NBER WORKING PAPER SERIES

DOES SAVING ANTICIPATE DECLINING LABOR INCOME? AN ALTERNATIVE TEST OF THE PERMANENT INCOME HYPOTHESIS

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Working Paper No. 1805

NATIONAL BUREAU OF ECONOMIC RESEARCH

1050 Massachusetts Avenue

Cambridge, MA 02138

January 1986

Lam grateful to Alan Blinder and Angus Deaton for generously supplying me with data, to Peter Rathjens for assistance with the data, and to Ben Bernanke, Alan Blinder, Angus Deaton, Doug Holtz-Eakin, Robert Shiller and participants in seminars at the University of Pennsylvania and the NBER Summer Institute workshop on intertemporal asset pricing for helpful comments. I am responsible for any remaining errors. I acknowledge support from the National Science Foundation, grant SES-8511070. The research reported here is part of the NBER's research program in Economic Fluctuations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

Does Saving Anticipate Declining Labor Income?

An Alternative Test of the Permanent Income Hypothesis

ABSTRACT

The permanent income hypothesis implies that people save because they rationally expect their labor income to decline; they save "for a rainy day". It follows that saving should be at least as good a predictor of declines in labor income as any other forecast that can be constructed from publicly available information.

The paper tests this hitherto ignored implication of the permanent income hypothesis, using quarterly aggregate data for the period 1953-84 in the U.S. A vector autoregression for saving and changes in labor income is used to generate an unrestricted forecast of declines in labor income. In the VAR, saving Granger causes labor income changes as one would expect if the PIH is true. The mean of the unrestricted forecast is far from the mean of saving, but the dynamics of the two series are quite similar.

The paper presents both formal test statistics and an informal evaluation of the "fit" of the permanent income hypothesis. By contrast with most of the recent literature, the results here are valid when income is nonstationary.

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DOES SAVING ANTICIPATE DECLINING LABOR INCOME?

AN ALTERNATIVE TEST OF THE PERMANENT INCOME HYPOTHESIS

I. Introduction

This paper reexamines the relationship between consumption and income at the aggregate level. The starting point for the paper is the old debate between the view that consumption is determined by current income in the manner suggested by Keynes, and the view that consumption is related to permanent income and is therefore smooth and noncyclical. Recently this debate has been revitalized by the application of rational expectations econometric techniques. Hall [1978] argued that the permanent income hypothesis (PIH) under rational expectations implies approximate unpredictability of consumption changes, since agents never plan to change consumption and in fact change it only in response to news about future income. Hall's idea has generated a large literature including papers by Flavin [1981], Hayashi [1982], Muellbauer [1983], Bernanke [1985] and others.

The papers which follow from Hall typically work with changes in consumption and pursue two objectives: to test the rational expectations version of the permanent income hypothesis, and if it is statistically rejected, to characterize its failure in economic terms. The implication which Hall drew from the PIH model is tested by regressing consumption changes on lagged variables and testing for joint significance of the coefficients. The same coefficients are used to characterize the failure of the model; Flavin [1981], for example, describes significant coefficients on lagged income as "excess sensitivity" of consumption to income. A few papers, such as Bernanke [1985], also try to interpret the contemporaneous correla-

tions of consumption changes with econometric estimates of innovations in other variables, but this line of research is hampered by the fact that the true innovations (to agents) are unobservable.

In this paper I propose a new way to test the PIH model and characterize any failure. I start from the obvious point that random walk behavior of consumption is only one implication of the PIH. A series can follow an approximate random walk and yet not be determined by permanent income. Tests which use only the random walk implication may not be powerful and will not yield a precise characterization of the strengths and weaknesses of the PIH model.

The approach taken here is to construct an econometric framework in which an alternative restriction of the PIH can be imposed and tested. This requires a tight specification of the model, and I use Flavin's [1981] formulation. However the basic intuition of the approach is almost as simple as that of Hall. If the PIH model is true, consumption is proportional to permanent income; it thus tends to be above current income when current income is relatively low and expected to rise, and below current income when current income is expected to fall. Put another way, dissaving anticipates rising income and saving anticipates falling income. People save "for a rainy day".

There are some subtle problems which arise in applying this idea.

First, it is important to distinguish between labor income and capital income. When saving occurs, wealth is increased and future capital income rises, partially offsetting the anticipated decline in labor income; this is the mechanism by which the random walk path of consumption is maintained. Sargent [1978] ignored the endogeneity of capital income and test-

ed an incorrect PIH formulation in which consumption does not follow a random walk, as pointed out by Flavin [1981]. In this paper I distinguish labor and capital income throughout.

Secondly, valid statistical tests must be carried out on stationary time series. The existing literature has often detrended the data on income and consumption before proceeding to formal analysis (e.g. Flavin [1981]). However Mankiw and Shapiro [1984], followed by Deaton [1985] and Nelson [1985], have recently pointed out that this may lead to spurious "cyclical" behavior of the residual and rejection of the PIH model if in fact income and consumption are stationary in first differences. In this paper I maintain the assumption that labor income is stationary in first differences. I derive the time series properties of consumption and capital income which are implied by the PIH given this behavior of labor income. It turns out that under the PIH, a linear combination of income and consumption - which can be thought of as saving - is stationary in its level even though neither income nor consumption are stationary. This observation can be used, along with the theory of "cointegrated" vectors in time series analysis, to help construct a test of the model.

The organization of the paper is as follows. In the next section I discuss the formal structure of the PIH model and show how tests of different implications are related to one another. In section 3 I summarize the theory of cointegrated processes and its implications for vector autoregressive (VAR) tests of the PIH. I also relate VAR tests to single-equation regression tests. In section 4 I describe the data and empirical results. The last section contains some conclusions.

II. The Permanent Income Hypothesis

The three variables which are studied in this paper are real capital income yk, real labor income yl, and real consumption c. All of these variables are measured as per capita aggregates. I will assume that yl is stationary in first differences, and use the restrictions of the PIH to characterize the time series behavior of capital income and consumption.

capital income is defined as the Hicksian income generated by real nonhuman wealth W_t . That is, capital income is the amount that can be consumed each period out of nonhuman wealth without changing its expected real value next period. The first assumption of the PIH model is that the expected real interest rate is constant at some level r. If this is the case $yk_t = rW_t$, wealth evolves according to $W_t = (1+r)W_t + yl_t - c_t + \eta_t$, and capital income obeys

(1) $yk_{t} - (1+r)yk_{t-1} - \frac{r[y_{t-1} - c_{t-1}]}{t-1} = r\eta_{t}$

where the error term η_t represents unanticipated capital gains and is unforecastable at time t-1. In general the conditional variance of η_t will be positively related (perhaps proportional) to the level of wealth ψ_{t-1} . Following Flavin [1981], I write the PIH model of consumption behavior

as

The timing convention in equation (1) is that of Flavin [1981]. In each period, the timing of events is as follows. A shock n to wealth occurs, and then wealth for the period is measured. Interest on wealth is paid over the period; labor earnings are received and consumption chosen at the end of the period. No interest is paid between the consumption decision and next period's wealth shock.

(2)
$$c_t = \delta[yk_t + (r/(1+r)) \sum_{i=0}^{\infty} (1/(1+r)) E_t y I_{t+i}]$$

and human wealth; I will assume that the proportionality factor %≤1. yk is the Hicksian income from nonhuman wealth, and the second term in square brackets is the Hicksian income from human wealth or r times the present discounted value of expected labor income. An error term representing "transitory consumption" may also be added to equation (2); the error is omitted for expositional simplicity at this stage.

The microfoundations of equation (2) are questionable, as has been pointed out by Hayashi [1982], King [1983], Deaton [1985] and others. Ignoring difficulties with aggregation, it is clear that (2) can describe an agent's behavior only if the agent is effectively infinitely lived. Even then, (2) is the solution of an optimization problem only under very special circumstances. First, if the variance of \$\eta_{\text{t+1}}\$ in equation (1) is constant and unrelated to the level of wealth, and an agent has quadratic utility with subjective rate of time discount equal to the riskfree market interest rate, then (2) holds with \$\eta\$ equal to unity. Secondly, if there is no uncertainty about future capital or labor income, and an agent has constant relative risk aversion utility, then (2) holds with \$\eta\$ determined by the relation between subjective time preference and market interest rates.

Although the conditions for (2) to hold exactly are very restrictive, it has often been used because it is a simple and tractable representation of the forward-looking consumption behavior postulated by the PIH, and be-

If the subjective rate of time discount does not equal the riskfree market interest rate, then consumption follows a random walk with drift, but (2) does not hold.

conditions. For example, Hayashi [1982] argues that (2) is a good approximation to the solution of the maximization problem with constant relative risk aversion utility under uncertainty. Accordingly I treat the system of equations (1) and (2) as representing the PIH model, while recognizing its limitations.

Rather than work directly with equation (2), I transform it as follows. Define $s = y - c_t/x$, where $y = \frac{yk_t + yl_t}{t}$ or total disposable in come. s_t is a measure of saving when t=1, and for simplicity it will be referred to as saving throughout the rest of this paper. Equation (2) can be rearranged so that it becomes a statement about saving.

$$= \frac{\sum_{i=1}^{\infty} (1/(1+r))^{1} E_{t} \Delta y 1}{i=1}$$

where A denotes a standard backward difference

Equation (3) says that saving equals the expected present value of future declines in labor income; this is the "saving for a rainy day" feature of the PIH model. It follows from (3) that

(4)
$$\frac{1}{t} - \frac{\Delta yl}{t} - \frac{1+r}{t-1} = -r\epsilon$$

Hayashi favors the use of a different discount rate in equations (1) and (2), which I do not allow here.

Even when $\delta=1$, note that the change in real wealth $\Delta W=s+\eta$. Unanticipated capital gains are distinguished from saving in this framework; thus in empirical work a time series similar to NIPA saving is more appropriate than series of the sort discussed by Auerbach [1984], which attempt to measure the change in real wealth.

where
$$\varepsilon$$
 = (1/(1+r)) Σ (1/(1+r)) $[E_t yl_{t+i} - E_{t-1} yl_{t+i}]$ $[i=0]$

is the unforecastable revision from t-1 to t in the expected value of human wealth.

Equations (1) and (4) neatly summarize the testable implications of the PIH. If we define a vector $\mathbf{x} = [\mathbf{y}\mathbf{k} \ \mathbf{y}\mathbf{l} \ \mathbf{c}]^{\dagger}$, the equations state that two linear combinations of \mathbf{x}_{t} and \mathbf{x}_{t-1} are unforecastable at time t-1. However recent empirical work on the PIH has focused not on (1) and (4), but on a linear combination of these equations. Subtracting (4) from (1) and using the definition of \mathbf{s}_{t} , we obtain the familiar result that consumption follows a first-order Markov process, or a random walk when %=1:

$$(5) c_{1}/\delta - [1+r(1-\delta)]c_{1-1}/\delta = r[\eta_{1}+\varepsilon_{1}]$$

Equation (5) is an appealing implication of the PIH. It captures the "consumption-smoothing" aspect of the model, which is likely to be a feature of optimal behavior even in circumstances where (1) and (2) fail. It can also be tested without having data on all elements of the vector x. However there are a number of reasons why it is worth going beyond exclusive concentration on (5).

First, the unpredictability of quasi-differenced consumption is only one implication of the PIH model. A data series might be unpredictable in quasi-differences yet not obey the model. This is particularly serious since many economic time series follow first-order univariate Markov processes with roots close to unity; obvious examples are stock prices and

iong-term interest rates. By contrast, unpredictability of the left hand side of (4) guarantees that (2) holds. More generally, since equations (1), (4) and (5) are linearly dependent, unpredictability of the left hand sides of any two of them establishes that a vector x obeys the PIH model. Some researchers have noted the partial nature of a test of equation (5). But the typical response has been to estimate proxies for x and \(\eta_t\) from univariate or other limited-information forecasting equations for income, and then to regress quasi-differenced consumption on these proxies. Since the proxies measure \(\eta_t\) and \(\eta_t\) with error, the coefficients in the second-stage regression are biased downwards; however if they are found to be larger than the theoretical value of x, this is taken as evidence against the model. Bernanke [1985] applies the above method to a model of durables and nondurables consumption. The use of equations (1) and (4) is an attractive alternative to Bernanke's approach, since one can test equality (zero) restrictions rather than merely inequality restrictions.

A second reason why it is unsatisfactory to concentrate solely on (5) is that an important time series property of the vector \mathbf{x} is not revealed by this equation. Consider the version of the PIH with $\mathbf{X}=1$. I have assumed that \mathbf{yl}_{t} is stationary in first differences. Then equations (1), (3) and (5) imply that \mathbf{yk}_{t} and \mathbf{c}_{t} are also stationary in first differences, but $\mathbf{s}_{t} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \mathbf{x}_{t}$ is stationary in its level. Intuitively, this is because saving is a discounted present value of expected changes in labor income; these changes are stationary, so saving is also.

This statement assumes that there are no "bubbles", so that difference equations like (4) can be solved forward to give expressions like (3). Unpredictability is meant here in a population sense, and with respect to the whole information set available to agents. In practice the model can be tested only with a limited information set and a finite sample, so it can not be directly verified.

A vector with the property that a linear combination of its elements is stationary in its level, even though the elements themselves are stationary only in differences, is an example of a cointegrated vector. Such vectors have a number of useful properties which are discussed in the next section.

A third problem with exclusive use of (5) is that it is hard to assess the economic significance of a statistical rejection of unforecastability in (5). Equation (3), by contrast, can be used to characterize the "fit" of the PIH model. One can compare the historical movements of saving with those of an unrestricted forecast of declines in labor income. This is better done with saving than consumption, because saving is stationary while consumption is not.

If %<1, s is still stationary, but yk and c are explosive rather than stationary in first differences. x no longer satisfies the formal definition of cointegration, but has many of the same properties. Both cases are analyzed in the next section.

III. Cointegration and Vector Autoregressions

In this section I summarize the theory of cointegrated processes, and show how it applies to the PIH model. I devote most attention to the PIH with %=1, discussing the case %<1 at the end of the section.

Definition (Granger and Engle [1985]). A vector \mathbf{x}_t is said to be cointegrated of order \mathbf{d} , \mathbf{b} , denoted \mathbf{x}_t CI(\mathbf{d} , \mathbf{b}), if (i) all components of \mathbf{x}_t are integrated of order \mathbf{d} (stationary in \mathbf{d} 'th differences), and (ii) there exists at least one vector $\mathbf{\alpha}$ ($\neq 0$) such that $\mathbf{z}_t = \mathbf{\alpha}' \mathbf{x}_t$ is integrated of order \mathbf{d} - \mathbf{b} , \mathbf{b} >0.

As noted at the end of the previous section, if the PIH holds with X=1 and changes in labor income are stationary, then consumption, capital and labor income are CI(1,1). Variables in a CI(1,1) vector share a common stochastic trend (a unit root), while diverging from one another in the short run (the divergence is stationary). This sort of behavior has been postulated for consumption and income by Davidson, Hendry, Srba and Yeo [1978], Davidson and Hendry [1981], and Granger and Engle [1985]. However these authors thought of cointegration as arising from disequilibrium adjustment of consumption to income, and used the idea to estimate relatively unrestricted consumption functions rather than to test the tight restrictions of the PIH.

Cointegrated systems have two important and unusual properties. These concern the estimation of unknown elements of the vector α , and the existence of vector time series representations for the cointegrated variables. Both properties turn out to be important in the context of the PIH.

The vector α is called the cointegrating vector; in the present example it is unique up to a scalar normalization, and is proportional to [1 1 -1]. Stock [1984] proves that if there is a single unknown element of α , a variety of methods provide estimates with a standard error which goes to zero at a rate proportional to the sample size T (rather than \sqrt{T} as in ordinary cases). The reason for this is that asymptotically all linear combinations of the elements of x_{τ} other than α and x_{τ} have infinite variance.

The practical implication is that an unknown element of α may be estimated in a first-stage regression and then treated as known in second-stage procedures, whose asymptotic standard errors will still be correct. As the PIH is stated above, if δ is known to equal 1 all elements of α are known a priori. However in one of the empirical applications of section 4 I will use data only on a subset of consumption c^* which is assumed to be a constant fraction of c_t : $c_t = \lambda c^*$. The scale factor λ must be estimated and Stock's theorem enables this to be done straightforwardly. It seems likely that Stock's theorem can also be extended to cover the case where δ is unknown, as discussed further below.

The second important property of cointegrating vectors arises when we consider a vector autoregressive (VAR) test of the PIH. An appealing way to evaluate the PIH is to set up a VAR, using stationary variables, and then to test cross-equation restrictions on the coefficients. Because x_t is cointegrated, the choice of stationary variables is critical. The most obvious choice, $\Delta x_t = |\Delta y_t| \Delta y_t \Delta y_t$, is a poor one for two reasons. First, the full set of restrictions of the PIH cannot be imposed on Δx_t since the PIH has implications for the level of c_t as well as its change. Even more seriously, the cointegration of x_t implies that no invertible

representation, exists for Δx . The reason is that if there were an invertible VMA representation, no linear combination of x could be stationary.

More formally, write $\Delta x = K(L)s_t = Is_t + K_s = I$ Invertibility requires that all roots of K(z) lie outside the unit circle, so $K(1) = I + K_s = I$ must be nonsingular. The variance covariance matrix of $s_t = I$ Ω , must also be nonsingular. Now if the variance of $\alpha' x$ exists, it will be given by

$$\frac{\mathbb{V}\operatorname{ar}(\alpha^{\dagger}x_{1}) = \sum_{i=0}^{\infty} \alpha^{\dagger}C_{i}\Omega_{i}^{c}\alpha^{\dagger}\alpha^{c} \quad \text{where } C_{i}}{i=0} = I + K_{1} + \ldots + K_{i}.$$

Since the limit of C as $i \to \infty$ is K(1), the terms in the summation above approach a nonzero limit and the variance of $\alpha^{\dagger}x$ will not be finite.

The importance of the above discussion is that if an economic theory imposes cointegration on a set of nonstationary variables, simple first differencing of all the variables does not lead to a well-behaved system for statistical modelling. This point is discussed further by Campbell and Shiller [1985] in the context of the term structure of interest rates and the work of Sargent [1979]. Fortunately, there is a simple solution to the difficulty which is to include α x in a VAR along with a subset of the elements of Δ x. An equation which relates the change in an element of x to its own lags and lags of α x is called an error-correction model for that element of x.

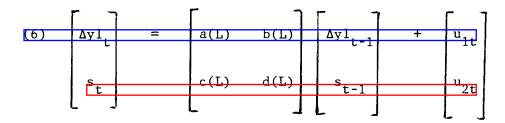
Error-correction models have been estimated for consumption by Davidson et al. [1978], Davidson and Hendry [1981] and Granger and Engle [1985].

Their equations predict changes in consumption using lagged changes in income and consumption, and lagged deviations of consumption from income

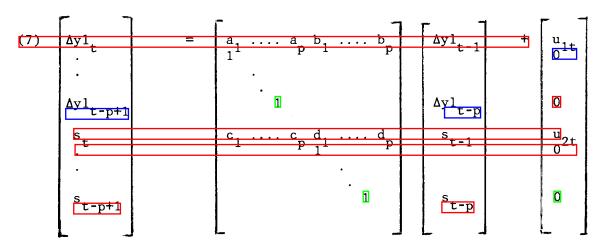
(roughly, lagged saving). Such a specification is motivated by a disequilibrium view of consumption behavior. The PIH, by contrast, implies that all coefficients should be zero in an equation predicting consumption change; the nontrivial error-correction equation describes labor income rather than consumption.

The above analysis suggests the use of a VAR with s and Ayl included. This system is well behaved in general, and all the restrictions of equation (3) can be imposed on it. I now go on to discuss the implications of the PIH for such a VAR.

First consider estimating



where the polynomials in the lag operator a(L), b(L), c(L) and d(L) are all of order p. (6) can be stacked into a first-order system



which can be written more succinctly as z = Az + v. The matrix A is called the companion matrix of the VAR. For all i,

$$E[z_{+}|H] = A^{i}z_{+},$$

where H is the information set $\{z, z, \ldots\}$. I assume that H is a subset of agents information set I; thus the VAR projects onto a limited information set.

It is straightforward to prove that under the PIH, s must Granger cause Δyl_t unless s is itself an exact linear function of current and lagged Δyl_t . For suppose that s does not Granger cause Δyl_t . Then $E[\Delta yl_{t+1}|H_t] = E[\Delta yl_{t+1}|\Delta yl_t, \Delta yl_{t-1}, \ldots]$ for all i. We know from (3) that

$$\begin{array}{c|c}
 & \infty \\
\hline
E[- \sum_{i=1}^{\infty} \frac{(1/(1+r))^{i}}{\Delta y_{1}} \frac{\Delta y_{1}}{t+i} \frac{|H_{t}|}{t} = E[s_{t}|H_{t}]
\end{array}$$

since H_t is a subset of I_t . Thus if s_t does not Granger cause Δyl_t , $E[s_t|H_t] = E[s_t|\Delta yl_t, \Delta yl_{t-1}, \ldots]$. But $s_t = E[s_t|H_t]$ so s_t is an exact linear function of current and lagged Δyl_t .

The intuitive explanation for this result is that s is an optimal to forecast of future declines in labor income, conditional on agents' full information set. s will therefore have incremental explanatory power for future labor income if agents have information useful for forecasting labor income beyond the history of that variable. If agents do not have such information, they form s as an exact linear function of current and lagged labor income

Equation (3) can be projected onto the information set H, and written as a set of restrictions on the VAR companion matrix A:

where g' and h' are row vectors with 2p elements, all of which are zero except for the p+1'st element of g' and the 1st element of h'.

The restrictions of equation (8) appear to be highly nonlinear crossequation restrictions of the type described by Hansen and Sargent [1981] as
the "hallmark" of rational expectations models. However it turns out that
equation (8) can be simplified so that its restrictions are linear and easily interpreted. The infinite sum on the right hand side of (8) is just

Fh'(1/(1+r))A [I-(1/(1+r))A] -1,

and postmultiplying (8) by [I-(1/(1+r))A], we obtain

(9) g'[1-(1/(1+r))A] = -h'(1/(1+r))A

Using the structure of the matrix A, laid out in equation (7), we can write out the restrictions of (9) on individual coefficients: $a_1 = c_1$, ..., $a_1 = c_1$, ..., $a_1 = c_1$, $a_1 = c_1$, $a_2 = c_2$, $a_1 = c_2$, $a_2 = c_3$. There is exactly one restriction for each column of A. To interpret these restrictions, we subtract the equation of the VAR from the standard equation to get $a_1 = c_2$ and $a_2 = c_3$ and $a_3 = c_4$ and $a_4 = c_3$ and $a_4 = c_4$ and $a_4 = c_4$ and $a_5 = c_5$ and $a_5 =$

The analysis can easily be extended to the case where there is a "transitory consumption" error in equation (2), so long as this error is

assumed to be orthogonal to all lagged information (including its own lagged values). This is a natural identifying assumption for transitory consumption. Then, although (3) and (8) no longer hold, we have

$$\underbrace{\text{(3)} \cdot \underbrace{E_{s}}_{t t+1} = -\underbrace{\Sigma}_{i=1}}^{\infty} \underbrace{\frac{(1/(1+r))}{E_{t}} \underbrace{E_{t} \Delta y 1}_{t+1+i}}_{i}$$

since the expectation of next period's transitory consumption is zero.

(3) can be tested by regressing $s - \Lambda y 1 - (1+r)s$ on information lagged two periods, or by testing the VAR restrictions

Comparing (8) and (8), it is clear that (8) can hold when (8) does not, only if the matrix A is singular (since otherwise (8) can be postmultiplied by the inverse of A to yield (8)).

The discussion above needs some modification if the PIH holds with 6<1. Then from equations (1) and (5), both yk and c are explosive rather than stationary in first differences. The vector x no longer satisfies the formal definition of cointegration.

However x still possesses the key property that a linear combination of its elements is stationary, when none of these elements is individually stationary. It seems likely that that linear combination can still be estimated precisely since it is still true that it is the only linear combination with asymptotically finite variance. In fact, one might expect an estimate of α to converge even faster in the explosive case.

The variables Ayl and s are still stationary when %<1. Thus one can test equation (4), or estimate the VAR system (6), and obtain well-behaved coefficient estimates and test statistics. However Ac and Ayk are not stationary, and inclusion of these variables in a VAR would jeopardize statistical inference.

As the PIH is written in section 2, the parameter % can be estimated from the cointegrating vector, and then used to obtain stationary quasidifferences of c and yk. However in one of the applications in the next section, I observe only a subset of consumption c^* such that $c = \lambda c^*$. With data only on c^* , the cointegrating vector identifies λ/X rather than 8. s can still be constructed from this information, but the stationary quasi-differences of c and yk cannot. Accordingly in the next section 1 confine my attention to the variables s and λyl , which are known to be stationary whether X=1 or X<1. I conduct a single-equation regression test of equation (4), and then use the VAR representation (6) to conduct an informal comparison of s with the optimal unrestricted forecast of declines in labor income.

Both of these procedures require that the expected real interest rate r be known or estimated. For simplicity I treat r as known, setting I/(1+r)=0.99 or r=4.04% on an annual basis. By inspection of (4), or the restrictions of (9) on individual coefficients, one can see that the test procedure is not very sensitive to a small error in the choice of r.

An important preliminary step is to decide on the appropriate order please for the vector autoregression. I use the Akaike Information Criterion (AIC) to choose p, that is I pick p to minimize [- In likelihood + number of parameters] in the vector autoregression. Sawa [1978] has argued that

the AIC tends to choose models of higher order than the true model, but states that the bias is small when p < T/10 as it is here.

One final technical point concerns the estimation of the variance-covariance matrix of the coefficient estimates in the VAR. The usual formula

-1 Σ & (X[†]X)

where I is the variance-covariance matrix of the equation residuals. When there is conditional heteroskedasticity, this estimate is no longer consistent and should be replaced by

-1 -1 (X⁺X) X⁺VX (X⁺X)

where V is a diagonal matrix with squared residuals on the diagonal (White [1984]). In this paper the variables are defined in levels rather than in logs so heteroskedasticity is potentially important and I report White standard errors throughout. In practice these are little different from the conventional standard errors, indicating that the equation error variances are not highly correlated with s_t or Δyl.

IV. Data and Empirical Results

The data used in this paper are taken from Blinder and Deaton [1985].

All data are seasonally adjusted quarterly series for the period

1953:2-1984:4, and are ultimately taken from the National Income and Product Accounts (NIPA). However Blinder and Deaton make several adjustments

to NIPA definitions and also break down real disposable income into capital

and labor components. Blinder and Deaton describe their transformations in

detail; here I merely provide a brief summary.

1) The series constructed are disposable total income, disposable labor income, total consumption and consumption of nondurables and services.

Disposable capital income is simply the difference between the first two of these.

- 2) Blinder and Deaton remove the 1975 tax rebate from the disposable income series. This can be justified on the grounds that the rebate was unanticipated and was not generated by the same stochastic process as the rest of the data. If the rebate is included, it tends to distort estimates of the time series process for income.
- 3) Consumer interest payments to business are subtracted from NIPA disposable income, thus treating these payments symmetrically with business interest payments to consumers.
- 4) Personal nontax payments to state and local governments are treated as part of both disposable income and consumption, since they include such things as state college tuition and state hospital payments.
- 5) Expenditures on clothing and shoes are treated as expenditures on durables.

6) The breakdown of disposable income into capital and labor components is carried out by completing the NIPA breakdown. Proprietors' income and personal income taxes, which are not broken down in NIPA, are attributed to labor and capital according to their overall factor shares, and social insurance contributions are deducted from labor income.

7) All series are on a real per-capita basis, divided by total population and a consumer spending deflator which is adjusted from NIPA in the same manner as consumption.

8) In this paper, all series are in units of thousand dollars.

The remainder of this section presents an analysis of the Rlinder-Dealton data, using the methods described in the previous section. All exercises are repeated twice: once for consumption of nondurables and services, which will be written c_t^* , and once for total consumption c_t^* . Most previous work on the PIH has used nondurables and services consumption, on the ground that this series is most likely to obey the random walk restriction of the model. However in the present context the c_t^* measure has the disadvantage that it is only a component of consumption; to use it, one must postulate that total consumption is unobservable and related to c_t^* as $c_t^{-\lambda}c_t^*$, where λ is estimated from the cointegrating vector. In order to ensure that empirical results are not sensitive to this procedure, total consumption is also used in this paper.

Blinder and Deaton report that the share of nondurables and services in total consumption expenditure has displayed a secular decline over the sample period. This casts some doubt on the practice of using nondurables and services consumption as a proxy for the total; nevertheless I follow this tradition and estimate a constant scale factor.

It begin in Table 1 by running two preliminary regressions for each consumption measure. These are designed as alternative ways to estimate the cointegrating vector, that is the parameter λ/δ for nondurables and services consumption and $1/\delta$ for total consumption.

Both Table 1 regressions are analyzed in Stock [1984]; they provide estimates of the cointegrating vector which can be treated as known in further analysis. The first regression is simply of total income y on c* or t* or t

Granger and Engle [1985] show how the residual from the first type of regression can be used to conduct tests of the hypothesis that two series are not cointegrated. They recommend the use of an "Augmented Dickey-Fuller" regression (Dickey and Fuller [1981]) in which the change in the residual is regressed on one lagged level of the residual, and one or more lagged changes. The t statistic on the level variable is biased upwards relative to the t distribution, but Granger and Engle provide significance levels based on a Monte Carlo study. If the t statistic is higher than 2.84, the hypothesis of no cointegration can be rejected at the 10% level,

Which the change in consumption is the dependent variable. As already noted, this is not a good way to identify the cointegrating vector, since under the PIH with X=1 all coefficients should be zero.

The Monte Carlo results are based on 10,000 replications of 100 observations of independent random walks, with 4 lagged residual changes included in the test. This setup is close to the one here, so Granger and Engle's significance levels should be fairly accurate.

while if it is higher than 3.17, the hypothesis is rejected at the 5% level.

The results of Table 1 can be summarized as follows. When nondurables and services consumption is used, the parameter λ/δ is estimated at 1.495 by the levels regression and 1.517 by the error-correction regression. When total consumption is used, the parameter $1/\delta$ is estimated at 1.062 by the levels regression and 1.083 by the error-correction regression. This reflects the fairly constant U.S. savings rate of a little over 6% in the sample period. If one is willing to combine the two sets of estimates, the limplied share of nondurables and services consumption in the total is 71%. The hypothesis of no cointegration is rejected at the 10% level but not at the 5% level for both consumption series.

It is encouraging that the two types of regression in Table 1 gives such similar estimates of the cointegrating vector. In the results which follow, I use the estimate from the levels regression, but the choice makes no difference to any of the statistical inference.

The next step in the analysis is to construct a saving series as $s^*_t = (v_t - 1.062c^*_t)$ or $s_t = (v_t - 1.495c_t)$, and to include this with the change in labor income in a single-equation regression test. The Akaike Information Criterion gives very similar results for choice of lag length in both cases, but these results are rather ambiguous. The value of the criterion is almost identical for a 1-lag model and for a 5-lag model, with much larger values for all other specifications. This reflects the fact that

Stock [1984] recommends the estimate from the error-correction regression, but Granger and Engle |1985| argue for the levels regression. I use the levels regression because it delivers an estimate under both the null (consumption change unpredictable, income change predictable), and an important alternative (income change unpredictable, disequilibrium error-correction behavior of consumption).

in labor income, whereas second, third and fourth lags do not contribute.

Blinder and Deaton [1985] argue for a first-order representation of these data, and this has obvious appeal. It is parsimonious and has more plausible dynamics than a model with large first and last lags and insignificant lags in between. However for completeness I conduct tests in both a first-order and a fifth-order model; the results are sensitive to this choice.

Table 2 presents a single-equation regression test of the PIH. The "saving for a rainy day" hypothesis is tested by forming $s_{t-\Delta y}^* - (1+r)s_{t-1}^*$ or $s_{t-\Delta y}^* - (1+r)s_{t-1}^*$, and regressing this variable on 1 or 5 lags of s_{t}^* or s_{t}^* and Δy_{t}^* . Under the PIH, all coefficients should equal zero. Two test statistics are calculated for each model; the first restricts all coefficients including the intercept (equivalently, the mean of s_{t}^* or s_{t}^*), while the second leaves the intercept free and restricts the other coefficients. Table 2 also shows individual coefficient values for the 1-lag model.

It is clear from the table that the PIH can be rejected at extremely high levels of confidence if it is taken to restrict the mean level of saving as well as the dynamics of saving. All the test statistics which restrict the mean are significant at the 0.001% level. Under the PIH, the mean of s* and s should be -(1/r) times the mean of Δyl_t . The mean of s* is indeed negative, while the mean of Δyl_t is positive, but the ratio is too small; mean s* is -0.310 and the mean change in labor income is

One might suspect that the effect of the fifth lag is due to seasonality which remains in the "deseasonalized" NIPA data. However when I included seasonal dummies in the estimated system the estimated coefficients and test statistics were almost unchanged; those results are therefore not reported.

0.014. For s, the problem is even worse because its mean is positive at

The PIH fares somewhat better if the intercept restriction is dropped.

In the 1-lag version, the PIH is rejected at the 1.2% level for s* and at the 0.1% level for s. The rejection is stronger, at the 0.1% and less than 0.001% levels respectively, in the 5-lag version of the PIH.

The coefficients from the 1-lag regression give some indication of the quantitative importance of this statistical rejection. The coefficients are small, particularly on lagged s* and s at -0.013 and -0.035 respectively. Even the lagged Ay coefficients are only -0.203 and -0.324. This suggests that it is worth examining informal measures of the "fit" of the model as well as formal statistical tests.

In Table 3 the single-equation regression tests are repeated, allowing for transitory consumption. The same dependent variable as in Table 2 is regressed on s* or s and Ay, lagged twice rather than once. The extra lag makes almost no difference if the intercept restriction is included, or in the fifth-order model. However the dynamic restrictions of the PIH with transitory consumption cannot be rejected at even the 10% level in the first-order model.

Table 4 presents estimates of a demeaned VAR system like equation (6). Coefficient estimates are reported for the 1-lag model, and summary statistics for both the 1-lag and 5-lag models. Although the VAR estimates yield exactly the same test statistics as are reported in Table 2, they can be used to characterize the data and the fit of the permanent income hypothesis.

A striking result for all models is that saving Granger causes changes in labor income at standard significance levels, and the first coefficient is negative. This negative effect is what one would expect if the PIH cross-equation restrictions hold, and the own coefficient of lagged saving on current saving is less than (1+r). Intuitively, the PIH claims that saving occurs because labor income is expected to decline in the future, and indeed a labor income decline follows in the next quarter. Own lags are also significant for saving and labor income changes, but labor income changes Granger cause only stand not s₊. 12

The restrictions of the PIH on the estimated VAR are that all coefficients in the cients in the first equation equal the corresponding coefficients in the second equation, except for the coefficients on once lagged saving, which must differ by (1+r). The deviations of the estimated coefficients from these restrictions are just the coefficients reported in the regression test of Table 2.

The VARs can be used to construct the optimal unrestricted forecast of declines in labor income, conditional on the information set H_t. Table 4 also presents the standard deviation of this forecast, the standard deviation of saving, and the correlation between the two. If the PIH is correct, the standard deviations should be the same and the correlation should be unity. It is clear from the table that the standard deviations are quite close (although saving tends to vary a little less than the optimal forecast). The correlations are extremely high for the 1-lag model, at 0.995 for s*t and 0.960 for st. In the 5-lag model, however, the correlation falls to 0.449 for s*t and is actually negative at -0.480 for st.

Note that the PIH allows but does not require saving to be Granger caused by labor income changes.

One can get a good feel for the fit of the PIH by plotting saving and the optimal unrestricted forecast of labor income declines against time. Four plots of this sort follow Table 4, for the 1-lag and 5-lag models and the variables stands. The 1-lag fit is extremely impressive; the PIH may be rejected statistically in this framework, but it appears to describe almost all of the variation in the data. The 5-lag plots are much less favorable to the model, but even here the two series move together at intermediate business cycle frequencies. In the plot for standard to the negative contemporaneous correlation between the two variables.

These results have some bearing on the common idea that consumption displays excess sensitivity to income, relative to the predictions of the PIH. As previously noted, "excess sensitivity" is usually inferred from correlation between consumption changes and lagged changes in disposable income, or from large regression coefficients of consumption changes on proxies for income innovations. Another interpretation of the phrase, however, would be that consumption displays excess sensitivity if it moves too closely with income - that is, if the difference between consumption and income, or saving, varies less than the optimal forecast of discounted declines in labor income.

The results of this paper give mild support to the idea that there is excess sensitivity in this sense. The variance of saving is always less than the variance of the optimal unrestricted forecast. 13 However, excess sensitivity is not the most striking feature of the time series plots fol-

The fact that the transitory consumption model cannot be rejected in the first-order case is not inconsistent with excess sensitivity since transitory consumption may have negative contemporaneous correlation with permanent income consumption.

lowing Table 4, which in the 1-lag version are dominated by the high correlation of saving with the optimal forecast of declines in labor income.

V. Conclusions

The permanent income hypothesis implies that people save because they rationally expect their labor income to decline; they save "for a rainy day". It follows that saving should be at least as good a predictor of rainy weather, or discounted declines in labor income, as any other fore-cast that can be constructed from publicly available information. Surprisfingly, this implication of the model seems to have been ignored in previous empirical work on consumption.

This paper tests the predictive power of saving for declines in labor income, using quarterly aggregate data for the period 1953-1984 in the U.S. Saving is measured in two alternative ways, both of which use Blinder and Deaton's [1985] adjustments to NIPA consumption data. The first savings measure is derived from consumption of nondurables and services, as is conventional in the literature on the PIH, while the second measure is derived from total consumption.

The paper compares these measures of saving with an unrestricted forecast of labor income declines based on a vector autoregression for lagged
saving and changes in labor income. The comparison is made both formally,
by calculating test statistics for the hypothesis that the two series are
the same, and informally, by plotting the series together and presenting
their sample moments.

The unrestricted forecast differs from saving most strongly in its mean, placing considerable weight on a constant term which captures the up-ward drift in labor income. The dynamics of the unrestricted forecast depend quite sensitively on the number of lags included in the vector autoregression. The Akaike Information Criterion suggested that one or five lags

should be used. The forecast from a first-order VAR moves very closely with saving, having a correlation of more than 0.95 for both saving measures; the formal test of the PIH is unable to reject in this case if a transitory error is allowed in the consumption function. The forecast from a fifth-order VAR moves much less closely with saving, and the PIH is more strongly rejected in this case. Even here, however, saving and the unrestricted forecast seem to move together at intermediate business cycle frequencies.

There are a number of objections that might be raised to the empirical work of this paper. I test a very tight formulation of the PIH model which ignores aggregation and many other difficulties, and I treat as known a key parameter (the discount rate) which should properly be estimated. Of course, these shortcomings of the model make it even more remarkable that it fits the movements of the first-order unrestricted forecast so closely. However this is probably due in part to the limited information set used in the first-order vector autoregression.

The results of this paper do suggest that the permanent income hypothesis is worth taking seriously as a description of the broad outlines

of aggregate consumption behavior. More generally, models which are

strongly rejected statistically may be good first approximations to the behavior of economic variables. Devices such as the time series plots presented here can be used to evaluate the performance of a model.

A natural next step is to apply the methods of this paper to larger and disaggregated data sets, and this is a priority for future research.

ESTIMATION OF THE COINTEGRATING VECTOR AND TEST FOR COINTEGRATION

Nondurables and services consumption

[1)
$$y = -0.309 + 1.495 c^*$$
 $R^2 = 0.996$ [0.023] (0.009) Estimate of $\lambda/\chi = 1.495$

$$\frac{\Delta y}{t} = \frac{-0.056 + 0.165 \Delta y}{(0.020) (0.098)} + \frac{0.638}{t-1} \Delta c^{*}t-1$$

Test of no cointegration from (1):

Augmented Dickey-Fuller Test with 1 lag 3.764

5 lags 3.010

Total consumption

3)
$$y = 0.040 + 1.062 c$$
 $R^2 = 0.997$ (0.014) (0.005) Estimate of $1/8 = 1.062$

$$\frac{\Delta y}{t} = \frac{0.007 + 0.086 \Delta y}{(0.010) (0.108)} + \frac{0.350}{(0.120)} \Delta c t - 1$$

Test of no cointegration from (3):

Augmented Dickey-Fuller Test with 1 lag 3.265

5 lags 3.078

Critical values (Granger and Engle [1985]): 10% 2.84, 5% 3.17.

Notes: y = total disposable real income per capita in thousands of dollars. $c^* = real$ consumption of nondurables and services per capita in thousands of dollars. c = total real consumption per capita in thousands of dollars. All data are quarterly from 1953:2 to 1984:4. Heteroskedasticity-consistent standard errors in parentheses.

REGRESSION TESTS OF THE PIH (NO TRANSITORY CONSUMPTION)

Nondurables and services consumption

1-lag model:

$$\frac{s^* - \Delta yl - (1+r)s^*}{t-1} = \frac{-0.004}{(0.013)} - \frac{0.203 \, \Delta yl}{t-1} - \frac{0.013}{(0.069)} s^* t-1$$

 $\mathbb{R}^2 = 0.074$

Test that all coefficients = 0: Chi-square(3) = 44.166, P-value < 0.001% Test excluding the intercept: Chi-square(2) = 8.783, P-value = 1.2%

5-lag model:

Test that all coefficients = 0: Chi-square(11) = 76.074, P-value < 0.001% Test excluding the intercept: Chi-square(10) = 29.819, P-value = 0.1%

Total consumption

1-lag model:

$$\frac{s - \Delta yl - (1+r)s}{t-1} = \frac{-0.009 - 0.324 \, \Delta yl - 0.035}{(0.004) \, \frac{(0.089)}{(0.089)}} s t - 1$$

$$\mathbb{R}^2 = 0.098$$

Test that all coefficients = 0: Chi-square(3) = 68.714, P-value < 0.001% Test excluding the intercept: Chi-square(2) = 13.213, P-value = 0.1%

5-lag model:

Test that all coefficients = 0: Chi-square(11) = 117.255, P-value < 0.001% Test excluding the intercept: Chi-square(10) = 54.191, P-value < 0.001%

Notes: y = total disposable real income per capita in thousands of dollars. s* = saving estimated from real consumption of nondurables and services per capita in thousands of dollars. s = saving estimated from total real consumption per capita in thousands of dollars. All data are quarterly from 1953:2 to 1984:4. Heteroskedasticity-consistent standard errors in parentheses.

REGRESSION TESTS OF THE PIH (WITH TRANSITORY CONSUMPTION)

Nondurables and services consumption

I-lag model:

$$\frac{s^* - \Delta yl - (1+r)s^* - 1}{t-1} = \frac{-0.006}{(0.013)} + \frac{0.127 \Delta yl}{(0.065)} + \frac{0.012 s^*}{(0.041)}$$

 $R^2 = 0.028$

Test that all coefficients = 0: Chi-square(3) = 41.189, P-value < 0.001% Test excluding the intercept: Chi-square(2) = 3.802, P-value = 14.9%

5-lag model:

Test that all coefficients = 0: Chi-square(11) = 74.722, P-value < 0.001% Test excluding the intercept: Chi-square(10) = 28.731, P-value = 0.1%

Total consumption

1-lag model:

$$\frac{s - \Delta yl - (1+r)s}{t - 1} = \frac{-0.012 - 0.194 \Delta yl}{(0.004) (0.111)} - \frac{0.013 s}{(0.078)} t^{-2}$$

 $R^2 = 0.033$

Test that all coefficients = 0: Chi-square(3) = 39.027, P-value < 0.001%Test excluding the intercept: Chi-square(2) = 3.047, P-value = 21.8%

5-lag model:

Test that all coefficients = 0: Chi-square(11) = 94.014, P-value < 0.001% Test excluding the intercept: Chi-square(10) = 39.791, P-value = 0.002%

Notes: y = total disposable real income per capita in thousands of dollars. s* = saving estimated from real consumption of nondurables and services per capita in thousands of dollars. s = saving estimated from total real consumption per capita in thousands of dollars. All data are quarterly from 1953:2 to 1984:4. Heteroskedasticity-consistent standard errors in parentheses.

MEASURING THE FIT OF THE PIH: ESTIMATED VAR SYSTEM AND SUMMARY STATISTICS

Nondurables and services consumption

1-lag model:

$$\Delta y l_{t} = \underbrace{0.478 \ \Delta y l_{t-1} \ (0.072)}_{(0.072)} s^{*}_{t-1}$$

$$s^{*}_{t} \text{ Granger causes } \Delta y l_{t} \text{ at } 0.01\% \text{ level.}$$

$$s^{*}_{t} = \underbrace{0.275 \ \Delta y l_{t-1} \ (0.092)}_{(0.042)} + \underbrace{0.844 \ s^{*}_{t-1}}_{(0.092)} + \underbrace{0.751}_{(0.042)}$$

$$\Delta y l_{t} \text{ Granger causes } s^{*}_{t} \text{ at } 0.3\% \text{ level.}$$
Summary statistics: $\sigma(s^{*}) = 0.048$

$$\sigma(unrestricted \text{ optimal forecast}) = 0.063$$

$$\rho(s^{*}, unrestricted \text{ forecast}) = 0.995$$

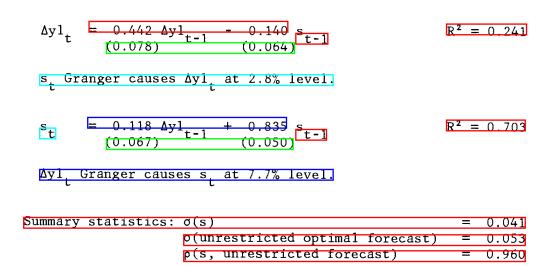
5-lag model:

Summary statistics: $\sigma(\text{unrestricted optimal forecast}) = 0.052$ $\rho(\text{s*, unrestricted forecast}) = 0.449$

(CONTINUED)

Total consumption

l-lag model:



5-lag model:

$$\Delta y l_t$$
 equation $R^2 = 0.357$
 s_t Granger causes $\Delta y l_t$ at 0.3% level.
 s_t equation $R^2 = 0.721$
 $\Delta y l_t$ Granger causes s_t at 19.0% level.

Summary statistics: $\sigma(\text{unrestricted optimal forecast}) = 0.051$ $\rho(\text{s, unrestricted forecast}) = -0.480$

Notes: y = total disposable real income per capita in thousands of dollars. s* = saving estimated from real consumption of nondurables and services per capita in thousands of dollars. s = saving estimated from total real consumption per capita in thousands of dollars. All data are quarterly from 1953:2 to 1984:4. Heteroskedasticity-consistent standard errors in parentheses.

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