

FAQs :

- * 2 Quizzes (15%)
- * 2 Exams (35%)

Mid Term + End Term

↳ Subj. to Modification
if mode → Online

Schedule: T, Th, F 10-12 (L 10)

Prof. Room: KD 223

Books / References :

- ① Macroeconomics by Blanchard (7th Ed.)
- ② " " Jones
- ③ " for Professionals by Lipschitz b Schadler

- ↑ the rates → Control Inflation } Changing the rates
↓ the rates → } influences aggregate
- Prosperity/Well being depends on Productivity
Prod. ↓ → Long Run Poorer Good Measure
of long term
- US → Caused GFC (00's) due to Mortgages
- EU → Euro [Common Currency in a lot of Nations]
- India → Demonitisation, Liberalisation (~80's/91), etc.
- Economies, like people, get sick -
High unemployment, recessions, financial crisis, low growth
- Macroeconomics is about why it happens,
and what can be done about it.
- 3 central **Macroeconomic variables**:

① Output ② Unemployment ③ Inflation

↳ GDP (Gross Domestic Product):

Standardized Value,
not intrinsic
value

Products:

Market value of all Final

Goods and Services produced in an
economy in a given time period.

Within the economy,
Not outside it

Don't consider
Intermediate
Revenues

specific
Time Period
Annually, Quarterly, etc.

Ex: Steel Company (F1)

Revenues from Sale: 1000

Expenses -

Wages → 800

Profit → 200

Car Company (F2)

Revenue: 2000

Expenses -

Wages → 700

Steel purchase → 1000

Profit → 300

GDP: Measure 1: Value of final goods & services

→ 2000

Measure 2: GDP is sum of value added
in the economy during a
given period

$$VA_{F_1} = 1000, VA_{F_2} = 2000 - 1000 = 1000$$

$$GDP = VA_{F_1} + VA_{F_2} = 2000$$

VA by a
firm is the
value of its
production
minus the
value of the
intermediate
goods used in production.

Measure 3: GDP is sum of incomes in the
economy during a given period.

$$(800 + 200) + (700 + 300) = 2000$$

Ex:

	Price	Quantity
Caps	10	5
Hats	20	15

$$\text{GDP}_{1999}: 10 \cdot 5 + 20 \cdot 15 = 50 + 300 = 350$$

{ Price ← (P · 10)

GDP₂₀₀₀

3500

"Production" has been the same but GDP ↑ due to Market Value.



Nominal

v/s.

Real

GDP

Sum of Quantities of Final Goods produced times their current price.

Sum of Quantities of Final Goods times

constant prices.

↳ Fix a particular year

Ex.	Y	Q	P	Used for RGDP
	2008	10	20000	
	2009	12	24000	
	2010	13	26000	

NGDP

200 000

288 000

338 000

RGDP

200 000

240 000

260 000

- GDP v/s. Per Capita Income : India Really ↑ in GDP due to size (population) but very ↓ in per capita income.

PPP : Purchasing Power Parity

Unemployment

- Employed: No. of people who have a job. (E)
- Unemployed: No. of people who do not have a job but are looking for one. (U)
- Labour Force : Sum of Employed & Unemployed. (L)

$$L = U + E$$

$$\text{Unemployment Rate} = \frac{U}{L}$$

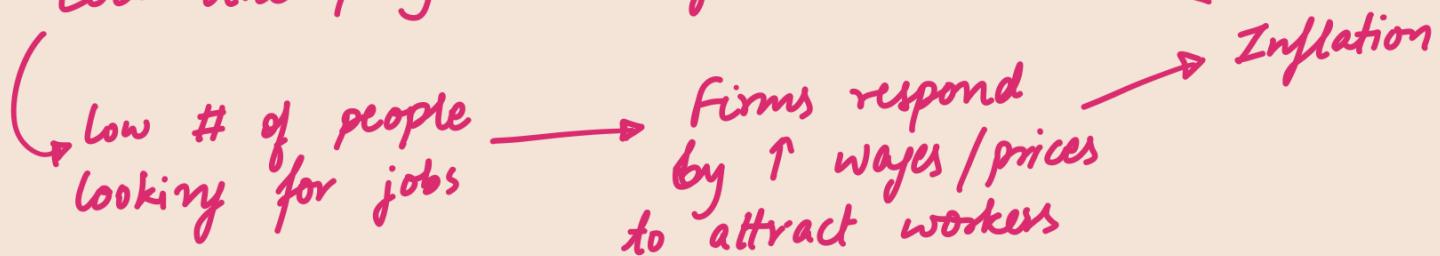
$$(\text{Labour Force}) = \frac{L}{\text{Participation}}$$

Generally,
only adults
are considered

(Labour force) participation rate is the ratio of labour force to the total population of working age

- High Unemployment: Bad → $\frac{\text{Economy} \downarrow}{\text{People} \rightarrow \text{Upset} \rightarrow \text{Economy: Aggregate of People}}$

Low Unemployment: Might be bad



Inflation: Sustained rise in the general level of prices in the economy.

Deflation: Sustained fall ...

Inflation Rate: Rate at which the price level increases.

GDP deflator:

Not as important as

$$P_t = \frac{\text{Nominal GDP at } t}{\text{Real GDP at } t}$$

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} \rightarrow \text{Rate of Inflation}$$

price ~ $\frac{\text{monetary output}}{\text{output}}$

CPI → Consumer Price Index

Fix a basket → Trace what's going to happen to the price of the basket overtime.
Update basket, consumer patterns change.

→ Quality of products change.

→ Not going to be the same for everyone.

CPI might diverge from GDP-deflator due to prices of imported goods (included in CPI).

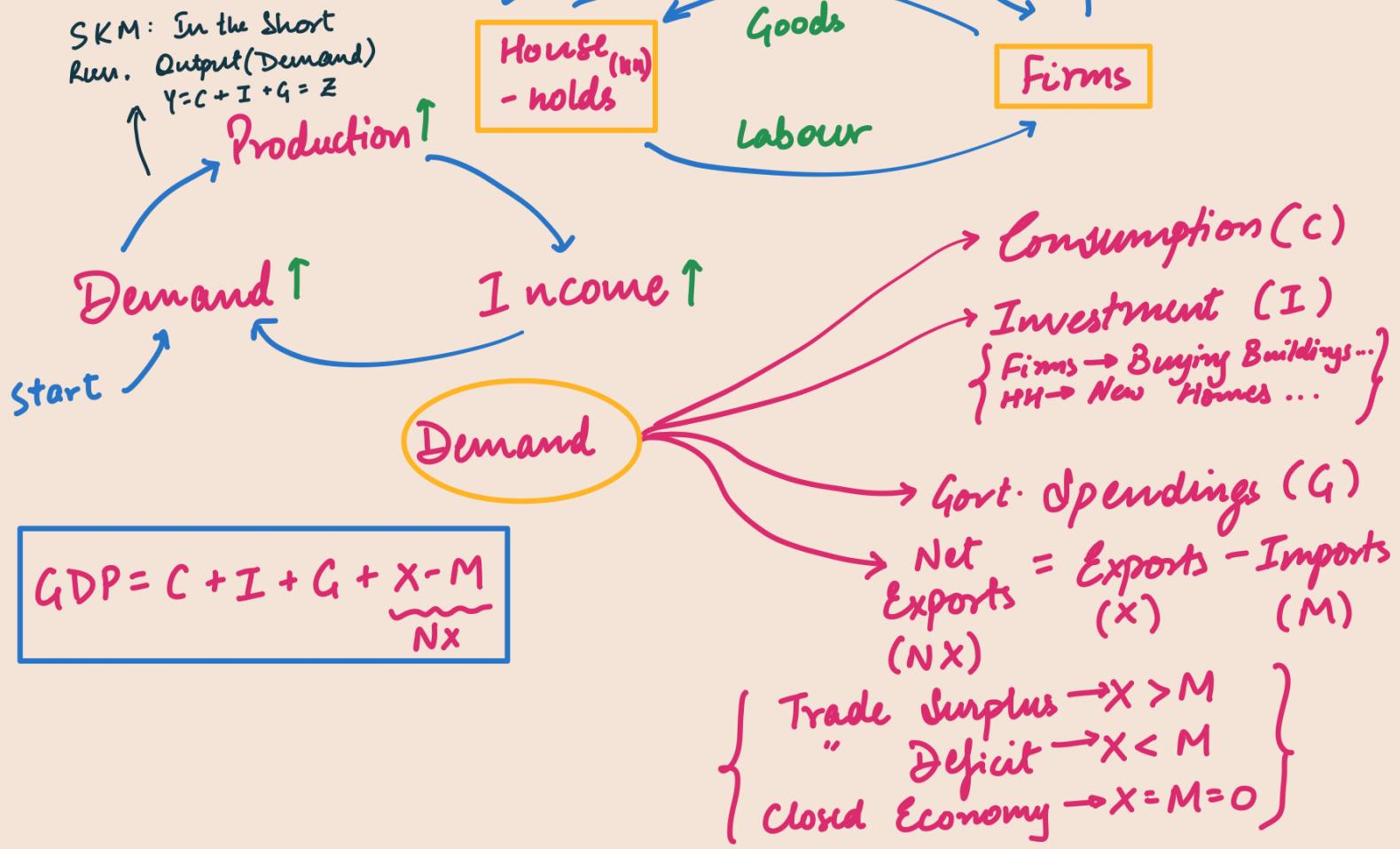
WPI → Wholesale Price Index

Core Inflation → Excludes food & energy from CPI basket.

What explains GDP? / Output

- Short run → Demand
 - Medium run → Supply-side factors
 - Long run → Roots, Geography, Culture, fundamental foundations, etc.
- labour.
Capital, etc.
- $\sim 1-2$ y.
- ~ 10 y.
- ~ 50 y.

Circular Flow of Income



What determines the Output of an Economy?

- Assumption: $M = X = 0 \leftarrow$ Closed Economy
- $C: C(Y_D) \rightarrow$ Disposable Income, $Y_D = Y - T$
- C is an Inc. function of Y_D
- Functional form $\rightarrow C = C_0 + C_1 Y_D, 0 < C_1 < 1, C_0 > 0$
- $C = C_0 + C_1 (Y - T)$
- Taxes - Govt. Transfers
- Income
- Endogenous vars.

Exogenous Vars.

"Given" to us from outside our model → Used to calculate

$I = \bar{I}, G = \bar{G}$

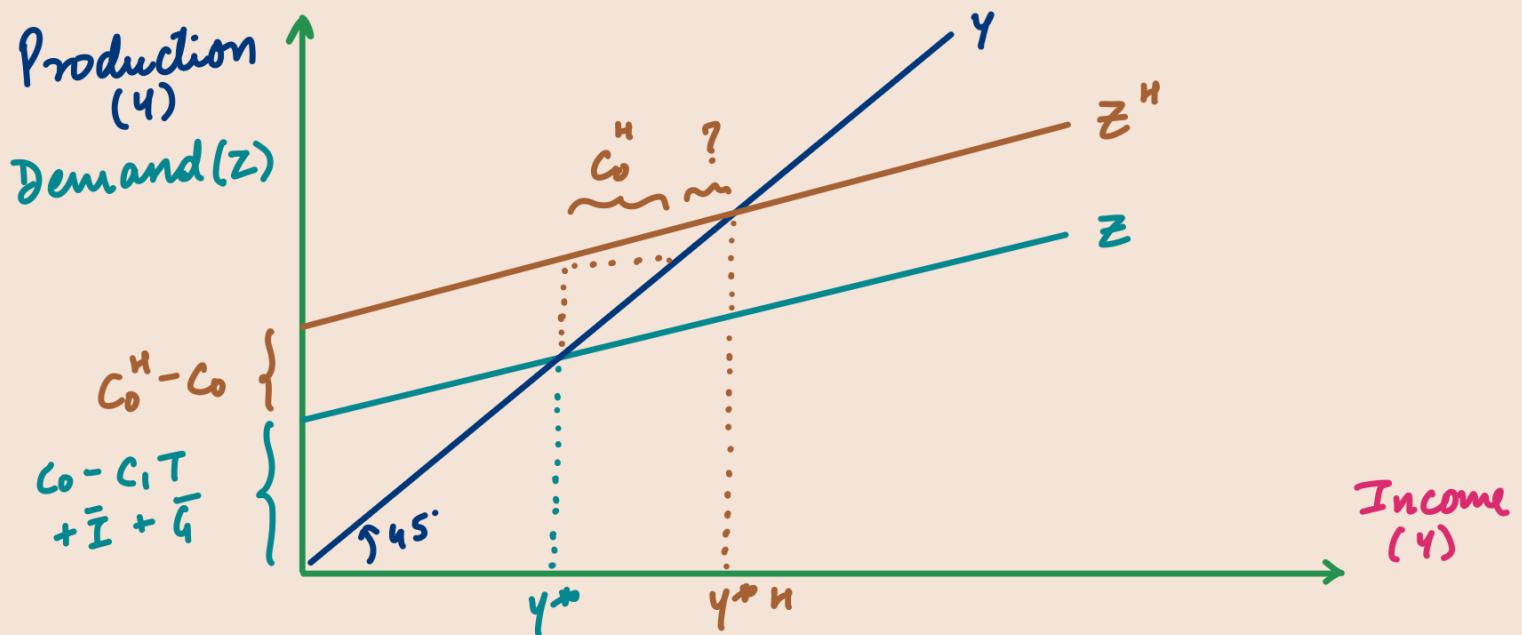
$$Z = \text{Demand} = C + I + G$$

$$= \underbrace{C_0}_{\text{Exo}} + \underbrace{C_1(Y-T)}_{\text{Exo}} + \underbrace{I+G}_{\text{Exo}}$$

Equilibrium: Demand (Z) = Production (Y)

$$Y = C_0 + C_1(Y-T) + \bar{I} + \bar{G}$$

$$\Rightarrow Y = \frac{C_0 - C_1 T + \bar{I} + \bar{G}}{1 - C_1}$$



- Suppose $C_0 \uparrow$ to C_0'' . How does Y^* change? It \uparrow to Y'' . But $Y'' - Y^* > C_0'' - C_0$, why? Because of Multiplier. Visible graphically, but mathematically,

$$Y = C_0 + C_1(Y-T) + \bar{I} + \bar{G} \rightarrow C_0 \uparrow \text{ by 1 unit} \quad \left. \begin{array}{l} \text{due to} \\ \text{Circular} \\ \text{Flow of} \\ \text{Income;} \\ \text{Known as the} \\ \text{Multiplier} \\ \text{Process} \end{array} \right\}$$

$$\frac{\Delta Y}{\Delta C_0} = \frac{1 + C_1 + C_1^2 + \dots}{1 - C_1} = \frac{1}{1 - C_1} > 0$$

$$\hookrightarrow \frac{\partial Y}{\partial C_0} \downarrow \quad \begin{array}{l} Y \uparrow \text{ by 1 unit} \\ Y \uparrow \text{ by } 1 \cdot C_1 \\ Y \uparrow \text{ by } C_1 \cdot C_1 \\ Y \uparrow \text{ by } C_1^2 \cdot C_1 \dots \end{array}$$

Savings + Investments

$$Y = C + I + G$$

$$= (G - T) + I + G + T \Rightarrow \underbrace{(Y - T - C)}_{S_P} + \underbrace{(T - G)}_{S_G} = I$$

Total Savings in an Economy

$y - T - C$: Private Savings (S_p)
 $T - G$: Govt. Savings (S_G)

$$S_p + T - G = I$$

- * $S_p = Y - T - C = Y - T - C_0 - C_1(Y - T) = (1 - c_1)(Y - T) - C_0$
- $\Rightarrow S_p = (1 - c_1)(Y - T) - C_0, I = S_p + T - G$
- If $C_0 \downarrow \Rightarrow I \sim, S_p \sim$ { ↓ consumption
↑ doesn't ↑ savings } Paradox of Saving
 $\frac{\partial S_p}{\partial C_0} = 0$ or "Paradox of Thrift":
 $C_0 \downarrow S_p \uparrow$ but saving more
of something less
(y)

* Goods market \rightarrow How output is determined via the [SKM] market in the short run.

TLDR: In the short run, output is determined by demand.

* : Comparative Statics : - Effect of parameters on Equilibrium.

Comparing \leftarrow Static \leftarrow
Expressions

E.g. $Y = Z = \underbrace{C(Y - T)}_{\text{Not necessarily linear}} + \bar{I} + \bar{G}$ i) Effect of $\uparrow \bar{G}$ on Y ?
ii) Effect of $\uparrow T, \bar{T}, \bar{G}$ by same amount on Y ?

i) $dY = C' dY + dG \Rightarrow \frac{dY}{dG} = \frac{1}{1 - C'}$

ii) $dY = C'(dY - dT) + dG$
 $\Rightarrow (1 - C') dY = dG - C' dT = dG (1 - C')$
 $\Rightarrow dY = dG \Rightarrow \uparrow \text{ by same amt.}$

Balanced Budget Multiplier
 $\Rightarrow T - G$: (Govt.) Budget Surplus
 $G - T$: Budget Deficit
[Simple Keynesian Model (SKM)]

Saving + Investments

Saving = Investment = \bar{I}

Personal Savings \swarrow Govt. Savings \searrow

$\Rightarrow S = \bar{I}$
 $\Rightarrow Y - T - C + T - G = I$
 $\Rightarrow Y - C - G = I$

$$(S_p) + I_S_G$$

$$Y - T - C \rightarrow T - G$$

$$\rightarrow C_0 + C_1(Y - T) = C$$

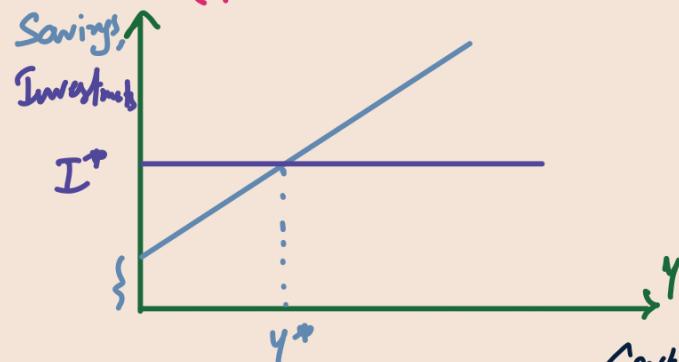
$$\Rightarrow \underbrace{(Y - T)(1 - C_1)}_{S_p} - C_0 + T - G = I$$

$C_1 = \frac{dC}{dY}$ → propensity to consume

(mpc) → marginal propensity to consume

$$\equiv Y = C + G + I$$

(Production = Demand)



$T, G \rightarrow$ Fiscal Policy
Monetary Policy

By Interest rates, etc.

Govt. managing market by taxes & investments

Some Eco. Jargon

- Money: What can be used to pay for transactions
- Income: What you earn (Flow)
- Saving: Part of after tax income that you do not spend (flow) (stock)
- Savings: Value of what you have accumulated over time.
- Financial wealth: Value of all your financial assets (or wealth) minus all your financial liabilities (stock)
- Investment: Purchase of new capital goods.
- Financial Investment: Purchase of shares or other financial assets.
- Flow: Over a certain period of time / per unit of time.
- Stock: Cumulative / at a certain instant of time / point of time

Central Bank and the Financial Market

Assume
Only 2 assets

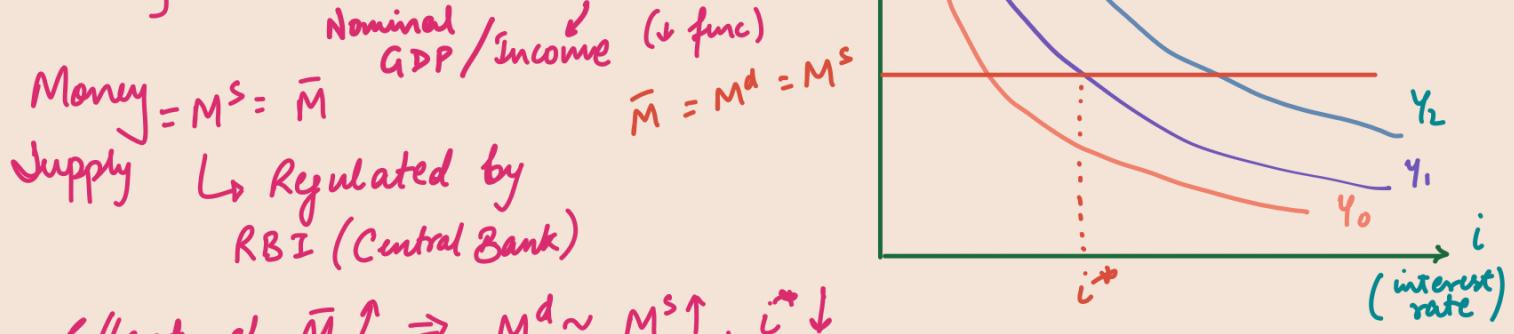
Money $\rightarrow 0$ interest rates

Bonds $\rightarrow i$ interest rates

$$\text{Money Demand} = M^d = Y L(i)$$

$$M^d \uparrow$$

$$Y_2 > Y_1 > Y_0$$



Supply \hookrightarrow Regulated by
RBI (Central Bank)

- Effect of $\bar{M} \uparrow \Rightarrow M^d \sim, M^S \uparrow, i^* \downarrow$
 $M^S \uparrow \Rightarrow M^{d*}$ has to adjust \rightarrow Can come only from $i^* \downarrow$

Central Bank's Balance Sheets

Assets	Liabilities
Bonds	Currencies



$i \downarrow \leftarrow P^* \uparrow \leftarrow RBI \uparrow \text{demand of bonds} \leftarrow$

$M_S \uparrow \Rightarrow i \downarrow$



$$i = \frac{100 - P_B}{P_B} \quad \begin{cases} \text{Interest rate} \\ v/s. \text{ Price of Bonds} \end{cases}$$

$$P_B = \frac{100}{1+i} \leftrightarrow P_B \uparrow \quad i \downarrow$$

CB

A	L
Bonds	Central Bank Money = Currencies + Reserves

Banks \longleftrightarrow Financial Intermediaries

A	L
Reserves, Loans, Bonds	Checkable Deposits

* Financial Market \rightarrow Demand for Money

Currencies \leftarrow Checking accounts

RBI controls Supply of Money

Liquidity Trap \rightarrow Zero Lower Bound



IS-LM \sim Hicks-Hansen Model

- Capitalism \rightarrow Everyone free to buy whatever they want
(subject to personal affordability, etc.)
 \hookrightarrow Western Countries
- Socialism \rightarrow Govt. dictates/controls the output / production

Soviet Countries

* **IS-LM** → Investment savings - liquidity preference - money supply

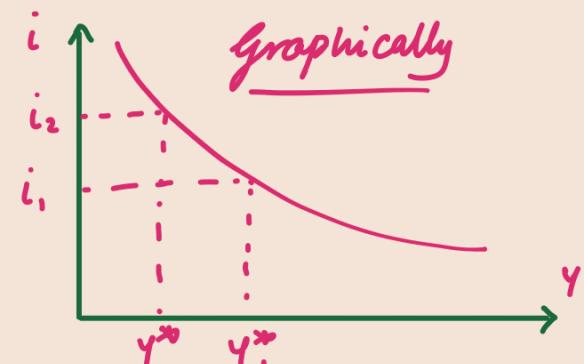
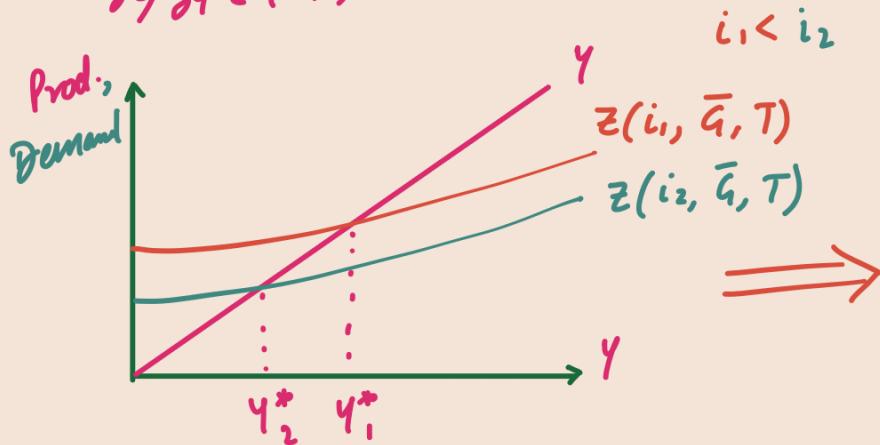
↳ Equilibrium in goods and financial markets
 IS (y, i) → goods market in equilibrium

$$y = C(y-T) + I(y, i) + \bar{G}$$

Axs. $0 < \frac{\partial C}{\partial y} + \frac{\partial I}{\partial y} < 1$

{ Similar to the prev. model }
 $\Rightarrow \frac{\partial y}{\partial i} \in (0, 1)$

$I(y, i)$
 $i \uparrow \Rightarrow$ Better to buy than invest
 More Money, More investment
 Opportunity Cost ↑ for invest
 Hence ↓ func.



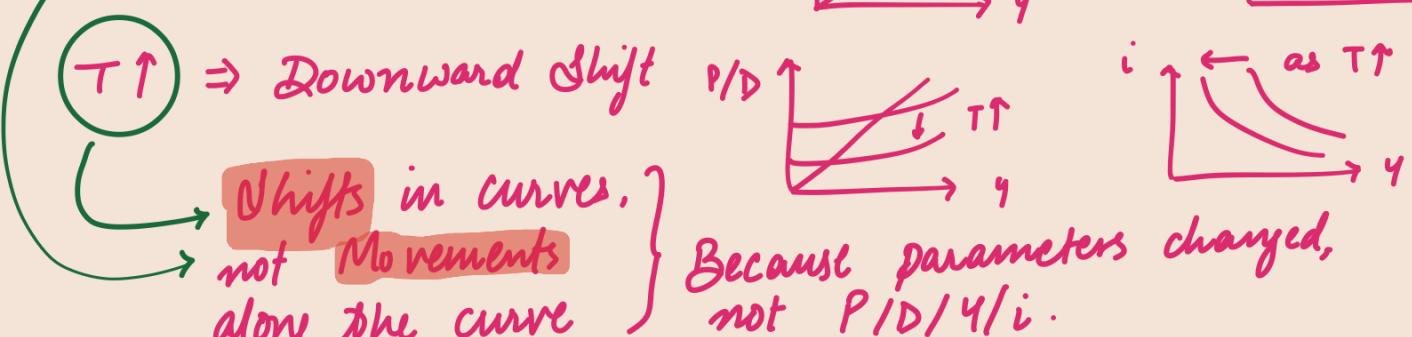
Algebraically. $dy = \frac{\partial C}{\partial y} dy + \frac{\partial I}{\partial y} dy + \frac{\partial I}{\partial i} di + 0$

$$\Rightarrow \frac{dy}{di} = \frac{\frac{\partial I}{\partial i}}{1 - \frac{\partial C}{\partial y} - \frac{\partial I}{\partial y}}$$

$0 < (+ve) < 1$

$\frac{-ve}{+ve} \Rightarrow \frac{dy}{di} = -ve$
 ↓ func. ↘

Words. $i \uparrow \Rightarrow I \downarrow \Rightarrow$ Consumption & Production ↓ $\Rightarrow y \downarrow$



LM: (i, Y) s.t. financial market is in equilibrium

$$M_d = Y L(i), M_s = \bar{M} \rightarrow \frac{\bar{M}}{P} = Y L(i) \rightarrow \ln(\bar{M}) = \ln Y + \ln L(i)$$

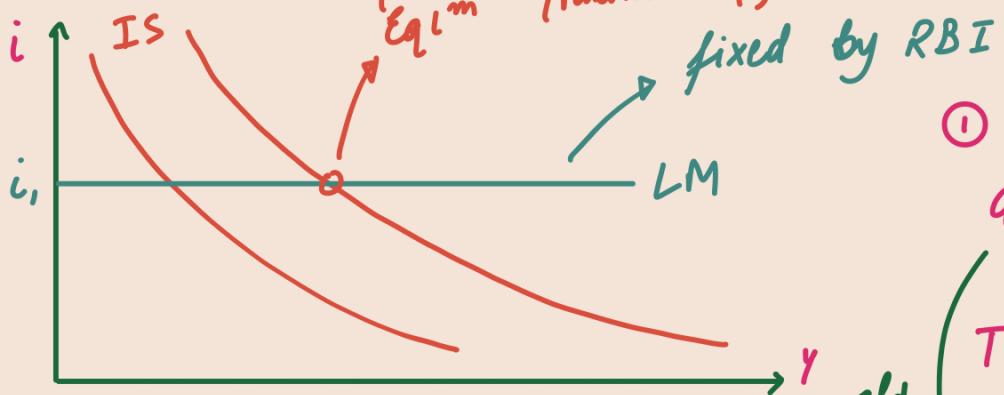
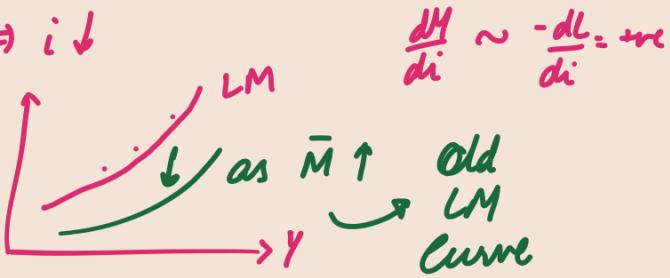
Real, not Y^m

$$\frac{\partial}{\partial i} = \frac{\partial Y}{\partial i} + \frac{\partial \ln L}{\partial i}$$



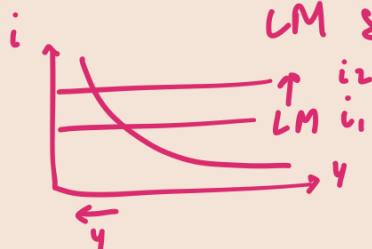
$Y_m \downarrow \Rightarrow M_d \downarrow$ (Shift) $\Rightarrow i \downarrow$

: General Eq^{l,m} : { Goods in eq^m }
Financial in eq^m



② Monetary Policy

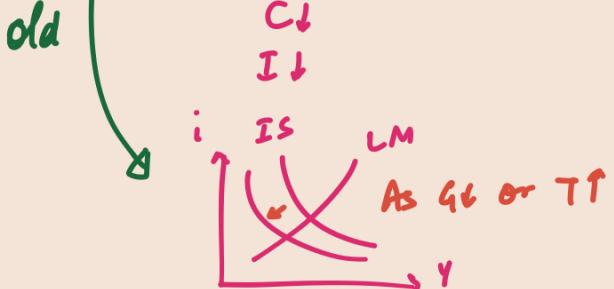
$i_1 \uparrow i_2 \rightarrow$ IS along the curve, LM shift
 $Y \downarrow C \downarrow I \downarrow$



① Fiscal Policy

$G \uparrow \Rightarrow Y \uparrow C \uparrow I \uparrow$

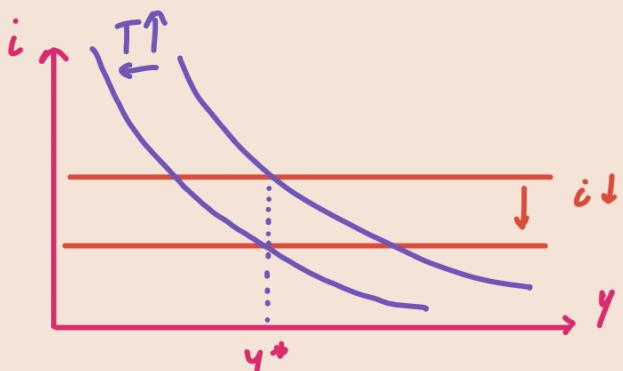
$T \uparrow \Rightarrow Y \downarrow C \downarrow I \downarrow$



IS shifts,
LM along

③ Fiscal + Monetary Policy

$T \uparrow, i \downarrow \Rightarrow Y \leftrightarrow, C \downarrow, I \uparrow$



★ In the Short run, output is determined by the equilibrium between the Goods Market and the Financial Market.



IS-LM → Expectations
→ Open Economy

→ $Y = f(Y^m, Demand-Supply)$

- (i) Fiscal policy (Changes in G, T) } (iii) Combinations of both
 (ii) Monetary policy (Changes in i) } Fiscal and Monetary

. Nominal interest rate is the interest rate in terms of rupees/dollars/euros.

Real interest rate is the interest rate in terms of a basket of goods.

→ Need to adjust nominal interest rates to take into account expected inflation.

Y-0

1 unit of good



P_t monetary value

$$1+r_t = \frac{P_t(1+i_t)}{P_{t+1}^e}$$



$$1+r_t = \left(\frac{1+i_t}{1+\pi_{t+1}^e} \right)$$

Expected price in $t+1$

Expected rate of inflation

Y-1

$1+r_t$ unit of good



$P_t(1+i_t)$ monetary value

$$\begin{aligned} \pi_{t+1}^e &= \frac{P_{t+1}^e - P_t}{P_t} = \frac{P_{t+1}^e}{P_t} - 1 \\ 1 + \pi_{t+1}^e &= \frac{P_{t+1}^e}{P_t} \end{aligned}$$

$$\Rightarrow r_t \approx i_t - \pi_{t+1}^e$$

. Risk Premium → (i) possibility of default

Govt. guarantees $(1+i)$
returns for 1

→ p : probability of default
↳ Company returns $(1+i+x)$, if no default, for 1

$$\Rightarrow (1+i) = \underbrace{p[0] + (1-p)[1+i+x]}_{\text{expected returns}} \Rightarrow$$

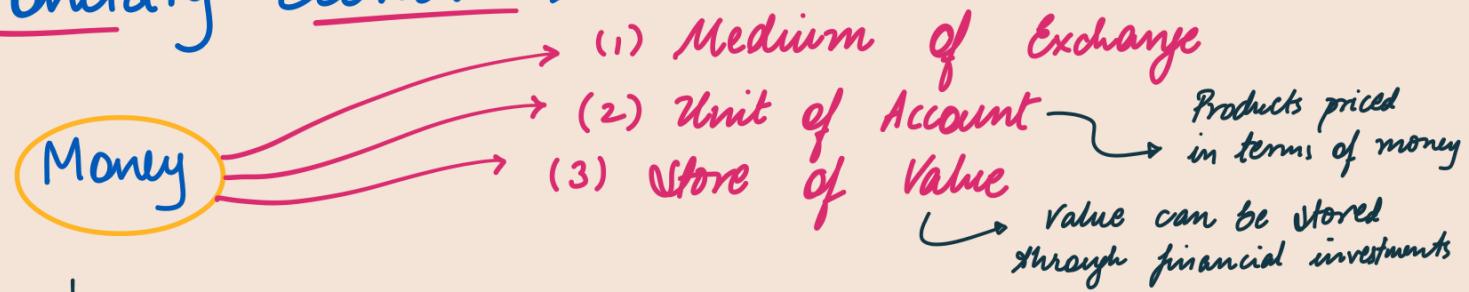
$$x = \frac{p}{1-p} (1+i)$$

$$\Rightarrow x - x_p = p + i_p \Rightarrow x \approx p$$

→ (ii) risk aversion → Govt. offers security, firms offer same expectation w/ risk. So, they need to ↑ their expected returns.

Modify IS-LM w/
 $i \leftarrow r+x$ to account
for these.

Monetary Economics



Barter Exchange: "Double Co-incidence of wants":
Not as prevalent as societies get more sophisticated.

Bread: Perishes, Baker might not always have sufficient supply.

- Characteristics of a currency:
- 1) Durable
 - 2) Easily measured
 - 3) Easily transferable
 - 4) Easily divisible
 - 5) Have Stable value

Viking Economy → No fixed money supply, No central bank

$$M^d = k_1 P_t T_t, \quad P_t T_t = k_2 P_t Y_t \Rightarrow M^d = k_2 P_t Y_t \quad \left. \begin{array}{l} Eq1^m \\ M^s = M \end{array} \right\} M^d = M^s$$

Price
Transactions

$$\Rightarrow M = k P_t Y_t$$

$$\Rightarrow M \cdot \frac{1}{k} = P_t Y_t \Rightarrow MV = P_t Y_t \Rightarrow \frac{M_1}{M_2} = \frac{P_1}{P_2}$$

Velocity of Money

M has no effect on output

① Money demand is an increasing function of value of transactions in the economy.

$$M^d = k_1 P_t T_t$$

② Value of transactions is an increasing function of value of final output in the economy.

$$P_t T_t = k_2 P_t Y_t \rightarrow M^d = k P_t Y_t$$

$$\Rightarrow \boxed{\frac{M^d}{P_t} = k Y_t}$$

$M_t^S = M = \text{No. of gold coins in circulation}$

$$\text{Eq1}^m \quad M^d = M^S \Rightarrow M_t^d = M$$

$$\rightarrow \frac{M}{P_t} = k Y_t \quad \text{Define } \boxed{\bar{V} = \frac{1}{k}} \rightarrow \text{Velocity of Money}$$

Quantity
Equation

~ How many times a coin is exchanged on average

★ $Y_t = A(L_t)$ → Amount of labour put in
 L^* : min./ideal/optimal amount of labour a/c to suppliers

Setting the price: $\frac{P_{t+1}}{P_t} = \left(\frac{L_t}{L^*}\right)^\theta, 0 < \theta < 1$

L_t : if labour $> L^*$, want to ↑ price

θ: don't want to ↑ price too much, might be bad for future

[1] Neo classical Economics

→ All prices rise immediately

→ Output unchanged

→ Money Neutral

$[\Delta M^S$ has no effect
on output]

[2] Keynesian Economics

→ Prices slow to rise
→ output responds to monetary shocks

If abrupt change in demand, prices not changed immediately. changed at e.o.d.
But, labour + output produced at old price.

$$M_t \bar{V} = P_t Y_t$$

$$Y_t = A L_t$$

$$L^* \rightarrow Y^*$$

$$\frac{P_{t+1}}{P_t} = \left(\frac{L_t}{L^*}\right)^\theta$$

$$\Rightarrow \frac{P_{t+1}}{P_t} = \left(\frac{Y_t}{Y^*}\right)^\theta$$

$P_t, Y_t \rightarrow$ endogenous variables

$M_t \rightarrow$ endogenous variables

$0 \rightarrow$ Date at which expedition come back w/ new gold coins

\bar{V}, A, θ : parameters

P_0 ?

$$\log M_t + \log \bar{V} = \log P_t + \log Y_t$$

$$\log P_{t+1} - \log P_t = \theta [\log Y_t - \log Y^*]$$

$$t=-1 \rightarrow Y_{-1} = Y^*$$

$$\log M_L + \log \bar{V} - \log Y^* = \log P_{-1}$$

$$\log P_0 - \log P_{-1} = \theta [\log Y^* - \log Y^*] = 0 \quad \left. \right\} \text{Day } -1$$

$$\Rightarrow \log P_0 = \log P_{-1} = \log M_L + \log \bar{V} - \log Y^* \quad \left. \right\} \text{Day } -1$$

$$M_0 = M_H (> M_L) \quad \left. \right\} \text{Day } 0$$

$$\log M_H + \log \bar{V} = \cancel{\log P_0 + \log Y_0} \quad \left. \right\} \text{Day } 0$$

$$\Rightarrow \log M_L + \log \bar{V} = \cancel{\log P_{-1} + \log Y^*}$$

$$\text{Subtract: } \log M_H - \log M_L = \log Y_0 - \log Y^* \Rightarrow \Delta \log Y_0 = \Delta \log M > 0$$

$$\log P_1 - \log P_0 = \theta [\log Y_0 - \log Y^*] = \theta \Delta \log Y_0 > 0 \\ = \theta \Delta \log M > 0$$

$\Rightarrow P \uparrow$ next day

$$\log M_1 + \log \bar{V} = \log P_1 + \log Y_1$$

$$\log M_0 + \log \bar{V} = \log P_0 + \log Y_0$$

$$\Rightarrow \log P_1 + \log Y_1 = \log P_0 + \log Y_0 \Rightarrow \log Y_1 - \log Y_0 = \log P_0 - \log P_1 \\ = -\theta [\log M_H - \log M_L] < 0$$

$$Y_1 < Y_0$$

$$\text{Also, } L Y_1 - L Y_0 = -\theta [L M_H - L M_L] \quad \left. \right\} L Y_1 - L Y^* = (1-\theta) [L M_H - L M_L] > 0$$

$$L Y_0 - L Y^* = L M_H - L M_L$$

$$\therefore Y^* < Y_1 < Y_0 \quad \& \quad P_1 > P_0$$

So,

$$P \rightarrow \frac{M_H \bar{V}}{Y^*}, \quad \Delta P \propto \Delta M, \quad \text{B} \quad Y \rightarrow Y^* \\ \xrightarrow{\text{Asymptotically}}$$

Financial Intermediaries

★ Capital Ratio: Ratio of Capitals to Assets. $\rightarrow 20/100$

★ Leverage Ratio: Ratio of Assets to Capitals $\rightarrow 100/20$

★ Assets < Liabilities \rightarrow Insolvency

(A) Bank	(L)
Assets $\rightarrow 100$	Liabilities $\downarrow 80$ Capital $\downarrow 20$

Leverage $\uparrow \Rightarrow$ Risk \uparrow , Profitability \uparrow
Ratio

Ex Rate of return on assets = 5% $(L, C) = (80, 20), (90, 10)$

Rate at which you are paying liabilities = 4%.

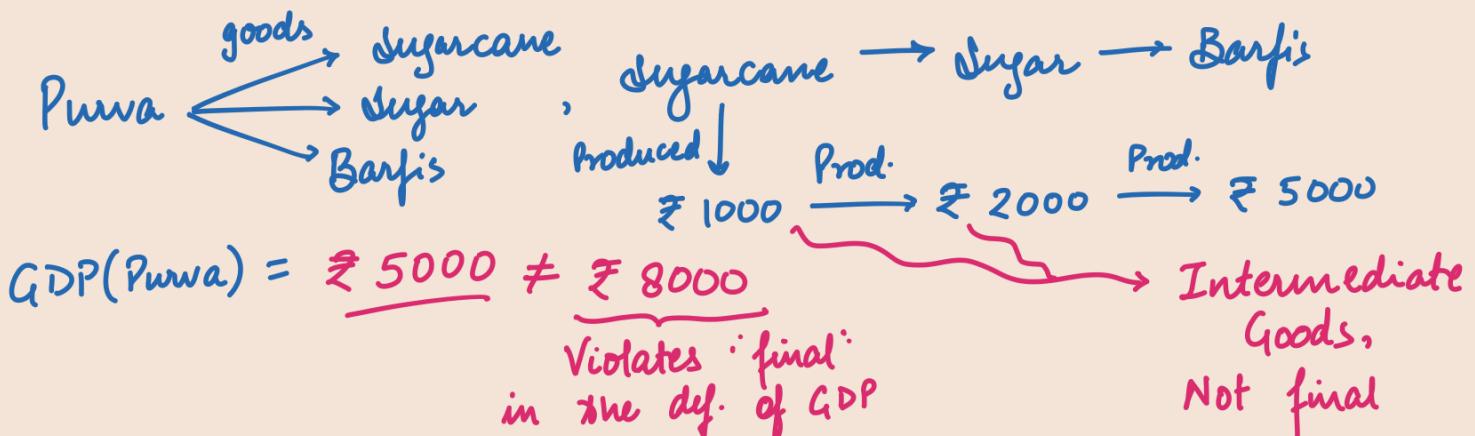
$$(L, C) = (80, 20) \rightarrow \text{Profit} = \frac{5 \cdot 100 - 4 \cdot 80}{100} = 1.8 \rightarrow \frac{1.8}{20} \cdot 100\% = 9\%$$

$$(L, C) = (90, 10) \rightarrow \text{Profit} = \frac{5 \cdot 100 - 4 \cdot 90}{100} = 1.4 \rightarrow \frac{1.4}{10} \cdot 100\% = 14\%$$

Low risk
High risk

Additional Practice Questions 1

Q1



Q2

Dakshin :

2020

↳ Base Year

Quantity/Price:

		BunRoti	Tea	Choco.	Lassi
2020	Q	100	200	50	50
	P	50	20	10	10
2021	Q	50	200	70	40
	P	70	30	20	10

1) Growth in nominal GDP from 2020 to 2021

$$\text{NGDP}_{2020} = 100 \cdot 50 + 200 \cdot 20 + 50 \cdot 10 + 50 \cdot 10 = 10,000$$

$$\text{NGDP}_{2021} = 50 \cdot 70 + 200 \cdot 30 + 70 \cdot 20 + 40 \cdot 10 = 11300$$

$$\text{Growth} = \frac{11300 - 10000}{10000} \cdot 100 = 13\%$$

2) Growth in real GDP from 2020 to 2021

$$\text{RGDP}_{2020} = \text{NGDP}_{2020} = 10000$$

$$\text{RGDP}_{2021} = 50 \cdot 50 + 20 \cdot 200 + 10 \cdot 70 + 10 \cdot 40 = 7600$$

$$\text{Growth} = \frac{7600 - 10000}{10000} \cdot 100 = -24\%$$

GDP deflator

$$P_t = \frac{\text{NGDP}_t}{\text{RGDP}_t}$$

$$P_{2020} = 1$$

$$P_{2021} = \frac{113}{76}$$

$$\Pi_{2021} = \frac{P_{2021} \cdot P_{2020}}{P_{2020}}$$

$$= \frac{113}{76} - 1$$

$$= \frac{37}{76} = 48.68\%$$

Rate of Infl.

$$P_{2021} = \frac{\text{NGDP}}{\text{RGDP}} \xrightarrow{\frac{\ln}{\frac{P_2}{P_1}}} g_p \approx g_{\text{NGDP}} - g_{\text{RGDP}} = 13 - (-24) = 37\%$$

Q3

Rural Urban ① Flood → Agricultural Output ↓

	Rural	Urban
Food	50	20
Housing	20	40
Energy	10	15
Transportation	5	15
Others	15	10

" Prices ↑ "
 Are both CPI's affected
 the same?

NO. Rural CPI is affected much more than Urban CPI.

② Agricultural reforms → Rural income ↑ → How does CPI adjust?

Empirical Observation: Food ↓, other things will take the share.

Q4 Consider a simple Keynesian model (SKM) for a closed economy w/o govt. Suppose Saving & Income, Marginal propensity to invest = 0.3, system initially in eq^l. Parallel downward shift of saving function, eq^{l'm} saving ↑ 12 units. Compute ΔY .

$$S_p = Y - C$$

$$S_T = T - G = 0$$

$$I = S_p + S_T = Y - C$$

$$I = S_p \Rightarrow \Delta I = \Delta S_p$$

$$0.3 \Delta Y = 12$$

$$S_p = Y - C \xrightarrow{\text{shift}} \Delta Y - \Delta C = 12$$

$$Y = I + C$$

$$\Rightarrow \Delta Y = \Delta I + \Delta C \Rightarrow \Delta Y = 0.3 \Delta Y + \Delta C$$

$$\Rightarrow \Delta Y - \Delta C = 12 = \Delta I = 0.3 \Delta Y$$

$$\Rightarrow \Delta Y = \frac{12}{0.3} = \frac{120}{3} = 40$$

Q5 IS-LM Model. Commodity market, Consumption: $C = a + bY$, $a > 0$, $b > 0$. I_0 , G_0 exogenous. In the money market, $M^d \rightarrow L = kY - gr$, ($k, g > 0$). Nominal $M^s = M_0$, Price level P_0 exogenously. C, Y, r : Consumption, Real GDP, interest rate.

① Set Up IS-LM equations.

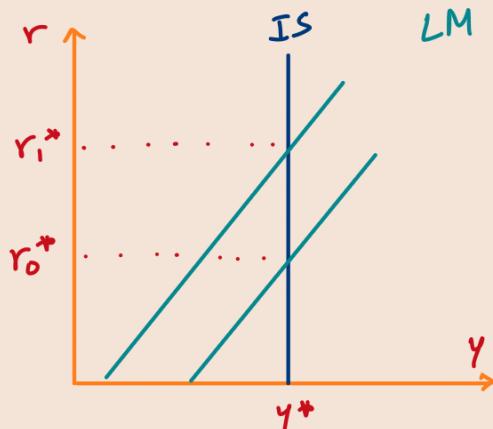
$$Y = C + G + I \Rightarrow$$

$$Y(1-b) = a + I_0 + G_0$$

$$M^d = M^s$$

$$\Rightarrow kY - gr = M_0/P_0 \Rightarrow$$

$$r = \frac{k}{g} Y - \frac{M_0}{g P_0}$$



② P: $P_0 \uparrow P_1$. GDP? r?

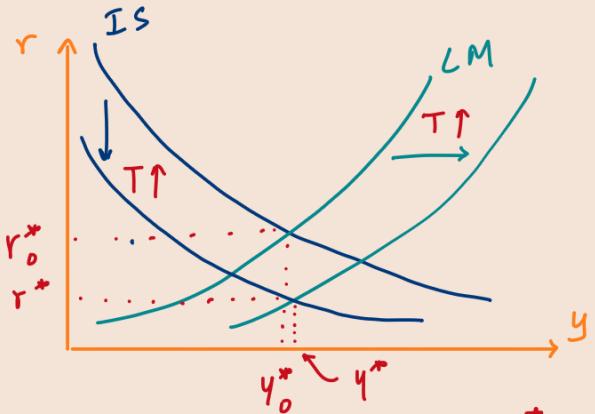
$P \uparrow \Rightarrow M^s \downarrow$ But $Y \sim \Rightarrow r \uparrow$

Q6 IS-LM model, closed economy. Private consumption

depends on disposable income. (1) are same. Both priv inv. & speculative M^d vary inversely w/ (r) . However, transaction M^d depends not on (y) but (y_d) . Argue how eq^l^m priv. inv., priv. saving, govt. saving, y_d , b y will change, if govt. $\uparrow T$.

$$C(y_d) = C(y - T), \quad I \propto \frac{1}{r}, M_{\text{spec}}^d \propto \frac{1}{r}, M_{\text{trans}}^d(y_d)$$

$$M^d = M_{\text{spec}}^d + M_{\text{trans}}^d = \frac{k_s}{r} + f(y_d) = \frac{k_s}{r} + f(y - T)$$



$$y = C + I + G \xrightarrow{T \uparrow} \uparrow \Rightarrow$$

$$y = C(y - T) + \frac{k_I}{r} + G \quad \textcircled{1}$$

$$M^d = \frac{k_s}{r} + f(y - T)$$

$$\begin{cases} y \uparrow \\ r \uparrow \\ y \uparrow \\ r \downarrow \end{cases} \xrightarrow{T \uparrow \Rightarrow \text{Shifts to the right}}$$

OR

$\frac{IS}{LM}$	$y = C(y - T) + I(r) + G$	Govt. Savings Same sign
	$M^s = M^d(y - T, r)$	
	$\rightarrow r^+ \downarrow, y^* \text{ ambiguous}, I \uparrow, (T - G) \uparrow, (y^* - T) \text{ ambiguous, Sp ambiguous}$	

Q7 Closed Economy:

$$\underbrace{\Delta Y}_{GDP} = \underbrace{\Delta C}_{\text{Consumption}} + \underbrace{\Delta I}_{\text{Private Investment}} + \underbrace{\Delta G}_{\text{Govt. Spending}}; \quad \Delta C = c \underbrace{\Delta Y_d}_{\text{Disposable Income}}; \quad \Delta Y_d = \Delta Y - \underbrace{\Delta T}_{\text{Tax Collections}}; \quad \Delta T = \underbrace{t \Delta Y}_{\substack{\text{tax rate} (0,1) \\ \text{Modifiable by Govt. in Fiscal Policy}}} - \Delta T_0$$

1) Find balanced budget multiplier (in terms of G & T_0)

$$\Delta Y = c \left(\Delta Y - \underbrace{t \Delta Y - \Delta T_0}_{\Delta T} \right) + \Delta I + \Delta G = c(1-t) \Delta Y + \cancel{\Delta I} + \Delta G - c \Delta T_0$$

$$\Rightarrow \Delta Y (1 - c(1-t)) = -c \Delta T_0 + \Delta G$$

$$\Delta G = \Delta T_0 \quad \uparrow L \quad \boxed{\frac{\Delta Y}{\Delta G} = \frac{1-c}{1-c(1-t)} < 1}$$

2) $c = 0.8, t = 0.375$. Govt. expenditure multiplier?

$$\Delta T_0 = 0 \rightarrow \frac{\Delta Y}{\Delta G} = \frac{1}{1-c(1-t)} = \frac{1}{1-\frac{4}{5}(1-\frac{3}{8})} = 2$$

3) $c = 0.8, t = 0.375$. Tax multiplier (w.r.t. T_0)?

$$\Delta G = 0 \rightarrow \frac{\Delta Y}{\Delta T_0} = \frac{-c}{1-c(1-t)} = \frac{-4/5}{1-\frac{4}{5}(1-\frac{3}{8})} = \frac{-8}{5} = -1.6$$



Medium Run

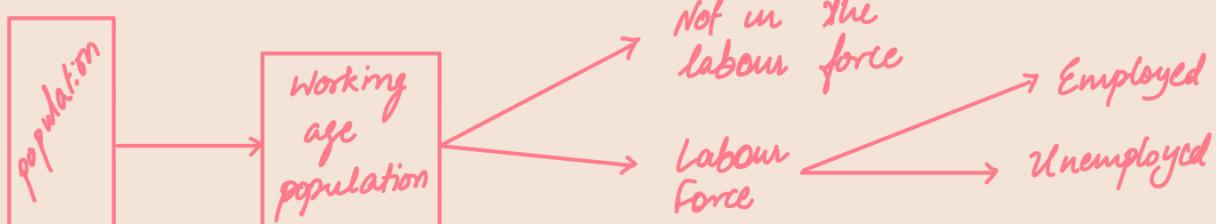
↳ Need to study supply-side issues.

Labour Market is a major component

prev. models do not account for it.

Some jargon

- Unemployment rate
- Labour force participation rate



* Thought Expt.: suppose you're negotiating your wage with a company, then, how would your wage argument depend on:

- 1) Expected Price Level (+)
- 2) Unemployment Rate (-)
- 3) Other things → Unemployment Benefits Rate (+)

$$\Rightarrow W = p^e F(U, z)$$

Wage-Setting Relationship / Equation

Nominal Wage Rate ← Expected Price Level →
↓ ↓
Other factors Unemployment rate

$$P = (1+m)W$$

Price ↓ Markup (m > 0) ↓ Wage ↓
↑
How much can they set

Price-Setting Relationship / Equation

$$y = N \xrightarrow{\text{Def } A=1} \text{OR} \xrightarrow{\text{Normalize}}$$

$$y = AN, \quad N \rightarrow \text{Labour Input}, \quad A \rightarrow \text{"Productivity"}, \quad y \rightarrow \text{Output}$$

} CRS production function,
Constant Returns to Scale

What people expect the prices to be = What the price is
(Medium Run)

the price
above w ?

$$p = (1+m)w \rightarrow \frac{w}{p} = \frac{1}{1+m}$$

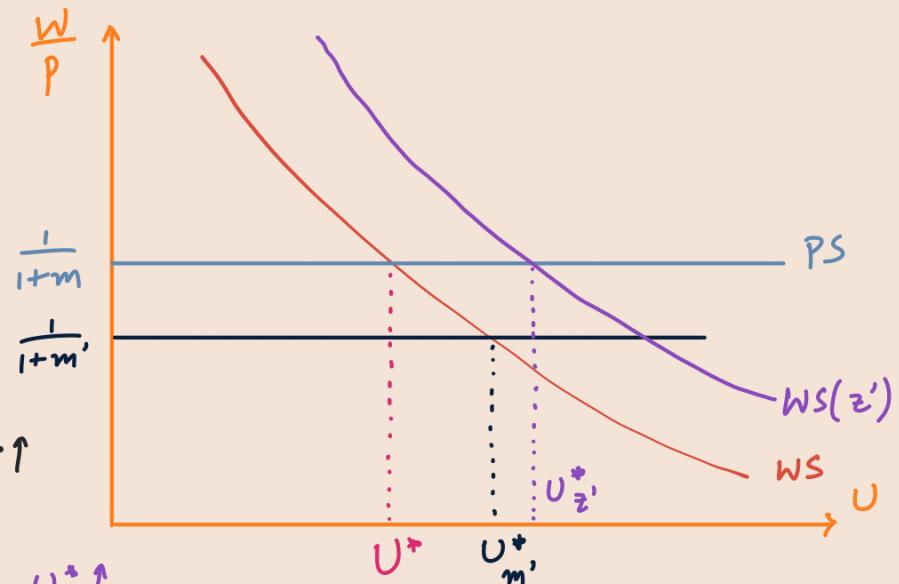
$$w = p^e F(U, z) \rightarrow \frac{w}{p^e} = F(U, z)$$

$$F(U, z) = \frac{1}{1+m}$$

U^* → U_m : Natural rate
of Unemployment

Comparative Statics:

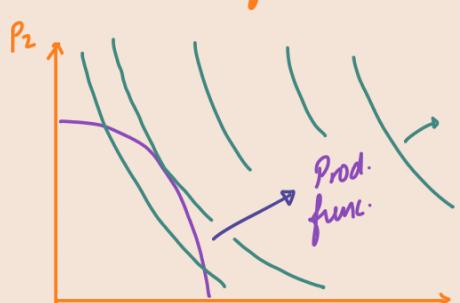
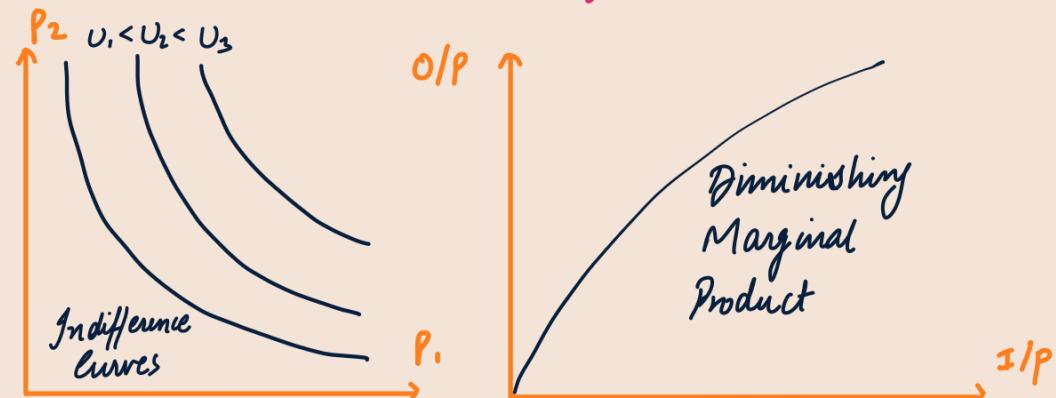
- What is the effect of higher market concentration?
≡ What's the effect of $\uparrow m$? $U^* \uparrow$
- What is the effect of $\uparrow z$? $U^* \uparrow$



- Production Function → How inputs translate into outputs holding other factors constant.
- Marginal Function → Change in output per unit change in input $\frac{\Delta \text{Income}}{\Delta \text{hrs. worked}}$ → Slope
- Average Product → Output per unit of input $\rightarrow I/P$
- Preferences and Choice Sets (Feasible Sets)

$$U = U(P_1, P_2)$$

↓ ↓
Utility Preferences



Dual

Economy

People

Total Prod.

Marg. Prod.

Avg. Prod.

family of people	1	50	50	50
	2	90	40	45
	3	120	30	40
	4	140	20	35
	5	150	10	30
	6	150	0	25
	7	150	0	150/7
	8	150	0	150/8
	9	150	0	150/9
	10	150	0	15

- 1) Company comes \rightarrow 4 people at ₹ 25 \rightarrow Accepted
- 2) After 4 people go $\uparrow \downarrow$ 1 person at ₹ 25 \rightarrow Not accepted
 1 person at ₹ 35 \rightarrow Accepted

★ We would like to account for inflation (and its -ve relation w/ Unemployment) using the equations

$$P = (1+m)W \quad \Rightarrow \quad P = p^e (1+m) F(U, z)$$

$$W = p^e F(U, z) \quad \begin{matrix} - \\ + \end{matrix} \quad \Rightarrow \quad P_t = P_t^e (1+m) F(U_t, z) \quad \begin{matrix} \text{Assume a linear function} \\ \text{for now} \end{matrix}$$

$$\Rightarrow \frac{P_t}{P_{t-1}} = \frac{P_t^e}{P_{t-1}^e} (1+m)(1+z - \alpha U_t)$$

$$\Rightarrow (1+\pi_t) = (1+\pi_t^e)(1+m)(1+z - \alpha U_t)$$

$$\Rightarrow \boxed{\pi_t = \pi_t^e + m + z - \alpha U_t} \quad \begin{matrix} \text{Expectation augmented} \\ \text{Phillips' curve} \end{matrix}$$

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

$$\pi_t^e = \frac{P_t^e - P_{t-1}^e}{P_{t-1}^e}$$

\hookrightarrow Different ways of expecting π_t^e

- 1) Inflation rate would be low \rightarrow
 | Past year's inflation was not a good predictor }
 | of next year's inflation }

$$\pi_t^e = \bar{\pi} \rightarrow \pi_t = \bar{\pi} + (m + z) - \alpha U_t \quad \begin{matrix} \text{Initial} \\ \text{Phillips' curve} \end{matrix}$$

$$\pi_t^e = (1-\theta) \bar{\pi} + \theta \pi_{t-1}$$

$$\hookrightarrow \theta=0 \rightarrow \pi_t^e = \bar{\pi} \rightarrow \text{: Anchored: inflation}$$

$$\hookrightarrow \theta=1 \rightarrow \pi_t^e = \pi_{t-1} \rightarrow \text{: Unanchored: inflation}$$

Initially, we had assumed that

$$p = p^e \Rightarrow \pi_t = \pi_t^e \Rightarrow 0 = m + z - \alpha u_n \Rightarrow u_n = \frac{m+z}{\alpha}$$

$$U_t = U_m$$

$$\pi_t = \pi_t^e + \alpha \left[\frac{m+z}{\alpha} - u_t \right] = \pi_t^e + \alpha [u_n - u_t]$$

Now, if people believe, $\pi_t^e = \pi_{t-1}$
then $\pi_t - \pi_{t-1} = -\alpha [u_t - u_n]$

$$\begin{cases} u_t = u_n \Rightarrow \pi_t = \pi_{t-1} \\ u_t > u_n \Rightarrow \pi_t < \pi_{t-1} \\ u_t < u_n \Rightarrow \pi_t > \pi_{t-1} \end{cases}$$

* If $\pi \uparrow$ then a lot of uncertainty,
people would want to play safe $\rightarrow \pi_t^e = (1-\lambda)\pi_{t-1} + \lambda \pi_t$
 $\Rightarrow \pi_t = \pi_t^e + (m+z) - \alpha u_t \Rightarrow \pi_t = \pi_{t-1} - \frac{\alpha}{1-\lambda} [u_t - u_n]$

Feedback Loop \Leftarrow Effect of u_t amplified now

* $u_n \rightarrow$ Natural rate / Non-accelerating of unemployment (If $u_t = u_n$, $\pi_t = \pi_{t-1}$)

• $\pi_t - \pi_{t-1} = -\alpha(u_t - u_n) \rightarrow$ If $u_t \uparrow$ like in Great Dep., deflation does not take place.

People not okay w/
having their nominal
wages being decreased

$$\rightarrow \pi_t = \pi_t^e - \alpha [u_t - u_n]$$

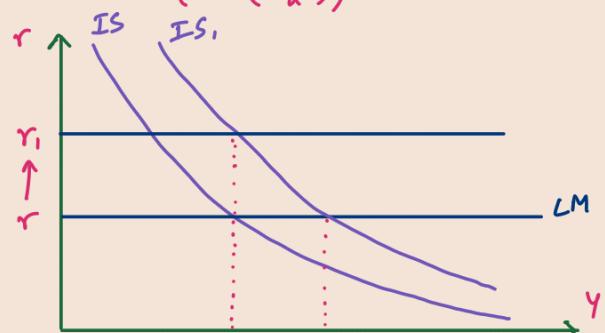
Unemployment Rate

$$\Rightarrow \boxed{\pi_L = \pi_t^e + \frac{\alpha}{L} [Y_t - Y_n]}$$

$$\begin{aligned} u &= \frac{U}{L} = \frac{U}{N+U} \quad \# \text{ employed} \\ \Rightarrow \frac{L-U}{L} &= u = 1 - \frac{N}{L} \Rightarrow N = L(1-u) \\ \Rightarrow N_t &= L(1-u_t), \quad N_n = L(1-u_n) \\ \Rightarrow N_t - N_n &= L(u_n - u_t) \\ \Rightarrow u_t - u_n &= -\frac{(N_t - N_n)}{L} \equiv -\frac{(Y_t - Y_n)}{L} \end{aligned}$$

$$\rightarrow Y = L(1-u)$$

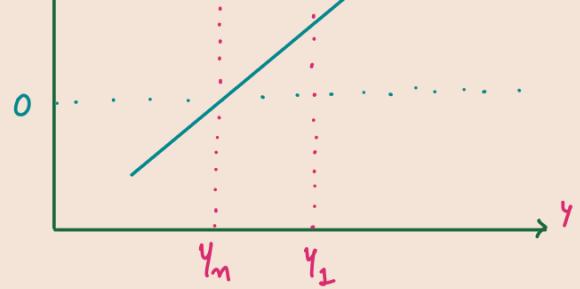
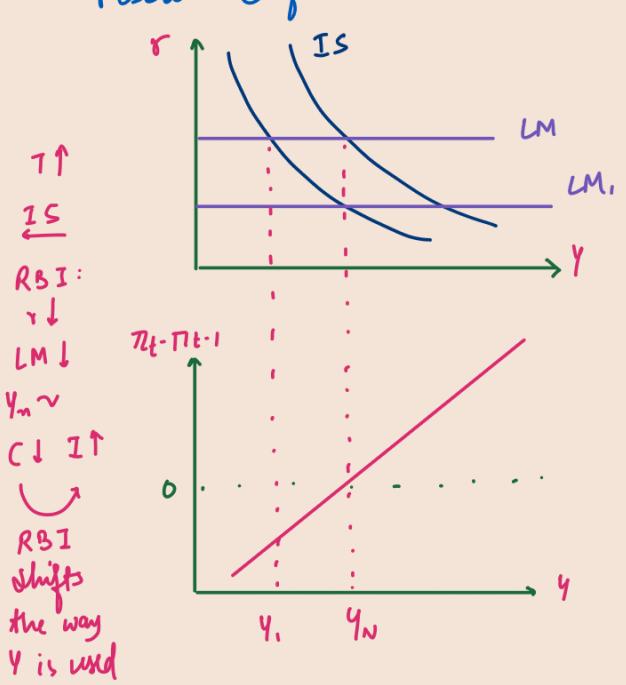
$$\begin{aligned} \Rightarrow Y_n &= L(1-u_n) \\ &= L\left(1 - \left(\frac{m+z}{\alpha}\right)\right) \end{aligned}$$



{

(PC)	$\pi_t - \pi_{t-1} = \frac{\alpha}{L} [Y_t - Y_n]$
(IS)	$Y = C(Y-T) + I(Y, r+x) + G$
(LM)	$r = \bar{r}$
(WS)	$W = p^e F(u, z)$
(PS)	$P = (1+m)W$

Reduction in:
Fiscal Deficit



Deflation Spiral:

Suppose Central Bank \downarrow LM but zero lower bound reached

$$\Rightarrow i=0 \Rightarrow r_t = -\pi_t^e = -\pi_{t-1}$$

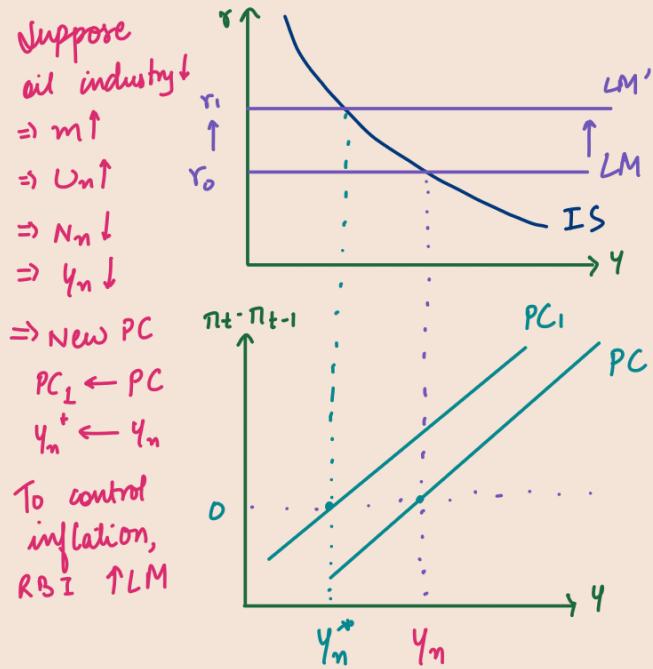
Also $\Rightarrow y: y_n \downarrow y_1 \uparrow y_2$ but $y_2 < y_n$

$$\Rightarrow \pi_t - \pi_{t-1} < 0 \Rightarrow \pi_t \downarrow \Rightarrow r \uparrow$$

$$\Rightarrow IS \text{ shifts left} \Rightarrow y \downarrow \Rightarrow \pi \downarrow \Rightarrow r \uparrow$$

Downward spiral of Inflation due to the loop (b zero lower bound).

Oil Price Shock



Expectations

→ Idea of "Present Value":

Ex. Suppose 5L cost → $\begin{array}{c} 1^{\text{st}} \text{ Year} \\ 2^{\text{nd}} \text{ Year} \\ 3^{\text{rd}} \end{array} \left. \begin{array}{l} 2L \\ 3L \\ 4L \end{array} \right\} \text{Profits}$

On the other hand, r.o.i. is 10% for some plan. Should you buy the 5L thingy.

$$\rightarrow 1 \xrightarrow{i_t} 1+i_t \Rightarrow \frac{1}{1+i_t} \xleftarrow{i_t} 1$$

$$\therefore \text{P.V. of } 1 \text{ (after 1 year)} \text{ is } \frac{1}{1+i_t}$$

$$1 \xrightarrow{i_t} 1+i_t \xrightarrow{i_t} (1+i_t)^2 \Rightarrow \left(\frac{1}{(1+i_t)^2} \right) \xleftarrow[2 \text{ years}]{} 1$$

$$\text{Do, buy it iff } \frac{2}{1.1} + \frac{3}{1.1^2} + \frac{4}{1.1^3} - 5 > 0$$

- Expected present discounted value of a sequence of future payments is the value today of the given sequence of expected payments.

(present) value \uparrow $\frac{1}{1+i_t}$: Discount Factor

$$V_t = Z_t + \frac{Z_{t+1}^e}{(1+i_t)} + \frac{Z_{t+2}^e}{(1+i_t)(1+i_{t+1}^e)} + \frac{Z_{t+3}^e}{(1+i_t)(1+i_{t+1}^e)(1+i_{t+2}^e)} + \dots$$

Special Case I: $i_t = i_{t+1}^e = i_{t+2}^e = \dots = i$ (say)

$$V_t = Z_t + \frac{Z_{t+1}}{(1+i)} + \frac{Z_{t+2}}{(1+i)^2} + \dots$$

Special Case II: (Const. interest rate b $Z_t = Z_{t+1}^e = \dots = Z$)

$$V_t = Z + \frac{Z}{1+i} + \frac{Z}{(1+i)^2} + \dots$$

$$\text{For example: } V_t = Z + \frac{Z}{1+i} + \dots + \frac{Z}{(1+i)^{10}}$$

Special Case III: Perpetuity (payments go on till forever) U SC II

$$V_t = \sum_{i=1}^{\infty} \frac{Z}{(1+i)^i} = \frac{Z}{1+i} \cdot \frac{1}{1 - \frac{1}{1+i}} = \frac{Z}{i} = \frac{Z}{1+i} + \frac{Z}{(1+i)^2} + \dots$$

- Would like to know "real" value (present)

Method I: $V_t \text{ (in real terms)} = \frac{V_t \text{ (in nominal terms)}}{P_t}$

$$Z_{rt} = \frac{Z_{nt}}{P_t}, Z_{rt+i}^e = \frac{Z_{nt+i}^e}{P_{t+i}^e} + \dots$$

$$1 + r_{t+j} = \frac{1 + i_{t+j}}{1 + P_{t+j}}$$

$$+ \frac{Z_{t+1}^e \text{ (in real terms)}}{(1+r_t)}$$

$$+ \frac{Z_{t+2}^e \text{ (in real terms)}}{(1+r_t)} + \dots$$

$$(1+r_t)(1+r_{t+1}^c)$$

Some Nomenclature

- Maturity: Length of time over which a bond promises to make the payment to the holder of the bond.
- Default rate: Risk that the issuer of the bond will not pay back the full amount promised by the bond.
- Price risk: Uncertainty about the price at which one can sell the bond before maturity.
- Yield to maturity or Yield: Interest rates associated with bonds of different maturities.
- Short term interest rate: Yields on bonds with a maturity < 1 year (typically)
- Long term interest rate: Yields on bonds with a maturity > 1 year (typically)
- Term structure of OR yield curves: Relation b/w interest rates & maturity & yield
- Govt. bonds, Corporate bonds
- Bond ratings: Ratings for default risk
- Risk premium: Difference b/w interest rate paid in a given bond & interest rate on a bond with the best ratings.
- Junk bonds: Bonds with high default risk
- Discount bonds: Bonds that promise a single payment at maturity (called the face value)
- Coupon bonds: Bonds that promise multiple payments before maturity & one payment at maturity.
- coupons payments coupon rate to Coupon yield

Coupon payments, coupon rate, coupon yield

- Life: Amount of time left until the bond matures.
- T-Bills, Indexed Bonds

Treasury Bonds

- Term Premium: Premium associated with longer maturities.

Strategy I: Take a 1-year bond: $1 \rightarrow (1+i_t)$

S II: Take a 2-year bond & sell it after 1-year.

$$1 \rightarrow \frac{1}{P_{2t}} P_{1,t+1}^e$$

$P_{nt} \rightarrow t^n$ year
↳ No. of years remaining

$$\frac{P_{1,t+1}^e}{P_{2t}} \text{ should equal } 1+i_t \Rightarrow P_{2t} = \frac{P_{1,t+1}^e}{1+i_t} = \frac{100}{(1+i_t)(1+i_{t+1}^e)}$$

$$\rightarrow P_{2t} = \frac{100}{(1+i_t)(1+i_{t+1}^e) + x}$$

Risk Premium

$$\frac{100}{(1+i_{2t})^2} = \frac{100}{(1+i_t)(1+i_{t+1}^e)}$$

$$\Rightarrow (1+i_{2t})^2 = (1+i_t)(1+i_{t+1}^e)$$

$$\Rightarrow i_{2t} \approx \gamma_2 (i_t + i_{t+1}^e)$$

* Yield to maturity on an n -year bond is the constant annual interest rate that makes bond price today equal to the present value of future payments on the bond.

Q What's the relation b/w $\underline{\delta}_t$, $\underline{\delta}_{t+1}^e$, \underline{D}_{t+1}^e & i_t ?

$$SI \rightarrow 1 \rightarrow 1+i_t$$

$$SII \rightarrow 1 \rightarrow \left(\frac{1}{\underline{\delta}_t}\right) (\underline{\delta}_{t+1}^e + \underline{D}_{t+1}^e) \Rightarrow$$

Price of Stock

Dividend

$$\underline{\delta}_t = \frac{\underline{\delta}_{t+1}^e + \underline{D}_{t+1}^e}{1+i_t}$$

• Equity Premium: Risk associated w/ stocks

Assumption: Interest rate is const. (i) & no risk premium.

$$\underline{\delta}_t = \frac{\underline{\delta}_{t+1}^e + \underline{D}_{t+1}^e}{1+i} = \frac{\underline{D}_{t+1}^e}{1+i} + \frac{1}{1+i} \left[\frac{\underline{\delta}_{t+2}^e}{1+i} + \frac{\underline{D}_{t+2}^e}{1+i} \right]$$

$$= \frac{D_t^e}{1+i} + \frac{D_{t+1}^e}{(1+i)^2} + \dots + \frac{D_{t+n}^e}{(1+i)^n} + \frac{Q_{t+n}^e}{(1+i)^n}$$

Additional Problem Set 2

Q1 P1 ℓ (labour) governed by maximisation of $u := C^{2/3} h^{1/3}$
 c : consumption, h : leisure, $\ell + h = 24$. w given,
house consumes complete wage ($w\ell$). $\ell = ?$

$$\text{M1: } L = C^{2/3} h^{1/3} + \lambda(24 - \ell - h) \rightarrow \frac{\partial L}{\partial \lambda} = 0 \Rightarrow 24 = \ell + h$$

$$\frac{\partial L}{\partial \lambda} = 0 = \frac{\partial L}{\partial w} = \frac{\partial L}{\partial h} \quad \left[\frac{\partial}{\partial c} \Rightarrow \frac{2}{3} \right] \rightarrow \frac{1}{3} h^{2/3} \cdot C^{2/3} = \lambda \Rightarrow \frac{C^{2/3} h^{1/3}}{3} = \lambda h$$

$$c = wl$$

$$\text{MII: Max } v = \log u = \frac{2}{3} \log c + \frac{1}{3} \log h$$

$$= \frac{2}{3} \log(w\ell) + \frac{1}{3} \log(24 - \ell)$$

$$\frac{dv}{d\ell} = 0 = \frac{2}{3} \cdot \frac{1}{\ell} + \frac{1}{3} \cdot \frac{-1}{24 - \ell} \Rightarrow 48 - 2\ell^* = \ell^* \Rightarrow \underline{\ell^* = 16}$$

Q1 P2 Keynesian (closed) economy. Two types of final expenditure
investment (36), c ($= wl$). Given $w=4$, $y = 24 \sqrt{\ell}$.
Find y^* & ℓ^* .

Q2 $y = U N^\alpha \rightarrow 0 < \alpha < 1$ N determined by equating marginal prod.
 \downarrow Shock \downarrow to $\frac{w}{P} \rightarrow$ Nominal Wage . $u = \log \alpha + \frac{1}{\alpha} \log U$,
 $P \rightarrow$ Price level $P = \log P$, $w = \log w$, $y = \log y$

1. Obtain y (aff. supply) in terms of p , w , u .

$$\log y = \log U + \alpha \log N \Rightarrow y = \log U + \alpha \log N$$

$$\frac{\partial y}{\partial N} = \frac{w}{P} = \alpha U \cdot N^{\alpha-1} \Rightarrow N = \left(\frac{w}{P} \cdot \frac{1}{\alpha U} \right)^{\frac{1}{\alpha-1}}$$

$$\therefore \log N = \log \left(\frac{w}{P} \cdot \frac{1}{\alpha U} \right)^{\frac{1}{\alpha-1}} = \log U + \frac{\alpha}{\alpha-1} (\log w - \log P - \log \alpha)$$

$$\begin{aligned}
 y &= \log U + \alpha \log \left(\frac{w}{p} \cdot \frac{u}{\alpha w} \right) - \alpha \log \frac{\alpha}{\alpha-1} (\theta - \log U) \\
 &= \frac{\log U}{1-\alpha} + \frac{\alpha w}{\alpha-1} - \frac{\alpha p}{\alpha-1} - \frac{\alpha \log \alpha}{\alpha-1} = \frac{\alpha}{\alpha-1} (w-p) + \frac{\alpha}{1-\alpha} \left(\log \alpha + \frac{1}{\alpha} \log U \right) \\
 &= \frac{\alpha}{1-\alpha} u + \frac{\alpha}{\alpha-1} (w-p) = \boxed{\frac{\alpha}{1-\alpha} (u+p-w) = y}
 \end{aligned}$$

2. (i) $w = \theta p$, $0 \leq \theta \leq 1$ & $y = \underline{m-p} \rightarrow \log \frac{\text{money}}{\text{policy var.}}$
 Find $y(m, u)$

$$\begin{aligned}
 p &= m-y \\
 w &= \theta p = \theta(m-y)
 \end{aligned}
 \quad \left. \begin{aligned}
 y &= \frac{\alpha}{1-\alpha} [u + p(1-\theta)] = \frac{\alpha}{1-\alpha} [u + (1-\theta)(m-y)] \\
 \Rightarrow (1-\alpha)y &= \alpha[u + (1-\theta)m] - \alpha(1-\theta)y \\
 \Rightarrow (1-\alpha + \alpha - \alpha\theta)y &= \alpha[u + (1-\theta)m]
 \end{aligned} \right\} \Rightarrow y = \frac{\alpha}{1-\alpha\theta} [u + (1-\theta)m]$$

3. Does m affect output if $\underbrace{0 < \theta < 1}_{\text{Yes}}, \underbrace{\theta = 1}_{\text{No}}$
 $\frac{\partial y}{\partial m} = \frac{\alpha(1-\theta)}{1-\alpha\theta}$ $\quad \partial \neq \frac{\partial y}{\partial m} = 0$

4. u affect y more when $\theta \uparrow$?

$$\begin{aligned}
 \frac{\partial y}{\partial u} &= \frac{\alpha}{1-\alpha\theta} \rightarrow \theta \uparrow, \alpha\theta \uparrow, 1-\alpha\theta \downarrow, \frac{1}{1-\alpha\theta} \uparrow, \frac{\partial y}{\partial u} \uparrow \\
 \text{OR} \quad \frac{\partial}{\partial \theta} \left(\frac{\partial y}{\partial u} \right) &= \frac{\alpha \cdot \alpha\theta}{(1-\alpha\theta)^2} > 0 \Rightarrow \theta \uparrow, \frac{\partial y}{\partial u} \uparrow
 \end{aligned}$$

Q3 Supply func.: $\log Y = \log Y^* + \lambda(\log P - \log P_e)$

$Y = N^\sigma$	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
Output	Natural level of output	Price Level	Expected price level	Wage (Nominal)	$\frac{w}{P_e} = N^\sigma$	Employment

Firms: $\underbrace{MP_N}_P = \frac{w}{P}$

$$\frac{\partial Y}{\partial N} = \frac{w}{P} \Rightarrow \alpha N^{\sigma-1} = \frac{w}{P} \Rightarrow N = \left(\frac{w}{\alpha P} \right)^{\frac{1}{\sigma-1}}, \quad N^\sigma = \frac{w}{P_e}$$

$$w = N^\sigma P_e = \alpha N^{\sigma-1} P \Rightarrow N^{1-\sigma+\sigma} = \frac{\alpha P}{P_e}$$

$$P = P_e \Rightarrow N = \alpha^{\frac{1}{1-\sigma+\sigma}}, \quad Y^* = \alpha^{\frac{\sigma}{1+\sigma-\sigma}}$$

$$(1-\sigma+\sigma) \log N = \log \alpha + (\log P - \log P_e) \rightarrow \log Y^* = \frac{\log \alpha}{1-\sigma+\sigma}$$

$$\log N = \frac{\log \alpha}{1-\sigma+\sigma} + \frac{1}{1-\sigma+\sigma} (\log P - \log P_e) \quad \lambda = \frac{1}{1-\sigma+\sigma}$$

Q4 $C = 200 + 0.5 Y_D$, $T = 200$, $\left(\frac{M}{P}\right)^d = 24 - 4000i$, $\left(\frac{M}{P}\right)^s = 1600$, $i = r - \pi^e$
 $I = 150 - 1000r$, $G = 250$,
P rigid.

1) $\pi^e = 0$. Determine eq¹ & r, i.

$$i = r, \quad Y = 200 + 0.5(Y - 200) + 250 + 150 - 1000r \\ \Rightarrow Y = 1000 - 2000r \quad \text{IS} \quad \left. \begin{array}{l} Y = 900 \\ r = \frac{1}{20} = 5\% = i \end{array} \right\}$$

$$24 - 4000i = 1600 \quad \text{LM}$$

2) $M_S \uparrow 2\%$ temp. Find Y, i, r.

$$\underbrace{M_S \uparrow 2\%}_{M_S = 1632} \quad \underbrace{\pi^e = 0}_{\pi^e = 0} \quad \left. \begin{array}{l} Y = 908 \\ r = \frac{92}{2000} = 4.6\% = i \end{array} \right\}$$

3) $M_S \uparrow 2\%$ permanently. Find Y, i, r

$$\underbrace{M_S \uparrow 2\%}_{M_S = 1632} \quad \underbrace{\pi^e = 2\%}_{\pi^e = 2\%} \quad \left. \begin{array}{l} i = 3.6\% \\ r = 5.6\% \\ Y = 888 \end{array} \right\}$$

$$i = r - 2 \\ \therefore i = r - \pi \quad \text{given}$$

Q6 Worker $\rightarrow W_1: 66$, $W_2 = 50$.

Wage $W_2 >$ Wage W_1 \nearrow Week 2

Sub. effect \rightarrow Wage \uparrow , Opp. Cost for not working \uparrow , Work More

Income effect \rightarrow Wage \uparrow , Richer \uparrow , Work Less
(full)

\therefore Worker works less \rightarrow IE dominates SE

Q7 Discount rate: 15% pa. Firms live 3 years.

F's cash flow starts w/ 500 in Y1, grow at 20% for two yrs
S's " " " " " " " " shrink at 20% " "

Prices of two firms? Which one better buy?

$$\text{Price of F} = \frac{500}{1.15} + \frac{500 \cdot 1.2}{1.15^2} + \frac{500 \cdot 1.2^2}{1.15^3} = 1361.88$$

$$\text{ " " S} = \frac{500}{1.15} + \frac{500 \cdot 0.8}{1.15^2} + \frac{500 \cdot 0.8^2}{1.15^3} = 947.64$$

Both equivalent at these prices.

2010 equations in blue paper.

Both return 15%.

Q8 Time 0 → initial investment 1 000 000.

Generate cash flow of 100 000 after time 1 & this grows by 4% / year thereafter.

What is the NPV? Assuming int rate to be const. at 10%.

$$NPV = -1\ 000\ 000 + \frac{100\ 000}{1.1} + \frac{100\ 000 \cdot 1.04}{(1.1)^2} + \frac{100\ 000 \cdot 1.04^2}{(1.1)^3} + \dots$$

$$= \frac{100\ 000}{1.1} \cdot \frac{1}{1 - \frac{1.04}{1.1}} - 1000\ 000 = \frac{100\ 000}{0.07} - 1000\ 000$$

= 428 571.43 > 0 Should take the opportunity.

Q9 Stock A → alternates b/w +20, -10%.

Stock B earned 4.5% pa.

On a 1 year basis, would a risk-neutral investor prefer A (equal prob.) or B?

How much would each rupee invested 10 yrs. ago in A have earned? In B?

$$A \rightarrow \frac{1}{2} \cdot 20 + \frac{1}{2} \cdot -10 = 5\% \rightarrow \text{Choose A.}$$

$$A_{10} \rightarrow (1.2)^5 (0.9)^5 \rightarrow 1.4693$$

$$B_{10} \rightarrow (1.045)^{10} \rightarrow 1.5530$$

Q10 $M_t \bar{Y} = P_t Y_t$, $Y_t = \bar{A} L_t$, $P_{t+1}/P_t = (L_t/L^*)^\theta$

1.) $\bar{Y} = 1 = \bar{A} = L^*$. Calculate steady state $M_t = 1$.

$$P_t Y_t = 1, Y^* = 1, P_{t+1} = P_t = 1 \quad L_t = L^*$$

2.) Take log & eliminate L_t .

3.) part (1) → till $t = -1$. At $t = 0 \rightarrow M_0 = 2$ ↗
const. for 20 yrs.
 $\theta = 0.2$

$$\log M_t = \log P_t + \log Y_t$$

$$\log Y_{-1} = 0$$

$$\log P_{t+1} - \log P_t = 0.2 [\log Y_t] \rightarrow \log P_0 = \log P_{-1} = 0$$

$$\log M_0 = \log P_0 + \log Y_0 \rightarrow \log 2 = \log M_0 = \log Y_0$$

$$\log P_1 = 0 + 0.2 \log 2 = 0.2 \log 2 \Rightarrow \log Y_1 = 0.8 \log 2$$

$$\Rightarrow \log P_2 = 0.2 \log 2 + 0.16 \log 2 = 0.36 \log 2, \log Y_2 = 0.64 \log 2$$



* $P_t = \frac{D_i^e}{1+r_{t+1}} + \frac{D_i^e}{1+r_{t+2}} + \dots \rightarrow D_i^e = 0 \Rightarrow P_t = 0$ but we can still buy it rationally if we expect that people would want to buy it in the future.

$Y \uparrow \rightarrow$ Did people expect it \rightarrow No change on P
 Did not $\rightarrow Y \uparrow \Rightarrow D \uparrow \Rightarrow P \uparrow$

Suppose banks $\uparrow r \rightarrow$ Expected? \rightarrow Yes \rightarrow No change in P
 No $\rightarrow P \downarrow$

Effects only exist if the underlying changes were not expected

* Expectations matter in both goods market & the financial market

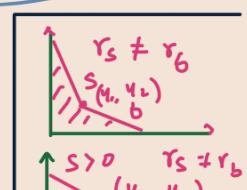


$$Ex: 20 \rightarrow 23 \dots 60 \rightarrow 79$$

$$0 \dots 100 \dots 100 \dots 0$$

$$C: 3800/60 \sim b^3$$

Might need to pay bills
 Loan required \rightarrow Given if income \uparrow
 Even if Present \leftarrow Present income matters
 Value is the same



2 period Consumption

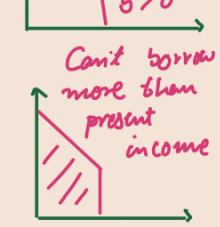
Model (Deterministic)

Can save or borrow
 (s) (b)
 at same interest rate (r)
 (say)

Household \rightarrow Y_1, Y_2
 (n.n.) $\rightarrow C_1, C_2$

Utility: $U(C_1, C_2)$

- $C_1 = Y_1 \Rightarrow C_2 = Y_2$
- $C_1 > 0, C_2 > 0$



(say)

s saved

$$C_1 = Y_1 - s$$

$$C_2 \leq Y_2 + s(1+r)$$

$$C_2 \leq Y_2 + (Y_1 - C_1)(1+r)$$

$$\Rightarrow C_1 + \frac{C_2}{1+r} \leq Y_1 + \frac{Y_2}{1+r}$$

$$\Rightarrow P.V.(\text{Consumption}) \leq P.V.(\text{Income})$$

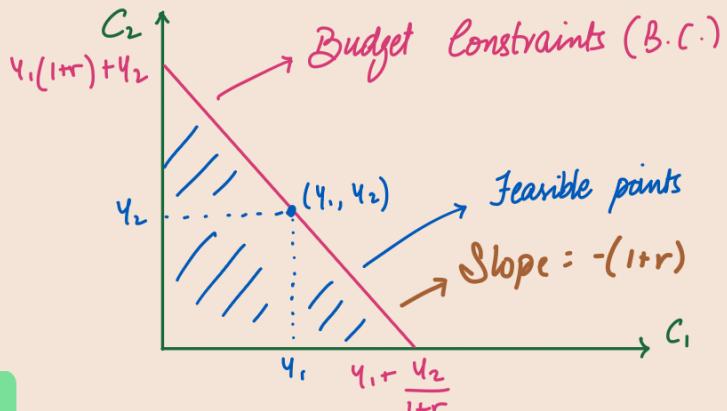
b borrowed

$$C_1 = Y_1 + b$$

$$C_2 \leq Y_2 - b(1+r)$$

$$C_2 \leq Y_2 - (C_1 - Y_1)(1+r)$$

$$C_1 + \frac{C_2}{1+r} \leq Y_1 + \frac{Y_2}{1+r}$$



$U(C_1, C_2) \rightarrow$ Maximise subject to B.C. b
Only need to analyse equality/boundary of B.C. ↗

$$L = U(C_1, C_2) + \lambda \left[Y_1 + \frac{Y_2}{1+r} - C_1 - \frac{C_2}{1+r} \right] \rightarrow \frac{\partial L}{\partial \lambda} = 0 \Rightarrow \text{Equality of B.C., i.e.}$$

$$\hookrightarrow \frac{\partial L}{\partial C_1} = 0 \Rightarrow \frac{\partial U}{\partial C_1} = \lambda, \quad \frac{\partial U}{\partial C_2} = \frac{\lambda}{1+r} \Leftarrow 0 = \frac{\partial L}{\partial C_2}$$

$$\Rightarrow -\frac{\partial U}{\partial C_1} = \frac{-\lambda}{1+r} \quad \begin{matrix} \text{Budget} \\ \text{Line} \end{matrix} \quad \Rightarrow \begin{matrix} \text{Indifference} \\ \text{Curve } (U = \bar{U}) \end{matrix} \quad \Rightarrow \begin{matrix} \text{should be tangent} \\ \text{to B.L.} \end{matrix}$$

$$U = \bar{U} \Rightarrow \frac{\partial U}{\partial C_1} dC_1 + \frac{\partial U}{\partial C_2} dC_2 = 0 \Rightarrow \frac{dC_2}{dC_1} = -\frac{\partial U / \partial C_1}{\partial U / \partial C_2}$$



Claim: $\frac{-\partial U / \partial C_1}{\partial U / \partial C_2} = -(1+r)$ at max. $U \rightarrow dU = \frac{\partial U}{\partial C_1} dC_1 + \frac{\partial U}{\partial C_2} dC_2$

$$dU = dC_1 \left[\frac{\partial U}{\partial C_1} - (1+r) \frac{\partial U}{\partial C_2} \right] \quad \begin{matrix} \frac{\partial U}{\partial C_1} < 1+r \Rightarrow (dC_1 < 0 \Rightarrow dU > 0) \\ \frac{\partial U}{\partial C_1} > 1+r \Rightarrow (dC_1 > 0 \Rightarrow dU > 0) \end{matrix} \quad \begin{matrix} \text{Contradict} \\ \text{maximality} \\ \Downarrow \\ \frac{\partial U / \partial C_1}{\partial U / \partial C_2} = 1+r \end{matrix}$$

$$\cdot U(C_1, C_2) = U(C_1) + \beta U(C_2)$$

$0 < \beta < 1 \rightarrow$ Measure of Impatience
Impatient patient → Money has more meaning when we are young, as does consumption
Consumption follows same utility function
Utility due to consumption today is independent of consumption tomorrow.

$$\text{Ex. } U(C_1, C_2) = \ln C_1 + \beta \ln C_2 \rightarrow (1+r) = \frac{C_2^*}{C_1^*} = \frac{C_2^*}{C_1^* \beta} \rightarrow (C_1^*, \beta(1+r)C_1^*)$$

$$\text{From B.C./B.L.} \rightarrow C_1^* = \frac{1}{1+\beta} \left[Y_1 + \frac{Y_2}{1+r} \right]; \quad C_2^* = \frac{\beta}{1+\beta} \left[Y_1(1+r) + Y_2 \right]$$

$\beta \uparrow C_1^* \uparrow, C_2^* \uparrow; \beta \downarrow C_1^* \uparrow, C_2^* \uparrow; r \uparrow C_1^* \downarrow, C_2^* \uparrow; r \downarrow C_1^* \uparrow, C_2^* \downarrow$

★ People respond more to permanent changes than temporary changes

★ Investment is much more volatile than consumption.

↪ If rates ↑, investment ↑
as we want

If rates ↑,
consumption changes in a
calculated manner

· Loans, etc., depend on present income ⇒ Present income matters even if PV same

$$\text{Consumption} = C(Y_t, Y_{t+1}, Y_{t+2}, \dots)$$

↗ Human Wealth $\leftrightarrow \frac{(1-T_{t+1}^e)Y_{t+1}^e}{1+r_t^e} + \dots$
 ↗ Non-Human Wealth
 ↗ Stocks ↓ Durable
 ↗ Bonds Goods
 ↘ $+ Y_t, Y_{t+1}^e, Y_{t+2}^e, \dots$
 ↘ $- r_{t+1}^e, r_{t+2}^e, r_{t+3}^e, \dots$
 ↘ $- T_t, T_{t+1}^e, T_{t+2}^e, \dots$

$$\text{Investment} = I\left(\frac{+ \pi_{t+1}^e, \pi_{t+2}^e, \dots}{- r_t, r_{t+1}^e, \dots, \delta}\right)$$

Firm → Profit (gross) → $\pi_t, \pi_{t+1}^e, \pi_{t+2}^e, \dots$ → Depreciation of Capital
 $\pi_t, \pi_{t+1}^e, (1-\delta)\pi_{t+2}^e, (1-\delta)^2\pi_{t+3}^e$

$$\begin{aligned} \text{NPV} &: \pi_t + \frac{\pi_{t+1}^e}{1+r_t} + \frac{\pi_{t+2}^e(1-\delta)}{(1+r_t)(1+r_{t+1}^e)} + \frac{\pi_{t+3}^e(1-\delta)^2}{(1+r_t)(1+r_{t+1}^e)(1+r_{t+2}^e)} \quad \left\{ \begin{array}{l} + \pi_i / \pi_i^e \\ - r_i / r_i^e, \delta \end{array} \right. \\ (\text{Net Present Value}) & \end{aligned}$$

$$* \pi_t = 0, I = I\left(\frac{\pi}{1+r} + \frac{\pi(1-\delta)}{(1+r)^2} + \dots\right) = I\left(\frac{\pi}{1+r} \cdot \frac{1}{1-\frac{1-\delta}{1+r}}\right) = I\left(\frac{\pi}{r+\delta}\right)$$

$\pi_i^e = \pi$
 $r_t^e = r$

* $\pi \rightarrow \text{Depends on } Y$

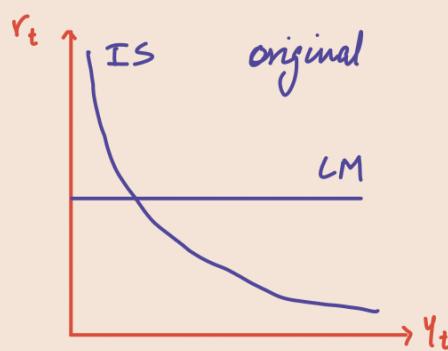
: user/
rental cost

★ $Y = C + I + G$

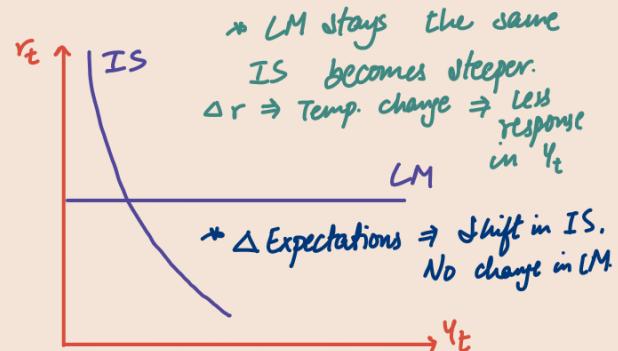
Expectations Model

$$\begin{aligned} &= C(Y_t - T_t) + \bar{I} + \bar{G} \rightsquigarrow \text{Simple Keynesian Model} \\ &= C(Y_t - T_t) + I(Y_t, r_t) + \bar{G} \rightsquigarrow \text{IS-LM Model} \\ &= C(Y_t - T_t) + I(Y_t, r_t + x) + \bar{G} \rightsquigarrow \text{Extended IS-LM Model} \\ &= C\left(\begin{array}{l} Y_t, Y_{t+1}^e, Y_{t+2}^e, \dots \\ T_t, T_{t+1}^e, T_{t+2}^e, \dots \\ r_t, r_{t+1}^e, r_{t+2}^e, \dots \end{array}\right) + I\left(\begin{array}{l} Y_t, Y_{t+1}^e, \dots \\ r_t, r_{t+1}^e, \dots \end{array}\right) + \bar{G} = \bar{G} + A\left(\begin{array}{l} + Y_t, Y_i^e, \\ - T_b, T_i^e, \\ - r_t, r_i^e \end{array}\right) \end{aligned}$$

$A = C + I$



Expectations



Long Run

$y_0 \rightarrow \text{growth rate } g$

$$y_t = y_0 e^{gt}$$

$y_t \rightarrow GDP$

$\frac{y_t}{N_t} \rightarrow GDP \text{ per capita}$

Output Technology Capital Labour

because cont. in time and not discretely.

continuously in time: $\frac{dy}{dt} = g$

discretely in time: $\frac{y_t - y_{t-1}}{g_{t-1}} = g$

$$y_t = F(K_t, N_t)$$

* How do we compare GDP of two countries?

1 \$ = ₹ 80

	Ind.	US	$\frac{GDP_{U,\$}}{GDP_{I,\₹}} = \frac{5500\$}{100k \₹} = 4.4$
Meal	1 → 20k ₹	1 → 500 \$	
Comp.	1 → 80k ₹	5 → 1000 \$	$\frac{GDP_{U,\$}}{GDP_{I,\$}} = \frac{5500\$}{500 \cdot 1 + 1000 \cdot 1} = \frac{5500\$}{1500\$} = 3.67$
Changes w/ changing currency			

Douglas (Swann) Model

Diminishing Rates
of Marginal Product

- $y = F(K, N) \rightarrow \frac{\partial F}{\partial K} > 0, \frac{\partial F}{\partial N} > 0, \frac{\partial^2 F}{\partial K^2} < 0, \frac{\partial^2 F}{\partial N^2} < 0$
- $F(xK, xN) = x F(K, N) \rightarrow$ Constant returns to scale
- $F(0, N) = F(K, 0) = 0$
- $\lim_{K \rightarrow 0} F_K(K, N) = \infty, \lim_{N \rightarrow 0} F_N(K, N) = \infty$
- $\lim_{K \rightarrow \infty} F_K(K, N) = 0, \lim_{N \rightarrow \infty} F_N(K, N) = 0$

Ex: $y = A K^\alpha N^{1-\alpha}, 0 < \alpha < 1,$

Cobb-Douglas
Production Function

Similarly for $\frac{\partial F}{\partial N}, \frac{\partial^2 F}{\partial N^2}$

$$\begin{aligned} \text{Also, } F(xK, xN) &= A \cdot x^\alpha K^\alpha \cdot x^{1-\alpha} N^{1-\alpha} \\ &= x \cdot A \cdot K^\alpha \cdot N^{1-\alpha} \\ &= x \cdot F(K, N) \end{aligned}$$

- * We were interested in $(Y/N)_t \rightsquigarrow N_t = N$

$$Y_t/N_t = Y_t/N = F(K_t, N)/N = \boxed{F\left(\frac{K_t}{N}, 1\right)} = \frac{y_t}{N} . \quad k_t = \frac{K_t}{N}$$

$$\frac{y_t}{N} = F\left(\frac{K_t}{N}, 1\right) = f\left(\frac{K_t}{N}\right) \text{ w/ } f' > 0 \text{ & } f'' < 0$$

$$K_{t+1} = K_t - \underbrace{\delta K_t}_{\text{Depreciation}} + \underbrace{I_t}_{\text{Investment}}$$

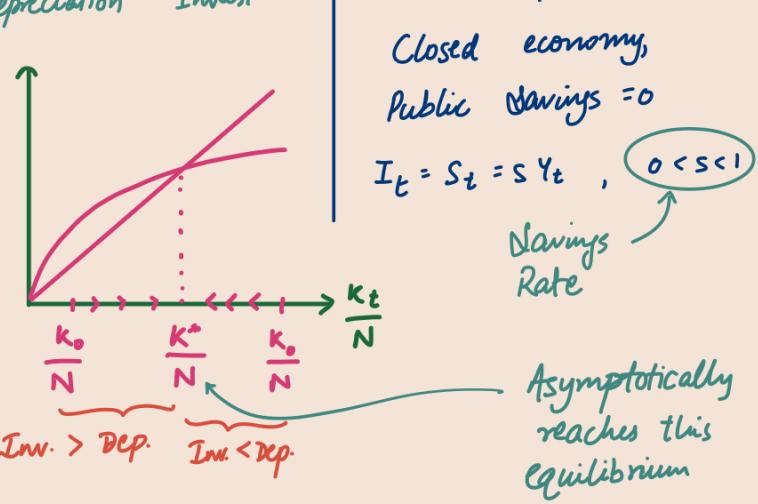


Assumption

$$K_{t+1} = K_t - \delta K_t + s Y_t$$

$$\Rightarrow \frac{K_{t+1}}{N} - \frac{K_t}{N} = s \frac{Y_t}{N} - \frac{\delta K_t}{N}$$

$$\Rightarrow \frac{K_{t+1}}{N} - \frac{K_t}{N} = \underbrace{s F\left(\frac{K_t}{N}\right)}_{\text{Investment Component}} - \underbrace{\frac{\delta K_t}{N}}_{\text{Depreciation Component}}$$



Not possible because
 $\lim_{K_t \rightarrow 0} f' = \infty$

- * $K/N \rightarrow$ Capital Labour ratio
- * $\Delta K > 0 \rightarrow$ Capital Accumulation

* If $s \uparrow$ then $\frac{K^*}{N} \uparrow \Rightarrow \frac{Y^*}{N} \uparrow$ but $\frac{\partial}{\partial t}$ (Growth Rate) still < 0

Also, $C_t = Y_t - s Y_t = (1-s) Y_t$ (s) $\begin{cases} s=0 \Rightarrow Y_t(0)=0 \Rightarrow C_t=0 \\ s=1 \Rightarrow C_t=0 \end{cases}$



* $\frac{Y_t}{N} = F\left(\frac{K_t}{N}, 1\right) = f\left(\frac{K_t}{N}\right)$ $\begin{cases} \text{Assumes } N_t = N \\ \text{Doesn't factor in productivity} \end{cases}$

\hookrightarrow Bring in new model $\begin{cases} 1) \text{Discrete time} \rightsquigarrow t = 0, 1, 2, \dots \\ 2) \text{Continuous time} \rightsquigarrow dt \end{cases}$

i) Discrete Time $t = 0, 1, 2, \dots$

$$Y_t = F(K_t, A_t N_t)$$

Assumption

Output Capital Labour-augmenting productivity

About F

- 1) $F_K(K, L) > 0, F_L(K, L) > 0$
- 2) $F_{KK}(K, L) < 0, F_{LL}(K, L) < 0$
- 3) CRS (Const. Returns to Scale):
 $F(cK, cL) = c F(K, L)$

4) $\lim_{K \rightarrow 0} F_K(K, L) = \infty, \lim_{K \rightarrow \infty} F_K(K, L) = 0$, 5) $F(K, 0) = F(0, L) = 0$

$\lim_{L \rightarrow 0} F_L(K, L) = \infty, \lim_{L \rightarrow \infty} F_L(K, L) = 0$

$$Y_t = F(K_t, A_t N_t) \xrightarrow{1/A_t N_t} \frac{Y_t}{A_t N_t} = \frac{1}{A_t N_t} F(K_t, A_t N_t) = F\left(\frac{K_t}{A_t N_t}, 1\right)$$

$$y_t = \frac{Y_t}{A_t N_t}, k_t = \frac{K_t}{A_t N_t}$$

$$y_t = F(k_t, 1) \equiv f(k_t)$$

$$K_t \rightarrow Y_t \downarrow$$

$$K_{t+1} \leftarrow I_t$$

$$\begin{aligned} N_{t+1} &= (1+n) N_t \\ A_{t+1} &= (1+g) A_t \\ K_{t+1} &= (1-\delta) K_t + I_t = (1-\delta) K_t + s Y_t \\ I_t &= S_t \quad (\text{Closed eco w/ no govt.}) \\ S_t &= s Y_t, \quad 0 < s < 1 \end{aligned}$$

$\frac{A_t N_t}{A_{t-1} N_{t-1}}$

$\frac{A_t N_t}{A_{t-1} N_{t-1}}$

$$1) f(0) = 0 \quad 2) f'(k) > 0 \quad 3) f''(k) < 0 \quad 4) \lim_{k \rightarrow 0} f' = \infty, \lim_{k \rightarrow \infty} f' = 0$$

$$f(k) = \frac{1}{A_t N_t} F(K_t, A_t N_t) = f\left(\frac{K_t}{A_t N_t}\right) \xrightarrow{\partial} \frac{F_k(K_t, A_t N_t)}{A_t N_t} = \frac{f_k(k)}{A_t N_t}$$

$$\Rightarrow f_k(k) = F_k(K_t, A_t N_t) > 0 \quad \text{Similarly for } f''$$

* We would like k in the difference equations:

$$K_{t+1} = (1-\delta) K_t + s F(K_t, A_t N_t)$$

$$\xrightarrow[A_t N_t]{\frac{K_{t+1}}{A_{t+1} N_{t+1}}} \frac{A_{t+1} N_{t+1}}{A_t N_t} = (1-\delta) k_t + s f(k_t) = k_{t+1} (1+g)(1+n)$$

$$\Rightarrow k_{t+1} = \frac{(1-\delta) k_t}{(1+g)(1+n)} + \frac{s f(k_t)}{(1+n)(1+g)} \quad \xrightarrow{\text{Steady state: } k_{t+1} = k_t = k^*}$$

$$k^* = \frac{(1-\delta)}{(1+g)(1+n)} k^* + \frac{s f(k^*)}{(1+n)(1+g)}$$

$$\Rightarrow k^* (n+g+gn+\delta) = s f(k^*)$$

$$1) k_t = k^* \Rightarrow k_{t+1} = k_t = k^*$$

$$2) k_t < k^* \Rightarrow k_{t+1} - k_t = \frac{s f(k_t)}{(1+n)(1+g)} + \frac{(1-\delta) k_t}{(1+n)(1+g)} - k_t$$

$$= \frac{1}{(1+n)(1+g)} [s f(k_t) - (\delta + n + g + gn) k_t]$$

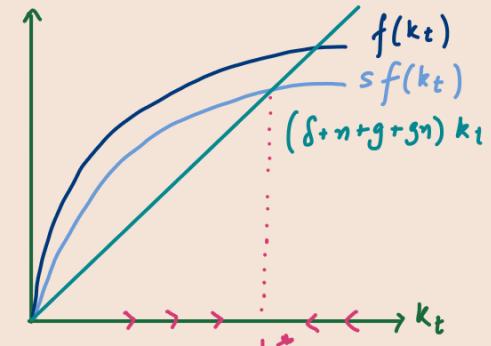
$$k_{t+1} > k_t \Leftrightarrow s f(k_t) > (\delta + n + g + gn) k_t$$

$$\Leftrightarrow k_t < k^*$$

$$3) k_t > k^* \Leftrightarrow k_{t+1} < k_t$$

* Steady state $\rightarrow k^* \rightarrow y^* = f(k^*)$

$$\Rightarrow y^* = \frac{y}{AN} = \text{const.} \Rightarrow \frac{y}{N} \text{ has a const. growth rate} (= \text{growth rate of } A)$$



2) Continuous dt

$$Y(t) = F(K(t), A(t) N(t)), \quad y(t) = \frac{Y(t)}{A(t) N(t)}, \quad F(K(t), 1) \equiv f(K(t)) = F\left(\frac{K(t)}{A(t) N(t)}, 1\right)$$

$$A_{t+1} = (1+g) A_t \leftrightarrow \dot{A}(t)/A(t) = g$$

$$N_{t+1} = (1+n) N_t \leftrightarrow \dot{N}(t) = n N(t)$$

$$\dot{K}(t) = I(t) - \delta K(t) = s Y(t) - \delta K(t)$$

$$K(t) \equiv \frac{K(t)}{A(t) N(t)} \Rightarrow \frac{\dot{K}}{K} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{N}}{N} = \frac{\dot{K}}{K} - g - n$$

$$\Rightarrow \dot{K} = \frac{\dot{K}}{K} \cdot K - (g+n) K = \frac{\dot{K}}{K} \cdot \frac{K}{AN} - (g+n) K = \frac{\dot{K}}{AN} - (g+n) K$$

$$\Rightarrow \dot{K} = \underline{s Y - \delta K} - (g+n) K = \underline{s F(K, AN)} - \delta \frac{K}{AN} - (g+n) K$$

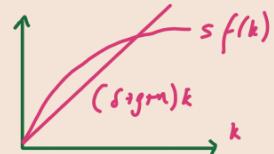
$$= s F\left(\frac{k}{AN}, 1\right) - \delta k - (g+n)k = s f(k) - (\delta + g + n)k$$

$$\Rightarrow \dot{k}(t) = s f(k(t)) - (\delta + g + n)k(t) \equiv \psi(k(t)) \rightarrow \psi'(k^*) < 0$$

↳ Steady State: $\dot{k} = 0 \Rightarrow s f(k^*) = (\delta + g + n)k^*$

- * $k > 0 \Leftrightarrow s f(k) > (\delta + g + n)k \Leftrightarrow k < k^*$
- $k < 0 \Leftrightarrow s f(k) < (\delta + g + n)k \Leftrightarrow k > k^*$

$k = 0 \Leftrightarrow$ steady state $\Leftrightarrow k = k^* ; y \rightarrow y^* \Rightarrow \frac{y(t)}{A(t)N(t)} \rightarrow f(k^*)$



- * Growth rate of $\frac{y(t)}{N(t)}$ in steady state = g
- " " " " = $g + n$

- * What happens if $s \uparrow$?

↳ Effect on steady state growth rate \rightarrow No effect

↳ values of k^* & y^* ? $k^* \uparrow, y^* \uparrow$

i) Graphically $\nearrow \partial(s f - (\delta + g + n)k)$

$$2) \frac{dk^*}{ds} = - \frac{\partial / \partial s}{\partial / \partial k} = \frac{-f'(k^*)}{sf'(k^*) - (\delta + g + n)} = - \frac{f'(k^*)}{\psi'(k^*)} > 0$$

$$s \uparrow \Rightarrow k^* \uparrow \Rightarrow y^* \uparrow = f(k^*) \uparrow$$

↳ Effect on short run growth? Yes

- * Consumption: $c = (1-s)y = f(k) - sy \rightarrow c^* = f(k^*) - sy^* = f(k^*) - (\delta + g + n)k^*$

$$\frac{dc^*}{ds} = [f'(k^*) - (\delta + g + n)]k^{**}(s), \quad k^{**}(s) > 0$$

$$\frac{dc^*}{ds} > 0 \Leftrightarrow f'(k^*) > (\delta + g + n) \quad * \exists s_0 \text{ s.t. } f'(k^*) = \delta + g + n \quad \text{which maximises } c$$

→ Suppose $s < s_0$ & $s \uparrow \Rightarrow \hat{k} \uparrow \Rightarrow f'(\hat{k}) \downarrow$
but $\hat{k} < k_{s_0}^*$ so $f'(\hat{k}) > f'(k_{s_0}^*)$
 $\Rightarrow dc^*/ds > 0 \Rightarrow c^* \uparrow$

→ Suppose $s > s_0$ & $s \uparrow \Rightarrow \hat{k} \uparrow$
 $\Rightarrow f'(\hat{k}) \downarrow$ & $f'(\hat{k}) < f'(k_{s_0}^*)$
 $\Rightarrow dc^*/ds < 0 \Rightarrow c^* \downarrow$

Depends on
the mbd. of s



s_0 s.t. $k^* = k_{s_0}^*$
for which
 $f'(k_{s_0}^*) = \delta + g + n$

- * What is the effect of A in the short run and the medium run?



$$Y = AN \rightarrow Y/N = A, A_0 \uparrow A_1$$

↑ *true expectations* ↓ *-ve expectations*

(IS \rightarrow IS₀) (IS_p \leftarrow IS)

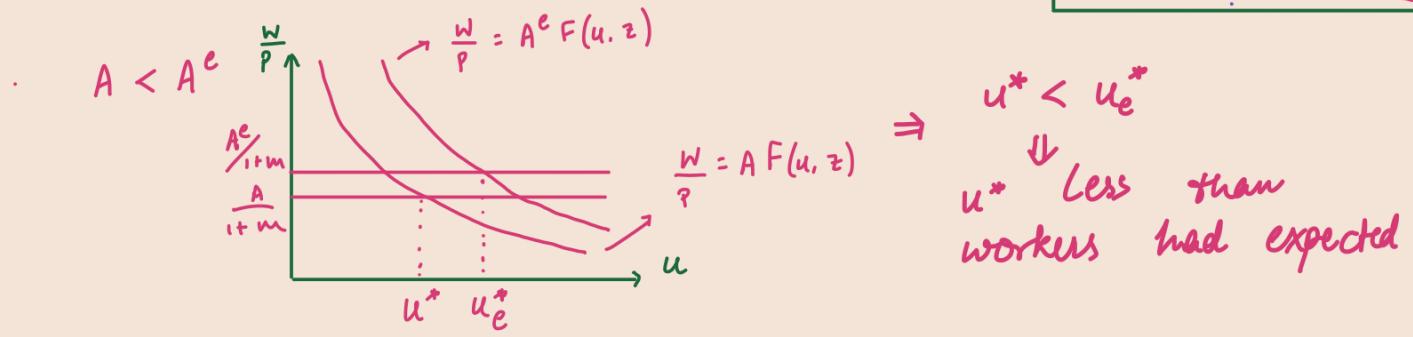
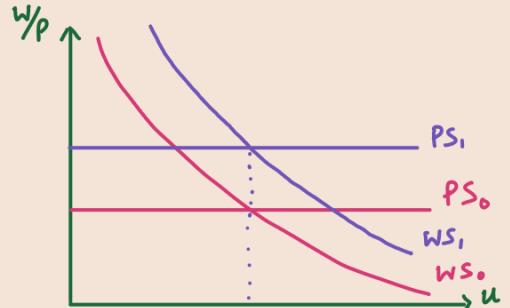
PS $P = (1+m)W \xrightarrow{Y=N \rightarrow Y=AN} \left\{ \begin{array}{l} P = (1+m) \frac{W}{A} \\ W = P^e F(u, z) \end{array} \right.$

WS $W = P^e F(u, z) \xrightarrow{Y=N \rightarrow Y=AN} \left\{ \begin{array}{l} W = P^e A^e F(u, z) \\ 1 \text{ unit of } N \rightarrow 1 \text{ unit of } Y \\ \frac{1}{A} \dots N \rightarrow 1 \dots Y \\ \text{Cost} = W/A \\ P = (1+m) W/A \end{array} \right.$

* $P = P^e, A = A^e$ $A = A_0 \uparrow A_1 \Rightarrow \frac{W}{P} \uparrow, u \leftarrow$

$\frac{W}{P} = \frac{A}{1+m}$

$\frac{W}{P} = A F(u, z)$



Govt. Budget Constraint $B_x \rightarrow$ Debt in year x . also called **ignorance**:
 Deficit in year t $= \text{Def}_t = r B_{t-1} + G_t - T_t$

$$\begin{aligned} B_t &= B_{t-1} + \text{Def}_t \\ &= (1+r) B_{t-1} + G_t - T_t \\ \Rightarrow B_t - B_{t-1} &= r B_{t-1} + \underline{G_t - T_t} \end{aligned}$$

Budget Deficit / Primary Deficit

Ex. $B_0 = G_0 = T_0 = 0$

$G_t \uparrow 1, T_t = 0, \text{Def}_t = 1, B_t = 1$

Suppose we want $B_2 = 0 \rightsquigarrow T_2 = 1+r > 1 > 0$

If $T_2 = G_2 = 0 \rightarrow B_2 = 1+r$

Suppose we want $B_3 = 0 \rightsquigarrow T_3 = (1+r)^2$

1) ↑ in tax

2) More one waits, higher the ↑ in tax

: interest :

- Ex. $B_0 = G_0 = T_0 = 0$
- $G_1 = 1, T_1 = 0, B_1 = 1$
- $G_2 = 0, T_2 = r, B_2 = 1$
- $G_3 = 0, T_3 = r, B_3 = 1$
- ★ Keep paying on
the debt does not increase

★ $B_t = (1+r)B_{t-1} + G_t - T_t, g = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$

$$\Rightarrow \frac{B_t}{Y_t} = (1+r) \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t} = \frac{(1+r)}{(1+g)} \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

$$\approx (1+r-g) \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

★ $\frac{B_t}{Y_t}$ is called
debt-GDP ratio

$$\Rightarrow \frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} \approx (r-g) \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Additional Practice Problems 3

- ★ Obs 1: Maximizing a Cobb-Douglas utility function

$$\max. U = X^\alpha Y^\beta \quad \text{s.t. } p_x X + p_y Y = m$$

From Lagrangian \rightarrow

$$\frac{\alpha}{\alpha+\beta} = \frac{p_x X}{m} ; -\frac{\beta}{\alpha+\beta} = \frac{p_y Y}{m}$$

$$X = \frac{m}{p_x} \cdot \frac{\alpha}{\alpha+\beta}, \quad Y = \frac{m}{p_y} \cdot \frac{\beta}{\alpha+\beta}$$

- ★ Obs. 2: Max. $U = X^\alpha Y^\beta \leftrightarrow \text{Max. } V = \alpha \log X + \beta \log Y$

Q1 $\max_{C_1, a, C_2} U(C_1) + \beta U(C_2)$ subject to $\begin{cases} U(c) = \frac{c^{1-\sigma}-1}{1-\sigma} \\ a = a_0 + y_1 - T_1 - C_1 \\ C_2 = y_2 - T_2 + (1+r)a \end{cases}$

P1 Solve for C_1^*, C_2^*, a^*

$$C_2 = y_2 - T_2 + (1+r)[a_0 + y_1 - T_1 - C_1]$$

$$\Rightarrow C_1 + \frac{C_2}{1+r} = a_0 + y_1 - T_1 + \frac{y_2 - T_2}{1+r}$$

$$\mathcal{L} = U(C_1) + \beta U(C_2) + \lambda \left[a_0 + y_1 - T_1 + \frac{y_2 - T_2}{1+r} - C_1 - \frac{C_2}{1+r} \right]$$

$$\frac{\partial \mathcal{L}}{\partial C_1} : \lambda = U'(C_1); \quad \frac{\partial \mathcal{L}}{\partial C_2} : \frac{\lambda}{1+r} = \beta U'(C_2); \quad \frac{\partial \mathcal{L}}{\partial \lambda} : BC \text{ again}$$

$$\frac{U'(c_1)}{\beta U'(c_2)} = (1+r) = \frac{c_1^{-\sigma}}{\beta c_2^{-\sigma}} \Rightarrow \frac{c_2}{c_1} = [\beta(1+r)]^{\frac{1}{\sigma}}$$

$$\Rightarrow c_1^* + \frac{c_2^*}{1+r} [\beta(1+r)]^{\frac{1}{\sigma}} = \dots \Rightarrow c_1^* = \frac{a_0 + y_1 - \tau_1 + \frac{y_2 - \tau_2}{1+r}}{1 + \frac{1}{1+r} [\beta(1+r)]^{\frac{1}{\sigma}}}$$

$$c_2^* = c_1^* [\beta(1+r)]^{\frac{1}{\sigma}}; \quad a^* = a_0 + y_1 - \tau_1 - c_1^*$$

P2 $\frac{c_1}{y_1}(y_2)$? What would happen if HHs become optimistic about their future? $y_2 \uparrow$

$$\frac{c_1^*}{y_1} = \frac{1}{y_1} \frac{(1+r)[a_0 + y_1 - \tau_1] + (y_2 - \tau_2)}{(1+r) + [\beta(1+r)]^{\frac{1}{\sigma}}} \longrightarrow y_2 \uparrow \left(\frac{c_1^*}{y_1}\right) \uparrow$$

P3 $\frac{c_1}{y_1}(a_0)$? $\rightsquigarrow a_0 \uparrow \left(\frac{c_1}{y_1}\right) \uparrow$

P4 $\frac{c_1}{y_1}(\beta)$? $\rightsquigarrow \beta \uparrow \Rightarrow$ More patient; $\left(\frac{c_1}{y_1}\right) \downarrow$

P5 $y_2 = \tau_1 = \tau_2 = 0$. $\frac{\partial c_1}{\partial r} = ?$ $\frac{\partial c_1}{\partial r}(\sigma) ?$

$$c_1^* = \frac{(1+r)(a_0 + y_1)}{(1+r) + [\beta(1+r)]^{\frac{1}{\sigma}}} = \frac{(a_0 + y_1)}{1 + \beta^{\frac{1}{\sigma}} \cdot (1+r)^{\frac{1}{\sigma}-1}}$$

$$\frac{\partial c_1^*}{\partial r} = -\frac{(a_0 + y_1) \cdot \beta^{\frac{1}{\sigma}} \cdot (\frac{1}{\sigma}-1) (1+r)^{\frac{1}{\sigma}-2}}{\left[1 + \beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}\right]^2} \rightsquigarrow \begin{matrix} \text{Sign is the} \\ \text{same as that of} \\ (1-\frac{1}{\sigma}) \end{matrix}$$

$$\sigma \rightarrow 1 \quad U(c) \rightarrow \ln c \quad \& \quad \frac{\partial c_1^*}{\partial r} = 0$$

Q2 $U(c_1, c_2) = \log c_1 + \beta \log c_2, \quad r > 0, \quad 0 < \beta < 1$

$$y_2 = y, \quad y_1 = 0; \quad BC = \{c_1 - a = 0, \quad c_2 = y - (1+r)a\}$$

borrowed \leftarrow

P1 $c_1^*, c_2^*, a^* = ? \quad c_2 = y - (1+r)c_1 \Rightarrow c_1 + \frac{c_2}{1+r} = \frac{y}{1+r}$

$$\frac{P_{c_1} c_1}{m} = \frac{1}{1+\beta}, \quad P_{c_1} = 1, \quad m = \frac{y}{1+r}, \quad \frac{P_{c_2} c_2}{m} = \frac{\beta}{1-\beta}$$

$$\Rightarrow c_1^* = \frac{y}{(1+r)(1+\beta)} = a^*, \quad c_2^* = \frac{\beta}{1+\beta} \frac{y}{1+r} \cdot (1+r) = \frac{y\beta}{1+\beta} = c_2^*$$

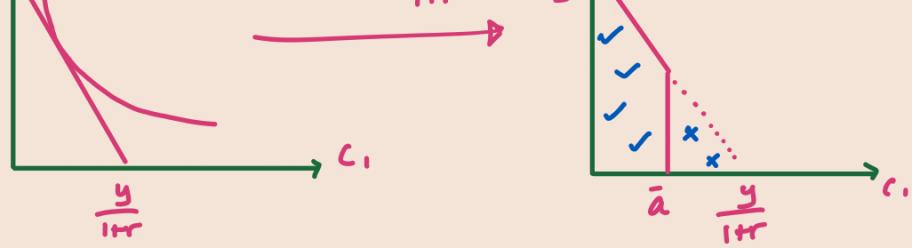
P2 $\frac{c_2}{c_1} = \beta(1+r), \quad \beta \uparrow \Rightarrow \frac{c_2}{c_1} \uparrow \Leftarrow r \uparrow$

P3 Max. you can $\frac{c_2}{y} \uparrow$

$\bar{a} < \frac{y}{1+r}$ $\frac{c_2}{y} \uparrow$

borrow: \bar{a}

$$\text{If } \bar{a} \geq \frac{y}{1+r} \\ \Rightarrow \text{No effect}$$



P4 $\bar{a} = \frac{y}{2(1+r)}$ Is $\bar{a} > a^*?$ $\Leftrightarrow \frac{y}{2(1+r)} > \frac{y}{(1+\beta)(1+r)} \Leftrightarrow 1+\beta > 2$
 $\bar{a} < a^* \Rightarrow c_1^* = a^* \rightsquigarrow c_1^* = \bar{a} = \frac{y}{2(1+r)} ; c_2^* = y - (1+r)c_1^* = \frac{y}{2}$

Q3 $U(c_1, c_2) = \log(c_1) + \beta \log\left(\frac{c_2}{c_1^\theta}\right), \theta \in [0, 1]. y_1, y_2, r$

P1 $\frac{c_2}{c_1}(\theta)?$

$$U(c_1, c_2) = (1-\beta\theta) \log c_1 + \beta \log c_2 \quad c_2^{**} = \frac{\beta}{1-\beta\theta} \left[y_1 + \frac{y_2}{1+r} \right] (1+r)$$

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \quad \hookrightarrow c_1^* = \frac{1-\beta\theta}{1-\beta\theta+\beta} \cdot \left[y_1 + \frac{y_2}{1+r} \right]$$

$$p_{c_1} = 1, p_{c_2} = \frac{1}{1+r} . \quad \frac{c_2^*}{c_1^*} = \frac{\beta(1+r)}{1-\beta\theta} ; \quad \theta \uparrow \quad 1-\beta\theta \downarrow \quad \frac{c_2^*}{c_1^*} \uparrow$$

P2 FOC (first order conditions) if govt. taxes interests at T .

$$\frac{c_2^{**}}{c_1^*} = \frac{\beta[1+r(1-T)]}{1-\beta\theta}$$

Q4 $U(c_1, c_2) = \log c_1 + \beta \log c_2, c_1 + B = y_1 - T_1, c_2 = y_2 + (1+R)B - T_2$

P1 $c_1, c_2?$ $c_1 + \frac{c_2}{1+R} = y_1 - T_1 + \frac{y_2 - T_2}{1+R}$

$$c_1^* = \frac{1}{1+\beta} \left[y_1 - T_1 + \frac{y_2 - T_2}{1+R} \right], \quad c_2^* = \beta \cdot \frac{1+R}{1+\beta} \left[y_1 - T_1 + \frac{y_2 - T_2}{1+R} \right]$$

- P2
- 1) $\Delta T_1 = G_1$
 - 2) Take loan of G_1 , b
raise tax in 2nd period
- How high does tax change need to be in 2nd period?
 $\Delta T_2 = G_1(1+R)$

P3 How do c_1, c_2 change w/ 1) / 2) ?

$$1) c_1 + \frac{c_2}{1+R} = y_1 - T_1 - \Delta T + \frac{y_2 - T_2}{1+R} = y_1 - T_1 - G_1 + \frac{y_2 - T_2}{1+R}$$

$$2) c_1 + \frac{c_2}{1+R} = y_1 - T_1 + \frac{y_2 - T_2 - \Delta T}{1+R} = y_1 - T_1 + \frac{y_2 - T_2 - G_1}{1+R}$$

BC same $\Rightarrow c_1^*, c_2^*$ same in 1 & 2)

Q5 $u = \log c_1 + \log c_2$. young $\rightarrow w$ wage, old $\rightarrow 0, r$

P1 level of saving? $\rightarrow c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} = w+0=w$

$$c_1^* = \frac{1}{1+r} \cdot \frac{w}{1} = \frac{w}{2}; s = y_1 - c_1^* = w - \frac{w}{2} = \frac{w}{2}$$

P2 $s(r)$? $\rightsquigarrow r$ has no effect on s
Generally the case when u has log functions.

Q6 Agent \rightarrow 3 periods \rightarrow consume in periods 2 & 3

$$u(c_2, c_3) = \log c_2 + \beta \log c_3$$

T₁: Invests e , borrowed at r $\xrightarrow{h \text{ is a concave function of } e}$

T₂: $w \cdot h(e)$, $\underbrace{h' > 0, h'' < 0}_{\text{repay loan from T}_1}$

T₃: No income, can save s in 2nd period $\rightarrow s(1+r)$

P1 BC in T₂ & T₃

$$e(1+r) + s + c_2 = w \cdot h(e) \xrightarrow{s(1+r) = c_3}$$

P2 Set up maximization problem w/ her choice variables.

$$\begin{array}{ll} \text{Max. } \log c_2 + \beta \log c_3 & \text{s.t. } c_2 + \frac{c_3}{1+r} = \underbrace{w \cdot h(e) - e(1+r)}_{\text{Max. this so}} \\ c_2, c_3, e & \text{that } c_2 \text{ & } c_3 \text{ are also max.} \end{array}$$

P3 FOC for the maxi problem?

$$\begin{aligned} \frac{\partial}{\partial c_2} &= \lambda \\ \frac{\partial}{\partial c_3} &= \frac{\lambda}{1+r} \end{aligned} \left\{ \begin{array}{l} \text{From} \\ \text{Lagrangian} \end{array} \right.$$

$$\Rightarrow \frac{c_3}{c_2} = \beta(1+r)$$

$$\text{From R.H.S. } \left\{ \begin{array}{l} w \cdot h'(e) = 1+r \\ \dots \end{array} \right.$$

P4 $\frac{c_2}{c_3} = \frac{1}{\beta(1+r)}$ P5 $e(\beta) \rightsquigarrow$ No effect of β on e
 $\therefore w \cdot h'(e) = 1+r$
 β only affects dist. of c_2 & c_3 .

Q7 Solow-Swan model w/ CD prod. $y = k^\alpha \cdot s, \delta, g, n$.

P1 k^*, y^*, c^*

$$k(t) = s f(k(t)) - (\delta + g + n) k(t)$$

$$\text{Steady-state} \rightarrow k^* = 0 \rightarrow s \cdot k^\alpha - (\delta + g + n)k = 0$$

$$\Rightarrow k^* = \left(\frac{s}{\delta + g + n} \right)^{\frac{1}{1-\alpha}} \Rightarrow y^* = \left(\frac{s}{\delta + g + n} \right)^{\frac{\alpha}{1-\alpha}}$$

$$c^* + s^* = y^* = c^* + s y^* \Rightarrow c^* = (1-s)y^* = (1-s)\left(\frac{s}{\delta + g + n}\right)^{\frac{\alpha}{1-\alpha}}$$

P2 $s \uparrow$ y^*, k^*, c^* ? For $c^* \rightsquigarrow$ focus on $(1-s)s^{\frac{\alpha}{1-\alpha}}$

$$s \uparrow \Rightarrow k^* \uparrow y^* \uparrow$$

$$\text{for } c^*, \frac{\partial}{\partial s} \rightarrow -s^{\frac{\alpha}{1-\alpha}} + (1-s) \cdot \frac{\alpha}{1-\alpha} \cdot s^{\frac{\alpha}{1-\alpha}} = s^{\frac{\alpha}{1-\alpha}} \left[\frac{\alpha}{1-\alpha} \cdot \frac{1}{s} - \frac{\alpha}{1-\alpha} - 1 \right]$$

$$= s^{\frac{\alpha}{1-\alpha}} \left[\frac{\alpha}{1-\alpha} \cdot \frac{1}{s} - \frac{1}{1-\alpha} \right] = \frac{s^{\frac{\alpha}{1-\alpha}}}{s(1-\alpha)} [\alpha - s]$$

$s \uparrow c^* \uparrow$ if $s < \alpha$, $c^* \downarrow$ if $s > \alpha$

P3 α on y^*, k^*, c^* ?

$$\uparrow \text{in } \alpha \Rightarrow \frac{1}{1-\alpha} \uparrow \Rightarrow k^* \uparrow \text{ iff } \frac{s}{\delta + g + n} > 1$$

$$\Rightarrow \frac{\alpha}{1-\alpha} \uparrow \Rightarrow y^* \uparrow \text{ iff } \frac{s}{\delta + g + n} > 1$$

$$c^* = (1-s)y^* \rightsquigarrow c^*, y^* \uparrow \text{ if } k^* \uparrow; k^* \uparrow \text{ if } \frac{s}{\delta + g + n} > 1$$

$$\text{Q8} \quad y(t) = k(t)^\alpha R(t)^\phi (A(t) L(t))^{1-\alpha-\phi}, \quad 0 < \alpha, \phi < 1$$

$R(t) \rightarrow$ Stock of Resources depleting at $\theta > 0$, $\dot{R}(t) = -\theta R(t)$

A growth rate: g b N : n

P1 $g_y(t)$ & $g_k(t)$ denote growth rates.

$$\text{Take log} \rightarrow \text{Diff.} \rightarrow g_y = \alpha \cdot g_k - \theta \phi + (1-\alpha-\phi)(g+n)$$

P2 g_y^* & g_k^* along a BGP. Show that $g_y^* = g_k^*$.

Solve for this rate \hookrightarrow All growth rates constant.

$$\dot{k}(t) = s y(t) - \delta k(t)$$

$$\Rightarrow \frac{\dot{k}}{k} = s \frac{y}{k} - \delta \xrightarrow{\text{LHS} = \text{const.}} \text{const.} \quad \because \text{BGP}$$

$$\Rightarrow \text{RHS} = \text{const.} \Rightarrow \frac{y}{k} = \text{const.} \Rightarrow g_y^* = g_k^*$$

$$\Rightarrow g^* = \alpha g^* - \theta \phi + (1-\alpha-\phi)(g+n)$$

$$\Rightarrow g^* = \frac{1}{1-\alpha} [(g+n)(1-\alpha-\phi) - \theta \phi]$$

P3 Does the economy necessarily converge to a BGP?

$$\delta + g_K(t) = s \frac{y(t)}{K(t)} \Rightarrow \frac{K(t)}{y(t)} = \frac{s}{\delta + g_K(t)}, \quad x(t) = \frac{K(t)}{y(t)}$$

$$\Rightarrow x^* = \frac{s}{\delta + g^*} \quad . \quad g_K(t) > g^* \Rightarrow x(t) < x^*(t)$$

Also, $g_y(t) - g^* = \alpha(g_K(t) - g^*) \Rightarrow g_K(t) - g_y(t) = (1-\alpha)(g_K - g^*)$

$$\& \quad x = K/y \Rightarrow g_x = g_K - g_y$$

So, $g_K(t) > g^* \Rightarrow x(t) < x^*$

$$\hookrightarrow g_K(t) > g_y(t) \Rightarrow g_x(t) > 0 \Rightarrow x(t) \uparrow$$

And if, $g_K(t) < g^* \Rightarrow x(t) > x^*$

$$\hookrightarrow g_K(t) < g_y(t) \Rightarrow g_x(t) < 0 \Rightarrow x(t) \downarrow$$

In both cases,
 $x \rightarrow x^*$
 $\Rightarrow g_K(t) \rightarrow g^*$
So, economy
does converge
towards
BGP.

P4 $\dot{R} = n R$

$$g_{\text{New}}^* = g^* = \frac{1}{1-\alpha} [(1-\alpha-\phi)(\phi+n) + \phi n]$$

$$\text{Gap} = \Delta g^* = g^* - g^* = \frac{\phi(n-\theta)}{1-\alpha}$$

Q9 $u(c_1, t, c_2, t+1) = \log(c_1, t) + \beta \log(c_2, t+1), \quad 0 < \beta < 1$

$$1^{\text{st}} \text{ period} \rightarrow w_t \rightarrow \text{B.C.}: c_{1,t} + s_t = w_t$$

$$2^{\text{nd}} \text{ period} \rightarrow 0 \rightarrow \text{B.C.}: c_{2,t+1} = (1+r_{t+1})s_t$$

P1 $\max_{s_t} \log [w_t - s_t] + \beta \log [s_t(1+r_{t+1})]$
 $\Leftrightarrow \log [w_t - s_t] + \beta \log [s_t]$

P2 $\frac{-1}{w_t - s_t} + \frac{\beta}{s_t} = 0 \Rightarrow s_t = \frac{\beta}{1+\beta} w_t \Rightarrow s_t \text{ does not depend on } r_{t+1}$

$$Y_t = A K_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1$$

P3 $w_t(K_t) = \frac{\partial Y_t}{\partial L_t} = A \cdot (1-\alpha) \cdot K_t^\alpha L_t^{-\alpha} \Big|_{L_t=1} = A (1-\alpha) K_t^\alpha = w_t$
(given)

P4 θ of savings lost. $K_{t+1}(K_t), \quad \delta = 1$.

$$K_{t+1} = (1-\delta)K_t + I_t = 0 + (1-\theta)s_t = (1-\theta) \frac{\beta}{1+\beta} w_t \text{ (from 2)}$$

$$K_{t+1} = (1-\theta) \cdot \frac{\beta}{1+\beta} \cdot A \cdot (1-\alpha) \cdot K_t^\alpha$$

P5 Steady State K ?

$$K_{t+1} = K_t = K^* \Rightarrow K^* = \left[(1-\theta) \cdot \frac{\beta}{1+\beta} \cdot A \cdot (1-\alpha) \right]$$

P6 $K(\theta)$? $\theta \uparrow \quad K^* \downarrow$.

Open

Economy

① Openness in Goods Markets

② Openness in Financial Markets

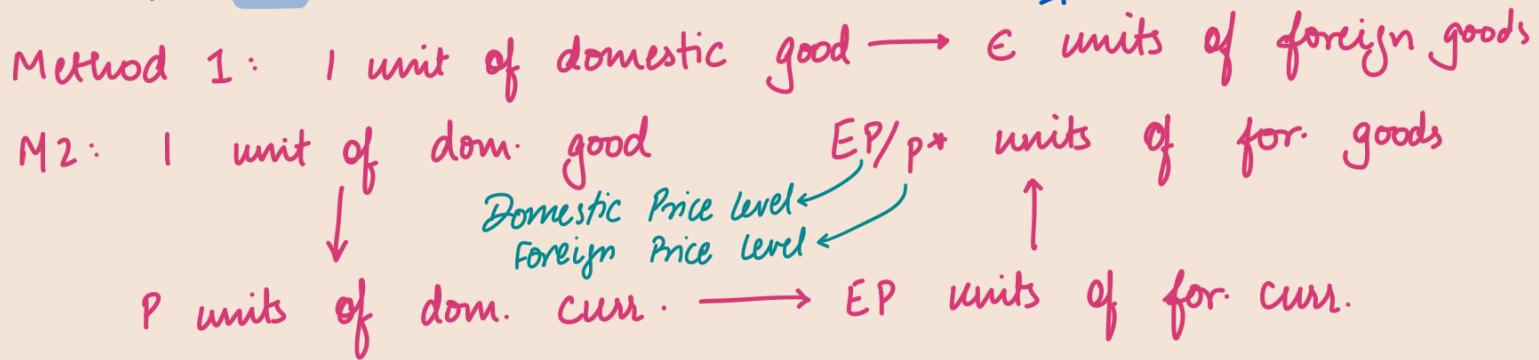
③ Openness in Factor Markets

$\left\{ \frac{\text{Export}}{\text{GDP}}, \frac{\text{Import}}{\text{GDP}} \right\}$

Goods → Tradables
Non-Tradables

Some jargon [Goods Markets]

- 1) Real Exchange Rate: Price of domestic goods relative to foreign goods
 - 2) Nominal Exchange Rate: Price of domestic currency in terms of foreign currency
 - 3) (Nominal) Appreciation: An increase in the price of domestic currency in terms of a foreign currency; \uparrow in exchange rate
 - 4) (Nominal) Depreciation: A decrease in the price of domestic currency in terms of a foreign currency → decrease in exchange rate
 - 5) Fixed Exchange Rate: A system in which two or more countries maintain a constant exchange rate b/w their currencies
 - 6) In fixed exchange rate system, revaluations are increases in the exchange rate and devaluations are decreases in the exchange rate
 - 7) (Real) Appreciation: Increase in real exchange rate
 - 8) (Real) Depreciation: Decrease in real exchange rate
- * Real Ex. Rate: 1 unit of domestic good $\Rightarrow E$ units of foreign goods
- * Nominal Ex. Rate: 1 unit of domestic currency $\Rightarrow E$ units of foreign currency



Some Jargon [Financial Markets]

- 1) Foreign Exchange: Buying & selling foreign currency
- 2) Balance of Payments: A set of accounts that summarize a country's transactions w/ the rest of the world. (BOP)
- 3) Current account: Transactions above the line \rightarrow records payments to and from the rest of the world (CA)
 - 3A) Exports and imports of goods and services (trade balance)
 - 3B) Net income balance b/w income received from rest of world and income paid to foreigners.
 - 3C) Net transfer received: Difference in foreign aid given and received
- 4) Current account balance: Sum of net payments to and from the world
- 5) Current account surplus: Positive net payments from the world.
- 6) Current account deficit: Negative net payments from the world
- 7) Capital account (\rightarrow Financial account): Transactions below the line \rightarrow records net foreign holdings of domestic assets
- 8) Net capital inflows / Capital account balance: Increase in net foreign indebtedness (holdings of domestic assets minus increase in domestic holdings of foreign assets)
- 9) Capital account surplus/deficit
- 10) Statistical Discrepancy + Current + Capital = 0

Quiz - 2

Q1 Cost = 100,000, Profits = 0, 20k, 20k(1-δ), 20k(1-δ)², ...
 $\delta = 0.1$, $r = ?$ So can buy?

$$\begin{aligned} NPV &= -100k + \frac{20k}{1+r} + \frac{20k(1-\delta)}{(1+r)^2} + \dots = -100k + \frac{20k}{1+r} \cdot \frac{1}{1-\frac{1-\delta}{1+r}} \\ &= \frac{20k}{r+\delta} - 100k \quad NPV > 0 \Leftrightarrow \frac{20k}{r+0.1} > 100k \Rightarrow r \leq 0.1 \\ &\boxed{r = 0.10} \end{aligned}$$

Q2 50 → Curr. Wealth = 5000; till 65 → 1000
After 65 → Sell for 10000; live till 75; $r = 0 = r_i^e$
Wants to leave 5000 behind

$$\frac{5k + 15 \cdot 1k + 10k - 5k}{25} = 1k = \boxed{1000}$$

Q3 $U = \sqrt{c_1 c_2}$, r, y_1, y_2 , can't borrow, $y_1 > \frac{y_2}{1+r}$, $c_2^* = ?$
 $c_1^* = \frac{1}{2} \left[y_1 + \frac{y_2}{1+r} \right] < y_1 \Rightarrow c_2^* = \frac{y_1(1+r) + y_2}{2}$

Q4 $GD_{2022} = 110, GD_{2024} = 100, BS_{2022} = 20, r_B = ?$
 $B_t = (1+r) B_{t-1} + G_t - T_t \Rightarrow 110 = (1+r) 100 - 20 \Rightarrow r = 30\%$

Q5 $5000 = y_t, y_{t+1} = 9000$: Bonds → r_s , Borrow → r_b
If $r_s = r_b = 10\%$. Then $C_t = 5000, C_{t+1} = 9000$ $c_2' = 9k$
Now, $r_s = 6\%, r_b = 12\%$. Then C_{t+1} ?

Q6 w young, o old, r, $U = \frac{\log c_1}{5} + 4 \frac{\log c_2}{5}$
 s^* ?

$$U = \frac{\log(w-s)}{5} + 4 \frac{\log s}{5} (1+r) \rightarrow \frac{dU}{ds} = 0 \Rightarrow \frac{4}{5s} - \frac{1}{5(w-s)} = 0 \Rightarrow 4w - 4s = s \Rightarrow s = \frac{4w}{5}$$

Q7,8 $y_t = \sqrt{K_t A_t N_t}, N_t = N, A_t = A$
 $d, s \in (0, 1)$ $y_t = \sqrt{k_t}$

$$k_{t+1} = (1-d) k_t + s y_t \Rightarrow k_{t+1} = k_t = k^* \Rightarrow \boxed{k^* = \left(\frac{s}{d}\right)^2}$$

s^* ? for $\frac{c}{c}$ max

$$C_t = A(c, s) \cdot S \Rightarrow \boxed{C = 1}$$

$$\frac{C_t}{A_t N_t} = (1-S) \frac{Y_t}{A_t N_t} = (1-S) \sqrt{k_t} \Rightarrow \frac{s_c}{N} = A(1-S) \cdot \frac{1}{d} \Rightarrow S = \frac{1}{2}$$

Q9 $Y = A F(K, L) = A L^\alpha K^\beta$, $F \rightarrow CRS \rightarrow \alpha + \beta = 1$

$$w = \partial Y / \partial L, r = \partial Y / \partial K, wL = rK \rightarrow \alpha = \beta = \frac{1}{2}$$

$$Y = A \sqrt{KL}, g_L = 4\%, g_K = 3\%, g_Y = 9\%, g_A = ?$$

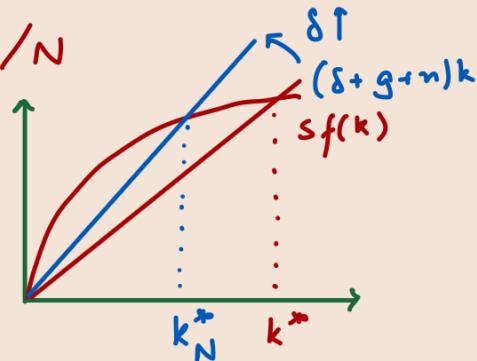
$$g_A = g_Y - \frac{1}{2}(g_K + g_L) = 9 - \frac{1}{2}(4+3) = \underline{5.5\%}$$

Q10 Solow Model \rightsquigarrow Cts. time, tech. progress
 $\delta \uparrow \rightarrow k^*$?

$$\dot{K} = I - \delta K, \dot{k}/k = \dot{K}/K - \dot{A}/A - \dot{N}/N$$

$$\frac{\dot{k}}{k} = \frac{sY}{K} - (\delta + g + n) \Rightarrow \delta \uparrow k^* \downarrow$$

$$\Rightarrow \dot{k} = sf(k) - (\delta + g + n)k$$



* $GNP = GDP + NI$

* Domestic Bond $\rightarrow i_t$
 Foreign Bond $\rightarrow i_t^*$

1 unit of domestic currency $\rightarrow 1 + i_t$ units of domestic currency Next year

$1 + i_t = \frac{E_t (1 + i_t^*)}{E_{t+1}^e}$

(Uncorrelated Interest Rate Parity)

$E_t = \frac{(1 + i_t)}{(1 + i_t^*)} E_{t+1}^e$

$i_t = i_t^* - \left(\frac{E_{t+1}^e - E_t}{E_t} \right)$

Open Economy

Goods Market $\rightsquigarrow Y = Output = C(Y - T) + I + G$ (SKM)



$y^* \rightarrow$ Foreign Demand

Net Exports/Trade Balance

domestic goods

$$Y = C + I + G + X - \frac{M}{E}$$

$$C(Y - T) \quad I(Y) \quad G \quad X - \frac{M}{E}$$

$$X(Y^*, E) \quad M(Y, E)$$

$$NX = X - \frac{M}{E} = Y - \frac{M}{E} - C(Y - T) - I(Y) - G$$

$$(Y, Y^*, E) \quad - + ?$$

★ $NX(Y, Y^*, E)$
 $- + -$

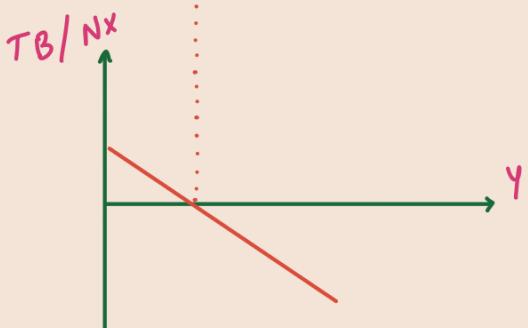
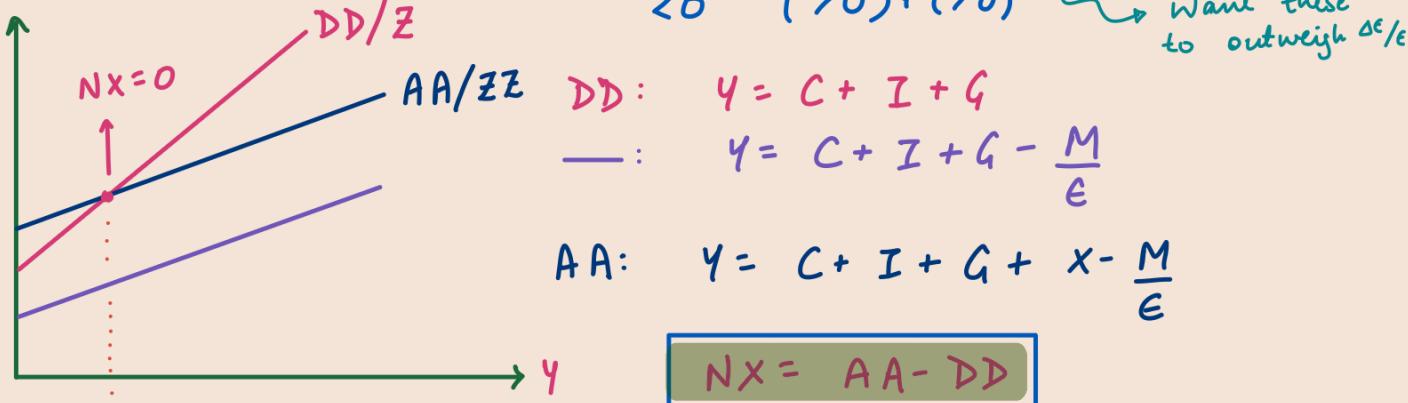
Under
: Marshall-Lerner
Conditions

Suppose $NX = X - \frac{M}{E} = 0$ initially

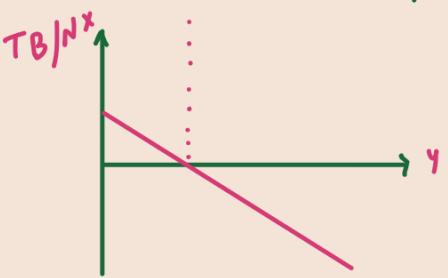
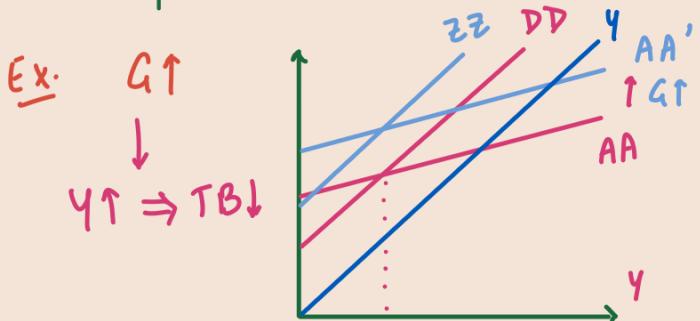
Suppose $\rightarrow E \rightarrow \Delta E < 0 \rightarrow eNX = eX - M \rightarrow eX = M$

$$\hookrightarrow (\Delta E)NX + e(\Delta X) = (\Delta e)X + e\Delta X - \Delta M$$

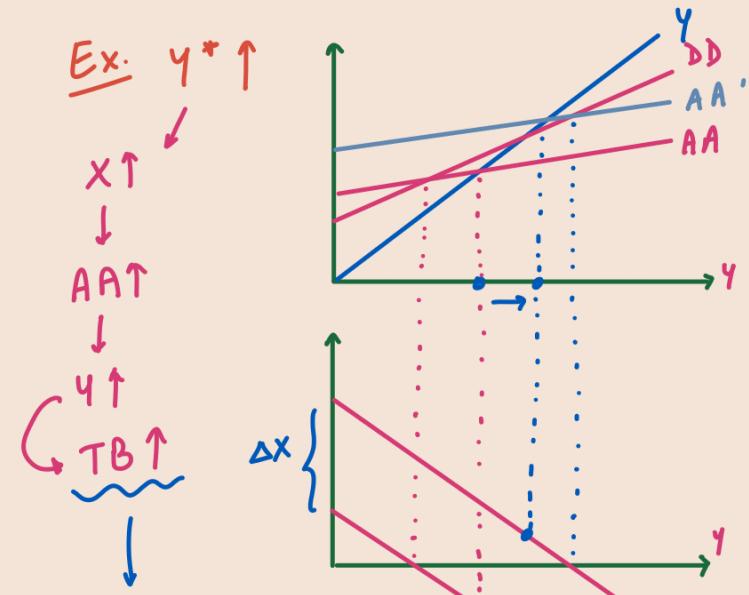
$$\frac{1}{eX} \Rightarrow \frac{\Delta NX}{X} = \frac{\Delta e}{e} + \frac{\Delta X}{X} - \frac{\Delta M}{M} > 0 \quad \text{for most countries}$$



- * $NX > 0 \leftrightarrow \text{Exports} > \text{Imports}$
 $\leftrightarrow \text{Trade Surplus}$
- * $NX < 0 \leftrightarrow \text{Exports} < \text{Imports}$
 $\leftrightarrow \text{Trade Deficit}$



Ex: $E \downarrow \rightsquigarrow \text{Similar to}$



$$Y = C + I + G + X - \frac{M}{E}$$

\downarrow
 AAT ↑ (under ML conditions)
 \downarrow
 $y \uparrow$
 $\rightarrow TB \uparrow ()$

$\rightarrow a_T = C a_T + I a_I + G + X dY^* - M' dY/E$
 $\Rightarrow NX = X - M/E$
 $\Rightarrow dNX = X' dY^* - M' dY/E$
 $= \underbrace{dY}_{>0} (\underbrace{1 - C' - I'}_{>0}) > 0 \Rightarrow NX \uparrow$
 {Related to slope}

Ex.

	Output too Low	Output too High
TB too low	$e \downarrow G?$	$G \downarrow e?$
TB too high	$G \uparrow e?$	$e \uparrow G?$

- * If a country depreciates ($e \downarrow \uparrow$) then in the short run, $X \& M$ fixed $\Rightarrow TB = X - M/E \rightsquigarrow TB \downarrow \downarrow$. Over time, TB improves, leading to a "J curve"

$$\begin{aligned}
 Y &= C + I + G + NX \\
 \Rightarrow (Y - T - C) &= I + (G - T) + NX \\
 \Rightarrow (Y + NT + NI - T - C) &= I + (G - T) + (NX + NT + NI) \\
 \Rightarrow S_{priv} &= I - S_{pub} + CA \\
 \Rightarrow S_{priv} + S_{pub} &= I + CA
 \end{aligned}$$

$$\begin{aligned}
 &\text{Before} \quad Y = C + I + G \\
 &Y - C - T = I + (G - T) \\
 &\Rightarrow S_{priv} = I - S_{pub} \\
 &\Rightarrow S_{priv} + S_{pub} = I
 \end{aligned}$$

Open Economy version of IS-LM Mundell-Fleming Model

$$\begin{aligned}
 IS \rightarrow Y &= C(Y - T) + I(Y, r) + G + NX(Y, Y^*, E) \\
 \left\{ \begin{array}{l} \text{Prices fixed} \Rightarrow \text{Inflation} = 0, \pi = 0 \Rightarrow r = i \\ \Rightarrow E = E P / P^* \Rightarrow \text{Replace } E \text{ by } E \end{array} \right.
 \end{aligned}$$

$$Y = C(Y - T) + I(Y, i) + G + NX(Y, Y^*, E)$$

$$LM \rightarrow i = \bar{i}$$

$$E_t = \frac{1+i_t}{1+i_t^*} \quad E_{t+1}^e = \bar{E} \quad \frac{1+i_t}{1+i_t^*}$$

$$i \uparrow \quad IS(T, G, Y^*)$$

LM

$$i \uparrow \quad i_t^*$$

...



Ex. $i \uparrow \Rightarrow LM \uparrow \Rightarrow Y \downarrow ; \Rightarrow \bar{E} \uparrow$

Ex. $G \uparrow \Rightarrow IS \uparrow \Rightarrow Y \uparrow ; \bar{E} \leftarrow e ; C \uparrow, I \uparrow, NX \downarrow$
(Shift rightward) $Y_0 \rightarrow Y_1, \text{ say}$

Ex. $G \uparrow, i = \bar{i} \uparrow$ so that $Y_0 < Y_1 < Y_2$,
 $Y \uparrow, i \uparrow, E \uparrow, C \uparrow, I \downarrow, G \uparrow, NX \downarrow$

* Govt's Disposal \rightarrow Fiscal, Monetary, Exchange Rate policy

Fixed v/s Flexible
Exchange Rate \rightarrow Govt. No longer has
1) Exchange Rate policy (E)
2) Monetary policy ($E(i)$)
Only Fiscal policy (G, T) can be used

• Assume $E_t = E_{t+1}^e = \bar{E}$ then $1+i = \frac{1+i_t^*}{E_{t+1}^e} E_t \Rightarrow i = i_t^*$

Now

$$Y = C(Y-T) + I(Y, \bar{\pi}) + G + NX(E, Y, Y^*) \leftarrow IS ; E = \frac{EP}{P^*}$$

$i^* - \pi^e (\because i = i^*)$

* $Y_t = Y_n$ $\pi_t - \bar{\pi} = \frac{\alpha}{L} (Y - Y_n) \Rightarrow \pi_t = \bar{\pi} \rightarrow E \text{ fixed}$
for both (constant)

* $Y_t < Y_n$ $\pi_t < \pi^* = \pi_t^* \Rightarrow \frac{P}{P^*} \downarrow \Rightarrow E \downarrow$
b $TB < 0$ " $\Rightarrow TB \uparrow$ b $Y \uparrow$



Additional Problem Set 4

Q1 $Y_1 \rightarrow$ Prod of 750, 150 exported,
rest unsold

$Y_2 \rightarrow$ No prod, unsold from Y_1 sold
domestically, Imports of 250

	GDP	Cons.	Inv.	Export	Import
1	750	0	600	150	0
2	850	-600	0	0	250

Investment = Inventory

Cons \rightarrow Overall, inc. of both

Q2 SKM w/o govt. w/ open eco. $C, M \propto Y$.

Avg. propensities to C & M are 0.8 & 0.3

$$I = 100 + 0.4Y, X = 100$$

P1 Agg. demand func. if $\max M = 450$. \exists eq^m?

$$Y = C + I + X - M, C = 0.8Y, M = \min\{0.3Y, 450\}$$

$$\downarrow \quad , \quad I = 100 + 0.4Y$$

$$1) M_a < 450 \Rightarrow 0.3Y < 450 \Rightarrow Y < 1500; M = 0.3Y$$

$$Y = 0.8Y + 100 + 0.4Y + 100 - 0.3Y = 200 + 0.9Y \Rightarrow Y_a = 2000 \Rightarrow \text{E}$$

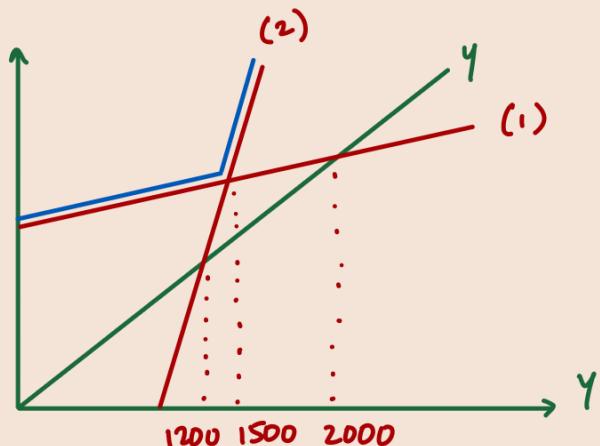
$$2) 0.3Y > 450 \Rightarrow Y > 1500$$

$$Y = 100 + 0.4Y + 0.8Y + 100 - 450 = 1.2Y - 250 \Rightarrow Y_a = 1250 \Rightarrow \text{E}$$

∅ an equilibrium

Graphically:

No int. b/w
/ b \



P2 $M = \min\{0.3Y, 615\}$, Stability of eq^m?

$$1) 0.3Y < 615 \Rightarrow Y < 2050$$

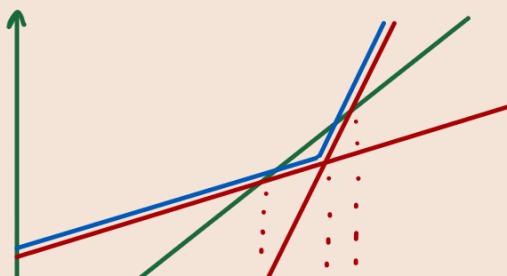
$$Y = 0.8Y + 100 + 0.4Y + 100 - 0.3Y \Rightarrow Y^* = 2000 \checkmark$$

$$2) 0.3Y > 615 \Rightarrow Y > 2050$$

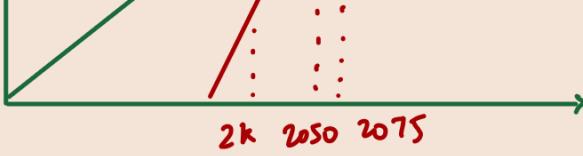
$$Y = 0.8Y + 100 + 0.4Y + 100 - 615 \Rightarrow Y^* = 2075 \checkmark$$

\exists 2 eq^m

In case of st.
lines, if



locally
 $\begin{cases} \text{Slope } < 1 \Rightarrow \text{Stable (2000)} \\ \text{Slope } > 1 \Rightarrow \text{Unstable (2075)} \end{cases}$



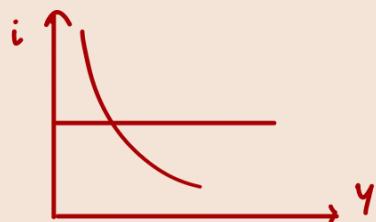
Globally, both unstable

- Q3 ISLM:
- $M/X > 0$ but no foreign capital transactions
 - e flexible P^*, P given exogenously.
 - No capital mobility & e adjusted to balance trade in eq^m. $TB = \bar{T} + \frac{\beta P^*}{P} - mY$, $\bar{T} > 0$, 4 GBP, β, m parameters. m is marginal propensity to import
 Foreigners cannot buy domestic asset.
 $CA = 0$ b others 0 $\Rightarrow TB = 0$

P1 TB eq^m & Commodity market eq^m. Y v/s r?
 Name as IS?

$$Y = C + I + G + TB, TB = 0 \quad [e \text{ changes accordingly}]$$

$$\Rightarrow Y = C + I + G = C(Y - T) + I(Y, r) + G \Rightarrow \text{Same as IS}$$



P2 $G \uparrow$ Y b e?

$$\hookrightarrow Y \uparrow \rightsquigarrow TB \downarrow \text{ so } e \uparrow$$

P3 $P^* \uparrow$ Y b e? $Y \leftrightarrow e \uparrow$

Q4 Indian exporter \rightarrow Sell 100 units at \$10/unit.
 Cost of each pen \rightarrow ₹9. Spot exchange today
 $E \text{ ₹}/\$ = 1$. Only cares about expected returns.

P1 Payment received instantaneously, should he export?
 Profit/loss?

$$\text{Revenue} = 100 \cdot 10 \$ = 1000 \$ = ₹1000$$

$$\text{Cost} = 100 \cdot 9 ₹ = ₹900 \xrightarrow{\text{Profit}} \text{Profit} = ₹100$$

Payment after 1 month now

export
 \uparrow

₹ today?

P2 Knows for sure that $E \xi/\$ = 1.1$. Export ~~today~~

Cost = $\xi 900$ \rightarrow Profit = $\xi 200 \rightarrow$ Should Export

Revenue = $1000 \$ = \xi 1100$

P3 $E \xi/\$ = 1.4$ w/ $p = \frac{1}{2}$ b 0.6 w/ $\frac{1}{2}$

Expected Revenue = $\frac{1}{2} \cdot 1400 + \frac{1}{2} \cdot 600 = 1000$

" Profit = $+100 \rightarrow$ Should export

P4 1.4 w/ $p = \frac{1}{3}$ b 0.6 w/ $\frac{2}{3}$

Expected Revenue = $\frac{1}{3} \cdot 1400 + \frac{2}{3} \cdot 600 = 2600/3$

" Profit = $2600/3 - 900 < 0 \rightarrow$ Should NOT export

P5 Can buy a forward contract for $E \xi/\$ = 1$ after one month. Same as (5), max. amount he is willing to pay for the contract?

Revenue $\rightarrow 1000$, Cost $\rightarrow 900$

Cost of contract $\rightarrow c$

$1000 - c - 900 > 0 \rightarrow \underline{\underline{c > 100}}$

