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THE SUBSTITUTABILITY OF DEBT AND EQUITY SECURITIES

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Abstract

This paper investigates empirically the degree of substitutability between debt and equity securities in the United States during 1960-1980. The analysis first applies fundamental relationships connecting portfolio choices with expected asset returns to infer key asset substitutabilities directly from the observed U.S. asset return experience. It then compares these implied substitutabilities with the observed portfolio behavior of U.S. households.

The resulting evidence provides little ground for any conclusion about even the sign, much less the magnitude, of the substitutability of short-term debt and equity. Although the implied optimal behavior indicates that these two assets are substitutes, the observed behavior indicates that households have treated them as complements. By contrast, the evidence consistently indicates that long-term debt and equity are substitutes. Moreover, with a few exceptions the empirical estimates of the associated substitution elasticity are quite closely clustered around the value -.035.

The conclusion that long-term debt and equity are substitutes with elasticity -.035 bears mixed implications for broader economic and financial questions. At one level, the finding that the two assets are indeed substitutes validates the standard assumption underlying a variety of familiar models in monetary economics and finance. At the same time, if the elasticity is only -.035, then many of these models' more important substantive conclusions do not follow.

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The substitutability of debt and equity securities in investors' portfolios is an old and important issue both in monetary economics and in the theory of finance. More than two decades ago, Tobin (1961) emphasized that the structure of macroeconomic models of the asset markets depends fundamentally upon investors' willingness to substitute debt and equity claims, with consequent strong implications for such familiar questions as the financing of capital formation, the economic impact of government deficits, and the potential efficacy of monetary policy. At the same time, following Modigliani and Miller (1958) the theory of corporate finance has focused heavily on the distinctions between debt and equity claims, and on the implications of the fact that corporations issuing these claims confront a competitive market in which investors price these forms of ownership according to their own, rather than the issuing corporations', objectives.

The basic reasons why debt and equity may be either close or distant substitutes are well known. Perhaps the most obvious distinction is that (non-indexed) debt is a claim on a fixed nominal payment stream, while equity is not, so that the two assets' risk properties with respect to changes in the economy's overall price level differ sharply.[1] Similarly, because of the residual nature of equity claims, the two assets also have different risk properties with respect to changes in relative prices — or equivalently, in a world in which not all markets are perfectly competitive, changes in supply-demand conditions in specific product and factor markets.[2] In comparison with money and other short-term instruments, however, debt and equity claims have much in common with one another. To the extent that both debt and equity represent claims to long-lived payment streams, their shared risk properties with respect to interest rate changes hold them apart from money and other short-term claims. Also, unlike money (and some money substitutes), conventional debt and equity claims are not normally acceptable as a means of payment.[3]

All of these factors affecting investors' willingness to substitute debt and equity securities are familiar enough at the qualitative level, but the actually prevailing debt-equity substitutability and its consequences for important issues of economic behavior remain questions that can only be resolved empirically. It is simply not possible, on the basis of a priori considerations alone, to say which risks or other factors are foremost in investors' minds, and hence how investors resolve the tug-of-war that pits the distinctions between debt and equity claims against their similarities. Moreover, because objective circumstances differ from one time and place to another, there is no reason to assume that the relative weights investors place on even the most important of these considerations are universal constants. As changes in the nonfinancial structure of an economy or in the posture of economic policy alter the character of the risks investors face, or as financial market practices and institutions evolve, debt and equity securities may become either closer or more distant substitutes.

The object of this paper is to investigate empirically the degree of substitutability between debt and equity securities in the United States, and to see whether the recent evidence indicates stability or change in this relationship. Section I applies fundamental relationships connecting portfolio choices to expected asset returns, based on the maximization of expected utility, to infer key asset substitutabilities from the experience of asset returns in the United States during 1960-1980. Section II compares these inferred substitutabilities with the observed portfolio behavior of U.S. households over this period. Section III performs analogous comparisons for two further alternative systems for grouping financial assets into the broad aggregates (debt, equity, etc.) that are necessary for formal analysis. Section IV focuses on whether there is reason to believe that asset substitutabilities have changed since 1960 — to anticipate, the answer is yes — and examines an extended model in light of this finding. Section V briefly summarizes the paper's principal conclusions and offers some concluding comments.

I. Implications of Asset Returns

The substitutability or complementarity of one asset for another is a way of describing how investors' portfolio choices respond to changes in expected asset returns. Because the data available for empirical applications necessarily indicate the composition of investor's portfolios only at specific intervals, it is useful to derive a discrete-time model of this aspect of portfolio behavior.

Following the familiar theory of expected utility maximization,[4] the investor's single-period objective as of time t, given initial wealth W_t, is

$$\max_{\underline{\alpha}_t} E[U(\tilde{W}_{t+1})] \tag{1}$$

subject to

$$\underline{\alpha}_t'\underline{1} = 1 \tag{2}$$

where E(•) is the expectation operator, U(W) is utility as a function of wealth, $\underline{\alpha}_t$ is a vector expressing the portfolio allocations in proportional form

$$\underline{\alpha}_t = \frac{1}{W_t} \cdot \underline{A}_t \tag{3}$$

for vector $\underline{A}$ of asset holdings, and wealth W evolves according to

$$\tilde{W}_{t+1} = W_t \cdot \underline{\alpha}_t'(\underline{1} + \tilde{\underline{r}}_t) \tag{4}$$

for perceived net asset returns $\underline{r}_t$ between time t and time t+1. As is well known, if U(W) is any power (or logarithmic) function such that the coefficient of relative risk aversion

$$\rho = -W \cdot \frac{U''(W)}{U'(W)} \tag{5}$$

is constant, and if the investor perceives asset returns $\tilde{\mathbf{r}}$ to be distributed as

$$\tilde{\mathbf{r}}_t \sim N(\mathbf{r}_t^e, \Omega) \tag{6}$$

then the resulting optimal asset demands exhibit the convenient properties of homogeneity in total wealth and linearity in the expected asset returns.[5]

If no asset in vector $\mathbf{A}$ bears a risk-free return, so that the variance-covariance matrix Ω is of full rank, then solution of the first-order condition for the maximization of (1) subject to (2) yields

$$\boldsymbol{\alpha}_t^* = B\,(\mathbf{r}_t^e + \mathbf{1}) + \boldsymbol{\pi} \tag{7}$$

where

$$B = \left\{\frac{-U'[E(\tilde{W}_{t+1})]}{W_t \cdot U''[E(\tilde{W}_{t+1})]}\right\} \cdot \left[\Omega^{-1} - (\mathbf{1}'\Omega^{-1}\mathbf{1})^{-1}\,\Omega^{-1}\,\mathbf{1}\,\mathbf{1}'\,\Omega^{-1}\right] \tag{8}$$

$$\boldsymbol{\pi} = (\mathbf{1}'\Omega^{-1}\mathbf{1})^{-1}\,\Omega^{-1}\mathbf{1}. \tag{9}$$

Alternatively, in the presence of a risk-free asset bearing return r^f, it is necessary to partition the asset demand system. The resulting solution, in which $\hat{\boldsymbol{\alpha}}$, $\hat{\mathbf{r}}^e$ and $\hat{\Omega}$ refer to the subset of risky assets only, is

$$\hat{\boldsymbol{\alpha}}_t^* = \hat{B}\,(\hat{\mathbf{r}}_t^e - r_t^f \cdot \mathbf{1} + \mathbf{1}) \tag{10}$$

where

$$\hat{B} = \left\{\frac{-U'[E(\tilde{W}_{t+1})]}{W_t \cdot U''[E(\tilde{W}_{t+1})]}\right\} \cdot \hat{\Omega}^{-1} \tag{11}$$

and the optimum portfolio share for the risk-free asset is just $(1-\hat{\boldsymbol{\alpha}}^{*\prime}\mathbf{1})$. In either case, if the time unit is sufficiently small to render W_t a good approximation to $E(\tilde{W}_{t+1})$ for purposes of the underlying expansion, then the scalar term within brackets in either (8) or (11) reduces to the reciprocal of the constant coefficient of relative risk aversion ρ.[6]

Because this system of asset demands provides the basic vehicle for the analysis that follows, it is useful at the outset to note explicitly several of its properties. First, because of the assumptions of constant relative risk aversion and normally distributed return assessments, the respective asset demands are each proportional to the investor's wealth, and they depend linearly on the associated expected returns. Second, as Brainard and Tobin (1968) have emphasized, the effect of the constraint (2) is to render the asset demands linearly dependent, so that matrix B (or $\hat{B}$) and vector $\underline{\pi}$ satisfy the "adding up" constraints

$$\underline{\beta}_j'\underline{1} = 0 \text{ , all } j \tag{12}$$

and

$$\underline{\pi}'\underline{1} = 1 \tag{13}$$

where vectors $\underline{\beta}_j$ are the columns of B. Third, because Ω is a variance-covariance matrix and therefore symmetrical, B (or $\hat{B}$) indicates symmetrical asset substitutions associated with cross-yield effects.[7] Fourth, B (or $\hat{B}$) is strictly proportional to a straightforward transformation of the variance-covariance matrix, with the factor of proportionality equal to the reciprocal of the coefficient of relative risk aversion. Each of these four properties figures importantly in the analysis presented below.

The primary focus of interest here is the specific off-diagonal elements (or, depending on the asset aggregation scheme employed, element) of B that describe the substitutability or complementarity of debt and equity securities — that is, the response of the demand for debt to changes in the expected return on equity, and vice versa. Following Brainard and Tobin, the standard assumption (at least in the macroeconomic literature) is that all assets are gross substitutes, so that the only question left to be resolved empirically is the absolute magnitude of the presumably negative off-diagonal β_{ij} elements measuring debt-equity substitutability. The β_{ij} in (8) and (11) are marginal responses, so that the associated elasticities of substitution, defined in the usual way as[8]

$$\varepsilon_{ij} = \frac{dA_i}{dr_j^e} \cdot \frac{r_j^e}{A_i} \tag{14}$$

simply follow from (7) and (3) as

$$\varepsilon_{ij} = \beta_{ij} \cdot \frac{r_j^e}{\alpha_i} . \tag{15}$$

In general, however, assets may or may not be gross substitutes. From (8) and (11) it is clear that not just the magnitude but also the sign of each asset demand response to variations in expected yields depend on the variance-covariance structure describing perceived asset returns. In the presence of a risk-free asset, Blanchard and Plantes (1977) have shown that a necessary (but not sufficient) condition for gross substitutability of all assets — that is, for all of the off-diagonal $\hat{\beta}_{ij}$ in (11) to be negative — is that the partial correlations among all asset returns be nonnegative.[9] In the absence of a risk-free asset, as is typically assumed here, no such straightforward condition on Ω to guarantee the negativity of all of the off-diagonal β_{ij} is apparent, and the most straightforward way to assess the question of gross substitutability is simply to inspect the elements of B directly.[10]

In financial markets as well developed as those in the United States, most investors confront a rich, and at times bewildering, variety of financial instruments. Different securities represent claims structured in sharply different ways, and therefore bear returns subject to different risks. Government securities differ from private securities. Even among private securities, claims against some obligors can differ importantly from identically structured claims against others. For purposes of the questions addressed here, however, it is important to focus on broadly defined asset categories, thereby disregarding much of this variety and implicitly treating as perfect substitutes many distinct claims among which investors are presumably not entirely indifferent.

Some aggregation among assets, therefore, is clearly necessary. Table 1 indicates an aggregation of the many forms of financial claims typically held by households in the United States into five broad categories: money, time and saving deposits, short-term debt, long-term debt, and equity. The table also indicates the amount of each asset category in the aggregate portfolio of the U.S. household sector as of yearend 1980.[11] The analysis here ignores entirely all nonfinancial assets, both because the available rate-of-return data are weak (nonexistent in many cases) and because a careful treatment of investment in nonfinancial assets lies beyond the scope of this paper.

The object of the aggregation shown in Table 1 is to preserve the fundamental distinctions among assets while at the same time reducing the number of separate categories to within manageable range for purposes of empirical analysis.[12] "Money," including currency and demand deposits, distinguishes assets that bear zero nominal rates of return and that provide means-of-payment services. "Time and saving deposits" distinguishes assets that bear (nonzero) nominal rates of return subject to fixed legal ceilings. "Short-term debt," including all other deposit instruments and

TABLE 1

DISAGGREGATION OF HOUSEHOLD SECTOR FINANCIAL ASSETS

Asset		1980:IV Value
Money (M)		$ 268.0
Time and Saving Deposits (T)		624.7
Short-Term Debt (S)		884.3
Money Market Fund Shares	74.4	
Competitive-Return Time Deposits	669.7	
U.S. Government Securities	102.0	
Open Market Paper	38.2	
Long-Term Debt (L)		464.3
U.S. Government Securities	180.2	
State and Local Obligations	74.2	
Corporate and Foreign Bonds	86.9	
Mortgages	122.5	
Equity (E)		1,215.6
Mutual Fund Shares	63.7	
Directly Held Equity Shares	1,151.8	
Total		3,456.9

Notes: Values in billions of dollars.
Detail may not add to total because of rounding.
Source: Board of Governors of the Federal Reserve System.

all open-market debt instruments maturing in less than one year, distinguishes assets that bear market-determined nominal rates of return but that are subject to little interest-rate risk. "Long-term debt," including all other debt instruments, distinguishes assets that bear nominal rates of return and that are subject to substantial interest-rate risk. "Equity" distinguishes assets that bear residual ownership risk.

The first column of Table 2 shows the annualized mean real returns observed on these five aggregate assets on a quarterly basis during 1960-1980. The nominal returns associated with these real returns are zero for money; a weighted-average yield for time and savings deposits; the four-to-six month prime commercial paper yield for short-term debt; the Moody's Baa corporate bond yield, plus annualized percentage capital gains or losses inferred by applying the consol pricing formula to changes in the Baa yield, for long-term debt; and the dividend-price yield, plus annualized percentage capital gains or losses, on the Standard and Poor's 500 index for equity. In each case the real return is just the respective nominal return minus the annualized percentage change in the consumer price index.[13]

The second column of Table 2 shows the corresponding after-tax returns on these five aggregate assets, computed by applying the marginal tax rates shown in Table 3 to each quarter's before-tax returns before subtracting the consumer price index change.[14] The marginal tax rates applied to interest and dividends are values estimated by Estrella and Fuhrer (1983), on the basis of Internal Revenue Service data, to reflect the marginal tax bracket of the average recipient of these two respective kinds of income in each year. The marginal tax rate applied to capital gains is an analogous estimate, including allowances for deferral and loss

TABLE 2

MEAN REAL RETURNS, 1960-1980

	Before-Tax	After-Tax
r_M	-5.43%	-5.43%
r_T	-1.24	-2.53
r_S	.78	-1.16
r_L	-1.60	-3.83
r_E	5.24	3.13

Note: Values in per cent per annum.

TABLE 3

MARGINAL TAX RATES, 1960-1979

	Interest	Dividends	Capital Gains
1960	.2955	.4949	.047
1961	.2989	.5022	.047
1962	.2905	.4968	.047
1963	.2893	.5022	.047
1964	.2631	.4597	.046
1965	.2552	.4359	.046
1966	.2599	.4376	.046
1967	.2695	.4476	.045
1968	.2745	.4500	.045
1969	.2803	.4423	.065
1970	.3064	.4536	.064
1971	.3360	.4721	.064
1972	.3128	.4559	.063
1973	.3220	.4614	.063
1974	.3341	.4967	.063
1975	.3341	.4759	.062
1976	.3407	.4834	.062
1977	.3243	.4786	.062
1978	.3353	.4874	.062
1979	.3442	.4744	.044

offset features, due to Feldstein et al. (1983).

As is clear in (7)-(11), the substitutability or complementarity among assets in investors' portfolios depends upon the variance-covariance structure of the returns that investors associate with those assets. Hence what matters in this context is not necessarily the actual experience of returns but investors' perceptions and expectations, which may or may not closely approximate the corresponding ultimate outcomes. Because expectations are not directly observable, arriving at values to use in their place for purposes of empirical analysis is always problematical. One solution to this problem, which is applicable in some isolated cases in which data are available, is to rely on survey information.[15] The most plausible alternative, which rests on the assumption of at least some form of "rationality" in investors' perceptions, is to infer the distribution of expected returns at least partly on the basis of the observed experience of actual returns.

Table 4 shows the variance-covariance matrix of the actual return experience corresponding to the mean after-tax real returns in Table 2.[16] As is familiar, these data show the large variation (even in real terms) associated with equity and, to a somewhat lesser extent, with long-term debt. As is also familiar, the variation associated with short-term debt is the smallest among any of the five aggregate assets.

What would these variance-covariance properties imply for the substitution properties among the five assets if they did accurately represent investors' assessments? Table 5 shows the transformation of Ω from the right-hand side of (8) computed on the basis of the Ω matrix in Table 4. To recall, these values indicate, to within a (positive) constant indicating the investor's relative risk aversion, the marginal responses of optimal

TABLE 4

VARIANCE-COVARIANCE STRUCTURE OF AFTER-TAX REAL RETURNS, 1960-1980

	r_M	r_T	r_S	r_L	r_E
r_M	15.78				
r_T	14.61	13.61			
r_S	9.99	9.28	7.09		
r_L	34.97	33.18	21.50	209.35	
r_E	32.79	31.43	22.58	161.77	597.96

TABLE 5

PORTFOLIO RESPONSES IMPLIED BY VARIANCE-COVARIANCE STRUCTURE

	r_M	r_T	r_S	r_L	r_E
M	7.48				
T	-8.51	10.29			
S	1.02	-1.74	.693		
L	.00842	-.0434	.0277	.00873	
E	.00178	-.00180	-.000559	-.00154	.00213

asset demands to changes in expected returns. For $\rho = 1$, a plausible and often assumed magnitude, these values are simply identical to the optimal marginal responses.[17]

What immediately stands out in Table 5 is that the implied system of optimal asset demands does not render all assets gross substitutes. Money is a complement for all assets except time and saving deposits, while short-term and long-term debt are complements for one another. Debt and equity securities are clearly substitutes, however. On the assumption that $\rho = 1$, so that the values in Table 5 represent the elements β_{ij} in (8), the corresponding elasticities of substitution follow as in (15). Table 6 shows the 1960-1980 mean asset shares which, together with the mean after-tax asset real returns in Table 2, facilitate calculating elasticities from the optimal marginal responses in Table 5. The results of such calculations are likely to be misleading in many cases, however, because four of the five mean net returns are negative. On the basis of the mean values as shown, the marginal response of the demand for short-term debt to the expected return on equity (which has a positive mean) implies an elasticity of substitution $\varepsilon_{SE} = -.025$, while the corresponding elasticity for equity and long-term debt is $\varepsilon_{LE} = -.037$.[18] Although both of these elasticities are negative, both are quite small in absolute value.

It is also useful to examine whether the assumption that no risk-free asset exists, as is implicit in using the values in Table 5 to imply whether assets are substitutes or complements, importantly affects these conclusions. In brief, the answer is no. The signs of all elements in (11) are identical to the corresponding signs shown in Table 5, except for that relating money and short-term debt. The respective elasticities of substitution of short-term and long-term debt for equity, calculated

TABLE 6

MEAN VALUES OF HOUSEHOLD FINANCIAL ASSET HOLDINGS, 1960-1980

	Value	Fraction
Money (M)	$ 137.2	.083
Time and Saving Deposits (T)	471.9	.275
Short-term Debt (S)	137.6	.069
Long-term Debt (L)	215.4	.129
Equity (E)	681.2	.444
Total	1,643.3	1.000

Note: Values in billions of dollars.
Detail may not add to total because of rounding.
Source: Board of Governors of the Federal Reserve System.

as above but using (11) instead of (8), are $\varepsilon_{SE} = -.270$ and $\varepsilon_{LE} = -.034$. Once again, even in the presence of a risk-free asset, not all of the five risky assets would be gross substitutes. As Table 7 shows, the partial correlations among the risky assets' after-tax real returns include a negative value and hence fail to satisfy the Blanchard-Plantes necessary condition.

Section II goes on to examine how the observed portfolio behavior of U.S. households has corresponded with the optimal behavior indicated in Table 5. Even before turning to the observed asset choices, however, it is helpful to focus on one aspect of the historical asset return experience that presents particular challenges for explaining investors' behavior. The optimal portfolio shares of the five asset aggregates, computed from (7) using the historical after-tax return means and variance-covariance matrix, indicate positive holdings of only two assets — time and saving deposits, and equity.[19] These two assets did have the largest shares of households' actual portfolios during this period, as Table 6 indicates, but holdings of the other three assets were of course positive as well. Hence the actual asset choices made by households clearly differed from the optimal choices implied by the simple model developed above from the basics of expected utility maximization. Either households' perceptions of returns systematically differed from the actual experience during this period, or else households were incorporating other factors in their portfolio decisions. The analysis that follows attempts to consider each of these possibilities.

TABLE 7

PARTIAL CORRELATIONS AMONG AFTER-TAX REAL RETURNS, 1960-1980

	r_M	r_T	r_S	r_L
r_T	.97			
r_S	.12	.11		
r_L	-.13	.21	.12	
r_E	.09	.07	.11	.32

II. Household Sector Portfolio Behavior

The model of portfolio behavior developed in Section I takes the maximization of expected utility as the sole objective guiding investors' asset choices. When one of the assets under consideration is money, however, the need for means-of-payment services constitutes another factor influencing asset selection. Following Tobin (1969), among other writers, a convenient way to represent the demand for such services in a model with asset demands homogenous in portfolio wealth is by the flow of transactions relative to wealth. In the linear model (7), the implied generalization is accordingly

$$\underline{\alpha}^*_t = B(\underline{r}^e_t + \underline{1}) + \underline{\delta}\left(\frac{Y_t}{W_t}\right) + \underline{\pi} \qquad (16)$$

where Y is the investor's transactions, $\underline{\delta}$ is a vector of coefficients, and all other terms are as before.[20] Because money provides means-of-payment services, the usual presumption is that $\delta_M > 0$. Moreover, $\underline{\delta}$ must satisfy an "adding up" constraint analogous to (17), so that this presumption also implies $\delta_i < 0$ for at least some asset $i \neq M$.

Table 8 presents results for the estimation of (16) by ordinary least squares, using quarterly U.S. data for 1960-1980. Data used for $\underline{\alpha}$ are seasonally adjusted shares of the U.S. household sector's aggregate portfolio during 1960-1980. As Lintner (1969) has explicitly shown, the linearity of asset demand relationships like (7) or (16) readily admits of aggregation across investors with diverse preferences (ρ), endowments (W) and assessments ($\underline{r}^e, \Omega$). The intended end result here of empirical analysis based on aggregate data is therefore an estimate of the relevant parameters describing the behavior of the collectivity of investors that together play a large role in determining the overall substitutability of debt and equity securities in the United States. Data for the household sector consist for the most part of the portfolio holdings of individual investors, and the household sector is the dominant holder of securities — and the ultimate holder of all wealth — in the U.S. economy.[21]

The data for $\underline{\alpha}$ are respective shares, and for W the aggregate level, of the household sector's portfolio of financial assets, constructed for each asset by decrementing backward from the reported 1980 yearend value using the corresponding seasonally adjusted quarterly flows.[22] In addition, for equities (the only financial asset for which the asset stock data are at market value), quarterly valuation changes are included without seasonal adjustment. As the discussion in Section I explains, the data used here omit holdings of nonfinancial assets, in part to avoid data inadequacies and in part simply to limit the scope of the analysis. The data also omit the household sector's outstanding liabilities, since the great bulk of household borrowing is tied to the ownership of nonfinancial assets.[23] The data for $\underline{r}^e$ are actual real return data for money, time and saving deposits, and short-term debt. For long-term debt and equity the data are actual real return data for the component of returns excluding capital gains, plus fitted values of the respective percentage capital gains from a simple univariate autoregressive process.[24] The data for Y are quarterly gross national product flows, seasonally adjusted.

Table 8 shows the estimated values and t-statistics for the elements of matrix B in (16), as well as summary statistics for each equation including the coefficient of determination (adjusted for degrees of freedom), the standard error of estimate, and the Durbin-Watson statistic.[25] A comparison of the estimated marginal response values β_{ij} with the implied optimal responses in Table 5 shows only modest congruence. The estimated values are predominantly smaller in absolute value than are the implied optimal values, as would ordinarily be the case in the presence of errors in measuring the unobservable expected returns. Nine of the estimated values differ in sign from the implied optimal values, however, although in only four cases are the differences statistically significant at the .05 level. Among the ten pairs of off-diagonal coefficients that (13) implies should be identical, four differ in sign; three of the four conflicting pairs are in the row and columns corresponding to money and its expected return.

On the key issue of substitutability of debt and equity securities, the estimated values indicate (without contradiction in signs of paired values) that short-term debt and equity are complements and that long-term debt and equity are substitutes — results that are, respectively, inconsistent and consistent with the solution in Table 5. Once again, it is necessary to base the corresponding elasticities of substitution on the short-term or long-term debt demand and the equity return in order to avoid sign changes due to negative mean net returns. Here, however, there are two separate estimates of each marginal response β_{ij} $(i \neq j)$, because the matrix of estimated coefficients is not symmetric. The respective pairs of implied elasticity estimates are $\varepsilon_{SE} = (.039, .116)$ and $\varepsilon_{LE} = (-.0004, -.006)$. Although both ε_{LE} values are negative, both are small in absolute value in comparison with $\varepsilon_{LE} = -.037$ implied by the solution in Table 5.

One immediately noticeable aspect of the summary statistics shown in Table 8 is the uniformly low Durbin-Watson statistics, indicating residuals in all five equations that unambiguously display significant serial correlation at the .01 level. This result is hardly surprising in a quarterly model, in light of the well known sluggishness of household portfolio behavior in the presence of (broadly defined) transactions costs. Especially in the context of occasional large moves in equity prices, which suddenly shift the relative portfolio shares of all assets, it is implausible to expect full re-alignment of asset holdings to expected returns within a calendar quarter.[26] Some model of portfolio adjustment out of equilibrium is therefore appropriate.

The most straightforward and familiar model of portfolio adjustment under transactions costs found in the asset demand literature is the multivariate partial adjustment form

$$\Delta \underline{A}_t = \Theta(\underline{A}^*_t - \underline{A}_{t-1}) \tag{17}$$

where $\underline{A}^*$ is the vector of equilibrium asset holdings corresponding to $\underline{\alpha}^*$ in (16) and Θ is a matrix of adjustment coefficients with columns satisfying an "adding up" constraint analogous to (12).[27] Applying (17) to (16) yields

$$\Delta \underline{A}_t = \Phi(\underline{r}^e_t + \underline{1})\cdot W_t + \underline{\xi}Y_t + \underline{\psi}W_t - \Theta \underline{A}_{t-1} \tag{18}$$

where

$$\Phi = \Theta B \tag{19}$$

$$\underline{\xi} = \Theta\underline{\delta} \tag{20}$$

$$\underline{\psi} = \Theta\underline{\pi} \tag{21}$$

so that the columns of matrix Φ and vector $\underline{\xi}$ all satisfy "adding up" constraints analogous to (12) while that for vector $\underline{\psi}$ is analogous to (13).

The top panel of Table 9 presents results for the estimation of (18) by ordinary least squares, using the same quarterly data for 1960-1980 described above. Because each term in (18) takes the dimension of nominal dollars, however — unlike the homogeneous form (16) — here care is necessary to avoid spurious correlations due to common time trends. Hence for purposes of estimation all nominal magnitudes ($\Delta \underline{A}$, W, Y and $\underline{A}$) are rendered in real per capita values.[28] In addition, both $\Delta \underline{A}_t$ and W_t exclude the current period's capital gains or losses (although the vector of lagged asset stocks $\underline{A}_{t-1}$ reflects previous periods' gains and losses), so that the estimated form focuses strictly on the household sector's aggregate net purchases or sales of each asset associated with the sector's net saving. Defining the asset flows in this way is equivalent to assuming that investors do not respond within the quarter to that quarter's changes in their holdings due to changing market valuations, but do respond to market valuations as of the beginning of each quarter.

The top panel of Table 9 reports summary statistics for each equation, and estimated values and t-statistics for the matrix Φ of immediate marginal responses of asset demands to expected returns. Not surprisingly, the use of the partial adjustment form sharply improves the overall fit properties of all five equations. Serial correlation remains significant in only two equations, and the standard errors, after conversion from real dollars per capita to portfolio shares as in Table 8, are uniformly smaller — in some cases by almost an order of magnitude.[29]

Although the immediate marginal portfolio responses Φ_{ij} may be useful for some purposes, what primarily matters in the context of the questions raised at the outset of this paper is the matrix of equilibrium marginal responses B, solved following (19) as $B = \Theta^{-1}\Phi$. The lower panel of Table 9 shows the implied matrix B, together with associated t-statistics found by using the full-information maximum likelihood method to re-estimate

(18) in a nonlinear form representing the elements of matrix Φ in the form (19) so as to derive direct estimates of the underlying β_{ij} values.[30] In addition, so as to derive t-statistics comparable to those shown in Table 8, in which the equivalent of an identity matrix is imposed a priori in place of the adjustment matrix Θ, for purposes of the maximum likelihood estimation the estimated Θ_{ij} values were taken as given.[31]

These estimated equilibrium asset demand responses again bear little resemblance overall to the implied optimal responses in Table 5. Of the twenty-five estimated β_{ij}, eleven differ in sign from the corresponding B elements in Table 5, including three negative values among the five on-diagonal β_{ii} indicating the "own" response of the demand for an asset to the expected return on that asset. Among the ten pairs of off-diagonal β_{ij}, all four pairs in the row and column corresponding to money and its expected return uniformly disagree in sign, while the remaining six pairs uniformly agree in sign. In light of the ample (but troubled) literature on the demand for money, it is hardly surprising that the "pure portfolio" approach followed here should meet only limited success in explaining money demand and/or the response of other asset demands to the expected return on money.[32]

These estimates for the partial adjustment model correspond to those shown in Table 8 for the equilibrium model in indicating that long-term debt and equity are substitutes, as in Table 5, but (unlike in Table 5) short-term debt and equity are complements. The associated pairs of implied elasticities (calculated, as usual, from the mean return on equity) are $\varepsilon_{SE} = (.154, .776)$ and $\varepsilon_{LE} = (-.005, -.034)$. Here it is interesting that one of the ε_{LE} values is almost identical to the corresponding optimal elasticity implied by the solution in Table 5.

Because the five equations comprising (18) have identical sets of regressors, either ordinary least-squares or (unconstrained) maximum-likelihood methods necessarily yield estimates satisfying the "adding up" constraints emphasized by Brainard and Tobin. By contrast, such estimates in general do not satisfy other cross-equation restrictions implied by the theory of portfolio choice outlined in Section I. In this context a further potential advantage of the nonlinear maximum-likelihood method underlying the β_{ij} estimates in Table 9 is the facility it provides for imposing such restrictions.

Table 10 presents an alternative set of maximum likelihood estimates for (18), subject to the restriction that the matrix of equilibrium marginal responses B be symmetric. (Familiar practice notwithstanding, there is no reason to assume symmetry of the matrix of immediate marginal responses Φ.) The table shows the usual summary statistics for each equation, and estimated values and t-statistics for the symmetric matrix of equilibrium portfolio responses.[33] Here two of the five on-diagonal β_{ij} elements — those indicating the respective "own" responses of long-term debt and equity — have negative estimated values, although neither differs significantly from zero. Among the ten off-diagonal β_{ij} elements, seven agree in sign with the implied optimal responses shown in Table 5 while three — all in the row and column corresponding to equity and its expected return — disagree.

As is the case for the unconstrained estimates shown in Tables 8 and 9, the constrained estimates indicate that short-term debt and equity are complements, while long-term debt and equity are substitutes. The associated implied elasticities (calculated in the usual way) are

ε_{SE} = .192 and ε_{LE} = -0.024. Once again, it is interesting that the implied ε_{LE} value not only agrees in sign but also comes close in magnitude to the corresponding optimal elasticity implied by the solution in Table 5, although in this case the underlying estimated β_{ij} is not significantly different from zero. As is to be expected, imposition of the symmetry constraint enlarges the standard error of each equation, and the appropriate test of the symmetry constraint itself yields $\chi^2(10) = 43.8$, warranting rejection of the implied restrictions at any plausible significance level.[34]

In addition to symmetry, the theory summarized in Section I implies that the matrix of equilibrium portfolio responses also be proportional to the transformation of the asset return variance-covariance matrix shown in (8), with the constant of proportionality (approximately) equal to the reciprocal of the coefficient of relative risk aversion. Nevertheless, estimating (18) by the same nonlinear maximum-likelihood method, subject to the further constraint that matrix B be proportional to the implied optimal response matrix in Table 5, yields $\chi^2(9) = 19.2$, warranting rejection at the .05 level (but not the .01 level) of the further restrictions imposed in addition to the symmetry restrictions.[35]

In sum, estimates for neither equilibrium model (16) nor partial adjustment model (17) yield a representation of the U.S. household sector's observed 1960-1980 portfolio behavior that is fully satisfactory in terms of the theory summarized in Section I. Moreover, these models do allow (albeit in a simple, though standard, way) for two potentially important influences on portfolio choice that the straightforward theory of expected utility maximization omits — the demand for means-of-payment services, and the transactions costs associated with portfolio adjustments. Sections III and IV therefore turn to examine whether it is plausible to assume that the stochastic structure of asset returns represented in Tables 2 and 4 accurately reflects the perceptions that guided investors' asset selection during this period.

III. Two Alternative Asset Aggregation Systems

As the discussion in Section I emphasizes, any scheme for reducing to analytically manageable terms the number of assets from which investors choose their portfolios is bound to be highly arbitrary. A possible explanation for the unsatisfactory estimates summarized in Tables 9 and 10, therefore, is that the five-asset aggregation system introduced in Table 1 either over- or under-states the important distinctions on which investors actually focus in making asset choices.

Tables 11 and 12 show the basic properties of realized asset returns, and the corresponding estimation results for asset demand system (18), under an alternative aggregation system that distinguishes only three separate asset categories: money plus time deposits, including all instruments bearing nominal returns subject to (zero or nonzero) fixed legal ceilings; short-term plus long-term debt, including all instruments bearing market-determined nominal rates of return; and equity, as before. The idea underlying this alternative is simply to group together assets bearing non-market nominal returns without distinguishing those that provide means of payment services, and to group together assets bearing market-determined nominal returns without distinguishing those subject to substantial interest rate risk. The returns associated with each composite asset category are just those of its two components, as described in Section I, weighted by their respective dollar magnitudes in each quarter.

According to the implied optimal portfolio responses shown in Table 11, the composite debt asset and equity are clearly substitutes, with elasticity of substitution $\varepsilon_{SL,E} = -.044$. If anything, however, the estimated portfolio responses shown in Table 12 are less satisfactory than those shown in Tables 9 and 10 for the five-asset classification. In the absence of the symmetry

TABLE 11

PROPERTIES OF REAL RETURNS UNDER FIRST ALTERNATIVE ASSET AGGREGATION

Mean Returns

	Before-Tax	After-Tax
Money plus Time and Saving Deposits (MT)	-2.22%	-3.21%
Short-term plus Long-term Debt (SL)	-.27	-2.44
Equity (E)	5.24	3.13

Variance-Covariance Structure (After-Tax)

	r_{MT}	r_{SL}	r_E
r_{MT}	13.83		
r_{SL}	22.16	90.85	
r_E	31.35	115.92	597.96

Portfolio Responses Implied by Variance-Covariance Structure

	r_{MT}	r_{SL}	r_E
MT	.0167		
SL	-.0173	.0201	
E	.000581	-.00279	.00221

TABLE 12

ESTIMATED PORTFOLIO RESPONSES UNDER FIRST ALTERNATIVE ASSET AGGREGATION

Unconstrained Estimates

Asset	$\beta_{\cdot MT}$	$\beta_{\cdot SL}$	$\beta_{\cdot E}$	$\bar{R}^2$	SE	DW
MT	.00850	-.00275	-.00357	.57	27.79	1.62
	(2.3)	(-2.0)	(-3.5)			
SL	-.0220	.00457	.00527	.70	28.43	1.47
	(-4.0)	(2.5)	(3.6)			
E	.0135	-.00182	-.00169	.23	3.47	1.74
	(6.2)	(-1.9)	(-3.4)			

Symmetric Estimates

Asset	$\beta_{\cdot MT}$	$\beta_{\cdot SL}$	$\beta_{\cdot E}$	$\bar{R}^2$	SE	DW
MT	-.0477			.41	33.92	1.15
	(-1.6)					
SL	.134	-.364		.55	34.52	1.10
	(1.7)	(-1.7)				
E	-.0867	.229	-.143	.03	3.89	1.56
	(-1.7)	(1.7)	(-1.7)			

constraint, the two estimated values corresponding to the substitutability of debt and equity differ in sign. With the symmetry constraint imposed, debt and equity are complements with elasticity $\varepsilon_{SL,E} = 3.62$, but the loss of fit associated with the restriction is clearly large, and the test statistic value $\chi^2(3) = 46.3$ warrants rejecting it at any plausible significance level.

Tables 13 and 14 present analogous asset return properties and estimation results for a second alternative aggregation scheme, again distinguishing only three asset categories: money plus time deposits plus short-term debt; long-term debt; and equity. The idea underlying this alternative is to group together all assets bearing nominal returns that are essentially fixed (and known in advance, at least on a quarterly basis) and hence subject to inflation risk only.[36] In effect, the application of the "pure portfolio" model to this set of aggregates is equivalent to assuming that investors first decide, on the basis of mean-variance utility maximization, how large a portfolio of liquid assets to hold, and secondarily divide their liquid assets among money, time deposits and short-term debt instruments on the basis of other considerations.[37] The return associated with the composite liquid asset category is a weighted average of the returns associated with its three components.[38]

The implied optimal portfolio responses shown in Table 13 for this aggregation scheme indicate that all three assets are gross substitutes, with elasticities $\varepsilon_{MTS,E} = -.005$ between liquid assets and equity, and $\varepsilon_{LE} = -.036$ between long-term debt and equity. The unconstrained estimates shown in Table 14, however, bear little apparent relation to these optimal responses. All three estimated on-diagonal "own" responses are negative, and the estimated off-diagonal responses indicate (without any sign

TABLE 13

PROPERTIES OF REAL RETURNS UNDER SECOND ALTERNATIVE ASSET AGGREGATION

Mean Returns

	Before-Tax	After-Tax
Money plus Time and Saving Deposits plus Short-Term Debt (MTS)	-1.62%	-2.80%
Long-Term Debt	-1.60	-3.83
Equity	5.24	3.13

Variance-Covariance Structure (After-Tax)

	r_{MTS}	r_L	r_E
r_{MTS}	11.18		
r_L	29.91	209.35	
r_E	30.24	161.77	597.96

Portfolio Responses Implied by Variance-Covariance Structure

	r_{MTS}	r_L	r_E
MTS	.00641		
L	-.00578	.00727	
E	-.000635	-.00150	.00213

TABLE 14

ESTIMATED PORTFOLIO RESPONSES UNDER SECOND ALTERNATIVE ASSET AGGREGATION

Unconstrained Estimates

Asset	$\beta_{\cdot MTS}$	$\beta_{\cdot L}$	$\beta_{\cdot E}$	$\bar{R}^2$	SE	DW
MTS	-.0192	.00283	.00575	.78	11.71	1.53
	(-1.7)	(1.3)	(2.7)			
L	.00201	-.000231	-.00117	.16	10.41	1.49
	(0.6)	(-0.3)	(-1.8)			
E	.0172	-.00260	-.00458	.25	3.43	1.81
	(2.2)	(-1.8)	(-3.0)			

Symmetric Estimates

Asset	$\beta_{\cdot MTS}$	$\beta_{\cdot L}$	$\beta_{\cdot E}$	$\bar{R}^2$	SE	DW
MTS	-.0135			.78	11.74	1.52
	(-2.5)					
L	.00266	-.000299		.16	10.42	1.48
	(2.0)	(-0.8)				
E	.0108	-.00237	-.00847	.18	3.58	1.73
	(2.6)	(-2.4)	(-2.7)			

contradictions) that liquid assets are a complement for both long-term debt and equity. With the symmetry constraint imposed, the implied elasticities are $\varepsilon_{MTS,E} = .079$ and $\varepsilon_{LE} = -.058$. Here the implied $\varepsilon_{MTS,E}$ value again indicates complementarity rather than substitutability, but the implied ε_{LE} value agrees in sign and approaches in magnitude the corresponding optimal ε_{LE} implied by the solution in Table 13. The test statistic value for the symmetry restriction is $\chi^2(3) = 8.0$, which warrants rejecting the restriction at the .05 level but not at the .01 level.

Comparison of these results with those presented in Sections I and II on the basis of a five-asset scheme hardly settles the question of which arbitrary asset aggregation system provides the best representation of how investors perceive the menu of assets confronting them. Nevertheless, it is instructive that the estimate of $\varepsilon_{SL,E}$ in the first three-asset alternative and ε_{LE} in the second are not all that different from the ε_{LE} estimates — or the implied optimal ε_{LE} — reported in Sections I and II. More broadly, however, on the basis of these results there is little ground for attributing the unsatisfactory properties of these models' empirical estimates in other respects to the asset aggregation system per se.

IV. Changes Over Time in the Structure of Asset Returns

The variance-covariance matrix exhibited in Table 4 reflects the stochastic structure of the after-tax asset returns actually realized during 1960-1980. Hence the portfolio responses to expected return variations exhibited in Table 5, which are implied from that variance-covariance matrix using (8) and $\rho = 1$, adequately describe investors' optimal behavior only to the extent that investors actually knew that the stochastic structure of asset returns was as it turned out to be. The question that immediately arises is how investors would have acquired this information.

The rationale for asserting that economic agents (on average) accurately know the relevant properties of the world in which they live usually rests on some presumption of stationarity: If the properties in question are economically relevant, agents will have an incentive to discover them; if the properties persist, agents will in fact do so. By contrast, if the relevant properties are changing over time, so that an appeal to economic incentives and (even sometimes quite astonishing) powers of observation is insufficient, how the representative agent comes to know these properties is highly problematical.[39]

The relevant question here, therefore, is how stable were the 1960-1980 sample properties of asset returns summarized in Section I. Tables 15 and 16 report mean returns and variance-covariance structures for the two sub-samples 1960:I-1979:II and 1970:III-1980:IV.[40] As is well known, investors confronted not only lower mean real returns but also more volatile real returns on all five categories of assets during the 1970s, and the data shown here clearly reflect these differences.

TABLE 15

SUB-SAMPLE MEAN REAL RETURNS

1960:I - 1970:II

	Before-Tax	After-Tax
r_M	-2.75%	-2.75%
r_T	.85	-.15
r_S	2.06	.72
r_L	-1.71	-3.07
r_E	5.89	4.16

1970:III - 1980:IV

	Before-Tax	After-Tax
r_M	-8.11%	-8.11%
r_T	-3.34	-4.92
r_S	-.51	-3.04
r_L	-1.48	-4.59
r_E	4.60	2.10

TABLE 16

SUB-SAMPLE VARIANCE-COVARIANCE STRUCTURE OF AFTER-TAX REAL RETURNS

1960:I - 1970:II

	r_M	r_T	r_S	r_L	r_E
r_M	3.59				
r_T	3.19	2.94			
r_S	1.68	1.56	1.16		
r_L	14.54	12.42	7.29	112.76	
r_E	20.84	18.86	10.30	112.61	444.46

1970:III - 1980:IV

	r_M	r_T	r_S	r_L	r_E
r_M	13.66				
r_T	13.30	12.97			
r_S	8.24	8.04	5.96		
r_L	52.10	51.05	33.33	309.88	
r_E	39.87	39.74	31.45	213.28	763.87

More importantly, Table 17 shows that the changes in the variance-covariance structure of asset returns that took place between the 1960s and the 1970s bore strong implications for optimal portfolio behavior, including implications for the substitutability of debt and equity securities. The table shows the implied optimal asset responses corresponding to the respective sub-sample variance-covariance matrices in Table 16, based again on (8) and $\rho = 1$. Many of the own- and cross-return responses changed by relatively large magnitudes, and the response indicating the substitutability of short-term debt and equity even changed sign, between the two sub-samples. For the 1960s the prevailing stochastic return structure implies that short-term debt and equity were complements, with $\varepsilon_{SE} = 1.28$. The analogous stochastic return structure for the 1970s implies that short-term debt and equity were substitutes, with $\varepsilon_{SE} = -.043$. By contrast, the stochastic return structure in both sub-samples implies that long-term debt and equity were substitutes, with $\varepsilon_{LE} = -.087$ and $\varepsilon_{LE} = -.019$ respectively.[41]

In light of these changes in the stochastic structure, and hence in the implied optimal portfolio responses, it is hardly surprising that straightforward estimation of either equilibrium model (16) or partial adjustment model (18) should yield unsatisfactory results, nor that the elasticity of substitution between short-term debt and equity be a particularly unsatisfactory aspect of these results. At a minimum, some allowance for these within-sample changes is necessary. Nevertheless, simply including fifteen moving-average variances and covariances in each equation is hardly likely to be an efficient approach.[42]

Some more compact way of summarizing the information contained in the evolving variance-covariance structure of asset returns is therefore

TABLE 17

SUB-SAMPLE IMPLIED PORTFOLIO RESPONSES

1960:I - 1970:II

	r_M	r_T	r_S	r_L	r_E
M	9.63				
T	-10.16	12.11			
S	.697	-2.06	1.33		
L	-.176	.134	.0267	.0181	
E	.00763	-.0231	.0148	-.00250	.00318

1970:III - 1980:IV

	r_M	r_T	r_S	r_L	r_E
M	54.32				
T	-56.63	59.57			
S	2.33	-2.94	.584		
L	-.0655	.0322	.0275	.00711	
E	.0397	-.0383	-.00184	-.00127	.00168

necessary. Following Sharpe (1964) and Lintner (1965), a plausible summary measure for this purpose is the ratio of the covariance of each asset's return with that on the "market" portfolio to the variance of the "market" return itself — that is, each asset's "beta." Figure 1 shows the 1960-1980 quarterly values of these "betas" computed on a trailing eight-quarter basis for the five aggregate assets defined in Table 1, with the "market" portfolio defined in each quarter simply as the total of the five aggregate assets.[43]

Generalizing equilibrium model (16) to allow for the changing "beta" values over time results in

$$\underline{\alpha}_t^* \quad = \quad B(\underline{r}_t^e + \underline{1}) + \Gamma\underline{x}_t + \underline{\delta}\left(\frac{Y_t}{W_t}\right) + \underline{\pi} \tag{22}$$

where $\underline{x}$ is a vector of "beta" values and Γ a matrix of coefficients with columns satisfying an "adding up" constraint analogous to (12). Applying partial adjustment process (16) to (22) then yields

$$\Delta\underline{A}_t \quad = \quad \Phi(\underline{r}_t^e + \underline{1})\cdot W_t + Z\underline{x}_t\cdot W_t + \underline{\xi}Y_t + \underline{\psi}W_t - \Theta\underline{A}_{t-1} \tag{23}$$

where

$$Z \quad = \quad \Theta\Gamma. \tag{24}$$

Table 18 summarizes the results of estimating (23), subject to the restriction that B be symmetric, for the full 1960-1980 sample. Here the values of $\underline{x}$ are as shown in Figure 1, and all other data and estimation procedures are as described in Section II. The table presents summary statistics and estimated values and t-statistics for the β_{ij}, corresponding to those in Table 10, as well as estimated values and t-statistics for the z_{ij}.

In comparison to the results in Table 10, those in Table 18 show that including the five moving-average "beta" terms typically does not

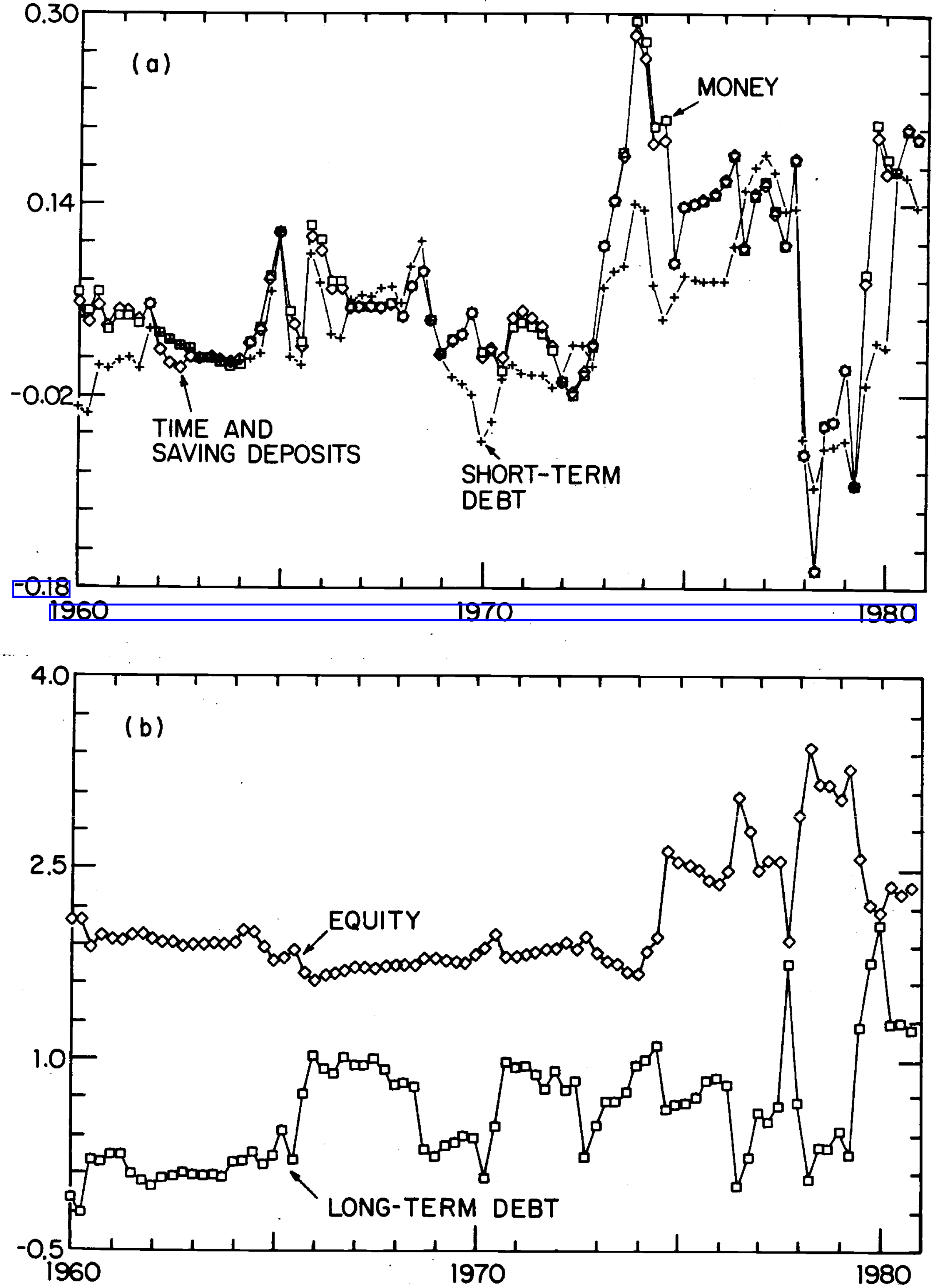

FIGURE 1

MOVING-AVERAGE "BETA" VALUES

improve the fit (after correction for degrees of freedom) of the estimated asset demand equations. Among the individual "betas," those for long-term debt and equity are each significant at the .05 level in two estimated equations, although in neither case is one of the two the "own" equation. The other three "betas" are rarely if ever significant. These values are at best difficult to interpret, however, because they represent the matrix of impact effects Z associated with the "betas," rather than the corresponding matrix of equilibrium effects Γ.[44] The usual β_{ij} coefficients on the expected returns appear to indicate that short-term debt and equity are substitutes while long-term debt and equity are complements, but in the presence of the "betas" these coefficients no longer bear the same structural interpretation as in (7) and (8). Moreover, the relevant test statistic value, $\chi^2(10) = 81.1$, once again warrants rejection of the symmetry restriction at any plausible significance level.

In sum, the inclusion in the analysis of summary information describing the changing stochastic structure of asset returns does apparently affect the estimated substitution properties of the asset demand system, but the properties of the expanded system do not necessarily represent an improvement and hence the associated estimates do not give grounds for much confidence.

V. Concluding Comments

Table 19 brings together the respective estimates of the elasticity of substitution between debt and equity securities developed throughout this paper, including values implied on the basis of optimal asset demand behavior in relation to actual asset return properties during 1960-1980, as well as values estimated on the basis of the observed portfolio behavior of the U.S. household sector over this period. As is clear from this set of comparisons, there is little ground here for drawing any conclusion at all about even the sign, much less the magnitude, of the substitutability of short-term debt and equity. Although the implied optimal behavior indicates that these two assets are substitutes, the observed behavior indicates that households have treated them as complements. By contrast, the values assembled here consistently indicate that long-term debt and equity are indeed substitutes. Moreover, with a few exceptions the various quantitative estimates of the associated substitution elasticity are quite closely clustered around the value -.035.[45]

For several reasons, it is difficult to know what (if any) broader economic and financial conclusions to draw from these results. Even for the one fairly consistent result that runs throughout the paper, the substitutability of long-term debt and equity, focusing on the sign leads to different implications than does focusing on the magnitude. At one level, the finding that (long-term) debt and equity are substitutes validates the standard assumption underlying a variety of familiar models in monetary economics and finance. At the same time, if the elasticity is really only -.035, then many of these models' more important substantive conclusions do not follow.

In addition, the analysis undertaken here indicates several conclusions at a more detailed level that also warrant caution: First, while the observed variance-covariance structure of real asset returns in the United States during 1960-1980 implies that debt and equity securities are substitutes, the variance-covariance structure changed between the 1960s and the 1970s, and the resulting differences imply sharply changed optimal substitution responses of the demand for debt and equity to their respective expected returns. For the relationship between short-term debt and equity, even the implied sign of the relevant optimal response differs between the two sub-samples.

Second, the estimated substitution properties of assets other than long-term debt and equity do not bear much systematic resemblance to the optimal responses implied by the observed variance-covariance structure. The system of asset aggregation employed does not appear to affect this conclusion in an important way, nor does taking explicit account of the nonstationary stochastic return structure appear to improve the relevant estimates.

Third, the data consistently warrant rejecting the hypothesis of symmetrical asset demand responses to variations in expected yields on alternative assets. This result does not contradict the theory of portfolio behavior based on expected utility maximization in general, but it does contradict the familiar specific form of that theory relying on joint normal (or lognormal) assessments of asset returns and on constant relative (or absolute) risk aversion.

To be sure, the empirical analysis presented here does not lack limitations to provide potential explanations for the more perplexing aspects of these results. The treatment of the aggregate household sector as if it were one individual's portfolio, the exclusion of nonfinancial assets (and hence of liabilities), the use of actual instead of instrumented returns except for capital gain and loss components, and the simplicity of the approaches taken to allow for means-of-payment services and transactions costs all constitute potential reasons for believing that there may well be substantial discrepancies between the behavioral parameters estimated here and the corresponding actual properties of household portfolio behavior.

Even so, the troubling possibility remains that the most important explanation for the problematical aspects of the results found here is instead that the expected asset returns and the associated variance-covariance structure inferred here do not closely correspond to the perceptions that investors actually held. One potential reason for suspecting descrepancies, of course, is the ever-present need for arbitrary assumptions in order to proxy unobservable expectations.[46] Even more troubling, however, is the possibility that investors systematically misperceived the real asset returns they confronted — in other words, that investors not only lacked perfect foresight about each quarter's capital gains and losses but, even over a substantial period of time, failed to understand the basic properties of the distributions generating real returns. With four of five assets exhibiting negative mean real returns over the entire two decades, and the implied optimal holdings consistent with those mean returns positive for only two of the five assets, it is difficult to reject out of hand the possibility that investors went through much of this period consistently anticipating more favorable returns than in fact materialized. That such behavior presents formidable obstacles to formal analysis, or even that it contradicts fashionable ideas about the formation of expectations, cannot rule it out.

Footnotes

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1. Tobin (1961) relied on this distinction in arguing that, if it were necessary to aggregate debt with either money or equity in a macroeconomic model, the former choice would be superior. Subsequent empirical work emphasizing the same distinction has included Fama and Schwert (1977) and Bodie (1982).

2. This line of reasoning also leads to a distinction, which lies beyond the scope of this paper, between default-free government debt and defaultable private debt.

3. Exceptions arise, however, as in corporate merger or acquisition transactions settled by direct exchanges of securities.

4. See, for example, Arrow (1965) or Cass and Stiglitz (1970).

5. For evidence supporting the assumption of constant relative risk aversion, see Friend and Blume (1975). Although Fama (1965) and others have shown that individual securities returns are not strictly normally (or lognormally) distributed, Lintner (1975) has shown that

the approximation involved here is close enough for most purposes, and more recently Fama and MacBeth (1973) have also relied on the normality assumption. See Friedman and Roley (1979b) for the explicit derivation of expressions (7)-(11) below under the assumptions of constant relative risk aversion and joint normally distributed return assessments. (These two assumptions are not strictly compatible, because normality in principle admits negative gross returns for which constant-relative-risk-aversion utility is not defined; but the approximation involved here is hardly troubling.)

6. The rationale for mean-variance analysis provided by Samuelson (1970) and Tsiang (1972), for example, suggests that mean-variance analysis per se is only an approximation that depends on (among other factors) a small time unit. The time unit used in the empirical work presented in this paper is a calendar quarter. Although the observed variation of some asset prices is large over this time unit, it is the expected variation that matters here.

7. More precisely, under constant relative risk aversion symmetry holds only as an approximation that is acceptable as the time unit is small; see again footnote 6. Symmetry would hold exactly in this model only under the alternative assumption of constant absolute risk aversion. See Roley (1983) for a thorough analysis of the conditions determining symmetry in asset demand systems.

8. Here and throughout this paper, elasticities of substitution ε_{ij} are defined in terms of net returns r_j^e as is more typical in the portfolio demand literature, rather than gross returns $(1 + r_j^e)$ as would be analogous to the consumer demand literature because of the reciprocal relationship between expected gross returns and asset

prices. Because the marginal responses β_{ij} are invariant to this distinction, the corresponding gross-return elasticities just equal the net-return elasticities as shown in (15) but with $(1 + r_j^e)$ in place of r_j^e.

9. Uniformly nonnegative partial correlations imply uniformly nonnegative simple correlations, of course, so that the latter is also a necessary (though weaker) condition for gross substitutability of all assets when a risk-free asset exists.

10. Even in the presence of a risk-free asset, it is just as easy to inspect the Ω^{-1} directly as to compute the partial correlations on which Blanchard and Plantes focus.

11. These data, from the Federal Reserve Board's flow-of-funds accounts, give market values for equity and par values for all other assets. Because interest rates exhibited an upward secular trend during 1960-1980, the period analyzed here, par-value data presumably overstate holdings of long-term debt. This problem does not arise for money or for time and saving deposits, and it is too small to be of consequence for short-term debt.

12. See Jones (1979) for a careful treatment of the conditions required for asset aggregation. Section III below briefly considers two further alternative asset aggregation schemes.

13. Some preliminary experimentation using the respective price deflators for gross national product and for personal consumption expenditures indicated that the results presented in this paper are not sensitive to the choice of specific inflation measure.

14. Because the Internal Revenue Service data needed to estimate these marginal tax rates for 1980 were unavailable as of the time of writing,

the 1979 rates were used to calculate 1980 after-tax returns. No major tax changes occurred in 1980.

15. For examples of work based on interest rate surveys, see Friedman (1979b) and Kane (1983).

16. As Smith points out (see his comments below), Table 4 shows the unconditional variance-covariance structure of returns. Hence it both overstates and understates investors' knowledge. The overstatement comes from implicitly giving investors, within the sample, knowledge of the full-sample return distribution parameters. The understatement comes from ignoring investors' use of the serial correlation properties of returns. Smith's suggestion of using regressions (perhaps with rolling samples) to derive conditional estimates is sensible, and I plan to implement it in further work along these lines. The use of the "beta" values derived in Section IV below is analogous.

17. The results found by Friend and Blume (1975) suggest a value of ρ between 1 and 2. More recent work by Grossman and Shiller (1981), using altogether different evidence, suggest a greater value. Bodie et al., in this volume, assume $\rho = 4$.

18. The corresponding gross-return elasticities are $\varepsilon_{SE} = -.836$ and $\varepsilon_{LE} = -1.23$. Because the gross-return means are positive, it is also possible to calculate the analogous gross-return elasticities of substitution referring to the response of the demand for equity to the expected return on either short-term or long-term debt. These elasticities are $\varepsilon_{ES} = -.124$ and $\varepsilon_{EL} = -.334$, respectively.

19. The finding that investors would not have held positive amounts of long-term debt under these assumptions is familiar; see Bodie (1982)

and Bodie et al. in this volume. What is surprising here is that, in the presence of money and time and saving deposits, as well as short-term debt, under these assumptions investors would not have held positive amounts of short-term debt either.

20. Deriving $\underline{\delta}$ directly from the underlying expected utility maximization would require an explicit representation of the transactions process and the associated role of means-of-payment services.

21. The analysis here includes only the assets that households own directly and via personal trusts. An alternative approach would be to include as well assets in which households have an interest via pension and insurance arrangements. Inferring the risk properties of pension and insurance assets would be highly problematical, however (unless, of course, the pension or insurance intermediary form were treated simply as a shell performing no risk-transformation services at all). In the limit, if all assets in the economy were aggregated together and imputed to the household sector as ultimate owner, there would be no basis for distinguishing the resulting "asset demand" equations from the corresponding "asset supply" equations.

22. The purpose of this procedure is to generate series of seasonally adjusted end-of-quarter asset stocks without any gaps or inconsistencies due to splicing of data series. (The Federal Reserve System does not construct such series.)

23. Out of $1,494 billion of household sector liabilities outstanding as of yearend 1980, $971 billion consisted of mortgage debt and $385 billion of installment and other consumer credit.

24. The two capital gain equations used are

$$cg_{Lt} = \underset{(-1.2)}{-1.63} + \underset{(5.0)}{0.567}\, cg_{L,t-1} - \underset{(-2.8)}{0.366}\, cg_{L,t-2}$$

$$+ \underset{(2.9)}{0.387}\, cg_{L,t-3} - \underset{(-0.0)}{.000615}\, cg_{L,t-4}$$

$$\bar{R}^2 = .28 \qquad SE = 11.25 \qquad DW = 1.99$$

$$cg_{Et} = \underset{(2.1)}{5.85} + \underset{(3.5)}{0.393}\, cg_{E,t-1} - \underset{(-2.2)}{0.268}\, cg_{E,t-2}$$

$$- \underset{(-0.0)}{0.00331}\, cg_{E,t-3} + \underset{(0.1)}{0.017}\, cg_{t-4}$$

$$\bar{R}^2 = .12 \qquad SE = 23.18 \qquad DW = 2.00$$

where the standard errors are in per cent per annum. In light of the familiar random-walk rendering of the efficient market hypothesis, it is interesting to note how much of the variance of observed capital gains (which are just transformations of observed price changes) even relatively simple autoregressive processes achieve — ex post. Multivariate analogs to these equations, including also lagged values of the associated coupon or dividend/price yields as well as short-term yields, produce $\bar{R}^2 = .47$ and SE = 9.66 for long-term debt capital gains, and $\bar{R}^2 = .36$ and SE = 19.77 for equity capital gains. These returns are based on month-average data for the last month in each quarter, so that Working's (1960) point about spurious autocorrelation applies. Even so, the fit of these (ex post) equations is striking.

25. The table excludes the estimated values of $\underline{\delta}$ and $\underline{\pi}$, so as to avoid diverting attention (and allocating space) to results not central to the paper's objectives. Subsequent tables of empirical results presented below reflect the same selectivity.

26. The expected returns evolve not independently, of course, but by

the market-clearing behavior of asset demanders and asset suppliers (including, to a limited extent, households). To the extent that households' behavior is a major element determining market-clearing returns, these returns are not really predetermined in (16), and an instrumental variables procedure is appropriate. Here only the capital gain component of the returns on long-term debt and equity are instrumented.

27. In previous work I have criticized this partial adjustment model for not adequately reflecting the greater sensitivity to expected returns of the allocation of new cash flows in comparison to the re-allocation of existing asset holdings under most transactions cost technologies, and have suggested an "optimal marginal adjustment" model as an alternative; see, for example, Friedman (1977). Applying the optimal marginal adjustment model in the context of the analysis presented below is an object left here for future work.

28. The price and population variables used to deflate the nominal magnitudes are the consumer price index (1967 = 1.00) and the total U.S. population (in millions). For purposes of comparison with the magnitudes shown in Tables 1 and 6, their respective 1980:IV and 1960-80 mean values are 2.658 and 1.322 for the price index, and 228.6 and 204.9 for population. Using the current period's price (and population) to deflate the vector of lagged asset stocks in (18) represents a multivariate generalization of the "nominal-adjustment" model suggested by Goldfeld (1976).

29. In terms of shares of the 1960-1980 mean portfolio, the five standard errors (in the order used in the tables) are .00136, .00377, .00482,

.00187, and .00061.

30. Because the five equations being estimated all share identical sets of regressors, full-information maximum likelihood is equivalent to ordinary least squares. As a check, the equations were actually estimated twice, once using each method. The corresponding sets of results were identical.

31. More precisely, each equation was estimated three times: twice as explained in footnote 28 (without any constrained values) and then a third time, by maximum likelihood, with the Θ_{ij} held fixed at the values estimated (identically) the first two times. Fixing the Θ_{ij} in this way does not affect the estimated values of the other coefficients, but does affect the associated t-statistics.

32. The literature associating an implicit nonpecuniary return to holding money balances is potentially instructive here; see, for example, Barro and Santomero (1972) and Klein (1974).

33. As is the case for the β_{ij} coefficients shown in Table 9, the t-statistics reported in Table 10 are derived by taking the Θ_{ij} values as given. Here the system of equations was first estimated by the nonlinear maximum-likelihood method, subject to the symmetry restriction on B but no restriction on Θ. The resulting Θ_{ij} values were those imposed on the final estimation. Once again, this procedure does not affect the estimated values of the coefficients, but it does affect the associated t-statistics. The summary statistics shown in Table 10 are comparable to those in Table 9, in that they refer to the initial joint estimation of the Θ_{ij} along with the other coefficients.

34. Roley (1983) also rejected the symmetry restriction. From an inspection of the pattern of signs among the off-diagonal β_{ij} values shown in Table 9, it appears as if only the coefficients in the row and column corresponding to money and its expected return are inconsistent with symmetry of matrix B. Nevertheless, an attempt to estimate (18) subject to a symmetry constraint applied only to the remaining four rows and columns of B yielded unsatisfactory results.

35. The implied coefficient of relative risk aversion is -1.68, with t-statistic -4.3.

36. This alternative aggregation scheme is the one proposed by Smith; see his comments in this volume.

37. See Ando and Shell (1975) for a theoretical justification for this two-part strategy. Several authors have investigated empirically the allocation of the household sector's liquid asset portfolio; see, for example, Fortune (1972).

38. Using simply the short-term debt return, as suggested by Smith, yields essentially identical results.

39. See Friedman (1979a) for a discussion of the information acquisition process in a parallel context in the macroeconomics literature.

40. A break at mid 1970 not only represents the sample midpoint but also roughly corresponds to several familiar changes in the objective circumstances determining real asset returns, including the Federal Reserve System's adoption of a monetary aggregate target in February 1970 and its suspension of Regulation Q interest ceilings on large time deposits in June 1970. More broadly, but also a good deal more roughly, the 1970s were a decade of slower real growth, more frequent

business recessions, faster price inflation, less capital formation, and larger federal government deficits than in the 1960s. There is also substantial evidence of a further break associated with the Federal Reserve's further change in operating procedures in October 1979 — see, for example, Friedman (1982) — but splitting the 1960-1980 sample at that point would serve little purpose here.

41. All implied substitution elasticities shown in Table 17 have unchanging sign across the two sub-samples except that between short-term debt and equity, but the elasticities of substitution between long-term debt and money and between long-term debt and time deposits differ in sign from the corresponding implied elasticities for the full sample shown in Table 5.

42. Friedman (1980) and Friedman and Roley (1979a) dealt with this problem by selectively including moving-average variances (but not covariances) in estimated asset demand equations.

43. Hence the "market" portfolio excludes nonfinancial assets; see again the discussion in Section I.

44. Programming the nonlinear estimation package to solve directly for the γ_{ij}, analogously to the β_{ij}, would make proper convergence of the nonlinear maximum likelihood estimation problematical.

45. On the basis of the mean equity return of 3.13%, used in calculating all of the full-sample elasticities shown in Table 19, a mean-return elasticity of -.035 corresponds to a gross-return elasticity of -1.15.

46. See again the discussion in Section I.

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