

$$\begin{aligned}
P_z^{iT} b \cdot \frac{T_x^i}{T_z^i} - P_x^{iT} b &= 0 \\
P_z^{iT} b \cdot \frac{T_y^i}{T_z^i} - P_y^{iT} b &= 0 \\
P_{z0}^i \frac{T_x^i}{T_z^i} b_x + P_{z1}^i \frac{T_x^i}{T_z^i} b_y + P_{z2}^i \frac{T_x^i}{T_z^i} b_z + P_{z3}^i \frac{T_x^i}{T_z^i} b_w - P_{x0}^i b_x + P_{x1}^i b_y + P_{x2}^i b_z + P_{x3}^i b_w &= 0 \\
P_{z0}^i \frac{T_y^i}{T_z^i} b_x + P_{z1}^i \frac{T_y^i}{T_z^i} b_y + P_{z2}^i \frac{T_y^i}{T_z^i} b_z + P_{z3}^i \frac{T_y^i}{T_z^i} b_w - P_{y0}^i b_x + P_{y1}^i b_y + P_{y2}^i b_z + P_{y3}^i b_w &= 0 \\
\left( P_{z0}^i \frac{T_x^i}{T_z^i} - P_{x0}^i \right) b_x + \left( P_{z1}^i \frac{T_x^i}{T_z^i} - P_{x1}^i \right) b_y + \left( P_{z2}^i \frac{T_x^i}{T_z^i} - P_{x2}^i \right) b_z + \left( P_{z3}^i \frac{T_x^i}{T_z^i} - P_{x3}^i \right) b_w &= 0 \\
\left( P_{z0}^i \frac{T_y^i}{T_z^i} - P_{y0}^i \right) b_x + \left( P_{z1}^i \frac{T_y^i}{T_z^i} - P_{y1}^i \right) b_y + \left( P_{z2}^i \frac{T_y^i}{T_z^i} - P_{y2}^i \right) b_z + \left( P_{z3}^i \frac{T_y^i}{T_z^i} - P_{y3}^i \right) b_w &= 0
\end{aligned} \tag{1}$$

$$\begin{bmatrix}
P_{z0}^1 \frac{T_x^1}{T_z^1} - P_{x0}^1 & P_{z1}^1 \frac{T_x^1}{T_z^1} - P_{x1}^1 & P_{z2}^1 \frac{T_x^1}{T_z^1} - P_{x2}^1 & P_{z3}^1 \frac{T_x^1}{T_z^1} - P_{x3}^1 \\
P_{z0}^1 \frac{T_y^1}{T_z^1} - P_{y0}^1 & P_{z1}^1 \frac{T_y^1}{T_z^1} - P_{y1}^1 & P_{z2}^1 \frac{T_y^1}{T_z^1} - P_{y2}^1 & P_{z3}^1 \frac{T_y^1}{T_z^1} - P_{y3}^1 \\
\vdots & \vdots & \vdots & \vdots \\
P_{z0}^m \frac{T_x^m}{T_z^m} - P_{x0}^m & P_{z1}^m \frac{T_x^m}{T_z^m} - P_{x1}^m & P_{z2}^m \frac{T_x^m}{T_z^m} - P_{x2}^m & P_{z3}^m \frac{T_x^m}{T_z^m} - P_{x3}^m \\
P_{z0}^m \frac{T_y^m}{T_z^m} - P_{y0}^m & P_{z1}^m \frac{T_y^m}{T_z^m} - P_{y1}^m & P_{z2}^m \frac{T_y^m}{T_z^m} - P_{y2}^m & P_{z3}^m \frac{T_y^m}{T_z^m} - P_{y3}^m
\end{bmatrix} \cdot \begin{bmatrix} b_x \\ b_y \\ b_z \\ b_w \end{bmatrix} = 0$$

$$K^i = \begin{bmatrix} f^i & 0 & u_0^1 \\ 0 & \alpha f^i & v_0^1 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} f^i & 0 & 0 \\ 0 & f^i & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$M^i = K^i \cdot R^i = \begin{bmatrix} f^i & 0 & 0 \\ 0 & f^i & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} R_x^{iT} \\ R_y^{iT} \\ R_z^{iT} \end{bmatrix} = \begin{bmatrix} f^i \cdot R_x^{iT} \\ f^i \cdot R_y^{iT} \\ -R_z^{iT} \end{bmatrix}$$

$$\begin{aligned}
M_x^{iT} \cdot M_x^{iT} &= M_y^{iT} \cdot M_y^{iT} \\
M_x^{iT} \cdot M_y^{iT} &= 0 \\
M_y^{iT} \cdot M_z^{iT} &= 0 \\
M_z^{iT} \cdot M_x^{iT} &= 0
\end{aligned}$$

$$Q = \begin{bmatrix} q_{00} & q_{01} & q_{02} & q_{03} \\ q_{01} & q_{11} & q_{12} & q_{13} \\ q_{02} & q_{12} & q_{22} & q_{23} \\ q_{03} & q_{13} & q_{23} & q_{33} \end{bmatrix} \\
MM^T = PQP^T$$

$$P_x^{iT} Q P_x^i - P_y^{iT} Q P_y^i = 0$$

$$P_x^{iT} Q P_y^i = 0$$

$$P_y^{iT} Q P_z^i = 0$$

$$P_z^{iT} Q P_x^i = 0$$

$$\begin{bmatrix} P_{x0}^i q_{00} + P_{x1}^i q_{01} + P_{x2}^i q_{02} + P_{x3}^i q_{03} \\ P_{x0}^i q_{01} + P_{x1}^i q_{11} + P_{x2}^i q_{12} + P_{x3}^i q_{13} \\ P_{x0}^i q_{02} + P_{x1}^i q_{12} + P_{x2}^i q_{22} + P_{x3}^i q_{23} \\ P_{x0}^i q_{03} + P_{x1}^i q_{13} + P_{x2}^i q_{23} + P_{x3}^i q_{33} \end{bmatrix}^T \begin{bmatrix} P_{x0}^i \\ P_{x1}^i \\ P_{x2}^i \\ P_{x3}^i \end{bmatrix} - \begin{bmatrix} P_{y0}^i q_{00} + P_{y1}^i q_{01} + P_{y2}^i q_{02} + P_{y3}^i q_{03} \\ P_{y0}^i q_{01} + P_{y1}^i q_{11} + P_{y2}^i q_{12} + P_{y3}^i q_{13} \\ P_{y0}^i q_{02} + P_{y1}^i q_{12} + P_{y2}^i q_{22} + P_{y3}^i q_{23} \\ P_{y0}^i q_{03} + P_{y1}^i q_{13} + P_{y2}^i q_{23} + P_{y3}^i q_{33} \end{bmatrix}^T \begin{bmatrix} P_{y0}^i \\ P_{y1}^i \\ P_{y2}^i \\ P_{y3}^i \end{bmatrix} = 0$$

$$\begin{aligned} & \left( P_{x0}^i P_{x0}^i - P_{y0}^i P_{y0}^i \right) q_{00} + \left( P_{x1}^i P_{x0}^i - P_{y1}^i P_{y0}^i \right) q_{01} + \left( P_{x2}^i P_{x0}^i - P_{y2}^i P_{y0}^i \right) q_{02} + \left( P_{x3}^i P_{x0}^i - P_{x3}^i P_{x0}^i \right) q_{03} + \\ & \left( P_{x0}^i P_{x1}^i - P_{y0}^i P_{y1}^i \right) q_{01} + \left( P_{x1}^i P_{x1}^i - P_{y1}^i P_{y1}^i \right) q_{11} + \left( P_{x2}^i P_{x1}^i - P_{y2}^i P_{y1}^i \right) q_{12} + \left( P_{x3}^i P_{x1}^i - P_{x3}^i P_{x1}^i \right) q_{13} + \\ & \left( P_{x0}^i P_{x2}^i - P_{y0}^i P_{y2}^i \right) q_{02} + \left( P_{x1}^i P_{x2}^i - P_{y1}^i P_{y2}^i \right) q_{12} + \left( P_{x2}^i P_{x2}^i - P_{y2}^i P_{y2}^i \right) q_{22} + \left( P_{x3}^i P_{x2}^i - P_{x3}^i P_{x2}^i \right) q_{23} + \\ & \left( P_{x0}^i P_{x3}^i - P_{y0}^i P_{y3}^i \right) q_{03} + \left( P_{x1}^i P_{x3}^i - P_{y1}^i P_{y3}^i \right) q_{13} + \left( P_{x2}^i P_{x3}^i - P_{y2}^i P_{y3}^i \right) q_{23} + \left( P_{x3}^i P_{x3}^i - P_{x3}^i P_{x3}^i \right) q_{33} = 0 \end{aligned}$$

$$\begin{bmatrix} q_{00} \\ q_{01} \\ q_{02} \\ q_{03} \\ q_{11} \\ q_{12} \\ q_{13} \\ q_{22} \\ q_{23} \\ q_{33} \end{bmatrix} \cdot \begin{bmatrix} P_{x0}^2 - P_{y0}^2 & 2P_{x0}P_{x1} - 2P_{y0}P_{y1} & 2P_{x0}P_{x2} - 2P_{y0}P_{y2} & 2P_{x0}P_{x3} - 2P_{y0}P_{y3} & P_{x1}^2 - P_{y1}^2 & 2P_{x1}P_{x2} - 2P_{y1}P_{y2} & 2P_{x1}P_{x3} - 2P_{y1}P_{y3} & P_{x2}^2 - P_{y2}^2 & 2P_{x2}P_{x3} - 2P_{y2}P_{y3} & P_{x3}^2 - P_{y3}^2 \\ P_{x0}P_{y0} & P_{x0}P_{y1} + P_{x1}P_{y0} & P_{x0}P_{y2} + P_{x2}P_{y0} & P_{x0}P_{y3} + P_{x3}P_{y0} & P_{x1}P_{y1} & P_{x1}P_{y2} + P_{x2}P_{y1} & P_{x1}P_{y3} + P_{x3}P_{y1} & P_{x2}P_{y2} & P_{x2}P_{y3} + P_{x3}P_{y2} & P_{x3}P_{y3} \\ P_{y0}P_{z0} & P_{y0}P_{z1} + P_{y1}P_{z0} & P_{y0}P_{z2} + P_{y2}P_{z0} & P_{y0}P_{z3} + P_{y3}P_{z0} & P_{y1}P_{z1} & P_{y1}P_{z2} + P_{y2}P_{z1} & P_{y1}P_{z3} + P_{y3}P_{z1} & P_{y2}P_{z2} & P_{y2}P_{z3} + P_{y3}P_{z2} & P_{y3}P_{z3} \\ P_{x0}P_{z0} & P_{x0}P_{z1} + P_{x1}P_{z0} & P_{x0}P_{z2} + P_{x2}P_{z0} & P_{x0}P_{z3} + P_{x3}P_{z0} & P_{x1}P_{z1} & P_{x1}P_{z2} + P_{x2}P_{z1} & P_{x1}P_{z3} + P_{x3}P_{z1} & P_{x2}P_{z2} & P_{x2}P_{z3} + P_{x3}P_{z2} & P_{x3}P_{z3} \\ P_{z0}^2 & 2P_{z0}P_{z1} & 2P_{z0}P_{z2} & 2P_{z0}P_{z3} & P_{z1}^2 & 2P_{z1}P_{z2} & 2P_{z1}P_{z3} & P_{z2}^2 & 2P_{z2}P_{z3} & P_{z3}^2 \end{bmatrix}.$$