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RISK-SHARING, ALTRUISM, AND THE FACTOR STRUCTURE OF CONSUMPTION

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ABSTRACT

We consider four models of consumption that differ with respect to efficient risk-sharing and altruism. They range from complete markets with altruism to family risk-sharing. We use a matched sample of parents and independent children available from the Panel Study of Income Dynamics to discriminate between the four models. Our testing procedure is designed to deal with the set of observed independent children being endogenously selected. The combined hypothesis of complete markets and altruism can be decisively rejected, while we fail to reject altruism and hence family risk-sharing for a subset of families.

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## 1. INTRODUCTION

An infinitely-lived representative agent is the central economic actor in a good many modern macroeconomic analyses. The justification for such an agent rests on complete markets and pure intergenerational altruism. With pure altruism, the family acts as a single immortal decision making agent. With complete markets, efficient risk-sharing leads heterogeneous agents to act in a manner equivalent to that of a single representative agent.<sup>1</sup> This paper uses micro data to test these two distinct hypothesis motivating the representative agent assumption.

If, as it turns out, the joint hypothesis of complete markets and pure altruism (our model a) fails empirically, we would like to know which component of the joint hypothesis is responsible for the failure. Therefore we also test pure altruism (model b) and complete markets (model c) separately. They are, in turn, special cases of an arguably more credible model, family risk-sharing (our model d), which posits that efficient risk-sharing implied by complete markets takes place within, but not necessarily between, families. Since family risk-sharing occurs automatically if family members are altruistically linked, the four models of consumption we test are partially nested, as shown in Figure 1.

It is well known that complete markets imply a very specific factor structure for the log-marginal utility.<sup>2</sup> We will show that each of the other three models of consumption also has its own factor structure. Since each factor structure can be translated into a set of restrictions on the same covariance matrix between consumption and endowments, we can test the four

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<sup>1</sup>See, e.g., Hansen and Sargent (1990) for an explicit statement of the aggregation procedure

<sup>2</sup>See, e.g., Altug and Miller (1990).

models sequentially using the same sample, starting with family risk-sharing and proceeding to the joint hypothesis of complete markets and altruism.

Two of the four models examined in this paper have been tested, independently from each other, by several researchers. A particular linear combination of the covariance restrictions implied by complete markets amounts to the zero restriction on the contemporaneous income growth coefficient in a cross-section regression of consumption growth. This has been tested extensively in the literature. Recent tests of complete markets using various micro data sets on households includes Abel and Kotlikoff (1988), Cochrane (1989), Mace (1989), and Townsend (1989).<sup>3</sup> Altug and Miller (1990) examined covariance among food consumption, leisure and wages and found no evidence against complete markets. The only available consumption-based test of altruism using micro data to our knowledge is in Altonji, Hayashi, and Kotlikoff (1989), who tested a particular linear combination of the covariance restriction implied by altruism.<sup>4,5</sup>

The data to be used in this paper come from the Panel Study of Income Dynamics which includes not only households originally sampled but also their

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<sup>3</sup>As argued in Hayashi (1987), all papers that have regressed the change in consumption against current and/or lagged income changes (see, e.g., Altonji and Siow (1987)) have tested efficient risk-sharing implicitly. Abel and Kotlikoff (1988) derived the regression from altruism, but the restriction tested is really an implication of complete markets.

<sup>4</sup>There is a strand of literature that tests for the existence of some altruism using data on wealth. See, e.g., Hurd (1989).

<sup>5</sup>Another model of consumption that figures prominently in the literature is self insurance, which delivers the well-known martingale property of the marginal utility. However, as first pointed out by Chamberlain (1984), this cannot be tested in a short panel unless one is willing to make a very specific assumption about how endowment depends on the state of the world. Available tests of self insurance (see Hayashi (1987) for a survey) require that endowment be the sum of an economy-wide and an idiosyncratic components. But then it is not clear why households do not share idiosyncratic risk. Previous tests of complete markets as well as our covariance-based test do not make any assumption about the endowment process.

split-offs, independent households headed by their children. We define a family to be a parent household plus its split-offs. The factor structures for the four models of consumption imply testable restrictions on the covariance matrix in consumption and endowment between member households of the family. Execution of the covariance-based test in the sample of families, however, requires a resolution of several difficulties. First, the composition of the family varies from one family to another, requiring us to adapt the test to each family type. Second, since the sample observation is "partial" in that it does not necessarily include all the member households of the family, our test has to take into account not only possible sample selection bias but also uncertainty due to the fact that the family's true type cannot be determined from data. Third, since the marginal utility is not directly observable, it has to be estimated. The second half of the paper is devoted to a resolution of these and other difficulties.

The contents of the paper are as follows. Section 2 contains an overview of our covariance-based test. In section 3, we formally derive the factor structure of the marginal utility from household optimization. This section generalizes the standard model of pure altruism to incorporate uncertainty. Section 4 shows how to translate the factor structure into a set of covariance restrictions for each family type. Section 5 provides a solution of the difficulties arising from the partial nature of the sample. Section 6 shows how to broaden the definition of the family type so that the proposed solution is feasible given our limited data. Section 7 describes and justifies our two-step procedure, which consists of regressing consumption and endowment on household characteristics (including leisure) and then calculating covariances using regression residuals. The sample is briefly described in section 8. Section 9 reports our results.

## 2. AN OVERVIEW

To see the main idea of the paper, consider the following simple situation. Take a sample of  $N$  families, each consisting of a parent household indexed by  $k = 0$  and its split-off (an independent household headed by the child of the parent household) indexed by  $k = 1$ , for three years, 1985, 86 and 87. Let  $(c_{ikt}, y_{ikt})$  be the log consumption and the log endowment for household  $k$  ( $= 0, 1$ ) of family  $i$  in year  $t$  ( $= 85, 86, 87$ ). The observation provided by the  $i$ -th family is a 12-dimensional vector  $\xi_i = (c_i, y_i)$  where  $c_i = (c_{i0,85}, c_{i0,86}, c_{i0,87}, c_{i1,85}, c_{i1,86}, c_{i1,87})$  and  $y_i$  is defined similarly.

The instantaneous utility function is the power function  $C^{1-\rho}/(1-\rho)$ . If  $\phi$  stands for measurement error in consumption, then  $(-\rho^{-1})$  times the log marginal utility ( $\theta$ ) is related to the log consumption ( $c$ ) as:

$$(2.1) \quad c_{ikt} = \theta_{ikt} + \phi_{ikt} \quad \text{for } k = 0, 1; t = 85, 86, 87$$

Assume the measurement error is uncorrelated in cross-section with endowment:

$$(2.2) \quad \text{Cov}(\phi_{ikt}, y_{i\ell s}) = 0 \quad \text{for all } k, \ell (= 0, 1); t, s (= 85, 86, 87)$$

The well-known implication of complete markets (our model (c) in Figure 1) is efficient risk-sharing, namely that the change in the log marginal utility is common to all households. Thus by (2.1) consumption growth differs across households only because of measurement error. Since measurement error is uncorrelated with endowment by (2.2), we have:

$$\text{Cov}(\Delta c_{ik,86}, y_{i\ell s}) = 0, \text{Cov}(\Delta c_{ik,87}, y_{i\ell s}) = 0 \quad \text{for all } k, \ell, s$$

or

$$(2.3) \quad \text{Cov}(c_{ik,85}, y_{i\ell s}) = \text{Cov}(c_{ik,86}, y_{i\ell s}) = \text{Cov}(c_{ik,87}, y_{i\ell s}) \quad \text{for all } k, \ell, s$$

where  $\Delta c_{ikt} = c_{ikt} - c_{ikt-1}$  is the consumption growth rate from year  $t-1$  to  $t$ . This is a set of equality restrictions on the  $6 \times 6$  cross covariance matrix  $\text{Cov}(c_{ikt}, y_{i\ell s})$  ( $k, \ell = 0, 1; t, s = 85, 86, 87$ ), which is displayed in Table I(i)

A weaker form of complete markets is family risk-sharing (our model (d)), which is that efficient risk-sharing occurs within, but not necessarily between, families. Then the change in log marginal utility differs between the parent household and the split-off only because of measurement error which is uncorrelated with endowment. Thus:

$$(2.4) \quad \text{Cov}(\Delta c_{10t} - \Delta c_{11t}, y_{1ts}) = 0 \quad \text{for all } t, s \text{ } (-86, 87), s \text{ } (-85, 86, 87),$$

which is a set of restrictions, not shown here but weaker than that in Table I(i), on the 6x6 covariance matrix.

Now suppose that the parent household and the split-off are altruistically linked (our model (b)). An extreme version of altruism (which will be somewhat relaxed later on) is that the level of marginal utility is equalized within the family. Consumption differs within the family only because of measurement error which is uncorrelated with endowment. Thus:

$$(2.5) \quad \text{Cov}(c_{10t} - c_{11t}, y_{1ts}) = 0 \quad \text{for all } t, s$$

The implied restrictions on the cross covariance matrix are displayed in Table I(ii). As is clear from comparing (2.5) with (2.4), altruism (model (b)) is stronger than family risk-sharing (model (d)). The joint hypothesis of complete markets and altruism (our model (a)) combines restrictions (2.3) and (2.5).

We have thus translated into the set of covariance restrictions the four partially nested models of consumption shown in Figure 1. The implied covariance restrictions are all linear and can easily be jointly tested. We emphasize that our covariance-based test of the four models requires neither assumptions about the endowment process nor data on Arrow-Debreu prices.

The 1987 tape release of the Panel Study of Income Dynamics (PSID) provide a sample of 192 families, each having one parent household and only

one split-off. There are two consumption components available in the PSID: food expenditure and housing consumption (actual and imputed rent). Figure 2A(i) is a plot of  $(c_{10,85} + c_{11,85} + c_{10,86} + c_{11,86} + c_{10,87} + c_{11,87})/6$ , the log food expenditure averaged over family members and over time, against the corresponding log average household endowment (labor income).<sup>6</sup> We verify that there is a highly significant correlation between food expenditure and endowment (the regression coefficient is 0.25 with a t value of 5.1 and  $R^2$  of 0.12). The corresponding plot for housing consumption is in Figure 2B(i). The regression coefficient is 0.61 with a t of 6.5 and  $R^2$  of 0.18.

To visually examine the impact of risk-sharing within families, consider the following linear combination of the set of covariance restrictions (2.4):

$$\text{Cov}\left((c_{10,87} - c_{10,85}) - (c_{11,87} - c_{11,85}), (y_{10,87} - y_{10,85}) - (y_{11,87} - y_{11,85})\right) = 0.$$

This is a covariance in differential 2-year changes. Figure 2A(ii) (Figure 2B(ii)) is a plot of this covariance for food (housing). The strong association apparent in Panel (i) mostly disappears, particularly for food. On the other hand, altruism does not hold in the data. To see this visually, consider the difference in time averages:

$$\text{Cov}\left(\frac{\sum_{t=85}^{87} c_{10t}}{3} - \frac{\sum_{t=85}^{87} c_{11t}}{3}, \frac{\sum_{t=85}^{87} y_{10t}}{3} - \frac{\sum_{t=85}^{87} y_{11t}}{3}\right) = 0,$$

which is a linear combination of (2.5). That this restriction does not hold is clear from Figure 2A(iii) and 2B(iii).

<sup>6</sup>Consumption (c) and endowment (y) are residuals from regressing c and y on household characteristics. For more details, see section 9.



### 3. FOUR MODELS OF CONSUMPTION

This section derives the factor structure of the marginal utility for the four models of consumption displayed in Table II. Readers not interested in the derivation can skip this section without losing continuity.

We begin by describing aspects of the economic environment that pertain to the properties of consumption we examine. Since households are finitely-lived, the set of existing households varies across states of the world. We index households that existed in some particular past state by "i". These households may give birth to *split-offs*, separate households headed by their children that exist in some states of the world. The k-th split-off of household i is indexed by "ik". To emphasize that household i is a parent household, we index it by "i0". A family is a collection of households consisting of a parent household and its split-offs. Whether it is significant that the household belongs to a particular family depends on the model we will consider below. For each household ik, there is a set  $A_{ik}$  of states in which the household is alive, which consists of a unique birth state ( $e_{ik}$ ) and a set of states that follow the birth state. The household's consumption in state  $e_t$  in date t is a scalar  $C_{ik,t}(e_t)$  defined over  $A_{ik}$ . (Extending the analysis to the multi-commodity case is straightforward.) Figure 3 gives an illustration of our notation. Household i0 exists in state a. It has five (potential) split-offs ( $k = 1, 2, \dots, 5$ ). Split-offs (k's) that are alive in each state are listed in braces in the figure. Thus, for example, the birth state for  $k = 1$  is state b, and  $A_{1k}$  for  $k = 1$  is (b,c,d).<sup>7</sup>

This example makes it clear that we distinguish between households with different birth states even if their utility function and the endowment process conditional on the birth state may be the same. We chose this convention, found in Wright (1987), because the notation (particularly (3.8) and (3.9) below) is simpler

### Complete Markets with Selfish Households

We first consider the well-known case of complete markets with selfish households. The household's objective is to maximize its expected lifetime utility:

$$(3.1) \quad U_{ik} = \sum_{e_t \in A_{ik}} \pi(e_t | e_{t-1}) u_{ik}(C_{ik}(e_t); e_t),$$

where  $\pi(e_t | e_{t-1})$  is the probability of  $e_t$  conditional on  $e_{t-1}$ . (If there is discounting, it is embedded in the dependence of  $u_{ik}$  on  $e_{t-1}$ .) Let  $P(e_t | e_s)$  be the price at state  $e_s$  of a claim to one unit of the good payable in state  $e_t$ . The first-order condition for optimality is

$$(3.2) \quad \frac{\pi(e_t | e_{t-1})}{\pi(e_{t-1} | e_{t-2})} \frac{\partial u_{ik}(C_{ik}(e_t); e_t)}{\partial C_{ik}} = \lambda_{ik} P(e_t | e_{t-1}),$$

where  $\lambda_{ik}$  is the Lagrange multiplier for  $ik$ 's lifetime budget constraint. Taking the log of the first-order condition (3.2) and using the arbitrage relation  $P(e_t | e_0) = P(e_t | e_{t-1})P(e_{t-1} | e_0)$ , we obtain

$$(3.3) \quad \theta_{ik}(e_t) = \lambda_{ik} + p(e_t),$$

where

$$(3.4) \quad \theta_{ik}(e_t) = \log \left( \frac{\partial u_{ik}(C_{ik}(e_t); e_t)}{\partial C_{ik}} \right).$$

is the log marginal utility in state  $e_t$  and

$$(3.5) \quad \lambda_{ik} = \log(\Lambda_{ik}) - \log[P(e_{ik} | e_0) / \pi(e_{ik} | e_0)],$$

$$(3.6) \quad p(e_t) = \log[P(e_t | e_0) / \pi(e_t | e_0)].$$

That is, as noted most recently by Townsend (1989) and Altug and Miller (1990), the log marginal utility for household  $ik$  has a simple factor structure, consisting of two "effects" or factors, one representing household  $ik$ 's endowment or lifetime resources  $\lambda_{ik}$  and the other an undiversifiable aggregate shock  $p(e_t)$ .

### Family Risk-Sharing between Selfish Households

Efficient risk-sharing between households within families is equivalent to the situation in which contingency claims are traded within but not necessarily between families. The security prices can differ across families. The first-order condition is now

$$(3.7) \quad \theta_{ik}(e_c) = \lambda_{ik} + p_i(e_c).$$

Note the difference from (3.3): the  $p$  factor can differ across  $i$ .

### Altruism

The alternative specification of preferences is that households within families are altruistically linked. Our specification of altruism is that household  $i0$ 's objective function  $V_{i0}$  is defined over its own consumption and that of its offspring.

$$(3.8) \quad V_{i0} = \sum_m \pi(e_{im} | e_{i0}) \Omega_{i0m} U_{im}$$

where  $U_{im}$  is defined in (3.1), and the summation is over household  $i0$  and all its potential offspring  $m$ . The summation can be rewritten as:

$$(3.9) \quad \sum_{e_c} \pi(e_c | e_{i0}) \left[ \sum_{m \in B_i(e_c)} \Omega_{i0m} u_{im}(C_{im}(e_c); e_c) \right],$$

where the summation  $\sum_{e_c}$  is over all states of the world emanating from  $e_{i0}$  and  $B_i(e_c)$  is the set of offspring (including the parent household) that exist in state  $e_c$ .

If the utility weight  $\Omega_{i0m}$  satisfies

$$(3.10) \quad \Omega_{i0m} = \Omega_{i0k} \Omega_{ikm} \text{ with } \Omega_{i00} = 1,$$

then  $V_{i0}$  can be rewritten recursively as

$$(3.11) \quad V_{i0} = U_{i0} + \sum_k \pi(e_{ik} | e_{i0}) \Omega_{i0k} V_{ik},$$

where the summation now is over all potential split-offs (i.e., direct offspring) of household  $i_0$  and where  $V_{ik}$  is defined by (3.8) with "0" replaced by "k". Therefore the collection of utility functions  $V_{ik}$  is intergenerationally consistent in the sense that each generation views its children's utility as a sufficient statistic for all its offspring's actions. This of course is a generalization to uncertainty of the Barro dynasty utility.

Now consider the problem of maximizing household  $i_0$ 's objective function (3.8). The maximization is over the actions by itself and all its offspring subject to a sequence of lifetime budget constraints, each pertaining to an offspring, under complete markets. Since the preferences are intergenerationally consistent, the solution will be self-enforcing in that future offspring, each acting to maximize its own objective  $V_{ik}$ , end up choosing the same action prescribed by the parent household.<sup>4</sup>

If the non-negativity constraint on bequests is not binding for the  $k$ -th split-off,<sup>5</sup> the first-order condition for maximization of (3.8) or (3.9) with respect to consumption in state  $e_t$  by the  $k$ -th split-off is:

$$(3.12) \quad \frac{\partial u_{ik}(C_{ik}(e_t); e_t)}{\partial C_{ik}} \pi(e_t | e_{i_0}) \Lambda_{i_0 k} = \Lambda_{i_0} p(e_t | e_{i_0})$$

where  $\Lambda_{i_0}$  is the Lagrange multiplier for the family's combined single budget constraint. Taking logs of both sides of (3.12) we obtain

$$(3.13) \quad \theta_{ik}(e_t) = \lambda_i + \omega_{ik} + p(e_t)$$

<sup>4</sup>The solution has this self-enforcing property even if the sequence of lifetime budget constraints are not integrated due to binding nonnegativity constraints. Streufert (1989) shows, for the case of certainty, that the solution is the outcome of a subgame-perfect equilibrium in Markov strategies in a game played by generations (with or without the non-negativity constraint).

<sup>5</sup>We will identify such split-offs in the data from information on the amount of expected bequests

where  $\theta_{ik}(e_t)$  is the log marginal utility (3.4),  $p(e_t)$  is as in (3.6), and

$$(3.14) \quad \lambda_i = \log(\Lambda_{i0}) - \log\{P(e_{i0}|e_0)/\pi(e_{i0}|e_0)\},$$

$$(3.15) \quad \omega_{ik} = -\log(\Omega_{i0k})$$

If complete markets do not necessarily exist, the  $p$  factor can depend on the family, so that the factor structure becomes:

$$(3.16) \quad \theta_{ik}(e_t) = \lambda_i + \omega_{ik} + p_i(e_t)$$

In order for this model of altruism to be empirically distinguishable from non-altruistic models of consumption, the utility weight  $\omega_{ik}$  must satisfy two requirements. First, it cannot depend on the endowment process  $Y_{ik}(e_t)$ ,  $e_t \in A_{ik}$ .<sup>10</sup> Consider distinguishing (3.13) from (3.3). The Lagrange multiplier  $\lambda_{ik}$  in (3.3) is a function of the value of  $ik$ 's endowments,  $\sum_{e_t \in A_{ik}} P(e_t|e_{ik}) Y_{ik}(e_t)$ . If the utility weight  $\omega_{ik}$  in (3.13) is allowed to be a function of the endowment process, then, for any non-altruistic allocation of consumption  $C_{ik}(\cdot)$  satisfying (3.3), there exists a function  $\omega_{ik}$  such that (3.13) holds for the same allocation. Thus, to make the model of altruism observationally different, the model must be specific about how the utility weight depends on the endowment process. But since we do not observe the endowment process (what we observe is a particular realization of the process for some dates), the only model of altruism empirically distinguishable from non-altruistic models is one in which the utility weight does not depend on the endowment process.

The second requirement for the model of altruism to be distinguishable is more subtle. Define the ex-ante type of a family to be a specification

<sup>10</sup>The standard model of altruism seems to be of this type. For example, the model in Chapter 8 of Becker (1981) apparently assumes that the beneficiary's income does not affect the altruist's preferences.

indicating which households are alive in which state. Figure 3 actually is an example of a particular ex-ante type: if there is another family for which the same figure can be drawn after suitably re-labelling  $k$ 's, then they are of the same type. Now suppose that the actual history is, say,  $(a, e, f)$  in Figure 3 and that we have observations on families with one split-off in date  $e$  and two in date  $f$ . There can be many ex-ante family types, not just the type in the figure, that have one split-off in date  $e$  and two in date  $f$ . If the utility weight depends on the family type, what could happen is that the utility weight appears to be correlated with endowments in the cross-section of families because certain family types may tend to have larger endowments.

The following utility weights satisfy the two requirements:

$$(3.17) \quad \omega_{ik} = \omega_{b_i(k)},$$

where  $b_i(k)$  is the birth order of  $k$ . For example for the family type depicted in Figure 3,  $b_1(1) = b_1(3) = 1$ ,  $b_1(2) = b_1(4) = b_1(5) = 2$ . Under this assumption the utility weight cannot depend on the number of existing split-offs, which is actually a desirable property. For, refer back to Figure 3. The number of split-offs is one in state  $e$  and two in state  $f$ . Clearly, if the utility weight is to depend on the number of split-offs, it must be state-dependent. For the rest of this paper we assume that (3.17) is part of our model of altruism.<sup>11</sup>

To summarize, we have derived the factor structure of the log marginal utility implied by the four models of consumption. They are displayed in

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<sup>11</sup> Assumption (3.17), which assumes that the utility weight is the same across families if the birth order is controlled for, is a sufficient but not necessary condition for the two requirements to hold. We could allow the utility weight to deviate randomly from (3.17) across families in the way specified in section 4.2 for  $\phi$ . We decided to take the more restrictive assumption (3.17) only because doing so substantially simplifies the exposition in the rest of the paper.

Table II. In the table and for the rest of the paper, we take as given a particular history of the state of the world, so that the dependence of any variable on  $e_t$  can be represented by subscript  $t$ . Also, since the split-off can be uniquely identified by its birth order given the particular history, hereafter the subscript " $k$ " will represent the birth order. For model (c), which is the union of the case of complete markets with selfish households and model (a) (complete markets with altruism), the factor structure in the table certainly is true for the case of complete markets with selfish households (see (3.3)) and is also implied by the factor structure for model (a). That is, the factor structure displayed for model (c) in the table is an implication of complete markets with or without altruism. Similarly, the factor structure for model (d) is an implication of risk-sharing *per se*.

#### 4. THE COVARIANCE RESTRICTIONS

In this section, we translate the factor structure of log marginal utility displayed in Table II into a set of restrictions on the cross covariance of consumption with endowments between parents and their split-offs. This task is rather complicated for two reasons. First, the population of families contains many family types with different household-year combinations. So, for example, the covariance of the parent's consumption with the third split-off's endowment cannot be defined for all families in the population. This section addresses this problem. The next section deals with the second problem, which is that the sample observation on a family does not necessarily include all member households of the family.

##### 4.1. Sub-population of Families

Consider the set of all U.S. families consisting of (parent) households that existed in 1968 and their split-offs (if any). For each family  $i$ , we assign  $k = 0$  to the parent household and index its split-offs (if any) by  $k$  ( $= 1, 2, \dots$ ) ordered by birth. We divide the population of families into sub-populations by the (ex-post) family type, indexed by  $\sigma$ , which is the set of household-year combinations ((k,t)'s). For the set  $\xi$  of variables pertaining to households, we define a vector of random variables,  $\xi_i^{(\sigma)}$ , defined over sub-population  $\sigma$  of families. For example, the family type  $\sigma$  displayed in Table III has the parent household ( $k = 0$ ) and the first split-off ( $k = 1$ ) in the first year, joined by the second split-off ( $k = 2$ ) in the second year yielding 5 household-year combinations with the associated set  $\sigma = \{(0,1), (1,1), (0,2), (1,2), (2,2)\}$ . For consumption we can define a 5-dimensional random vector  $C_i^{(\sigma)} = (C_{ikt}^{(\sigma)}; (k,t) \in \sigma) = (C_{i01}^{(\sigma)}, C_{i11}^{(\sigma)}, C_{i02}^{(\sigma)}, C_{i12}^{(\sigma)}, C_{i22}^{(\sigma)})$  over the sub-population



#### 4.2. The Maintained Hypothesis

The two consumption items available from the PSID are food expenditure and housing consumption (actual and imputed rent). We assume that the utility from the commodity is additively separable from the rest of the commodities (except for leisure, see below). The utility from the commodity in year  $t$  for household  $k$  in family  $i$  of type  $\sigma$  is

$$(4.1) \quad \frac{1}{1-\rho} \left( C_{ikt}^{(\sigma)} \right)^{1-\rho} \left( \Phi_{ikt}^{(\sigma)} \right)^{\rho}, \quad \rho > 0,$$

where  $C_{ikt}^{(\sigma)}$  is now either food or housing consumption and  $\Phi_{ikt}^{(\sigma)}$  is a taste shifter that affects the utility of the commodity. If  $\theta_{ikt}^{(\sigma)}$  is  $(-\rho^{-1})$  times the log marginal utility of the commodity in question, we have:

$$(4.2) \quad \log(C_{ikt}^{(\sigma)}) = \theta_{ikt}^{(\sigma)} + \log(\Phi_{ikt}^{(\sigma)}).$$

The taste shifter is not directly observable, but there is available a vector of observed household characteristics,  $x_{ikt}^{(\sigma)}$ , that affects the taste shifter. Also, let  $y_{ikt}^{(\sigma)}$  be household  $ik$ 's endowment. We make the following assumption on the joint distribution of  $(\log(\Phi_{ikt}^{(\sigma)}), x_{ikt}^{(\sigma)}, y_{ikt}^{(\sigma)})$ ,  $(k,t) \in \sigma$ , over sub-population  $\sigma$  of families

$$(4.3a) \quad E \left( \log(\Phi_{ikt}^{(\sigma)}) \mid (x_{ile}^{(\sigma)}, y_{ile}^{(\sigma)}), (l,s) \in \sigma \right) = \alpha_{kt}^{(\sigma)} x_{ikt}^{(\sigma)},$$

$$(4.3b) \quad \alpha_{kt}^{(\sigma)} = \alpha_{kt}.$$

Assumption (4.3a) has three parts, each of which seems a reasonable assumption. First, the household characteristics of other households within the same family do not help to predict the taste shifter of the household, given its own characteristics  $x_{ikt}^{(\sigma)}$ . Second, the residual taste shifter,  $\log(\Phi_{ikt}^{(\sigma)}) - \alpha_{kt}^{(\sigma)} x_{ikt}^{(\sigma)}$ , which would include things like the average physical body weight of the individuals within the household, is unrelated to endowments  $y_{ikt}^{(\sigma)}$ . This

is an implication of the basic premise in economics that preference and endowments are unrelated. This is an identifying assumption; once one permits unrestricted correlation between unobserved taste shifters and endowments, no theory can deliver any testable restrictions on the correlation between consumption and endowments. Third, the conditional expectation of the taste shifter conditional on the household characteristics is linear. This is not restrictive because our choice of  $x_{ikt}^{(\sigma)}$  consists of dummy variables and numerical variables with their squares. Assumption (4.3b), which will be needed for proving the Proposition in the next section, says that the relationship between the taste shifter and the characteristics of the household does not depend on what relatives this household has within the family.

The maintained hypothesis (4.2) and (4.3) can be rewritten as

$$(4.4a) \quad c_{ikt}^{(\sigma)} = \theta_{ikt}^{(\sigma)} + \phi_{ikt}^{(\sigma)}, \quad E\left(\phi_{ikt}^{(\sigma)} | (x_{its}^{(\sigma)}, Y_{its}^{(\sigma)}), (\ell, s) \in \mathcal{G}\right) = 0, \quad (k, t) \in \mathcal{G},$$

where

$$(4.4b) \quad c_{ikt}^{(\sigma)} = \log(c_{ikt}^{(\sigma)}) - \alpha_{kt} x_{ikt}^{(\sigma)}, \quad \phi_{ikt}^{(\sigma)} = \log(\phi_{ikt}^{(\sigma)}) - \alpha_{kt} x_{ikt}^{(\sigma)}.$$

We will include in  $x_{ikt}^{(\sigma)}$  not only the usual demographics (family size, age, etc.) but also leisure of the head and the spouse, thus allowing for the possible interaction between consumption and leisure in the utility function. Measurement error in consumption can be included as part of  $\phi_{ikt}^{(\sigma)}$  as long as it satisfies (4.4a). It can be correlated with true consumption. Also, endowments can have measurement error that is independent of the taste shifter. We must not make any assumption about the relationship between  $\theta_{ikt}^{(\sigma)}$  and  $(x_{ikt}^{(\sigma)}, Y_{ikt}^{(\sigma)}, \phi_{ikt}^{(\sigma)})$ , because consumption demand, and hence the log marginal utility  $\theta_{ikt}^{(\sigma)}$ , depend on  $x_{ikt}^{(\sigma)}$ ,  $Y_{ikt}^{(\sigma)}$ , and  $\phi_{ikt}^{(\sigma)}$ .

#### 4.3. The Covariance Restrictions

The residual consumption  $c_{ikt}^{(\sigma)}$  defined in (4.4b) is not directly observable.

ble unless we know the coefficient vector  $\alpha_{kt}$ ; until section 7, we temporarily assume that we know  $\alpha_{kt}$ . Since we do not make any assumptions about the correlation between elements of  $\phi_{ikt}^{(\sigma)}$ ,  $(k,t) \in \sigma$ , no restriction can be placed on the consumption covariances  $\text{Cov}(c_{ikt}^{(\sigma)}, c_{ils}^{(\sigma)})$  defined over sub-population  $\sigma$ . However, each of the four models of consumption does imply a set of restrictions on the cross covariance matrix between consumption  $c_{ikt}$  and any trans- form of endowment  $y_{ils}^{(\sigma)} = h(Y_{ils}^{(\sigma)})$  for  $(k,t), (\ell,s) \in \sigma$ . To see this, we note that

$$(4.6) \quad \text{Cov}(\phi_{ikt}^{(\sigma)}, y_{ils}^{(\sigma)}) = 0, \quad (k,t) \in \sigma, (\ell,s) \in \sigma$$

so that the cross covariance can be written as

$$(4.7) \quad \text{Cov}(c_{ikt}^{(\sigma)}, y_{ils}^{(\sigma)}) = \text{Cov}(\theta_{ikt}^{(\sigma)}, y_{ils}^{(\sigma)}), \quad (k,t) \in \sigma, (\ell,s) \in \sigma$$

Combining this with the factor structure displayed in Table II leads to the cross covariances in Panel (i) of Table IV. For example, consider the factor structure for model (a) in Table II. Since  $\omega_k$  and  $p_t$  are constant across families of all types, only the  $\lambda_t$  factor contributes to the covariance of  $\theta_{ikt}^{(\sigma)}$  with  $y_{ils}^{(\sigma)}$ . An equivalent representation of the covariance restrictions in Panel (i) is the set of equality restrictions displayed in Panel (ii) of Table IV. For example, the equality restrictions for the family type which has  $k = 0.1$  for three years ( $t=85,86,87$ ) have already been shown in Table I for models (b) (altruism) and (c) (complete markets).

## 5. TESTING COVARIANCE RESTRICTIONS ON "PARTIAL" DATA

If we had observations on all the household-year combinations for each family in the sample, then testing the restriction displayed in Table IV for sub-sample  $\sigma$  would be straightforward. This is not the case for our data, where the observation on the family is only partial: some households may not have been in the sample from the start, and others may be initially in the sample but have temporarily or permanently dropped out. Therefore, an observed type  $r$ , which is a set of observed household-year combinations, is only a subset of  $\sigma$ . This section describes the problems arising from this partial nature of the data and how their resolution. Readers not interested in technical details can skip to the last paragraph of this section.

Let  $\xi_i^{(\sigma)} = ((\log(C_{ikt}^{(\sigma)}), \theta_{ikt}^{(\sigma)}, \log(\Phi_{ikt}^{(\sigma)}), x_{ikt}^{(\sigma)}, y_{ikt}^{(\sigma)}), (k,t) \in \sigma)$ , be a random vector, defined over sub-population  $\sigma$  of families, of the variables pertaining to each household in the sample family  $i$ . The maintained hypothesis is the restriction (4.3) on the joint distribution of  $\xi_i^{(\sigma)}$ . We have shown in section 4.3 that, under the maintained hypothesis, the covariance restrictions in Table IV are immediately implied by the factor structure for  $\theta_{ikt}^{(\sigma)}$  in Table II. Let  $\xi_{ir}^{(\sigma)}$  be the sub-vector of  $\xi_i^{(\sigma)}$  corresponding to  $r \subset \sigma$ . The partial nature of the data creates two problems. The first is the familiar selectivity bias: if the selection rule determining which  $(k,t)$ 's are observable is not random, the distribution of  $\xi_{ir}^{(\sigma)}$  is not the marginal of  $\xi_i^{(\sigma)}$  so that the covariance restrictions may not hold on the (conditional) distribution of  $\xi_{ir}^{(\sigma)}$ . The second problem is less standard: we do not know the set of missing  $(k,t)$ 's, so that there is no way of telling which sub-population the partial observation was drawn from. That is, if the sample family is of type  $r$ , it can come from any sub-population  $\sigma$  that includes  $r$ . If  $\xi_i^{(r)} = (\log(C_{ikt}^{(r)}), \theta_{ikt}^{(r)}, \log(\Phi_{ikt}^{(r)}), x_{ikt}^{(r)}, y_{ikt}^{(r)}), (k,t) \in r$ , is the sample observation of

observed type  $r$ , the distribution of  $\xi_i^{(r)}$  is a mixture of conditional distributions.<sup>12</sup>

$$(5.1) \quad \xi_i^{(r)} = \xi_{ir}^{(\sigma)} \text{ with probability } \mu_{\sigma r} \text{ where } \sum_{\sigma} \mu_{\sigma r} = 1.$$

The question is: does the maintained hypothesis also hold on the distribution of  $\xi_i^{(r)}$ ? If so, then the covariance restrictions immediately follow from the factor structure. The following proposition says that the answer is yes if the selection rule is not based on consumption.

**PROPOSITION** Suppose the sample selection is such that the set  $r$  of household-year combinations is observed for family  $i$  from sub-population  $\sigma$  if  $(x_i^{(\sigma)}, y_i^{(\sigma)}) \in \Gamma_{\sigma r}$ , where  $(x_i^{(\sigma)}, y_i^{(\sigma)}) = (x_{ikt}^{(\sigma)}, y_{ikt}^{(\sigma)})$ ,  $(k, t) \in \sigma$ . Then the maintained hypothesis holds on  $\xi_i^{(r)}$ , that is, (4.4) holds for  $r = \sigma$ .

**PROOF.** From (5.1) and the assumed sample selection rule, we have, for any function  $g(\cdot)$  of  $\xi_i^{(r)}$ ,

$$(5.2) \quad E[g(\xi_i^{(r)})] = \sum_{\sigma} \pi_{\sigma r} E[g(\xi_{ir}^{(\sigma)}) | (x_i^{(\sigma)}, y_i^{(\sigma)}) \in \Gamma_{\sigma r}].$$

Now define  $\phi_{ikt}^{(r)} = \log(\phi_{ikt}^{(r)}) - \alpha_{kt} x_{ikt}^{(r)}$  and let  $(x_{ir}^{(\sigma)}, y_{ir}^{(\sigma)})$  be the elements corresponding to  $(x_i^{(r)}, y_i^{(r)})$ . Let  $h(\cdot)$  be an arbitrary function of  $(x_i^{(r)}, y_i^{(r)})$ . Then  $\phi_{ikt}^{(r)} \cdot h(x_i^{(r)}, y_i^{(r)})$  is a function of  $\xi_i^{(r)}$ . Thus by (5.2)

$$(5.3) \quad E[\phi_{ikt}^{(r)} \cdot h(x_i^{(r)}, y_i^{(r)})] = \sum_{\sigma} \pi_{\sigma r} E[\phi_{ikt}^{(\sigma)} \cdot h(x_i^{(r)}, y_i^{(r)}) | (x_i^{(\sigma)}, y_i^{(\sigma)}) \in \Gamma_{\sigma r}].$$

<sup>12</sup>An analogy to a different situation may help to understand the notation here. Suppose the population consists of singles ( $\sigma = 1$ ) and couples ( $\sigma = 2$ ). Let  $(y_h, y_w)$  be the head's and the wife's income. For singles,  $\xi_i^{(1)} = y_{hi}^{(1)}$ , and for couples  $\xi_i^{(2)} = (y_{hi}^{(2)}, y_{wi}^{(2)})$ . The sample selection rule is that couples are in the sample if  $y_w^{(2)} > 0$ . For couples in the sample, all we observe is the head's income and we do not know whether the head is married or not. So there is only one observed type, which consists of the household head only. The sample observation  $\xi_i^{(r)}$  is either  $y_h^{(1)}$  or  $y_w^{(2)}$ .

(This would not be true if  $\alpha_{kt}$ , which defines  $\phi_{ikt}^{(\sigma)}$  in (4.4b), depended on  $\sigma$ ). The term  $E[\phi_{ikt}^{(\sigma)} \cdot h(x_{ir}^{(\sigma)}, y_{ir}^{(\sigma)}) | (x_i^{(\sigma)}, y_i^{(\sigma)}) \in \Gamma_{\sigma r}]$  on the right hand side of (5.3) equals (with  $f(x, y)$  here being the density of  $(x, y)$ )

$$\int_{(x, y) \in \Gamma_{\sigma r}} E[\phi_{ikt}^{(\sigma)} \cdot h(x_{ir}^{(\sigma)}, y_{ir}^{(\sigma)}) | x, y] f(x, y) dx dy / \text{Prob}((x_i^{(\sigma)}, y_i^{(\sigma)}) \in \Gamma_{\sigma r}),$$

which is zero because the integrand is zero by (4.4a). Therefore

$E[\phi_{ikt}^{(\tau)} \cdot h(x_i^{(\tau)}, y_i^{(\tau)})] = 0$  for any function  $h(\cdot)$  for which the expectation exists. But since

$$(5.4) \quad E[\phi_{ikt}^{(\tau)} \cdot h(x_i^{(\tau)}, y_i^{(\tau)})] = E[h(x_i^{(\tau)}, y_i^{(\tau)}) \cdot E[\phi_{ikt}^{(\tau)} | (x_i^{(\tau)}, y_i^{(\tau)})]],$$

it is necessary that  $E[\phi_{ikt}^{(\tau)} | (x_i^{(\tau)}, y_i^{(\tau)})] = 0$  for all  $(x_i^{(\tau)}, y_i^{(\tau)})$ . Q.E.D.

REMARK. The reader may have noticed that the relationship between  $\sigma$  and  $r$  is analogous to that between ex-ante types and  $\sigma$ . There are many ex-ante family types that could have produced an ex-post type  $\sigma$ . Thus assuming (4.3) for  $\sigma$  amounts to assuming the same for ex-ante types as well.

Thus, as far as testing goes, we can proceed as if the observed type is the true type  $\sigma$ . The intuition here is that even if the values of cross covariances are contaminated by the sample selection and mixing of distributions, the equality restrictions between cross covariances remain valid.

## 6. BROADENING THE DEFINITION OF FAMILY TYPE

In order to carry out our covariance-based tests for each sub-sample defined by the observed type, we need a sample variance-covariance matrix of cross covariances, requiring the sample size be greater than the square of the number of household-year combinations. Unfortunately, our sample is not large enough to allow us to strictly adhere to the definition of the observed type in dividing the sample into sub-samples. We therefore restrict our attention to the most popular family types, which are "balanced" families, families in which the same set of member households including the unique parent household are continuously in the sample throughout the sample period.

Still, sub-samples of "balanced" families are still too small relative to the number of cross covariances. This difficulty can be largely resolved if we decide not to use the information on the birth order: for each family, we take a random permutation of split-offs to re-assign the household index  $k$  ( $= 1, 2, \dots$ ). That this broadens the definition of the type can be seen from an example provided in Table V. The two families in Panel (i) of the table are "balanced" because the set of split-offs is the same over time. They are, however, different types because the composition of the split-offs differs between the two families. But the number of split-offs is the same, so they are of the same broader type after the random permutation of split-offs. For the rest of the paper we use  $M$  (the number of ever-present split-offs) to index the broader type. Even under this broader definition of the observed type, there are not sufficiently many families with  $M > 2$ . We henceforth focus on sub-samples with  $M = 0, 1$  or  $2$ .

Since the permutation of split-offs is random, the distribution of the sample observation of type  $M = 2$  satisfies symmetry

$$(6.1a) \quad \text{Cov}(c_{ikt}^{(2)}, y_{ils}^{(2)}) = \text{Cov}(c_{ilt}^{(2)}, y_{iks}^{(2)}) \quad \text{for } k, l = 1, 2,$$

$$(6.1b) \quad \text{Cov}(c_{i0t}^{(2)}, y_{iks}^{(2)}) = \text{Cov}(c_{i0t}^{(2)}, y_{ils}^{(2)}) \quad \text{for } k, l = 1, 2,$$

$$(6.1c) \quad \text{Cov}(c_{ikt}^{(2)}, y_{i0s}^{(2)}) = \text{Cov}(c_{ilt}^{(2)}, y_{i0s}^{(2)}) \quad \text{for } k, l = 1, 2$$

These symmetry restrictions are displayed in Table V(ii); for any given year combinations (t,s), the 9 covariances are described by 5 unique cross covariances. If symmetry is imposed on the cross covariance matrix, the random permutation of split-offs does not affect the test statistics numerically. So there is no need to literally take a random permutation of split-offs in the first place. Symmetry is a logical consequence of the construction of broader types, and we will incorporate it in our testing procedure. Table VI lists the parameters (unique cross covariances) to be estimated under symmetry. We use  $\delta^{(M)}$  for the stacked vector of unique cross covariances, ordered as indicated in Table VI, for type  $M = 0, 1, 2$ .

The same argument we made for the Proposition in section 5 establishes that the maintained hypothesis holds for any  $M$ . Thus, the covariance matrix  $\text{Cov}(c_{ikt}^{(M)}, y_{ils}^{(M)})$ , for  $k, l = 0, \dots, M$  and  $t, s = 1, \dots, T$ , where  $c_{ikt}^{(M)}$  is defined as

$$(6.2) \quad c_{ikt}^{(M)} = \log(c_{ikt}^{(M)}) - \alpha_{kt} x_{ikt}^{(M)},$$

satisfies the covariance restrictions in Table IV for  $\sigma = M$ . The covariance restrictions can be easily translated into a set of linear restrictions on the unique cross covariances  $\delta^{(M)}$ :

$$(6.3) \quad \delta^{(M)} = W_M^{(M)}, \quad \zeta^{(M)} \text{ free parameters.}$$

The appropriate matrix  $W_M$  for each of the four models of consumption is displayed in Table VII.



## 7. ESTIMATION AND TESTING

So far we have assumed that the  $\alpha_{kt}$  vector necessary for calculating the residual consumption  $c_{ikt}^{(M)}$  in (6.2) is known. This section describes our two-step procedure for estimation and testing of the covariance restrictions whose first step estimates  $\alpha_{kt}$  by regressing the log of consumption and endowment on household characteristics. The second step uses the residual for consumption and endowment from the first-stage regression to calculate the cross covariances and tests the restriction (6.3) on the covariance matrix. Readers not interested in econometric details can skip the rest of this section without losing continuity.

Let  $E^*(\log(c_{ikt}^{(M)})|x_{ikt}^{(M)})$  be the population least squares projection of  $\log(c_{ikt}^{(M)})$  on  $x_{ikt}^{(M)}$  and let  $\hat{c}_{ikt}^{(M)} = \log(c_{ikt}^{(M)}) - E^*(\log(c_{ikt}^{(M)})|x_{ikt}^{(M)})$ . Define  $\hat{y}_{ikt}^{(M)}$  similarly. By (6.2),

$$(7.1) \quad \hat{c}_{ikt}^{(M)} = c_{ikt}^{(M)} - E^*(c_{ikt}^{(M)}|x_{ikt}^{(M)}).$$

It is important to note that  $\hat{c}_{ikt}^{(M)}$  and  $\hat{y}_{ils}^{(M)}$  are not equal because the log marginal utility ( $\theta_{ikt}^{(M)}$ ), which is a component of  $c_{ikt}^{(M)}$  (see (4.4a) for  $\sigma = M$ ), can be correlated with  $x_{ikt}^{(M)}$ . This raises two questions. First, do the covariance restrictions for  $\hat{c}_{ikt}^{(M)}$  and  $\hat{y}_{ils}^{(M)}$  nevertheless hold for  $\hat{c}_{ikt}^{(M)}$  and  $\hat{y}_{ils}^{(M)}$ ? Second, if so, can we carry out the covariance-based test for  $\hat{c}_{ikt}^{(M)}$  and  $\hat{y}_{ils}^{(M)}$  while ignoring the sampling error that arises from the fact that the population least squares projection has to be estimated from the sample by OLS (ordinary least squares) to obtain  $\hat{c}_{ikt}^{(M)}$  and  $\hat{y}_{ils}^{(M)}$ ? This is sometimes referred to as the problem of generated regressors.

The answer to the first question is yes. Since the maintained hypothesis is (4.4) holds for  $\sigma = M$  and since  $\hat{y}_{ils}^{(M)}$  is a linear function of  $x_{ils}^{(M)}$  and  $\hat{y}_{ils}^{(M)}$ , we have  $\text{Cov}(\hat{c}_{ikt}^{(M)}, \hat{y}_{ils}^{(M)}) = 0$ . Thus the same set of covariance restricti-

ons (6.3) holds for the unique elements of  $\text{Cov}(c_{ikt}^{(M)}, y_{ils}^{(M)})$ . But  $\text{Cov}(c_{ikt}^{(M)}, y_{ils}^{(M)}) = \text{Cov}(c_{ikt}^{(M)}, y_{ils}^{(M)})$ , because  $y_{ikt}^{(M)}$  is orthogonal to  $x_{ikt}^{(M)}$  and  $c_{ikt}^{(M)}$  depends linearly on  $x_{ikt}^{(M)}$ . Therefore, (6.3) also holds for the unique elements of  $\text{Cov}(c_{ikt}^{(M)}, y_{ils}^{(M)})$  under the respective models of consumption. Hereafter we use  $\delta^{(M)}$  for the unique elements of  $\text{Cov}(c_{ikt}^{(M)}, y_{ils}^{(M)})$ .

If we have data on  $\hat{c}_{ikt}^{(M)}$  and  $\hat{y}_{ils}^{(M)}$  testing the covariance restriction (6.3) is quite straightforward. Let  $\hat{z}_1^{(M)}$  be the stacked vector of cross products  $\hat{c}_{ikt}^{(M)} \hat{y}_{ils}^{(M)}$ . For  $M = 0$  or 1, the symmetry restriction is absent and the optimal estimator  $\hat{\delta}^{(M)}$  of unique covariances is simply the sample mean of  $\hat{z}_1^{(M)}$  because  $\text{Cov}(c_{ikt}^{(M)}, y_{ils}^{(M)}) = E(c_{ikt}^{(M)} y_{ils}^{(M)})$ . Its asymptotic covariance matrix  $\hat{\Delta}^{(M)}$  is consistently estimated by the sample variance of  $\hat{z}_1^{(M)}$ . For  $M = 2$ , let  $D$  be the selection matrix representing symmetry (6.1) that maps the stacked vector of covariances into the unique covariances  $\delta^{(2)}$ . A consistent and asymptotically efficient estimator  $\hat{\delta}^{(2)}$  of  $\delta^{(2)}$  and a consistent estimate  $\hat{\Delta}^{(2)}$  of its asymptotic variance are given by

$$(7.2) \quad \hat{\delta}^{(2)} = (D' V^{-1} D)^{-1} (D' V^{-1} z), \quad \hat{\Delta}^{(2)} = (D' V^{-1} D)^{-1}.$$

where  $z$  and  $V$  are, respectively, the sample mean and the sample variance of  $\hat{z}_1^{(2)}$ . Given  $(\hat{\delta}^{(M)}, \hat{\Delta}^{(M)})$ , the linear restriction (6.3) on  $\delta^{(M)}$  can be easily tested for each  $M$  using the fact that the minimum over  $\zeta^{(M)}$  of

$$(7.3) \quad N^{(M)} (\hat{\delta}^{(M)} - W_M \zeta^{(M)})' (\hat{\Delta}^{(M)})^{-1} (\hat{\delta}^{(M)} - W_M \zeta^{(M)})$$

where  $N^{(M)}$  is the number of observations in sub-sample  $M$ , is asymptotically distributed  $\chi^2(q)$  where  $q$  equals the dimension of  $\delta^{(M)}$  minus the rank of  $W_M$ .

The answer to the second question we posed above is also yes: estimation and testing can proceed exactly as described in the previous paragraph while ignoring the problem of generated regressors. The proof, which is straightforward but tedious, is in Appendix 1.

### 8. SAMPLE

This section briefly describes the construction of our sample. The PSID began in 1968 with 4,802 original (parent) households, consisting of a sample representative of the U.S. population (2,930 households) and a supplementary sample of low income households (1,872 households). The PSID has reinterviewed not only the parent households but also new households which arise when members of the parent households split off to form an independent household (a split-off). Hence, the PSID provides matched data on parents together with at least a subset of their independent children.

The tapes for the 1987 wave contain 36,580 individuals who ever existed in the survey.<sup>13</sup> Our sample construction has two steps. First, we identify the head, the spouse (if any) and children (including those yet to be born as of 1968) for each of the 4,802 parent households. There are 24,945 such individuals; virtually all the rest consists of those who married parent household members after 1968. Second, for every year we check whether they are a household head or spouse. If so, they are either in a parent household or in a split-off household. Since each individual record also contains information on the household the individual inhabits, we can get information on the household from the record on the head. This generates, for each year and for each parent household, a set of households (i.e., a family) whose head or spouse was a member of the parent household. We performed these steps for the period 1985-87 to produce 20,519 household-year combinations for 3,070 families.<sup>14</sup> (The rest of the 4,802 original households disappeared from the survey by 1985 and did not re-emerge by 1987.) We choose to focus

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<sup>13</sup>See Survey Research Center (1989), Part 9 of Section 1 for details.

<sup>14</sup>A complete documentation of the sample along with the SAS programs is available upon request from the first author at a nominal charge.

on the most recent three years to maximize the number of split-offs.

From this sample we exclude observations in the supplementary sample of low income households, because doing so makes the sample more representative. As discussed in the next section, the effect of including the low income sample on our results is small. This reduced the sample size to 11,060 observations (household-year combinations) for 1,819 families. We then drop 119 split-off observations for which 1985 was the first year of existence, because their reported income for the first year may be for the parent household from which they split off. This should not bias our results; it only forces us to focus on a particular type of family. We then delete 27 observations with top-coded values for relevant consumption and income components. We also delete 39 observations whose food expenditure (food at home and away from home, plus the value of food stamps) is zero, and 223 observations whose food figure was assigned by the Survey Research Center. Although the exclusion of these 262 ( $\sim 223 + 39$ ) observations does not materially change our results, this is nonetheless a sample selection based on consumption, which may bias our results. Our justification is that these cases are distinguished because of measurement errors.<sup>15</sup> We then delete 196 observations for which the sum of annual labor income and pension income, which is our measure of endowment, is 0. Since this is a sample selection based on endowments, it should not bias our results. This leaves us with a sample of 10,446 household-year combinations for 1,793 families. Table VIII contains simple statistics for this sample by parent/split-off and by year.

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<sup>15</sup> There are 4 clear outlying observations whose food expenditure at home is greater than or equal to \$52,000. Inspection of their record led us to believe that their number is off by one digit, so we decided to divide their food expenditure at home by 10. This made very little difference to our results. For housing consumption, there was no observations with zero consumption or obvious extreme values.

## 9. RESULTS

Although there is no technical difficulty in testing the covariance restrictions jointly, we do the test for food and housing consumption separately. The reason is that the unique cross covariances of endowments with both food and housing consumption are too numerous for some sub-samples. Appendix 2 provides the definition of the variables used in analysis. In particular, endowment is the sum of labor income, pension income, and social security benefits. Asset income is not a good measure of endowment because it reflects past savings behavior, which can be related to a permanent component of the taste shifter. The vector of household characteristics includes leisure and health dummies, for head and spouse.

We created two samples of balanced families from the sample of 10,446 household-year combinations described in section 8. The difference between the two samples lies in the set of observations (household-year combinations) to be deleted before balanced families are extracted: the first sample excludes no observations, while the second sample excludes split-off observations whose head in the 1984 survey expected to receive no bequests.<sup>16</sup> We wish to examine this second sample because family members will not be effectively linked by altruism if the non-negativity constraint on bequests is binding.<sup>17</sup> Since whether the constraint binds or not depends on the distribution of endowments within the family, our Proposition in section 5 guaran-

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<sup>16</sup>The 1984 wave of the PSID contains information on expected amount of bequests. Its tape code is V10950. Since split-offs that did not exist in 1984 were already excluded from the sample (see section 8), this variable in the split-off's record is for that split-off, not for its parent household.

<sup>17</sup>We are thus excluding in the second sample those split-offs whose expected bequests are zero but who are nevertheless altruistically linked through other form of transfers. This is not a problem: what is important is that those in the sample are altruistically linked under altruism, and those with positive expected bequests are certainly so.

tees that the covariance restrictions hold for this sample as well. Table IX gives an enumeration of balanced families by number of split-offs ( $M$ ). Since about 80% of split-offs expect to receive no bequests, balanced families with two or more split-offs in the second sample are scarce. Since the sample size must be sufficiently greater than the number of unique cross covariances (which equals  $5 \times T^2 = 45$  for  $M \geq 2$ ) for the Chi-squared statistic (7.3) to be reliably calculated, we are forced to restrict the application of our two-step procedure to sub-samples  $M = 0, 1, 2$  for the first sample and  $M = 0, 1$  for the second sample.<sup>18</sup>

Panel (i) of Table X reports our results from the first sample. The lower half of the panel displays the Chi-squared statistics, derived from the upper panel, appropriate for sequential testing of the four partially nested hypotheses. It only reports the statistics aggregated over subsamples, which are also Chi-squared. The Chi-square statistics aggregated over family types point to a rejection of all four models of consumption. Only on sub-sample  $M = 1$  for food, neither model (c) (complete markets) nor (d) (family risk-sharing) can be rejected, which suggests that our food data are noisier than data on housing. One possible reason for the relatively weak evidence from food consumption against complete markets and family risk-sharing is that its measurement error is not persistent so that the signal-to-noise ratio gets very small in first differences.<sup>19</sup>

Panel (ii) of Table X shows results from the second sample of balanced

<sup>18</sup> The first-stage regression of log consumption and endowment on household characteristics is for each  $(k, t)$  and for each  $M$ . However, by symmetry (6.1), split-offs with different birth order can be pooled in the first-stage regression.

<sup>19</sup> As made clear in section 2 (see (2.3) and (2.4)), the covariance restrictions implied by complete markets and family risk-sharing reduce to restrictions on first differences in consumption.

families, which excludes split-offs with zero expected bequests. Again, the results for food are weak for complete markets and family risk-sharing, but if data on housing are to receive more weights, the evidence as a whole is that all the four models can be rejected

In order to check the robustness of our results, we perturbed the two samples in the following ways: (i) drop leisure from the vector of household characteristics; (ii) conduct the test in levels rather than in logs; (iii) delete observations whose food consumption or endowment is less than \$520 in 1987 dollars before extracting balanced samples; (iv) delete split-offs inhabited by married daughters of parent households; and (v) include the low income supplementary sample. Overall, our results are robust to these perturbations. The only notable exceptions are the following. The Chi-squared statistic for models (c) and (d) based on food is sensitive but only slightly so; the overall p-value for model (d) for food in the basic sample is slightly above 5% when the low income sample is included. There are only three instances where the overall p-value based on housing is above 1% for any model, and they occur in the second sample: if married daughters are excluded, the p-value for model (b) (altruism) is about 6% and the p for model (d) is about 1%; if the low income sample is included, the p for model (d) is about 9%

Our reading of the evidence can be summarized as follows. First, clearly, complete markets fails empirically. Its failure is obscured by the low signal-to-noise ratio for food changes and the additional noise from the low income sample. Second, altruism may hold but it is operative only for families for which the non-negativity constraint on intergenerational transfers is apparently not binding

# APPENDIX 1: PROOF OF THE CLAIM IN THE END OF SECTION 7

We prove that the problem of generated regressors does not arise in the context of our covariance-based test. Since the proof can be easily adapted to include symmetry (6.1), we focus on the case with  $M = 0$  or 1. To simplify the notation, we drop the superscript "(M)" in the following proof.

Let  $(\beta_{kt}, \gamma_{kt})$  be the coefficients in the (population) least square projection of  $(\log(C_{ikt}), y_{ikt})$  on  $x_{ikt}$ , and let  $(\tilde{\beta}_{kt}, \tilde{\gamma}_{kt})$  be their OLS counterparts. As in the text,  $\hat{z}_i$  is the stacked vector of cross products calculated from  $\hat{c}_{ikt} = \log(C_{ikt}) - \beta'_{kt} x_{ikt}$  and  $\hat{y}_{ils} = y_{ikt} - \gamma'_{kt} x_{ikt}$ , and  $(\hat{\delta}, \hat{\Delta})$  is the sample mean and the sample variance matrix of  $\hat{z}_i$ . We write  $(\tilde{z}_i, \tilde{c}_{ikt}, \tilde{y}_{ikt}, \tilde{\delta}, \tilde{\Delta})$  for the OLS counterpart of  $(\hat{z}_i, \hat{c}_{ikt}, \hat{y}_{ikt}, \hat{\delta}, \hat{\Delta})$ . In particular,  $\tilde{\Delta}$  is the sample variance of  $\tilde{z}_i$ . We wish to show that  $\tilde{\delta}$  has the same asymptotic distribution and that  $\tilde{\Delta}$  and  $\hat{\Delta}$  converge in probability to the same limit. The latter is immediate from the fact that the OLS estimator  $(\tilde{\beta}_{kt}, \tilde{\gamma}_{kt})$  is consistent for  $(\beta_{kt}, \gamma_{kt})$ .

To evaluate the asymptotic distribution of  $\tilde{\delta}$ , we note that the typical element of  $\sqrt{N}(\tilde{\delta} - \delta)$  is

$$\begin{aligned} & \sqrt{N} \left( \text{sample mean of } \hat{c}_{ikt} \hat{y}_{ils} - \text{sample mean of } \tilde{c}_{ikt} \tilde{y}_{ils} \right) \\ &= \sqrt{N} (\tilde{\beta}_{kt} - \beta_{kt})' \left[ \frac{\sum_{i=1}^N x_{ikt} \hat{y}_{ils}}{N} \right] + \sqrt{N} (\tilde{\gamma}_{ts} - \gamma_{ts})' \left[ \frac{\sum_{i=1}^N x_{ikt} \hat{c}_{its}}{N} \right] \\ & \quad + \sqrt{N} (\tilde{\beta}_{kt} - \beta_{kt})' \left( \frac{\sum_{i=1}^N x_{ikt} x_{ils}'}{N} \right) (\tilde{\gamma}_{ts} - \gamma_{ts}). \end{aligned}$$

Clearly, the last term converges to 0 in probability. The first two terms on the right hand side also converge to 0, because the probability limits of the bracketed terms are zero by construction of  $\hat{c}_{ikt}$  and  $\hat{y}_{ils}$ , and  $\sqrt{N}(\tilde{\beta}_{kt} - \beta_{kt})$  and  $\sqrt{N}(\tilde{\gamma}_{ts} - \gamma_{ts})$  are bounded in probability



## APPENDIX 2: MAPPING OF THE PSID CODES TO VARIABLES IN THE ANALYSIS

This appendix provides an explicit mapping from the PSID tape codes (see Survey Research Center (1989) for the definition of the coded variables) to the variables used in this study.

### 1. Consumption and Endowments

The following is our definition of food (FOOD), housing consumption (HOUSE), and endowment (Y).

$$\text{FOOD} = \text{FDEXHO} + \text{FDEXRE} + 12 * \text{FDSPMO}$$

$$\text{HOUSE} = 0.08 * \text{HOUSEV} + \text{RENT} + \text{IRENT}$$

$$Y = \text{EARNH} + \text{EARNW} + \text{SSRHW} + \text{PENNET} + \text{PENNET}$$

Table A1 provides the mapping from the PSID tape codes to the components of food, housing, and endowments. The labor income (EARNH, EARNW) available from the PSID includes a labor component of business income imputed by the Survey Research Center.

### 2. Household Characteristics: Categorical Variables

Table A2 provides the mapping along with the number of dummies created from each categorical variable. The dummies relating to the spouse are zero if the spouse does not exist.

### 3. Household Characteristics: Numerical Variables

The first three variables in Table A3 along with their squares are also included in the vector of household characteristics. To calculate leisure, we assume that time endowment is 24 hours a day. The log of leisure and its square are included in the vector of characteristics. If the head is not married, the spouse's leisure is zero.

TABLE A1: VARIABLES USED FOR CONSUMPTION AND ENDOWMENT

variable name	description	1985	PSID tape code	
			1986	1987
FDEXHO	food at home	11375	12774	13876
FDEXRE	food away from home	11377	12776	13878
FDSPMO	value of food stamps last month	11373	12772	13874
HOUSEV	value of house	11125	12524	13724
RENT	annual rent	11133	12532	13732
IRENT	rental value of free housing	11135	12534	13734
EARNH	labor income of head	12372	13624	14671
EARNW	labor income of spouse	11404	12803	13905
SSBHW	social security benefits	11433	12832+12853	13934+13955
PENVET	veteran's pension for head	11436	12835	13937
PENNV	other pension income for head	11438	12837	13939

TABLE A2: CATEGORICAL VARIABLES OF HOUSEHOLD CHARACTERISTICS

description	#dummies	PSID tape code		
	created	1985	1986	1987
head's sex	1	11607	13012	14113
head's race	2	11938	13565	14612
head's marital status	1	12426	13665	14712
head's health condition	4	11991	13417	14513
spouse's health condition	4	12344	13452	14524

TABLE A3: NUMERICAL VARIABLES OF HOUSEHOLD CHARACTERISTICS

description	PSID tape code		
	1985	1986	1987
head's age	11606	13011	14114
number of adults in the household	11605	13010	14113
number of people under age 18	11609	13014	14117
head's annual hours worked	11146	12545	13745
spouse's annual hours worked	11258	12657	13809

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TABLE

## EQUALITY RESTRICTIONS ON CROSS COVARIANCES

## (i) Complete Markets

			endowment (y)					
			t = 85		t = 86		t = 87	
			k=0	k=1	k=0	k=1	k=0	k=1
c	t=85	k=0	x	x	x	x	x	x
		k=1	x	x	x	x	x	x
	t=86	k=0	x	x	x	x	x	x
		k=1	x	x	x	x	x	x
	t=87	k=0	x	x	x	x	x	x
		k=1	x	x	x	x	x	x

## (ii) Altruism

			endowment (y)					
			t = 85		t = 86		t = 87	
			k=0	k=1	k=0	k=1	k=0	k=1
c	t=85	k=0	x	x	x	x	x	x
		k=1	x	x	x	x	x	x
	t=86	k=0	x	x	x	x	x	x
		k=1	x	x	x	x	x	x
	t=87	k=0	x	x	x	x	x	x
		k=1	x	x	x	x	x	x

TABLE II

## FACTOR STRUCTURE OF LOG MARGINAL UTILITY OF INCOME

Model	$\theta_{ikt}$
(a) complete markets with altruism:	$\lambda_i + \omega_k + p_t$
(b) altruism:	$\lambda_i + \omega_k + p_{it}$
(c) complete markets:	$\lambda_{ik} + p_t$
(d) family risk-sharing:	$\lambda_{ik} + p_{it}$

Note:  $i$  is the family index and  $k$  the member household index ordered by birth

TABLE III

## EXAMPLE OF A TYPE

$\begin{matrix} t \\ k \end{matrix}$	1	2
0	✓	✓
1	✓	✓
2	n.a.	✓

TABLE IV

## REPRESENTATION OF RESTRICTIONS

## (i) Representation of Restrictions on Covariances based on Factors

Model	
(a) complete markets with altruism:	$\text{Cov}(c_{ikt}^{(\sigma)}, y_{ils}^{(\sigma)})$
(b) altruism:	$\text{Cov}(\lambda_1^{(\sigma)}, y_{ils}^{(\sigma)}) + \text{Cov}(p_{it}^{(\sigma)}, y_{ils}^{(\sigma)})$
(c) complete markets:	$\text{Cov}(\lambda_{ik}^{(\sigma)}, y_{ils}^{(\sigma)})$
(d) family risk-sharing:	$\text{Cov}(\lambda_{ik}^{(\sigma)}, y_{ils}^{(\sigma)}) + \text{Cov}(p_{it}^{(\sigma)}, y_{ils}^{(\sigma)})$

## (ii) Equivalent Representation of Restrictions based on Equalities

Model	Equality Restrictions
(a) complete markets with altruism:	$\text{Cov}(c_{ikt}^{(\sigma)}, y_{ils}^{(\sigma)}) = \text{Cov}(c_{imv}^{(\sigma)}, y_{ils}^{(\sigma)})$ for all $(k, t), (m, v), (\ell, s) \in \sigma$
(b) altruism:	$\text{Cov}(c_{ikt}^{(\sigma)}, y_{ils}^{(\sigma)}) = \text{Cov}(c_{imt}^{(\sigma)}, y_{ils}^{(\sigma)})$ for all $(k, m, \ell, t, s)$ such that $(k, t), (m, t), (\ell, s) \in \sigma$
(c) complete markets:	$\text{Cov}(c_{ikt}^{(\sigma)}, y_{ils}^{(\sigma)}) = \text{Cov}(c_{ikv}^{(\sigma)}, y_{ils}^{(\sigma)})$ for all $(k, \ell, t, v, s)$ such that $(k, t), (k, v), (\ell, s) \in \sigma$
(d) family risk-sharing:	$\text{Cov}(c_{ikt}^{(\sigma)}, y_{ils}^{(\sigma)}) - \text{Cov}(c_{ikv}^{(\sigma)}, y_{ils}^{(\sigma)})$ $= \text{Cov}(c_{imt}^{(\sigma)}, y_{ils}^{(\sigma)}) - \text{Cov}(c_{imv}^{(\sigma)}, y_{ils}^{(\sigma)})$ for all $(k, m, \ell, t, v, s)$ such that $(k, t), (k, v), (m, t), (m, v), (\ell, s) \in \sigma$

TABLE V

BROADENING THE FAMILY TYPE

(i) Birth Order and the Type

$\begin{matrix} t \\ k \end{matrix}$	1	2
0	✓	✓
1	✓	✓
2	n.a.	n.a.

$\begin{matrix} t \\ k \end{matrix}$	1	2
0	✓	✓
1	n.a.	n.a.
2	✓	✓

(ii) Symmetry Reduces the Number of Unique Covariances

Covariance Matrix for  $M = 2$

$\begin{matrix} y \\ c \end{matrix}$	$y_{10s}$	$y_{11s}$	$y_{12s}$
$c_{i0t}$	1	3	3
$c_{i1t}$	2	4	3
$c_{i2t}$	2	5	4

TABLE VI

LIST OF UNIQUE CROSS COVARIANCES ( $\delta^{(M)}$ )

$M = 0$	$M = 1$	$M = 2$
$\text{vec}[\text{Cov}(c_{i0}^{(0)}, y_{i0}^{(0)})]$	$\text{vec}[\text{Cov}(c_{i0}^{(1)}, y_{i0}^{(1)})]$	$\text{vec}[\text{Cov}(c_{i0}^{(M)}, y_{i0}^{(M)})]$
	$\text{vec}[\text{Cov}(c_{i0}^{(1)}, c_{i1}^{(1)})]$	$\text{vec}[\text{Cov}(c_{i0}^{(M)}, y_{i1}^{(M)})]$
	$\text{vec}[\text{Cov}(c_{i1}^{(1)}, c_{i0}^{(1)})]$	$\text{vec}[\text{Cov}(c_{i1}^{(M)}, y_{i0}^{(M)})]$
	$\text{vec}[\text{Cov}(c_{i1}^{(1)}, c_{i1}^{(1)})]$	$\text{vec}[\text{Cov}(c_{i1}^{(M)}, y_{i1}^{(M)})]$
		$\text{vec}[\text{Cov}(c_{i1}^{(M)}, y_{i2}^{(M)})]$

Note:  $c_{ik}^{(M)} = (c_{ik1}^{(M)}, \dots, c_{ikt}^{(M)})$ ,  $y_{ik} = (y_{ik1}^{(M)}, \dots, y_{ikt}^{(M)})$ .

TABLE VII

REPRESENTATION OF RESTRICTIONS ON CROSS COVARIANCES  $\delta^{(M)}$ 

Model	$W_M$	#restrictions
$M = 0$		
(a) complete markets with altruism:	$I_T \otimes I_T$	$T^2 - 1$
(b) altruism:	$I_{T^2}$	0
(c) complete markets:	$I_T \otimes I_T$	$T^2 - T$
(d) family risk-sharing:	$I_{T^2}$	0
$M = 1$		
(a) complete markets with altruism:	$I_2 \otimes I_2 \otimes I_T \otimes I_T$	$4T^2 - 2T$
(b) altruism:	$I_{2^2} \otimes I_{T^2}$	$4T^2 - 2T^2$
(c) complete markets:	$I_{4T} \otimes I_T$	$4T^2 - 4T$
(d) family risk-sharing:	$(I_2 \otimes I_2 \otimes I_{T^2}, I_{4T} \otimes I_T)$	$4T^2 - (2T^2 + 2T)$
$M = 2$		
(a) complete markets with altruism:	$G \otimes I_T \otimes I_T$	$5T^2 - 2T$
(b) altruism:	$G \otimes I_{T^2}$	$5T^2 - 2T^2$
(c) complete markets:	$I_{5T} \otimes I_T$	$5T^2 - 5T$
(d) family risk-sharing:	$(G \otimes I_{T^2}, I_{5T} \otimes I_T)$	$5T^2 - (2T^2 + 3T)$

Note:  $i_T$  is a  $T$  dimensional column vector of ones.

$$G_{5 \times 2} = \begin{pmatrix} I_2 \otimes I_2 \\ 0 \quad 1 \end{pmatrix}$$

$W_1$  for model (d) has  $2T^2 + 4T$  columns, but only  $(2T^2 + 2T)$  of them are linearly independent. Similarly,  $W_M$  for  $M \geq 2$  for model (d) has  $2T^2 + 5T$  columns, but only  $2T^2 + 3T$  of them are linearly independent.



TABLE VII

## SIMPLE STATISTICS FOR PARENT HOUSEHOLDS AND SPLIT-OFFS BY YEAR

1985		Mean	Std Dev	Mean	Std Dev
		parents (N = 1728)		split-offs (N = 1637)	
MARRIED	1 if married	0.65	0.48	0.71	0.46
HAGE	head's age	58.66	13.91	31.17	5.98
FSZ	family size	2.43	1.28	3.01	1.46
NKID	#people under 18	0.45	0.88	1.24	1.19
FOOD	food consumption	4362.2	2952.6	4250.0	2137.7
HOUSE	housing consumption	5805.6	4750.7	5137.7	4144.7
Y	endowment	27051.1	25268.3	30129.8	23385.0
INCOME	household income	35835.5	33912.3	33470.0	25220.1

1986		parents (N = 1712)		split-offs (N = 1824)	
MARRIED	1 if married	0.65	0.48	0.70	0.46
HAGE	head's age	59.39	13.70	31.31	6.33
FSZ	family size	2.38	1.26	2.95	1.45
NKID	#people under 18	0.41	0.82	1.18	1.20
FOOD	food consumption	4230.4	2600.9	4182.7	2141.9
HOUSE	housing consumption	5812.8	4931.6	5138.4	4507.9
Y	endowment	26992.9	26580.1	30150.2	23900.6
INCOME	household income	35721.1	31994.3	33271.7	25459.9

1987		parents (N = 1649)		split-offs (N = 1896)	
MARRIED	1 if married	0.64	0.48	0.70	0.46
HAGE	head's age	59.80	13.44	31.90	6.48
FSZ	family size	2.35	1.23	2.99	1.45
NKID	#people under 18	0.38	0.79	1.21	1.20
FOOD	food consumption	4071.3	2461.4	4229.6	2153.3
HOUSE	housing consumption	6068.3	5481.3	5437.9	5095.2
Y	endowment	28038.9	32402.3	31386.9	22931.4
INCOME	household income	36935.1	38492.5	34887.9	24946.6

Note: Consumption, endowment and income are in 1987 dollars. See Appendix 2 for the definition of FOOD, HOUSE and Y. INCOME is household pretax income.

TABLE IX

## ENUMERATION OF BALANCED FAMILIES BY NUMBER OF SPLIT-OFFS

Description of the sample	Number of split-offs (M)						total
	0	1	2	3	4	5+	
basic sample	643	192	121	67	17	11	1,051
no split-offs with zero bequests	1,066	162	27	2	2	0	1,259

TABLE X

## CHI-SQUARE TESTS OF THE FOUR MODELS OF CONSUMPTION

## Panel (i): Basic Sample

		food		housing		
subsample	#families	d.f.	$\chi^2$	p-value	$\chi^2$	p-value
Model (a): Complete Markets with Altruism						
M = 0	643	6	12.0	0.0612	24.0	0.0005
M = 1	192	30	68.8	0.0001	84.6	0.0000
M = 2	121	39	227.4	0.0000	217.4	0.0000
total:	956	75	308.2	0.0000	326.0	0.0000
Model (b): Altruism						
M = 0	643	0	0.0	1.0000	0.0	1.0000
M = 1	192	18	46.6	0.0002	56.2	0.0000
M = 2	121	27	136.5	0.0000	181.6	0.0000
total:	956	45	183.1	0.0000	237.8	0.0000
Model (c): Complete Markets						
M = 0	643	6	12.0	0.0612	24.0	0.0005
M = 1	192	24	32.0	0.1266	61.7	0.0000
M = 2	121	30	113.0	0.0000	140.0	0.0000
total:	956	60	157.0	0.0000	225.7	0.0000
Model (d): Family Risk-sharing						
M = 0	643	0	0.0	1.0000	0.0	1.0000
M = 1	192	12	18.8	0.0926	34.7	0.0005
M = 2	121	18	61.5	0.0000	121.2	0.0000
total:	956	30	80.4	0.0000	155.9	0.0000

## Chi-square Statistics for Sequential Testing

null hypothesis	maintained hypothesis	d.f.	food		housing	
			$\chi^2$	p-value	$\chi^2$	p-value
Model (b)	model (d)	15	102.7	0.0000	81.9	0.0000
Model (c)	model (d)	30	76.7	0.0000	69.8	0.0001
Model (a)	model (b)	30	125.1	0.0000	88.2	0.0000
Model (a)	model (c)	15	151.2	0.0000	100.2	0.0000

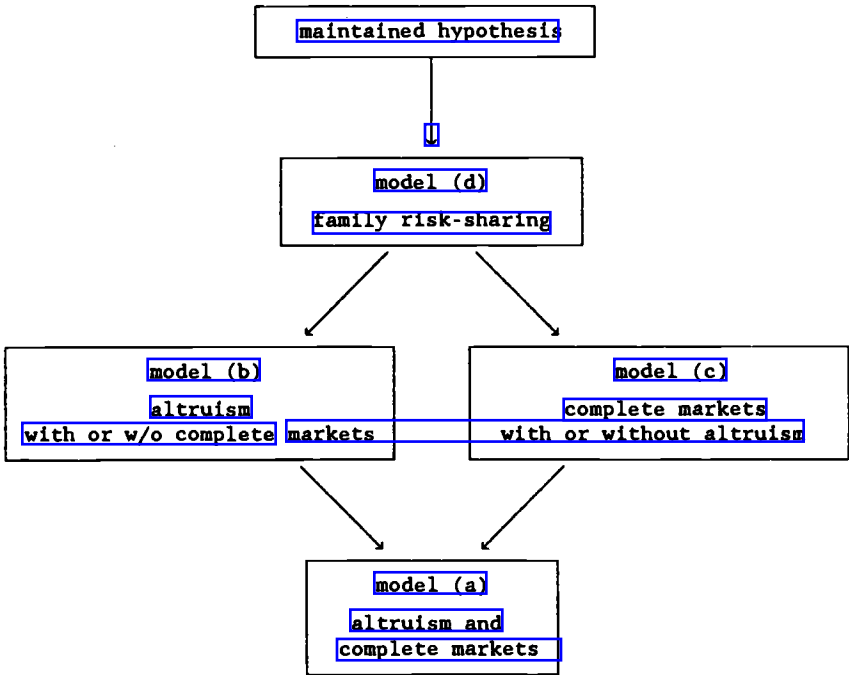
TABLE X

## CHI-SQUARE TESTS OF THE FOUR MODELS OF CONSUMPTION

Panel (ii): No Split-offs with Zero Expected Requests

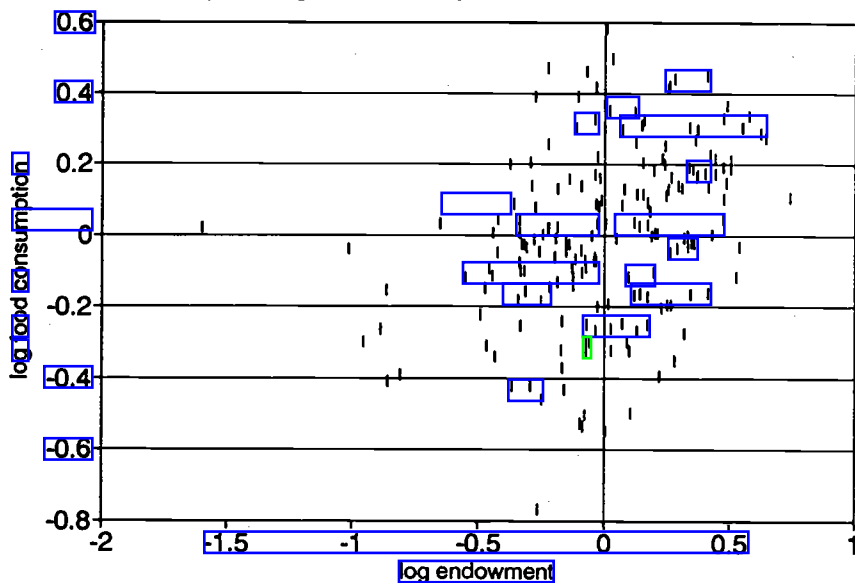
		food				housing	
subsample	#families	d.f.	$\chi^2$	p-value	$\chi^2$	p-value	
<b>Model (a): Complete Markets with Altruism</b>							
M = 0	1066	6	11.5	0.0736	43.0	0.0000	
M = 1	162	30	68.0	0.0001	89.1	0.0000	
total:	1228	36	79.5	0.0000	132.2	0.0000	
<b>Model (b): Altruism</b>							
M = 0	1066	0	0.0	1.0000	0.0	1.0000	
M = 1	162	18	31.1	0.0282	49.7	0.0001	
total:	1228	18	31.1	0.0282	49.7	0.0001	
<b>Model (c): Complete Markets</b>							
M = 0	1066	6	11.5	0.0736	43.0	0.0000	
M = 1	162	24	29.9	0.1869	59.0	0.0001	
total:	1228	30	41.5	0.0796	102.0	0.0000	
<b>Model (d): Family Risk-sharing</b>							
M = 0	1066	0	0.0	1.0000	0.0	1.0000	
M = 1	162	12	4.2	0.9793	29.5	0.0034	
total:	1228	12	4.2	0.9793	29.5	0.0034	
<b>Chi-square Statistics for Sequential Testing</b>							
		food				housing	
null hypothesis	maintained hypothesis	d.f.	$\chi^2$	p-value	$\chi^2$	p-value	
Model (b)	model (d)	6	26.9	0.0002	20.2	0.0025	
Model (c)	model (d)	18	37.2	0.0049	72.5	0.0000	
Model (a)	model (b)	18	48.4	0.0001	82.5	0.0000	
Model (a)	model (c)	6	38.1	0.0000	30.2	0.0000	

FIGURE 1



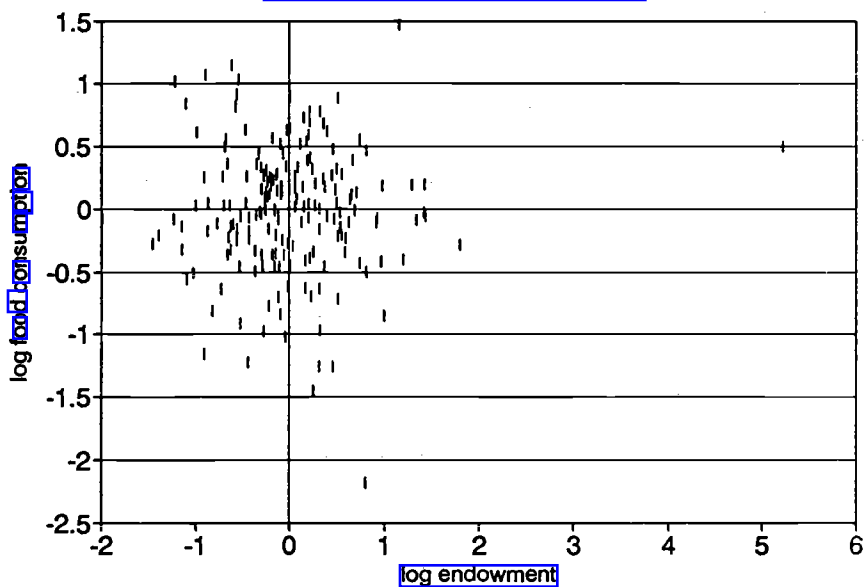
**FIGURE 2A - Food and Endowment**

**(i) Average over Family Members and Time**



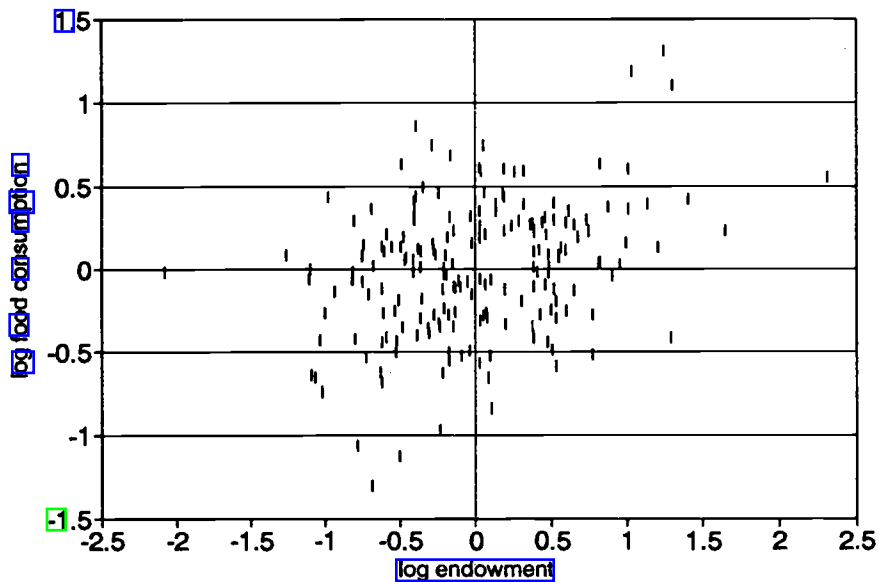
**FIGURE 2A - Food and Endowment**

**(ii) Differential Changes in Logs**



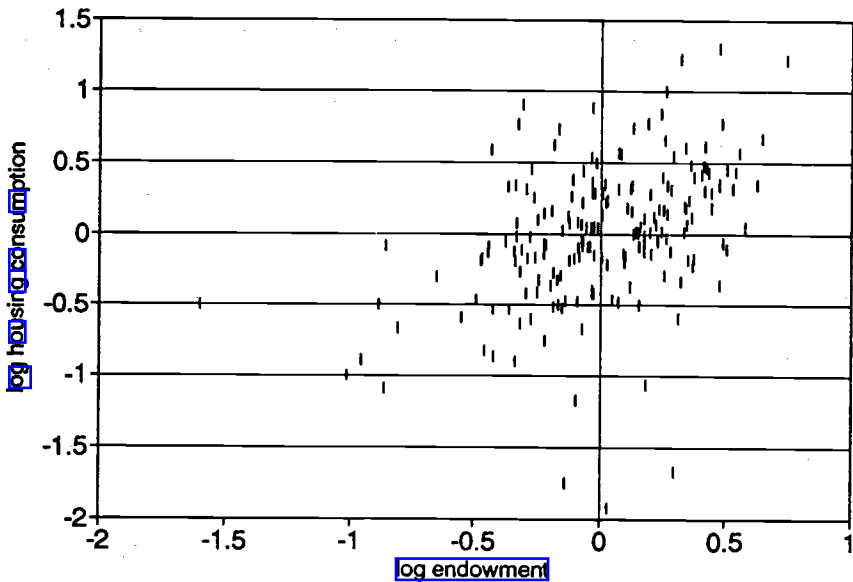
## FIGURE 2A - Food and Endowment

(iii) Differences in Time Averages



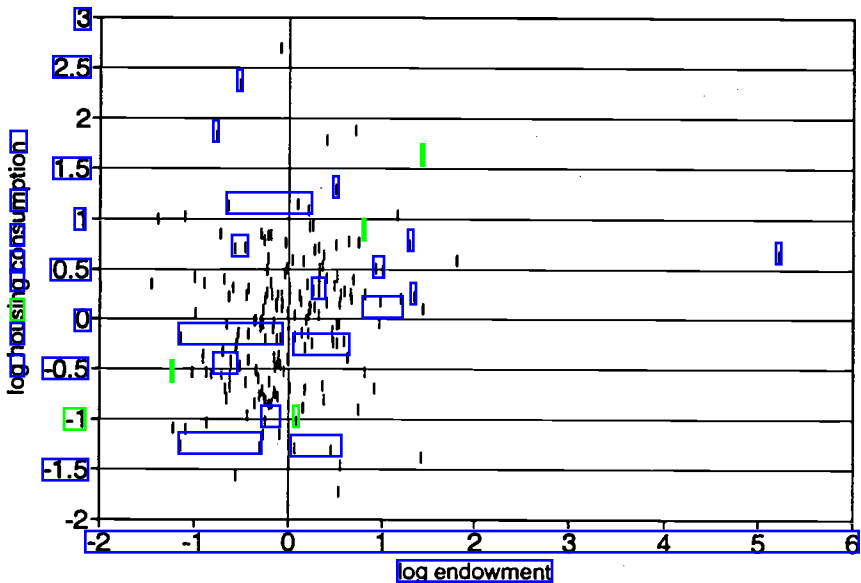
**FIGURE 2B - Housing and Endowment**

**(i) Average over Family Members and Time**



**FIGURE 2B - Housing and Endowment**

**(ii) Differential Changes in Logs**



**FIGURE 2B - Housing and Endowment**  
**(iii) Differences in Time Averages**

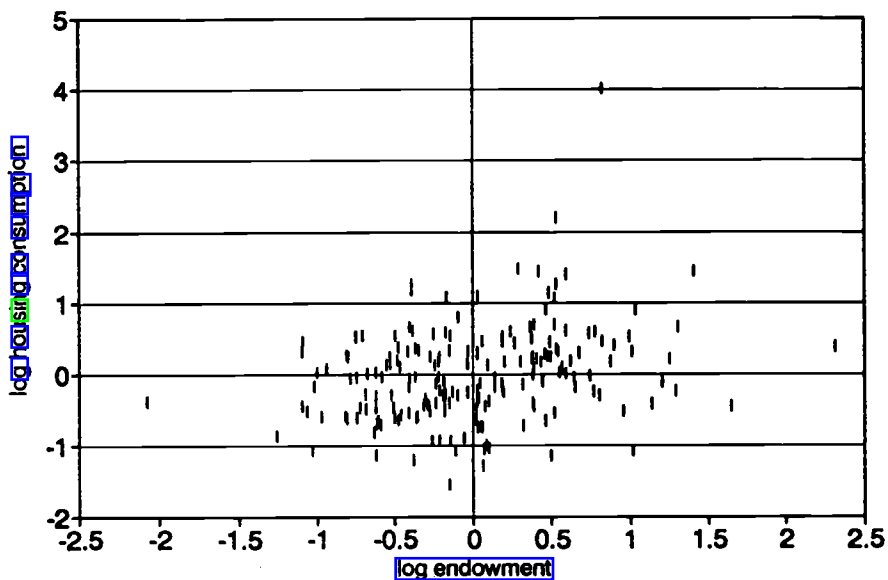




FIGURE 3

