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AND INTERMEDIARIES

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A Macroeconomic Model with Financially Constrained Producers and Intermediaries

Vadim Elenev, Tim Landvoigt, and Stijn Van Nieuwerburgh

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**ABSTRACT**

How much capital should financial intermediaries hold? We propose a general equilibrium model with a financial sector that makes risky long-term loans to firms, funded by deposits from savers. Government guarantees create a role for bank capital regulation. The model captures the sharp and persistent drop in macro-economic aggregates and credit provision as well as the sharp change in credit spreads observed during the Great Recession. Policies requiring intermediaries to hold more capital reduce financial fragility, reduce the size of the financial and non-financial sectors, and locally increase macro-economic volatility. They redistribute wealth from savers to the owners of banks and non-financial firms. Current capital requirements are close to optimal.

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# 1 Introduction

The financial crisis and Great Recession of 2007-09 underscored the importance of the financial system for the broader economy. Borrower default rates, bank insolvencies, government bailouts of financial institutions, and credit spreads all spiked while real interest rates were very low. The disruptions in financial intermediation fed back on the real economy. Consumption, investment, and output all fell substantially and persistently.

These events have prompted a vigorous yet unresolved debate among policymakers and academics on whether the economy would be better off with stricter bank capital requirements. The December 2017 Minneapolis Plan reflects the Federal Reserve's view and proposes raising bank capital requirements to 23.5% of risk-weighted assets, with further increases to 38% for banks that remain systemically important. In their seminal book, Admati and Hellwig (2013) propose raising capital requirements to 25% of assets. Larger equity capital buffers would result in less risk-taking, lower risk of bank failure and concomitant government bailouts, but also in a smaller banking sector that lends less to the real economy, depressing investment and output. Considering this trade-off, Admati and Hellwig argue that "for society, there are in fact significant benefits and essentially no cost from much higher equity requirements." The authors of the Minneapolis Plan agree, writing that their plan "will have paid for itself many times over if it avoids one financial crisis." This argument is not without controversy in the academy (Calomiris, 2013) and heavily contested by the industry.

What is missing in this debate is a quantitative general equilibrium model that embeds a financial sector in a model of the macro-economy, and that can capture infrequent but large financial crises. Our paper proposes such a model. In the model, banks extend long-term loans to firms who invest and are subject to aggregate and idiosyncratic productivity shocks. Firm default results in losses for their lenders, which can trigger bank default. Even banks that remain standing become fragile and cut lending to firms. A large and persistent decline in credit depresses output persistently. The high leverage of banks, which far exceeds that of firms, amplifies modest credit losses into financial disasters. The nonlinear behavior of credit spreads reflects this financial distress. The government bails out the creditors of the banks that fail by issuing government debt, gradually repaid through future taxation. Because financial intermediaries are constrained in their ability to re-lever and raising new equity from their

shareholders is expensive, the banking sector shrinks substantially and persistently. Real interest rates must fall to induce savers to accommodate the reduction in deposits. Banks' reduced ability to absorb aggregate risk in financial crises results in a deterioration of risk sharing and higher macro-economic volatility. The intermediary-driven dynamics arise in equilibrium since all aggregate shocks emerge from the real sector.

The calibrated model matches many features of the data, both in terms of macro-economic quantities and prices. It matches the average credit spread and its volatility. Faced with a realistic corporate bond rate, firms choose the observed amount of leverage. The non-financial leverage ratio is 37%, close to the U.S. data. The model delivers a 93% leverage ratio for financial firms, a key moment not directly targeted by the calibration, which is close to the data. Debt is attractive to banks for four reasons. First, debt enjoys a tax shield. Second, the government guarantees the liabilities of the bank. This guarantee captures not only deposit insurance but also broader too-big-to-fail guarantees to banks and the rest of the levered financial system.<sup>1</sup> Third, banks face equity adjustment costs which increase the cost of equity relative to debt. Fourth, banks provide a safe asset to patient households with a strong preference for holding such risk-free assets. While the first motive for debt financing also applies to non-financial firms, the other three do not. The large wedge between financial and non-financial sector leverage is a key feature of many developed economies and crucial to understanding systemic risk in society. The equilibrium fully takes into account that the cost of bank debt changes endogenously with the safety of the financial sector.

Our main exercise is to study macro-prudential policy in this environment. We study increasing the minimum bank equity capital requirement from its pre-crisis level of 6% of assets. Higher capital requirements reduce financial fragility but at the cost of shrinking the economy and, in some cases, increasing macroeconomic volatility. They are successful at reducing financial leverage and the bank failure rate. Banks that hold more equity capital become effectively more risk averse and stay away farther from their regulatory constraint. They also become smaller, shrinking both assets and liabilities. The charter value of a bank shrinks because banks with more onerous equity capital requirements have reduced ability to take advantage of

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<sup>1</sup>We use the labels intermediaries and banks interchangeably to mean the entire levered financial sector. That sector also includes broker-dealers and insurance companies, which are subject to macro-prudential regulation and enjoy explicit or implicit government guarantees on their liabilities. Appendix C.7 provides a detailed definition of our intermediary sector.

the low cost of debt.

Corporate debt also becomes safer and loss rates fall. Firms borrow less and reduce leverage. Equilibrium credit spreads are higher despite a reduction in loss rates. In other words, the price of credit increases strongly with bank capital requirements since intermediaries are forced to move away from cheap deposit financing. Higher credit spreads are consistent with lower non-financial leverage.

The reduction in firm and bank bankruptcies is good news for the economy, as it frees up resources otherwise spent on deadweight losses from bankruptcy. However, firms' reduced ability to borrow from smaller banks reduces investment, the capital stock, and output. The reduced size of the economy is the first adverse effect from tighter macro-prudential policy.

A second adverse effect is that macro-economic volatility rises, locally, with tighter bank capital constraints. Two offsetting effects determine macro volatility. First, a reduction in financial fragility lowers macro-economic volatility. Second, a reduction in risk sharing increases macro-economic volatility. The latter effect dominates the former as minimum bank equity increases from 6% to 15% of assets, and macro-economic volatility increases. Increasing bank equity capital further gradually lowers volatility. Loosening capital requirements from 6% downward also increases volatility as the fragility effect dominates.

To rank economies that differ in capital requirement, we calculate welfare for the two types of households in the model: patient savers who invest in risk-free assets and impatient borrowers who are the equity holders of the non-financial and financial firms. Tighter capital requirements redistribute wealth from savers to borrowers. A smaller banking sector reduces deposits and thereby the wealth of the savers. Borrower-equity holders receive a larger fraction of aggregate income as banks and firms shift their capital structure towards equity. Thus, perversely, the owners of the banks gain from tighter regulation. Depending on the aggregation scheme, welfare maximizing capital requirements are either slightly higher or slightly lower than the pre-crisis level. A utilitarian social welfare function generates modest positive aggregate welfare gains from tighter bank equity capital requirements, with gains reaching a maximum at a 9% equity capital-asset ratio. A tax-and-transfer scheme that induces Pareto improvements instead suggests that slightly looser capital requirements, at 4%, would be optimal. Current capital requirement levels are straddled by these two numbers. An alternative counter-cyclical capital requirement policy that tightens capital requirements in good times and relaxes them in times

of financial stress allows for a larger financial sector and for improved risk sharing. Out of the policies we evaluate, it admits the largest Pareto improvement.

Our work is at the intersection of macro-economics, asset pricing, corporate finance, and banking. We contribute to the literature on the role of credit constraints in models of the macro-economy. Building on early work that emphasized the importance of credit markets in amplifying business cycle shocks, notably Bernanke, Gertler, and Gilchrist (1996) and Kiyotaki and Moore (1997), a second generation of models has explored nonlinear dynamics, notably Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013). A first modeling contribution is to separate out the role of producers and banks, while the prior literature usually combines their roles. Combining balance sheets implicitly assumes that financial intermediaries hold equity claims in productive firms, while in reality, banks hold debt-like claims. These debt contracts are subject to default risk of the borrowers.<sup>2</sup> Intermediaries help to allocate risk between borrowers and savers, and their risk-bearing capacity is a key state variable. The separation of producers and intermediaries not only allows us to generate the large wedge between financial and non-financial leverage, it also activates a second financial accelerator in addition to the traditional financial accelerator mechanism. Losses on corporate loans reduce intermediary net worth, reduce banks' ability and willingness to extend loans to producers, which hurts investment and output in the real economy.

The second new model ingredient is to introduce the possibility of default for financial intermediaries, with the government guaranteeing bank debt for savers (deposit insurance). As Reinhart and Rogoff (2009) and Jorda, Schularick, and Taylor (2014) make clear, financial intermediaries frequently become insolvent. When they do, their creditors (mostly depositors) are typically bailed out by the government. The combination of limited liability and government guarantees affects banks' risk taking incentives and creates scope for regulation that limits bank leverage.<sup>3</sup> We model a Basel-style regulatory capital requirement that limits intermediary

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<sup>2</sup>It is well understood that debt-like contracts arise in order to reduce the cost of gathering information and to mitigate principal-agent problems. See the costly state verification models in the tradition of Townsend (1979) and Gale and Hellwig (1985), and the work on the information insensitivity of debt by Dang, Gorton, and Holmstrom (2015). Our debt is non-state contingent which confers the advantage that loan defaults induce losses for the intermediaries. Costly state verification models also justify the existence of financial intermediaries who avoid the duplication of verification costs, as in Williamson (1987), Krasa and Villamil (1992), Diamond (1984). Recent work by Klimenko, Pfeil, Rochet, and Nicolo (2016), Rampini and Viswanathan (2017), and Gale and Gottardi (2017) also models intermediaries separately from producers. The setting is simpler since their focus is theoretical; ours is quantitative.

<sup>3</sup>See Kareken and Wallace (1978), Van den Heuvel (2008), Farhi and Tirole (2012), or Gomes, Grotteria, and

liabilities to a certain fraction of their assets. The minimum regulatory capital that banks must hold is the key macro-prudential policy parameter. Banks optimally trade off the costs and benefits of default for their shareholders. Our equilibrium features a counter-cyclical fraction of banks defaulting, consistent with the data. In case of bank default, the government steps in, liquidates the bank's assets and makes whole their creditors. By allowing for the possibility of bank insolvencies, our model helps explain how a corporate default wave triggers financial fragility. Intermediaries perform the traditional role of maturity and risk transformation. Most models in the intermediary literature feature no default on corporate loans.<sup>4</sup> Those that do feature default employ short-term debt, abstracting from a key source of risk associated with financial intermediation. Long-term debt allows us to realistically model liquidity-based default of non-financial firms.

The third key model element is the inclusion of savers who do not participate in risky asset markets and the endogenous determination of safe asset rates. The data reveal that a large fraction of households indeed do not participate in risky asset markets. These savers are the marginal agents in the market for safe debt. With risk averse savers and endogenous safe asset rates, the dynamics of the model change substantially. In a crisis, intermediaries contract the size of their balance sheet, reducing the supply of safe assets. This is only partially offset by an increase in government debt due to counter-cyclical fiscal policy. To clear the market, the equilibrium real interest rates must fall. The low cost of debt allows the intermediaries to recapitalize more quickly, dampening the effect of the crisis. More generally, a key question in the literature is how tighter bank capital regulation affects bank profitability. To answer this question, one needs to understand how supply and demand in both major markets banks operate in, the loan market and the market for safe debt, respond in general equilibrium. Our model endogenously determines supply, demand, and risk in both markets. While the safe debt issued by banks is guaranteed by the government and therefore risk-free for savers, it is not risk-free to society due to the possibility of bank default.

Our paper belongs to the literature on quantitative models of optimal bank regulation, including Wachter (2018). Others justify the presence of bank leverage or net worth constraints by the ability of banks to divert cash flows, as in Gertler, Kiyotaki, and Queralto (2012).

<sup>4</sup>For example, Cúrdia and Woodford (2008), Goodfriend and McCallum (2007), Meh and Moran (2010), and Christiano, Motto, and Rostagno (2014). A few exceptions are Gertler and Kiyotaki (2010), Angeloni and Faia (2013), Hirakata, Sudo, and Ueda (2013), Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis (2015), and Gesteira (2016).

ing Van den Heuvel (2008), Nguyen (2015), Begenau (2016), Begenau and Landvoigt (2017), Corbae and D’Erasmus (2017), and Davydiuk (2017). Relative to the previous literature, we study a general equilibrium model that features severe financial recessions arising due to the nonlinear interaction of financial constraints in the production and intermediation sectors. This allows us to quantify the benefit of preventing such financial crises using regulation. A different branch of the normative literature instead studies the interactions between conventional and unconventional monetary policy and financial intermediation.<sup>5</sup>

Our work also contributes to the intermediary-based asset pricing literature.<sup>6</sup> The model features a market for long-term defaultable bonds and for short-term risk-free bonds. It generates the unconditional credit spread, a puzzle in the asset pricing literature (Chen, 2010). It also generates the volatility and counter-cyclical of that spread, consistent with patterns documented by Krishnamurthy and Muir (2017). For the observed amount of credit risk, substantial variation in intermediary wealth generates a high enough price of credit risk. The (shadow) stochastic discount factor of the intermediaries, driven by the intermediary net worth dynamics, is volatile and counter-cyclical.

We also contribute to the literature on second-moment shocks to firm productivity, since a key shock in our model is an increase in the cross-sectional dispersion of firm-level productivity growth.<sup>7</sup> In our setup, these shocks cause costly firm defaults and intermediary losses, which in turn lead to lower credit supply and investment. This is a complementary mechanism to the inaction effect of Bloom (2009), where increased uncertainty causes depressed investment because of fixed costs. Alfaro, Bloom, and Lin (2016) study how financial frictions amplify the effect of uncertainty shocks on investment and hiring.

More broadly, our model creates room for macro-prudential regulation due to incomplete

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<sup>5</sup>See Gertler and Karadi (2011), Angeloni and Faia (2013), Drechsler, Savov, and Schnabl (2017b, 2017a), Cúrdia and Woodford (2016), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2016), Elenev (2016), and De Fiore, Hoerova, and Uhlig (2017).

<sup>6</sup>In addition to the work cited above, notable contributions are He and Krishnamurthy (2012, 2013, 2014), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), Maggiori (2013), and Moreira and Savov (2016). On the empirical side, He, Kelly, and Manela (2017) develop a risk factor which captures the systematic risk associated with declines in intermediary equity capital, and Adrian, Etula, and Muir (2014) document that intermediary leverage performs well in pricing the cross-section of stock returns.

<sup>7</sup>The shock is calibrated from firm-level evidence in Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012). Christiano, Motto, and Rostagno (2014) introduce a similar “risk” shock and argue it is an important driver of business cycle dynamics. Jermann and Quadrini (2012) study how financial shocks affect balance sheet variables. One interpretation of our uncertainty shocks is as aggregate misallocation shocks, as in Hsieh and Klenow (2009). Ai, Li, and Yang (2016) study the role of intermediaries in reducing capital misallocation.



markets and borrowing constraints.<sup>8</sup> Our model is set up to study imperfect risk sharing between borrower, saver, and intermediation sectors. The only purpose of heterogeneity within the borrower sector and within the intermediary sector is to generate fractional default.

Finally, our paper contributes in its solution technique. The model has two exogenous and persistent sources of aggregate risk and five endogenous aggregate state variables, which capture the wealth distribution. It features default and occasionally binding borrowing constraints in both non-financial and financial sectors. To solve this problem, we provide a nonlinear global solution method. The algorithm, detailed in computational appendix B, solves for a set of nonlinear equations including the Euler, Kuhn-Tucker, and market clearing equations.

The rest of the paper is organized as follows. Section 2 discusses the model setup. Section 3 presents the calibration. Section 4 contains the main results. Section 5 uses the model to study various macro-prudential policies. Section 6 concludes. All model derivations, some details on the calibration, and some additional quantitative results are relegated to the appendix.

## 2 The Model

### 2.1 Preferences, Technology, Timing

**Preferences** The model features two groups of households: borrower-entrepreneurs (denoted by superscript B) and savers (denoted by S). Savers are more patient than borrower-entrepreneurs, implying for the discount factors that  $\beta_B < \beta_S$ . All agents have Epstein-Zin preferences over utility streams  $\{u_t^j\}_{t=0}^{\infty}$  with intertemporal elasticity of substitution  $\nu_j$  and risk aversion  $\sigma^j$

$$U_t^j \equiv \left\{ (1 - \beta_j) (u_t^j)^{1-1/\nu_j} + \beta_j \left( \mathbb{E}_t \left[ (U_{t+1}^j)^{1-\sigma_j} \right] \right)^{\frac{1-1/\nu_j}{1-\sigma_j}} \right\}^{\frac{1}{1-1/\nu_j}} \quad (1)$$

for  $j = B, S$ . Agents derive utility from consumption of the economy's sole good, such that  $u_t^j = C_t^j$ , for  $j = B, S$ .

<sup>8</sup>Papers that have studied the qualitative role of these frictions in determining optimal policy are Lorenzoni (2008), Mendoza (2010), Korinek (2012), Bianchi and Mendoza (2013), Guerrieri and Lorenzoni (2015), and Clerc et. al. (2015).

**Technology** Non-financial firms, or firms for short, operate the production technology, which turns capital and labor into aggregate output:

$$Y_t \equiv Z_t^A K_t^{1-\alpha} L_t^\alpha, \quad (2)$$

where  $K_t$  is capital,  $L_t$  is labor, and  $Z_t^A$  is total factor productivity (TFP). Shocks to  $Z^A$  are the first source of aggregate risk in the model. In addition to the technology for producing consumption goods, firms also have access to a technology that turns consumption into capital goods subject to adjustment costs.

Firms are funded by long-term corporate debt issued by intermediaries, and by equity provided by borrower-entrepreneurs. There are no frictions associated with changing the equity capital of non-financial firms. This is equivalent to assuming that borrower-entrepreneurs hold the firms' capital stock directly.

Financial intermediaries, or banks for short, are profit-maximizing firms that extend loans to non-financial firms. They fund these loans through deposits that they issue to savers and equity capital that they raise from borrower-entrepreneurs. Banks face equity issuance costs, an important financial friction described in detail below.

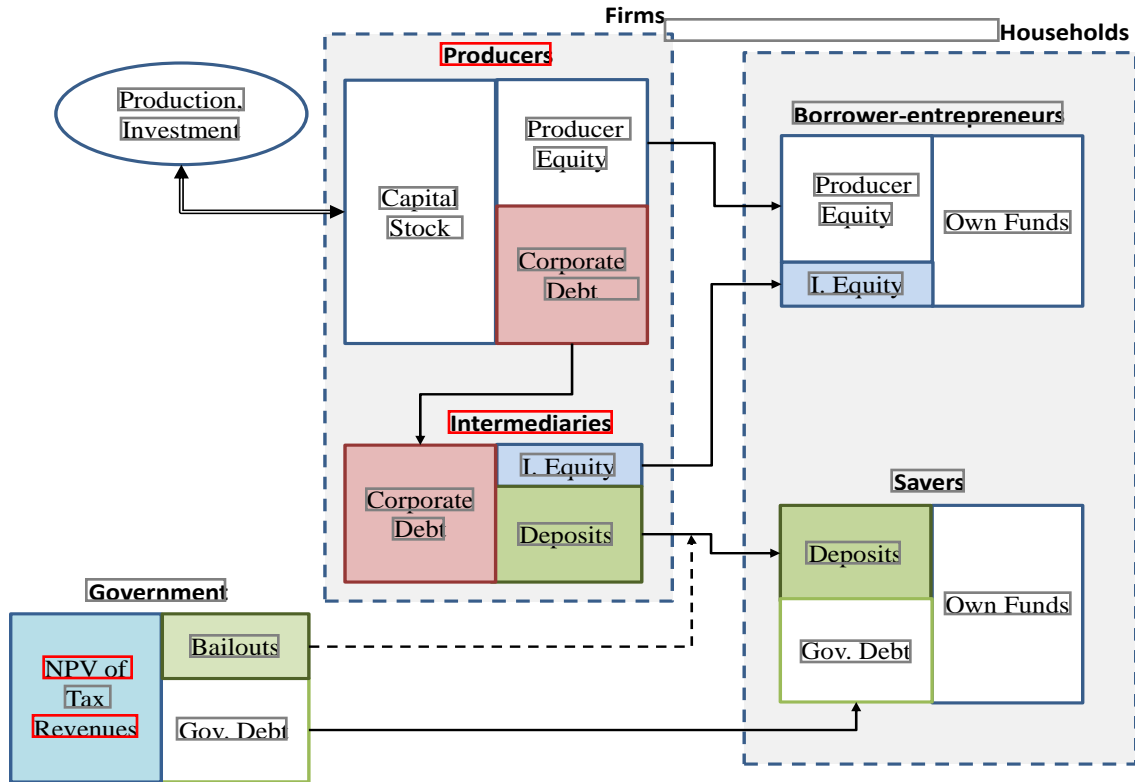
We assume that savers only hold risk-free assets to capture the reality of limited participation in risky asset markets. The provision of safe assets to savers is an important function of the intermediary sector. Borrower-entrepreneurs and savers are endowed with  $\bar{L}^B$  and  $\bar{L}^S$  units of labor, respectively. Both types of households supply their labor endowment inelastically.

As explained below, both firms and banks face idiosyncratic shocks. This within-type heterogeneity allows us to capture fractional default. Perfect within-type risk sharing implies no further implications from within-type heterogeneity, and allows us to focus on incomplete risk-sharing between types.

Figure 1 illustrates the balance sheets of the model's agents and their interactions. Each agent's problem depends on the wealth of others; the entire wealth distribution is a state variable. Each agent must forecast how that state variable evolves, including the bankruptcy decisions of borrowers and intermediaries.

**Timing** The timing of agents' decisions at the beginning of period  $t$  is as follows:

Figure 1: Overview of Balance Sheets of Model Agents



1. Aggregate and idiosyncratic productivity shocks for firms are realized. Production occurs. Idiosyncratic profit shocks for banks are realized.

2. Firms with low idiosyncratic productivity realizations default. Banks assume ownership of bankrupt firms.

3. Individual intermediaries decide whether to declare bankruptcy. The government liquidates bankrupt intermediaries. If intermediary assets are insufficient to cover the amount owed to depositors, the government provides the shortfall (deposit insurance).

4. All agents solve their consumption and portfolio choice problems. Markets clear. Households consume.

We now describe the saver, borrower-entrepreneur, and intermediary problems in more detail.

A full set of Bellman equations and first order conditions is relegated to appendix A.

## 2.2 Savers

Savers can invest in one-period risk free bonds (deposits and government debt) that trade at price  $q_t^f$ . They inelastically supply their unit of labor  $\bar{L}^S$  and earn wage  $w_t^S$ . Entering with wealth  $W_t^S$ , the saver's problem is to choose consumption  $C_t^S$  and short-term bonds  $B_t^S$  to maximize life-time utility  $U_t^S$  in (1), subject to the budget constraint:

$$C_t^S + (q_t^f + \tau^D r_t^f) B_t^S \leq W_t^S + (1 - \tau_t^S) w_t^S \bar{L}^S + G_t^{T,S} + O_t^S, \quad (3)$$

where saver wealth is simply given by the face value of last period's bond purchases  $W_t^S = B_{t-1}^S$ . The budget constraint (3) shows that savers use beginning-of-period wealth, after-tax labor income, transfer income from the government ( $G_t^{T,S}$ ), and transfer income from bankruptcy proceedings ( $O_t^S$ ) to be defined below, to pay for consumption and purchases of short-term bonds. Savers are taxed on interest rate income at the time they purchase the bonds at rate  $\tau^D$ . The risk-free interest rate is the yield on risk free bonds,  $r_t^f = 1/q_t^f - 1$ .

## 2.3 Borrower-Entrepreneurs and Firms

There is a unit-mass of identical borrower-entrepreneurs indexed by  $i$ . The borrowers form a collective ("family") that provides insurance against idiosyncratic shocks.

Each entrepreneur owns a technology, a firm, that creates consumption goods  $Y_{i,t}$  from capital  $K_{i,t}$  and labor  $L_{i,t}$ . At the beginning of the period, each firm receives an idiosyncratic productivity shock  $\omega_{i,t} \sim F_{\omega,t}$ . Output depends on aggregate productivity  $Z_t^A$  and idiosyncratic productivity  $\omega_{i,t}$ :

$$Y_{i,t} = \omega_{i,t} Z_t^A K_{i,t}^{1-\alpha} L_{i,t}^\alpha$$

The  $\omega_{i,t}$ -shocks are uncorrelated across firms and time. However, the cross-sectional dispersion of the  $\omega$ -shocks varies over time; specifically,  $\sigma_{\omega,t}$  follows a first-order Markov process. Productivity dispersion is the second exogenous source of aggregate risk in the model. We refer to changes in  $\sigma_{\omega,t}$  as uncertainty shocks.

While each individual entrepreneur manages her own firm's production, the family manages the allocation of production inputs and consumption and issues debt to intermediaries. Because all firms are identical at the start of each period, they are given the same capital, labor, and

debt allocation. Corporate debt is long-term, modeled as perpetuity bonds. Bond coupon payments decline geometrically,  $\{1, \delta, \delta^2, \dots\}$ , where  $\delta$  captures the duration of the bond. We define a “face value”  $F = \frac{\theta}{1-\delta}$  as a fixed fraction  $\theta$  of all repayments for each bond issued. Per definition, interest payments are the remainder  $\frac{1-\theta}{1-\delta}$ .

At the beginning of the period, the family jointly holds  $K_t^B$  units of capital, and has  $A_t^B$  bonds outstanding. Producers jointly hire their own labor and the labor of savers, denoted by  $L_t^j$ , with  $j = B, S$ . During production, the labor inputs of the two types are combined into aggregate labor:

$$L_t = (L_t^B)^{1-\gamma_S} (L_t^S)^{\gamma_S}.$$

Before idiosyncratic productivity shocks are realized, each producer is given the same amount of capital and labor for production, such that  $K_{i,t} = K_t^B$  and  $L_{i,t} = L_t$ . Further, each producer is responsible for repaying the coupon on an equal share of the total debt,  $A_{i,t} = A_t^B$ .

The individual profit of producer  $i$  is therefore given by:

$$\pi_{i,t} = \omega_{i,t} Z_t^A (K_t^B)^{1-\alpha} L_t^\alpha = \sum_j w_t^j L_t^j - A_t^B. \quad (4)$$

After production, each producer who achieves a sufficiently high profit,  $\pi_{i,t} \geq \underline{\pi}$ , returns this profit to the family, where  $\underline{\pi}$  is a parameter. Further, capital depreciates during production by fraction  $\delta_K$ , and individual members with profit above the threshold return the depreciated capital after production. Producers with  $\pi_{i,t} < \underline{\pi}$  default on the share of debt they were allocated. The debt is erased, and the intermediary takes ownership of the bankrupt firm, including its share of the capital stock. The intermediary liquidates the bankrupt firms' capital, seizes their output, and pays their wage bill. The remaining funds are the intermediary's recovery value.<sup>9</sup> In return for production, each family member receives the same amount of consumption goods  $C_{i,t} = C_t^B$ .

From (4), it immediately follows that there exists a cutoff productivity shock:

$$\omega_t^* \equiv \frac{\underline{\pi} + \sum_{j=B,S} w_t^j L_t^j + A_t^B}{Z_t^A (K_t^B)^{1-\alpha} (L_t)^\alpha}, \quad (5)$$

<sup>9</sup>Our model of liquidity default of firms fits the data much better than a model of strategic default for firms, which we explored in an earlier version of this paper.

such that all entrepreneurs receiving productivity shocks below this cutoff default on their debt.

Using the threshold level  $\omega_t^*$ , we define  $\Omega_A(\omega_t^*)$  to be the fraction of debt *repaid* to lenders and  $\Omega_K(\omega_t^*)$  to be the average productivity of the firms that do not default:

$$\Omega_A(\omega_t^*) = \Pr[\omega_{i,t} \geq \omega_t^*], \quad (6)$$

$$\Omega_K(\omega_t^*) = \Pr[\omega_{i,t} \geq \omega_t^*] E[\omega_{i,t} | \omega_{i,t} \geq \omega_t^*]. \quad (7)$$

After making a coupon payment of 1 per unit of remaining outstanding debt, the amount of outstanding debt declines to  $\delta \Omega_A(\omega_t^*) A_t^B$ .

The total profit of the producers' business is subject to a corporate profit tax with rate  $\tau_{\Pi}^B$ . The profit for tax purposes is defined as sales revenue net of labor expenses, capital depreciation, and interest payments of non-bankrupt producers:<sup>10</sup>

$$\Pi_t^{B,\tau} \equiv \Omega_K(\omega_t^*) Z_t^A (K_t^B)^{1-\alpha} (L_t)^\alpha - \Omega_A(\omega_t^*) \left( \sum_j w_t^j L_t^j + \delta_K p_t K_t^B + (1-\theta) A_t^B \right)$$

The fact that interest expenditure  $(1-\theta)A_t^B$  is deducted from taxable profit creates a “tax shield” and hence a preference for debt funding.

In addition to producing consumption goods, firms jointly create capital goods from consumption goods. In order to create  $X_t$  new capital units, the required input of consumption goods is  $X_t + \Psi(X_t/K_t^B) K_t^B$ , with adjustment cost function  $\Psi(\cdot)$  which satisfies  $\Psi''(\cdot) > 0$ ,  $\Psi(\delta_K) = 0$ , and  $\Psi'(\delta_K) = 0$ .

Borrower-entrepreneurs further own all equity shares of the intermediary sector. Each period, they receive an effective dividend  $D_t^I$  from intermediaries, to be defined below in equation (14).

The borrower-entrepreneur family's problem is to choose consumption  $C_t^B$ , capital for next period  $K_{t+1}^B$ , new debt  $A_{t+1}^B$ , investment  $X_t$  and labor inputs  $L_t^j$  to maximize life-time utility

<sup>10</sup> Aggregate producer profit is the integral over the idiosyncratic profit (4) of non-defaulting producers, net of capital depreciation expenses and adding back principal payments  $\theta A_t^B$  which are not tax deductible.

$U_t^B$  in (1), subject to the budget constraint:

$$\begin{aligned}
& C_t^B + X_t + \Psi(X_t/K_t^B)K_t^B + \Omega_A(\omega_t^*)A_t^B(1 + \delta q_t^m) + p_t K_{t+1}^B + \Omega_A(\omega_t^*) \sum_{j=B,S} \bar{w}_t^j L_t^j + \tau_{\Pi}^B \Pi_t^{B,\tau} \\
& \leq \Omega_K(\omega_t^*)Z_t^A(K_t^B)^{1-\alpha}(L_t)^\alpha + (1 - \tau_t^B)w_t^B \bar{L}^B + p_t(X_t + \Omega_A(\omega_t^*)(1 - \delta_K)K_t^B) \\
& + D_t^I + q_t^m A_{t+1}^B + G_t^{T,B} + O_t^B, \tag{8}
\end{aligned}$$

The borrower household receives output, after-tax labor income, sales of old ( $K_t^B$ ) and newly produced ( $X_t$ ) capital units, dividends from the intermediation sector ( $D_t^I$ ), new debt raised ( $q_t^m A_{t+1}^B$ ), where  $q_t^m$  is the price of one bond in terms of the consumption good, transfer income from the government ( $G_t^{T,B}$ ), and transfer income from bankruptcy proceedings ( $O_t^B$ ). These resources are used to pay for consumption, investment including adjustment costs, debt service, new capital purchases, wages, and corporate taxes.

Costly defaults of individual borrowers who receive bad idiosyncratic shocks endogenously limit the optimal leverage of the borrower family. Borrowers take into account that each marginal unit of debt issued in  $t$  increases costly defaults in  $t + 1$ .<sup>11</sup> Corporate leverage is driven by the classic trade-off between costs of financial distress and benefits from the tax shield.

## 2.4 Intermediaries

### 2.4.1 Setup

Intermediaries (“banks”) are financial firms that buy long-term risky corporate debt issued by producers ( $A_t^I$ ) and use this debt as collateral to issue short-term debt to savers ( $B_t^I$ ). They maximize the present discounted value of net dividend payments  $d_t^I$  to their shareholders, the borrower-entrepreneurs. There are two important frictions in the banking sector:

1. Moving equity into or out of banks is costly, i.e. paying a (positive or negative) dividend  $d_t^I$  is subject to a cost  $\Sigma(d_t^I)$  that is convex in deviation of  $d_t^I$  from a target level. The total cost of paying out dividend  $d_t^I$  is  $d_t^I + \Sigma(d_t^I)$  for the intermediary.

<sup>11</sup>The full model in the appendix adds a hard borrowing constraint for firms. The model is calibrated so that this constraint is rarely binding; the constraint plays a minor role for the results. See Appendix C.3.

2. **Limited liability.** Intermediaries receive idiosyncratic profit shocks  $\epsilon_t^I$ , realized at the time of dividend payouts. The net dividend received by the shareholders is  $d_t^I - \epsilon_t^I$ . The profit shocks are i.i.d. across banks and time with  $E(\epsilon_t^I) = 0$  and c.d.f.  $F_\epsilon$ .<sup>12</sup> Intermediaries optimally decide to default on their liabilities. Intermediary debt is guaranteed by the government (deposit insurance or TBTF guarantees) and therefore risk-free.

The coupon payment on performing loans in the current period is  $A_t^I \Omega_A(\omega_t^*)$ . For firms that default and enter into foreclosure, banks repossess the firms, sell current period's output, pay current period's wages, and sell off the assets. Payments on defaulted bonds are:

$$M_t = (1-\zeta) \left[ ((1-\Omega_A(\omega_t^*))(1-\delta_K)p_t K_t^B + (1-\Omega_K(\omega_t^*))Z_t^A(K_t^B)^{1-\alpha} L_t^\alpha) \right] = (1-\Omega_A(\omega_t^*)) \sum_j w_t^j L_t^j \quad (9)$$

where  $\zeta$  is the fraction of capital value and output lost in bankruptcy.

At the beginning of the period, after aggregate and idiosyncratic productivity shocks are realized and a fraction  $1 - \Omega_A(\omega_t^*)$  of firms has defaulted, the wealth (net worth) of a bank is:

$$W_t^I = \Omega_A(\omega_t^*)(1 + \delta q_t^m) A_t^I + M_t + B_{t-1}^I. \quad (10)$$

Each intermediary optimally decides on bankruptcy, conditional on the realization of  $W_t^I$  and the idiosyncratic profit shock  $\epsilon_t^I$ . Bankrupt intermediaries are liquidated by the government, which redeems deposits at par value. Immediately thereafter, shareholders (borrower-entrepreneurs) replace all bankrupt intermediaries with new banks that receive initial equity equal to that of the non-defaulting banks,  $W_t^I$ . This ensures that at the time of the dividend payout and portfolio decisions, all banks have identical wealth and face identical decision problems. In appendix A.2.1, we show more formally that given our assumptions, the problem reduces to that of a representative intermediary with wealth  $W_t^I$ .

In addition to making loans, intermediaries can trade in short-term bonds with savers and the government. They are allowed to take a short position in these bonds (issuing deposits), using their loans to borrower-entrepreneurs as collateral. Intermediary debt is subject to a leverage

<sup>12</sup>The idiosyncratic shocks to bank profitability capture unmodeled heterogeneity in bank portfolios, such as that resulting from differences in credit quality across banks' loan portfolios or from differences in consumer lending. Technically, the assumption guarantees that there is always a fraction of banks which defaults. The shocks only affect the dividend payout, but have no effect on bank net worth going forward.



constraint:

$$-q_t^f B_t^I \leq \xi q_t^m A_{t+1}^I. \quad (11)$$

A negative position in the short-term bond must be collateralized by banks' loan portfolio. The parameter  $\xi$  determines how much debt can be issued against a dollar of assets. The constraint (11) is a Basel-style regulatory bank capital constraint. The parameter  $\xi$  is the key macro-prudential policy parameter in the paper. We have chosen to have market prices on the right-hand side of (11) because levered financial intermediaries face regulatory constraints that depend on market prices.<sup>13</sup>

Intermediaries are subject to corporate profit taxes at rate  $\tau_H^I$ . Their profit for tax purposes is defined as the net interest income on their loan business:

$$\Pi_t^I \equiv (1 - \theta) \Omega_A(\omega_t^*) A_t^I + r_t^f B_t^I$$

They need to pay a deposit insurance fee  $\kappa$  to the government that is proportional to the amount of short-term bonds they issue. Banks' leverage choice is affected by the same tax benefit and cost of distress trade-off faced by firms. Additionally, banks enjoy deposit insurance, face costly equity issuance, and provide safe assets to patient households.

## 2.4.2 Recursive Intermediary Problem

Denote by  $\mathcal{S}_t^I$  the vector of aggregate state variables exogenous to the problem of intermediaries. After default decisions and recapitalizations have taken place, all intermediaries face the same optimization problem (see appendix A.2.1 for details):

$$V_t^I(W_t^I, \mathcal{S}_t^I) = \max_{d_t^I, B_t^I, A_{t+1}^I} [d_t^I + E_t] \left[ \mathcal{M}_{t,t+1}^B \max \{V_{t+1}^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I, 0\} \right], \quad (12)$$

subject to the budget constraint:

$$d_t^I + \Sigma(d_t^I) + q_t^m A_{t+1}^I + (q_t^f - \mathbb{I}_{\{B_t^I < 0\}} \kappa) B_t^I + \tau_H^I \Pi_t^I \leq W_t^I, \quad (13)$$

<sup>13</sup>Insurance companies face such constraints as part of the Solvency II regime, broker-dealers face value-at-risk constraints, and market prices affect bank regulation through their effect on risk weights. Further, note that bank loans are marked-to-market each period in the model

the regulatory capital constraint (11), and the definition of wealth (10). The continuation value in the objective function (12) reflects that the value of the bank in case of default is zero. Intermediaries discount future payoffs by  $\mathcal{M}_{t,t+1}^B$ , which is the stochastic discount factor of borrowers, their equity holders

### 2.4.3 Aggregation and Government Bailouts

The aggregate net dividend paid by the banking sector is:

$$\begin{aligned}
 D_t^I &= \underbrace{F_{\epsilon,t}(d_t^I - \epsilon_t^{I,-})}_{\text{Dividend of non-defaulters}} + \underbrace{(1 - F_{\epsilon,t})(d_t^I - W_t^I)}_{\text{Dividend of defaulters net of initial equity}} \\
 &\equiv d_t^I - F_{\epsilon,t}\epsilon_t^{I,-} - (1 - F_{\epsilon,t})W_t^I, \tag{14}
 \end{aligned}$$

where  $F_{\epsilon,t}$  is the mass of non-defaulting banks and  $\epsilon_t^{I,-} = \mathbb{E}_\epsilon(\epsilon | \epsilon < V^I(W_t^I, \mathcal{S}_t^I))$ , is the expected idiosyncratic loss conditional on not defaulting. The last term represents the cost to shareholders of recapitalizing defaulted banks, from zero net worth post-bailout to the same positive net worth of the non-defaulted banks.

Defaulting intermediaries are liquidated by the government. During the bankruptcy process, a fraction  $\zeta$  of the asset value of a bank is lost. Hence the aggregate bailout payment of the government is:

$$\text{bailout}_t = (1 - F_{\epsilon,t}) \left[ \epsilon_t^{I,+} - W_t^I + \zeta(\Omega_A(\omega_t^*)(1 + \delta q_t^m)A_t^I + M_t) \right]. \tag{15}$$

The conditional expectation,  $\epsilon_t^{I,+} = \mathbb{E}_\epsilon(\epsilon | \epsilon \geq V^I(W_t^I, \mathcal{S}_t^I))$ , is the expected idiosyncratic loss of defaulting intermediaries.

### 2.4.4 Aggregate Bankruptcy Costs

Default of producing firms and intermediaries causes bankruptcy losses. When firms default, a fraction  $\zeta$  of their capital value and output is lost to banks, see equation (9). Similarly, when banks default, a fraction  $\zeta$  of their asset value is lost to the government, see equation (15). We assume that only a fraction  $\eta$  of this total loss from bankruptcy is a deadweight loss to society while the remainder is rebated to the households in proportion to their population

shares. These are the  $O_t^i$  terms in the budget constraints (3) and (8):

$$O_t^B + O_t^S = \zeta(1-\eta) \left[ ((1 - \Omega_A(\omega_t^*)) (1 - \delta_K) p_t K_t^B + (1 - \Omega_K(\omega_t^*)) Z_t^A (K_t^B)^{1-\alpha} L_t^A) + \zeta(1-\eta)(1 - F_{\epsilon,t}) [\Omega_A(\omega_t^*)(1 + \delta q_t^m) A_t^I + M_t^I] \right] \quad (16)$$

This can be interpreted as income payments to the actors involved in bankruptcy cases. We avoid the strong assumption that all bankruptcy costs are deadweight losses to society.

## 2.5 Government

The actions of the government are determined via fiscal rules: taxation, spending, bailout, and debt issuance policies. Government tax revenues,  $T_t$ , are labor income tax, non-financial and financial profit tax, deposit income tax, and deposit insurance fee receipts:

$$T_t \equiv \sum_{j=B,S} [\tau_t^j w_t^j L_t^j + \tau_H^B \Pi_t^B + \tau_H^I \Pi_t^I + \tau^B r_t^f B_t^S - \mathbb{I}_{\{B_t^I < 0\}} \kappa B_t^I]$$

Government expenditures,  $G_t$  are the sum of exogenous government spending,  $G_t^o$ , transfer spending  $G_t^T$ , and financial sector bailouts:

$$G_t \equiv G_t^o + G_t^{T,B} + G_t^{T,S} + \text{bailout}_t$$

The government issues one-period risk-free debt. Debt repayments and government expenditures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_{t-1}^G + G_t \leq q_t^f B_t^G + T_t \quad (17)$$

We impose a transversality condition on government debt:

$$\lim_{u \rightarrow \infty} \mathbb{E}_t [\mathcal{M}_{t,t+u}^S B_{t+u}^G] \equiv 0$$

where  $\mathcal{M}^S$  is the SDF of the saver. Because of its unique ability to tax, the government can spread out the cost of default waves and financial sector rescue operations over time.

Government policy parameters are  $\Theta_t \equiv (\tau_t^i, \tau_H^i, \tau^B, G_t^o, G_t^{T,i}, \xi, \kappa)$ . The capital requirement

$\xi$  in equation (11) and the deposit insurance fee  $\kappa$  are macro-prudential policy tools.

## 2.6 Equilibrium

Given a sequence of aggregate productivity shocks  $\{Z_t^A, \sigma_{\omega,t}\}$ , idiosyncratic productivity shocks  $\{\omega_{t,i}\}_{i \in B}$ , and idiosyncratic intermediary profit shocks  $\{\epsilon_{t,i}\}_{i \in I}$ , and given a government policy  $\Theta_t$ , a competitive equilibrium is an allocation  $\{C_t^B, K_{t+1}^B, X_t, A_{t+1}^B, L_t^j\}$  for borrower-entrepreneurs,  $\{C_t^S, B_t^S\}$  for savers,  $\{d_t^I, A_{t+1}^I, B_t^I\}$  for intermediaries, and a price vector  $\{p_t, q_t^m, q_t^f, w_t^B, w_t^S\}$ , such that given the prices, borrower-entrepreneurs and savers maximize life-time utility, intermediaries maximize shareholder value, the government satisfies its budget constraint, and markets clear. The market clearing conditions are:

$$\text{Risk-free bonds: } B_t^G = B_t^S + B_t^I \quad (18)$$

$$\text{Loans: } A_{t+1}^B = A_{t+1}^I \quad (19)$$

$$\text{Capital: } K_{t+1}^B = (1 - \delta_K) K_t^B + X_t \quad (20)$$

$$\text{Labor: } L_t^j = \bar{L}^j \text{ for all } j = B, S \quad (21)$$

$$\text{Consumption: } Y_t = C_t^B + C_t^S + G_t^o + X_t + K_t^B \Psi(X_t/K_t^B) + \Sigma(d_t^I) + DWL_t \quad (22)$$

The last equation is the economy's resource constraint. It states that total output (GDP) equals the sum of aggregate consumption, discretionary government spending, investment including capital adjustment costs, bank equity adjustment costs, and aggregate resource losses from corporate and intermediary bankruptcies. The  $DWL_t$  term equals  $\frac{\eta}{1-\eta}(O_t^B + O_t^S)$ , as defined in (16).

## 2.7 Welfare

In order to compare economies that differ in the policy parameter vector  $\Theta_t$ , we must take a stance on how to weigh the two households, borrowers and savers. We propose two different measures of aggregate welfare. First, we compute an ex-post utilitarian social welfare function summing value functions of the agents:

$$\mathbb{W}_t^{pop}(\cdot; \Theta_t) \equiv V_t^B + V_t^S$$

where the  $V^j(\cdot)$  functions are the value functions defined in the appendix. The value functions already incorporate the mass of agents of each type (population shares  $\ell^i$ ).

Second, we compute an ex-ante measure of welfare based on compensating variation similar to Alvarez and Jermann (2005). Consider the equilibrium of two different economies  $k = 0, 1$ , characterized by policy vectors  $\Theta^0$  and  $\Theta^1$ , and denote expected lifetime utility at time 0 for agent  $j$  in economy  $k$  by  $\bar{V}^{j,k} = E_0[V_1^j(\cdot; \Theta^k)]$ . Denote the time-0 price of the consumption stream of agent  $j$  in economy  $k$  by:

$$\bar{P}^{j,k} = E_0 \left[ \sum_{t=0}^{\infty} M_{t,t+1}^{j,k} C_{t+1}^{j,k} \right],$$

where  $M_{t,t+1}^{j,k}$  is the SDF of agent  $j$  in economy  $k$ . The percentage welfare gain for agent  $j$  from living in economy  $\Theta^1$  relative to economy  $\Theta^0$ , in expectation, is:

$$\Delta \bar{V}^j \equiv \frac{\bar{V}^{j,1}}{\bar{V}^{j,0}} - 1.$$

Since the value functions are expressed in consumption units, we can multiply these welfare gains with the time-0 prices of consumption streams in the  $\Theta^0$  economy and add up:

$$\mathcal{W}^{cv} \equiv \Delta \bar{V}^B \bar{P}^{B,0} + \Delta \bar{V}^S \bar{P}^{S,0}.$$

This measure is the minimum one-time wealth transfer in the  $\Theta^0$  economy (the benchmark) required to make agents at least as well off as in the  $\Theta^1$  economy (the alternative). If this number is positive, a transfer scheme can be implemented to make the alternative economy a Pareto improvement. If this number is negative, such a scheme cannot be implemented because it would require a bigger transfer to one agent than the other is willing to give up.

We solve the model using projection-based numerical methods and provide a detailed description of the globally nonlinear algorithm in appendix B.

### 3 Calibration

The model is calibrated at annual frequency. The parameters of the model and their targets are summarized in Table 1. Appendix C.1 conducts a parameter sensitivity analysis of the type suggested by Andrews, Gentzkow, and Shapiro (2017) that helps clarify what moments structurally identify what parameters.

**Aggregate Productivity** Following the macro-economics literature, the TFP process  $Z_t^A$  follows an AR(1) in logs with persistence parameter  $\rho_A$  and innovation volatility  $\sigma^A$ . Because TFP is persistent, it becomes a state variable. We discretize  $Z_t^A$  into a 5-state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points and the transition probabilities between them to match the volatility and persistence of HP-detrended GDP. GDP is endogenously determined but heavily influenced by TFP. Consistent with the model, our measurement of GDP excludes net exports and government investment. We define the GDP deflator correspondingly. Observed real per capita HP-detrended GDP has a volatility of 2.53% and its persistence is 0.55. The model generates a volatility of 2.43% and a persistence of 0.55.

**Idiosyncratic Productivity** We calibrate the firm-level productivity risk directly to the micro evidence. We normalize the mean of idiosyncratic productivity at  $\mu_\omega = 1$ . We let the cross-sectional standard deviation of idiosyncratic productivity shocks  $\sigma_{t,\omega}$  follow a 2-state Markov chain, with four parameters. Fluctuations in  $\sigma_{t,\omega}$  govern aggregate corporate credit risk since high levels of  $\sigma_{t,\omega}$  cause a larger left tail of low-productivity firms to default in equilibrium. We refer to periods in the high  $\sigma_{t,\omega}$  state as *high uncertainty* periods. We set  $(\sigma_{L,\omega}, \sigma_{H,\omega}) = (0.095, 0.175)$ . The value for  $\sigma_{L,\omega}$  targets the unconditional mean corporate default rate. The model-implied average default rate of 2.2% is similar to the data.<sup>14</sup> The high value,  $\sigma_{H,\omega}$ , is chosen to match the time-series standard deviation of the cross-sectional interquartile range of firm productivity, which is 4.9% according to Bloom, Floetotto, Jaimovich,

<sup>14</sup>We look at two sources of data: corporate loans and corporate bonds. From the Federal Reserve Board of Governors, we obtain delinquency and charge-off rates on Commercial and Industrial loans and Commercial Real Estate loans by U.S. Commercial Banks for the period 1991-2015. The average delinquency rate is 3.1%. The second source of data is Standard & Poors' default rates on publicly-rated corporate bonds for 1981-2014. The average default rate is 1.5%; 0.1% on investment-grade bonds and 4.1% on high-yield bonds. The model is in between these two values.

Saporta-Eksten, and Terry (2012) (their Table 6). The transition probabilities from the low to the high uncertainty state of 9% and from the high to the low state of 20% are also taken directly from Bloom et al. (2012).<sup>15</sup> The model spends 31% of periods in the high uncertainty regime. Like in Bloom et al., our uncertainty process is independent of the first-moment shocks. About 10% of periods feature both high uncertainty and low TFP realizations. We will refer to those periods as financial recessions or financial crises, since those periods will feature (endogenous) financial fragility in the equilibrium of the model. Using a long time series for the U.S., Reinhart and Rogoff (2009) find the same 10% frequency of financial crises.

**Production** Adjustment costs are quadratic. We set the marginal adjustment cost parameter  $\eta = 2$  in order to match the observed volatility of the ratio of investment to GDP,  $X/Y$ , of 1.58%. The model generates a value of 1.56%. The adjustment costs on average amount to 0.04% of GDP. We set the parameter  $\alpha$  in the Cobb-Douglas production function equal to 0.71, which yields an overall labor income share of 65%, the standard value in the business cycle literature. We choose an annual depreciation of capital  $\delta_K$  of 8% to match the investment-to-output ratio of 18% observed in the data.

**Population and Labor Income Shares** To pin down the population shares of our two different types of households we turn to the Survey of Consumer Finance (SCF). We use all survey waves from 1995 until 2013 and average across them. We compute for each SCF household the share of assets (excluding real estate) held in stocks or private business equity, considering both direct and indirect holdings of stock. Using this definition of the risky share, we then calculate the fraction of households whose risky share is less than one percent. This amounts to 69% of SCF households. These are the savers in our model who hold only safe assets ( $\ell^S$ ). The remaining  $\ell^B = 31\%$  of households have a nontrivial risky asset share. The labor income share of savers in the SCF is 60%. The income share of the borrower-entrepreneurs is the remaining 40%. The income shares determine the Cobb-Douglas parameters  $\gamma_B$  and  $\gamma_S$ .

**Corporate Loans** In the model, a corporate loan is a geometric bond. The issuer of one unit of the bond at time  $t$  promises to pay 1 at time  $t+1$ ,  $\delta$  at time  $t+2$ ,  $\delta^2$  at time  $t+3$ , and

<sup>15</sup>They estimate a two-state Markov chain for the cross-sectional standard deviation of establishment-level productivity using annual data for 1972-2010 from the Census of Manufactures and Annual Survey of Manufactures. We annualize their quarterly transition probability matrix.

Table 1: Calibration

| Par   | Description                           | Value        | Target   |
|---|---------------------------------------|--------------|--|
| Exogenous Shocks                            |                                       |              |  |
| $\rho_A$                                    | persistence TFP                       | 0.7          | AC(1) HP-detr GDP 53-14 of 0.55                        |
| $\sigma_A$                                  | innov. vol. TFP                       | 2.0%         | Vol HP-detr GDP 53-14 of 2.56%                         |
| $\sigma_{\omega, L}$                        | low uncertainty                       | 0.095        | Avg. corporate default rate of 2%                      |
| $\sigma_{\omega, H}$                        | high uncertainty                      | 0.175        | Avg. IQR firm-level productivity (Bloom et al. (2012)) |
| $p_{LL}^{\omega}, p_{HH}^{\omega}$          | transition prob                       | {0.91, 0.80} | Bloom et al. (2012)                                    |
| Production, Population, Labor Income Shares |                                       |              |  |
| $\psi$                                      | marginal adjustment cost              | 2            | Vol. investment-to-GDP ratio 53-14 of 1.58%            |
| $\alpha$                                    | labor share in prod. fct.             | 0.71         | Labor share of output of 2/3                           |
| $\delta_K$                                  | capital depreciation rate             | 8%           | Investment-to-capital ratio, 53-14                     |
| $\rho^i$                                    | pop. shares $i \in \{S, B\}$          | {69,31}%     | Population shares SCF 95-13                            |
| $\gamma^i$                                  | inc. shares $i \in \{S, B\}$          | {60,40}%     | Labor inc. shares SCF 95-13                            |
| Corporate loans and Intermediation          |                                       |              |  |
| $\delta$                                    | average life loan pool                | 0.937        | Duration fcn. in App. C.2                              |
| $\theta$                                    | principal fraction                    | 0.582        | Duration fcn. in App. C.2                              |
| $\zeta$                                     | Losses in bankruptcy                  | 0.6          | Corporate loan and bond severities 81-15 of 44%        |
| $\eta$                                      | % bankr. loss is DWL                  | 0.2          | Bris, Welch, and Zhu (2006)                            |
| $\Phi$                                      | maximum LTV ratio                     | 0.45         | App. C.3   |
| $\pi$                                       | profit default threshold              | 0.04         | FoF non-fin sector leverage 85-14 of 37%               |
| $\sigma_{\epsilon}$                         | cross-sect. dispersion $\epsilon_t^i$ | 0.025        | FDIC failure rate of deposit. inst. of 0.5%            |
| $\sigma^I$                                  | marg. dividend payout cost            | 5            | avg. credit spread of 2.05%                            |
| Preferences                                 |                                       |              |  |
| $\beta^B$                                   | time discount factor B                | 0.931        | Capital-to-GDP ratio 53-14 of 2.24                     |
| $\beta^S$                                   | time discount factor S                | 0.982        | Mean risk-free rate 76-14 of 2.2%                      |
| $\sigma^B = \sigma^S$                       | risk aversion B & S                   | 1            | Log utility  |
| $\nu^B = \nu^S$                             | IES B & S                             | 1            | Log utility  |
| Government Policy                           |                                       |              |  |
| $G^o$                                       | discr. spending                       | 17.17%       | BEA discr. spending to GDP 53-14 of 17.58%             |
| $G^T$                                       | transfer spending                     | 2.42%        | BEA transfer spending to GDP 53-14 of 3.18%            |
| $\tau$                                      | labor income tax rate                 | 29.5%        | BEA pers. tax rev. to GDP 53-14 of 17.30%              |
| $\tau_H^B = \tau_H^I$                       | corporate tax rate                    | 20.0%        | BEA corp. tax rev. to GDP 53-14 of 3.41%               |
| $\tau^D$                                    | interest rate income tax rate         | 13.2%        | tax code; see text                                     |
| $b_o$                                       | cyclicality discr. spending           | -2.5         | slope log discr. sp./GDP on GDP growth                 |
| $b_T$                                       | cyclicality transfer spending         | -25          | slope log transfer sp./GDP on GDP growth               |
| $b_{\tau}$                                  | cyclicality lab. inc. tax             | 2            | slope log discr. sp./GDP on GDP growth                 |
| $\kappa$                                    | deposit insurance fee                 | 0.0084       | Deposit insurance revenues/bank assets                 |
| $\xi$                                       | max. intermediary leverage            | 0.94         | Basel II reg. capital charge for C&I loans & bonds     |



so on. Given that the present value of all payments ( $1/(1-\delta)$ ) can be thought of as the sum of a principal (share  $\theta$ ) and an interest component (share  $1-\theta$ ), we define the book value of the debt as  $F = \theta/(1-\delta)$ . We set  $\delta = 0.937$  and  $\theta = 0.582$  ( $F = 9.238$ ) to match the observed duration of corporate bonds. Appendix C.2 contains the details. The model's corporate loans have a duration of 6.8 years on average.

We set the bankruptcy cost parameter  $\zeta = 0.6$  to match the observed average severity rate of 44% on corporate bonds rated by S&P and Moody's rated during 1985-2004. The model produces a similar unconditional loss-given-default rate of 43%. Combined with the average default rate, this LGD number implies a loss rate on corporate loans of 1.0%. Our baseline model generates a modest quantity of corporate default risk, consistent with the data.

A fraction  $\eta$  of the cost of distress to intermediaries is a deadweight loss to the economy. The remainder  $1-\eta$  is transfer income that enters in the budget constraint of the agents. We set  $\eta = 0.2$  based on evidence in Bris, Welch, and Zhu (2006) showing that firms lose on average 20% of assets between the beginning and end of a Chapter 7 procedure.

We set the profit default threshold to  $\pi = 0.04$  to target average non-financial leverage. The higher this threshold, the more firms will default on average for a given level of firm debt. Since defaults are costly to the borrower family, borrower leverage is decreasing in  $\pi$ . The model generates a ratio of borrower book debt-to-assets of 36%. In the Flow of Funds data, the average ratio of loans and debt securities of the nonfinancial corporate and non-financial non-corporate businesses to their non-financial assets is 37%.

**Intermediary Parameters** The intermediary profit shocks are distributed Gaussian with mean zero. The cross-sectional standard deviation  $\sigma_\epsilon = \text{Var}(\epsilon_t^I)^{0.5}$  governs the average intermediary failure rate. The benchmark model with  $\sigma_\epsilon = .025$  generates an average failure rate of 0.54%, which is exactly the asset-weighted failure rate of depository institutions in the FDIC data.

We adopt the following functional form for the dividend payout cost of intermediaries:

$$\Sigma(d_t^I) \equiv \frac{[\sigma_I]}{2} [(d_t^I - \bar{d})^2]$$

The marginal dividend payout cost for intermediaries is set to  $\sigma_I = 5$  to match the average

credit spread. The higher  $\sigma^I$ , the more costly it becomes for intermediaries to deviate from their dividend target  $\bar{d}$ . We set the target to the dividend level in the deterministic steady state of the model. A higher adjustment cost causes a higher risk premium in the corporate loan market and thus increases the credit spread. We define the credit spread in the data as a weighted average of the Moody's Aaa and Baa yields and subtract the one-year constant maturity Treasury rate. To determine the portfolio weights on the Aaa versus Baa grade bonds, we use market values of the amounts outstanding from Barclays. The weights are 80% and 20%, respectively. The mean spread over the 1953-2015 period is 2.08%, while the model generates a mean spread of 2.05%.

The intermediary borrowing constraint parameter  $\xi$  can be interpreted as a minimum regulatory equity capital requirement. Under Basel II and III, corporate loans have a 100% risk weight and corporate bonds have a risk weight that depends on their credit rating. The risk weight on corporate bonds under the standardized approach of Basel II ranges from 20% for AAA to AA-, 50% for A+ to A-, to 100% for BBB+ to B-. A blended regulatory capital requirement of 6% (8% times a blended risk weight of 75%) seems appropriate given the assets of the levered financial sector.<sup>16</sup> This implies that  $\xi = 0.94$ . This is the key parameter we vary in our macro-prudential policy experiments.

We set the deposit insurance fee parameter  $\kappa$  to 8.4 basis points. To compute this number, we divide the total assessment revenue reported by the FDIC for 2016, \$10 billion, by the total short-term debt of U.S. chartered financial institutions from the Flow of Funds, \$11,849 billion.<sup>17</sup>

**Preference Parameters** Preference parameters affect many equilibrium quantities and prices simultaneously, and are harder to pin down directly by data. For simplicity, we assume that both borrowers and savers have log utility:  $\sigma_B = \nu_B = 1$  and  $\sigma_S = \nu_S = 1$ .<sup>18</sup> The subjective time discount factor of borrowers  $\beta_B = 0.931$  targets the capital-to-GDP ratio, as it governs

<sup>16</sup>Corporate loans are \$7.6 trillion and corporate bonds are \$5.1 trillion as of 2016 year-end. Given the observed 40%-40%-20% split of corporate bonds in the three ratings categories (reflecting the same 80-20 split between investment grade and high yield bonds from Barclays), the risk weight for corporate bonds is 48%, and the overall risk weight is 79%.

<sup>17</sup>Banks pay 14.2 cents per \$100 dollar of insured deposits but 8.4 cents per \$100 of insured and uninsured deposits. Since the model has only insured deposits, we use the latter number.

<sup>18</sup>We have solved the model for Epstein-Zin preferences with a range of risk aversion and EIS parameter choices. Results are qualitatively similar and available upon request.

borrowers' desire to accumulate wealth. The capital-to-output ratio is 2.25 in the model, and 2.24 in the data. The time discount factor of the saver disproportionately affects the mean of the short-term interest rate. We set  $\beta_S = 0.982$  to generate a low average real rate of interest of 2.2%.

**Government Parameters** To add quantitative realism to the model, we match both the unconditional average and cyclical properties of discretionary spending, transfer spending, labor income tax revenue, and corporate income tax revenue.

Discretionary and transfer spending as a fraction of GDP are modeled as follows:  $G_t^i/Y_t = G^i \exp\{b_i(g_t - g)\}$ ,  $i = o, T$ . The scalars  $G^o$  and  $G^T$  are set to match the observed average discretionary spending to GDP of 17.58% and transfer spending to GDP of 3.18%, respectively, in the 1953-2014 NIPA data.<sup>19</sup> The model produces 17.56% and 3.19%. We set  $b_o = -2.5$  and  $b_T = -25$  in order to match the slope in a regression of log discretionary/transfer spending-to-GDP on GDP growth and a constant. We match these slopes: -0.75 and -7.26 in the model versus -0.71 and -7.14 in the 1953-2014 data.

Similarly, we model the labor income tax rate as  $\tau_t = \tau \exp\{b_\tau(g_t - g)\}$ . We set the tax rate  $\tau = 29.5\%$  in order to match observed average income tax revenue to GDP of 17.3%. Appendix C.4 details how labor income tax revenue is computed in the data. The model generates an average of 18.6%. We set the sensitivity of the tax rate to aggregate productivity growth  $b_\tau = 2$  to match the observed sensitivity of log income tax revenue to GDP to GDP growth. The regression slope of log income tax revenue to GDP on GDP growth and a constant produces similar pro-cyclicality: 0.86 in the model and 0.70 in the data.

Fourth, we set the corporate tax rate that both financial and non-financial corporations pay to a constant  $\tau_H^B = \tau_H^I = 20\%$  to match observed corporate tax revenues of 3.41% of GDP. The model generates an average of 3.62%. The tax shield of debt and depreciation that firms and banks enjoy in the model substantially reduces the effective tax rate corporations pay, both in the model and in the data. We set the tax rate on financial income for savers (interest on short-term debt) equal to  $\tau^D = 13.2\%$ . Appendix C.5 contains the details of the calculation.

Government debt to GDP averages 60% of GDP in a long simulation of the benchmark model.

<sup>19</sup>We divide by  $\exp\{b_i/2\sigma_g^2/(1 - \rho_g^2)(b_i - 1)\}$ , a Jensen correction, to ensure that average spending means match the targets.

While it fluctuates meaningfully over prolonged periods of time (standard deviation of 50%), the government debt to GDP ratio remains stationary as explained in Appendix C.6

## 4 Results

This section studies the behavior of key macro-economic and balance sheet variables. The model captures important features of macro-economic quantities, corporate and bank balance sheets, and asset prices in normal times and in crises. The benchmark model's fit lends credibility to the macro-prudential policy experiments in Section 5.

### 4.1 Macro Quantities

We report means and standard deviations from a long simulation of the model (10,000 years), as well as averages conditional on being in a good state (high TFP, low uncertainty, i.e.  $\sigma_{\omega,L}$ ), non-financial recession (low TFP, low uncertainty), and financial recession (low TFP, high uncertainty  $\sigma_{\omega,H}$ ).

Table 2 reports the standard deviation of aggregate quantities, their correlation with GDP, and their autocorrelation. Moments in the data are computed from HP-detrended log series. Moments in the model are by assumption stationary, and are also computed from log series. The model matches the volatility of GDP and its autocorrelation. TFP shocks with 2% volatility are amplified and lead to 2.43% GDP volatility. The model further matches the volatility of the investment to GDP ratio and the investment rate. The latter series display modest procyclicality in both data and model. Investment rates are insufficiently persistent in the model. The model somewhat overstates consumption volatility. Consumption in the model exhibits the right cyclicalities, but is slightly too persistent relative to the data. We return to the source of the consumption volatility in the model below.

We present impulse-response graphs to explore the behavior of macro-economic quantities conditional on the state of the economy. We start off the model in year 0 in the average TFP state (the middle of the five points on the TFP grid) and in the low uncertainty state ( $\sigma_{\omega,L}$ ). The five endogenous state variables are at their ergodic averages. In period 1, the model undergoes a change to a lower TFP grid point. In one case (red line), the recession is

**Table 2: Unconditional Macroeconomic Quantity Moments**

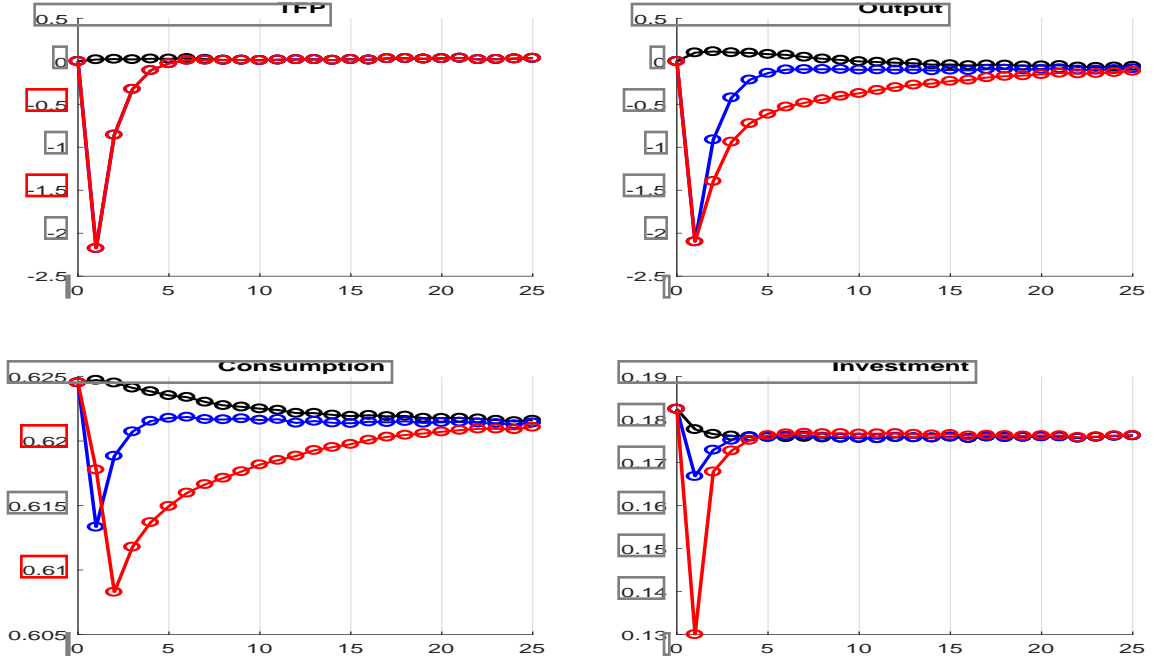
|      | Data  |              |      | Model |              |      |
|------|-------|--------------|------|-------|--------------|------|
|      | stdev | output corr. | AC   | stdev | output corr. | AC   |
| GDP  | 2.53% | 1.00         | 0.55 | 2.43% | 1.00         | 0.55 |
| CONS | 1.75% | 0.88         | 0.42 | 2.43% | 0.87         | 0.60 |
| X/Y  | 1.58% | 0.73         | 0.57 | 1.56% | 0.55         | 0.18 |
| X/K  | 0.82% | 0.63         | 0.72 | 0.81% | 0.60         | 0.21 |

accompanied by a switch to the high uncertainty state ( $\sigma_{\omega,H}$ ); a financial recession. In the second case, the economy remains in the low uncertainty state; a non-financial recession (blue line). From period 2 onwards, the two exogenous state variables follow their stochastic laws of motion. For comparison, we also show a series that does not undergo any shock in period 1 but where the exogenous states stochastically mean revert from the low-uncertainty state in period 0 (black line). For each of the three scenarios, we simulate 10,000 sample paths of 25 years and average across them. Figure 2 plots the macro-economic quantities. The top left panel is for the productivity level  $Z^A$ . By construction, it falls by the same amount in financial and non-financial recessions; a 2% drop. Productivity then gradually mean reverts over the next decade. The black line shows how productivity would have evolved absent a shock in period 1.

The other three panels show impulse-responses for output, consumption, and investment. In the initial period of the shock, the drop in output is the same when the economy is additionally hit by an uncertainty shock (red line) and when it is not (blue line). This has to be the case because capital is a state variable, labor is supplied inelastically, and productivity is identical. In financial recessions, the economy suffers from a second period of decline in consumption, despite the rebound in productivity. Output remains lower for longer in a financial recession. The added persistence resembles the slow recovery that typically follows a financial crisis. The bottom right panel shows a 28% drop in investment in financial recessions but only a modest drop in non-financial recessions. Despite the bounce back in period 2, investment remains depressed for a prolonged period of time. Aggregate consumption partially offsets the initial decline in investment in a financial recession: the initial drop in consumption is smaller than in a non-financial recession. The low rate of return on savings induces the saver to consume more in a financial crisis.<sup>20</sup> Consumption drops subsequently and remains below the non-financial

<sup>20</sup>Since output in the first period is by construction identical for both types of recessions, the approximately 2.5% of output that are not reflected in consumption and investment in a financial recession are accounted for by deadweight losses from firm and intermediary bankruptcies.

Figure 2: Financial vs. Non-financial Recessions: Macro Quantities



The graphs show the average path of the economy through a recession episode which starts at time 1. In period 0, the economy is in the average TFP state. The recession is either accompanied by high uncertainty (high  $\sigma_{\omega}$ ), a financial recession plotted in red, or low uncertainty (low  $\sigma_{\omega}$ ), a non-financial recession plotted in blue. From period 2 onwards, the economy evolves according to its regular probability laws. The black line plots the dynamics of the economy absent any shock in period 1. We obtain the three lines via a Monte Carlo simulation of 10,000 paths of 25 periods, and averaging across these paths. **Blue line:** non-financial recession, **Red line:** financial recession, **Black line:** no shocks.

recession level for the remaining periods, as the capital stock remains depressed. It is these protracted declines in consumption and investment in financial recessions that macro-prudential policy aims to remedy. In appendix D.1, we include IRF graphs that compare a financial recession to a pure uncertainty shock, which is a switch to  $\sigma_{\omega,H}$  with TFP remaining constant. This comparison demonstrates that the combination of both shocks leads to significant amplification, i.e., the financial recession triggered by the combination is much larger than the sum of the effects of each individual shock.<sup>21</sup>

<sup>21</sup>This feature of our model is consistent with the empirical finding that uncertainty shocks alone have at most moderate negative effects on output and investment, see for example Bachmann and Bayer (2013) or Vavra (2014).

## 4.2 Balance Sheet Variables

Next, we turn to the key balance sheet variables in Table 3. The first two columns report the unconditional mean and volatility. The last three columns report conditional averages in expansions, non-financial recessions, and financial recessions, respectively.

**Non-financial Corporate Sector** The first panel focuses on firms. Rows 1 and 2 display the market value of assets ( $p_t K_t^B$ ) and the market value of liabilities ( $q_t^m A_t^B$ ), both scaled by GDP. Their difference is the market value of firm equity scaled by GDP. Their ratio is the market leverage ratio (row 4). Book leverage, defined as the book value of debt to the book value of assets in row 5, is 35.2%, and matches the low observed corporate leverage ratio in the data. Entrepreneurs own 64.8% of firms in the form of corporate equity. Total credit to non-financial firms amounts to 79.1% of GDP (row 3).

Firms default when their profits fall below the threshold  $\pi$ . This is more likely when uncertainty  $\sigma_\omega$  is high, as the mass of firms with productivity shocks below the threshold  $\omega_t^*$  increases. The model generates average corporate default and loss rates of 2.25% (row 6) and 0.96% points (row 8), respectively, implying an average loss-given-default rate of 43.1% (row 7). Default and loss rates are 6-7 times higher in financial recessions (5.50% and 2.31%) than in non-financial recessions and expansions (about 0.9% and 0.4% in both). The model generates the right amount of corporate credit risk on average (as discussed in the calibration section). It also generates the strong cyclicity in the quantity of risk observed in the data.<sup>23</sup>

Firms reduce their reliance on debt financing in financial recessions. They face a higher cost of debt in these periods (rows 19 and 20), reflecting both a higher expected loss rate (row 8) and a higher risk premium charged by banks. The latter reflects financial fragility, as discussed below. Hence, financial sector fragility feeds back on the real economy and amplifies the initial shock emanating from the real sector, a second financial accelerator. Firms do not pursue the investment projects they would otherwise undertake. Relative to expansions, output falls by 4.5% and investment by 20% in financial recessions (row 9).

<sup>22</sup>In the 1991 recession, the delinquency rate spiked at 8.2% and the charge-off rate at 2.2%. For the 2007-09 crisis, the respective numbers are 6.8% and 2.7%. These are far above the unconditional averages of 3.1% and 0.7% cited in footnote 14. Similarly, during the 2001 recession, the default rate on high-yield bonds was 9.9%, far above the 1981-2014 average of 4.1%.

Table 3: Balance Sheet Variables and Prices

|                                     | Unconditional |       | Expansions | Non-fin Rec. | Fin Rec. |
|-------------------------------------|---------------|-------|------------|--------------|----------|
|                                     | mean          | stdev | mean       | mean         | mean     |
| <b>Firms</b>                        |               |       |            |              |          |
| 1. Mkt val of capital / Y (in %)    | 225.0         | 4.3   | 226.5      | 227.3        | 219.9    |
| 2. Mkt val of corp debt / Y (in %)  | 80.6          | 4.7   | 81.6       | 80.4         | 78.0     |
| 3. Book val of corp debt / Y (in %) | 79.1          | 4.5   | 79.2       | 80.2         | 78.8     |
| 4. Market corp leverage (in %)      | 35.8          | 1.9   | 36.0       | 35.4         | 35.5     |
| 5. Book corp leverage (in %)        | 35.2          | 1.8   | 35.3       | 34.9         | 34.9     |
| 6. Default rate (in %)              | 2.25          | 2.07  | 0.85       | 0.91         | 5.50     |
| 7. Loss-given-default rate (in %)   | 43.1          | 3.2   | 44.0       | 42.6         | 41.7     |
| 8. Loss Rate (in %)                 | 0.96          | 0.89  | 0.38       | 0.39         | 2.31     |
| 9. Investment / Y (in %)            | 18.0          | 1.58  | 18.9       | 17.3         | 15.3     |
| <b>Banks</b>                        |               |       |            |              |          |
| 10. Mkt fin leverage (in %)         | 93.3          | 3.2   | 93.2       | 93.6         | 92.9     |
| 11. Book fin leverage (in %)        | 97.1          | 4.5   | 98.4       | 97.7         | 92.2     |
| 12. % leverage constr binds         | 61.3          | 48.7  | 31.5       | 89.7         | 91.1     |
| 13. Bankruptcies (in %)             | 0.54          | 1.12  | 0.10       | 0.81         | 2.23     |
| 14. Dividends / Y (in %)            | 0.52          | 1.57  | 1.14       | 0.06         | -1.26    |
| <b>Savers</b>                       |               |       |            |              |          |
| 15. Deposits / Y (in %)             | 76.9          | 5.9   | 78.0       | 78.4         | 72.8     |
| 16. Government Debt / Y (in %)      | 60.2          | 49.8  | 57.8       | 69.2         | 64.8     |
| <b>Prices</b>                       |               |       |            |              |          |
| 17. Tobin's q                       | 1.00          | 0.017 | 1.01       | 0.99         | 0.97     |
| 18. Risk-free rate (in %)           | 2.19          | 2.86  | 2.45       | 4.10         | 0.26     |
| 19. Corporate bond rate (in %)      | 4.24          | 0.20  | 4.13       | 4.40         | 4.55     |
| 20. Credit spread (in %)            | 2.05          | 2.94  | 1.68       | 0.30         | 4.28     |
| 21. Excess ret. corp. bonds (in %)  | 1.09          | 3.44  | 1.87       | -0.15        | -0.49    |

**Financial Intermediaries** The second panel of Table 3 focuses on banks. Intermediary leverage is 93.3% on average in market values (row 10) and 93.3% in market values. The average ratio of total intermediary book debt-to-assets in the 1953-2014 data is 91.5%, close to the intermediaries in our model; see Appendix C.7 for the data calculations. Financial leverage was not directly targeted in the calibration, yet is close to the data. Several model ingredients contribute to the high financial leverage. Like the non-financial firms, they are owned by impatient shareholders. They enjoy a tax shield. Unlike firms, they benefit from deposit insurance and they produce safe assets for patient savers, both of which keep down their cost of funding (2.19%, row 18). Finally, they face dividend adjustment costs, which makes debt issuance relatively attractive. We explore the various drivers of leverage in Appendix D.2 and find that the dominant force is the wedge between the time discount factor of borrowers and savers.



Banks suffer losses on their credit portfolio in financial recessions (row 21), reducing their book value of assets. At the same time risk is high. Low prices (high yields, row 19) of corporate loans reflect the higher default risk (row 9) and the higher credit risk premium (row 20). This reduces the market value of intermediary assets, amplifying the decline in the book value of assets. A lower value of bank assets in turn tightens the regulatory bank capital constraint. The constraint binds in 91.1% of the financial crises compared to 61.3% unconditionally and 31.5% in expansions (row 12). When binding, intermediaries must reduce liabilities to meet capital requirements, as measured by deposits to GDP in row 15, which explains the drop in banks' debt. The decline in debt exceeds the decline in book assets, so that bank book leverage is pro-cyclical. Book leverage falls from 98.4% in expansions to 92.2% in financial recessions. The decline in the price of loans, i.e., the larger fall in the market value of bank assets, results in a-cyclical market leverage. This pattern is directionally consistent with the data. Adrian, Boyarchenko, and Shin (2015) show that book leverage is pro-cyclical for commercial banks and broker-dealers, while market leverage is counter-cyclical.

While banks are roughly equally likely to be constrained in financial and non-financial recessions, the reasons for the tightness of the bank capital constraint are fundamentally different. In financial recessions, banks suffer large credit losses and are forced to shrink, delever, and issue equity. Going forward, banks earn high risk yields on corporate debt (4.55%) and face low borrowing costs (0.26%), making lending very profitable. They would like to expand lending, but are up against their borrowing constraint which prevents raising new debt. They can and do raise outside equity (negative dividends of 1.26% of GDP, line 14), but are held back by the cost of raising equity.

In contrast, in non-financial recessions, banking becomes much less profitable due to the shrinking net interest margin (0.3%). These recessions resemble standard TFP-induced recessions in real business cycle models: as productivity and labor income are temporarily low, savers reduce their demand for safe assets in order to smooth consumption. In addition, the supply of government debt goes up due to increased government spending and lower tax revenue (row 16). The risk free rate has to rise to 4.1% to clear the market for short-term debt. At the same time, low productivity reduces corporate loan demand. In response to the drop in profits which depletes equity, banks lower dividend payments to 0.06% of GDP. To avoid raising costly equity, banks exhaust their borrowing constraint.

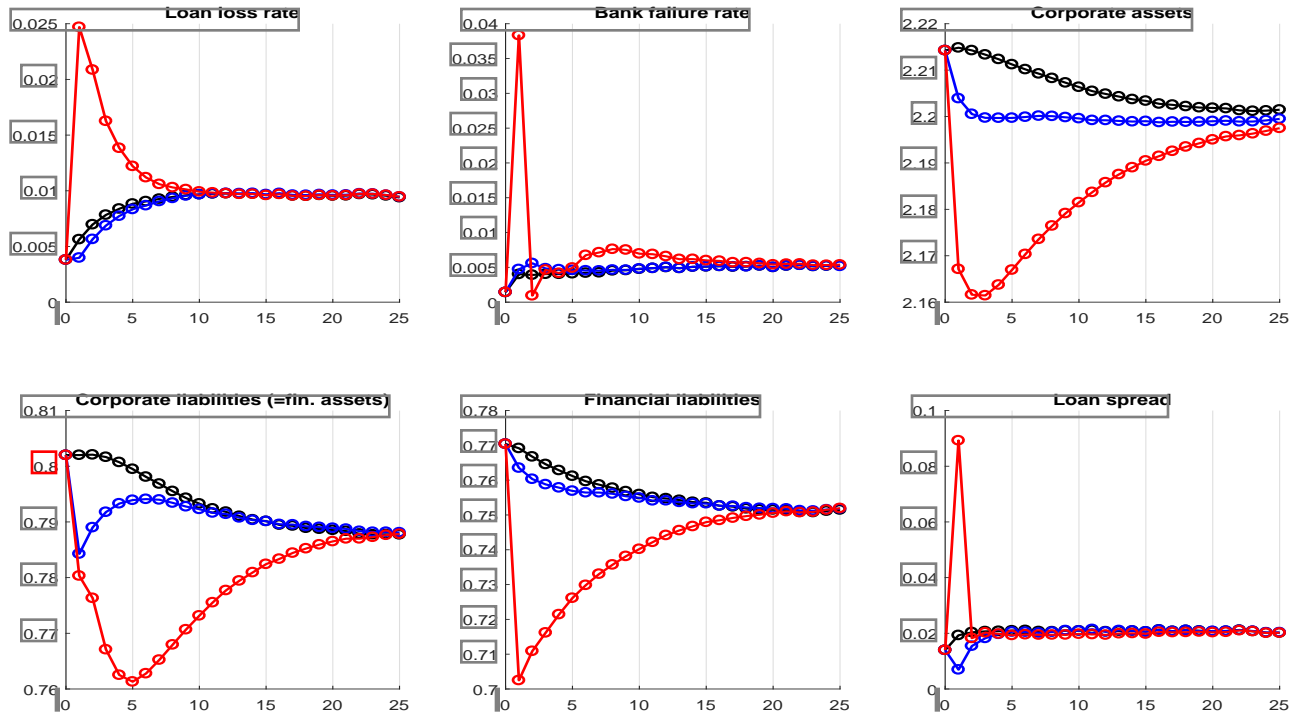
The size of the intermediary sector, relative to GDP, shrinks in financial recessions. Bank assets shrink about 3-5% relative to their levels in expansions (rows 2 and 3). At the same time, their liabilities shrink from 78% of GDP in expansions to 72% of GDP in financial recessions (row 15). Since GDP itself falls, bank deposits fall by 11%.

In the equilibrium of our model, 0.54% of banks are insolvent in a typical year (row 13). Intermediary failures are concentrated in financial recessions (hence the name), when 2.23% of banks are insolvent, 22 times as many as in expansions and 2.8 times as many as in non-financial recessions. The model generates rare financial disasters when a non-trivial fraction of the banking system is insolvent and needs to be bailed out. These banking crises result from the balance of two forces. Bank equity owners try to avoid low intermediary net worth states because they are risk averse and because of the cost of equity issuance. But when net worth is sufficiently low they have an incentive to shift the risk onto the tax payer.

An important quantitative success of the model is that it can generate the large observed credit spread for the small observed amount of credit risk, a challenge in standard macro-finance models; see Chen (2010). The model generates not only the average spread, but also high volatility and counter-cyclicalities. As Appendix D.3 explains in more detail, the credit spread is high when intermediary net worth is low. Intermediaries' marginal value of equity capital (shadow SDF) is high in those states of the world. Like in Santos and Veronesi (2017), our model generates a large credit spread in a model where all aggregate shocks emanate from the real economy.

Figure 3 shows the impulse-response functions for assets and liabilities of both non-financial firms and banks. Loan loss rates spike in financial recessions and take several more years to return to normal. The high loan losses cause a spike in intermediary failures. Both the asset and liability side of corporate and intermediary balance sheet shrinks. The credit spread also spikes in the first period of a financial crisis. The increased cost of credit is consistent with a reduced loan demand from firms. Credit declines persistently. In contrast, asset prices recover quickly. This pattern is consistent with the data, where the credit spread normalizes fairly quickly after the initial spike, but the quantity of credit takes a long time to recover. The behavior of quantities and prices in financial crises is in sharp contrast to the much milder changes in non-financial recessions (blue lines).

Figure 3: Financial vs. Non-financial Recessions: Balance Sheet Variables



**Blue line: non-financial recession, Red line: financial recession, Black line: no shocks.**

**Savers** Risk averse savers only hold safe debt, provided both by the intermediaries and the government. On average, these two sources of safe assets account for 77% (row 15) and 60% of GDP (row 16). In financial recessions, intermediaries have to delever due to their equity losses. An increase in government debt due to counter-cyclical fiscal policy partially offsets the reduction in the supply of safe assets from bank deleveraging. A drop in the interest rate is required to induce the savers to reduce their demand for safe assets and consume instead. On average the real interest rate is close to 2 percentage points lower in a financial recession compared to the unconditional average (row 18).<sup>23</sup> A substantial reduction in the real interest rate is consistent with the experience in the Great Recession. This reduction in interest rates benefits banks and helps them to rebuild their net worth. This in turn restores their ability to lend to firms.

<sup>23</sup>The magnitude of the drop in real rates depends on the EIS of the saver. For example, when  $\nu_S = 50 \gg 1$ , the risk-free interest rate volatility approaches zero. Intermediaries no longer benefit from low, even negative interest rates in crises. The absence of cheap funding in crises makes them more reluctant to take on leverage ex-ante

### 4.3 Importance of Intermediary Frictions

Intermediary frictions are essential to generate financial crises. To see this, we successively relax two key intermediary frictions. First, we turn off the equity adjustment cost by setting  $\sigma^I = 0$ . Then we turn off limited liability (deposit insurance) for intermediaries, so that they cannot go bankrupt. Figure 4 compares financial recessions in these economies to those in the benchmark (black line). The underlying shocks in all three economies are the same. Clearly, the disruption in credit is not nearly as severe, and the credit spread does not increase nearly as much. Macro-economic quantities are not nearly as adversely affected as in the benchmark economy.

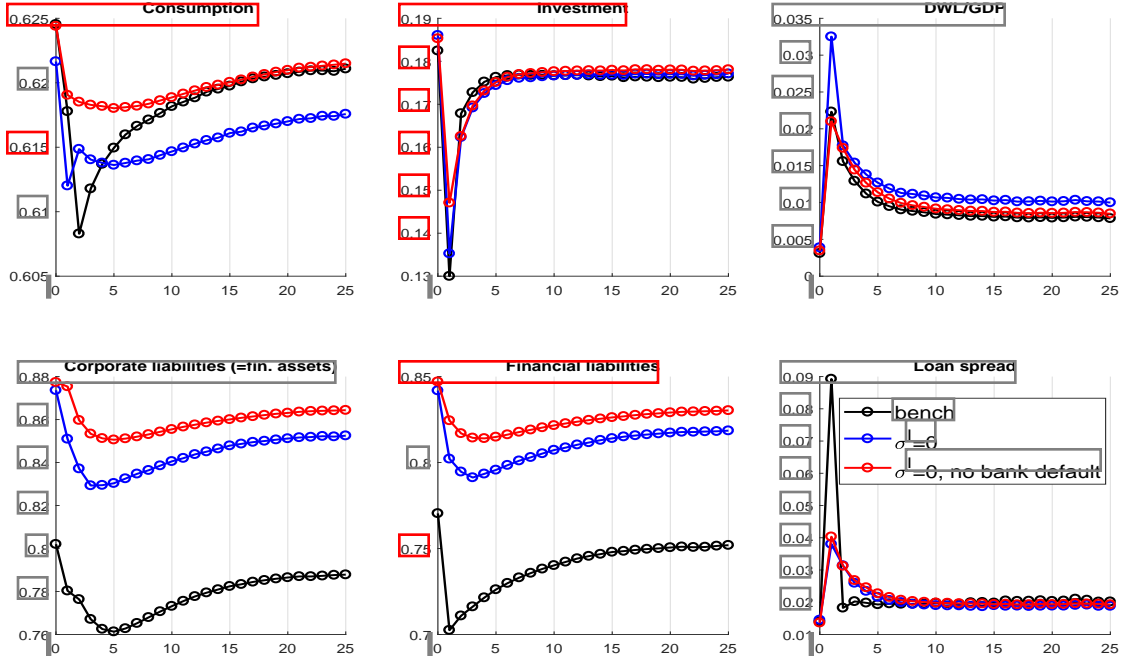
Eliminating the equity adjustment cost ( $\sigma^I = 0$ , blue line) means that shareholders can quickly recapitalize intermediaries in crises, which results in less shrinkage of the financial sector. However, limited liability still causes a suboptimally slow recapitalization when intermediaries are closer to default, as they have incentives to take on debt and shift the risk onto the taxpayer. Turning off limited liability in addition (red line) eliminates the option value of bankruptcy and banks recapitalize more quickly. The end result is an economy that allows for more corporate debt and intermediary leverage, and suffers fewer DWL from bankruptcies. Consumption dynamics during financial recessions in this counterfactual economy look similar to non-financial recessions.

This special case without equity issuance frictions and deposit insurance approaches the model of Brunnermeier and Sannikov (2014) where the balance sheets of firms and banks are merged. The results show substantial amplification from the decoupling of the balance sheets.

## 5 Macro-prudential Policy

We use our calibrated model to investigate the effects of macro-prudential policy choices. Our main experiment is a variation in the intermediaries' leverage constraint. In the benchmark model, intermediaries can borrow 94 cents against every dollar in assets; they must hold 6% equity capital. We explore tighter constraints ( $\xi < .94$ ), as well as looser constraints ( $\xi > .94$ ). We also study a time-varying capital requirement conditional on the uncertainty state. Our third macro-prudential policy experiment is to charge intermediaries  $\kappa = 1.0\%$  for deposit

Figure 4: The Role of Intermediary Frictions in Generating Financial Crises



**Black line:** responses of various outcome variables to a financial recession in the benchmark economy; **blue line:** responses of same outcome variables to a financial recession in an economy that only differs from the benchmark economy in its equity adjustment cost ( $\sigma^I = 0$ ); **red line:** responses of same outcome variables to a financial recession in an economy that differs from the benchmark economy in its equity adjustment cost ( $\sigma^I = 0$ ) and in the ability of intermediaries to default (no intermediary bankruptcy). The underlying shock sequences are identical across experiments.

insurance, a much higher tax on leverage than in the benchmark (0.084%). Tables 4 and 5 show the results. Table 5 reports moments in percentage deviation from the benchmark.

## 5.1 Changing maximum intermediary leverage

**Effect on lending and intermediation** Rows 10 and 11 of Table 4 show that a policy that constrains bank leverage is indeed successful at bringing down that leverage. Banks reduce the size of their assets, both in book and market value terms (rows 2 and 3) and the size of their liabilities (row 16). On net, intermediary equity capital increases sharply as  $\xi$  is lowered (row 14).

With intermediaries better capitalized, financial fragility falls. Intermediary bankruptcies (row 13) drop rapidly from 0.54% to 0.01% first (at  $\xi = .90$ ) and are eradicated for even tighter

capital requirements. Interestingly, with tighter regulation, intermediaries' leverage constraints bind *less* frequently (row 12). Intermediaries become more cautious when they are better capitalized, since equity capital adjustments become effectively more costly (as explained below) and the option to default (limited liability) is farther out-of-the-money. Tighter regulation leads to a safer intermediary sector, but also to a smaller one.

The increased safety of the financial sector as  $\xi$  falls is also reflected in lower corporate default, loss-given-default, and loss rates on loans to non-financial firms (rows 6-8). Firms choose to reduce leverage (rows 4-5). Firms' reluctance to undertake more leverage despite the safer environment may be understood from the higher interest rate they face on debt (row 19). When intermediary capacity shrinks (with lower  $\xi$ ), the reward for providing intermediation services increases. The modest 25bps increase in the credit spread at  $\xi = 0.75$  is the result of large offsetting movements in the expected loss rate, which halves, and in the excess return intermediaries earn on their asset holdings, which doubles (row 21).

To understand the effect of tighter regulation on intermediary profitability, we compute the franchise value of intermediation, defined as the market value of banks to shareholders per dollar of equity capital  $V^I/W^I - 1$  (row 15). If the value of banks were simply equal to the difference between the market value of assets and liabilities, franchise value would be zero ( $V^I = W^I$ ). The franchise value declines from 34% in the benchmark to 4% at  $\xi = .75$ . The drop can be understood from the additional measures of bank profitability reported in rows 15a-c. First, banks become less profitable for shareholders as measured by the return on bank equity (in an accounting sense), reported in row 15a. We compute accounting ROE as

$$[AROE = \text{Excess ret. on loans (row 21)} \times \text{book value of loans (row 3)} / W^I \text{ (row 14)}]$$

Tighter regulation requires more bank equity (row 14) to operate a smaller banking sector (rows 3). This shrinkage effect dominates the rise in profitability per dollar of loans issued reflected in the greater excess return (row 21), causing a decline in accounting ROE by 55% at  $\xi = .75$ .

As Admati, DeMarzo, Hellwig, and Pfleiderer (2013) point out, this calculation may not tell the whole story. The required return on equity will decline as banks are forced to hold more capital and shareholders are exposed to less risk. This force is also present in our model, and we can compute the equilibrium market return on equity for bank shareholders to measure its

magnitude. The cum-dividend market value of intermediary equity to borrower-entrepreneurs is given by  $V_t^I = V^I(W_t^I, S_t^I)$  as defined in (12). Using this market price, we can compute the expected market return on equity as

$$\text{MROE}_t \equiv \frac{\mathbb{E}_t[\max\{V_{t+1}^I - \epsilon_{t+1}^I, 0\}]}{V_t^I - d_t^I}$$

Indeed we find that the market return on equity declines as we tighten regulation and banks become less risky (row 15b). However, we also compute the weighted average cost of capital (WACC) for banks in row 15c, using the market ROE from row 15b as cost of equity. To compute WACC, first calculate the effective cost of debt finance to banks as  $r_t^{debt} = (q_t + \tau_\Pi r_t^f - \kappa)^{-1} - 1$ . The total firm value of the bank is  $V_t^{bank} = V_t^I + q_t \times \text{Deposits}_t$ . Hence

$$\text{WACC}_t \equiv \frac{V_t^I}{V_t^{bank}} \text{MROE}_t + \frac{q_t \times \text{Deposits}_t}{V_t^{bank}} r_t^{debt}$$

The decline in the cost of less-risky equity is not able to offset the large change in the composition of funding, which shifts from deposits towards equity with lower  $\xi$ . As a result, WACC rises by almost 50% relative to the benchmark at  $\xi = 75\%$ . The reduced franchise value is a direct result of this sharp rise in WACC and consistent with bankers' argument that tighter regulation destroys shareholder value. The main driver of this value in the benchmark economy is access to cheap deposit funding from savers. As intermediaries are forced to fund each dollar of loans with a greater proportion of equity, the value created for bank shareholders per dollar of capital invested declines. Banks charge higher spreads in the loan market, but can only partially pass through the higher funding cost to borrowers in general equilibrium.

**Effect on production and macroeconomic volatility** A first major adverse effect of tighter macro-prudential policy is that the economy's output shrinks (row 27 of Table 5). The capital stock shrinks sizeably (row 28). The reduction in output arises because firms are smaller and borrow less from a smaller intermediary sector, since debt finance became more costly. Even though GDP shrinks, aggregate consumption increases slightly (row 29) thanks to lower deadweight losses from firm and bank failures (row 26).

A second adverse effect of tighter capital regulation is that it reduces the risk absorption capacity of the intermediary sector. Maintaining a larger equity buffer means that intermediaries

need to incur larger deviations from the dividend target (in absolute value). At lower  $\xi$ , the dividend adjustment cost makes reacting to economic fluctuations by varying dividends more expensive, relative to the benchmark. Intermediaries adjust assets and debt instead of equity (rows 32-34): lower  $\xi$  increases the volatility of asset and debt growth, but reduces the volatility of dividend growth. When going from the benchmark to modestly tighter regulation, the reduced ability of intermediaries to absorb aggregate risk spills over to higher investment growth volatility (35) and consumption growth volatility (rows 36-38). The reduced risk-sharing role of the intermediary is reflected in a more volatile ratio of marginal utility of borrowers and savers, a marker of increased market incompleteness (row 39).

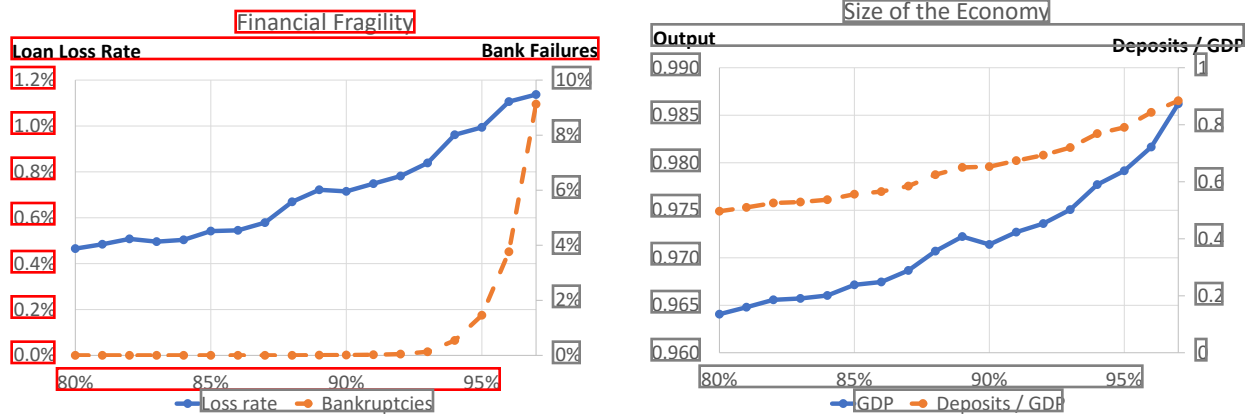
The overall effect on macroeconomic volatility is non-monotonic: tighter regulation makes financial recessions less severe, but also diminishes the intermediary's willingness to absorb aggregate risk. Volatility of investment and consumption growth peak at  $\xi = .85$ . For capital requirements higher than 15%, volatility starts to decline as the difference between financial and non-financial recessions becomes smaller. At a capital requirement of 25% ( $\xi = .75$ ), volatilities are lower than in the benchmark. Interestingly, looser regulation than benchmark *also* raises macroeconomic volatility. The economy with  $\xi = .97$  experiences more severe financial recessions as the indicated by the higher default rates of firms and banks. This increased fragility raises macro-economic volatility. The subtle, hard-to-predict, pattern in macro-economic volatility underscores the need for a rich structural model.

In summary, tightening the capital requirement has two key effects: (1) it shrinks the economy and lowers leverage of firms and banks, reducing macroeconomic volatility and bankruptcy-related losses; and (2), it effectively makes banks more risk averse, reducing their willingness to absorb aggregate risk and causing higher macroeconomic volatility. Figure 5 summarizes the effects of macro-prudential policy on financial fragility (left panel) and the size of the economy (right panel).

**Welfare** Population-weighted aggregate welfare  $\mathcal{W}^{pop}$  (row 24) is maximized at  $\xi = .90$ ; a 4 percentage point higher equity capital requirement than in the benchmark. The policy change leads to a modest aggregate welfare gain of 32 bps. Higher equity requirements than 10% reduce the gains. Locally, they increase volatility and reduce risk sharing without additional benefits from fewer defaults. At a 25% capital requirement, ex-post welfare is back to the benchmark



Figure 5: Effect of tighter capital requirement on size and fragility of the economy



The left panel plots the loss rate on the loans held by banks and the failure rate of banks as a function of the macro-prudential policy parameter  $\xi$ . The right panel plots output and the ratio of deposits to output as a function of the macro-prudential policy parameter  $\xi$ . Each dot represents a different economy where all parameters are the same as in the benchmark, except for  $\xi$ . The benchmark economy has  $\xi = .94$ .

value. The green line in the left panel of Figure 6 shows modest welfare gains for a large range of tighter policies.

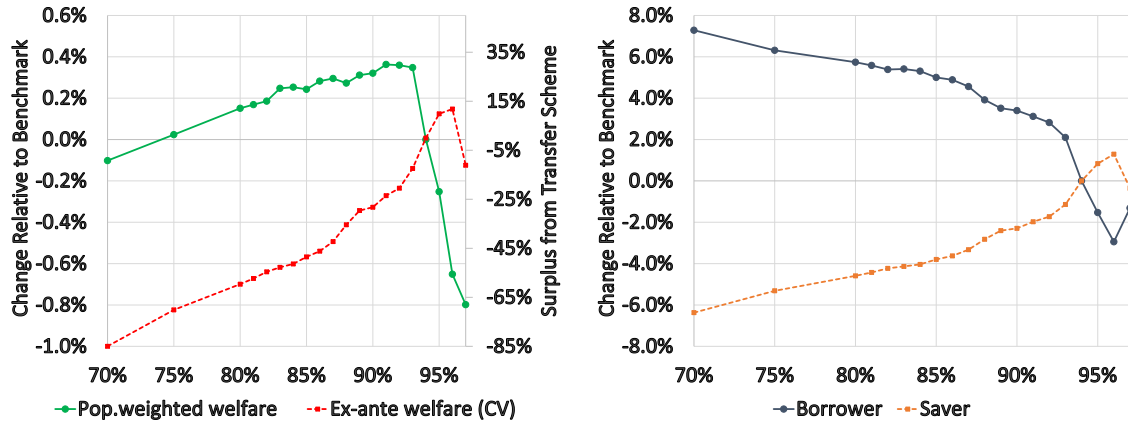
The small aggregate gain masks large heterogeneity in gains and losses among borrowers and savers. Tighter regulation redistributes wealth from savers to borrowers. It both reduces the supply of safe assets and makes it less reliable. As debt finance becomes more expensive, borrowers rely more on equity finance and a larger share of firm earnings accrues to them. At  $\xi = .90$ , borrower consumption is 3.4% higher than in the benchmark and saver consumption is 2.7% lower (row 30 and 31), leading to welfare gains and losses of similar size (row 24 and 25); see also the right panel of Figure 6. Looser capital requirements have the opposite distributional effect and increase saver wealth at the expense of borrowers. However, DWL from firm and bank failures are so large at  $\xi = .97$  that consumption of both agents is lower than in the benchmark. The increased financial fragility makes both agents worse off. Maybe surprisingly at first, forcing banks to hold more equity ends up benefiting their shareholders. In sum, tighter capital requirements lead to small (population-weighted) aggregate welfare gains, they increase consumption and wealth inequality. Policy makers have signaled concern about redistributive implications of monetary policies adopted after the Great Financial Crisis. We show that macro-prudential policy has similar implications.

While none of the variations of  $\xi$  considered in Table 5 allow for a Pareto improving transfer

scheme (row 23), the red line in the left panel of Figure 6 makes clear that there is a Pareto improvement possible at  $\xi = .95$  and  $\xi = .96$ . In those cases, the welfare gains to savers are sufficient to compensate borrowers for their losses and still have about 12% of GDP left over. Intuitively, borrowers are less patient than savers and require lower compensation for the same permanent reduction in consumption. In the  $\xi = .96$  economy, the policy maker could levy a tax on savers that reduces their utility to the level of the benchmark. The *present value* of the tax revenue stream would be 23% of benchmark GDP. The revenue could be used to pay a subsidy to borrowers that makes them exactly as well off as in the benchmark; the present value of this subsidy stream would be 11.2% of GDP. The remaining revenue has a present value of 11.8% of GDP and measures the Pareto improvement. By the same logic, a 25% capital requirement would require a massive transfer to the savers, much more than the impatient borrowers are willing to give up.

The two different ways of aggregating welfare lead to optimal capital requirements of 10% and 4%, straddling the 6% in the benchmark. The results suggest that the status quo capital requirements are close to optimal.

Figure 6: Welfare Across Macro-Prudential Policy Experiments



The left panel plots the ex-post population-weighted aggregate welfare function  $\mathcal{W}^{pop}$  in green and the ex-ante consumption equivalent variation welfare function  $\mathcal{W}^{cev}$  in red as a function of the macro-prudential policy parameter  $\xi$ . The right panel plots the value function of Borrower (black) and Saver (orange) as a function of the macro-prudential policy parameter  $\xi$ . Each dot represents a different economy where all parameters are the same as in the benchmark, except for  $\xi$ . The benchmark economy has  $\xi = .94$ .

## 5.2 Other Macro-prudential Policy Measures

**Time-varying capital requirement** The 7th column of tables 4 and 5 show an experiment with a capital requirement that varies conditional on the uncertainty state  $\sigma_{\omega,t}$ . When uncertainty is low, banks' constraint is tightened ( $\xi = .93$ ) compared to the benchmark, whereas it is loosened ( $\xi = .95$ ) when uncertainty is high. This counter-cyclical capital requirement causes a moderate expansion in corporate and financial leverage, leading to slightly higher loan losses (1.06% vs .96%) and substantially more frequent bank defaults (1.13% vs. .54%). However, the higher DWL are offset by a greater capital stock (+1.1%) and higher GDP (+.3%), such that aggregate consumption remains unchanged. Even though credit risk increases, the credit spread shrinks due to a smaller credit risk premium (+.89% vs. 1.09%). Since intermediaries are less constrained in financial crises now, they require less compensation for carrying aggregate risk and macroeconomic volatility decreases. Risk sharing among borrowers and savers improves, as indicated by the lower volatility of the MU ratio (row 41, -34.2%). The larger financial sector distributes wealth from borrowers to savers: saver consumption increases by 2.2% and welfare by 1.5%. Borrower welfare declines by 2.4%, implying an ex-post aggregate welfare loss of 31 bps. However, since the experiment makes the more patient savers significantly better off, it allows for Pareto improving wealth transfers. The compensating variation wealth residual (row 23) is 17% of GDP.

Appendix Figure 13 compares financial recessions in the world with counter-cyclical capital requirements to financial recessions in the benchmark economy. The former feature much smaller reductions in credit extension, in part due to a much smaller increase in the credit spread, as well as a shallower recession in terms of consumption and investment.

**Increasing cost of deposit insurance** The last column of tables 4 and 5 shows the result of an experiment that increases the cost of deposit insurance  $\kappa$  from 0.08% to 1% per unit of deposit. While this is a direct tax on bank leverage, its incidence falls on non-financial firms and savers in equilibrium. Firms bear most of the cost through a significantly higher credit risk premium, paying a 2.39% credit spread (vs. 2.05%), despite a reduction in the loan loss rate from 0.96% to 0.50%. As a result of the higher cost of debt, equilibrium firm leverage is much lower. Contrary to the presumed intention of the policy, banks lever up due to the high credit spread they earn and become more fragile (1.13% bank failures, twice the benchmark level).

The capital stock shrinks by 4.2% and GDP by 1.2%. Since DWL from corporate defaults fall, aggregate consumption is unaffected. Overall, the policy redistributes wealth from savers to borrowers, as the overall supply of risk free debt shrinks. While there is an ex-post aggregate welfare gain of .49%, the policy does not allow for Pareto improving transfers.

## 6 Robustness

Appendix D studies the robustness of the results. Section D.5 shows transition dynamics from the benchmark economy to one with tighter capital requirements. Along the transition path, there is a consumption boom as the economy gradually adjusts to a permanently lower capital stock.

Appendix D.6 studies a range of values for the equity issuance cost around the benchmark. The effects of a greater adjustment cost are similar to those of tighter capital requirements. Higher  $\sigma^I$  limits banks' ability to absorb aggregate risk and increases the cost of debt finance to firms. In equilibrium, this leads to substantially lower corporate leverage and hence reduces financial fragility for both producers and intermediaries. At the same time, it causes a smaller capital stock and lower output. The exercise highlights the importance of the equity adjustment cost for generating a large credit risk premium. When we raise  $\sigma^I$  to 7 from a benchmark value of 5, the loss rate on corporate loans is cut in half, while the risk premium earned by intermediaries doubles. This implies that the price of risk earned by intermediaries increases by factor 4.

Appendix D.7 finds that the conclusions of our main macro-prudential policy experiment are robust to changes in key model parameters. Neither the presence of the bankruptcy option, nor the equity adjustment costs, nor the tax shield for banks are crucial for the qualitative macro-pru implications of the model. In every case, borrowers gain from tighter policy and savers loose. The trade-off between less financial fragility and a smaller economy is present in every model variant.

## 7 Conclusion

We provide the first calibrated macro-economic model which features intermediaries who extend long-term defaultable loans to firms producing output and raise deposits from risk averse savers,

and in which both banks and firms can default. The model incorporates a rich set of fiscal policy rules, including deposit insurance, and endogenizes the risk-free interest rate.

Like in the standard accelerator model, shocks to the economy affect entrepreneurial net worth. Since firm borrowing is constrained by net worth, macroeconomic shocks are potentially amplified by tighter borrowing constraints. For realistic firm leverage ratios, this traditional accelerator is not very powerful. A second, much more powerful financial accelerator arises from explicitly modeling the financial intermediaries' net worth dynamics. Intermediaries are subject to regulatory capital constraints. Macroeconomic shocks that lead to binding intermediary borrowing constraints amplify the shocks through their direct effect on intermediaries' net worth and the indirect effect on borrowers to whom the intermediaries lend.

We find that tighter restrictions on bank leverage reduce financial fragility, but also shrinks the size of the intermediation sector and its risk absorption capacity. The size of the economy shrinks and macro-economic volatility may increase. Counter-cyclical capital equity requirements centered around the current level generate a Pareto improvement. Tighter macro-prudential policy has sizeable effects on inequality; the incidence of policies designed to limit the riskiness of the financial sector may fall on other sectors of the economy.

Several extensions are fruitful directions for future research. One could study monetary policy in a model that has endogenous labor supply and New-Keynesian ingredients. Modeling heterogeneity within the financial sector would be interesting, splitting institutions into levered and unlevered for example. One can add mortgage borrowers and study interactions between stress in mortgage and corporate loans and their implications for the real economy. Finally, our model assumed perfect alignment between management and shareholders. The presence of management's incentives to gamble arising from compensation structures may affect optimal equity capital requirements.

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## A Model Appendix

### A.1 Borrower-entrepreneur problem

#### A.1.1 Technology

The exogenous laws of motion for the TFP level  $Z_t^A$  is (lower case letters denote logs):

$$\log Z_t^A = (1 - \rho_A)z^A + \rho_A \log Z_{t-1}^A + \epsilon_t^A \quad \epsilon_t^A \sim iid \mathcal{N}(0, \sigma^A)$$

Denote  $\mu_{Z^A} = e^{z^A + \frac{(\sigma^A)^2}{2(1-\rho_A^2)}}$ .

Idiosyncratic productivity of borrower-entrepreneur  $i$  at date  $t$  is denoted

$$\omega_{i,t} \sim iid \text{Gamma}(\gamma_{0,t}, \gamma_{1,t}),$$

where the parameters  $\gamma_{0,t}$  and  $\gamma_{1,t}$  are chosen such that

$$\begin{aligned} \mathbb{E}(\omega_{i,t}) &\equiv 1 \\ \text{Var}(\omega_{i,t}) &\equiv \sigma_{\omega,t}^2 \end{aligned}$$

Individual output is

$$Y_{i,t} = \omega_{i,t} Z_t^A K_t^{1-\alpha} L_t^\alpha$$

Aggregate production is

$$Y_t \equiv \int_{\Omega} Y_{i,t} dF(\omega_i) = \int_{\Omega} \omega dF(\omega) Z_t^A K_t^{1-\alpha} (L_t)^\alpha = Z_t^A K_t^{1-\alpha} (L_t)^\alpha$$

Individual producer profit is

$$\pi_{i,t} = Y_{i,t} - \sum_j w^j L_t^j - A_t$$

Therefore, the default cutoff at  $\pi_{i,t} = 0$  is

$$\omega_t^* \equiv \frac{\pi + \sum_j w_t^j L_t^j + A_t}{Y_t} \quad (23)$$

#### A.1.2 Preliminaries

We start by defining some preliminaries.

#### Borrower Defaults

$$\begin{aligned} \Omega_A(\omega_t^*) &\equiv 1 - F_{\omega,t}(\omega_t^*) \\ \Omega_K(\omega_t^*) &\equiv \int_{\omega_t^*}^{\infty} \omega dF_{\omega,t}(\omega) \end{aligned}$$

where  $F_{\omega,t}(\cdot)$  is the CDF of  $\omega_{i,t}$

It is useful to compute the derivatives of  $\Omega_K(\cdot)$  and  $\Omega_A(\cdot)$ :

$$\begin{aligned}\frac{\partial \Omega_K(\omega_t^*)}{\partial \omega_t^*} &\equiv \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} \omega f_{\omega}(\omega) d\omega = -\omega_t^* f_{\omega}(\omega_t^*), \\ \frac{\partial \Omega_A(\omega_t^*)}{\partial \omega_t^*} &\equiv \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} f_{\omega}(\omega) d\omega = -f_{\omega}(\omega_t^*),\end{aligned}$$

where  $f_{\omega}(\cdot)$  is the p.d.f. of  $\omega_{i,t}$ .

**Capital Adjustment Cost** Let

$$\Psi(X_t, K_t^B) \equiv \frac{\psi}{2} \left( \frac{X_t}{K_t^B} - \delta_K \right)^2 K_t^B$$

Then partial derivatives are

$$\Psi_X(X_t, K_t^B) \equiv \psi \left( \frac{X_t}{K_t^B} - \delta_K \right) \quad (24)$$

$$\Psi_K(X_t, K_t^B) \equiv -\frac{\psi}{2} \left( \left( \frac{X_t}{K_t^B} \right)^2 - \delta_K^2 \right) \quad (25)$$

### A.1.3 Optimization Problem

We consider the producers's problem in the current period after aggregate TFP and idiosyncratic productivity shocks have been realized.

Let  $\mathcal{S}_t^B \equiv (Z_t^A, \sigma_{\omega,t}, W_t^I, W_t^S, B_{t-1}^G)$  represent state variables exogenous to the borrower-entrepreneur's decision.

Then the borrower problem is

$$\begin{aligned}V^B(K_t^B, A_t^B, \mathcal{S}_t^B) &= \max_{\{C_t^B, K_{t+1}^B, X_t, A_{t+1}^B, L_t^I\}} \left\{ [(1 - \beta_B)(C_t^B)^{1-1/\nu}] \right. \\ &\quad \left. + \beta_B E_t \left[ (V^B(K_{t+1}^B, A_{t+1}^B, \mathcal{S}_{t+1}^B))^{1-\sigma_B} \right]^{\frac{1-1/\nu}{1-\sigma_B}} \right\}\end{aligned}$$

subject to

$$\begin{aligned}C_t^B &\equiv (1 - \tau_{\Pi}^B) \Omega_K(\omega_t^*) Y_t + (1 - \tau_t^B) w_t^B \bar{L}^B + G_t^{T,B} + p_t [X_t + \Omega_A(\omega_t^*) (1 - \tilde{\delta}_K) K_t^B] \\ &\quad + q_t^m A_{t+1}^B - \Omega_A(\omega_t^*) A_t^B (1 - (1 - \theta) \tau_{\Pi}^B + \delta q_t^m) \\ &\equiv p_t K_{t+1}^B - X_t - \Psi(X_t, K_t^B) - (1 - \tau_{\Pi}^B) \Omega_A(\omega_t^*) \sum_{j=B,S} w_t^j L_t^j + D_t^I\end{aligned} \quad (26)$$

$$FA_t^B \leq \Phi p_t \Omega_A(\omega_t^*) (1 - \tilde{\delta}_K) K_t^B, \quad (27)$$

where we have define after-tax depreciation  $\tilde{\delta}_K \equiv (1 - \tau_{\Pi}^B) \delta_K$ .

Denote the value function and the partial derivatives of the value function as:

$$\begin{aligned} V_t^B &\equiv V(K_t^B, A_t^B, S_t^B) \\ V_{A,t}^B &\equiv \frac{\partial V(K_t^B, A_t^B, S_t^B)}{\partial A_t^B} \\ V_{K,t}^B &\equiv \frac{\partial V(K_t^B, A_t^B, S_t^B)}{\partial K_t^B} \end{aligned}$$

Denote the certainty equivalent of future utility as:

$$CE_t^B \equiv E_t \left[ \left( V^B(K_{t+1}^B, A_{t+1}^B, S_{t+1}^B) \right)^{1-\sigma_B} \right]^{\frac{1}{1-\sigma_B}}$$

**Marginal Cost of Default** Before deriving optimality conditions, it is useful to compute the marginal consumption loss due to an increased default threshold  $\omega_t^*$

$$\begin{aligned} \frac{\partial C_t^B}{\partial \omega_t^*} &\equiv \frac{\partial \Omega_K(\omega_t^*)}{\partial \omega_t^*} (1 - \tau_{\Pi}^B) Y_t \\ &\equiv \frac{\partial \Omega_A(\omega_t^*)}{\partial \omega_t^*} \left[ (1 - \delta_K) p_t K_t^B - A_t^B (1 - (1 - \theta) \tau_{\Pi}^B + \delta q_t^m) - (1 - \tau_{\Pi}^B) \sum_j w_t^j L_t^j \right] \\ &\equiv -f_{\omega}(\omega_t^*) \left[ (1 - \tau_{\Pi}^B) \omega_t^* Y_t + (1 - \delta_K) p_t K_t^B - A_t^B (1 - (1 - \theta) \tau_{\Pi}^B + \delta q_t^m) - (1 - \tau_{\Pi}^B) \sum_j w_t^j L_t^j \right] \\ &\equiv -f_{\omega}(\omega_t^*) Y_t \underbrace{\left[ \frac{(1 - \tilde{\delta}_K) p_t K_t^B - A_t^B (\theta \tau_{\Pi}^B + \delta q_t^m)}{Y_t} \right]}_{\equiv F_t} \\ &\equiv -f_{\omega}(\omega_t^*) Y_t F_t \end{aligned}$$

The function  $F_t$  has an intuitive interpretation as the marginal loss, expressed in consumption units per unit of aggregate output, to producers from an increase in the default threshold. The first term is the loss of capital due to defaulting members. The second term represents gains due to debt erased in foreclosure

#### A.1.4 First-order conditions

**Loans** The FOC for loans  $A_{t+1}^B$  is

$$\begin{aligned} q_t^m \frac{[(u_t^B)^{1-1/\eta}]}{C_t^B} [(1 - \beta_B)(V_t^B)^{1/\nu}] &\equiv \\ \lambda_t^B F - \beta_B E_t[(V_{t+1}^B)^{-\sigma_B} V_{A,t+1}^B] (CE_t^B)^{\sigma_B-1/\nu} (V_t^B)^{1/\nu} &\equiv \end{aligned} \quad (28)$$

where  $\lambda_t^B$  is the Lagrange multiplier on the constraint in (27).

**Capital** Similarly, the FOC for new capital  $K_{t+1}^B$  is

$$\frac{p_t [(1 - \beta_B)(V_t^B)^{1/\nu} (u_t^B)^{1-1/\nu}]}{C_t^B} \equiv \beta_B E_t [(V_{t+1}^B)^{-\sigma_B} V_{K,t+1}^B] (CE_t^B)^{\sigma_B - 1/\nu} (V_t^B)^{1/\nu} \quad (29)$$

**Investment** The FOC for investment  $X_t$  is

$$[1 + \Psi_X(X_t^B, K_t^B) - p_t] \frac{[(1 - \beta_B)(U_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}]}{C_t^B} \equiv 0,$$

which simplifies to

$$1 + \Psi_X(X_t^B, K_t^B) = p_t. \quad (30)$$

**Labor Inputs** Defining  $\gamma_B = 1 - \gamma_I - \gamma_S$ , aggregate labor input is

$$L_t \equiv \prod_{j=B,I,S} (L_t^j)^{\gamma_j}$$

We further compute

$$\frac{\partial \omega_t^*}{\partial L_t^j} \equiv \left( \frac{w_t^j}{Y_t} - \omega_t^* \frac{\text{MPL}_t^j}{Y_t} \right) \frac{1}{Y_t}$$

defining the marginal product of labor of type  $j$  as

$$\text{MPL}_t^j \equiv \alpha \gamma_j Z_t^A \frac{L_t}{L_t^j} \left( \frac{K_t^B}{L_t} \right)^{1-\alpha}$$

The FOC for labor input  $L_t^j$  is then

$$\frac{[(1 - \beta_B)(u_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}]}{C_t^B} \left[ [(1 - \tau_{\Pi}^B) \Omega_K(\omega_t^*) \text{MPL}_t^j - (1 - \tau_{\Pi}^B) \Omega_A(\omega_t^*) w_t^j] + \frac{\partial \omega_t^*}{\partial L_t^j} \frac{\partial C_t^B}{\partial \omega_t^*} \right] \equiv 0,$$

which yields

$$[(1 - \tau_{\Pi}^B) \Omega_K(\omega_t^*) \text{MPL}_t^j - (1 - \tau_{\Pi}^B) \Omega_A(\omega_t^*) w_t^j + f_{\omega}(\omega_t^*) \left( w_t^j - \omega_t^* \text{MPL}_t^j \right)] \mathcal{F}_t. \quad (31)$$

### A.1.5 Marginal Values of State Variables and SDF

**Loans** Taking the derivative of the value function with respect to  $A_t^B$  gives

$$\begin{aligned} V_{A,t}^B &\equiv \left[ (1 - (1 - \theta) \tau_{\Pi}^B + \delta q_t^B) \Omega_A(\omega_t^*) + \frac{\partial \omega_t^*}{\partial A_t^B} \frac{\partial C_t^B}{\partial \omega_t^*} \right] \frac{[(1 - \beta_B)(u_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}]}{C_t^B} \\ &\equiv \left[ (1 - (1 - \theta) \tau_{\Pi}^B + \delta q_t^B) \Omega_A(\omega_t^*) + f_{\omega}(\omega_t^*) \mathcal{F}_t \right] \frac{(1 - \beta_B)(u_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}}{C_t^B}, \end{aligned} \quad (32)$$

where we used the fact that  $\frac{\partial \omega_t^*}{\partial A_t^B} = \frac{1}{Y_t}$ .

**Capital** Taking the derivative of the value function with respect to  $K_t^B$  gives:

$$V_{K,t}^B = \left[ p_t \Omega_A(\omega_t^*) (1 - (1 - \tau_H^B) \delta_K) + (1 - \tau_H^B)(1 - \alpha) \Omega_K(\omega_t^*) Z_t^A \left( \frac{K_t^B}{L_t} \right)^{-\alpha} - \Psi_K(X_t^B, K_t^B) + \frac{\partial C_t^B}{\partial \omega_t^*} \frac{\partial \omega_t^*}{\partial K_t^B} \right. \\ \left. + \tilde{\lambda}_t^B \Phi p_t (1 - \tilde{\delta}_K) \left( \Omega_A(\omega_t^*) + K_t^B \frac{\partial \Omega_A(\omega_t^*)}{\partial \omega_t^*} \frac{\partial \omega_t^*}{\partial K_t^B} \right) \right] \frac{[(1 - \beta_B)(u_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}]}{C_t^B}$$

where  $\tilde{\lambda}_t^B$  is the original multiplier  $\lambda_t^B$  divided by the marginal value of wealth. Taking the derivative

$$\frac{\partial \omega_t^*}{\partial K_t^B} \equiv - \frac{\omega_t^*}{Y_t} [(1 - \alpha) Z_t^A \left( \frac{K_t^B}{L_t} \right)^{-\alpha}]$$

we get

$$V_{K,t}^B = \left\{ p_t \Omega_A(\omega_t^*) (1 - \tilde{\delta}_K) (1 + \Phi \tilde{\lambda}_t^B) + (1 - \tau_H^B)(1 - \alpha) \Omega_K(\omega_t^*) Z_t^A \left( \frac{K_t^B}{L_t} \right)^{-\alpha} - \Psi_K(X_t^B, K_t^B) \right. \\ \left. + (1 - \alpha) f_\omega(\omega_t^*) \omega_t^* \left[ Z_t^A \left( \frac{K_t^B}{L_t} \right)^{-\alpha} (F_t + \tilde{\lambda}_t^B \Phi p_t (1 - \tilde{\delta}_K)) \right] \right\} \frac{[(1 - \beta_B)(u_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}]}{C_t^B} \quad (33)$$

**SDF** We can define the stochastic discount factor (SDF) from  $t$  to  $t+1$  of borrowers:

$$\mathcal{M}_{t,t+1}^B = \beta_B \left( \frac{C_{t+1}^B}{C_t^B} \right)^{-1/\nu_B} \left( \frac{V_{t+1}^B}{C E_t^B} \right)^{1/\nu_B - \sigma_B} \quad (34)$$

### A.1.6 Euler Equations

**Loans** Substituting in for  $V_{A,t+1}^B$  in (28) and using the SDF expression, we get the recursion:

$$q_t^m \equiv \tilde{\lambda}_t^B F + E_t \left\{ \mathcal{M}_{t,t+1}^B \left[ \Omega_A(\omega_{t+1}^*) (1 - (1 - \theta) \tau_H^B + \delta q_{t+1}^m) + f_\omega(\omega_{t+1}^*) F_{t+1} \right] \right\} \quad (35)$$

**Capital** Substituting in for  $V_{K,t+1}^B$  and using the SDF expression, we get the recursion:

$$p_t \equiv E_t \left\{ \mathcal{M}_{t,t+1}^B \left\{ p_{t+1} \Omega_A(\omega_{t+1}^*) (1 - \tilde{\delta}_K) (1 + \Phi \tilde{\lambda}_{t+1}^B) + (1 - \tau_H^B)(1 - \alpha) \Omega_K(\omega_{t+1}^*) Z_{t+1}^A \left( \frac{K_{t+1}^B}{L_{t+1}} \right)^{-\alpha} \right. \right. \right. \\ \left. \left. - \Psi_K(X_{t+1}^B, K_{t+1}^B) + (1 - \alpha) f_\omega(\omega_{t+1}^*) \omega_{t+1}^* \left[ Z_{t+1}^A \left( \frac{K_{t+1}^B}{L_{t+1}} \right)^{-\alpha} (F_{t+1} + (1 - \tilde{\delta}_K) \Phi \tilde{\lambda}_{t+1}^B p_{t+1}) \right] \right\} \right\} \quad (36)$$

## A.2 Intermediaries

### A.2.1 Aggregation

Here we show that three assumptions we make are sufficient to obtain aggregation to a representative intermediary. These assumptions are (i) that the intermediary objective is linear in the idiosyncratic profit shock  $\epsilon_{t,i}$ , (ii) that idiosyncratic shocks only affect the contemporaneous payout (but not net

worth), and (iii) that defaulting intermediaries are replaced by new intermediaries with equity equal to that of non-defaulting intermediaries.

Denote by  $w_{t,i}^I$  the beginning-of-period wealth of intermediary  $i$  which did not default. Further denote by  $\mathcal{S}_t^I = (Z_t^A, \sigma_{\omega,t}, K_t^B, A_t^B, W_t^I, W_t^S, B_{t+1}^G)$  all aggregate state variables exogenous to the individual intermediary problem, where  $W_t^I$  is aggregate intermediary wealth.

In this case, we can define the optimization problem of the non-defaulting intermediary with profit shock realization  $\epsilon_{t,i}$  recursively as

$$\hat{V}_{ND}^I(w_{t,i}^I, \epsilon_{t,i}, \mathcal{S}_t^I) = \max_{\{d_{t,i}^I, B_{t,i}^I, A_{t+1,i}^I\}} [d_{t,i}^I - \epsilon_{t,i} + E_d] \left[ \mathcal{M}_{t,t+1}^B \max \left\{ \hat{V}_{ND}^I(w_{t+1,i}^I, \epsilon_{t+1,i}, \mathcal{S}_{t+1}^I), 0 \right\} \right] \quad (37)$$

subject to the budget constraint (13), the regulatory capital constraint (11), and the definition of wealth (10). Since the objective function is linear (assumption (i)) in the profit shock  $\epsilon_{t,i}$ , we can equivalently define a value function  $V^I(w_{t,i}^I, \mathcal{S}_t^I) = \hat{V}_{ND}^I(w_{t,i}^I, \epsilon_{t,i}, \mathcal{S}_t^I) + \epsilon_{t,i}$ , and write the objective as

$$V^I(w_{t,i}^I, \mathcal{S}_t^I) = \max_{\{d_{t,i}^I, B_{t,i}^I, A_{t+1,i}^I\}} [d_{t,i}^I + E_d] \left[ \mathcal{M}_{t,t+1}^B \max \left\{ V^I(w_{t+1,i}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1,i}, 0 \right\} \right] \quad (38)$$

subject to the same set of constraints. Conditional on the same state variables  $(w_{t,i}^I, \mathcal{S}_t^I)$ , the objective functions in (37) and (38) imply the same optimal choices  $(d_{t,i}^I, B_{t,i}^I, A_{t+1,i}^I)$ , independent of the realization of the current profit shock  $\epsilon_{t,i}$ . Thus conjecturing that all non-defaulting banks start the period with identical wealth  $w_{t,i} = W_t^I$ , these banks will also have identical wealth at the beginning of the next period,  $W_{t+1}^I$ , since idiosyncratic shocks do not affect next-period net worth directly (assumption (ii)). Hence absent default, all banks have identical wealth  $W_t^I$ .

What about defaulting banks? By construction, the realization of the profit shock is irrelevant for banks that defaulted and were reseeded with initial capital. Here we assume that equity holders (borrower households) seed all newly started banks with identical capital  $W_t^{Def}$ . Therefore, all banks newly started to replace defaulting banks are identical and solve the problem

$$V^I(W_t^{Def}, \mathcal{S}_t^I) = \max_{\{d_t^{Def}, B_t^{Def}, A_{t+1}^{Def}\}} [d_t^{Def} + E_d] \left[ \mathcal{M}_{t,t+1}^B \max \left\{ V^I(W_{t+1}^{Def}, \mathcal{S}_{t+1}^I) - \epsilon_{t+1,i}, 0 \right\} \right] \quad (39)$$

again subject to the same set of constraints, conformably rewritten for the different choice variables. Clearly, if  $W_t^{Def} = W_t^I$ , which is assumption (iii), then the new banks will choose the same portfolio  $(d_t^{Def}, B_t^{Def}, A_{t+1}^{Def}) = (d_t^I, B_t^I, A_{t+1}^I)$  as the non-defaulting banks. This means that new banks replacing defaulted banks will also have the same wealth at the beginning of next period,  $W_{t+1}^I$ . Together, this means that all banks have the same beginning-of-period wealth  $W_t^I$ .

### A.2.2 Statement of stationary problem

Wealth  $W_t^I$  is the wealth of all intermediaries after firm and intermediary bankruptcies and recapitalization of defaulting intermediaries by borrowers.

At the end of each period, all intermediaries face the following optimization problem over dividend payout and portfolio composition (see equation (12) in the main text):

$$V^I(W_t^I, \mathcal{S}_t^I) = \max_{\{d_t^I, B_t^I, A_{t+1}^I\}} [d_t^I + E_d] \left[ \mathcal{M}_{t,t+1}^B E_{\epsilon,t+1} \left( V_{t+1}^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I \right) \right] \quad (40)$$

subject to:

$$W_t^I \geq d_t^I + \Sigma(d_t^I) + q_t^m A_{t+1}^I + (q_t^f + \tau_H^I r_t^f - \mathbb{1}_{\{B_t^I < 0\}} \kappa) B_t^I, \quad (41)$$

$$W_{t+1}^I \equiv \left[ \left( \tilde{M}_{t+1} + \Omega_A(\omega_{t+1}^*) \delta q_{t+1}^m \right) [A_{t+1}^I + B_t^I] \right], \quad (42)$$

$$q_t^f B_t^I > -\xi q_t^m A_{t+1}^I, \quad (43)$$

$$A_{t+1}^I > 0, \quad (44)$$

$$\mathcal{S}_{t+1}^I = h(\mathcal{S}_t^I). \quad (45)$$

For the evolution of intermediary wealth in (42), we have defined the total after-tax payoff per unit of the bond

$$\tilde{M}_{t+1} = (1 - (1 - \theta)\tau_{II}^I)\Omega_A(\omega_{t+1}^*) + M_{t+1}/A_{t+1}^B,$$

where  $M_{t+1}$  is the total recovery value of bankrupt borrower firms seized by intermediaries, as defined in (9).

Since the idiosyncratic bank profit shocks are independent of the aggregate state of the economy, an individual bank's probability of continuing (i.e. not defaulting) conditional on the aggregate state, but before realization of the idiosyncratic shock is:

$$\text{Prob} \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I > 0 \right) \equiv \text{Prob} \left( \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \equiv F \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right)$$

By the law of large numbers,  $F \left( V^I(W_t^I, \mathcal{S}_t^I) \right)$  is also the aggregate survival rate of intermediaries, i.e.  $1 - F \left( V^I(W_t^I, \mathcal{S}_t^I) \right)$  is the intermediary default rate.

Hence we can express the intermediary problem as:

$$V_t^I(W_t^I, \mathcal{S}_t^I) = \max_{d_t^I, B_t^I, A_{t+1}^I} \left[ d_t^I + E \left[ \left( M_{t,t+1}^B F \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I \right) \right) \right] \right]$$

The conditional expectation,  $\epsilon_t^I = E_\epsilon(\epsilon | \epsilon \leq V^I(W_t^I, \mathcal{S}_t^I))$ , is the expected idiosyncratic loss conditional on not defaulting.

### A.2.3 First-order conditions

#### Dividend Adjustment Cost Let

$$\Sigma(d_t^I) \equiv \frac{\sigma^I}{2} [(d_t^I - \bar{d})^2]$$

The derivative is

$$\Sigma'(d_t^I) = \sigma^I (d_t^I - \bar{d})$$

**Dividend Payout** To take the FOC for dividends  $d_t^I$ , eliminate  $B_t^I$  by substituting the budget constraint into the transition law for wealth to get

$$W_{t+1}^I = (\tilde{M}_{t+1} + \delta \Omega_A(\omega_{t+1}^*) q_{t+1}^m) A_{t+1}^I + \frac{W_t^I - d_t^I - \Sigma(d_t^I) - q_t^m A_{t+1}^I}{q_t^f + \tau_H^I r_t^f - \kappa}, \quad (46)$$



and for the leverage constraint

$$\frac{W_t^I - d_t^I - \Sigma(d_t^I) - q_t^m A_{t+1}^I}{q_t^f + \tau_{\Pi}^I r_t^f - \kappa} q_t^f < \xi q_t^m A_{t+1}^I. \quad (47)$$

Now we can differentiate the objective function with respect to  $d_t^I$

$$\frac{\mathbb{I}}{\mathbb{I} + \Sigma'(d_t^I)} \equiv \frac{\mathbb{I}}{q_t^f + \tau_{\Pi}^I r_t^f - \kappa} \left[ q_t^f \lambda_t^I + \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1}^B \frac{\partial}{\partial W_{t+1}^I} \left( F_{\epsilon,t+1} \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I \right) \right) \right\} \right]$$

where  $\lambda_t^I$  denotes the Lagrange multiplier on the leverage constraint.

To compute the derivative in the expectation, rewrite the expression as

$$F_{\epsilon,t+1} \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I \right) \equiv F_{\epsilon,t} V_t^I(W_t^I, \mathcal{S}_t^I) = \int_{-\infty}^{V_t^I(W_t^I, \mathcal{S}_t^I)} \epsilon dF_{\epsilon}(\epsilon)$$

Differentiating with respect to  $W_t^I$  gives (by application of Leibniz' rule)

$$V_t^I V_{W,t}^I F_{\epsilon,t} + V_{W,t}^I F_{\epsilon,t} - V_t^I V_{W,t}^I F_{\epsilon,t} \equiv V_{W,t}^I F_{\epsilon,t}$$

Substituting in this result, the FOC becomes

$$\frac{\mathbb{I}}{\mathbb{I} + \Sigma'(d_t^I)} \equiv \frac{\mathbb{I}}{q_t^f + \tau_{\Pi}^I r_t^f - \kappa} \left[ q_t^f \lambda_t^I + \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1} \right\} \right]$$

**Loans** Using the same approach as for the dividend payout FOC, the FOC for loans  $A_{t+1}^I$  is

$$\begin{aligned} \frac{q_t^m}{q_t^f + \tau_{\Pi}^I r_t^f - \kappa} & \left[ q_t^f \lambda_t^I + \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1} \right\} \right] \\ & \equiv \frac{\mathbb{I}}{q_t^f + \tau_{\Pi}^I r_t^f - \kappa} \left[ \xi q_t^m \lambda_t^I + \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1} \left( M_{t+1} + \delta \Omega_A(\omega_{t+1}^*) q_{t+1}^m \right) \right\} \right] \end{aligned}$$

Noting that the LHS is equal to the RHS of the dividend FOC above, this can be written more compactly as

$$\frac{\mathbb{I}}{\mathbb{I} + \Sigma'(d_t^I)} \equiv \frac{\mathbb{I}}{q_t^f + \tau_{\Pi}^I r_t^f - \kappa} \left[ \xi q_t^m \lambda_t^I + \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1} \left( M_{t+1} + \delta \Omega_A(\omega_{t+1}^*) q_{t+1}^m \right) \right\} \right]$$

#### A.2.4 Marginal value of wealth and SDE

First take the envelope condition

$$V_{W,t}^I \equiv \frac{\mathbb{I}}{q_t^f + \tau_{\Pi}^I r_t^f - \kappa} \left[ q_t^f \lambda_t^I + \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1} \right\} \right]$$

Combining this with the FOC for dividends above yields

$$V_{W,t}^I \equiv \frac{\mathbb{I}}{\mathbb{I} + \Sigma'(d_t^I)} \quad (48)$$

We can define a stochastic discount factor for intermediaries as

$$\mathcal{M}_{t,t+1}^I \equiv \mathcal{M}_{t,t+1}^B \frac{1 + \Sigma^I(d_t^I)}{1 + \Sigma^I(d_{t+1}^I)} E_{\epsilon,t+1}. \quad (49)$$

### A.2.5 Euler Equations

Using the definition of the SDF  $\mathcal{M}_{t,t+1}^I$  above, we can write the FOC for dividend payout and new loans more compactly as:

$$q_t^f + \tau^H r_t^f - \kappa = q_t^f \tilde{\lambda}_t^I + E_t \left[ \mathcal{M}_{t,t+1}^I \right] \quad (50)$$

$$q_t^m = \xi \tilde{\lambda}_t^I q_t^m + E_t \left[ \mathcal{M}_{t,t+1}^I \left( M_{t+1} + \delta q_{t+1}^m \Omega_A(\omega_{t+1}^*) \right) \right] \quad (51)$$

where  $\tilde{\lambda}_t^I$  is the original multiplier  $\lambda_t^I$  divided by the marginal value of wealth.

## A.3 Savers

### A.3.1 Statement of stationary problem

Let  $\mathcal{S}_t^S \equiv (Z_t^A, \sigma_{\omega,t}, K_t^B, A_t^B, W_t^I, B_{t-1}^G)$  be the saver's state vector capturing all exogenous state variables. The problem of the saver is:

$$V^S(W_t^S, \mathcal{S}_t^S) = \max_{\{C_t^S, B_t^S\}} \left\{ (1 - \beta_S) [C_t^S]^{1-1/\nu} + \beta_S E_t \left[ (V^S(W_{t+1}^S, \mathcal{S}_{t+1}^S))^{1-\sigma_S} \right]^{\frac{1-1/\nu}{1-\sigma_S}} \right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$C_t^S = (1 - \tau_t^S) w_t^S \bar{L}^S + G_t^{T,S} + W_t^S - q_t^f B_t^S \quad (52)$$

$$W_{t+1}^S = B_t^S \quad (53)$$

$$B_t^S \geq 0 \quad (54)$$

$$\mathcal{S}_{t+1}^S = h(\mathcal{S}_t^S) \quad (55)$$

As before, we will drop the arguments of the value function and denote the marginal value of wealth as:

$$V_t^S \equiv V_t^S(W_t^S, \mathcal{S}_t^S)$$

$$V_{W,t}^S \equiv \frac{\partial V_t^S(W_t^S, \mathcal{S}_t^S)}{\partial W_t^S}$$

Denote the certainty equivalent of future utility as:

$$CE_t^S = E_t \left[ (V^S(W_t^S, \mathcal{S}_t^S))^{1-\sigma_S} \right]^{\frac{1}{1-\sigma_S}}$$

### A.3.2 First-order conditions

The first-order condition for the short-term bond position is:

$$q_t^f (C_t^S)^{-1/\nu} (1 - \beta_S) (V_t^S)^{1/\nu} = \lambda_t^S + \beta_S E_t[(V_{t+1}^S)^{-\sigma_S} V_{W,t+1}^S] (CE_t^S)^{\sigma_S - 1/\nu} (V_t^S)^{1/\nu} \quad (56)$$

where  $\lambda_t^S$  is the Lagrange multiplier on the no-borrowing constraint (54).

### A.3.3 Marginal Values of State Variables and SDF

The marginal value of saver wealth is:

$$V_{W,t}^S = (C_t^S)^{-1/\nu} (1 - \beta_S) (V_t^S)^{1/\nu}, \quad (57)$$

Defining the SDF in the same fashion as we did for borrowers, we get:

$$M_{t,t+1}^S = \beta_S \left( \frac{V_{t+1}^S}{CE_t^S} \right)^{1/\nu_S - \sigma_S} \left( \frac{C_{t+1}^S}{C_t^S} \right)^{1/\nu_S}$$

### A.3.4 Euler Equations

Combining the first-order condition for short-term bonds (56) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q_t^f = \tilde{\lambda}_t^S + E_t[M_{t,t+1}^S] \quad (58)$$

where  $\tilde{\lambda}_t^S$  is the original multiplier  $\lambda_t^S$  divided by the marginal value of wealth

## A.4 Equilibrium

The optimality conditions describing the problem are (26), (30), (35), (36), and (31) for borrowers, (41), (50), and (51) for intermediaries, and (52) and (58) for depositors. We add complementary slackness conditions for the constraints (27) for borrowers, (43) and (44) for intermediaries, and (54) for depositors. Together with the market clearing conditions (18), (19), (20), and (21) these equations fully characterize the economy

## B Computational Method

The equilibrium of dynamic stochastic general equilibrium models is usually characterized recursively. If a stationary Markov equilibrium exists, there is a minimal set of state variables that summarizes the economy at any given point in time. Equilibrium can then be characterized using two types of functions: transition functions map today's state into probability distributions of tomorrow's state, and policy functions determine agents' decisions and prices given the current state. Brumm, Kryczka, and Kubler (2018) analyze theoretical existence properties in this class of models and discuss the literature. Perturbation-based solution methods find local approximations to these functions around the "deterministic steady-state". For applications in finance, there are often two problems with local solution methods. First, portfolio restrictions such as leverage constraints may be occasionally binding in the true stochastic equilibrium. Generally, a local approximation around the steady state (with a binding or slack constraint) will therefore inaccurately capture nonlinear dynamics when constraints go from slack to binding. Guerrieri and Iacoviello (2015) propose a solution using local methods. Secondly, the portfolio allocation of agents across assets with different risk profiles is generally indeterminate at the non-stochastic steady state. This means that it is generally impossible to solve for equilibrium dynamics using local methods since the point around which to perturb the system is not known.

Global projection methods (Judd (1998)) avoid these problems by not relying on the deterministic steady state. Rather, they directly approximate the transition and policy functions in the relevant area of the state space. Additional advantages of global nonlinear methods are greater flexibility in dealing with highly nonlinear functions within the model such as probability distributions or option-like payoffs.

### B.1 Solution Procedure

The projection-based solution approach used in this paper has three main steps:

- Step 1. **Define approximating basis for the policy and transition functions.** To approximate these unknown functions, we discretize the state space and use multivariate linear interpolation. Our general solution framework provides an object-oriented MATLAB library that allows approximation of arbitrary multivariate functions using linear interpolation, splines, or polynomials. For the model in this paper, splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation.
- Step 2. **Iteratively solve for the unknown functions.** Given an initial guess for policy and transition functions, at each point in the discretized state space compute the current-period optimal policies. Using the solutions, compute the next iterate of the transition functions. Repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose. This step is completely parallelized across points in the state space within each iterate.
- Step 3. **Simulate the model for many periods using approximated functions.** Verify that the simulated time path stays within the bounds of the state space for which policy and transition functions were computed. Calculate relative Euler equation errors to assess the computational accuracy of the solution. If the simulated time path leaves the state space boundaries or errors are too large, the solution procedure may have to be repeated with optimized grid bounds or positioning of grid points.

We will now provide a more detailed description for each step.

**Step 1** The state space consists of

- two exogenous state variables  $[Z_t^A, \sigma_{\omega,t}]$ , and
- five endogenous state variables  $[K_t^B, A_t^B, W_t^I, W_t^S, B_t^G]$

We first discretize  $Z_t^A$  into a  $N^{ZA}$ -state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points  $\{Z_j^A\}_{j=1}^{N^{ZA}}$  and the  $N^{ZA} \times N^{ZA}$  Markov transition matrix  $\Pi_{ZA}$  between them to match the volatility and persistence of HP-detrended GDP. The dispersion of idiosyncratic productivity shocks  $\sigma_{\omega,t}$  can take on two realizations  $\{\sigma_{\omega,L}, \sigma_{\omega,H}\}$  as described in the calibration section. The  $2 \times 2$  Markov transition matrix between these states is given by  $\Pi_{\sigma_\omega}$ . We assume independence between both exogenous shocks. Denote the set of the  $N^x = 2N^{ZA}$  values the exogenous state variables can take on as  $S_x = \{Z_j^A\}_{j=1}^{N^{ZA}} \times \{\sigma_{\omega,L}, \sigma_{\omega,H}\}$ , and the associated Markov transition matrix  $\Pi_x = \Pi_{ZA} \otimes \Pi_{\sigma_\omega}$ .

One endogenous state variable can be eliminated for computational purposes since its value is implied by the agents' budget constraints, conditional on any four other state variables. We eliminate saver wealth  $W_t^S$ , which can be computed as

$$W_t^S = \Omega_A(\omega_t^*)(1 + \delta q_t^m)A_t^B + M_t - W_t^I + B_t^G$$

Our solution algorithm requires approximation of continuous functions of the endogenous state variables. Define the “true” endogenous state space of the model as follows: if each endogenous state variable  $S_t \in \{K_t^B, A_t^B, W_t^I, B_t^G\}$  can take on values in a continuous and convex subset of the reals, characterized by constant state boundaries,  $[\bar{S}_l, \bar{S}_u]$ , then the endogenous state space  $S_n \equiv [\bar{K}_l^B, \bar{K}_u^B] \times [\bar{A}_l^B, \bar{A}_u^B] \times [\bar{W}_l^I, \bar{W}_u^I] \times [\bar{B}_l^G, \bar{B}_u^G]$ . The total state space is the set  $S = S_x \times S_n$ .

To approximate any function  $f : S \rightarrow \mathcal{R}$ , we form an univariate grid of (not necessarily equidistant) strictly increasing points for each endogenous state variables, i.e., we choose  $\{K_j^B\}_{j=1}^{N^K}$ ,  $\{A_k^B\}_{k=1}^{N^A}$ ,  $\{W_m^I\}_{m=1}^{N^W}$ , and  $\{B_n^G\}_{n=1}^{N^G}$ . These grid points are chosen to ensure that each grid covers the ergodic distribution of the economy in its dimension, and to minimize computational errors, with more details on the choice provided below. Denote the set of all endogenous-state grid points as  $\hat{S}_n = \{K_j^B\}_{j=1}^{N^K} \times \{A_k^B\}_{k=1}^{N^A} \times \{W_m^I\}_{m=1}^{N^W} \times \{B_n^G\}_{n=1}^{N^G}$ , and the total discretized state space as  $\hat{S} = S_x \times \hat{S}_n$ . This discretized state space has  $N^S = N^x \cdot N^K \cdot N^A \cdot N^W \cdot N^G$  total points, where each point is a  $5 \times 1$  vector as there are 5 distinct state variables. We can now approximate the smooth function  $f$  if we know its values  $\{f_j\}_{j=1}^{N^S}$  at each point  $\hat{s} \in \hat{S}$ , i.e.  $f_j = f(\hat{s}_j)$  by multivariate linear interpolation.

Our solution method requires approximation of three sets of functions defined on the domain of the state variables. The first set of unknown functions  $\mathcal{C}_P : S \rightarrow \mathcal{P} \subseteq \mathcal{R}^{N^P}$ , with  $N^P$  being the number of policy variables, determines the values of endogenous objects specified in the equilibrium definition at every point in the state space. These are the prices, agents' choice variables, and the Lagrange multipliers on the portfolio constraints. Specifically, the 12 policy functions are bond prices  $q^m(S)$ ,  $q(S)$ , investment  $X(S)$ , consumption  $c^B(S)$ ,  $c^S(S)$ , the bank dividend  $d^I(S)$ , wages  $w^B(S)$ ,  $w^S(S)$ , the Lagrange multipliers for the bank leverage constraint  $\lambda^I(S)$  and no-shorting constraint  $\mu^I(S)$ , the multiplier for borrowers' leverage constraint  $\lambda^B(S)$ , and finally the multiplier on the savers no-shorting constraint  $\mu^S(S)$ . There is an equal number of these unknown functions and nonlinear functional equations, to be listed under step 2 below.

The second set of functions  $\mathcal{C}_T : S \times S_x \rightarrow S_n$  determine the next-period endogenous state variable realizations as a function of the state in the current period and the next-period realization of exogenous

shocks. There is one transition function for each endogenous state variable, corresponding to the transition law for each state variable, also to be listed below in step 2.

The third set are forecasting functions  $\mathcal{C}_F : \mathcal{S} \rightarrow \mathcal{F} \subseteq \mathcal{R}^{N^F}$ , where  $N^F$  is the number of forecasting variables. They map the state into the set of variables sufficient to compute expectations terms in the nonlinear functional equations that characterize equilibrium. They partially coincide with the policy functions, but include additional functions. In particular, the forecasting functions for our model are the bond price  $q^m(\mathcal{S})$ , investment  $X(\mathcal{S})$ , consumption  $c^B(\mathcal{S})$ ,  $c^S(\mathcal{S})$ , the bank dividend  $d^I(\mathcal{S})$ , the value functions of households  $V^S(\mathcal{S})$ ,  $V^B(\mathcal{S})$ , and banks  $V^I(\mathcal{S})$ , the wage bill  $w(\mathcal{S}) = w^B(\mathcal{S}) + w^S(\mathcal{S})$  and the Lagrange multiplier on the borrowers' leverage constraint  $\lambda^B(\mathcal{S})$ .

**Step 2** Given an initial guess  $\mathcal{C}^0 = \{\mathcal{C}_P^0, \mathcal{C}_T^0, \mathcal{C}_F^0\}$ , the algorithm to compute the equilibrium takes the following steps.

A. **Initialize** the algorithm by setting the current iterate  $\mathcal{C}^m = \{\mathcal{C}_P^m, \mathcal{C}_T^m, \mathcal{C}_F^m\} \equiv \{\mathcal{C}_P^0, \mathcal{C}_T^0, \mathcal{C}_F^0\}$

B. **Compute forecasting values.** For each point in the discretized state space,  $s_j \in \hat{\mathcal{S}}$ ,  $j = 1, \dots, N^S$ , perform the steps:

i. Evaluate the transition functions at  $s_j$  combined with each possible realization of the exogenous shocks  $x_i \in \mathcal{S}_x$  to get  $\hat{s}_j^i(x_i) = \mathcal{C}_T^m(s_j, x_i)$  for  $i = 1, \dots, N^x$ , which are the values of the endogenous state variables given the current state  $s_j$  and for each possible future realization of the exogenous state.

ii. Evaluate the forecasting functions at these future state variable realizations to get  $f_{i,j}^0 \equiv \mathcal{C}_F^m(\hat{s}_j^i(x_i), x_i)$

The end result is a  $N^x \times N^S$  matrix  $\mathcal{F}^m$ , with each entry being a vector

$$f_{i,j}^m = [q_{i,j}^m, c_{i,j}^B, c_{i,j}^S, d_{i,j}^I, V_{i,j}^B, V_{i,j}^S, V_{i,j}^I, X_{i,j}, w_{i,j}, \lambda_{i,j}^B] \quad (\text{F})$$

of the next-period realization of the forecasting functions for current state  $s_j$  and future exogenous state  $x_i$ .

C. **Solve system of nonlinear equations.** At each point in the discretized state space,  $s_j \in \hat{\mathcal{S}}$ ,  $j = 1, \dots, N^S$ , solve the system of nonlinear equations that characterize equilibrium in the equally many “policy” variables, given the forecasting matrix  $\mathcal{F}^m$  from step B. This amounts to solving a system of 12 equations in 12 unknowns

$$\hat{P}_j = [\hat{q}_j^m, \hat{q}_j, \hat{X}_j, \hat{c}_j^B, \hat{c}_j^S, \hat{d}_j^I, \hat{w}_j^B, \hat{w}_j^S, \hat{\lambda}_j^I, \hat{\mu}_j^I, \hat{\lambda}_j^B, \hat{\mu}_j^S] \quad (\text{P})$$

at each  $s_j$ . The equations are

$$\hat{q}_j^m = \hat{\lambda}_j^B F + E_{s_{i,j}|s_j} \left\{ \hat{\mathcal{M}}_{i,j}^B [\Omega_A(\omega_{i,j}^*) (1 - (1 - \theta)\tau_\Pi + \delta q_{i,j}^m) + f_\omega(\omega_{i,j}^*) \mathcal{F}_{i,j}] \right\} \quad (\text{E1})$$

$$\begin{aligned} \hat{p}_j = E_{s_{i,j}|s_j} \left[ \hat{\mathcal{M}}_{i,j}^B \left\{ p_{i,j} \Omega_A(\omega_{i,j}^*) (1 - \tilde{\delta}_K) (1 + \Phi \lambda_{i,j}^B) + (1 - \tau_\Pi)(1 - \alpha) \Omega_K(\omega_{i,j}^*) Z_i^A \left( \frac{K_{i,j}^B}{L_{i,j}^B} \right)^\alpha \right. \right. \\ \left. \left. - \Psi_K(X_{i,j}, K_{i,j}^B) + (1 - \alpha) f_\omega(\omega_{i,j}^*) \omega_{i,j}^* \left( Z_i^A \left( \frac{K_{i,j}^B}{L_{i,j}^B} \right)^\alpha \mathcal{F}_{i,j} + (1 - \tilde{\delta}_K) \Phi \lambda_{i,j}^B p_{i,j} \right) \right\} \right] \quad (\text{E2}) \end{aligned}$$

$$(1 - \tau_\Pi^B) \Omega_K(\omega_j^*) \hat{\text{MPL}}_j^B = (1 - \tau_\Pi^B) \Omega_A(\hat{\omega}_j^B) \hat{w}_j^B + f_\omega(\hat{\omega}_j^B) \left( \hat{w}_j^B - \hat{\omega}_j^* \hat{\text{MPL}}_j^B \right) \hat{\mathcal{F}}_j \quad (\text{E3})$$

$$(1 - \tau_\Pi^B) \Omega_K(\omega_j^*) \hat{\text{MPL}}_j^S = (1 - \tau_\Pi^B) \Omega_A(\hat{\omega}_j^S) \hat{w}_j^S + f_\omega(\hat{\omega}_j^S) \left( \hat{w}_j^S - \hat{\omega}_j^* \hat{\text{MPL}}_j^S \right) \hat{\mathcal{F}}_j \quad (\text{E4})$$

$$\hat{q}_j^f + \tau_\Pi^f \hat{r}_j^f - \kappa = \hat{q}_j^f \hat{\lambda}_j^I + E_{s_{i,j}|s_j} \left[ \hat{\mathcal{M}}_{i,j}^I \right] \quad (\text{E5})$$

$$\hat{q}_j^m = \xi \hat{\lambda}_j^I \hat{q}_j^m + E_{s_{i,j}|s_j} \left[ \hat{\mathcal{M}}_{i,j}^I \left( \hat{M}_{i,j} + \delta q_{i,j}^m \Omega_A(\omega_{i,j}^*) \right) \right] \quad (\text{E6})$$

$$\hat{q}_j^f = \hat{\mu}_j^S + E_{s_{i,j}|s_j} \left[ \hat{\mathcal{M}}_{i,j}^S \right] \quad (\text{E7})$$

$$\left( \Phi p_j \Omega_A(\omega_j^*) (1 - \tilde{\delta}_K) K_j^B - F \hat{A}_j^B \right) \hat{\lambda}_j^B = 0 \quad (\text{E8})$$

$$\left( \xi q_j^m \hat{A}_j^I + \hat{q}_j^f \hat{B}_j^I \right) \hat{\lambda}_j^I = 0 \quad (\text{E9})$$

$$\hat{A}_j^I \hat{\mu}_j^I = 0 \quad (\text{E10})$$

$$\hat{B}_j^S \hat{\mu}_j^S = 0 \quad (\text{E11})$$

$$\hat{B}_j^C = \hat{B}_j^S + \hat{B}_j^I \quad (\text{E12})$$

(E1) and (E2) are the Euler equations for borrower-entrepreneurs from (35) and (36). (E3) and (E4) are the intratemporal optimality conditions for labor demand by borrower-entrepreneurs from (31). (E5) and (E6) are the Euler equations for banks from (50) and (51). (E7) is the savers' Euler equation (58). (E8) and (E9) are the leverage constraints ((27) and (43)) for borrowers and banks, respectively. (E10) and (E11) are the no-shorting constraints ((44) and (54)) for banks and savers, respectively. Finally, (E12) is the market clearing condition for riskfree debt, (18).

Expectations are computed as weighted sums, with the weights being the probabilities of transitioning to exogenous state  $x_i$ , conditional on state  $s_j$ . Hats ( $\hat{\cdot}$ ) in (E1) – E(12) indicate variables that are direct functions of the vector of unknowns (P). These are effectively the choice variables for the nonlinear equation solver that finds the solution to the system (E1) – (E12) at each point  $s_j$ . All variables in the expectation terms with subscript  $i,j$  are direct functions of the forecasting variables (F).

These values are *fixed* numbers when the system is solved, as they we pre-computed in step B. For example, the stochastic discount factors  $\hat{\mathcal{M}}_{i,j}^h$ ,  $h = B, I, S$ , depend on both the solution and the forecasting vector, e.g. for savers

$$\hat{\mathcal{M}}_{i,j}^S = \beta_S \left( \frac{V_{i,j}^S}{CE_j^S} \right)^{1/\nu_S - \sigma_S} \left( \frac{c_{i,j}^S}{c_j^S} \right)^{1/\nu_S} \quad \square$$

since they depend on future consumption and indirect utility, but also current consumption. To

compute the expectation of the right-hand side of equation (E7) at point  $s_j$ , we first look up the corresponding column  $j$  in the matrix containing the forecasting values that we computed in step B,  $\mathcal{F}^m$ . This column contains the  $N^x$  vectors, one for each possible realization of the exogenous state, of the forecasting values defined in (F). From these vectors, we need saver consumption  $c_{i,j}^S$  and the saver value function  $V_{i,j}^S$ . Further, we need current consumption  $\hat{c}_j^S$ , which is a policy variable chosen by the nonlinear equation solver. Denoting the probability of moving from current exogenous state  $x_j$  to state  $x_i$  as  $\pi_{i,j}$ , we compute the certainty equivalent

$$CE_j^S \equiv \sum_{x_i | x_j} \pi_{i,j} (V_{i,j}^S)^{1-\sigma_S} \quad \square$$

and then complete expectation of the RHS of (E7)

$$\mathbb{E}_{s_{i,j} | s_j} [\mathcal{M}_{i,j}^S] \equiv \sum_{x_i | x_j} \pi_{i,j} \beta_S \left( \frac{V_{i,j}^S}{CE_j^S} \right)^{1/\nu_S - \sigma_S} \left( \frac{c_{i,j}^S}{\hat{c}_j^S} \right)^{-1/\nu_S} \quad \square$$

The mapping of solution and forecasting vectors (P) and (F) into the other expressions in equations (E1) – E(12) follows the same principles and is based on the definitions in model appendix A. For example, the borrower default threshold is a function of current wages and state variables based on (23)

$$\hat{\omega}_j^* \equiv \frac{\bar{\pi} + \bar{w}_j^B L^B + \bar{w}_j^S L^S + A_j^B}{Z_t^A (K_j^B)^{1-\alpha} (L)^\alpha} \quad \square$$

and the capital price is a linear function of investment from the first-order condition (30)

$$\bar{p}_j = 1 + \eta \left( \frac{\bar{K}_j}{\bar{K}_j^B} - \delta_K \right) \quad \square$$

The system (E1) – (E12) implicitly uses the budget constraints of all agents, and the market clearing condition for corporate debt. First, one can solve for new debt issued by borrowers from their budget constraint (26)

$$\begin{aligned} \bar{A}_j^B &= \frac{1}{\bar{q}_j^n} \left[ \bar{c}_j^B - \left( (1 - \tau_H^B) \Omega_K(\hat{\omega}_j^*) \hat{Y}_j + (1 - \tau^B) \bar{w}_j^B \bar{L}^B + \hat{G}_j^{T,B} + \bar{p}_j [\hat{X}_j + \Omega_A(\hat{\omega}_j^*) (1 - \delta_K) K_j^B] \right) \right. \\ &\quad \left. - \Omega_A(\hat{\omega}_j^*) A_j^B (1 - (1 - \theta) \tau_H^B + \delta \hat{q}_j^m) \right. \\ &\quad \left. - \hat{p}_j \hat{K}_j^B - \hat{X}_j - \Psi(\hat{X}_j, K_j^B) - (1 - \tau_H^B) \Omega_A(\hat{\omega}_j^*) \sum_{n=B,S} \left( \bar{w}_j^n \hat{L}_t^n + \hat{D}_j^n \right) \right] \quad \square \end{aligned}$$

All expressions on the right-hand side of the above equation are direct functions of the state or policy variables. Market clearing for corporate debt implies  $\hat{A}_j^I = \hat{A}_j^B$ , and thus deposits issued by banks follow from their budget constraint (46)

$$\bar{B}_j^I = \frac{1}{\bar{q}_j^f + \tau_H^f r_j^f - R} \left[ \bar{W}_j^I - \left( \bar{d}_j^I + \Sigma(\hat{d}_j^I) + \hat{q}_j^m \hat{A}_j^I \right) \right] \quad \square$$



Similarly, deposits bought by savers follow from their budget constraint (52)

$$B_j^S = \frac{1}{\bar{q}_j^S} \left[ \hat{c}_j^S - \left( (1 - \tau^S) \bar{w}_j^S L^S + \hat{G}_j^{T,S} + W_j^S \right) \right]$$

Note that we could exploit the linearity of the market clearing condition in (E12) to eliminate one more policy variable,  $\hat{c}_j^S$ , from the system analytically. However, in our experience the algorithm is more robust when we explicitly include consumption of all agents as policy variables, and ensure that these variables stay strictly positive (as required with power utility) when solving the system. To solve the system in practice, we use a nonlinear equation solver that relies on a variant of Newton's method, using policy functions  $\mathcal{C}_P^m$  as initial guess. More on these issues in subsection B.2 below.

The final output of this step is a  $N^S \times 12$  matrix  $\mathcal{P}^{m+1}$ , where each row is the solution vector  $\bar{P}_j$  that solves the system (E1)–E(12) at point  $s_j$ .

**D. Update forecasting, transition and policy functions.** Given the policy matrix  $\mathcal{P}^{m+1}$  from step B, update the policy function directly to get  $\mathcal{C}_P^{m+1}$ . All forecasting functions with the exception of the value functions are also equivalent to policy functions. Value functions are updated based on the recursive definitions

$$\bar{V}_j^S = \left\{ \left[ (1 - \beta_S) \left[ \hat{c}_j^S \right]^{1-1/\nu} + \beta_S E_{s_{i,j}|s_j} \left[ \left( V_{i,j}^S \right)^{1-\sigma_S} \right]^{\frac{1-1/\eta}{1-\sigma_S}} \right] \right\}^{\frac{1}{1-1/\nu}} \quad (V1)$$

$$\bar{V}_j^B = \left\{ \left[ (1 - \beta_B) \left[ \hat{c}_j^B \right]^{1-1/\nu} + \beta_B E_{s_{i,j}|s_j} \left[ \left( V_{i,j}^B \right)^{1-\sigma_B} \right]^{\frac{1-1/\eta}{1-\sigma_B}} \right] \right\}^{\frac{1}{1-1/\nu}} \quad (V2)$$

$$\bar{V}_j^I = d_j^I + E_{s_{i,j}|s_j} \left[ \mathcal{M}_{i,j}^B F_{\epsilon,i,j} \left( V_{i,j}^I - \epsilon_{i,j}^I \right) \right] \quad (V3)$$

using the same notation as defined above under step C. Note that each value function combines current solutions from  $\mathcal{P}^{m+1}$  (step C) for consumption and dividend with forecasting values from  $\mathcal{F}^m$  (step B). Using these updated value functions, we get  $\hat{\mathcal{C}}_F^{m+1}$

Finally, update transition functions for the endogenous state variables using the following laws of motion, for current state  $s_j$  and future exogenous state  $x_i$  as defined above:

$$K_{i,j}^B = (1 - \delta_K) K_j^B + \hat{X}_j \quad (T1)$$

$$A_{i,j}^B = \hat{A}_j^B \quad (T2)$$

$$W_{i,j}^I = \left( \bar{M}_{i,j} + \delta q_{i,j}^m \Omega_A(\omega_{i,j}^*) \right) \hat{A}_j^I + \hat{B}_j^I \quad (T3)$$

$$B_{i,j}^G = \frac{1}{\bar{q}_j^G} \left( B_j^G + \hat{G}_j - \hat{T}_j \right) \quad (T4)$$

(T1) is simply the law of motion for aggregate capital, and (T2) follows trivially from the direct mapping of policy into state variable for borrower debt. (T3) is the law of motion for bank net worth (42), which again combines inputs from old forecasting functions  $\mathcal{F}^m$  and new policy solutions  $\mathcal{P}^{m+1}$ . (T4) is the government budget constraint (17). Updating according to (T1)–(T4) gives the next set of functions  $\hat{\mathcal{C}}_F^{m+1}$

**E. Check convergence.** Compute distance measures  $\Delta_F = \|\mathcal{C}_F^{m+1} - \mathcal{C}_F^m\|$  and  $\Delta_T = \|\mathcal{C}_T^{m+1} - \mathcal{C}_T^m\|$ . If  $\Delta_F < \text{ToI}_F$  and  $\Delta_T < \text{ToI}_T$ , stop and use  $\mathcal{C}^{m+1}$  as approximate solution. Otherwise

reset policy functions to the next iterate i.e.  $\mathcal{P}^m \rightarrow \mathcal{P}^{m+1}$  and reset forecasting and transition functions to a convex combination of their previous and updated values i.e.  $\mathcal{C}^m \rightarrow \mathcal{C}^{m+1} \equiv D \times \mathcal{C}^m + (1 - D) \times \hat{\mathcal{C}}^{m+1}$ , where  $D$  is a dampening parameter set to a value between 0 and 1 to reduce oscillation in function values in successive iterations. Next, go to step B.

**Step 3** Using the numerical solution  $\mathcal{C}^* = \mathcal{C}^{m+1}$  from step 2, we simulate the economy for  $\bar{T} \equiv T_{ini} + T$  period. Since the exogenous shocks follow a discrete-time Markov chain with transition matrix  $\Pi_x$ , we can simulate the chain given any initial state  $x_0$  using  $\bar{T} - 1$  uniform random numbers based on standard techniques (we fix the seed of the random number generator to preserve comparability across experiments). Using the simulated path  $\{x_t\}_{t=1}^{\bar{T}}$ , we can simulate the associated path of the endogenous state variables given initial state  $s_0 = [x_0, K_0^B, A_0^B, W_0^I, W_0^S, B_0^G]$  by evaluating the transition functions

$$[K_{t+1}^B, A_{t+1}^B, W_{t+1}^I, W_{t+1}^S, B_{t+1}^G] \equiv \mathcal{C}_T^*(s_t, x_{t+1}),$$

to obtain a complete simulated path of model state variables  $\{s_t\}_{t=1}^{\bar{T}}$ . To remove any effect of the initial conditions, we discard the first  $T_{ini}$  points. We then also evaluate the policy and forecasting functions along the simulated sample path to obtain a complete sample path  $\{s_t, P_t, f_t\}_{t=1}^{\bar{T}}$ .

To assess the quality and accuracy of the solution, we perform two types of checks. First, we verify that all state variable realizations along the simulated path are within the bounds of the state variable grids defined in step 1. If the simulation exceeds the grid boundaries, we expand the grid bounds in the violated dimensions, and restart the procedure at step 1. Secondly, we compute relative errors for all equations of the system (E1) – E(12) and the transition functions (T1) – (T4) along the simulated path. For equations involving expectations (such as (E1)), this requires evaluating the transition and forecasting function as in step 2B at the current state  $s_t$ . For each equation, we divide both sides by a sensibly chosen endogenous quantity to yield “relative” errors; e.g., for (E1) we compute

$$1 - \frac{\mathbb{E} \left( \frac{\mathcal{M}_{i,j}^B [\Omega_A(\omega_{i,j}^*) (1 - (1 - \theta)\tau_H + \delta q_{i,j}^m) + f_\omega(\omega_{i,j}^*) \mathcal{F}_{i,j}]}{\mathcal{M}_{i,j}^B [\Omega_A(\omega_{i,j}^*) (1 - (1 - \theta)\tau_H + \delta q_{i,j}^m) + f_\omega(\omega_{i,j}^*) \mathcal{F}_{i,j}]} \right)}{\mathcal{M}_{i,j}^B [\Omega_A(\omega_{i,j}^*) (1 - (1 - \theta)\tau_H + \delta q_{i,j}^m) + f_\omega(\omega_{i,j}^*) \mathcal{F}_{i,j}]}$$

using the same notation as in step 2B. These errors are small by construction when calculated at the points of the discretized state grid  $\hat{\mathcal{S}}$ , since the algorithm under step 2 solved the system exactly at those points. However, the simulated path will likely visit many points that are between grid points, at which the functions  $\mathcal{C}^*$  are approximated by interpolation. Therefore, the relative errors indicate the quality of the approximation in the relevant area of the state space. We report average, median, and tail errors for all equations. If errors are too large during simulation, we investigate in which part of the state space these high errors occur. We then add additional points to the state variable grids in those areas and repeat the procedure.

## B.2 Implementation

**Solving the system of equations.** We solve system of nonlinear equations at each point in the state space using a standard nonlinear equation solver (MATLAB’s `fsolve`). This nonlinear equation solver uses a variant of Newton’s method to find a “zero” of the system. We employ several simple modifications of the system (E1) – E(12) to avoid common pitfalls at this step of the solution procedure. Nonlinear equation solver are notoriously bad at dealing with complementary slackness conditions associated with constraint, such as (E8) – E(11). Judd, Kubler, and Schmedders (2002) discuss the reasons for this and also show how Kuhn-Tucker conditions can be rewritten as additive equations for this purpose. For example, consider the bank’s Euler Equation for risk-free bonds and the Kuhn-Tucker

condition for its leverage constraint:

$$\begin{aligned} \hat{q}_j^f(1 - \hat{\lambda}_j^I) + \tau^H \hat{r}_j^f - \kappa &= \mathbb{E}_{s_{i,j}|s_j} \left[ \mathcal{M}_{i,j}^I \right] \\ \left( \xi \hat{q}_j^m \hat{A}_j^I + \hat{q}_j^f \hat{B}_j^I \right) \hat{\lambda}_j^I &= 0 \end{aligned}$$

Now define an auxiliary variable  $h_j \in \mathcal{R}$  and two functions of this variable, such that  $\hat{\lambda}_j^{I,+} = \max\{0, h_j\}^3$  and  $\hat{\lambda}_j^{I,-} = \max\{0, -h_j\}^3$ . Clearly, if  $h_j < 0$ , then  $\hat{\lambda}_j^{I,+} = 0$  and  $\hat{\lambda}_j^{I,-} > 0$ , and vice versa for  $h_j > 0$ . Using these definitions, the two equations above can be transformed to:

$$\hat{q}_j^f(1 - \hat{\lambda}_j^{I,+}) + \tau^H \hat{r}_j^f - \kappa = \mathbb{E}_{s_{i,j}|s_j} \left[ \mathcal{M}_{i,j}^I \right] \quad (\text{K1})$$

$$\xi \hat{q}_j^m \hat{A}_j^I + \hat{q}_j^f \hat{B}_j^I - \hat{\lambda}_j^{I,-} = 0 \quad (\text{K2})$$

The solution variable for the nonlinear equation solver corresponding to the multiplier is  $h_j$ . The solver can choose positive  $h_j$  to make the constraint binding ( $\hat{\lambda}_j^{I,-} = 0$ ), in which case  $\hat{\lambda}_j^{I,+}$  takes on the value of the Lagrange multiplier. Or the solver can choose negative  $h_j$  to make the constraint non-binding ( $\hat{\lambda}_j^{I,+} = 0$ ), in which case  $\hat{\lambda}_j^{I,-}$  can take on any value that makes (K2) hold.

Similarly, certain solution variables are restricted to positive values due to the economic structure of the problem. For example, with power utility consumption must be positive. To avoid that the solver tries out negative consumption values (and thus utility becomes ill-defined), we use  $\log(\hat{c}_j^n)$ ,  $n = B, S$ , as solution variable for the solver. This means the solver can make consumption arbitrarily small, but not negative.

The nonlinear equation solver needs to compute the Jacobian of the system at each step. Numerical central-difference (forward-difference) approximation of the Jacobian can be inaccurate and is computationally costly because it requires  $2N+1$  ( $N+1$ ) evaluations of the system, with  $N$  being the number of variables, whereas analytically computed Jacobians are exact and require only one evaluation. We follow Elenev (2016) in “pre-computing” all forecasting functions in step 2B of the algorithm, so that we can calculate the Jacobian of the system analytically. To do so, we employ the Symbolic Math Toolbox in MATLAB, passing the analytic Jacobian to fsolve at the beginning of step 2C. This greatly speeds up calculations.

**Grid configuration.** We choose to include borrower wealth  $W_t^B$  as state variable instead of borrower debt  $A_t^B$ , defined as

$$W_t^B \equiv p_t K_t^B - q_t^m A_t^B$$

such that the total set of endogenous state variables is  $[K_t^B, W_t^B, W_t^I, W_t^S, B_t^G]$ . Keeping track of borrower wealth  $W_t^B$  instead of debt  $A_t^B$  turns out to have better properties for numerical approximation and the same information content. The reason is that borrower wealth is much more stable in the dynamics of the model than borrower debt, since borrower debt and capital are strongly correlated reflecting borrowers’ optimal investment and leverage choices. Recall that one endogenous state variable can be eliminated because of the adding-up property of budget constraints in combination with market clearing. We choose to eliminate saver wealth  $W^S$ . The grid points in each state dimension are as follows

- $Z^A$ : We discretize  $Z_t^A$  into a 5-state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points  $\{Z_j^A\}_{j=1}^5$  and the  $5 \times 5$  Markov transition matrix  $\Pi_{Z^A}$  between them to match the volatility and persistence of HP-detrended GDP. This yields the possible realizations:  $[0.957, 0.978, 1.000, 1.022, 1.045]$

- $\sigma_w$ : [0.095, 0.175] (see calibration)

- $K^B$ : [1.84, 1.98, 2.05, 2.10, 2.26, 2.45, 2.70]

- $W^B$ : [1.00, 1.16, 1.20, 1.23, 1.24, 1.285, 1.33, 1.35, 1.375, 1.41, 1.50, 1.60, 1.70]

- $W^I$ :

[−0.02, −0.01, 0, 0.005, 0.01, 0.015, 0.02, 0.025, 0.03, 0.035, 0.04, 0.045, 0.05, 0.055, 0.06, 0.065, ...  
 ..., 0.07, 0.075, 0.08, 0.10, 0.125, 0.15, 0.25, 0.3, 0.38]

- $B^G$ : [−0.2000, −0.02, 0, 0.1833, 0.4667, 0.7500, 1.0333, 1.3167, 1.4000]

The total state space grid has 204,750 points. As pointed out by several previous studies such as Kubler and Schmedders (2003), portfolio constraints lead to additional computational challenges since portfolio policies may not be smooth functions of state variables due to occasionally binding constraints. Hence we cluster grid points in areas of the state space where constraints transition from slack to binding. Our policy functions are particularly nonlinear in bank net worth  $W_t^I$ , since the status of the bank leverage constraint (binding or not binding) depends predominantly on this state variable. To achieve acceptable accuracy, we have to specify a very dense grid for  $W^I$ , as can be seen above. Also note that the lower end of the  $W^I$  grid includes some negative values. Negative realizations of  $W^I$  can occur in severe financial crisis episodes. Recall that  $W^I$  is the beginning-of-period net worth of all banks. Depending on the realization of their idiosyncratic payout shock, banks decide whether or not to default. Thus the model contains two reasons why banks may not default despite initial negative net worth: (i) positive idiosyncratic shocks, and (ii) positive franchise value. The lower bound of  $W^I$  needs to be low enough such that bank net worth is not artificially truncated during crises, but it must not be so low that, given such low initial net worth, banks cannot be recapitalized to get back to positive net worth. Thus the “right” lower bound depends on the strength of the equity issuance cost and other parameters. Finding the right value for the lower bound is a matter of experimentation.

**Generating an initial guess and iteration scheme.** To find a good initial guess for the policy, forecasting, and transition functions, we solve the deterministic “steady-state” of the model under the assumption that the bank leverage constraint is binding and government debt/GDP is 40%. We then initialize all functions to their steady-state values, for all points in the state space. Note that the only role of the steady-state calculation is to generate an initial guess that enables the nonlinear equation solver to find solutions at (almost) all points during the first iteration of the solution algorithm. In our experience, the steady state delivers a good enough initial guess.

In case the solver cannot find solutions for some points during the initial iterations, we revisit such points at the end of each iteration. We try to solve the system at these “failed” points using as initial guess the solution of the closest neighboring point at which the solver was successful. This method works well to speed up convergence and eventually find solutions at all points.

To further speed up computation time, we run the initial 100 iterations with a coarser state space grid (19,500 points total). After these iterations, the algorithm is usually close to convergence; however, the accuracy during simulation would be too low. Therefore, we initialize the finer (final) solution grid using the policy, forecasting, and transition function obtained after 100 coarse grid iterations. We then run the algorithm for at most 40 more iterations on the fine grid.

To determine convergence, we check absolute errors in the value functions of households and banks,  $(V1) - V(3)$ . Out of all functions we approximate during the solution procedure, these exhibit the

slowest convergence. We stop the solution algorithm when the maximum absolute difference between two iterations, and for all three functions and all points in the state space, falls below  $1e-3$  and the mean distance falls below  $1e-4$ . For appropriately chosen grid boundaries, the algorithm will converge within the final 40 iterations.

In some cases, our grid boundaries are wider than necessary, in the sense that the simulated economy never visits the areas near the boundary on its equilibrium path. Local convergence in those areas is usually very slow, but not relevant for the equilibrium path of the economy. If the algorithm has not achieved convergence after the 40 additional iterations on the fine grid, we nonetheless stop the procedure and simulate the economy. If the resulting simulation produces low relative errors (see step 3 of the solution procedure), we accept the solution. After the 140 iterations described above, our simulated model economies either achieve acceptable accuracy in relative errors, or if not, the cause is a badly configured state grid. In the latter case, we need to improve the grid and restart the solution procedure. Additional iterations, beyond 100 on the coarse and 40 on the fine grid, do not change any statistics of the simulated equilibrium path for any of the simulations we report.

We implement the algorithm in MATLAB and run the code on a high-performance computing (HPC) cluster. As mentioned above, the nonlinear system of equations can be solved in parallel at each point. We parallelize across 28 CPU cores of a single HPC node. From computing the initial guess and analytic Jacobian to simulating the solved model, the total running time for the benchmark calibration is about 2 hours and 40 minutes. Calibrations that exhibit more financial fragility and/or macro volatility converge up to 15% slower.

**Simulation.** To obtain the quantitative results, we simulate the model for 10,000 periods after a “burn-in” phase of 500 periods. The starting point of the simulation is the ergodic mean of the state variables. As described in detail above, we verify that the simulated time path stays within the bounds of the state space for which the policy functions were computed. We fix the seed of the random number generator so that we use the same sequence of exogenous shock realizations for each parameter combination.

To produce impulse response function (IRF) graphs, we simulate 10,000 different paths of 25 periods each. In the initial period, we set the endogenous state variables to several different values that reflect the ergodic distribution of the states. We use a clustering algorithm to represent the ergodic distribution non-parametrically. We fix the initial exogenous shock realization to mean productivity ( $Z^A = 1$ ) and low uncertainty ( $\sigma_{\omega, low}$ ). The “impulse” in the second period is either only a bad productivity shock ( $Z^A = 0.978$ ) for non-financial recessions, or both low  $Z^A$  and a high uncertainty shock ( $\sigma_{\omega, hi}$ ) for financial recessions. For the remaining 23 periods, the simulation evolves according to the stochastic law of motion of the shocks. In the IRF graphs, we plot the median path across the 10,000 paths given the initial condition.

### B.3 Evaluating the solution

**Equation errors.** Our main measure to assess the accuracy of the solution are relative equation errors calculated as described in step 3 of the solution procedure. Table 6 reports the median error, the 95<sup>th</sup> percentile of the error distribution, the 99<sup>th</sup>, and 100<sup>th</sup> percentiles during the 10,000 period simulation of the model. Median and 75th percentile errors are small for all equations. Equations (E5) – (E6) and (E9) have elevated maximum errors. These errors are caused by a bad approximation of the bank’s Lagrange multiplier  $\lambda^I$  in rarely occurring states. It is possible to reduce these errors by placing more grid points in those areas of the state space. In our experience, adding points to eliminate the tail errors has little to no effect on any of the results we report. Since it increases computation

Table 6: Computational Errors

| Equation |           | Percentile |        |        |        |        |
|----------|-----------|------------|--------|--------|--------|--------|
|          |           | 50th       | 75th   | 95th   | 99th   | Max    |
| E1       | (35)      | 0.0004     | 0.0009 | 0.0019 | 0.0033 | 0.0316 |
| E2       | (36)      | 0.0003     | 0.0005 | 0.0011 | 0.0017 | 0.0051 |
| E3       | (31), $B$ | 0.0001     | 0.0002 | 0.0002 | 0.0003 | 0.0004 |
| E4       | (31), $S$ | 0.0001     | 0.0002 | 0.0002 | 0.0003 | 0.0004 |
| E5       | (50)      | 0.0038     | 0.0079 | 0.0140 | 0.0180 | 0.1302 |
| E6       | (51)      | 0.0042     | 0.0091 | 0.0185 | 0.0212 | 0.1389 |
| E7       | (58)      | 0.0007     | 0.0014 | 0.0026 | 0.0036 | 0.0119 |
| E8       | (27)      | 0.0041     | 0.0065 | 0.0137 | 0.0228 | 0.0581 |
| E9       | (43)      | 0.0005     | 0.0011 | 0.0027 | 0.0048 | 0.1069 |
| E10      | (44)      | 0.0002     | 0.0006 | 0.0010 | 0.0015 | 0.0079 |
| E11      | (54)      | 0.0055     | 0.0080 | 0.0181 | 0.0288 | 0.0783 |
| E12      | (18)      | 0.0005     | 0.0006 | 0.0007 | 0.0009 | 0.0369 |

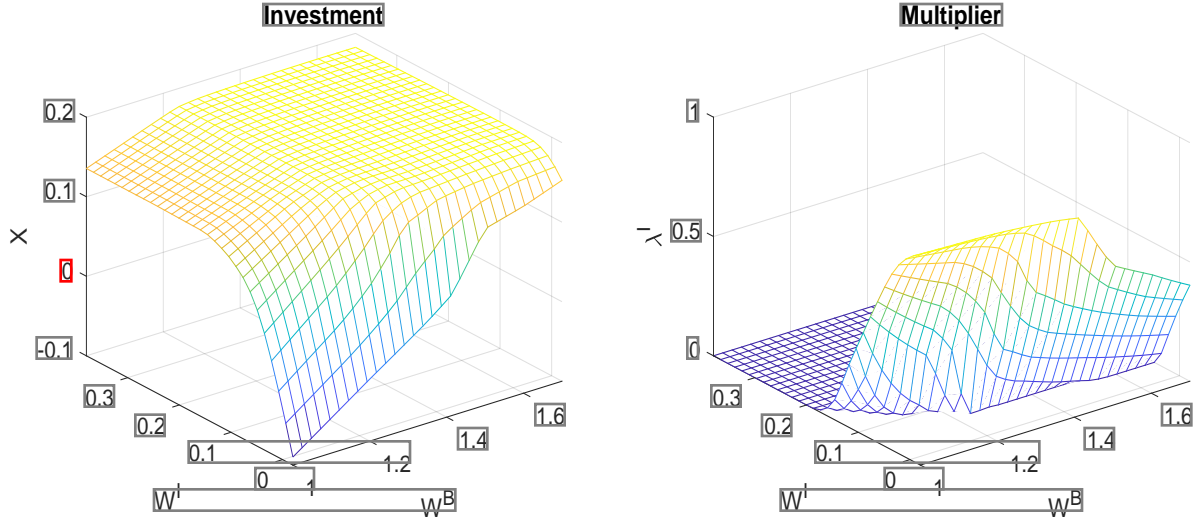
The table reports median, 75th percentile, 95th percentile, 99th percentile, and maximum absolute value errors, evaluated at state space points from a 10,000 period simulation of the benchmark model. Each row contains errors for the respective equation of the nonlinear system (E1) – (E12) listed in step 2 of the solution procedure. The table’s second column contains corresponding equation numbers in the main text and appendix A.

times nonetheless, we chose the current grid configuration.

**Policy function plots.** We further visually inspect policy functions to gauge whether the approximated functions have the smoothness and monotonicity properties implied by our choices of utility and adjustment cost functions. Such plots also allow us to see the effect of binding constraints on prices and quantities. For example, figure 7 shows investment by firms and the Lagrange multiplier on the bank’s leverage constraint. It is obvious from the graphs that a binding intermediary constraint restricts investment. The intermediary constraint becomes binding for low values of intermediary net worth. Further note the interaction with borrower-entrepreneur net worth: holding fixed intermediary net worth, the constraint is more likely to become binding for low borrower wealth.

**State space histogram plots.** We also create histogram plots for the endogenous state variables, overlaid with the placement of grid points. These types of plots allow us to check that the simulated path of the economy does not violate the state grid boundaries. It further helps us to determine where to place grid points. Histogram plots for the benchmark economy are in figure 8.

Figure 7: Plot of optimal investment and Lagrange multiplier on bank leverage constraint



The left panel plots investment by borrower-entrepreneurs as function of borrower-entrepreneur wealth  $W^B$  and bank net worth  $W^I$ . The right panel plots the Lagrange multiplier on the bank leverage constraint for the same state variables. Both plots are for the benchmark economy. The other state variables are fixed to the following values:  $Z^A = 1$ ,  $\sigma_\omega = \bar{\sigma}_{\omega,L}$ ,  $K^B = 2.3$ ,  $B^G = 0.5$ .

## C Calibration Appendix

### C.1 Parameter Sensitivity Analysis

In a complex, non-linear structural general equilibrium model like ours, it is often difficult to see precisely which features of the data drive the ultimate results. This appendix follows the approach advocated by Andrews, Gentzkow, and Shapiro (2017) to report how key moments are affected by changes in the model's key parameters, in the hope of improving the transparency of the results. Structural identification of parameters and sensitivity of results are two sides of the same coin.

Consider a generic vector of moments  $\mathbf{m}$  which depends on a generic parameter vector  $\theta$ . Let  $\iota_i$  be a selector vector of the same length as  $\theta$  taking a value of 1 in the  $i$ 'th position and zero elsewhere. Denote the parameter choices in the benchmark calibration by a superscript  $b$ . For each parameter  $\theta_i$ , we solve the model once for  $\theta^b \circ e^{\iota_i \varepsilon}$  and once for  $\theta^b \circ e^{-\iota_i \varepsilon}$ . We then report the symmetric finite difference:

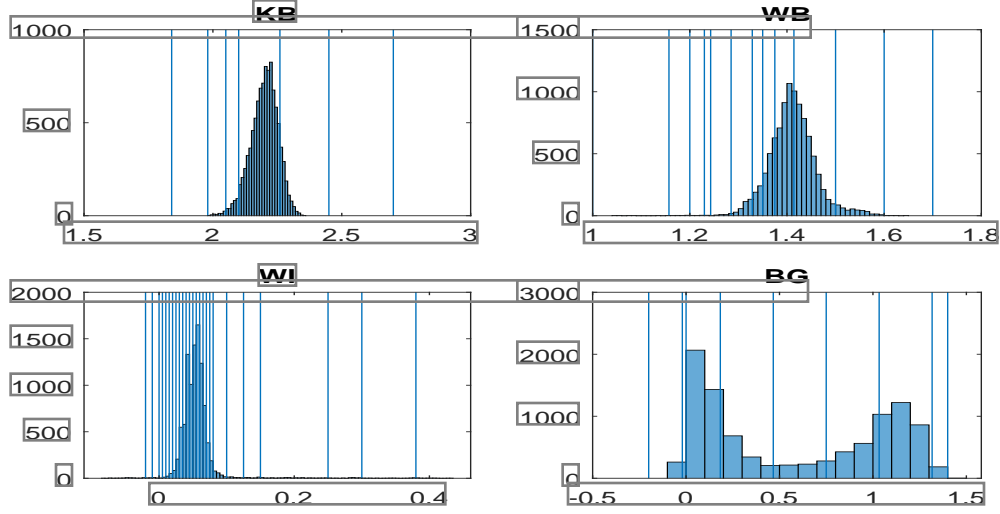
$$\frac{\log(\mathbf{m}(\theta^b \circ e^{\iota_i \varepsilon})) - \log(\mathbf{m}(\theta^b \circ e^{-\iota_i \varepsilon}))}{2\varepsilon}$$

We set  $\varepsilon = .01$ , or 1% of the benchmark parameter value. The resulting quantities are elasticities of moments to structural parameters.

To avoid excessive reporting, we focus on 8 key parameters and 13 key moments. The parameters are: (1) the equity adjustment cost parameter  $\sigma_I$ , (2) the cost of default parameter  $\zeta$ , (3) the mortgage duration parameter  $\delta$ , (4) the capital adjustment cost parameter  $\phi$ , (5) the idiosyncratic bank profit risk  $\sigma_\varepsilon$ , (6) the dispersion of TFP shocks in the normal state  $\sigma_{\omega,L}$ , and (7) the dispersion of TFP shocks in the crisis state  $\sigma_{\omega,H}$ , and (8) the risk aversion coefficient (of both borrowers and savers,  $\sigma_B = \sigma_S$ ). Each panel of Figure 9 lists the same 14 moments and shows the elasticity of the moments to one of the eight parameters. As an aside, the movements in the excess bond return in response to



Figure 8: Histogram plots of endogenous state variables



The plots show histograms for capital and borrower-entrepreneur wealth in the top row, and intermediary net worth and government debt in the bottom row, for the 10,000 period simulation of the benchmark economy. The vertical lines indicate the values of grid points.

multiple parameters appear to be large but they are only large relative to a fairly small baseline level of excess returns of 30 basis points per year. For consistency, we report percentage changes, which are unit-free, in every moment.

A higher equity adjustment cost in the first panel, strongly increases the excess return on corporate bonds, the moment chosen to pin down this parameter. It also strongly decreases bank bankruptcies. Increasing  $\sigma^I$  is akin to an increase in the risk aversion of banks, consistent with the discussion in Section D.6 below. Higher risk aversion naturally results in a larger equilibrium compensation for bearing credit risk and a tendency for banks to stay farther away from their borrowing constraint.

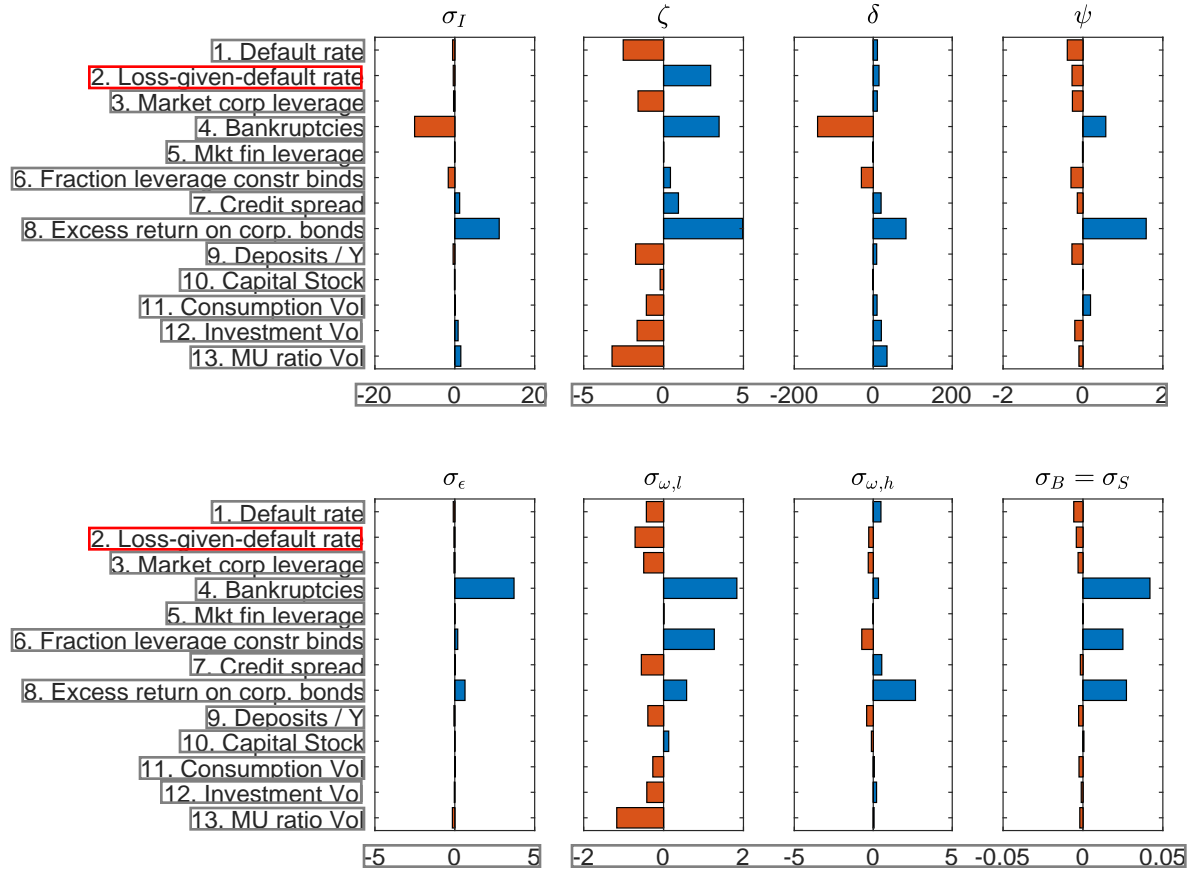
A higher value for the bankruptcy cost parameter  $\zeta$  naturally results in higher losses given default, the moment chosen to pin down this parameter. While there is a modest decline in the default rate, the overall loss rate still goes up. There are more bank bankruptcies and a higher excess return on corporate bonds, given the increased quantity of credit risk. Corporate leverage declines in the wake of costlier credit. With less corporate debt and unchanged financial sector leverage, the banking sector shrinks (Deposits/Y). Lower corporate debt also results in a lower capital stock and a less volatile economy, which improves risk sharing (MU vol goes down).

An increase in the corporate debt maturity parameter  $\delta$  most directly affects bond duration, the elasticity of corporate bond prices to interest rates (not reported). An increase in bond duration increases the excess return on bonds. With increased duration, firms become better duration-matched since the duration of their capital assets is high. As a result, firm leverage slightly increases despite the higher cost of debt.

The fourth panel explores changes in the capital adjustment cost parameter  $\psi$ . Higher capital adjustment costs naturally reduce investment volatility. They raise consumption volatility. The increase in capital adjustment costs increases the volatility in the price of capital (not reported), which causes risk-averse firms to de-lever. Lower leverage reduces the quantity of default risk as well as the credit spread unconditionally, but makes realized excess returns lower in crises, eroding bank capital and increasing expected excess returns enough to increase them unconditionally as well. With more risk



Figure 9: Parameter Sensitivity Analysis



in bad times, the banking sector shrinks (deposits/Y).

Panel five increases the volatility of idiosyncratic bank profit shocks  $\sigma_\epsilon$ . That most directly affects bank bankruptcies, which is how the parameter is calibrated. The banks' leverage constraint binds more frequently. It increases the credit spread and excess bond return. A riskier banking sector shrinks.

Panel six (seven) studies an increase in the idiosyncratic productivity dispersion in normal (crisis) times. The elasticities tend to have an opposite pattern since the former change narrows the gap between the low and the high state thereby reducing the aggregate risk in the economy, while the latter change increases the gap. The reduction in aggregate risk is consistent with a reduction in macroeconomic volatility and an improvement in risk sharing (a reduction in MU vol). Financial firms respond to the safer macro-economic environment and the higher excess bond returns by increasing their risk taking, which results in higher financial sector leverage and bankruptcies.

Panel eight studies an increase in the risk aversion of both types of households in the economy, from the benchmark value of one. The intertemporal elasticity of substitution stays unchanged at one. The effect of this change is orders of magnitude smaller than the effect of other parameter changes. Corporate leverage and defaults go down. The financial leverage constraint becomes binding more frequently, as intermediating has become more profitable as witnessed by the increase in the excess

bond return

## C.2 Long-term Corporate Bonds

Our model's corporate bonds are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time  $t$  promises to pay the holder 1 at time  $t+1$ ,  $\delta$  at time  $t+2$ ,  $\delta^2$  at time  $t+3$ , and so on. Issuers must hold enough capital to collateralize the face value of the bond, given by  $F = \frac{\theta}{1-\delta}$ , a constant parameter that does not depend on any state variable of the economy. Real life bonds have a finite maturity and a principal payment. They also have a vintage (year of issuance), whereas our bonds combine all vintages in one variable. This appendix explains how to map the geometric bonds in our model into real-world bonds by choosing values for  $\delta$  and  $\theta$ .

Our model's corporate loan/bond refers to the entire pool of all outstanding corporate loans/bonds. To proxy for this pool, we use investment-grade and high-yield indices constructed by Bank of America Merrill Lynch (BofAML) and Barclays Capital (BarCap). For the BofAML indices (Datastream Codes LHYIELD and LHCCORP for investment grade and high-yield corporate bonds, respectively) we obtain a time series of monthly market values, durations (the sensitivity of prices to interest rates), weighted-average maturity (WAM), and weighted average coupons (WAC) for January 1997 until December 2015. For the BarCap indices (C0A0 and H0A0 for investment grade and high-yield corporate bonds, respectively), we obtain a time series of option-adjusted spreads over the Treasury yield curve.

First, we use market values of the BofAML investment grade and high-yield portfolios to create an aggregate bond index and find its mean WAC  $c$  of 5.5% and WAM  $T$  of 10 years over our time period. We also add the time series of OAS to the constant maturity treasury rate corresponding to that period's WAM to get a time series of bond yields  $r_t$ . Next, we construct a plain vanilla corporate bond with a semiannual coupon and maturity equal to the WAC and WAM of the aggregate bond index, and compute the price for \$1 par of this bond for each yield:

$$P^c(r_t) = \sum_{i=1}^{2T} \frac{c/2}{(1+r_t)^{i/2}} + \frac{1}{(1+r_t)^T}$$

We can write the steady-state price of a geometric bond with parameter  $\delta$  as

$$P^G(r_t) = \frac{1}{1+r_t} [1 + \delta P^G(r_t)]$$

Solving for  $P^G(r_t)$ , we get

$$P^G(r_t) = \frac{1}{1+r_t-\delta}$$

The calibration determines how many units  $X$  of the geometric bond with parameter  $\delta$  one needs to sell to hedge one unit of plain vanilla bond  $P^c$  against parallel shifts in interest rates, across the range of historical yields:

$$\min_{\delta, X} \sum_{t=1997.1}^{2015.12} [P^c(r_t) - X P^G(r_t; \delta)]^2$$

We estimate  $\delta = 0.937$  and  $X = 12.9$ , yielding an average pricing error of only 0.41%. This value for  $\delta$  implies a time series of durations  $D_t = -\frac{1}{r_t} \frac{dP_t^G}{dr_t}$  with a mean of 6.84.

To establish a notion of principal for the geometric bond, we compare it to a duration-matched zero-coupon bond i.e. borrowing some amount today (the principal) and repaying it  $D_t$  years from now. The principal of this loan is just the price of the corresponding  $D_t$  maturity zero-coupon bond

We set the “principal”  $F$  of one unit of the geometric bond to be some fraction  $\theta$  of the undiscounted sum of all its cash flows  $\frac{\theta}{1-\delta}$ , where

$$\theta \equiv \frac{1}{N} \sum_{t=1997.1}^{2015.12} \frac{1}{(1+r_t)^{D_t}}$$

We get  $\theta = 0.582$  and  $F = 9.18$

### C.3 LTV constraint

The cost of bankruptcy induces banks to limit leverage. In the computation of the model solution, we additionally impose a hard constraint on leverage. This is a standard leverage constraint:

$$FA_{t+1}^B \leq \Phi p_t (1 - (1 - \tau_H^B) \delta_K) \Omega_A(\omega_t^*) K_t^B. \quad (59)$$

The borrowing constraint in (59) caps the face value of debt at the end of the period,  $FA_{t+1}^B$ , to a fraction of the market value of the available capital units after default and depreciation,  $p_t(1 - (1 - \tau_H^B) \delta_K) \Omega_A(\omega_t^*) K_t^B$ , where  $\Phi$  is the maximum leverage ratio. With such a constraint, declines in capital prices (in bad times) tighten borrowing constraints, as in Kiyotaki and Moore (1997). The constraint (59) imposes a hard upper bound on borrower leverage.

We set the maximum LTV ratio parameter  $\Phi = 0.45$ . This value is just large enough so that the LTV constraint never binds during expansions and non-financial recessions. In the simulation of the benchmark model, the borrower’s LTV constraint binds in 3% of financial recessions. The LTV constraint limits corporate borrowing as a fraction of the market value of capital. We set  $\Phi$  to match the volatility of corporate debt-to-GDP of the non-financial sector, which is 5.2% in the data and 4.3% in the model

We have verified that relaxing this constraint to the extent that it is never binding does not significantly affect the results. For example, setting the maximum leverage ratio to  $\Phi = .55$  yields almost identical results. We include the constraint for comparability with the existing literature that has emphasized the financial accelerator operating through capital prices. In our setup the main force limiting corporate leverage is a standard trade-off between the benefits and costs of debt finance.

### C.4 Measuring Labor Income Tax Revenue

We define income tax revenue as current personal tax receipts (line 3) plus current taxes on production and imports (line 4) minus the net subsidies to government sponsored enterprises (line 30 minus line 19) minus the net government spending to the rest of the world (line 25 + line 26 + line 29 - line 6 - line 9 - line 18). Our logic for adding the last three items to personal tax receipts is as follows. Taxes on production and export mostly consist of federal excise and state and local sales taxes, which are mostly paid by consumers. Net government spending on GSEs consists mostly of housing subsidies received by households which can be treated equivalently as lowering the taxes that households pay. Finally, in the data, some of the domestic GDP is sent abroad in the form of net government expenditures

to the rest of the world rather than being consumed domestically. Since the model has no foreigners, we reduce personal taxes for this amount, essentially rebating this lost consumption back to domestic agents.

## C.5 Taxation of Savers' Financial Income

Savers earn financial income from two sources. First, they earn interest on their private lending i.e. deposits in the financial intermediaries. This income is ultimately a claim on the capital rents in the economy and should be taxed at the same rate  $\tau_K$  as borrowers' and intermediaries' net income.

Second, they earn interest on their public lending i.e. government bonds. In the data, Treasury coupons are taxed at the household's marginal tax rate,  $\tau$  in the model. However, the tax revenue collected by the government from interest income on its own bonds is substantially lower than  $\tau B_f^G$  because (a) Treasury coupons are exempt from state and local taxes, and (b) more than half of privately owned Treasury debt is held by foreigners, who also do not pay federal income taxes.

In the model, there is one tax rate  $\tau_D$  at which all of the saver's interest income is taxed. We choose  $\tau_D$  to satisfy

$$\tau_D(\hat{B}^I + \hat{B}^G) = \tau^K(\hat{B}^I - \hat{B}_{\text{pension}}^I) + \tau \frac{\tau^{\text{federal}}}{\tau^{\text{total}}}(\hat{B}^G - \hat{B}_{\text{foreign}}^G - \hat{B}_{\text{pension}}^I)$$

where hats denote quantities in the data. Specifically, the revenue from taxes collected at rate  $\tau_D$  on all private safe debt and government debt must equal the sum of tax revenues collected on taxable private safe debt (private safe debt not held in tax-advantaged pension funds) at rate  $\tau_K$ , and tax revenues collected on taxable public debt (Treasury debt not held by foreigners, the Fed, or pension funds) taxed at rate  $\tau \frac{\tau^{\text{federal}}}{\tau^{\text{total}}}$ .

We measure all quantities at December 31, 2014. Private debt stocks are taken from the Financial Accounts of the United States. Treasury debt stocks are taken from the Treasury Bulletin. Federal and total personal tax revenues are taken from the BEA's National Income and Product Accounts. There is approximately \$13 trillion each outstanding of private and public debt. Almost all private debt is taxable, but only \$4 trillion of public debt is. Federal taxes constitute approximately 80% of all personal income tax revenue. Using the calibration for  $\tau_K$  and  $\tau$ , we get

$$\tau_D \approx \frac{20\% \times \$13T + 29.5\% \times 0.8 \times \$4T}{\$13T + \$13T}$$

or  $\tau_D = 13.4\%$  precisely.

## C.6 Stationarity of Government Debt

In our numerical work, we guarantee the stationarity of the ratio of government debt to GDP by gradually decreasing personal tax rates  $\tau_t$  when debt-to-GDP falls below  $\bar{b}^G = 0.1$  –the profligacy region– and by gradually increasing personal tax rates when debt-to-GDP exceed  $\bar{b}^G = 1.2$  –the austerity region. Specifically, taxes are gradually and smoothly lowered with a convex function until they hit zero at debt to GDP of -0.1. Tax rates are gradually and convexly increased until they hit 60% at a debt-to-GDP ratio of 150%. Our simulations never reach the -10% and +150% debt/GDP states. The simulation spends 24% of the time in the profligacy and 15% of the time in the austerity region. The fraction of time spent in these regions has no effect on the overall resources of the economy.

Achieving stationarity of government debt requires primary surpluses, since the government must also service the debt. Generating primary surpluses requires slightly overshooting on personal and corporate tax revenue relative to the data, since the U.S. government has historically had an average primary surplus of (just about) zero. Put differently, the actual U.S. fiscal path is unsustainable, i.e., incompatible with a stationary model.

## C.7 Measuring Intermediary Sector Leverage

Our notion of the intermediary sector is the levered financial sector. We take book values of assets and liabilities of these sectors from the Financial Accounts of the United States (formerly Flow of Funds). We subtract holding and funding company equity investments in subsidiaries from those subsidiaries' liabilities. Table 7 reports the assets, liabilities, and leverage of each sector as of 2014, as well as the average leverage from 1953 to 2014. We find that the average leverage ratio of the levered financial sector was 91.5%. This is our calibration target.

Krishnamurthy and Vissing-Jorgensen (2015) identify a similar group of financial institutions as net suppliers of safe, liquid assets. Their financial sector includes money market mutual funds (who do not perform maturity transformation) and equity REITS (who operate physical assets) but excludes life insurance companies (which are highly levered). The financial sector definition of Krishnamurthy and Vissing-Jorgensen (2015) suggests a similar ratio of 90.9%. As an aside, we note that Krishnamurthy and Vissing-Jorgensen (2015) report lower total assets and liabilities than in our reconstruction of their procedure because they net out positions within the financial sector by instrument while we do not.

Table 7: Balance Sheet Variables and Prices

| Table   | Sector                                     | Dec 2014  |             |          | Avg 53-14    |
|---------|--|-----------|-------------|----------|--------------|
|         |  | Assets    | Liabilities | Leverage | Leverage     |
| L.111   | U.S.-Chartered Depository Institutions     | \$ 13,647 | \$ 12,161   | 0.891    | 0.921        |
| L.112   | Foreign Banking Offices in U.S.            | \$ 2,093  | \$ 2,086    | 0.996    | 1.065        |
| L.113   | Banks in U.S.-Affiliated Areas             | \$ 92     | \$ 88       | 0.953    | 1.080        |
| L.114   | Credit Unions                              | \$ 1,066  | \$ 958      | 0.899    | 0.916        |
|         | <i>Subtotal: Banks</i>                     | \$ 16,898 | \$ 15,292   | 0.905    | 0.928        |
| L.125   | Government-Sponsored Enterprises (GSEs)    | \$ 6,400  | \$ 6,387    | 0.998    | 0.971        |
| L.126   | Agency- and GSE-Backed Mortgage Pools      | \$ 1,649  | \$ 1,649    | 1.000    | 1.000        |
| L.127   | Issuers of Asset-Backed Securities (ABS)   | \$ 1,424  | \$ 1,424    | 1.000    | 1.003        |
| L.129.m | Mortgage Real Estate Investment Trusts     | \$ 568    | \$ 483      | 0.851    | 0.955        |
| L.128   | Finance Companies                          | \$ 1,501  | \$ 1,376    | 0.916    | 0.873        |
| L.130   | Security Brokers and Dealers               | \$ 3,255  | \$ 1,345    | 0.413    | 0.808        |
| L.131   | Holding Companies                          | \$ 4,391  | \$ 2,103    | 0.479    | 0.441        |
| L.132   | Funding Corporations                       | \$ 1,305  | \$ 1,305    | 1.000    | 1.000        |
|         | <i>Subtotal: Other Liquidity Providers</i> | \$ 20,492 | \$ 16,070   | 0.784    | 0.872        |
| L.116   | Life Insurance Companies                   | \$ 6,520  | \$ 5,817    | 0.892    | 0.932        |
|         | <b>Total</b>                               | \$ 43,910 | \$ 37,179   | 0.847    | <b>0.915</b> |
| L.121   | Money-Market Mutual Funds                  | \$ 2,725  | \$ 2,725    | 1.000    | 1.000        |
| L.129.e | Equity Real Estate Investment Trusts       | \$ 157    | \$ 539      | 3.427    | 2.577        |
|         | <i>Total (K-V.I Definition)</i>            | \$ 40,271 | \$ 33,549   | 0.833    | 0.909        |

## D Results Appendix

### D.1 Pure Uncertainty Shock

Figure 10 compares the dynamics of important macro-economic aggregates and balance sheet variables in a financial recession (red lines) to the effect of a pure second-moment shock. The IRF plots are generated as explained in the main text. The red line in the plots of figure 10 is identical to the red lines in figures 2 and 3 in the main text, as both are caused by the same combination of a low TFP realization and an increase in  $\sigma_\omega$  in period 1. The blue lines in figure 10 show dynamics after the economy is hit only by the increase in  $\sigma_\omega$ , with stable TFP. The plots show that this pure uncertainty shock has much smaller negative effects on output, consumption and investment than the combination that causes a financial recession. This feature of our model is consistent with the empirical finding that uncertainty shocks alone have at most moderate negative effects on output and investment, see for example Bachmann and Bayer (2013) or Vavra (2014).

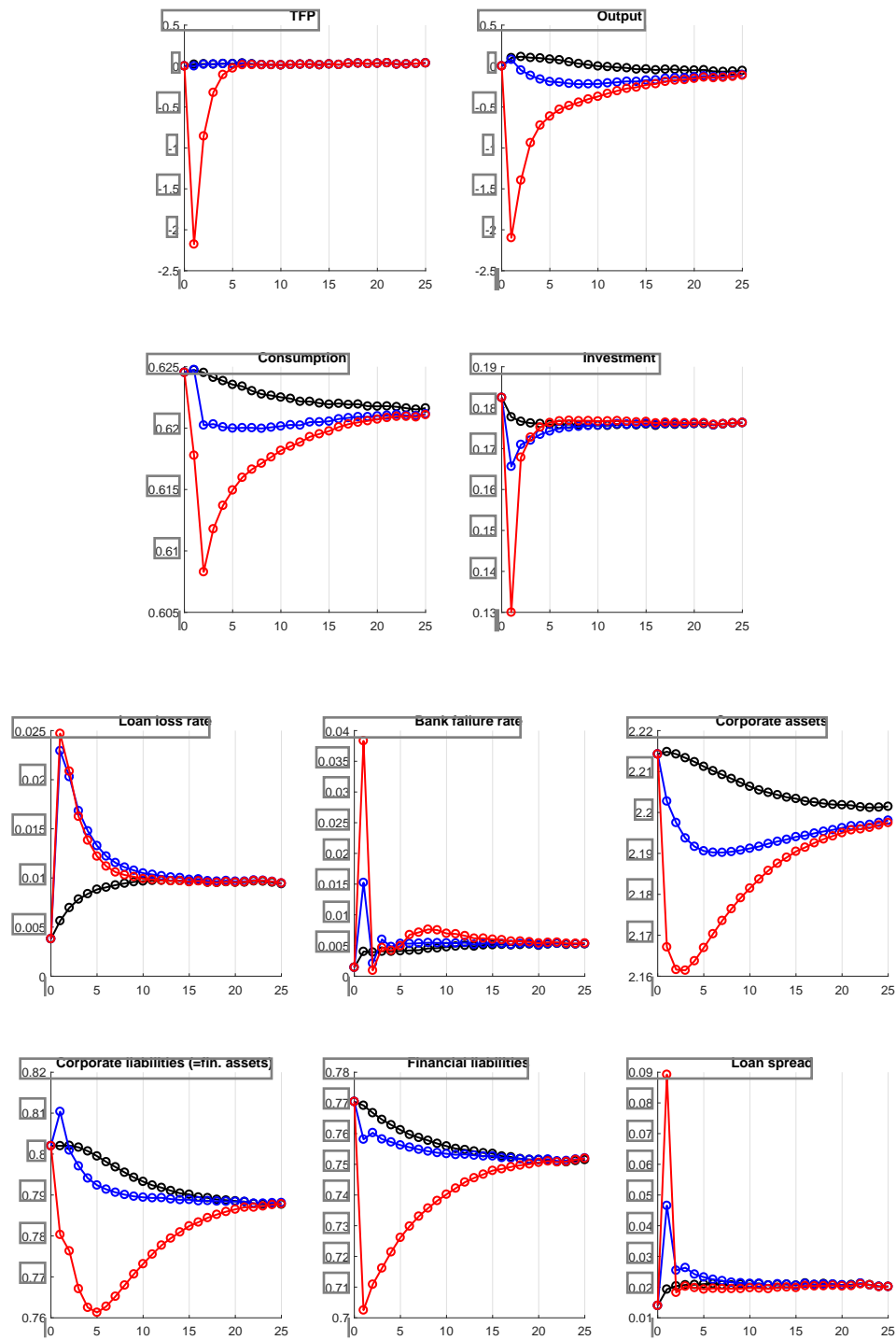
A closer look at the balance sheet variables in the bottom panel reveals that the fundamental difference between both types of shocks lies in the response of intermediaries. The losses suffered on loans during a financial crisis are only marginally larger than those from the uncertainty shock. However, the financial sector does not shrink after the uncertainty shock. Rather, firms raise more debt (bottom left panel) despite a temporarily smaller capital stock (top right panel), effectively increasing leverage. Banks reduce deposit funding only marginally (bottom middle). The spikes in bank failure rate and credit spread are less than half of those experienced in a financial recession. We can conclude that only the combination of TFP and uncertainty shock activates the intermediary-based financial accelerator.

Why are financial recessions so much worse despite similar losses from borrower defaults for banks? Figure 11 shows that the dynamics of the corporate bond price (top right) are the key amplifying force. This price drops sharply in financial recessions, causing large market value losses for intermediaries. This large drop in price is driven by two main forces. First, the negative TFP shock reduces bank demand for corporate bonds, as seen in Figure 3 in the main text. Second, the losses on corporate bonds caused by the uncertainty shock reduce bank capital, and thus amplify the first effect of reduced demand on prices. The stronger financial accelerator means that intermediary net worth falls only half as much in an uncertainty shock episode compared to a financial recession (bottom right). As a result, intermediaries are not forced to shrink as they are in a financial recession. Continuity in lending to borrower-entrepreneurs prevents a sharp reduction in investment and the capital price (bottom left) despite intermediary losses on loans. In the third period of a financial recession, intermediary wealth overshoots as banks earn large spreads due to the sharp drop in the risk-free rate. Intermediaries deplete this extra wealth to gradually expand lending again as the production sector recovers to normal levels of capital. These dynamics are not present in an uncertainty shock episode, since lending never contracted to begin with.

### D.2 Drivers of Financial Leverage

This appendix explores what model ingredients contribute quantitatively to the high financial leverage that the benchmark model is able to generate. Specifically, we turn off the three financial frictions, one at the time: (1) the bankruptcy option for banks, (2) equity adjustment costs ( $\sigma^I = 0$ ), and (3) the tax shield for financial firms. Table 8 contains the results. The main finding is that financial leverage is affected very little by these financial frictions. In other words, the main driver of the high financial leverage is the wedge between the subjective time discount rate of borrowers and savers. This wedge

Figure 10: Financial Recession vs. Uncertainty Shock: Macro Quantities and Balance Sheets

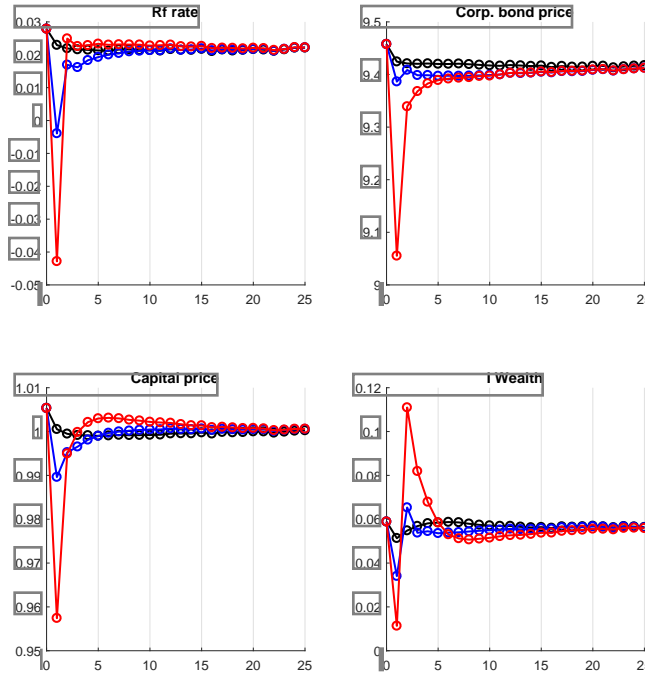


Blue line: uncertainty shock, red line: financial recession, black line: no shock

creates a strong incentive to channel savings from depositors to non-financial firms, i.e., for financial intermediation



Figure 11: Financial Recession vs. Uncertainty Shock: Prices



**Blue line:** uncertainty shock, **red line:** financial recession, **black line:** no shock

Furthermore, we see that when banks are not allowed to default, they stay away from their leverage constraint more often. Without equity adjustment costs, it becomes much cheaper to recapitalize banks for their shareholders. This acts like a reduction in risk aversion for bank shareholders and their leverage constraint becomes binding all the time. The slightly higher financial leverage results in significantly more bank bankruptcies. The effective reduction in risk aversion also lowers the required compensation for risk banks receive, as shown in the lower credit spread and excess return on corporate bonds, despite a slightly higher loss rate on corporate loans. The cheaper cost of debt in turn incentivizes non-financial firms to increase leverage. In sum, a reduction in the cost of equity finance for banks has a stronger effect on non-financial leverage than on financial leverage.

The model without tax shield features higher credit spread and excess return and lower loss rates. The banks manage to pass through the loss of their tax shield to their customers, the non-financial firms. Their compensation per unit of risk increases, providing incentives to increase financial leverage (modestly). The increased cost of credit coincides with lower corporate leverage.

### D.3 Credit Spread and Risk Premium

One important quantitative success of the model is its ability to generate a high unconditional credit spread while matching the observed amount of default risk. The credit spread is also highly volatile (2.94% standard deviation) and more than twice as high in financial recessions than in expansions. The rise in the credit spread in financial recessions to 4.28% reflects not only the increase in the quantity default risk but also an increase in the price of credit risk. The model generates a high and counter-cyclical price of credit risk, which itself comes from the high and counter-cyclical “shadow

Table 8: Drivers of Financial Sector Leverage

|                                | Bench | No bankruptcy | $\sigma^I = 0$ | No tax shield |
|--------------------------------|-------|---------------|----------------|---------------|
| Mkt fin leverage (in %)        | 93.3  | 93.3          | 94.0           | 93.9          |
| Book fin leverage (in %)       | 97.1  | 97.3          | 99.7           | 96.6          |
| % fin leverage constr binds    | 61.3  | 50.7          | 100.0          | 78.8          |
| Bankruptcies (in %)            | 0.54  | 0.00          | 1.71           | 1.21          |
| Credit spread (in %)           | 2.05  | 2.05          | 1.88           | 2.16          |
| Excess ret. corp. bonds (in %) | 1.09  | 1.01          | 0.74           | 1.43          |
| Loss Rate (in %)               | 0.96  | 1.05          | 1.17           | 0.79          |
| Market corp leverage (in %)    | 35.8  | 36.9          | 38.8           | 33.6          |
| Book corp leverage (in %)      | 35.2  | 36.2          | 37.2           | 33.3          |

SDF” for the intermediary sector

The intermediary SDF is given by:

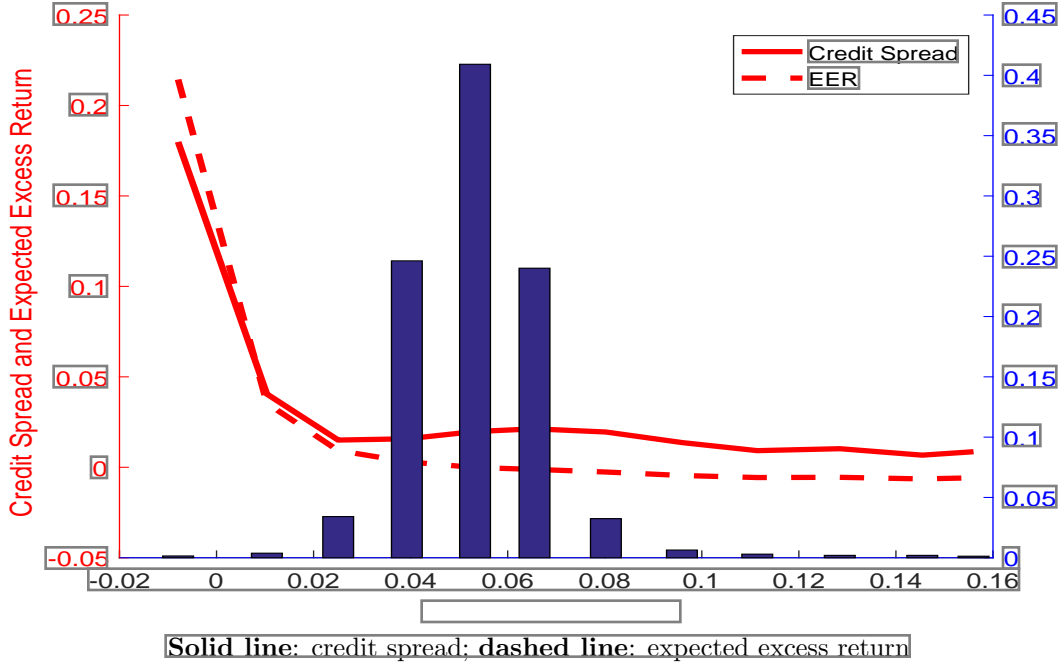
$$\mathcal{M}_{t,t+1}^I = \mathcal{M}_{t,t+1}^B \left( \frac{1 + \sigma^I(d_{t+1}^I - \bar{d})}{1 + \sigma^I(d_t^I - \bar{d})} \right)^{-1} F_{\epsilon,t+1}$$

where  $\mathcal{M}_{t,t+1}^B$  is the borrower SDF,  $F_{\epsilon,t+1}$  is the probability of intermediary failure in  $t + 1$ , and  $\frac{1}{1 + \sigma^I(d_t^I - \bar{d})}$  is the marginal value of wealth to intermediaries in  $t$ .

Figure 12 shows the histogram of the intermediary wealth share plotted against two different measures of credit risk compensation earned by intermediaries. The solid red line plots the credit spread, the difference between the yield  $r_t^m$  on corporate bonds and the risk-free rate. We compute the bond yield as  $r_t^m = \log \left( \frac{1}{d_t^m} + \delta \right)$ . This is a simple way of transforming the price of the long-term bond into a yield; however, note that this definition assumes a default-free payment stream  $(1, \delta, \delta^2, \dots)$  occurring in the future. Consistent with the result in He and Krishnamurty (2013), the credit spread is high when the financial intermediary’s wealth share is low. Since our model has defaultable debt, the increase in the credit spread reflects both risk-neutral compensation for expected defaults and a credit risk premium.

To shed further light on the source of the high credit spread, we compute the expected excess return (EER) on corporate loans earned by the intermediary. The EER consists both of the credit risk premium, defined as the (negative) covariance of the intermediary’s stochastic discount factor with the corporate bond’s excess return, and an additional component that reflects the tightness of the intermediary’s leverage constraint. This component arises because the marginal agent in the market for risk-free debt is the saver household, while corporate bonds are priced by the constrained intermediary. The market risk free rate is lower than the “shadow” risk free rate implied by the intermediary SDF. Given log preferences, most of the action in the EER comes from the constraint tightness component. When intermediary wealth is relatively high, the leverage constraint is not binding and the EER is approximately zero. Low levels of intermediary wealth result from credit losses, and the lowest levels occur during financial crises. At these times, credit risk increases and the intermediary becomes constrained. In the worst crisis episodes when intermediary wealth reaches zero or drops below zero, the EER reaches 20 percent.

Figure 12: The Credit Spread and the Financial Intermediary Wealth Share



#### D.4 Counter-cyclical Capital Requirements

#### D.5 Policy transitions

The tables above only compare the ergodic distributions of economies with different policy parameters. How does an unanticipated policy change to a tighter or looser capital requirement affect output, consumption, and the welfare of borrowers and savers in the short term? Figure 14 plots the evolution of these variables after a policy change from the benchmark to either a higher ( $\xi = .90$ ) or a lower ( $\xi = .97$ ) capital requirement. In the long run, output, consumption, and agent welfare converge to their ergodic means in tables 4 and 5. In the short run, consumption “overshoots” in both cases. Tightening the capital requirement by 4 p.p. leads a contraction in GDP as investment drops. But lower investment also causes a consumption boom in the short run as the economy transitions to a permanently lower capital stock.

#### D.6 Effect of Equity Adjustment Cost

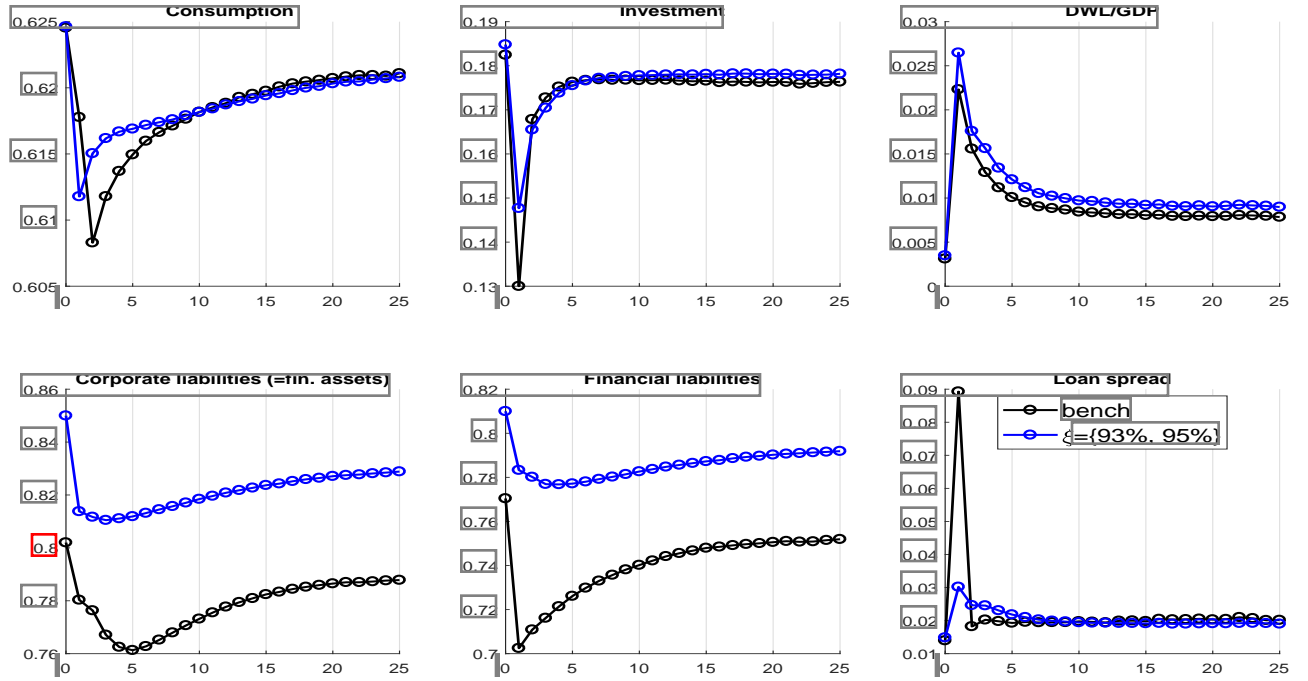
Table 9 shows the effect of larger or smaller equity adjustment costs ( $\sigma^I$ ) relative to the benchmark economy. The overall take-away from this comparison is that larger equity frictions in the intermediation have a similar effect to tightening the intermediaries’ capital requirement. Higher marginal equity adjustments costs (columns  $\sigma^I = 6$ ,  $\sigma^I = 7$ ) lead to a smaller non-financial sector, both in terms of assets and liabilities. Corporate leverage declines, and as a result, fewer firms default, causing an overall decline in loss rates on corporate loans.

Even though intermediaries face less credit risk, they reduce their own leverage and their constraint becomes binding much less frequently as  $\sigma^I$  is increased. Consequently, intermediary failures are almost completely eliminated at  $\sigma^I = 7$ . An important difference to the macro-prudential policy exercise with tighter capital requirements is the effect on bank profitability. In both cases (tighter

Table 9: Effect of Varying  $\sigma^I$ 

|                                       | Bench ( $\sigma^I = 5$ ) | $\sigma^I = 3$ | $\sigma^I = 4$ | $\sigma^I = 6$ | $\sigma^I = 7$ |
|---------------------------------------|--------------------------|----------------|----------------|----------------|----------------|
| Borrowers                             |                          |                |                |                |                |
| 1. Mkt value of capital / Y (in %)    | 225.0                    | 226.6          | 226.4          | 221.4          | 219.0          |
| 2. Mkt value of corp debt / Y (in %)  | 80.6                     | 84.4           | 83.9           | 72.2           | 64.5           |
| 3. Book val of corp debt / Y (in %)   | 79.1                     | 82.1           | 81.7           | 72.2           | 65.1           |
| 4. Market corp leverage (in %)        | 35.8                     | 37.3           | 37.1           | 32.6           | 29.4           |
| 5. Book corp leverage (in %)          | 35.2                     | 36.2           | 36.1           | 32.6           | 29.7           |
| 6. % leverage constr binds            | 0.32                     | 0.77           | 0.62           | 0.01           | 0.00           |
| 7. Default rate (in %)                | 2.25                     | 2.40           | 2.37           | 1.96           | 1.70           |
| 8. Loss-given-default rate (in %)     | 43.09                    | 45.10          | 44.86          | 38.66          | 32.86          |
| 9. Loss Rate (in %)                   | 0.96                     | 1.07           | 1.06           | 0.75           | 0.53           |
| Intermediaries                        |                          |                |                |                |                |
| 10. Mkt fin leverage (in %)           | 93.3                     | 93.9           | 93.8           | 92.7           | 91.6           |
| 11. Book fin leverage (in %)          | 97.1                     | 98.7           | 98.5           | 94.9           | 92.7           |
| 12. % leverage constr binds           | 61.30                    | 93.61          | 82.39          | 30.66          | 20.70          |
| 13. Bankruptcies (in %)               | 0.54                     | 1.45           | 1.26           | 0.07           | 0.03           |
| 14. Wealth I / Y (in %)               | 5.6                      | 5.2            | 5.3            | 5.7            | 5.8            |
| 15. Franchise value (in %)            | 33.9                     | 20.0           | 21.1           | 75.8           | 90.2           |
| Savers                                |                          |                |                |                |                |
| 16. Deposits / Y (in %)               | 76.9                     | 81.1           | 80.5           | 68.5           | 60.5           |
| 17. Government debt / Y               | 60.2                     | 115.7          | 110.6          | 19.0           | 15.4           |
| Prices                                |                          |                |                |                |                |
| 18. Risk-free rate (in %)             | 2.19                     | 2.24           | 2.24           | 2.23           | 2.23           |
| 19. Corporate bond rate (in %)        | 4.24                     | 4.15           | 4.16           | 4.42           | 4.52           |
| 20. Credit spread (in %)              | 2.05                     | 1.91           | 1.92           | 2.19           | 2.30           |
| 21. Excess ret. corp. bonds (in %)    | 1.09                     | 0.87           | 0.90           | 1.45           | 1.72           |
| Welfare                               |                          |                |                |                |                |
| 22. Aggr. welfare $\mathcal{W}^{pop}$ | 0.620                    | -0.38%         | -0.39%         | +0.28%         | +0.52%         |
| 23. Aggr. welfare $\mathcal{W}^{cev}$ | 0%                       | +16.20%        | +13.62%        | -24.24%        | -31.92%        |
| 24. Value function, B                 | 0.285                    | -2.50%         | -2.28%         | +2.84%         | +4.29%         |
| 25. Value function, S                 | 0.336                    | +1.43%         | +1.22%         | -1.89%         | -2.69%         |
| 26. DWL/GDP                           | 0.008                    | +18.89%        | +15.31%        | -18.68%        | -30.84%        |
| Size of the Economy                   |                          |                |                |                |                |
| 27. GDP                               | 0.978                    | +0.29%         | +0.25%         | -0.65%         | -1.09%         |
| 28. Capital stock                     | 2.199                    | +1.00%         | +0.87%         | -2.24%         | -3.72%         |
| 29. Aggr. Consumption                 | 0.621                    | -0.07%         | -0.05%         | +0.02%         | -0.00%         |
| 30. Consumption, B                    | 0.291                    | -2.71%         | -2.45%         | +2.70%         | +4.42%         |
| 31. Consumption, S                    | 0.343                    | +2.17%         | +1.98%         | -2.25%         | -3.75%         |
| Volatility                            |                          |                |                |                |                |
| 32. Mkt value corp debt gr            | 0.029                    | -1.62%         | -6.25%         | +10.61%        | +78.99%        |
| 33. Deposits gr                       | 0.049                    | -56.63%        | -56.80%        | -1.82%         | +86.46%        |
| 34. Dividend gr                       | 2.370                    | +7.71%         | +3.93%         | -30.67%        | -38.58%        |
| 35. Investment gr                     | 29.56%                   | -63.82%        | -63.66%        | -31.95%        | +40.37%        |
| 36. Consumption gr                    | 2.17%                    | -12.69%        | -14.68%        | -0.94%         | +27.08%        |
| 37. Consumption gr, B                 | 3.12%                    | -5.24%         | -6.17%         | -6.34%         | +8.37%         |
| 38. Consumption gr, S                 | 4.08%                    | -40.84%        | -40.96%        | -5.96%         | +45.08%        |
| 39. log (MU B / MU S)                 | 0.052                    | -25.85%        | -27.18%        | -9.44%         | +29.32%        |

Figure 13: Financial Recessions with Counter-cyclical Capital Requirements



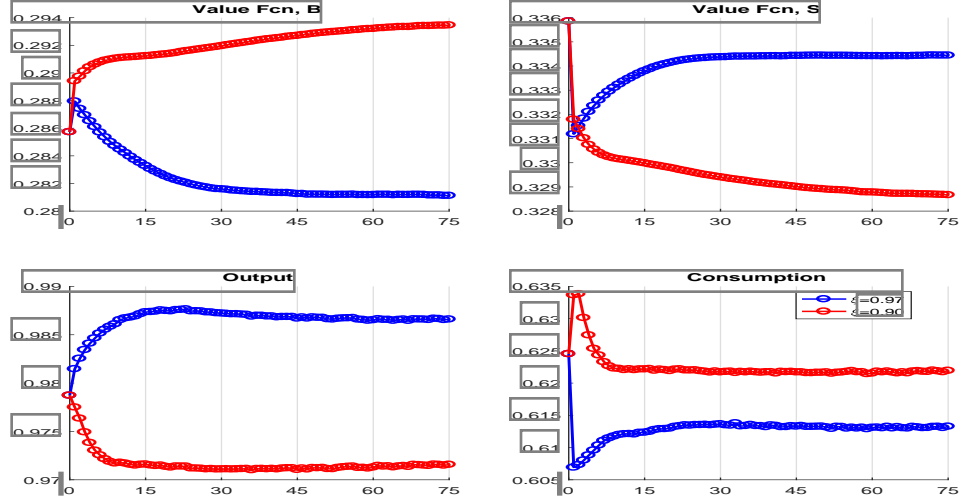
**Blue line:** responses to financial recession in economy with counter-cyclical capital requirements; **Black line:** responses to financial recession in benchmark economy. The underlying shocks in the two cases are identical.

capital constraint and higher equity adjustment cost), intermediaries effectively become more risk averse and require larger compensation for bearing risk, as evidenced by the large increase in the excess return on loans (row 21). However, increasing  $\sigma^I$  increases the franchise value of intermediaries (row 15), since it raises the risk premium while at the same time not requiring banks to raise more equity. Hence greater  $\sigma^I$  raises the return on bank equity, while lower  $\xi$  does not.

The overall welfare effects of larger equity adjustment frictions are comparable to the effects of tighter  $\xi$ . Locally the reduction in bankruptcies of producers and intermediaries dominates the reduction in the size of the capital stock, leading to a small aggregate welfare gain based on the population-weighted measure (row 22). Like tighter capital regulation, greater  $\sigma^I$  benefits equity owners of producers at the expense of savers.

The effects on macroeconomic volatility are nonlinear based on the same opposing forces that are at play with tighter capital constraints: since higher intermediation frictions increase the cost of debt funding, producers reduce the debt share of financing, which makes financial recessions less severe. At the same time, greater intermediation frictions hamper banks' ability to absorb aggregate risk through their balance sheet. The net effect, at least locally around the benchmark level of  $\sigma^I$ , is that aggregate investment and consumption growth become less volatile with lower equity adjustment costs ( $\sigma^I = 3$ ,  $\sigma^I = 4$ ), and risk sharing improves (MU ratio in row 40 becomes less volatile). Interestingly, this is also the case for slightly higher adjustment costs ( $\sigma^I = 6$ ). However, as we increase  $\sigma^I$  to 7, the impairment of banks' risk-bearing capacity dominates the reduction in risk: both aggregate consumption and investment growth are more volatile, and risk sharing between borrowers and savers becomes worse (row 39).

Figure 14: Transition Dynamics After Change in Capital Requirement



Blue line: lower capital requirement; red line: higher capital requirement

## D.7 Sensitivity of Macro-prudential Policy

In this appendix we study how sensitive the macro-prudential policy conclusions are to specific model ingredients/parameter constellations. In each experiment, we compare the effects of relaxing bank capital requirements by two percentage points versus tightening them by two percentage points, around the benchmark model. In other words, we study a four percentage point relaxation from  $\xi = .92$  to  $\xi = .96$ . The first column of Table 10 reports the results from this particular relaxation for the benchmark model. Firm loss rates and bank bankruptcies both increase, the size of the banking sector and the economy as a whole increase, investment volatility falls modestly while consumption growth volatility rises modestly, and aggregate welfare falls since the gains to the savers are insufficient to offset the losses to the borrowers. All these results are in line with our discussion in the main text.

Table 10: Sensitivity of Macro-prudential Policy Experiment

|                            | Benchmark | No bankruptcy | $\sigma^I = 0$ | No tax shield | Higher $\beta_B$ | Lower $\beta_S$ |
|----------------------------|-----------|---------------|----------------|---------------|------------------|-----------------|
| <b>Financial Fragility</b> |           |               |                |               |                  |                 |
| Loss rate                  | +0.32%    | +0.42%        | +0.07%         | +0.03%        | +0.15%           | +0.20%          |
| Bankruptcies               | +3.72%    | 0.00%         | +5.24%         | +3.01%        | +2.92%           | +3.21%          |
| <b>Size of the Economy</b> |           |               |                |               |                  |                 |
| GDP                        | +0.83%    | +0.72%        | +0.54%         | +0.21%        | +0.26%           | +0.30%          |
| Deposits / GDP             | +21.61%   | +24.24%       | +9.58%         | +5.72%        | +14.04%          | +14.17%         |
| <b>Macro Volatility</b>    |           |               |                |               |                  |                 |
| Investment vol             | -0.05%    | -0.33%        | +0.50%         | +0.20%        | +0.16%           | -0.04%          |
| Consumption vol            | +0.05%    | -0.15%        | +0.37%         | +0.13%        | +0.17%           | +0.02%          |
| MU vol                     | -1.65%    | -1.64%        | +0.54%         | -0.07%        | -0.19%           | -1.55%          |
| <b>Welfare</b>             |           |               |                |               |                  |                 |
| Borrower                   | -5.61%    | -3.16%        | -2.36%         | -3.69%        | -0.94%           | -4.74%          |
| Saver                      | +3.08%    | +2.14%        | +0.91%         | +1.95%        | +0.22%           | +2.53%          |
| Aggregate                  | -1.01%    | -0.35%        | -0.59%         | -0.69%        | -0.30%           | -0.87%          |

The other columns of Table 10 study the same change in macro-prudential policy in a model without bankruptcy option (column 2), in a model without equity issuance costs (column 3), in a model without tax shield for banks (column 4), in a model with more patient borrowers (column 5,  $\beta_B$  increases by 0.15), and less patient savers (columns 6,  $\beta_S$  decreases by 0.15). The latter two changes decrease the wedge between the patience of borrowers and savers and reduce the need for intermediation services.

The main finding is that the aggregate welfare changes from macro-prudential policy are robust to these parameter variations. In all experiments, welfare decreases in response to the four percentage point increase in maximum allowable financial sector leverage from 92% to 96%. The range of estimates is -0.35%, when banks are not allowed to fail (and hence cannot be bailed out), to -1.01%. In all cases, we see more fragility in the form of higher corporate loss rates and higher bank bankruptcies (except of course when banks are not allowed to go bankrupt). When it is easier and cheaper for shareholders to recapitalize banks, the size of the banking sector is naturally less sensitive to a change in macro-prudential regulation. The basic trade-off between a larger banking sector and size of the economy and more financial fragility is also present in every model. The quantitative slope of that trade-off depends on the model details. The only results that are more fragile are those on macro-economic volatility. That should not come as a surprise since, even in the benchmark model, macro-economic volatility is non-monotonic in  $\xi$ . Risk sharing tends to improve (MU vol falls) as macro-prudential policy is relaxed, reflecting the financial sector's improved ability to absorb aggregate risk when it is larger. The one exception is when bank equity can be costlessly adjusted, which is also the model when the banking sector size changes the least and macro-economic volatility increases the most.