Authors' Responses to the Review of "Large-Scale Constrained Multiobjective Optimization based on Variable Adaptive Optimization and Population Reconstruction" (SMCA-23-08-2471)

Dear Editors,

Thank you for coordinating the review process and the reviewers for providing comments on our paper. We hereby submit a revised version of the paper, where the revision is in blue colour. The detailed responses to reviewers' comments are listed as follows, where reviewers' comments are written in *italic* with our response in a plain font. We believe we have addressed all the comments from the reviewers, and are looking forward to hearing from you soon.

Best regards,
Kunjie Yu, Zhenyu Yang, Jing Liang, Kangjia Qiao*, Boyang Qu, Qiao Peng, and Ponnuthurai Nagaratnam Suganthan
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Associate Editor

Reviewers provide many useful and detailed comments. It is encouraged to revise this paper by carefully following the review report's comments and addressing them adequately. The response file as well as the revised paper should be uploaded for the next round of review.

Response: Thank you very much for your time. We have carefully taken all the comments into consideration in our revised manuscript. Specifically, we have provided more details and explanations of the proposed method, and enriched the analysis of the performance of the proposed method.

Reviewer 1

In this paper, the authors investigate on large-scale constrained multiobjective optimisation based on variable adaptive optimisation and population reconstruction. The paper is well organized and written. The results are of great interest.

Reviewer Point 1.1 — *However, some improvements should be included in the revised version of the paper. Namely, more background on multiobjective optimization theory is needed.*

Response: Thank you very much for your suggestions. In the revised manuscript, we have given more theoretical background on multiobjective optimization. Details are as follows (Please also see Section I on page 1 and Section II-A on page 2 of the revised manuscript).

I. Introduction

Constrained multiobjective optimization problems (CMOPs) involve the optimization of multiple objectives and the simultaneous satisfaction of constraints [1]. CMOPs are challenging to solve and have wide applicability in various domains. Examples include the optimization problems in welded beam design [2], ethylene cracking furnace design [3] and airline staffing [4]. Researchers have developed evolutionary algorithms (EAs) based on multiobjective optimization theory to handle CMOPs and obtain the well-distributed Pareto front. These solutions enable decision makers to make informed decisions, taking into account conflicting objectives and constraints.

II-A Concept of constrained multiobjective optimization

Without loss of generality, a CMOP can be formulated as [21]:

$$\begin{aligned} &\min \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_M(\mathbf{x}))^T \\ &s.t. \begin{cases} \mathbf{x} = (x_1, x_2, ..., x_D)^T \in \mathbb{S}^n \\ g_i(\mathbf{x}) \leqslant 0, i = 1, ..., p_g \\ h_i(\mathbf{x}) = 0, i = p_g + 1, ..., p_g + p_h \end{cases} \end{aligned} \tag{1}$$

where $\mathbf{F}(\mathbf{x})$ indicates conflicting objective functions; $\mathbb{S}^n$ represents the D-dimensional decision space; $\mathbf{x}$ is a candidate solution; $g_i(\mathbf{x})$ and $h_i(\mathbf{x})$ denote inequality and equality constraints, respectively; and p_g and p_h are the numbers of inequality and equality constraints, respectively. The degree of total constraint violation (CV) of a solution $\mathbf{x}$ is defined as:

$$CV(\mathbf{x}) = \sum_{i=1}^{p_g} \max(0, g_i(\mathbf{x})) + \sum_{i=p_g+1}^{p_g+p_h} \max(0, |h_i(\mathbf{x})| - \varphi) \tag{2}$$

where φ is a small positive value to relax the equality constraints into inequality constraints. $\mathbf{x}$ is called a feasible solution if $CV(\mathbf{x}) = 0$. Since in multiobjective optimisation each solution has multiple objectives, dominance relations are often used to compare solutions [22]. Given two feasible solutions $\mathbf{x}_1$ and $\mathbf{x}_2$, for each $j \in (1, ..., M)$, $f_j(\mathbf{x_1}) \leqslant f_j(\mathbf{x_2})$, and there exists $k \in (1, ..., M)$, such that $f_k(\mathbf{x_1}) < f_k(\mathbf{x_2})$, then it is said that $\mathbf{x}_1$ Pareto-dominates $\mathbf{x}_2$. A solution is said to be Pareto-optimal if no other solution can dominate it. When constraints are considered, all feasible Pareto-optimal solutions form the constrained Pareto set (CPS), whose mapping vector in the objective space is the constrained Pareto front (CPF). When no constraints are considered, all Pareto optimal solutions form the unconstrained Pareto set (UPS), and the mapping of the UPS in the objective space is called the unconstrained Pareto front (UPF).

[1] J. Wang, Y. Li, Q. Zhang, Z. Zhang, and S. Gao, “Cooperative multiobjective evolutionary algorithm with propulsive population for constrained multiobjective optimization,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 6, pp. 3476–3491, 2021.
[2] H. Chen, W. Li, W. Song, P. Yang, and W. Cui, “Grid feature-based weighted simulation method for multi-objective reliability-based design optimization,” *International Journal of Computational Intelligence Systems*, vol. 15, no. 1, p. 81, 2022.
[3] X. Lin, L. Zhao, W. Du, W. He, and F. Qian, “Data-driven modeling and cyclic scheduling for ethylene cracking furnace system with inventory constraints,” *Industrial & Engineering Chemistry Research*, vol. 60, no. 9, pp. 3687–3698, 2021.
[4] S. Z. Zhou, Z. H. Zhan, Z. G. Chen, S. Kwong, and J. Zhang, “A multi-objective ant colony system algorithm for airline crew rostering problem with fairness and satisfaction,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 22, no. 11, pp. 6784–6798, 2020.
[21] K. Yu, J. Liang, B. Qu, Y. Luo, and C. Yue, “Dynamic selection preference-assisted constrained multiobjective differential evolution,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 5, pp. 2954–2965, 2021.
[22] Q. Kang, X. Song, M. Zhou, and L. Li, “A collaborative resource allocation strategy for decomposition-based multiobjective evolutionary algorithms,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 12, pp. 2416–2423, 2018.

Reviewer Point 1.2 — *** Please note that according to IEEE publications policy guideline, no specific articles/journals are recommended for the authors to avoid the appearance of conflict of interest. ** Consequently, a minor revision is requested.*

Response: Thank you very much for your comments to help us improve the paper. We have carefully revised the article based on the comments of the associate editor and all reviewers.

Reviewer 2

In this manuscript, the authors present a large-scale constrained multi-objective optimization algorithm called LCMVAPR. This algorithm provides a variable adaptive optimization strategy to balance convergence and diversity. The decision variables are divided into convergence-related variables and diversity-related variables. In the local search stage, the algorithm proposes a population reconstruction strategy based on archives to generate initial population, which helps in rapid convergence. The manuscript evaluates LCMVAPR through experiments on several benchmark problems and three real-world dynamic economic emission dispatch problems, and the results show that the proposed method performs well.

Reviewer Point 2.1 — *There are some grammar errors or typos in the manuscript. For example,*

(1) In Section I, in the sentence "Various constraint handling techniques (CHTs) has been designed and embed into. . . ", "has" should be "have".

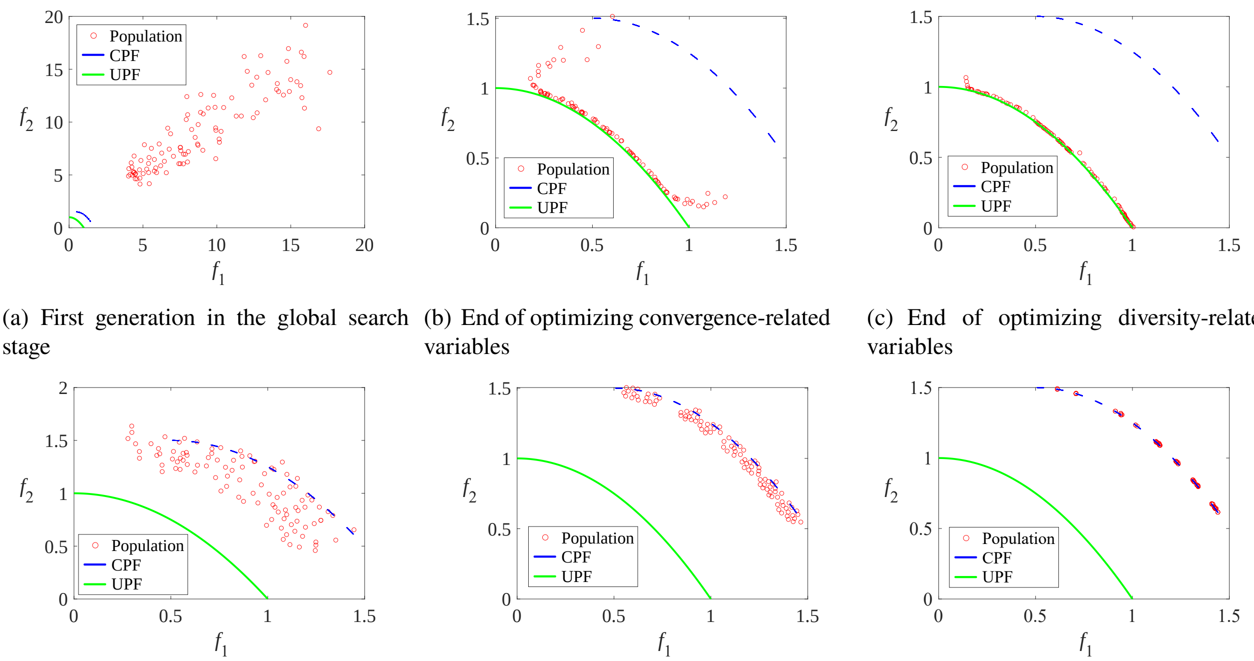
(2) In Section I, the sentence "When it comes to LSCMOPs, the effectiveness of these CMOEAs can decrease dramatically due to the "curse of dimensionality"[21]." uses two closing double quotes.

(3) In line 27 of page 10, in the sentence "Additionally, the results of the Wilcoxon test for for multiple-problems [22] are shown in Table IV.", the word 'for' is repeated.

Response: Thank you very much for pointing out these errors. In the revised manuscript, we have corrected these errors to improve the quality of the paper.

Reviewer Point 2.2 — *In the legends of Fig. 4, UPF should be represented by the blue dashed line instead of dots.*

Response: Thank you very much for pointing out this error. I think what you mean is "CPF should be represented by the blue dashed line instead of dots". In the revised manuscript, we have modified the legend of Fig. 4, in which CPF and UPF are represented by the blue and green dashed lines, respectively. The details are shown as follows (Please also see Section III on Page 7 of the revised manuscript).

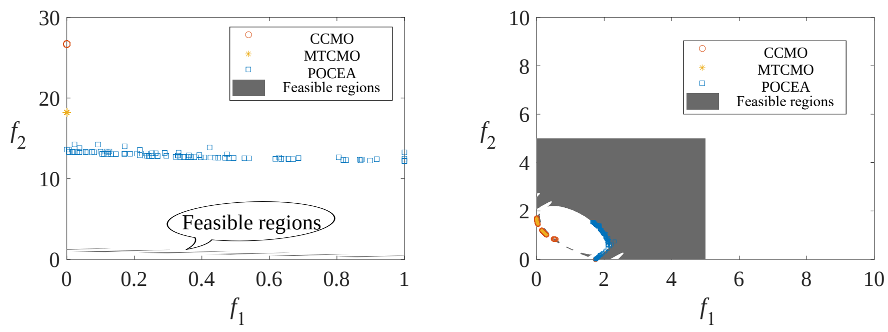


(a) First generation in the global search stage
(b) End of optimizing convergence-related variables
(c) End of optimizing diversity-related variables
(d) First generation in the local search stage
(e) Middle generation in the local search stage
(f) Last generation in the local search stage

Fig. 4. Population distribution of LCMVAPR at different stages on the 100-dimensional LIRCMOP3.

Reviewer Point 2.3 — *Some figures seem not clear enough. For example, the purple line in Fig. 2 means the distance but it isn't indicated in the legend. And the yellow line for MTCMO in Fig. 6 is too light in color to be* *observed clearly. Besides, the font of the text "Feasible regions" in the lower part of Fig. 1 (a) is too small to be observed clearly.*

Response: Thank you very much for your valuable comments. In the revised manuscript, we have modified Fig. 1 (a), Fig. 2, and Fig. 6, so that the figures can be observed more clearly. Please note that due to space limitations, Fig. 6 is not provided in the manuscript, but is shown in Fig. S-V in the supplementary file. The details are shown as follows (Please also see section I on page 2 and section III on page 6 of the revised manuscript, and page 14 of the Supplementary file).



(a) MW1 (b) LIRCMOP10

Fig. 1. Population distribution of three representative algorithms on MW1 [16] and LIRCMOP10 [17] with 100 decision variables.

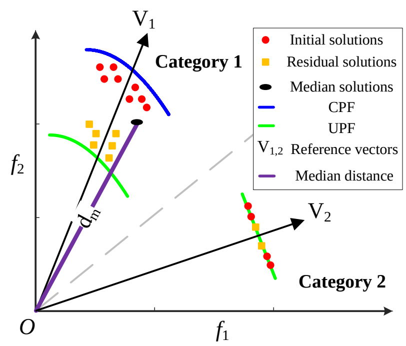
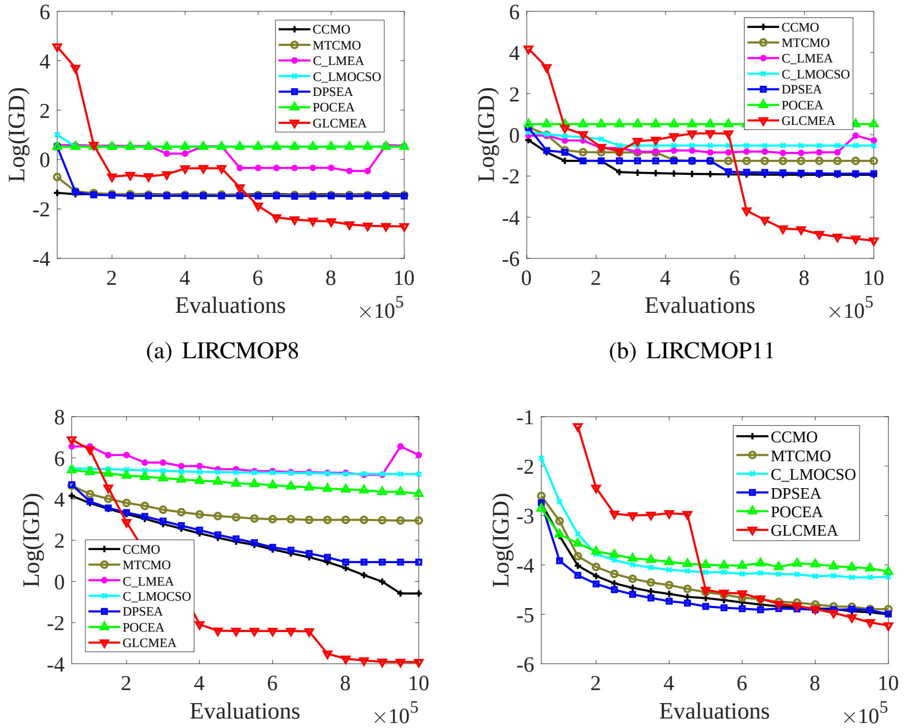


Fig. 2. Soclution selection procedure for the two types of situations in which V_1 and V_2 are respectively two reference vectors.



(a) LIRCMOP8 (b) LIRCMOP11 (c) DC3_DTLZ3 (d) MW3

Fig. S-V. Convergence curves of LCMVAPR and other algorithms for 100-dimensional IGD median values for selected problems. Note that there is no curve for C_LMEA in (d) because it does not find a feasible solution.

Reviewer Point 2.4 — *The styles of the tables are not consistent in the manuscript. For example, TABLE V lacks the relevant explanation of symbols at the bottom of the table similar to those of TABLE II and TABLE VI.*

Response: Thank you very much for your comments. In the manuscript, we have reviewed all Tables and supplemented the bottom of Table 5. The details are shown as follows (Please also see Section IV on Page 10 of the revised manuscript).

Table V: Statistics of the performance comparison of LCMVAPR and its variants according to the IGD metric.

Verification	Algorithm Comparison	IGD (+/-/=)
Effectiveness of the Variable Adaptive optimization Strategy	LCMVAPR_LMEA vs LCMVAPR	2/36/0
Effectiveness of the Archive	LCMVAPR_UPF vs LCMVAPR	0/9/29
Effectiveness of the Archive-Assisted Population Reconstruction Method	LCMVAPR_random1 vs LCMVAPR	0/8/30
	LCMVAPR_random2 vs LCMVAPR	2/11/25

The symbols "+", "-", and "=" indicate that the variant is significantly better than, worse than, and similar to the LCMVAPR, respectively.

Reviewer Point 2.5 — *The styles of names of the ordinate axes in the figures should be unified in the manuscript. For example, the name f2 of the ordinate axis in Fig. 2 is written normally, but the name f2 of the ordinate axis Fig. 1 is rotated ninety degrees counterclockwise. It is recommended that unless there are special circumstances, the name f2 of vertical axis should be written horizontally as usual.*

Response: Thank you very much for your suggestions. In the manuscript, we have unified the naming style of the ordinate axes in the figures. The vertical axis name f_2 is uniformly written normally. Details of Fig. 2 are as follows (Please also see Section IV on Page 10 of the revised manuscript).

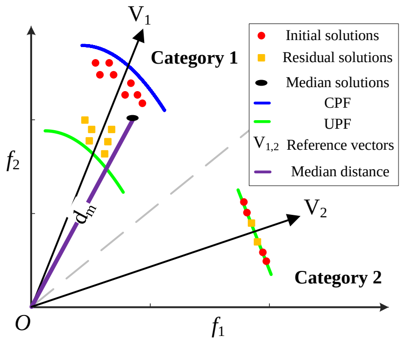


Fig. 2. Solution selection procedure for the two types of situations in which V_1 and V_2 are respectively two reference vectors.

Reviewer Point 2.6 — *Terminology in the paper should be used in a consistent way. For example, the words "optimize" and "optimise" are used interchangeably in the manuscript, and it is recommended to use one of them consistently.*

Response: Thank you very much for your comments. We have revised the manuscript to uniformly use the word "optimize" throughout the manuscript.

Reviewer Point 2.7 — *There are so many symbol names of the in the manuscript like the variable dimension D, the objective number M, the archive size N, the population size NP, and the population P, the offspring O. It's difficult to remember their meanings especially during reading Section III-F. It's suggested to put their definitions in a table.*

Response: Thank you very much for your suggestions. Section III-F in the original manuscript is an analysis of computational complexity and contains some symbols. Due to page limitations, in the revised manuscript, we have placed it in Section S-I of the Supplementary file. The symbol names and definitions are placed in Table S-I to enhance the readability. The details are shown as follows (Please also see Section S-I on Page 3 of the Supplementary file).

S-II Computational Complexity

The symbols and names commonly used in this article are shown in Table S-I. The main complexity of LCMVAPR comes from the variable grouping, the global search stage, and the local search stage. The worst complexity of variable grouping is $O(D^2)$. The global search stage includes variable optimization, archive updating, switching conditions, and environmental selection. Their worst complexities are $O(M \cdot NP^2)$, $O(M \cdot NP^2)$, $O(M \cdot N)$, $O(1)$, and $O(M \cdot NP^2)$, respectively. The local search stage involves population reconstruction and population updating, and their worst complexities are both $O(M \cdot NP^2)$. For a LSCMOP, D generally is much larger than M, NP, and N, thus the computational complexity of LCMVAPR is $O(D^2)$.

Table S-I: Related symbols and names commonly used in this article

Symbol	Name
D	Decision variable dimension
M	Objective number
N	Archive size
NP	Population size
P	Population
O	Offspring

Reviewer Point 2.8 — *The second half of Section I is too long and includes some repetitive contents. It is suggested to simplify it.*

Response: Thank you very much for your suggestion. We have simplified the second half of the section I in the revised manuscript and removed some repetitive content. Moreover, in order to enhance readability, the concepts and definitions of constrained multiobjective optimization in Section I have been placed in Section II-A. Specific details are provided on pages 1-2 of the manuscript.

Reviewer Point 2.9 — *It is mentioned in Section IV-B, "Although LCMVAPR achieves good FR results for most functions, it is not able to achieve good FR results for higher dimensional functions.". Here the logic of this statement is inconsistent since all functions are high dimensional functions in Table III.*

Response: Thank you very much for your comments. Yes, all functions are high-dimensional in this study. The "higher dimensional functions" in this sentence means that the functions with D = 500 and 1000 compared to functions with D = 100 and 200. In the revised manuscript, we have corrected this sentence to improve its readability. The details are shown as follows (Please also see Section IV on Page 10 of the revised manuscript).

Although LCMVAPR achieves good FR results on functions with D =100 and 200, the FR values decrease for higher dimensional functions with D = 500 and 1000.

Reviewer Point 2.10 — *In Section I, the logic of the sentence "Moreover, solving LSCMOPs requires more computational resources than CMOPs, so better convergence ability of the algorithm should be considered." is not clear. The reason why the causal relationship is established should be supplemented. For example, when computational resources are insufficient, individuals are too slow to converge to UPF, so the better convergence ability of the algorithm should be considered.*

Response: Thank you very much for your suggestion. In the revised manuscript, we have edited this sentence in the manuscript to make it more logical. The details are shown as follows (Please also see Section I on Page 2 of the revised manuscript).

Moreover, the individuals' convergence speed on LSCMOPs is slower than that on CMOPs due to the huge search space. When computational resources are insufficient, individuals are too slow to converge to Pareto front. Therefore, better convergence ability of the algorithm should be considered under limited computing resources.

Reviewer Point 2.11 — *The manuscript mentions the variable coupling relationship many times. But it does not explain clearly what is the coupling of variables? Moreover, in the example of Fig1(b), the authors statement that there is the coupling of variables especially for problems with discrete and small feasible regions. But, on the contrary, the example of Fig1(b) has continuous and large feasible regions. The logic is not rigorous. Besides, the reason why the proposed method can solve or reduce the variable coupling problem is rarely mentioned in the manuscript. The authors should give the definition of the variable coupling relationship and explain clearly the reason why the proposed method can solve the variable coupling problem.*

Response: Thank you very much for your comments. We have compiled your questions and answered them one by one.

(1) The manuscript mentions the variable coupling relationship many times. But it does not explain clearly what is the coupling of variables?

The variable coupling relationship in the manuscript is mainly defined as follows: on the one hand, different variables have different objective control relationships; on the other hand, there is a correlation relationship between variables, which means that the related variables need to be optimized simultaneously to obtain the optimal value. In the revised manuscript, we have added a more detailed definition of variable coupling. The details are shown as follows (Please also see Section S-I-A on Page 2 of the supplementary file).

S-I-A Definition of Variable Coupling

To make it easier to understand, we use a problem with two objective and four decision variables to explain the two coupling relationships, as shown in Eq. (1).

$$\begin{cases} \min f_1(\mathbf{x}) = x_1 - \sin(0.3\pi x_2 x_3) + {x_4}^2 \\ \min f_2(\mathbf{x}) = 1 - x_1 + \cos(0.6\pi x_2 x_3) + {x_4}^2 \\ s.t. \quad x_i \in [0,1], i = 1,2,3,4 \end{cases} \tag{1}$$

First, x_1 and x_4 are sampled multiple times, and the values of the remaining variables remain unchanged. The generated solutions are shown in Fig. S-1. It can be seen that the solutions generated after sampling variable x_1 are basically non-dominated by each other, that is, when optimizing x_1, it will have a greater impact on the diversity of the population. On the contrary, the solutions generated after sampling variable x_4 basically dominate each other, that is, when optimizing x_4, it has a greater impact on the convergence of the population. To sum up, x_1 and x_4 should be optimized separately due to their different control relationships with the objective.

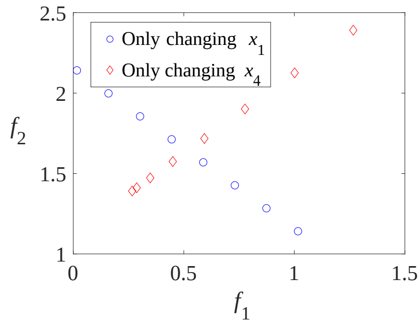


Fig. S-1. Plot of the sampling points by changing x_1, x_4 and the other variables are fixed as 0.5.

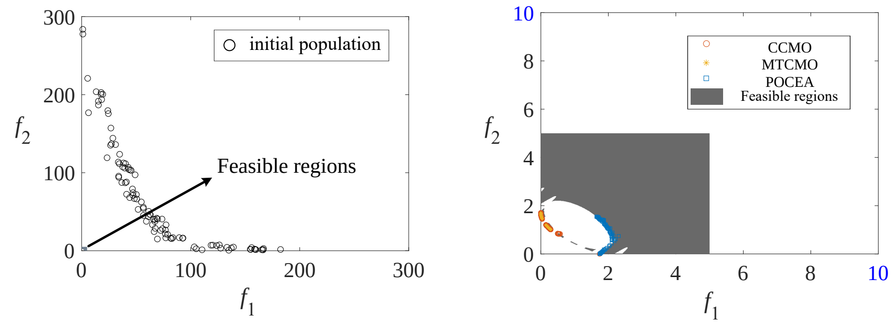
Second, We take f_1 in the Eq. (1) as an example to illustrate the interactive relationship between variables. It can be expressed as:

$$f_1 = \underbrace{(x_1 + {x_4}^2)}_{f_{11}} + \underbrace{(-\sin(0.3\pi x_2 x_3))}_{f_{12}}. \tag{2}$$

For f_{11}, the minimum value $f_{11} = 0$, at this time $x_1 = x_4 = 0$. That is, $\arg\min_{(x_1,x_2)} f_{11}(x_1,x_2) = [0,0]$, $\arg\min_{(x_1,x_2)} f_{11}(x_1,x_2)$ represents the optimal solution of $f_{11}(x_1,x_2)$ in the decision space. If a certain variable is fixed, the other variable will only minimize f_{11} when it takes 0. That is, $\arg\min_{x_1} f_{11}(x_1,x_2) = \arg\min_{x_2} f_{11}(x_1,x_2) = 0$. Obviously, $\arg\min_{(x_1,x_2)} f_{11}(x_1,x_2) = [0,0] = [\arg\min_{x_1} f_{11}(x_1,x_2), \arg\min_{x_2} f_{11}(x_1,x_2)]$. So x_1 and x_4 can be optimized independently, and they are not related to each other. For f_{12}, it is obvious that x_2 and x_3 cannot be optimized independently. The minimum value of $f_{12} = -1$, then $x_2 x_3 = \frac{5}{3} + \frac{20}{3}k, k \in Z$ (Z represents all integers). There is a correlation between x_2 and x_3.

(2) Moreover, in the example of Fig. 1 (b), the authors statement that there is the coupling of variables especially for problems with discrete and small feasible regions. But, on the contrary, the example of Fig1(b) has continuous and large feasible regions. The logic is not rigorous.

We are sorry that Fig. 1 (b) is indeed easily misunderstood. In fact, the feasible region of the LIRCMOP10 is not large when the decision variables are high-dimensional. To facilitate explanation, the initial population on LIRCMOP10 is given in Fig. R2 (a). It can be seen that the initial objective space of this problem is very large, while the feasible region is very small. In addition, from Fig. R2 (b), it can be seen that the most feasible regions of the problem are not continuous and smaller. It is therefore difficult for populations to converge into these regions. To avoid misunderstanding, we have expanded the coordinate axis range of Fig. 1 (b) in the manuscript for better understanding. Originally it was $[0, 5]$, now it is $[0, 10]$.



(a) Initial population (b) The final population obtained

Fig. R2. The initial population and the populations obtained by the three algorithms are on LIRCMOP10 with D = 100.

(3) Besides, the reason why the proposed method can solve or reduce the variable coupling problem is rarely mentioned in the manuscript. The authors should give the definition of the variable coupling relationship and explain clearly the reason why the proposed method can solve the variable coupling problem.

Our variable adaptive optimization strategy is designed based on the variable grouping method in LMEA [23] (Numbered as [28] in the original manuscript). In order to decouple, it first divides variables into two categories: convergence-related and diversity-related variables based on their control relationships with the objectives. Then, further grouping is performed based on the interaction between variables. In addition, the proposed strategy first optimizes the convergence-related variables to speed up the convergence process and quickly explore the huge objective space. Diversity-related variables are then optimized to improve the distribution of solutions. The adaptive optimization method can not only reduce the difficulty of variable optimization but also save resources.

In the revised manuscript, we have added Section II-B to explain the variable grouping method used, including the definition of variable coupling and how to solve it. The details are shown as follows (Please also see section II-B on page 2 of the revised manuscript and Section S-I on pages 2-3 of the Supplementary File.)

II-B Variable grouping technique

When solving LSCMOPs, a major problem is how to deal with large-scale variables to speed up the convergence. This is because the coupling between large-scale variables is severe. Specifically, different variables have different control relationships on the objective, and there is a correlation between variables. In this study, we design an adaptive variable optimization method to handle large-scale variables according to the decision variable grouping technique in LMEA [23], which has been proven to be very powerful in LSMOP. This technology consists of two parts: analysis of decision variables and correlations. The former divides variables according to the different contributions of decision variables to the objective, while the latter further groups the variables according to the interrelationships between them. Through the above analyses, the decision variables can be fully decoupled to greatly reduce the difficulty of optimization.

Due to space limitation, details of the variable grouping techniques used are provided in Section S-I of the Supplementary File.

S-I-B Decision Variable Analysis

The main objective of decision variable analysis is to find the coupling relationships between variables so that the coupling variables can be split into the same group to be optimized. A bi-objective optimization problem consisting of four decision variables is taken as an example, and the method is illustrated in Fig. S-I. For a solution $\mathbf{x}$, each dimension is perturbed ten times to obtain ten solutions. Then the objective values of these ten solutions are normalised to form a line (L) in the objective space. For example, if the second dimension is perturbed, the line obtained is represented by L_2. Next, the angle (θ_1, θ_2, θ_3 and θ_4) between each L and the hyperplane normal is calculated. Using the angle, the variables of the first and second dimensions are considered as variables related to diversity ($\mathbf{x_d}$), since the solutions on L_1 and L_2 have a good distribution between two objectives. Additionally, the variables of the third and fourth dimension are considered as variables related to convergence ($\mathbf{x_c}$), since the solutions on L_3 and L_4 point in a convergence direction.

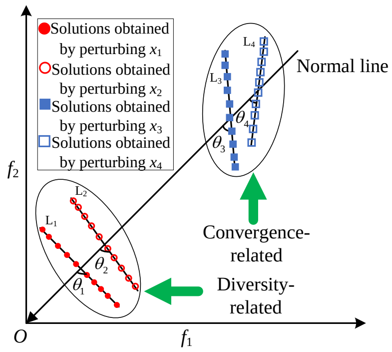


Fig. S-I. Example of a variable analysis for a bi-objective minimisation problem with four decision variables, where x_1 and x_2 are identified as diversity-related variables and x_3 and x_4 as convergence-related variables.

S-I-C Correlation analysis

After conducting the decision variable analysis, the following grouping technique is used to further divide $\mathbf{x_c}$ into different groups, since some $\mathbf{x_c}$ are correlated with each other. Given a MOP: $\min \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_M(\mathbf{x}))^T$, the two different dimensions x_i and x_j are interactive, if there exists $\mathbf{x}, a_1, a_2, b_1, b_2$ and at least one $f_s(\mathbf{x})$ $(1 \leqslant s \leqslant M)$ satisfying the following condition and they are in the same group:

$$\begin{aligned} &f_s(\mathbf{x})|_{x_i=a_2,x_j=b_1} < f_s(\mathbf{x})|_{x_i=a_1,x_j=b_1} \\ &\wedge f_s(\mathbf{x})|_{x_i=a_2,x_j=b_2} > f_s(\mathbf{x})|_{x_i=a_1,x_j=b_2} \end{aligned} \tag{3}$$

where $f_s(\mathbf{x})|_{x_i=a_2,x_j=b_1} \triangleq f_s(x_1, ..., x_{i-1}, a_2, ..., x_{j-1}, b_1, \ldots, x_D)$.

Then, ns $(1 \leqslant ns \leqslant |\mathbf{x_c}|)$ groups of $\mathbf{x_c}$ ($\mathbf{subx}_c = (sub\mathbf{x}_{c,1}, sub\mathbf{x}_{c,2}, ..., sub\mathbf{x}_{c,ns})$) is obtained. If there is no interaction between the variables, each variable belongs to an isolated group and ns is $|\mathbf{x_c}|$. If, on the other hand, all variables interact with each other, there is only one group and ns is 1.

[23] X. Zhang, Y. Tian, R. Cheng, and Y. Jin, “A decision variable clustering-based evolutionary algorithm for large-scale many-objective optimization.” *IEEE Transactions on evolutionary Computation*, vol. 22, no. 1, pp. 97–112, 2016.

Reviewer Point 2.12 — *The Variable Adaptive Optimization Strategy is one of the main contributions of the manuscript. However, its operation mainly relies on the variable grouping method from the other reference [28]. The authors should explain clearly that why this variable grouping method is helpful for solving the variable coupling, what are the contributions of reference [28] and what is the real contributions of this manuscript, respectively.*

Response: Thank you very much for your comments. Please note that in the revised manuscript, the original reference number has changed, and the reference [28] mentioned in comments has been changed to [23]. Regarding why literature [23] can solve the problem of variable coupling, a reply has been made in Reviewer point 2.11.

In [23], the diversity-related and convergence-related variables are iteratively optimized for multiple rounds, which requires a large amount of computing resources. If it is used to search for UPF, it is easy to cause resource waste. Furthermore, it incorporates variable optimization techniques throughout the evolution process. If it is directly used to solve LSCMOPs, it is difficult to maintain feasibility due to constraints that may be related to all variables. We also conducted corresponding experimental verification in Sections II-B and II-C, and the results also confirmed the above-mentioned defects.

Although the variable adaptive optimization strategy we proposed is designed based on literature [23], the optimization strategy for grouped variables is different. In our strategy, the two types of variables adaptively switch to optimize and stop according to the designed switching conditions. Moreover, the variable grouping optimization technology only takes effect in the global search stage. This design is beneficial for the population to quickly explore the entire objective space. In addition, while making the population evenly distributed on the UPF, sufficient resources are reserved for local search to explore the CPF to further enhance feasibility.

According to the above analysis, the proposed strategy is essentially different from the method in literature [23]. In the revised manuscript, we have modified the contribution so that readers can more clearly understand the novelty of the variable adaptive optimization strategy. Besides, we have also added more descriptions to better distinguish the proposed method and the method in literature [23]. The details are shown as follows (Please also see Section I on Page 2 and Section III-B on Page 5 of the revised manuscript).

A variable adaptive optimization strategy based on the variable analysis method is proposed in the global search stage. The convergence-related variables and the diversity-related variables are adaptively selected for optimization to balance convergence and diversity. In this way, not only the difficulty of variable coupling is reduced, but also the promising regions that are close to the CPF can be found quickly. Further, sufficient resources are reserved for local search stage to explore CPF to enhance feasibility.

In particular, the proposed strategy is fundamentally different from previous variable grouping optimization techniques [23, 27]. Specifically, in [23], multiple rounds of iterative optimization are performed on diversity-related and convergence-related variables, which requires a large amount of computing resources. In [27], diversity-related variables are optimized only in late stages of evolution, and most resources are spent on optimizing convergence-related variables. Furthermore, they incorporate variable optimization techniques throughout the evolution process for solving LSMOPs. In LSCMOP, constraints may be related to all variables. If these methods are directly embedded into our algorithm to solve LSCMOPs, it will lead to the loss of feasibility, which is also verified in paragraph IV. In our strategy, the two types of variables adaptively switch optimization and stopping according to the designed switching conditions, so that the population is evenly distributed on the UPF. This strategy reserves enough resources for local search to explore CPF, enhancing feasibility. At the same time, the archived population is more evenly distributed on UPF and CPF to enhance the selection effect of the archive-assisted population reconstruction.

[23] X. Zhang, Y. Tian, R. Cheng, and Y. Jin, "A decision variable clustering-based evolutionary algorithm for large-scale many-objective optimization." *IEEE Transactions on evolutionary Computation*, vol. 22, no. 1, pp. 97–112, 2016.

[27] X. Ma, F. Liu, Y. Qi, X. Wang, L. Li, L. Jiao, M. Yin, and M. Gong, "A multiobjective evolutionary algorithm based on decision variable analyses for multiobjective optimization problems with large-scale variables." *IEEE Transactions on evolutionary Computation*, vol. 20, no. 2, pp. 275–298, 2015.

Reviewer 3

Handling constrained multiobjective optimisation problems (CMOPs) is a hot research topic in the evolutionary computation community. However, CMOPs with large-scale decision variables (usually not less than 100 dimensions) have posed great challenges for current constrained multi-objective evolutionary algorithms (CMOEAs) *due to the complex coupling relationships among variables and the huge search space. To address these issues, this paper proposes a large-scale constrained multiobjective evolutionary algorithm based on variable adaptive optimisation and population reconstruction (LCMVAPR) to better solve large-scale CMOPs (LSCMOPs). First, a variable adaptive optimisation strategy is developed to adaptively select convergence-related and diversityrelated variables in the global search stage. Second, an archive-assisted population reconstruction strategy is proposed to generate an initial population by using promising solutions from the global search stage to rapidly guide the population towards the CPF. Experiments on 43 benchmark problems with up to 1000 decision variables and three real-world problems demonstrate the competitiveness of the proposed algorithm. Generally speaking, the research topic is worth studying and the proposed algorithm is novel and effective. The authors have made a great effort to demonstrate the effectiveness of the proposed algorithm in the experimental section. However, there are still some issues that need to be taken care of before the manuscript is accepted for publication.*

Reviewer Point 3.1 — *In the literature review, the authors missed some recently published papers on LSMOPs.*

Response: Thank you very much for your suggestions. In the revised manuscript, we have added some recently published papers on LSMOP in Section II-D. The details are shown as follows (Please also see Section II-D on Page 3 of the revised manuscript).

Furthermore, Xu *et al*. [28] group convergence-related variables according to their contribution to each objective.

Liu *et al*. [32] design three competitive learning strategies with different competitions to enhance the search ability of failed particles. In [33], the search mechanism of Monte Carlo tree is used to improve the performance insensitivity of the algorithm.

In [36], a self-guided problem transformation method is designed to strike a balance between computational burden and convergence speed. Liu *et al*. [37] utilize discriminative reconstruction neural networks to effectively optimize multiple LSMOPs simultaneously.

[28] Y. Xu, C. Xu, H. Zhang, L. Huang, Y. Liu, Y. Nojima, and X. Zeng, "A multi-population multi-objective evolutionary algorithm based on the contribution ofdecision variables to objectives for large-scale multi/many-objective optimization," *IEEE Transactions on Cybernetics*, vol. 53, no. 11, pp. 6998-7007, 2023.
[32] S. Liu, Q. Lin, Q. Li, and K. C. Tan, "A comprehensive competitive swarm optimizer for large-scale multiobjective optimization," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 9, pp. 5829-5842, 2022.
[33] H. Hong, M. Jiang, and G. G. Yen, "Improving performance insensitivity of large-scale multiobjective optimization via monte carlo tree search," IEEE Transactionson Cybernetics, 2023, doi:10.1109/TCYB.2023.3265652.
[36] S. Liu, M. Jiang, Q. Lin, and K. C. Tan, "Evolutionary large-scale multiobjective optimization via self-guided problem transformation," in *2022 IEEE Congress on Evolutionary Computation (CEC)*. IEEE, 2022, pp. 1–8.
[37] S. Liu, Q. Lin, L. Feng, K.-C. Wong, and K. C. Tan, "Evolutionary multitasking for large-scale multiobjective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 27, no. 4, pp. 863–877, 2023.

Reviewer Point 3.2 — *What is the CPU time cost of the proposed algorithm? It would be interesting to know if the proposed strategy is efficient enougth.*

Response: Thank you very much for your comments. In the revised manuscript, we have added comparative experiments on CPU runtime in Section S-IV-B of the supplementary file. From the results, the proposed algorithm ranks second in average runtime. It shows that our method not only achieves the best results, but also has a competitive runtime. The details are shown as follows (Please also see Section S-IV-B on Pages 14-15 of the supplementary file).

S-IV-B Runtime Comparison

Fig. S-VI shows the average runtime ranking of LCMVAPR and the six comparison algorithms. It is obvious that C_LMOCSO has the shortest runtime, while LCMVAPR ranks second in terms of runtime. This shows that the proposed algorithm not only consumes shorter runtime, but also achieves more outstanding effects.

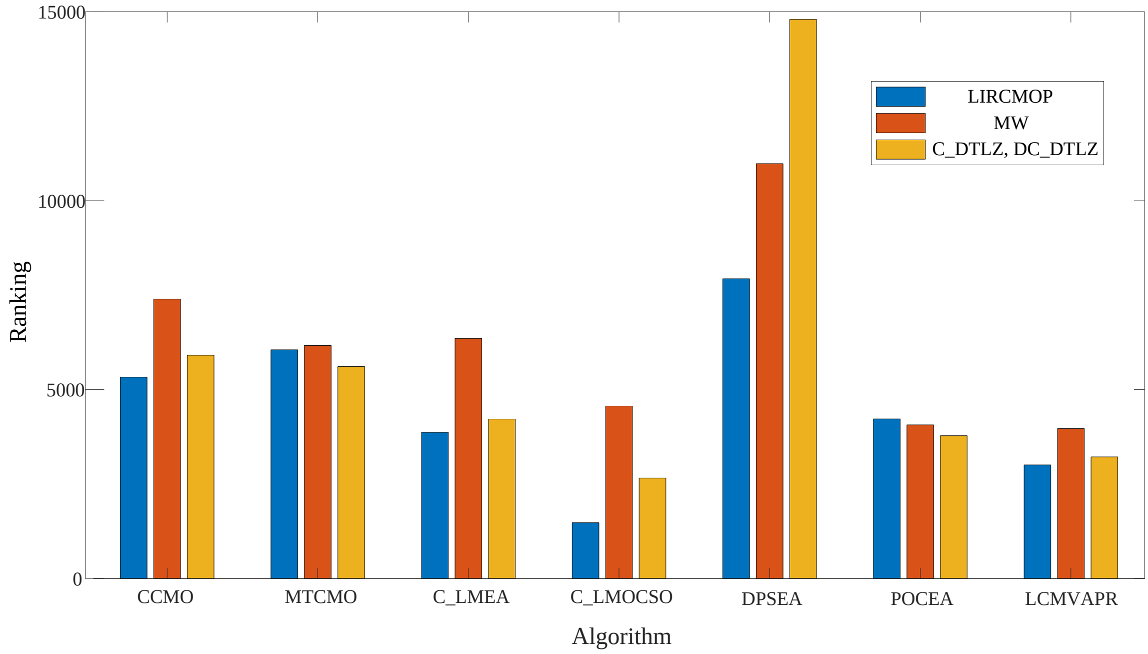


Fig. S-VI. Average runtime consumed by 7 algorithms under three test suites with all decision variables.

Reviewer Point 3.3 — *Identifying convergence-related and divserity-related variables has been discussed in the communtiy for a long time. What are the differences and advantanges of the proposed strategy compared with existing approaches (for example [27])?*

Response: Thank you very much for your comments. Indeed, identifying convergence-related and diversity-related variables has been widely discussed in solving LSMOPs. We applied it for the first time to solve LSCMOP. The proposed algorithm has obvious differences and advantages compared with existing methods [27]. Specifically, literature [27] incorporated variable optimization techniques throughout the evolution process. Diversity-related variables are optimized only late in evolution, so most resources are devoted to optimizing convergence-related variables. Adequate optimization of convergence-related variables can speed up the convergence speed of the population, which is important for solving LSMOPs. However, the constraints of LSCMOPs may be related to all variables, and the distribution of CPFs is also complex. Spending most of the resources on optimizing convergence-related variables throughout the evolution process would result in a severe loss of feasibility and diversity.

In our strategy, variable grouping optimization technology plays a role in the global search stage, and the local search stage will optimize all variables as a whole to enhance feasibility. In addition, the two types of variables adaptively switch to optimize and stop according to the designed switching conditions. This strategy increases the diversity of the population on UPF. At the same time, sufficient resources are set aside in local search to explore CPF.

In the revised manuscript, we have made detailed additions in section III. A comparative analysis with literature [23] is also added. The details are shown as follows (Please also see Section III-B on Page 5 of the revised manuscript).

In particular, the proposed strategy is fundamentally different from previous variable grouping optimization techniques [23, 27]. Specifically, in [23], multiple rounds of iterative optimization are performed on diversity-related and convergence-related variables, which requires a large amount of computing resources. In [27], diversity-related variables are optimized only in late stages of evolution, and most resources are spent on optimizing convergence-related variables. Furthermore, they incorporate variable optimization techniques throughout the evolution process for solving LSMOPs. If these methods are directly embedded into our algorithm to solve LSCMOPs, it will lead to the loss of feasibility, which is also verified in paragraph VI. In our strategy, the two types of variables adaptively switch optimization and stopping according to the designed switching conditions, so that the population is evenly distributed on the UPF. This strategy reserves enough resources for local search to explore CPF, enhancing feasibility. At the same time, the archived population is more evenly distributed on UPF and CPF to enhance the selection effect of the archive-assisted population reconstruction.

[23] X. Zhang, Y. Tian, R. Cheng, and Y. Jin, "A decision variable clustering-based evolutionary algorithm for large-scale many-objective optimization." *IEEE Transactions on evolutionary Computation*, vol. 22, no. 1, pp. 97–112, 2016.
[27] X. Ma, F. Liu, Y. Qi, X. Wang, L. Li, L. Jiao, M. Yin, and M. Gong, "A multiobjective evolutionary algorithm based on decision variable analyses for multiobjective optimization problems with large-scale variables." *IEEE Transactions on evolutionary Computation*, vol. 20, no. 2, pp. 275–298, 2015.

Reviewer Point 3.4 — *More in-depth discussions on the experimental results should be included in the next version of manuscript.*

Response: Thank you very much for your suggestion. In the revised manuscript, we have provided a more in-depth discussion of the comparative experimental results. The details are shown as follows (Please also see Section IV-B on Pages 8-10 of the revised manuscript).

1) ***LIRCMOP Test Suite:*** LIRCMOP functions have very small feasible regions, which presents the algorithm with a major challenge of maintaining diversity in huge infeasible regions. From Table II, the number of functions that six comparison algorithms are significantly worse than LCMVAPR on 56 functions ranges from 42 to 54. More detailed results can be found in Tables S-II and S-III of the supplementary file. Among the remaining functions, C_LMOCSO achieves the best results on LIRCMOP1-2. This may be due to the effectiveness of the "acceleration" search mechanism designed to deal with problems with simple CPFs. The larger population size in the early stage of DPSEA is conducive to fully exploring functions with larger-dimensional objective spaces, thus achieving competitive results on LIRCMOP13-14.

Fig. 5 shows the estimated Pareto-optimal solutions obtained from the seven algorithms with median IGD values on the 100-dimensional LIRCMOP4 function. LIRCMOP4 has several very small feasible regions, which makes it difficult for the algorithms to find a balance between convergence and diversity in the high-dimensional search space. C_LMEA uses CDP to deal with constraints that result in insufficient selection pressure on the population in infeasible regions to prevent it from escaping from infeasible regions. C_LMOCSO and POCEA do not achieve CPF in marginal regions because the environmental selection methods used do not generate sufficient selection pressure in the marginal regions. Although CCMO, MTCMO and DPSEA use infeasible information to obtain diversity, the effectiveness of these promising infeasible solutions decreases in high-dimensional search spaces. As a result, they can only find one feasible region and miss other feasible regions. LCMVAPR finds more feasible regions because it uses the information from the archive to support population evolution in the local search stage, which contributes to a balance between diversity and convergence.

2) ***MW Test Suite:*** MW functions have strong variable coupling relations that reduce the convergence rate of algorithms in infeasible regions. Therefore, algorithms should be able to handle complex constraints and variable coupling relations simultaneously. Tables S-IV and S-V of the supplementary file display the comparison results on IGD and HV indicators, respectively. As shown in these two Tables, LCMVAPR achieves optimal performance in most functions with different dimensions. LCMVAPR obtains 100% FR results on 37 out of 56 functions, while the FR values of the other algorithms are very low. Moreover, GLCMEA obtains very small IGD values. Therefore, only GLCMEA is able to solve the large-scale MW functions effectively. In addition, when the dimension increases, the performance of algorithms decreases. For example, when the dimension is changed from 100 to 1000, the FR values of all algorithms decrease on MW13, and the IGD values of all algorithms become worse on MW3, which shows that the dimension has a serious impact on the difficulty of functions and performance of algorithms.

In the supplementary file, Fig. S-III plots the Pareto-optimal solutions obtained by the seven algorithms with median IGD values on 1000-dimensional MW3 function. MW3 has an irregular CPF and moderate overlap between UPF and CPF. The algorithm should maintain diversity in infeasible regions close to CPF. CCMO and MTCMO do not have mechanisms to tackle large-scale variables, thus they are unable to maintain good diversity to find the complete CPF. The constraints of MW3 are related to all variables, and C_LMEA does not optimize all variables as a whole in the later generations, so it cannot arrive at the feasible regions. For C_LMOCSO and POCEA, the environmental selection method proposed in [47] is adopted, which makes the convergence ability of the population in the marginal regions inadequate in the high-dimensional search space. DPSEA and LCMVAPR obtain a better population distribution to cover the complete CPF, but the convergence performance of LCMVAPR is better than that of DPSEA. This is because the variable adaptive optimization strategy and the archive-assisted population reconstruction method accelerate the convergence rate and the employed improved ε-based environmental selection effectively preserves diversity.

However, the FR values of all algorithms for the MW functions are poor from Table III. For the MW functions, a small change in a decision variable does not lead to a significant change in the objective value. The population is difficult to converge when large-scale decision variables are optimized as a whole. This is also the reason why other compared algorithms do not achieve high FR values for most MW functions. Although LCMVAPR achieves good FR results on functions with D =100 and 200, the FR values decreased for higher dimensional functions with D = 500 and 1000. Therefore, it remains a challenge for existing algorithms to solve large-scale MW problems.

2) ***C_DTLZ and DC_DTLZ Test Suite:*** The main difficulties of DTLZ functions include non-monotonic convergence (DC_DTLZs) and the huge objective space for high-dimensional decision variables (C_DTLZs and DC_DTLZs). Table S-VI presents the IGD results of seven algorithms on 40 DTLZ functions, and Table S-VII of the supplementary file presents the HV results. As shown in Table S-VI and Table S-VII, LCMVAPR obtains the maximum number of best IGD results (32 out of 40) and HV results (34 out of 40). Furthermore, GLCMEA achieves 100% FR results on all functions. Especially, only GLCMEA finds feasible solutions on DC2_DTLZs, while the other algorithms fail to find any feasible solution except for CCMO. In addition, most of the IGD values obtained by GLCMEA are smaller than 0.1, indicating that the population of GLCMEA is close to CPF. However, the IGD values of the other algorithms are relatively large, which indicates that the populations might be far away from CPF and fall into local regions.

Fig. S-IV in the supplementary file plots the Pareto-optimal solutions with the median IGD values obtained by seven algorithms on 1000-dimensional DC1_DTLZ3 function. DC1_DTLZ3 has three objectives and two discrete feasible regions. Moreover, it has a very large objective space. The population should keep a fast convergence ability to solve this function. The populations of DPSEA, C_LMOCSO, and POCEA are unable to converge into feasible regions. The population size of DPSEA gradually decreases in the later stage, thus it is unable to continuously maintain diversity to find promising regions. For C_LMOCSO and POCEA, the populations are still far away from the CPF, which may be due to the insufficient search efficiency of the competitive swarm optimizer on large-scale constraint search space. The populations of C_LMEA and LCMVAPR cover two CPFs. For LCMVAPR, it is important to use the variable grouping technique so that the population can quickly traverse the infeasible regions and approach the feasible regions.

Reviewer Point 3.5 — *The width of TABLE II on Page 8 might be too large.*

Response: Thank you very much for your comments. In the revised manuscript, we have adjusted the width of Table II. The details are shown as follows (Please also see Section IV on Page 9 of the revised manuscript).

Table II: Comparison of IGD and HV results based on the Wilcoxon rank-sum test under different D for all three benchmark test suites.

Problem	D	CCMO IGD (HV) +/-/=	MTCMO IGD (HV) +/-/=	C_LMEA IGD (HV) +/-/=	C_LMOCSO IGD (HV) +/-/=	DPSEA IGD (HV) +/-/=	POCEA IGD (HV) +/-/=
LIRCMOP	100	0/13/1 (0/13/1)	0/14/0 (0/14/0)	1/13/0 (1/13/0)	2/12/0 (2/12/0)	2/12/0 (2/12/0)	0/13/1 (1/13/0)
	200	0/13/1 (0/13/1)	0/14/0 (0/14/0)	1/13/0 (1/13/0)	2/12/0 (2/12/0)	2/12/0 (2/12/0)	1/13/0 (1/13/0)
	500	3/10/1 (1/12/1)	1/13/0 (0/14/0)	1/13/0 (1/13/0)	2/12/0 (2/12/0)	4/10/0 (3/11/0)	0/13/1 (0/13/1)
	1000	2/10/2 (2/12/0)	0/13/1 (0/14/0)	1/13/0 (1/12/1)	0/13/1 (0/13/1)	4/8/2 (4/9/1)	0/13/1 (0/14/0)
	Total	**5/46/5 (3/50/3)**	**1/54/1 (0/56/0)**	**4/52/0 (4/51/1)**	**6/49/1 (6/49/1)**	**12/42/2 (11/44/1)**	**1/52/3 (2/53/1)**
MW	100	0/13/1 (1/13/0)	0/13/1 (1/13/0)	0/14/0 (0/14/0)	0/14/0 (0/14/0)	1/13/0 (1/13/0)	0/14/0 (0/14/0)
	200	0/13/1 (1/13/0)	0/13/1 (1/13/0)	0/14/0 (0/14/0)	0/14/0 (0/14/0)	0/13/1 (0/13/1)	0/14/0 (0/14/0)
	500	0/12/2 (0/12/2)	0/13/1 (1/13/0)	0/14/0 (0/14/0)	0/14/0 (0/14/0)	0/12/2 (1/12/1)	0/14/0 (0/14/0)
	1000	1/9/4 (1/9/4)	0/10/4 (0/10/4)	0/11/3 (0/11/3)	1/10/3 (1/10/3)	0/9/5 (0/9/5)	0/10/4 (1/10/3)
	Total	**1/47/8 (3/47/6)**	**0/49/7 (3/49/4)**	**0/53/3 (0/53/3)**	**1/52/3 (1/52/3)**	**1/47/8 (2/47/7)**	**0/52/4 (1/52/3)**
C_DTLZ, DC_DTLZ	100	0/9/1 (0/9/1)	0/8/2 (0/8/2)	0/9/1 (0/10/0)	0/10/0 (1/9/0)	1/8/1 (2/8/0)	1/9/0 (1/9/0)
	200	0/8/2 (0/9/1)	0/8/2 (0/8/2)	0/9/1 (0/10/0)	1/9/0 (1/9/0)	0/8/2 (1/8/1)	0/10/0 (0/10/0)
	500	0/9/1 (0/9/1)	0/8/2 (0/8/2)	1/8/1 (0/9/1)	0/10/0 (0/10/0)	0/8/2 (1/8/1)	0/10/0 (0/10/0)
	1000	1/8/1 (1/8/1)	0/8/2 (0/8/2)	1/8/1 (1/9/0)	1/9/0 (1/9/0)	0/8/2 (2/8/0)	1/9/0 (1/9/0)
	Total	**1/34/5 (1/35/4)**	**0/32/8 (0/32/8)**	**2/34/4 (1/38/1)**	**2/38/0 (3/37/0)**	**1/32/7 (6/32/2)**	**2/38/0 (2/38/0)**

The symbols "+", "-", and "=" indicate that the compared algorithm is significantly better than, worse than, and similar to the LCMVAPR, respectively. The overall results in each test suite are shown in bold.

Reviewer Point 3.6 — *The authors should double-check the manuscript to avoid typos and grammar issues.*

Response: Thank you very much for your suggestions. In the revised manuscript, we have checked the manuscript and corrected grammatical errors.

Reviewer Point 3.7 — *Is the source code available online for the ease of replication of the results?*

Response: Thank you very much for your comments. We have uploaded the source code to Github. For detailed information please visit the website: `https://github.com/cilabzzu/Codes`. We have also noted this on page 11 of the revised manuscript