5 mixing times and 3 aggressiveness levels seems more than enough. All in all, we have obtained a semantic action space whose size is $\approx 10^4$.

It is worth mentioning that the variable time required for fulfilling these semantic actions is not the same as the frequency of the decision making process. To be reactive to the dynamic world, we should make decisions at a high frequency — in our implementation, every 100ms. In contrast, each such decision is based on constructing a trajectory that fulfills some semantic action, which will have a much longer time horizon (say, 10 seconds). We use the longer time horizon since it helps us to better evaluate the short term prefix of the trajectory. In the next subsection we discuss the evaluation of semantic actions, but before that, we argue that semantic actions induce a sufficient search space.

Is this sufficient: We have seen that a semantic action space induces a subset of all possible geometrical curves, whose size is exponentially smaller (in T) than enumerating all possible geometrical curves. The first immediate question is whether the set of short term prefixes of this smaller search space contains all geometric commands that we will ever want to use. We argue that this is indeed sufficient in the following sense. If the road is free of other agents, then there is no reason to make changes except setting a lateral goal and/or absolute acceleration commands and/or speed constraints on certain positions. If the road contains other agents, we may want to negotiate the right of way with the other agents. In this case, it suffices to set longitudinal goals relatively to the other agents. The exact implementation of these goals in the long run may vary, but the short term prefixes will not change by much. Hence, we obtain a very good cover of the relevant short term geometrical commands.

4.2 Constructing an evaluation function for semantic actions

We have defined a semantic set of actions, denoted by A^s. Given that we are currently in state s, we need a way to choose the best $a^s \in A^s$. To tackle this problem, we follow a similar approach to the *options mechanism* of [6]. The basic idea is to think of a^s as a meta-action (or an option). For each choice of a meta-action, we construct a geometrical trajectory $(s_1, a_1), \ldots, (s_T, a_T)$ that represents an implementation of the meta-action, a^s. To do so we of course need to know how other agents will react to our actions, but for now we are still relying on (the non-realistic) assumption that $s_{t+1} = f(s_t, a_t)$ for some known deterministic function f. We can now use $\frac{1}{T}\sum_{t=1}^{T} \rho(s_t, a_t)$ as a good approximation of the quality of performing the semantic action a^s when we are at state s_1.

Most of the time, this simple approach yields a powerful driving policy. However, in some situations a more sophisticated quality function is required. For example, suppose that we are following a slow truck before an exit lane, where we need to take the exit lane. One semantic option is to keep driving slowly behind the truck. Another one is to overtake the truck, hoping that later we can get back to the exit lane and make the exit on time. The quality measure described previously does not consider what will happen after we will overtake the truck, and hence we will not choose the second semantic action even if there is enough time to make the overtake and return to the exit lane. Machine

learning can help us to construct a better evaluation of semantic actions, that will take into account more than the immediate semantic actions. Previously, we have argued that learning a Q function over immediate geometric actions is problematic due to the low signal-to-noise ratio (the lack of advantage). This is not problematic when considering semantic actions, both because there is a large difference between performing the different semantic actions and because the semantic time horizon (how many semantic actions we take into account) is very small (probably less than three in most cases).

Another advantage of applying machine learning is for the sake of *generalization*: we can probably set an adequate evaluation function for *every* road, by a manual inspection of the properties of the road, and maybe some trial and error. But, can we automatically generalize to *any* road? Here, a machine learning approach can be trained on a large variety of road types so as to generalize to unseen roads as well.

To summarize, our semantic action space allows to enjoy the benefits of both worlds: semantic actions contain information on a long time horizon, hence we can obtain a very accurate evaluation of their quality while being resource efficient.

4.3 The dynamics of the other agents

So far, we have relied on the assumption that s_{t+1} is a deterministic function of s_t and a_t. As we have emphasized previously, this assumption is completely not realistic as our actions affect the behavior of other road users. While we do take into account some reactions of other agents to our actions (for example, we assume that if we will perform a safe cut-in than the car behind us will adjust its speed so as not to hit us from behind), it is not realistic to assume that we model all of the dynamics of other agents.

The solution to this problem is to re-apply our decision making at a high frequency, and by doing this, we constantly adapt our policy to the parts of the environment that are beyond our modeling. In a sense, one can think of this as a Markovization of the world at every step. This is a common technique that tends to work very good in practice as long as the balance between modeling error and frequency of planning is adequate.

5 Sensing

In this section we describe the sensing state, which is a description of the relevant information of the scene, and forms the input to the driving policy module. By and large, the sensing state contains static and dynamic objects. The static objects are lanes, physical road delimiters, constraints on speed, constraints on the right of way, and information on occluders (e.g. a fence that occludes relevant part of a merging road). Dynamic objects are vehicles (bounding box, speed, acceleration), pedestrians (bounding box, speed, acceleration), traffic lights, dynamic road delimiters (e.g. cones at a construction area), temporary traffic signs and police activity, and other obstacles on the road (e.g. an animal, a mattress that fell from a truck, etc.).

In any reasonable sensor setting, we cannot expect to obtain the exact sensing state, s. Instead, we view raw sensor and mapping data, which we denote by $x \in X$, and there is a sensing system that takes x and produces an approximate sensing state. Formally,

Definition 16 (Sensing system) *Let S denotes the domain of sensing state and let X be the domain of raw sensor and mapping data. A sensing system is a function $\hat{s} : X \to S$.*

It is important to understand when we should accept $\hat{s}(x)$ as a reasonable approximation to s. The ultimate way to answer this question is by examining the implications of this approximation on the performance of our driving policy in general, and on the safety in particular. Following our safety-comfort distinction, here again we distinguish between sensing mistakes that lead to non-safe behaviour and sensing mistakes that affect the comfort aspects of the ride.

Before we dive into the details, let us first describe the type of errors a sensing system might make:

- False negative: the sensing system misses an object
- False positive: the sensing system indicates a "ghost" object

- Inaccurate measurements: the sensing system correctly detects an object but incorrectly estimates its position or speed
- Inaccurate semantic: the sensing system correctly detects an object but misinterpret its semantic meaning, for example, the color of a traffic light

5.1 Comfort

Recall that for a semantic action a, we have used $Q(s,a)$ to denote our evaluation of a given that the current sensing state is s. Our policy picks the action $\pi(s) = \operatorname{argmax}_a Q(s,a)$. If we inject $\hat{s}(x)$ instead of s then the selected semantic action would be $\pi(\hat{s}(x)) = \operatorname{argmax}_a Q(\hat{s}(x),a)$. Clearly, if $\pi(\hat{s}(x)) = \pi(s)$ then $\hat{s}(x)$ should be accepted as a good approximation to s. But, it is also not bad at all to pick $\pi(\hat{s}(x))$ as long as the quality of $\pi(\hat{s}(x))$ w.r.t. the true state, s, is almost optimal, namely, $Q(s,\pi(\hat{s}(x))) \geq Q(s,\pi(s)) - \epsilon$, for some parameter ϵ. We say that $\hat{s}$ is ϵ-accurate w.r.t. Q in such case. Naturally, we cannot expect the sensing system to be ϵ-accurate all the time. We therefore also allow the sensing system to fail with some small probability δ. In such a case we say that $\hat{s}$ is Probably (w.p. of at least $1-\delta$), Approximately (up to ϵ), Correct, or PAC for short (borrowing Valiant's PAC learning terminology [7]).

We may use several (ϵ,δ) pairs for evaluating different aspects of the system. For example, we can choose three thresholds, $\epsilon_1 < \epsilon_2 < \epsilon_3$ to represent mild, medium, and gross mistakes, and for each one of them set a different value of δ. This leads to the following definition.

Definition 17 (PAC sensing system) *Let $((\epsilon_1,\delta_1),\ldots,(\epsilon_k,\delta_k))$ be a set of (accuracy,confidence) pairs, let S be the sensing state domain, let X be the raw sensor and mapping data domain, and let D be a distribution over $X \times S$. Let A be an action space, $Q : S \times A \to \mathbb{R}$ be a quality function, and $\pi : S \to A$ be such that $\pi(s) \in \operatorname{argmax}_a Q(s,a)$. A sensing system, $\hat{s} : X \to S$, is Probably-Approximately-Correct (PAC) with respect to the above parameters if for every $i \in \{1,\ldots,k\}$ we have that $\mathbb{P}_{(x,s)\sim D}[Q(s,\pi(\hat{s}(x))) \geq Q(s,\pi(s)) - \epsilon_i] \geq 1-\delta_i$.*

Few remarks are in order:

- The definition depends on a distribution D over $X \times S$. It is important to emphasize that we construct this distribution by recording data of many human drivers but not by following the particular policy of our autonomous vehicle. While the latter seems more adequate, it necessitates online validation, which makes the development of the sensing system impractical. Since the effect of any reasonable policy on D is minor, by applying simple data augmentation techniques we can construct an adequate distribution and then perform offline validation after every major update of the sensing system.
- The definition provides a sufficient, but not necessary, condition for comfort ride using $\hat{s}$. It is not necessary because it ignores the important fact that short term wrong decisions have little effect on the comfort of the ride. For example, suppose that there is a vehicle 100 meters in front of us, and it is slower than the host vehicle. The best decision would be to start accelerating slightly now. If the sensing system misses this vehicle, but will detect it in the next time (after 100 mili-seconds), then the difference between the two rides will not be noticeable. To simplify the presentation, we have neglected this issue and required a stronger condition. The adaptation to a multi-frame PAC definition is conceptually straightforward, but involves more technicality and therefore we omit it.

We next derive design principles that follow from the above PAC definition. Recall that we have described several types of sensing mistakes. For mistakes of types false negative, false positive, and inaccurate semantic, either the mistakes will be on non-relevant objects (e.g., a traffic light for left turn when we are proceeding straight), or they will be captured by the δ part of the definition. We therefore focus on the "inaccurate measurements" type of errors, which happens frequently.

Somewhat surprisingly, we will show that the popular approach of measuring the accuracy of a sensing system via ego-accuracy (that is, by measuring the accuracy of position of every object with respect to the host vehicle) is not sufficient for ensuring PAC sensing system. We will then propose a different approach that ensures PAC sensing system, and will show how to obtain it efficiently. We start with some additional definitions.

For every object o in the scene, let $p(o), \hat{p}(o)$ be the positions of o in the coordinate system of the host vehicle according to $s, \hat{s}(x)$, respectively. Note that the distance between o and the host vehicle is $\|p\|$. The *additive* error of $\hat{p}$ is $\|p(o) - \hat{p}(o)\|$.The *relative* error of $\hat{p}(o)$, w.r.t. the distance between o and the host vehicle, is the additive error divided by $\|p(o)\|$, namely $\frac{\|p(o)-\hat{p}(o)\|}{\|p(o)\|}$.

We first argue that it is not realistic to require that the additive error is small for far away objects. Indeed, consider o to be a vehicle at a distance of 150 meters from the host vehicle, and let ϵ be of moderate size, say $\epsilon = 0.1$. For additive accuracy, it means that we should know the position of the vehicle up to 10cm of accuracy. This is not realistic for reasonably priced sensors. On the other hand, for relative accuracy we need to estimate the position up to 10%, which amounts to 15m of accuracy. This is feasible to achieve (as we will describe later).

We say that a sensing system, $\hat{s}$, positions a set of objects, O, in an ϵ-ego-accurate way, if for every $o \in O$, the (relative) error between $p(o)$ and $\hat{p}(o)$ is at most ϵ. The following example shows that an ϵ-ego-accurate sensing state does not guarantee PAC sensing system with respect to every reasonable Q. Indeed, consider a scenario in which the host vehicle drives at a speed of $30m/s$, and there is a stopped vehicle 150 meters in front of it. If this vehicle is in the ego lane, and there is no option to change lanes in time, we must start decelerating now at a rate of at least $3m/s^2$ (otherwise, we will either not stop in time or we will need to decelerate strongly later). On the other hand, if the vehicle is on the side of the road, we don't need to apply a strong deceleration. Suppose that $p(o)$ is one of these cases while $\hat{p}(o)$ is the other case, and there is a 5 meters difference between these two positions. Then, the relative error of $\hat{p}(o)$ is

$$\frac{\|\hat{p}(o) - p(o)\|}{\|p(o)\|} \;=\; \frac{5}{150} = \frac{1}{30} \leq 0.034\,.$$

That is, our sensing system may be ϵ-ego-accurate for a rather small value of ϵ (less than 3.5% error), and yet, for any reasonable Q function, the values of Q are completely different since we are confusing between a situation in which we need to brake strongly and a situation in which we do not need to brake strongly.

The above example shows that ϵ-ego-accuracy does not guarantee that our sensing system is PAC. Is there another property that is sufficient for PAC sensing system? Naturally, the answer to this question depends on Q. We will describe a family of Q functions for which there is a simple property of the positioning that guarantees PAC sensing system. Intuitively, the problem of ϵ-ego-accuracy is that it might lead to semantic mistakes — in the aforementioned example, even though $\hat{s}$ was ϵ-ego-accurate with $\epsilon < 3.5\%$, it mis-assigned the vehicle to the correct lane. To solve this problem, we rely on *semantic units* for lateral position.

Definition 18 (semantic units) *A lane center is a simple natural curve, namely, it is a differentiable, injective, mapping $\ell : [a, b] \to \mathbb{R}^3$, where for every $a \leq t_1 < t_2 \leq b$ we have that the length $\mathrm{Length}(t_1, t_2) := \int_{\tau=t_1}^{t_2} |\ell'(\tau)| d\tau$ equals to $t_2 - t_1$. The width of the lane is a function $w : [a, b] \to \mathbb{R}_+$. The projection of a point $x \in \mathbb{R}^3$ onto the curve is the point on the curve closest to x, namely, the point $\ell(t_x)$ for $t_x = \mathrm{argmin}_{t\in[a,b]} \|\ell(t) - x\|$. The semantic longitudinal position of x w.r.t. the lane is t_x and the semantic lateral position of x w.r.t. the lane is $\ell(t_x)/w(t_x)$. Semantic speed and acceleration are defined as first and second derivatives of the above.*

Similarly to geometrical units, for semantic longitudinal distance we use relative error: if $\hat{s}$ induces a semantic longitudinal distance of $\hat{p}(o)$ for some object, while the true distance is $p(o)$, then the relative error is $\frac{|\hat{p}(o)-p(o)|}{\max\{p(o),1\}}$ (where the maximum in the denominator deals with cases in which the object has almost the same longitudinal distance (e.g., a car next to us on another lane). Since semantic lateral distances are small we can use additive error for them. This leads to the following defintion:

Definition 19 (error in semantic units) *Let ℓ be a lane and suppose that the semantic longitudinal distance of the host vehicle w.r.t. the lane is 0. Let $x \in \mathbb{R}^3$ be a point and let $p_{\mathrm{lat}}(x), p_{\mathrm{lon}}(x)$ be the semantic lateral and longitudinal distances to the point w.r.t. the lane. Let $\hat{p}_{\mathrm{lat}}(x), \hat{p}_{\mathrm{lon}}(x)$ be approximated measurements. The distance between $\hat{p}$ and p w.r.t. x is defined as*

$$d(\hat{p}, p; x) \;=\; \max\left\{ |\hat{p}_{\mathrm{lat}}(x) - p_{\mathrm{lat}}(x)|\,,\; \frac{|\hat{p}_{\mathrm{lon}}(x) - p_{\mathrm{lon}}(x)|}{\max\{p_{\mathrm{lon}}(x), 1\}} \right\}$$

The distance of the lateral and longitudinal velocities is defined analogously.

Equipped with the above definition, we are ready to define the property of Q and the corresponding sufficient condition for PAC sensing system.

Definition 20 (Semantically-Lipschitz Q) *A Q function is L-semantically-Lipschitz if for every a, $s, \hat{s}$, $|Q(s,a) - Q(\hat{s}(x),a)| \leq L \max_o d(\hat{p}, p; o)$, where $\hat{p}, p$ are the measurements induced by $s, \hat{s}$ on an object o.*

As an immediate corollary we obtain:

Lemma 8 *If Q is L-semantically-Lipschitz and a sensing system $\hat{s}$ produces semantic measurements such that with probability of at least $1-\delta$ we have $d(\hat{p}, p; o) \leq \epsilon/L$, then $\hat{s}$ is a PAC sensing system with parameters ϵ, δ.*

5.2 Safety

We now discuss sensing mistakes that lead to non-safe behavior. As mentioned before, our policy is provably safe, in the sense that it won't lead to accidents of the AV's blame. Such accidents might still occur due to hardware failure (e.g., a break down of all the sensors or exploding tire on the highway), software failure (a significant bug in some of the modules), or a sensing mistake. Our ultimate goal is that the probability of such events will be extremely small — a probability of 10^{-9} for such an accident per hour. To appreciate this number, the average number of hours an american driver spend on the road is (as of 2016) less than 300. So, in expectation, one needs to live 3.3 million years to be in an accident.

We first define what is a safety-critic sensing mistake. Recall that at every step, our policy picks the value of a that maximizes $Q(s,a)$, namely, $\pi(s) = \operatorname{argmax}_a Q(s,a)$. We ensure safety by letting $Q(s,a) = -\infty$ for every action a which is not cautious (see Definition 14). Therefore, the first type of safety-critic sensing mistake is if our sensing system leads to picking a non-safe action. Formally, letting $\pi(\hat{s}(x)) = \operatorname{argmax}_a Q(\hat{s}(x), a)$ be the decision according to $\hat{s}$, we say that $\hat{s}$ leads to a *safety-critic miss* if $Q(s, \pi(\hat{s}(x))) = -\infty$. The second type of safety-critic sensing mistake is if all the actions are non safe according to $\hat{s}(x)$, and we must apply the standard emergency policy (brakeing hard), while according to s there is a safe action, namely, $\max_a Q(s,a) > -\infty$. This is dangerous when our speed is high and there is a car behind us. We call such mistake a *safety-critic ghost*.

Usually, a safety-critic miss is caused by a false negative while a safety-critic ghost is caused by a false positive. Such mistakes can also be caused from significantly incorrect measurements, but in most cases, our comfort objective ensures we are far away from the boundary of the safety definitions, and therefore reasonable measurements errors are unlikely to lead to a safety-critic mistake.

How can we ensure that the probability of safety-critic mistakes will be very small, say, smaller than 10^{-9} per hour? As followed from Lemma 1, without making further assumptions we need to check our system on more than 10^9 hours of driving. This is unrealistic (or at least extremely challenging) — it amounts to recording the driving of 3.3 million cars over a year. Furthermore, building a system that achieves such a high accuracy is a great challenge. Our solution for both the system design and validation challenges is to rely on several sub-systems, each of which is engineered independently and depends on a different technology, and the systems are fused together in a way that ensures boosting of their individual accuracy.

Suppose we build 3 sub-systems, denoted, s_1, s_2, s_3 (the extension to more than 3 is straightforward). Each sub-system receives a and should output safe/non-safe. Actions for which the majority of the sub-systems (2 in our case) accept as safe are considered safe. If there is no action that is considered safe by at least 2 sub-systems then we apply the default emergency policy.

Let us now analyze the performance of this fusion scheme. We rely on the following definition:

Definition 21 (One side c-approximate independent) *Two Bernoulli random variables r_1, r_2 are called one side c-approximate independent if*

$$\mathbb{P}[r_1 \wedge r_2] \leq c \ \mathbb{P}[r_1] \ \mathbb{P}[r_2] \ .$$

For $i \in \{1,2,3\}$, denote by e_i^m, e_i^g the Bernoulli random variables that indicate if sub-system i performs a safety-critic miss/ghost respectively. Similarly, e^m, e^g indicate a safety-critic miss/ghost of the fusion system. We rely on the assumption that for any pair $i \neq j$, the random variables e_i^m, e_j^m are one sided c-approximate independent, and the same holds for e_i^g, e_j^g. Before explaining why this assumption is reasonable, let us first analyze its implication. We can

bound the probability of e^m by:

$$\begin{aligned}\mathbb{P}[e^m] &= \mathbb{P}[e_1^m \wedge e_2^m \wedge e_3^m] + \sum_{j=1}^{3} \mathbb{P}[\neg e_j^m \wedge \wedge_{i \neq j} e_i^m] \\ &\leq 3\,\mathbb{P}[e_1^m \wedge e_2^m \wedge e_3^m] + \sum_{j=1}^{3} \mathbb{P}[\neg e_j^m \wedge \wedge_{i \neq j} e_i^m] \\ &= \sum_{j=1}^{3} \mathbb{P}[\wedge_{i \neq j} e_i^m] \\ &\leq c \sum_{j=1}^{3} \prod_{i \neq j} \mathbb{P}[e_i^m].\end{aligned}$$

Therefore, if all sub-systems have $\mathbb{P}[e_i^m] \leq p$ then $\mathbb{P}[e^m] \leq 3\,c\,p^2$. The exact same derivation holds for the safety-critic ghost mistakes. By applying a union bound we therefore conclude:

Corollary 2 *Assume that for any pair $i \neq j$, the random variables e_i^m, e_j^m are one sided c-approximate independent, and the same holds for e_i^g, e_j^g. Assume also that for every i, $\mathbb{P}[e_i^m] \leq p$ and $\mathbb{P}[e_i^g] \leq p$. Then,*

$$\mathbb{P}[e^m \vee e^g] \leq 6\,c\,p^2\ .$$

This corollary allows us to use significantly smaller data sets in order to validate the sensing system. For example, if we would like to achieve a safety-critic mistake probability of 10^{-9}, instead of taking order of 10^9 examples, it suffices to take order of 10^5 examples and test each system separately.

It is left to reason about the rational behind the one sided independence assumption. There are pairs of sensors that yield completely non-correlated errors. For example, radar works well in bad weather conditions but might fail due to non-relevant metallic objects, as opposed to camera that is affected by bad weather but is not likely to be affected by metallic objects. Seemingly, camera and lidar have common sources of mistakes — both are affected by foggy weather, heavy rain or snow. However, the type of mistake for camera and lidar would be different — camera might miss objects due to bad weather while lidar might detect a ghost due to reflections from particles in the air. Since we have distinguished between the two types of mistakes, the approximate independency is still likely to hold.

Remark: Our definition of safety-critic ghost requires that *all* actions are non-safe by at least two sensors. We argue that even in difficult conditions (e.g. heavy fog), this is unlikely to happen. The reason is that in such situations, systems that are affected by the difficult conditions (e.g. the lidar), will dictate a very defensive driving, as they can declare high velocity and lateral maneuver to be non-safe. As a result, we will drive slowly, and then even if we require an emergency stop, it is not dangerous due to the low speed of driving. Therefore, our definition yields an adaptation of the driving style to the conditions of the road.

5.3 Building a scalable sensing system

We have described the requirements from a sensing system, both in terms of comfort and safety. We now briefly suggest our approach for building a sensing system that meets these requirements while being scalable.

There are three main components of our sensing system. The first is long range, 360 degrees coverage, of the scene based on cameras. The three main advantages of cameras are: (1) high resolution, (2) texture, (3) price. The low price enables a scalable system. The texture enables to understand the semantics of the scene, including lane marks, traffic light, intentions of pedestrians, and more. The high resolution enables a long range of detection. Furthermore, detecting lane marks and objects in the same domain enables excellent semantic lateral accuracy. The two main disadvantages of cameras are: (1) the information is 2D and estimating longitudinal distance is difficult, (2) sensitivity to lighting conditions (low sun, bad weather). We overcome these difficulties using the next two components of our system.

The second component of our system is a semantic high-definition mapping technology, called Road Experience Management (REM). A common geometrical approach to map creation is to record a cloud of 3D points (obtained by a lidar) in the map creation process, and then, localization on the map is obtained by matching the existing lidar points to the ones in the map. There are several disadvantages of this approach. First, it requires a large memory per kilometer of mapping data, as we need to save many points. This necessitates an expensive communication infrastructure. Second, only few cars are equipped with lidar sensors, and therefore, the map is updated very infrequently. This is problematic as changes in the road can occur (construction zones, hazards), and the "time-to-reflect-reality" of lidar-based mapping solutions is large. In contrast, REM follows a semantic-based approach. The idea is to leverage the large number of vehicles that are equipped with cameras and with software that detects semantically meaningful objects in the scene (lane marks, curbs, poles, traffic lights, etc.). Nowadays, many new cars are equipped with ADAS systems which can be leveraged for crowd source based creation of the map. Since the processing is done on the vehicle side, only a small amount of semantic data should be communicated to the cloud. This allows a very frequent update of the map in a scalable way. In addition, the autonomous vehicles can receive the small sized mapping data over existing communication platforms (the cellular network). Finally, highly accurate localization on the map can be obtained based on cameras, without the need for expensive lidars.

REM is used for three purposes. First, it gives us a foresight on the static structure of the road (we can plan for a highway exit way in advance). Second, it gives us another source of accurate information of all of the static information, which together with the camera detections yields a robust view of the static part of the world. Third, it solves the problem of lifting the 2D information from the image plane into the 3D world as follows. The map describes all of the lanes as curves in the 3D world. Localization of the ego vehicle on the map enables to trivially lift every object on the road from the image plane to its 3D position. This yields a positioning system that adheres to the accuracy in semantic units described in Section 5.1.

The third component of our system is a complementary radar and lidar systems. These systems serve two purposes. First, they enable to yield an extremely high accuracy for the sake of safety (as described in Section 5.2). Second, they give direct measurements on speed and distances, which further improves the comfort of the ride.

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A Technical Lemmas

A.1 Technical Lemma 1

Lemma 9 *For all* $x \in [0, 0.1]$*, it holds that* $1 - x \geq e^{-2x}$.

Proof Let $f(x) = 1 - x - e^{-2x}$. Our goal is to show it is ≥ 0 for $x \in [0, 0.1]$. Note that $f(0) = 0$, and it is therefore sufficient to have that $f'(x) \geq 0$ in the aforementioned range. Explicitly, $f'(x) = -1 + 2e^{-2x}$. Clearly, $f'(0) = 1$, and it is monotonically decreasing, hence it is sufficient to verify that $f'(0.1) > 0$, which is easy to do numerically, $f'(0.1) \approx 0.637$. ■

B Efficient Cautiousness Verification - Occluded Objects

As we do with non occluded objects, we check whether giving the current command, and after it, the DEP, is RSS. For this, we unroll our future until t_{brake}, when assuming the exposure time is 1 and we then command DEP, which suffices for cautiousness by definition. For all $t' \in [0, t_{\text{brake}}]$, we check whether a blameful accident can occur - when assuming worst case over the occluded object. We use some of our worst case manoeuvres and safe distance rules. We take an occluder based approach to find interest points - namely, for each occluding object, we calculate the worst case. This is a crucial efficiency-driven approach - a pedestrian, for example, can be hidden in many positions behind a car, and can perform many manoeuvres, but there's a single worst case position and manoeuvre it can perform.

Let us deal here with the more elaborate case, that of the occluded pedestrian. Consider an occluded area behind a parked car. We start off by noting that it is very easy to find the closest points in an occluded area and the front/side of our car c, by their simple geometrical properties (triangles, rectangles[7]). Furthermore, it is easy to see that a pedestrian can run into the front of the car (under the v_{limit} constraint) from the occluded area IFF he can do it using the shortest path possible. Using the fact that the maximal distance which can be travelled by the pedestrian is $v_{limit} \cdot t'$, we obtain a simple check for a frontal hit possibility. As for a side hit, we note that in case the path is shorter than $v_{limit} \cdot t'$, we are responsible IFF our lateral velocity is greater than μ, in the direction of the hit. See Figure 4 for examples for shortest paths. We now obtain a simple algorithm for cautiousness verification w.r.t. occluded pedestrians, which we describe here in free pseudo code. The crucial part, that of checking existence of possibility of blameful accident with a pedestrian occluded by a vehicle, is done in the simple manner described above.

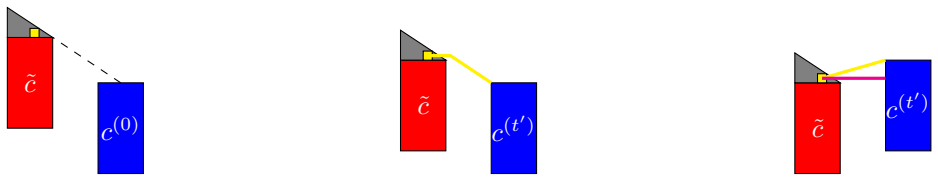
Algorithm 2: Check cautiousness w.r.t. an occluded pedestrian

for $t' \in [0, t_{\text{brake}}]$
  Roll self future until t'
  if Exists possibility of blameful accident with a pedestrian occluded by a vehicle
    return "non-cautious"
return "cautious"

C On the Problem of Validating a Simulator

Multi-agent safety is hard to validate statistically as it should be done in an "online" manner. One may argue that by building a simulator of the driving environment, we can validate the driving policy in the "lab". The problem with this argument is that validating that the simulator faithfully represents reality is as hard as validating the policy itself. To see why this is true, suppose that the simulator has been validated in the sense that applying a driving policy π in the simulator leads to a probability of an accident of $\hat{p}$, and the probability of an accident of π in the real world is p,

[7] Formally, we can consider the occluded area as a union of a small number of convex regions of simple shape, and treat each of them separately.



(a) (b) (c)

Figure 4: Worst case position (for time 0), and shortest paths, for frontal hit at time t', in yellow. In Figure 4c, in magenta, the only place where there's difference between the side hit's shortest path and the frontal hit's path.

with $|p - \hat{p}| < \epsilon$. (We need that ϵ will be smaller than 10^{-9}.) Now we replace the driving policy to be π'. Suppose that with probability of 10^{-8}, π' performs a weird action that confuses human drivers and leads to an accident. It is possible (and even rather likely) that this weird action is not modeled in the simulator, without contradicting its superb capabilities in estimating the performance of the original policy π. This proves that even if a simulator has been shown to reflect reality for a driving policy π, it is not guaranteed to reflect reality for another driving policy.

D The Lane-Based Coordinate System

The most basic simplifying assumption we made in the RSS definition was that the road is comprised by adjacent, straight lanes, of constant width. The distinction between lateral and longitudinal axes, along with an ordering of longitudinal position, play a significant role in RSS, and in common-sense driving. Moreover, the definition of those directions is clearly based on the lane shape. We propose a transformation from (global) positions on the plane, to a lane-based coordinate system, reducing the problem yet again to the original, "straight lane of constant width", case.

Assume that the lane's center is a smooth directed curve r on the plane, where all of its pieces, denoted $r^{(1)}, \ldots, r^{(k)}$, are either linear, or an arc. Note that smoothness of the curve implies that no pair of consecutive pieces can be linear. Formally, the curve maps a "longitudinal" parameter, $Y \in [Y_{\min}, Y_{\max}] \subset \mathbb{R}$, into the plane, namely, the curve is a function of the form $r : [Y_{\min}, Y_{\max}] \to \mathbb{R}^2$. We define a continuous lane-width function $w : [Y_{\min}, Y_{\max}] \to \mathbb{R}_+$, mapping the longitudinal position Y into a positive lane width value. For each Y, from smoothness of r, we can define the normal unit-vector to the curve at position Y, denoted $r^{\perp}(Y)$. We naturally define the subset of points on the plane which reside in the lane as follows:

$$R = \{r(Y) + \alpha w(Y) r^{\perp}(Y) \mid Y \in [Y_{\min}, Y_{\max}],\ \alpha \in [\pm 1/2]\} .$$

Informally, our goal is to construct a transformation ϕ of R into $\mathbb{R}^2$,[8] such that for two cars which are on the lane, their "logical ordering" will be preserved:

- If c_r is "behind" c_f on the curve, then $\phi(c_r)_y < \phi(c_f)_y$.
- If c_l is "to the left of" c_r on the curve, then $\phi(c_l)_x < \phi(c_r)_x$.

In order to define ϕ, we rely on the assumption that for all i, if $r^{(i)}$ is an arc of radius ρ, then the width of the lane throughout $r^{(i)}$ is $\leq \rho/2$. Note that this assumption holds for any practical road. The assumption trivially implies that for all $(x', y') \in R$, there exists a unique pair $Y' \in [Y_{\min}, Y_{\max}]$, $\alpha' \in [\pm 1/2]$, s.t. $(x', y') = r(Y') + \alpha' w(Y') r^{\perp}(Y')$. We can now define $\phi : R \to \mathbb{R}^2$ to be $\phi(x', y') = (Y', \alpha')$, where (Y', α') are the unique values that satisfy $(x', y') = r(Y') + \alpha' w(Y') r^{\perp}(Y')$.

This definition captures the notion of a "lateral manoeuvre" in lane's coordinate system. Consider, for example, a widening lane, with a car driving exactly on one of the lane's boundaries (see Figure 5 for an illustration). The widening of the lane means that the car is moving away from the center of the lane, and therefore has lateral velocity w.r.t. it. However, this doesn't mean it performs a lateral manoeuvre. Our definition of $\phi(x', y')_x = \alpha'$, namely, the lateral distance to the lane's center in $w(Y')$-units, implies that the lane boundaries have a fixed lateral position of

[8] Where, as in RSS, we will associate the y-axis with the "longitudinal" axis, and the x-axis with the "lateral".

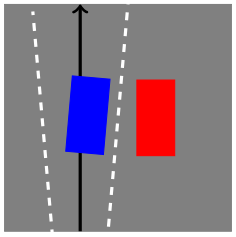


Figure 5: Changing lane width. Although the red car drives in parallel to the lane's center (black arrow), it clearly makes lateral movement towards the lane. The blue car, although getting further away from the lane's center, stays in the same position w.r.t. the lane boundary.

$\pm 1/2$, hence, a car which sticks to one of the lane's boundaries is not considered to perform any lateral movement. Finally, it is easy to see that ϕ is a homomorphism. We will use the term *lane-based coordinate system* when discussing $\phi(R) = [Y_{\min}, Y_{\max}] \times [\pm 1/2]$. We have thus obtained a reduction from a general lane geometry to a straight, longitudinal/lateral, coordinate system.

E Extending RSS to General Road Structure

In this section we give a complete definition of RSS, that holds for all road structures.[9] We do this by introducing the concept of *route priority*. This concept captures any situation in which more than a single lane geometry exists, for example junctions. The concept of route priority is defined in Section E.1.

The second generalization we make deals with two-way roads, in which there can be two cars driving at opposite directions. For this case, we show that the already established RSS definition is still valid, with the minor generalization of "safe distance" to oncoming traffic. This is defined in Section E.2.

In Section E.3 we show how controlled junctions (that use traffic lights to dictate the flow of traffic), are fully handled by the concepts of route priority and two-way roads.

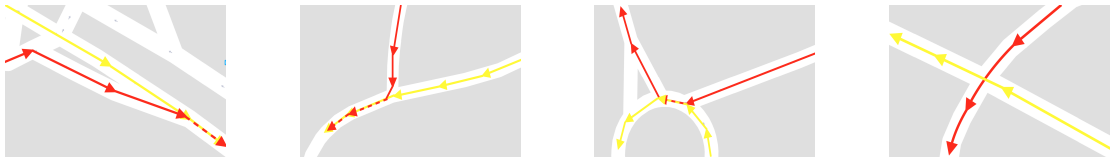
Finally, in Section E.4 we deal with unstructured roads (for example parking areas), where there is no clear route definition. We show that RSS is still valid for this case, where the only needed modification is a way to define virtual routes and to assign each car to (possibly several) routes.

E.1 Route Priority

We now introduce the concept of *route priority*. This concept enables us to deal with scenarios in which there are multiple different road geometries in one scene that overlap in a certain area. Examples include roundabouts, junctions, and merge into highways. See Figure 6 for illustration.

In Section D we have shown a way to transform general lane geometry into a lane-based one, with coherent meaning for longitudinal and lateral axes. We now face scenarios in which multiple routes of different road geometry exists. It follows that when two vehicles approach the overlap area, both perform a cut-in to the frontal corridor of the other one. This phenomenon cannot happen when two routes have the same geometry (as is the case of two adjacent highway lanes). Roughly speaking, the principle of *route priority* states that if routes r_1, r_2 overlap, and r_1

[9] This section deals with the definition of RSS and not on how to efficiently ensure that a policy adheres to RSS. The latter will be available in a separate document.



(a) (b) (c) (d)

Figure 6: Different examples for multiple routes scenarios. In yellow, the prioritized route. In red, the secondary route.

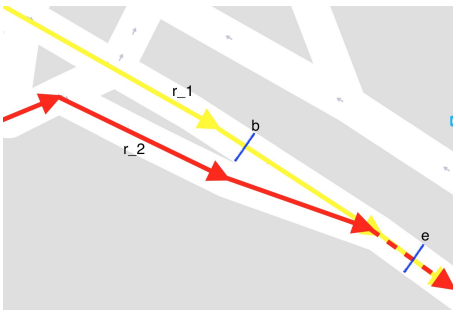
has priority over r_2, then a vehicle coming from r_1 that enters into the frontal corridor of a vehicle that comes from r_2 is not considered to perform a cut-in.

To explain the concept formally, recall that the blame of an accident depends on geometrical properties which are derived from the lane's coordinate system, and on worst case assumptions which rely on it too. Let $r_1, \ldots, r_k$ be the routes defining the road's structure. As a simple example, consider the merge scenario, as depicted in Figure 6a. Assume two cars, c_1, c_2, are driving on routes r_1, r_2 respectively, and r_1 is the prioritized route. For example, suppose that r_1 is a highway lane and r_2 is a merging lane. Having defined, in Section D, the route-based coordinate systems for each route, a first observation is that we can consider any manoeuvre in any route's coordinate system. For example, if we use r_2's coordinate system, driving straight on r_1 seems like a merge into r_2's left side. A naive approach to definition of RSS could be that each of the cars can perform a manoeuvre IFF $\forall i \in \{1, 2\}$, it is safe w.r.t. r_i. However, this implies that c_1, driving on the prioritized route, should be very conservative w.r.t. r_2, the merging route, as c_2 can drive exactly on the route, and hence can win by lateral position. This is of course unnatural, as cars on the highway have the right-of-way in this case. To overcome this problem, we define certain areas in which route priority is defined, and only some of the routes are considered as relevant for safety.

Definition 22 (Accident Blame with Route Priority) *Suppose r_1, r_2 are two routes with different geometry that overlap. We use $r_1 >_{[b,e]} r_2$ to symbolize that r_1 has priority over r_2 in the longitudinal interval $[b, e]$ of r_1 coordinate system. Suppose there is an accident between cars c_1, c_2, driving on routes r_1, r_2. For $i \in \{1, 2\}$, let $b_i \subset \{1, 2\}$ indicate the cars to blame for the accident if we consider the coordinate system of r_i. The blame for the accident is as follows:*

- *If $r_1 >_{[b,e]} r_2$ and on the blame time w.r.t. r_1, one of the cars was in the interval $[b, e]$ of the r_1-system's longitudinal axis, then the blame is according to b_1.*
- *Otherwise, the blame is according to $b_1 \cup b_2$.*

To illustrate the definition, consider again the merge into highway example. The blue lines in the figure below indicate the values of b, e for which $r_1 >_{[b,e]} r_2$.



Thus, we allow cars to drive naturally on the highway, while implying merging cars must be safe w.r.t. it. In particular, observe that in the case a car c_1 drives at the center of the prioritized lane, with no lateral velocity, it will not be blamed for an accident with a car c_2 driving on a non-prioritized lane, unless c_2 has cut-in into c_1's corridor at a safe distance. Note, that the end result is very similar to the regular RSS - this is exactly the same as a case where a car, on a straight road, tries to perform a lane change.

Remark: Note that there are cases where the route used by another agent is unknown: for example, see Figure 7. In such case, RSS is obtained by simply checking all possibilities.

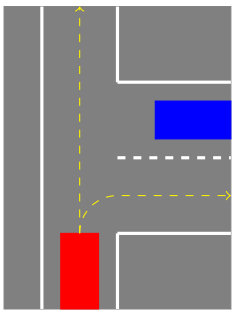


Figure 7: The blue car cannot know for sure what is the route of the red one.

E.2 Two-Way Traffic

To deal with two-way traffic, the modification to the blame definition comes through sharpening the parts which rely on rear/front relationships, as those are of a slightly different meaning in such cases. Consider two cars c_1, c_2 driving on some straight two lane road, in opposite longitudinal directions, namely, $v_{1,\text{long}} \cdot v_{2,\text{long}} < 0$. The direction of driving w.r.t. a lane may be negative in reasonable urban scenarios, such as a car deviating to the opposite lane to overtake a parked truck, or a car reversing into a parking spot. It is therefore required to extend the definition of the safe longitudinal distance which we have introduced for cases where negative longitudinal velocity was assumed to be un-realistic. Recall that a distance between c_r, c_f was safe if a maximal brake by c_f would allow enough of response time for c_r to brake before crashing into c_f. In our case, we again consider the "worst-case" by the opposite car, in a slightly different manner: of course we do not assume that the "worst case" is that it speeds up towards us, but that it indeed will brake to avoid a crash - but only using some reasonable braking power. In order to capture the difference in responsibility between the cars, when one of them clearly drives at the opposite direction, we start by defining a "correct" driving direction.

Recall that in the RSS definition for parallel lanes, we have considered the relevant lane to be the one whose center is closest to the cut-in position. We can now reduce ourselves to consideration of this lane (or, in the case of symmetry, deal with the two lanes separately, as in Definition 22)

In the definition below, we use the term "heading" to denote the arc tangent (in radians) of the lateral velocity divided by the longitudinal velocity.

Definition 23 ((μ_1, μ_2, μ_3)-Winning by Correct Driving Direction) *Assume c_1, c_2 are driving in opposite directions, namely $v_{1,\text{long}} \cdot v_{2,\text{long}} < 0$. Let x_i,h_i be their lateral positions and headings w.r.t. the lane. We say that c_1 (μ_1, μ_2, μ_3)-*Wins by Correct Driving Direction *if all of the following conditions hold:*

- $|h_1| < \mu_1$,
- $|h_2 - \pi| < \mu_2$,
- $|x_1| < \mu_3$.

We denote the indicator of this event by $W_{CDD}(i)$.

In words, c_1 wins if it drives close to the lane canter, in the correct direction, while c_2 takes the opposite direction. We note that at most one car can win, and it can be that none of the cars does so. Intuitively, assume there is a crash in the discussed situation. It is reasonable to put more responsibility over a car c_1, that loses by Correct Driving Direction. We do this by re-defining $a_{\text{max,brake}}$ for the case that a car wins by correct driving direction.

Definition 24 (Reasonable Braking Power) *Let $a_{\text{max,brake,wcdd}} > 0$ be a constant, smaller than $a_{\text{max,brake}}$. Assume c_1, c_2 are driving in opposite directions. The* Reasonable Braking Power *of each car c_i, denoted RBP_i is $a_{\text{max,brake,wcdd}}$ if c_i (μ_1, μ_2, μ_3)-Wins by Correct Driving Direction and $a_{\text{max,brake}}$ otherwise.*

The exact values of $a_{\max,\text{brake,wcdd}}, a_{\max,\text{brake}}$, for the cases of winning/not-winning by correct driving direction, are constants which should be defined, and can depend on the type of road and the lanes driven by each car. For example, in a narrow urban street, it may be the case that winning by correct driving direction does not imply a much lesser brake value: in dense traffic, we do expect a car to brake at similar forces, either when someone clearly deviated into its lane or not. However, consider an example of a rural two way road, where high speeds are allowed. When deviating to the opposite lane, we cannot expect cars which drive at the correct direction to apply a very strong braking power in order to avoid hitting us - we will have more responsibility than them. Different constants can be defined for the case when two cars are at the same lane, with one of them reversing into a parking spot.

We can now define the safety distance between cars which are driving in opposite directions, and immediately derive its exact value.

Definition 25 (Safe Longitudinal Distance - Two-Way Traffic) *A* longitudinal distance *between a car c_1 and another car c_2 which are driving in opposite directions and are both in the frontal corridors of each other, is* safe *w.r.t. a response time ρ if for any acceleration command a, $|a| < a_{\max,\text{accel}}$, performed by c_1, c_2 until time ρ, if c_1 and c_2 will apply their Reasonable Braking Power from time ρ until a full stop then they won't collide.*

Lemma 10 *Let c_1, c_2 as in Definition 25. Let $RBP_i, a_{\max,\text{accel}}$ be the reasonable braking (for each i) and acceleration commands, and let ρ be the cars' response time. Let v_1, v_2 be the longitudinal velocities of the cars, and let l_1, l_2 be their lengths. Define $v_{i,\rho,\max} = |v_i| + \rho \cdot a_{\max,\text{accel}}$. Let $L = (l_r + l_f)/2$. Then, the minimal safe longitudinal distance is:*

$$d_{\min} = L + \sum_{i=1}^{2} \left(\frac{|v_i| + v_{i,\rho,\max}}{2} \rho + \frac{v_{i,\rho,\max}^2}{2RBP_i} \right)$$

Proof It is clear that the term in the sum is the maximal distance travelled by each car until it reaches full stop, when performing the manoeuvre from Definition 25. Therefore, in order for the full stop to be at a distance greater than L, the initial distance must be larger than this sum and an additional term of L. ■

We now use the same blame time definition of RSS, with the non safe longitudinal distance as defined in Definition 25, to define the blame for a two-way traffic scenario.

Definition 26 (Blame in Two-Way Traffic) *The* Blame in Two-Way Traffic *of an accident between cars c_1, c_2 driving in opposite directions, is a function of the state at the* Blame Time*, and is defined as follows:*

- *If the Blame Time is also a cut-in time, the blame is defined as in the regular RSS definition.*
- *Otherwise, for every i, the blame is on c_i if at some t that happens after the blame time, c_i was not braking at a power of at least RBP_i.*

In words, assume a safe cut-in occurred before the blame time. For example, c_1 has deviated to the opposite lane, performed a cut-in into c_2's corridor, at a safe distance. Note that c_2 wins by correct driving direction, and hence this distance can be very large - we do not expect c_2 to perform strong braking power, but only the Reasonable Braking Power. Then, both cars have responsibility not to crash into each other. However, if the cut-in was not in a safe-distance, we use the regular definition, noting that c_2 will not be blamed if it drove in the center of its lane, without lateral movement. The blame will be solely on c_1. This allows a car to drive naturally at the center of its lane, without worrying about traffic which may unsafely deviate into its corridor. On the other hand, safe deviation to the opposite lane, a common manoeuvre required in dense urban traffic, is allowed. Considering the example of a car which initiates a reverse parking manoeuvre, it should start reversing while making sure the distance to cars behind it is safe.

E.3 Traffic Lights

We next discuss intersections with traffic lights. One might think that the simple rule for traffic lights scenarios is "if one car's route has the green light and the other car's route has a red light, then the blame is on the one whose route has the red light". However, this is not the correct rule. Consider for example the scenario depicted in Figure 8. Even

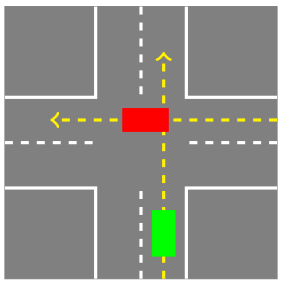


Figure 8: The red car's route has a red light and it is stuck in the intersection. Even though the green car's route has a green light, since it has enough distance, it should brake so as to avoid an accident.

(a)

(b)

Figure 9: Unstructured roads. (a) a wide roundabout around arc-de-triomphe. (b) parking lot.

if the green car's route has a green light, we do not expect it to ignore the red car that is already in the intersection. The correct rule is that the route that has a green light have a priority over routes that have a red light. Therefore, we obtain clear reduction from traffic lights to the route priority concept we have described previously.

E.4 Unstructured Road

Finally, we turn to consideration of roads where no clear route geometry can be defined. For example, see Figure 9. Consider first a scenario where there is no lane structure at all (e.g. the parking lot given in Figure 9 (b)). A simple way to ensure that there will be no accidents can be to require that every car will drive in a straight line, while if change of heading occur, it must be done when there are no close cars in my surrounding. The rational behind this is that a car can predict what other cars will do, and behave accordingly. If other cars deviate from this prediction (by changing heading), it is done with a long enough distance and therefore we have enough time to correct the prediction. When there is lane structure, it determines smarter predictions on what other cars will do. If there is no lane structure at all, we revert back to assuming that a car will continue according to its current heading. Technically speaking, this is equivalent to assigning every car to a virtual straight route according to its heading.

Next, consider the scenario given in Figure 9b(b). Here, the more sensible naive prediction is to assume that a car will continue according to the geometry of the roundabout, while keeping its offset. Technically, this is equivalent to assigning every car to a virtual arc route according to its current offset from the center of the roundabout.