A Matlab and CasADi-based Implementation of RICE Dynamic Game

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Abstract

The most widely used integrated assessment model for studying the economics of climate change is the dynamic/regional integrated model of climate and economy (DICE/RICE) [2, 3]. In this document, we first represent the RICE-2010 model as a dynamic game, termed the RICE-GAME model. Then, both cooperative and non-cooperative solutions to the RICE-GAME model are considered. Next, a description of how to use the repository *RICE-GAME* on GitHub is provided [6]. The repository *RICE-GAME* is a Matlab and CasADi-based implementation of the RICE-GAME model and its cooperative and non-cooperative solutions.

1 Description of RICE as a Dynamic Game

In what follows, we represent the RICE-2010 model as a dynamic game, termed the RICE-GAME model. Our presentation of the RICE-GAME model is based on the RICE-2010 model with slight modifications, but the nature of being a dynamic game is preserved.

Regions. There are 12 regions in the RICE-GAME model. Each region is considered a player and the regions are indexed in $\mathcal{V} = \{1, 2, \dots, n\}$ with $n = 12$.

Time Steps and Calendar Years. The RICE-GAME model operates in periods of $\Delta = 5$ years, starting from the year 2020 as the initial year[1]. Taking the discrete time step index $\mathcal{T} = \{0, 1, \dots, T\}$, the relation between an actual calendar year and the corresponding discrete time step is determined by

$$year(t) = year(0) + 5t, \quad year(0) = 2020, \tag{1}$$

yielding calendar year $2020, 2025, 2030, \dots$ as desired.

State Variables and Control Decisions. The RICE-GAME model has $n+5$ state variables: two variables to model the temperature dynamics in the form of the temperature deviation in the atmosphere and in the lower ocean from the reference year 1750 (T^{AT} and T^{LO}, respectively), three variables to model the carbon dynamics in the form of average carbon mass in the atmosphere, the upper ocean and biosphere, and the deep ocean ($M^{\mathrm{AT}}, M^{\mathrm{UP}}$ and M^{LO}, respectively), and n variables to model the economic dynamics in the form of

[1] The RICE-2010 model operates in periods of 10 years starting from 2005.

each region's capital ($K_i, i \in \mathcal{V}$). Control decisions of the RICE-GAME model are each region's emission-reduction rate μ_i and each region's saving rate s_i where the former represents the ratio of investment to the net economic output in each region, and the latter represents the rate at which CO_2 emissions are reduced in each region.

We denote the dynamical state of the RICE-GAME model at time step $t \in \mathcal{T}$ as

$$\mathbf{x}(t) = [T^{\mathrm{AT}}(t); T^{\mathrm{LO}}(t); M^{\mathrm{AT}}(t); M^{\mathrm{UP}}(t); M^{\mathrm{LO}}(t); K_1(t); \ldots; K_n(t)] \in \mathbb{R}^{n+5}. \tag{2}$$

We also denote control decisions of region $i \in \mathcal{V}$ at time step $t \in \mathcal{T}$ by

$$\mathbf{u}_i(t) = [\mu_i(t); s_i(t)]^\top := [\mathbf{u}_{i[1]}(t); \mathbf{u}_{i[2]}(t)]^\top \in [0,1]^2. \tag{3}$$

Consequently, control decisions of the RICE dynamic game at time step $t \in \mathcal{T}$ of all players are

$$\mathbf{u}(t) = [\mu_1(t); \ldots, \mu_N(t); s_1(t); \ldots; s_N(t)] \in [0,1]^{24}. \tag{4}$$

Further denote region i's control decisions over the time horizon by $\mathbf{U}_i := [\mu_i(0); \ldots, \mu_i(T); s_i(0); \ldots; s_i(T)]$. The control decisions of all regions excluding region i at time step t is denoted by $\mathbf{u}_{-i}(t)$, and the control decisions of all regions excluding region i over the time horizon is represented by $\mathbf{U}_{-i}$.

RICE Dynamics. The RICE-GAME model is also driven by several exogenous and time-varying signals that evolve independently such as radiative forcing caused by greenhouse gases other than CO_2 emissions F^{EX}, each region's natural CO_2 emissions due to land use E_i^{land}, each region's carbon intensity σ_i, each region's total factor productivity A_i and each region's population L_i. The dynamics of $\mathbf{x}(t)$ can be written as

$$\mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)), \quad \mathbf{x}(0) = \mathbf{x}_0, \quad t \in \mathcal{T}, \tag{5}$$

where $\mathbf{x}_0$ represents the initial state of the underlying dynamical system. The transition functions $\mathbf{f} := [f_1; f_2; \ldots; f_{n+5}]$ follow the RICE dynamics:

$$\begin{bmatrix} T^{\mathrm{AT}}(t+1) \\ T^{\mathrm{LO}}(t+1) \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} T^{\mathrm{AT}}(t) \\ T^{\mathrm{LO}}(t) \end{bmatrix} + \begin{bmatrix} \xi_2 \\ 0 \end{bmatrix} F(t), \tag{6}$$

$$\begin{bmatrix} M^{\mathrm{AT}}(t+1) \\ M^{\mathrm{UP}}(t+1) \\ M^{\mathrm{LO}}(t+1) \end{bmatrix} = \begin{bmatrix} \zeta_{11} & \zeta_{12} & 0 \\ \zeta_{21} & \zeta_{22} & \zeta_{23} \\ 0 & \zeta_{32} & \zeta_{33} \end{bmatrix} \begin{bmatrix} M^{\mathrm{AT}}(t) \\ M^{\mathrm{UP}}(t) \\ M^{\mathrm{LO}}(t) \end{bmatrix} + \begin{bmatrix} \xi_1 \\ 0 \\ 0 \end{bmatrix} E(t), \tag{7}$$

$$\begin{aligned} K_i(t+1) = {} & (1-\delta_i^{\mathrm{K}})^5 K_i(t) + 5\Big(1 - a_i^{[1]} T^{\mathrm{AT}}(t) - a_i^{[2]} T^{\mathrm{AT}}(t)^{a_i^{[3]}}\Big) \\ & \cdot \Big(1 - \theta_i^{[1]}(t)\mu_i(t)^{\theta_i^{[2]}}\Big) A_i(t) K_i(t)^{\gamma_i} \Big(\frac{L_i(t)}{1000}\Big)^{1-\gamma_i} s_i(t), \end{aligned} \tag{8}$$

where total radiative forcing $F(t)$ at time step t, each region's total emissions $E_i(t)$ including natural emissions and industrial emissions at time step t and global emission $E(t)$ at time step t are given by

$$F(t) = \eta \log_2\Big(\frac{M^{\mathrm{AT}}(t)}{M^{\mathrm{AT},1750}}\Big) + F^{\mathrm{EX}}(t), \tag{9}$$

$$E_i(t) = \sigma_i(t)(1-\mu_i(t)) A_i(t) K_i(t)^{\gamma_i}(t) \Big(\frac{L_i(t)}{1000}\Big)^{1-\gamma_i} + E_i^{\mathrm{land}}(t), \tag{10}$$

$$E(t) = \sum_{i=1}^{n} E_i(t), \tag{11}$$

$$\theta_i^{[1]}(t) = \frac{pb_i}{1000 \cdot \theta_i^{[2]}} (1 - \delta_i^{pb})^t \cdot \sigma_i(t). \tag{12}$$

Payoff Functions. For each region, the social welfare at time step t of the population $L_i(t)$ consuming the consumption $C_i(t)$ is defined by the population-weighted utility of per capita consumption

$$g_i(C_i(t), L_i(t)) = L_i(t) \cdot \frac{(\frac{C_i(t)}{L_i(t)})^{1-\alpha_i} - 1}{1 - \alpha_i}, \tag{13}$$

where each region's consumption at time step t is given by

$$C_i(t) = \left(1 - a_i^{[1]} T^{\mathrm{AT}}(t) - a_i^{[2]} T^{\mathrm{AT}}(t)^{a_i^{[3]}}\right)\left(1 - \theta_i^{[1]}(t)\mu_i(t)^{\theta_i^{[2]}}\right) A_i(t) K_i(t)^{\gamma_i} \left(\frac{L_i(t)}{1000}\right)^{1-\gamma_i} s_i(t). \tag{14}$$

Each region's payoff function is defined as the cumulative social welfare of region i across the time horizon:

$$\begin{aligned}
\mathsf{J}_i &= \sum_{t=0}^{T} \frac{g_i(C_i(t), L_i(t))}{(1+\rho_i)^{5t}} \\
&= \sum_{t=0}^{T} \left(\Big[\frac{A_i(t) L_i(t)^{1+\alpha_i-\gamma_i}}{(1-\alpha_i)(1+\rho_i)^{5t}} \Big(1 - \mathbf{u}_{i[2]}(t)\Big)\Big(1 - a_i^{[1]}\mathbf{x}_1(t) - a_i^{[2]}\mathbf{x}_1(t)^{a_i^{[3]}}\Big)\Big(1 - \theta_i^{[1]}(t)\mathbf{u}_{i[1]}(t)^{\theta_i^{[2]}}\Big)\mathbf{x}_{5+i}(t)^{\gamma_i}\Big] \right. \\
&\quad \left. - \frac{L_i(t)}{(1-\alpha_i)(1+\rho_i)^{5t}} \right).
\end{aligned} \tag{15}$$

For each region $i \in \mathcal{V}$, naturally it will attempt to maximize its cumulative social welfare.

We have now formally presented the RICE-GAME model where the regions as players seek to plan their control decisions in emission-reduction rate and saving rate for the entire time horizon

$$\mathbf{U}_i := [\mu_i(0); \ldots, \mu_i(T); s_i(0); \ldots; s_i(T)]$$

so as to maximize their payoff functions (15) subject to the underlying dynamical system (5), represented in (6)-(8).

The Social Cost of CO_2. The regional SCC is then given by the ratio of the regional marginal welfare with respect to regional emissions and with respect to regional consumption

$$\begin{aligned}
\mathrm{SCC}_i(t) &= -1000 \cdot \frac{\partial \mathsf{J}_i}{\partial E_i(t)} \Big/ \frac{\partial \mathsf{J}_i}{\partial C_i(t)} \\
&= -1000 \cdot \frac{\partial C_i(t)}{\partial E_i(t)}.
\end{aligned} \tag{16}$$

Parameters, Exogenous Signals and Initial State. The values for the parameters and initial state can be found in the tables at the end of this document. In the RICE-GAME model, the initial state are calibrated to match the data in the starting year 2020. The values for the parameters in the geophysical sector uses the latest updated values in the DICE-2016 model. The values for the exogenous and time-varying signals can be found in the Excel file named `exogenous_states_long.mat` in the repository *RICE-GAME* .

2 Cooperative Decisions

In this section, we study the solutions to the RICE-Game model under cooperative settings. First of all, we revisit the classical RICE solution concept defined by a system-level social welfare maximization. Next, we move to Pareto solutions to the RICE-Game model. Finally, we introduce a receding horizon solution to the classical RICE solution.

2.1 Solution to RICE Social Welfare Maximization

RICE social welfare maximization is for a centralized climate policy planner to compute the $\mathbf{U}_i, i \in \mathcal{V}$ that maximize the sum of the weighed regional social welfare across all regions for a given initial condition $\mathbf{x}_0$:

$$\begin{aligned} \max_{\mathbf{U}_i, i\in\mathcal{V}} \quad & \sum_{i=1}^{n} c_i \mathsf{J}_i, \\ \text{subject to} \quad & \mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)), \quad \mathbf{x}(0) = \mathbf{x}_0, t \in \mathcal{T} \\ & \mathbf{u}(t) \in [0,1]^{24}, \quad t \in \mathcal{T}. \end{aligned} \tag{17}$$

where the values of $c_i, i \in \mathcal{V}$ can be found in Table 3.

2.2 Pareto Frontier between Developed and Developing Regions

The regions in the RICE-GAME model are classified into two board categories: developed regions and developing regions. We denote $\mathcal{V}_{developed} = \{1, 2, 3, 11\}$ and $\mathcal{V}_{developing} = \{4, 5, 6, 7, 8, 9, 10, 12\}$. We also denote the sum of the social welfare across the developed regions by $\mathsf{W}_{developed} = \sum_{i\in\mathcal{V}_{developed}} \mathsf{J}_i$, and the sum of the social welfare across the developing regions by $\mathsf{W}_{developing} = \sum_{i\in\mathcal{V}_{developing}} \mathsf{J}_i$,.

A centralized climate policy planner can find Pareto solutions by computing the $\mathbf{U}_i, i \in \mathcal{V}$ that maximize the following optimization problem with $0 \le p \le 1$ for a given initial condition $\mathbf{x}_0$:

$$\begin{aligned} \max_{\mathbf{U}_i, i\in\mathcal{V}} \quad & p \cdot \mathsf{W}_{developed} + (1-p) \cdot \mathsf{W}_{developing} \\ \text{subject to} \quad & \mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)), \quad \mathbf{x}(0) = \mathbf{x}_0, t \in \mathcal{T} \\ & \mathbf{u}(t) \in [0,1]^{24}, \quad t \in \mathcal{T}. \end{aligned} \tag{18}$$

By then trying various values of p and solving the respective optimization problem (18) under each p, the social welfare Pareto frontier is approximated.

2.3 Receding Horizon Solution to RICE Social Welfare Maximization

A centralized climate policy planner can use the receding horizon approach to approximately solve RICE social welfare maximization problem.

For the receding horizon approach, we denote the prediction horizon by T_{rh} and the simulation horizon by T_{sim}. We introduce $l(t, \mathbf{x}(t), \mathbf{u}(t)) := \sum_{i=1}^{N} c_i \cdot \frac{g_i(C_i(t), L_i(t))}{(1+\rho_i)^{5t}}$. We assume a full measurement of the estimate of the state $\mathbf{x}(t)$ is available at each time step $t \in \mathcal{T}_{sim} := \{0, 1, \ldots, T_{sim}\}$. We present the receding horizon approximation of (17) in Algorithm 1:

After Algorithm 1, the control decision profile $\mathbf{U}^{\text{rhw}} := [\mathbf{u}^{\text{rhw}}(0)^\top, \ldots, \mathbf{u}^{\text{rhw}}(T_{sim})^\top]^\top$ is the receding horizon solution to RICE social welfare maximization problem (17).

Algorithm 1 MPC-RICE

Input: simulation horizon T_{sim}; prediction horizon T_{rh}.

$t \leftarrow 0$
while $t \leq T_{sim}$ **do**
  observe $\mathbf{x}(t)$
  compute the optimal solution $\mathbf{u}^*(s), s \in \mathcal{S} := \{t, t+1, \ldots, t+T_{rh}\}$, to the following optimization problem over the receding horizon $\mathcal{S}$

$$\begin{aligned} \max_{\mathbf{u}(s), \forall s \in \mathcal{S}} \quad & \sum_{s=t}^{t+T_{rh}} l(s, \mathbf{x}(s), \mathbf{u}(s)) \\ \text{subject to} \quad & \mathbf{x}(s+1) = \mathbf{f}(s, \mathbf{x}(s), \mathbf{u}(s)), \quad s \in \mathcal{S}, \\ & \mathbf{u}(s) \in [0,1]^{24}, \quad s \in \mathcal{S}. \end{aligned} \tag{19}$$

  apply $\mathbf{u}^{\text{rhw}}(t) := \mathbf{u}^*(t)$ to RICE dynamic game
end while

3 Non-cooperative Decisions

In this section, we study the solutions to the RICE-GAME model under non-cooperative settings. We introduce two planning process: dynamic best-response planning and decentralized myopic receding horizon feedback planning.

3.1 Dynamic Best-response Planning

Suppose that RICE dynamic game is repeatedly played for K iterations. In each iteration, each player's control decisions is a sequence of control over the entire time horizon. The dynamic best-response planning is for each player to plan a new sequence of control over the entire time horizon in each iteration that maximizes its payoff function given other players' control sequences in the last iteration.

Denote the aggregated control decisions of all players over the entire time horizon in iteration $k = 1, \cdots, K$ in dynamic best-response planning by $\mathbf{U}^{(k)}$. We present the dynamic best-response planning in Algorithm 2.

3.2 Decentralized Myopic Receding Horizon Feedback Planning

The decentralized myopic receding horizon feedback planning is for each player at each time step to plan control decisions for the next time step that maximize its payoff function assuming that other players' control decisions at current time step would continue being taken for a receding horizon.

Assume that at each time step, each player can observe the game state at the current time step, and all other players' control decisions that have been taken. It is also assumed that each player would myopically consider that all other players would continue taking the control decisions at current time step for a receding time horizon. Denote $\mathbf{U}_i^{fb}(t)$ as player i's receding horizon feedback control decision at time step t. We state the decentralized myopic receding horizon feedback planning in Algorithm 3.

Algorithm 2 The Dynamic Best-response Algorithm

Initialize: any cooperative solution $[\mathbf{U}_1^c; \cdots; \mathbf{U}_N^c]$.

let $\mathbf{U}^{(0)} = [\mathbf{U}_1^c; \cdots; \mathbf{U}_N^c]$

$k \leftarrow 0$

while `k` $\leq$ `K` **do**

  for `each player` $i \in \mathcal{V}$ **do**

    observe $\mathbf{U}_{-i}^{(k)}$

    compute $\mathbf{U}_i^{(k+1)}$ by solving the problem

$$\begin{aligned} \max_{\mathbf{U}_i} \quad & \mathsf{J}_i(\mathbf{X}, \mathbf{U}_i, \mathbf{U}_{-i}), && \text{(20a)} \\ s.t. \quad & \mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)), && \text{(20b)} \\ & \mathbf{U}_{-i} = \mathbf{U}_{-i}^{(k)}, && \text{(20c)} \\ & \mathbf{x}(0) = \mathbf{x}_0. && \text{(20d)} \end{aligned}$$

  end for

  $\mathbf{U}^{(k+1)} = [\mathbf{U}_1^{(k+1)}; \cdots; \mathbf{U}_N^{(k+1)}]$

  $k \leftarrow k + 1$

end while

4 Description of Code

In what follows, a description of how to use the repository *RICE-GAME* on GitHub is provided [6]. The repository *RICE-GAME* is a Matlab and CasADi-based implementation of the RICE-GAME model and its cooperative and non-cooperative solutions. A Matlab implementation of DICE and receding horizon solution to DICE can be found in [7, 8].

The repository *RICE-GAME* consists of two folders:

- `/RICE-GAME/cooperative` provides cooperative solutions to the RICE-GAME model under cooperative settings;
- `/RICE-GAME/non-cooperative` provides non-cooperative solutions to the RICE-GAME model under non-cooperative settings.

4.1 Cooperative Solutions

Under cooperative settings, the following cooperative solutions are considered:

- Solution to RICE social welfare maximization under the directory

  `/RICE-GAME/cooperative/RICE SWM`;
- Pareto frontier between developed and developing regions under the directory

  `/RICE-GAME/cooperative/Pareto Frontier`;
- Receding horizon solution to RICE social welfare maximization under the directory

Algorithm 3 The Decentralized Myopic Receding Horizon Feedback Algorithm

Initialize: any cooperative solution $[\mathbf{U}_1^c; \cdots; \mathbf{U}_N^c]$.

Input: simulation horizon T_{sim}; prediction horizon T_{rh}.

let $\mathbf{U}_i^{fb}(0) = \mathbf{U}_i^c(0), \forall i \in \mathcal{V}$
$t \leftarrow 0$
while $t \leq T_{sim}$ **do**
  for `each player` $i \in \mathcal{V}$ **do**
    Take action $\mathbf{U}_i^{fb}(t)$
  end for
  for `each player` $i \in \mathcal{V}$ **do**
    observe $\mathbf{x}(t)$ and $\mathbf{U}^{fb}(t)$
    compute $\mathbf{x}(t+1)$ according to (5)
    assume all players $j \in \mathcal{V}/\{i\}$ will continue to play $\mathbf{U}_{-i}^{fb}(t)$, $t \in \mathcal{S} := \{t+1, \cdots, t+T_{rh}\}$.
    compute the optimal solution $\{\mathbf{u}_i^*(s), \forall s \in \mathcal{S}\}$ to the following decentralized receding horizon optimization problem

$$\max_{\mathbf{u}_i(s), \forall s \in \mathcal{S}} \quad \sum_{s=t+1}^{t+T_{rh}} g_i(s, \mathbf{x}(s), \mathbf{u}_i(s), \mathbf{u}_{-i}(s)) \tag{21a}$$

$$s.t. \quad \mathbf{x}(s+1) = \mathbf{f}(s, \mathbf{x}(s), \mathbf{u}(s)), \tag{21b}$$

$$\mathbf{u}_{-i}(s) = \mathbf{U}_{-i}^{fb}(t), s \in \mathcal{S}. \tag{21c}$$

    plan $\mathbf{U}_i^{fb}(t+1) = \mathbf{u}_i^*(t+1)$
  end for
end while

🗁 /RICE-GAME/cooperative/MPC-RICE.

Solution to RICE Social Welfare Maximization. It consists of seven files and a folder:

- `RICE_SWM.m` is the top-level file and calls the subsequent files. It provides an implementation of RICE social welfare maximization.
- `specify_paramaters.m` specifies and returns the parameters and exogenous terms for the RICE-GAME model, in the structure `Params`.
- `solve_swm_problem.m` is a function that solves RICE social welfare maximization problem (17) given initial condition `x0` and time horizon `problem_horizon`.
- `test_rice_dynamics.m` is a function that takes all players' control decisions of `double` data type as one of input arguments and calculates the dynamical states of the RICE-Game model (all dynamical states belong to `double` data type). In addition, it also calculates the value of the payoff functions (belonging to `double` data type) and the quantities for each region's emissions and consumption (belonging to `double` data type).
- `rice_dynamics.m` is a function that takes all players' control decisions of `SX` data type from CasADi as one of input arguments and calculates the dynamical states of the RICE-Game model (all dynamical states belong to `SX` data type). In addition, it also calculates the value of the payoff functions (belonging to `SX` data type) and the quantities for each region's emissions and consumption (belonging to `SX` data type).
- `try_a_guess.m` returns an initial guess as a starting point for RICE social welfare maximization problem (17).
- `plot_results.m` generates plots of the trajectories of the emission-reduction rate, saving rate, atmospheric temperature deviation and social cost of CO_2.
- 🗁 `./results` is a folder where the results computed from `RICE_SWM.m` are saved.

Pareto Frontier between Developed and Developing Regions. It consists of six files and a folder:

- `Pareto.m` is the top-level file and calls the subsequent files. It takes 1001 linearly spaced values between 0 and 1 as the values of p, and solves the respective optimization problem (18) under each p.
- `specify_paramaters.m`, `rice_dynamics.m` and `test_rice_dynamics.m` are the same as those in RICE social welfare maximization.
- `plot_pareto.m` calls the subsequent file `zoomplot.m` and generates plots of the social welfare Pareto frontier between developed regions and developing regions, and the atmospheric temperature deviation at the final time step versus the parameter p.
- `zoomplot.m` adds a zoomed plot inset to a plot for highlighting a subarea.
- 🗁 `./results` is a folder where the results computed from `Pareto.m` are saved.

Receding Horizon Solution to RICE Social Welfare Maximization. It consists of seven files and a folder:

- `MPC_RICE.m` is the top-level file and calls the subsequent files. It provides an implementation of Algorithm 1. The simulation horizon is specified in `T_simulation`. The prediction horizon is set in `T_prediction` and `problem_horizon`.
- `specify_paramaters.m`, `solve_swm_problem.m`, `rice_dynamics.m`, `test_rice_dynamics.m` and `try_a_guess.m` are the same as those in RICE social welfare maximization.
- `comparison_dg_mpc.m` generates the plots of the comparison of the atmospheric temperature deviation trajectories and optimal emission-reduction rates under the optimal control decisions solved from (17) and Algorithm 1 with different receding horizons.
- `./results` is a folder where the results computed from `MPC_RICE.m` are saved.

4.2 Non-cooperative Solutions

Under non-cooperative settings, the following planning processes are considered:

- Dynamic best-response planning under the directory
  `/RICE-GAME/non-cooperative/BR`;
- Decentralized myopic receding horizon feedback planning under the directory
  `/RICE-GAME/non-cooperative/Decentralized RH`.

Dynamic Best-response Planning. There are nine files and a folder:

- `BR_Algorithm.m` is the top-level file and calls the subsequent files. It provides an implementation of Algorithm 2.
- `specify_paramaters.m` and `test_rice_dynamics.m` are the same as those in RICE social welfare maximization.
- `ith_rice_dynamics.m` is a function that takes player i's control decisions of `SX` data type from CasADi and other players' control decisions of `double` data type as two of input arguments and calculates the dynamical states of the RICE-Game model (all dynamical states belong to `SX` data type). In addition, it also calculates the value of the payoff functions (belonging to `SX` data type) and the quantities for each region's emissions and consumption (belonging to `SX` data type).
- `solve_ith_problem.m` is a function that solves the optimization problem (20) in the iteration $k+1$ of the dynamic best-response planning given initial condition `x0`, time horizon `problem_horizon` and other players' control decisions `U_br(:,:,k)` in the iteration k.
- `comparison_br.m` generates the plots of the comparison of the atmospheric temperature deviation trajectories and optimal emission-reduction rates under the optimal control decisions solved from (17) and Algorithm 2 in the final iteration K.

- `plot_br_convergence.m` call the subsequent file `createaxes.m` and generates the plot of the convergence of $\|\mathbf{U}_i^{(k)} - \mathbf{U}_i^{(k-1)}\|$ versus iterations.
- `createaxes.m` is a function that adds a zoomed plot inset to a plot for highlighting a subarea.
- 🗀 `./results` is a folder where the results computed from `BR_Algorithm.m` are saved.

Decentralized Myopic Receding Horizon Feedback Planning. There are six files and a folder:

- `Decentralized_RH_Feedback.m` is the top-level file and calls the subsequent files. It provides an implementation of Algorithm 3.
- `specify_paramaters.m` and `test_rice_dynamics.m` are the same as those in RICE social welfare maximization
- `solve_ith_problem.m` and `ith_rice_dynamics.m` are the same as those in the dynamic best-response planning.
- `comparison_DRH.m` generates the plots of the comparison of the atmospheric temperature deviation trajectories and optimal emission-reduction rates under the optimal control decisions solved from (17), Algorithm 2 in the final iteration K, and Algorithm 3 with different prediction horizons.
- 🗀 `./results` is a folder where the results computed from `Decentralized_RH_Feedback.m` are saved.

5 Software Requirements

This implementation of *RICE-GAME* is implemented in the platform of Matlab [5] along with the CasADi framework for automatic differentiation and numeric optimization [1]. Version 3.5.5 of CasADi is used. The compatible Matlab version for different operating systems can be found on the CasADi website [4]. The repository *RICE-GAME* is distributed under the GNU General Public License v3.0.

References

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[4] CasADi v3.5.5, available at `https://web.casadi.org/get/`.

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[7] T. Faulwasser, C. M. Kellett, and S. R. Weller, “DICE2013R-mc: Dynamic Integrated model of Climate and Economy 2013R - Matlab and CasADi”, available at `https://github.com/cmkellett/DICE2013R-mc`.

[8] T. Faulwasser, C. M. Kellett, and S. R. Weller, “MPC-DICE: Model Predictive Control – Dynamic Integrated model of Climate and Economy”, available at `https://github.com/cmkellett/MPC-DICE`.

Table 1: Variables definitions for the RICE-GAME model. State variables and control variables are marked with asterisks $*$ and stars $\star$, respectively.

Variable	Definition	Unit
Regions:		
i	Index of a region	-
Time steps and Calendar years:		
t	Discrete time step	-
$year(t)$	Calendar year	-
Temperature dynamics:		
${}^*T^{\mathrm{AT}}(t)$	Atmospheric temperature deviation from the reference year 1750	°C
${}^*T^{\mathrm{LO}}(t)$	Temperature deviation in lower ocean from the reference year 1750	°C
$F(t)$	Total radiative forcing	$\mathrm{W/m^2}$
$F^{\mathrm{EX}}(t)$	Radiative forcing caused by greenhouse gases other than $\mathrm{CO_2}$	$\mathrm{W/m^2}$
Carbon dynamics:		
${}^*M^{\mathrm{AT}}(t)$	Carbon mass in reservoir for atmosphere	GtC
${}^*M^{\mathrm{UP}}(t)$	Carbon mass in reservoir for upper ocean	GtC
${}^*M^{\mathrm{LO}}(t)$	Carbon mass in reservoir for lower ocean	GtC
$\sigma_i(t)$	Region i's ratio of uncontrolled industrial emissions to gross economic output	GtC/trillions USD
${}^\star\mu_i(t)$	Region i's emission-reduction rate	-
$E_i^{\mathrm{land}}(t)$	Region i's natural $\mathrm{CO_2}$ emissions from land use	$\mathrm{GtCO_2}$
$E_i(t)$	Region i's $\mathrm{CO_2}$ emissions including industrial emissions and natural emissions	$\mathrm{GtCO_2}$
$E(t)$	Global $\mathrm{CO_2}$ emissions across all regions	$\mathrm{GtCO_2}$
Economic dynamics:		
$L_i(t)$	Region i's population	millions people
$A_i(t)$	Region i's total productivity factor	-
${}^*K_i(t)$	Region i's capital	trillions USD
$g_i(t)$	Region i's utility	trillions USD
$C_i(t)$	Region i's consumption	trillions USD
J_i	Region i's cumulative social welfare across the time horizon	trillions USD
${}^\star s_i(t)$	Saving rate	-

Table 2: Parameters for the RICE-GAME model.

Parameter	Description	Value
Regions:		
n	Number of regions	12
Time steps and Calendar years:		
Δ	Sampling rate	5
T	Horizon length	120
$year(0)$	Starting year	2020
Temperature dynamics:		
ϕ_{11}	Diffusion coefficient between temperature layers	0.871810629
ϕ_{12}	Diffusion coefficient between temperature layers	0.008844
ϕ_{21}	Diffusion coefficient between temperature layers	0.025
ϕ_{22}	Diffusion coefficient between temperature layers	0.975
η	Forcing associated with equilibrium of carbon doubling (W/m^2)	3.6813
ξ_2	Multiplier for η	0.1005
$M_{\mathrm{AT},1750}$	Atmospheric mass of carbon in the year 1750 (GtC)	588
Carbon dynamics:		
ζ_{11}	Diffusion coefficient between carbon reservoirs	0.88
ζ_{12}	Diffusion coefficient between carbon reservoirs	0.196
ζ_{21}	Diffusion coefficient between carbon reservoirs	0.12
ζ_{22}	Diffusion coefficient between carbon reservoirs	0.797
ζ_{23}	Diffusion coefficient between carbon reservoirs	0.001465
ζ_{32}	Diffusion coefficient between carbon reservoirs	0.007
ζ_{33}	Diffusion coefficient between carbon reservoirs	0.99853488
ξ_1	Conversion factor from emissions to carbon mass (GtC / $GtCO_2$)	5*0.27272727
Economic dynamics:		
δ_i^{K}	Region i's depreciation rate on capital per year	See Table 3
$a_i^{[1]}$	Region i's damage coefficient on temperature	
$a_i^{[2]}$	Region i's damage exponent	
$a_i^{[3]}$	Region i's damage coefficient on temperature squared	
$\theta_i^{[2]}$	Region i's exponent of emission-reduction cost function	
γ_i	Region i's capital elasticity in gross economic output	
pb_i	Region i' backstop technology price in the year 2020 (USD/tCO_2)	
δ_i^{pb}	Region i's decline rate of backstop cost	
α_i	Region i's elasticity of marginal utility of consumption	
ρ_i	Region i's rate of social time preference per year	
c_i	Region i's Negishi parameter in RICE social welfare function	

Table 3: Parameter Values for the economic dynamics.

	US	EU	JN	RS	EUR	CN	IN	ME	AF	LA	OHI	OA
δ_i^{K}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
$a_i^{[1]}$	0	0	0	0	0	0.0008	0.0044	0.0028	0.0034	0.0006	0	0.0018
$a_i^{[2]}$	0.0014	0.0016	0.0016	0.0011	0.0013	0.0013	0.0017	0.0016	0.0020	0.0014	0.0016	0.0017
$a_i^{[3]}$	2	2	2	2	2	2	2	2	2	2	2	2
$\theta_i^{[2]}$	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
γ_i	0.141	0.159	0.162	0.115	0.130	0.126	0.169	0.159	0.198	0.135	0.156	0.173
pb_i	1051	1635	1635	701	701	817	1284	1167	1284	1518	1284	1401
δ_i^{pb}	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
α_i	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45
ρ_i	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
c_i	0.2010	0.1030	0.1300	0.0300	0.0080	0.0040	0.0020	0.0156	0.0013	0.0157	0.1187	0.0031

Table 4: Default initial condition for the geophysical dynamics.

	Value
$T^{\mathrm{AT}}(0)$	1.15
$T^{\mathrm{LO}}(0)$	0.05
$M^{\mathrm{AT}}(0)$	979
$M^{\mathrm{UP}}(0)$	485
$M^{\mathrm{LO}}(0)$	1741

Table 5: Default initial condition for the economic dynamics.

	US	EU	JN	RS	EUR	CN	IN	ME	AF	LA	OHI	OA
$K_i(0)$	36.59	37.11	9.60	4.96	2.61	28.47	11.94	14.46	6.81	17.49	11.61	11.09