FAQs :

* 2 Quizzes (15%)
* 2 Exams (35%) Mid Term + End Term

↳ Subj. to Modification if mode → Online

Schedule: T, Th, F 10-12 (L10)

Prof. Room: KD 223

Books/References:

1. MacroEconomics by Blanchard (7th Ed.)
2. " " Jones
3. " for Professionals by Lipschitz & Schadler

- ↑ the rates → Control Inflation
  ↓ the rates →
  Changing the rates influences aggregates
- Prosperity/Well being depends on Productivity
  Prod. ↓ → Long Run Poorer
  Good Measure of long term
- US → Caused GFC (00's) due to Mortgages
  EU → Euro [Common Currency in a lot of Nations]
  India → Demonetisation, Liberalisation (~80's/'91), etc.
- ∵ Economies, like people, get sick —
  High unemployment, recessions, financial crisis, low growth ∴
  ∴ MacroEconomics is about why it happens, and what can be done about it.
- 3 central Macroeconomic variables:

① **Output** ② **Unemployment** ③ **Inflation**

→ GDP (Gross Domestic Product):

Market value of all **Final** **Goods and Services** produced in an **economy** in a given **time period**.

- Market → Standardized Value, not intrinsic value
- Final → Don't consider Intermediate Revenues
- Goods and Services → Products
- economy → Within the economy, Not outside it
- time period → Specific Time Period Annually, Quarterly, etc.

Ex. **Steel Company (F1)**

Revenues from Sale : 1000

Expenses -

Wages → 800

Profit → 200

Car Company (F2)

Revenue : 2000

Expenses -

Wages → 700

Steel purchase → 1000

Profit → 300

GDP: Measure **1**: Value of final goods & services → 2000

Measure **2**: GDP is sum of **value added** in the economy during a given period

VA by a firm is the value of its production minus the value of the intermediate goods used in production.

→ $VA_{F_1} = 1000$, $VA_{F_2} = 2000 - 1000 = 1000$

$GDP = VA_{F_1} + VA_{F_2} = 2000$

Measure **3**: GDP is sum of incomes in the economy during a given period.

→ $(800 + 200) + (700 + 300) = 2000$

Ex.

	Price	Quantity
Caps	10	5
Hats	20	15

GDP_{1999}: $10 \cdot 5 + 20 \cdot 15 = 50 + 300 = 350$

{ Price ← (P · 10)

GDP_{2000} : 3500

∴ Production ∴ has been the same but GDP ↑ due to Market Value.

Nominal v/s. Real GDP

Nominal → Sum of Quantities of Final Goods produced times their current price.

Real → Sum of Quantities of Final Goods times constant prices.
→ Fix a particular Year

Ex.

Y	Q	P	NGDP	RGDP
2008	10	20000 (Used for RGDP)	200 000	200 000
2009	12	24000	288 000	240 000
2010	13	26 000	338 000	260 000

- GDP v/s. Per Capita Income : India Really ↑ in GDP due to size (population) but very ↓ in per capita income.

PPP : Purchasing Power Parity

Unemployment

- Employed: No. of people who have a job. (E)
- Unemployed: No. of people who do not have a job but are looking for one. (U)
- Labour Force : Sum of Employed & Unemployed. (L)

(Generally, only adults are considered)

$$L = U + E$$

$$\text{Unemployment Rate} = U/L$$

$$\text{(Labour Force) Participation} = \frac{L}{(\#\text{ of working age})}$$

- (Labour force) Participation rate is the ratio of labour force to the total population of working age

- High Unemployment: Bad → Economy ↓ ←
  → People Upset → Economy: Aggregate of People

Low Unemployment: Might be bad ← Inflation

low # of people looking for jobs → Firms respond by ↑ wages/prices to attract workers → Inflation

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Inflation: Sustained rise in the general level of prices in the economy.

* Deflation: Sustained fall ...
* Inflation Rate: Rate at which the price level increases.
* GDP deflator: $\boxed{P_t = \dfrac{\text{Nominal GDP at } t}{\text{Real GDP at } t}}$ → price ~ $\dfrac{\text{monetary}}{\text{output}}$

Not as important as

$$\boxed{\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} \rightarrow \text{Rate of Inflation}}$$

* CPI → Consumer Price Index
  - ↳ Fix a basket → Trace what's going to happen to the price of the basket overtime.
    - ↳ Update basket, consumer patterns change.
  - → Quality of products change.
  - → Not going to be the same for everyone.
* CPI might diverge from GDP-deflator due to prices of imported goods (included in CPI).

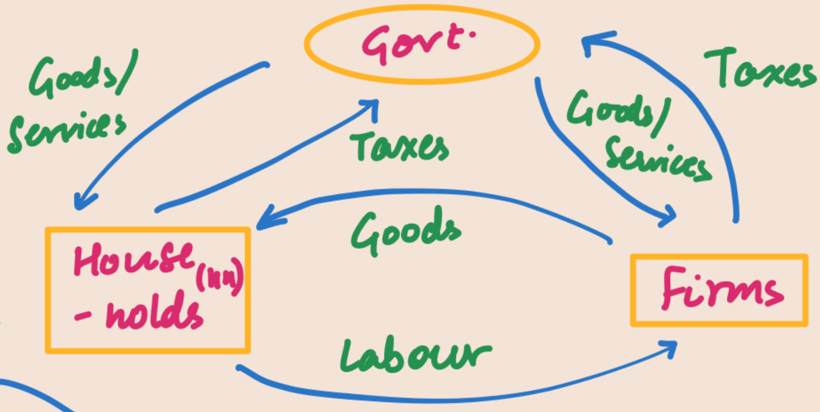
- WPI → Wholesale Price Index
- Core Inflation → Excludes food & energy from CPI basket.

GDP / Output

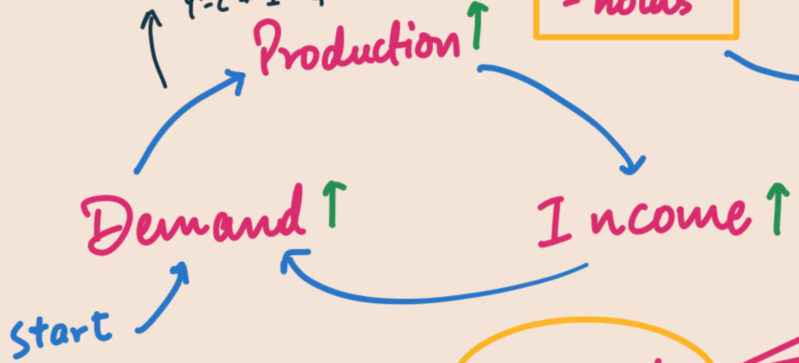
What explains GDP? / Output

- Short run → Demand ~ 1-2 y.
- Medium run → Supply-side factors ~ 10 y. (Labour, Capital, etc.)
- Long run → Roots, Geography, Culture, fundamental foundations, etc. ~ 50 y.

Circular Flow of Income



SKM: In the Short Run, Output (Demand)
$Y = C + I + G = Z$



Demand →
- Consumption (C)
- Investment (I) { Firms → Buying Buildings... ; HH → New Homes ... }
- Govt. Spendings (G)
- Net Exports (NX) = Exports (X) − Imports (M)

$$\boxed{GDP = C + I + G + \underbrace{X - M}_{NX}}$$

- Trade Surplus → $X > M$
- Trade Deficit → $X < M$
- Closed Economy → $X = M = 0$

What determines the Output of an Economy?

- Assumption : $M = X = 0$ ← Closed Economy
- C : $C(\overset{+}{Y_D})$ → Disposable Income, $Y_D = Y - T$ (Y → Income; T → Taxes − Govt. Transfers)

C is an Inc. function of Y_D

Functional form → $C = c_0 + c_1 Y_D$, $0 < c_1 < 1$, $c_0 > 0$

$C = c_0 + c_1 (Y - T)$ → Endogenous vars.

Exogenous Vars:

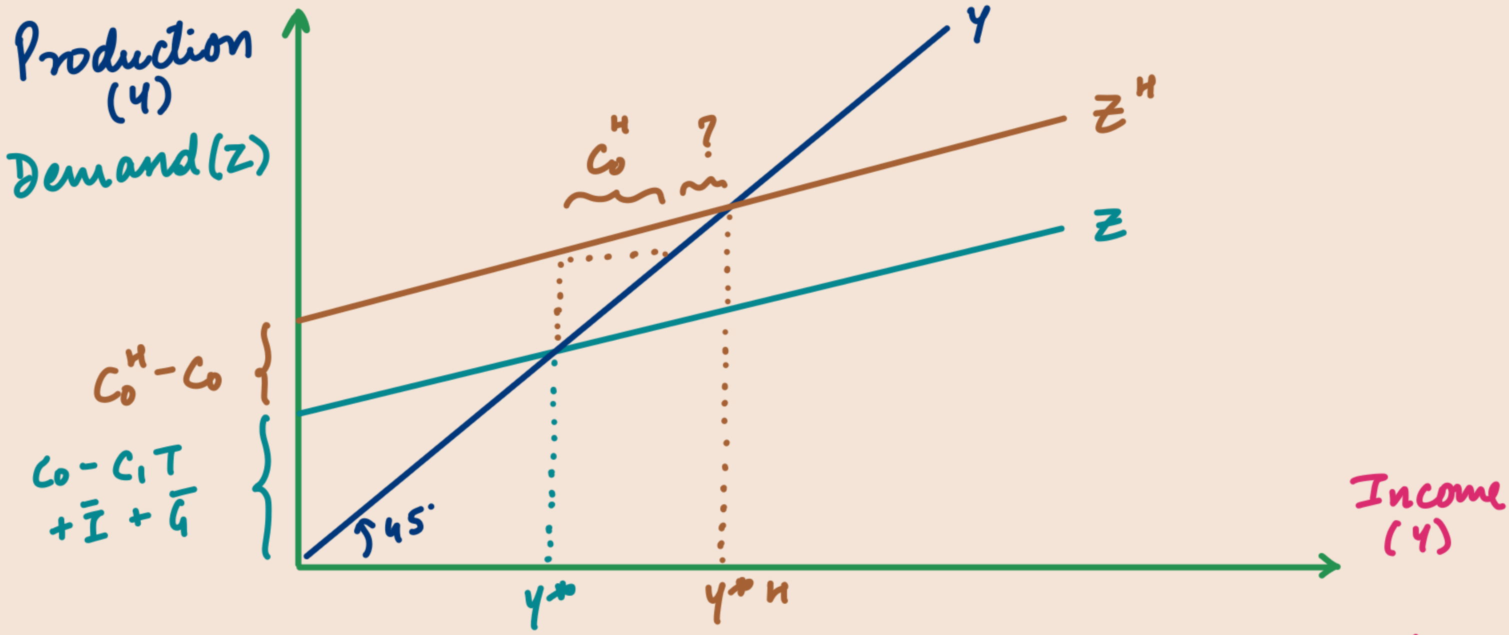
"Given" to us from outside our model → Used to Calculate

$I = \bar{I}, \; G = \bar{G}$

$$Z = \text{Demand} = C + I + G$$
$$= \underbrace{C_0}_{} + \underbrace{C_1}_{}(Y - T) + \underbrace{I}_{} + \underbrace{G}_{} \quad \text{Exo}$$

Equilibrium: Demand (Z) = Production (Y)

$$Y = C_0 + C_1(Y - T) + \bar{I} + \bar{G}$$
$$\Rightarrow \quad Y = \frac{C_0 - C_1 T + \bar{I} + \bar{G}}{1 - C_1}$$



- Suppose $C_0 \uparrow$ to C_0^H. How does Y^* change? It $\uparrow$ to Y^{*H}.
  But $Y^{*H} - Y^* > C_0^H - C_0$, why? Because of Multiplier.
  Visible graphically, but mathematically,

$$Y = C_0 + C_1(Y - T) + \bar{I} + \bar{G} \rightarrow C_0 \uparrow \text{ by } 1 \text{ unit}$$

$Y \uparrow$ by 1 unit

$Y \uparrow$ by $1 \cdot C_1$

$Y \uparrow$ by $C_1 \cdot C_1$

$Y \uparrow$ by $C_1^2 \cdot C_1 \ldots$

due to circular flow of Income; Known as the "Multiplier" Process

$$\frac{\Delta Y}{\Delta C_0} = \frac{1 + C_1 + C_1^2 + \ldots}{1} = \frac{1}{1 - C_1} > 0$$

$\hookrightarrow \dfrac{\partial Y}{\partial C_0}$ $\qquad \Downarrow$ $\; Y \uparrow, C_0 \uparrow$

Savings + Investments

$$Y = C + I + G$$
$$= (G - T) + I + C + T \Rightarrow \underbrace{\underbrace{(Y - T - C)}_{S_P} + \underbrace{(T - G)}_{S_G}}_{\text{Total Savings in an Economy}} = I$$

$Y-T-C$: Private Savings (S_P)

$T-G$: Govt. Savings (S_G)

$$\boxed{S_P + T - G = I}$$

* $S_P = Y-T-C = Y-T-C_0-C_1(Y-T) = (1-c_1)(Y-T)-C_0$

$\Rightarrow S_P = (1-c_1)(Y-T) - C_0$, $I = S_P + T - G$

If $C_0 \downarrow \Rightarrow I \sim$, $S_P \sim$ $\left\{ \begin{array}{l} \downarrow \text{consumption} \\ \text{doesn't} \uparrow \text{savings} \end{array} \right.$ → "Paradox of Saving" or "Paradox of Thrift"

$\partial S_P / \partial C_0 = 0$

$C_0 \downarrow$ $S_P \uparrow$ but saving more of something less (Y)

* Goods market [SKM] → How output is determined via the market in the short run.

TLDR: In the short run, output is determined by demand.

* "Comparative Statics" :- Effect of parameters on Equilibrium.

Comparing ← Comparative; Static ← Statics (Static Expressions)

Eg. $Y = Z = \underbrace{C(Y-T)}_{\text{Not necessarily linear}} + \bar{I} + \bar{G}$

i) Effect of $\uparrow \bar{G}$ on Y?

ii) Effect of $\uparrow T, \bar{G}$ by same amount on Y? → Balanced-Budget Multiplier

$T-G$: (Govt.) Budget Surplus

$G-T$: Budget Deficit

1) $dY = C' dY + dG \Rightarrow \frac{dY}{dG} = \frac{1}{1-C'}$

2) $dY = C'(dY - dT) + dG$

$\Rightarrow (1-C')dY = dG - C'dT = dG(1-C')$

$\Rightarrow dY = dG \Rightarrow \uparrow$ by same amt.

[Simple Keynesian Model (SKM)]

Saving + Investments

Saving = Investment = $\bar{I}$ → $S = \bar{I}$

Saving → Personal Savings, Govt. Savings

$\Rightarrow Y-T-C+T-G = I$

$\Rightarrow Y-C-G = I$

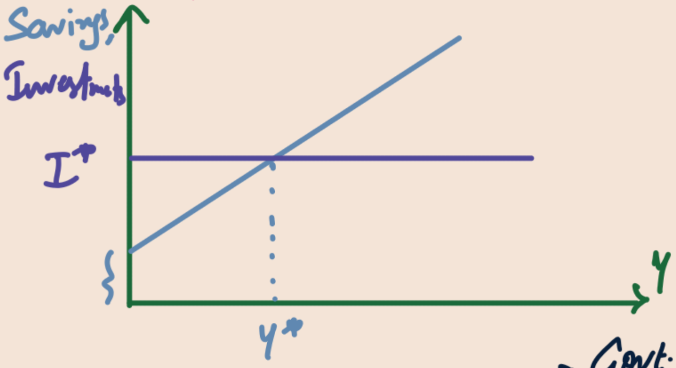
$(S_p) + (S_G)$
$\hookrightarrow Y-T-C \quad \hookrightarrow T-G$

$\equiv \quad Y = C+G+I$

(Production = Demand)

$\rightarrow C_0 + C_1(Y-T) = C$

$\Rightarrow \underbrace{(Y-T)(1-C_1)-C_0}_{S_p} + T - G = I$



$C_1 = \frac{dC}{dY} \rightarrow$: propensity to consume

mpc → marginal propensity to consume

T, G → Fiscal Policy → Govt. managing market by taxes & investments

Monetary Policy
↳ By Interest rates, etc.

Some Eco. Jargon

- **Money**: What can be used to pay for transactions
- **Income**: What you earn (Flow)
- **Saving**: Part of after tax income that you do not spend (flow)
- **Savings**: Value of what you have accumulated over time. (Stock)
- ⇕ **Financial wealth (or wealth)**: Value of all your financial assets minus all your financial liabilities (Stock)
- **Investment**: Purchase of new capital goods.
- **Financial Investment**: Purchase of shares or other financial assets.
- **Flow**: Over a certain period of time / per unit of time.
- **Stock**: Cumulative / at a certain instant of time / point of time

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Central Bank and the Financial Market

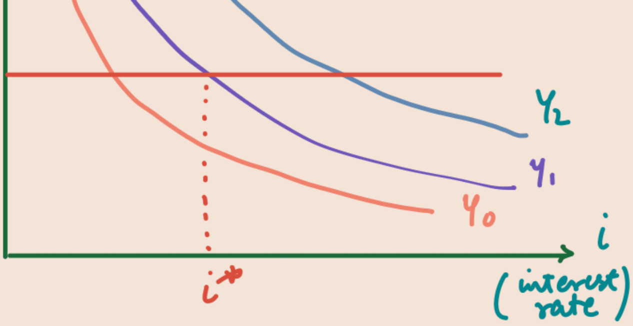
Assume
↓
Only 2 assets → Money → 0 interest rates
→ Bonds → i interest rates

Money Demand = $\boxed{M^d = Y L(\underline{i})}$

Nominal GDP/Income (↓ func)

Money Supply $= M^s = \bar{M}$
↳ Regulated by RBI (Central Bank)

$\bar{M} = M^d = M^s$



- Effect of $\bar{M}\uparrow \Rightarrow M^d \sim, M^s\uparrow, i^*\downarrow$
  $M^s\uparrow \Rightarrow M^{d*}$ has to adjust → Can come only from $i^*\downarrow$

- Central Bank's Balance Sheets

Assets	Liabilities
Bonds	Currencies

Expansionary open Market Operation → Exchange currency for Bonds ($M_s\uparrow$)
↳ Buy Bonds → RBI ↑ demand of bonds → $P^*\uparrow$ → $i\downarrow$

$M_s\uparrow \Rightarrow i\downarrow$

$i = \frac{100 - P_B}{P_B}$ } Interest rate v/s. Price of Bonds

$P_B = \frac{100}{1+i} \longleftrightarrow P_B\uparrow \; i\downarrow$

- CB

A	L
Bonds	Central Bank Money = Currencies + Reserves

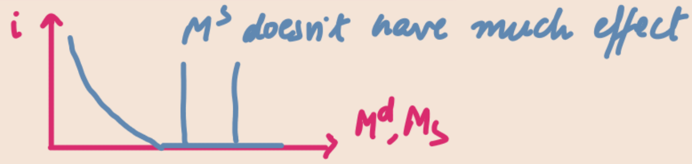
Banks ← Financial Intermediaries

A	L
Reserves, Loans, Bonds	Checkable Deposits

* Financial Market → Demand for Money
Currencies ← ↳ Checking accounts

RBI controls supply of Money

- Liquidity Trap → Zero Lower Bound



IS-LM ~ Hicks-Hansen Model

- Capitalism → Everyone free to buy whatever they want (subject to personal affordability, etc.)
  ↳ Western Countries
- Socialism → Govt. dictates/controls the output/production

↳ Soviet Countries

* **IS-LM** → Investment savings - liquidity preference - money supply

↳ Equilibrium in goods and financial markets

↳ $IS(Y, i)$ → goods market in equilibrium

$$\boxed{Y = \underset{+}{C(Y-T)} + I(\underset{+}{Y}, \underset{-}{i}) + \bar{G}}$$

$I(\underset{+}{Y}, \underset{-}{i})$

$+$ → More Money, More investment

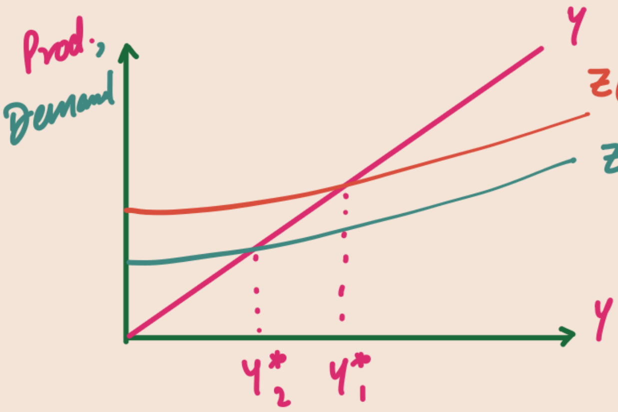
$-$ → $i\uparrow \Rightarrow$ Better to buy than invest ⇓ Opportunity Cost ↑ for invest Hence ↓ func.

Ax. $0 < \frac{\partial C}{\partial Y} + \frac{\partial I}{\partial Y} < 1$

{Similar to the prev. model}

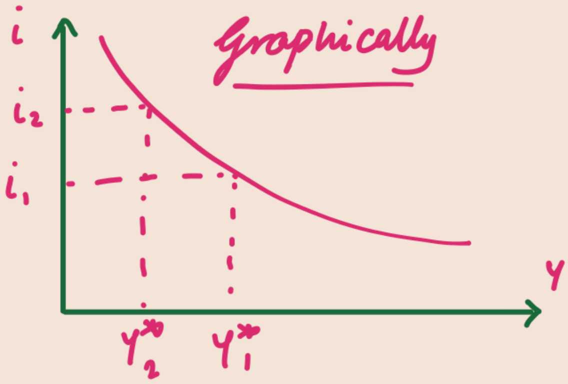
$\partial Y / \partial Y \in (0,1)$

$i_1 < i_2$



$\Rightarrow$

Graphically



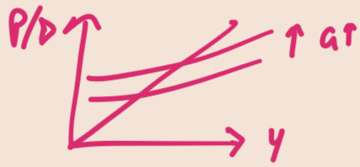
Algebraically, $dY = \frac{\partial C}{\partial Y} dY + \frac{\partial I}{\partial Y} dY + \frac{\partial I}{\partial i} di + 0$

$$\Rightarrow \frac{dY}{di} = \frac{\partial I / \partial i}{\underbrace{1 - \frac{\partial C}{\partial Y} - \frac{\partial I}{\partial Y}}_{0 < (+ve) < 1}} \rightarrow \frac{-ve}{+ve} \Rightarrow \frac{dY}{di} = -ve$$

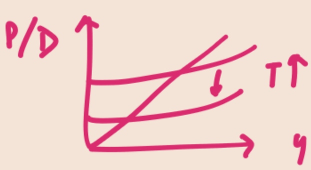
↓ func. ⇙

Words. $i\uparrow \Rightarrow I\downarrow \Rightarrow$ Consumption & Production ↓ $\Rightarrow Y\downarrow$

$G\uparrow \Rightarrow$ Parallel Shifts



$T\uparrow \Rightarrow$ Downward Shift



Shifts in curves, not Movements along the curve } Because parameters changed, not P/D/Y/i.

LM : (i, Y) s.t. financial market is in equilibrium

$M^d = YL(i)$, $M_s = \bar{M} \rightsquigarrow \frac{\bar{M}}{P} = YL(i) \rightarrow \ln(\,) = \ln Y + \ln L(i)$

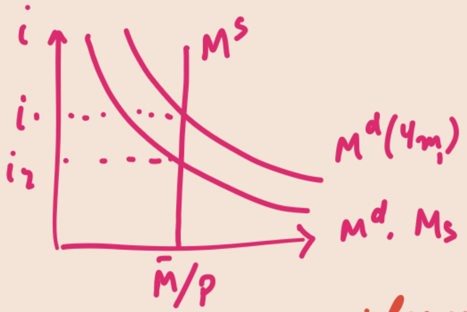
$P \rightarrow$ Real, not eq^m

$\Downarrow$

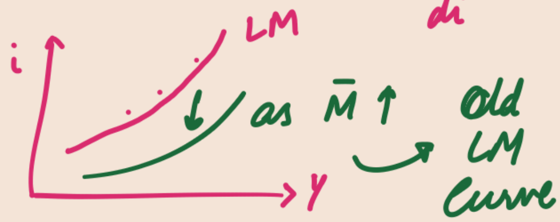
$0 = \frac{dY}{Y} + \frac{dL}{L\,di}$

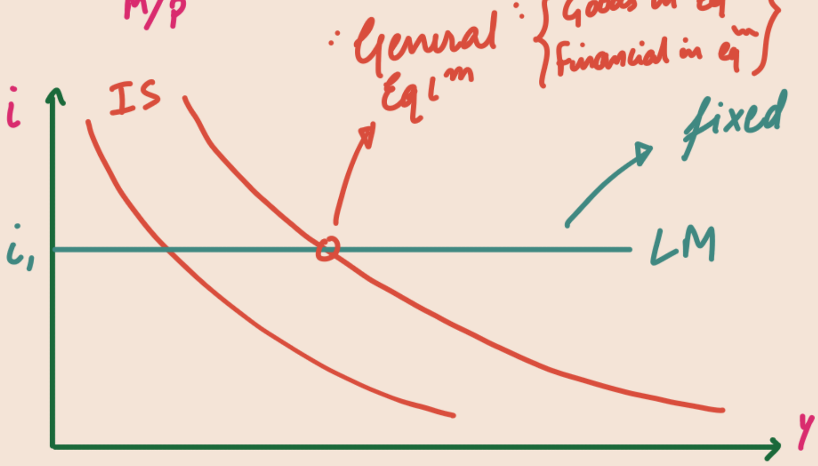
$\Downarrow$

$\frac{dY}{di} \sim \frac{-dL}{di} = +ve$



$Y_m \downarrow \Rightarrow M_d \downarrow \Rightarrow i \downarrow$
(shift)



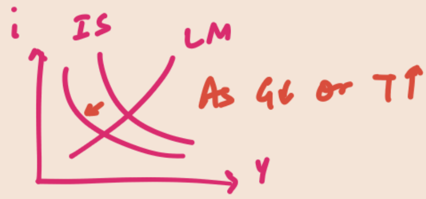


① Fiscal Policy

$G\downarrow \Rightarrow Y\downarrow, C\downarrow, I\downarrow$ $\quad i \leftrightarrow$

$T\uparrow \Rightarrow Y\downarrow, C\downarrow, I\downarrow$ $\quad i \leftrightarrow$

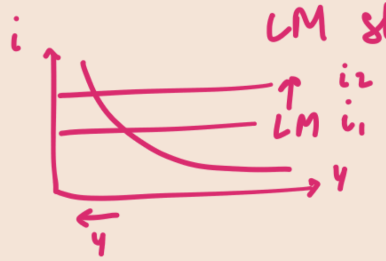
old

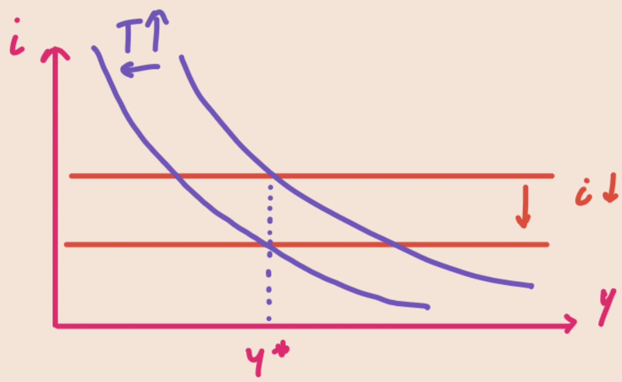


IS shifts,
LM along

② Monetary Policy

$i_1 \uparrow i_2 \rightarrow$ IS along the curve, LM shift $\quad Y\downarrow, C\downarrow, I\downarrow$





③ Fiscal + Monetary Policy

$T\uparrow, i\downarrow \Rightarrow Y\leftrightarrow, C\downarrow, I\uparrow$

☆ In the short run, output is determined by the equilibrium between the Goods Market and the Financial Market.

☆ IS-LM $\rightarrow$ Expectations
$\rightarrow$ Open Economy
$\rightarrow$ AD-AS (Aggregate Demand-Supply)

→ AD-AS (Aggregate Demand Supply)

(i) Fiscal policy (Changes in G, T)
(ii) Monetary policy (Changes in $\bar{i}$)
(iii) Combinations of both Fiscal and Monetary

- **Nominal interest rate** is the interest rate in terms of rupees/dollars/euros.

Real interest rate is the interest rate in terms of a basket of goods.

→ Need to adjust nominal interest rates to take into account expected inflation.

Y-0		Y-1
1 unit of good	⟶	$1+r_t$ unit of good
↓		
P_t monetary value	⟶	$P_t(1+i_t)$ monetary value

$$1+r_t = \frac{P_t(1+i_t)}{P^e_{t+1}}$$

Expected price in Y_{t+1}

$$\pi^e_{t+1} = \frac{P^e_{t+1}-P_t}{P_t} = \frac{P^e_{t+1}}{P_t} - 1$$

expected rate of inflation

$$1+\pi^e_{t+1} = \frac{P^e_{t+1}}{P_t}$$

⇓

$$\boxed{1+r_t = \left(\frac{1+i_t}{1+\pi^e_{t+1}}\right)} \Rightarrow (1+r_t)(1+\pi^e_{t+1}) = (1+i_t)$$

$$\Rightarrow r_t + \pi^e_{t+1} + \cancel{r_t\pi^e_{t+1}} = i_t \qquad \sim 0$$

$$\Rightarrow \boxed{r_t \approx i_t - \pi^e_{t+1}}$$

- **Risk Premium** → (i) possibility of default

Govt. guarantees $(1+i)$ returns for 1

p: probability of default
↳ Company returns $(1+i+x)$, if no default, for 1

$$\Rightarrow (1+i) = \underbrace{p[0] + (1-p)[1+i+x]}_{\text{expected returns}} \Rightarrow \boxed{x = \frac{p}{1-p}(1+i)}$$

$\Rightarrow x - xp = p + ip \Rightarrow x \approx p$

→ (ii) risk aversion → Govt. offers security, firms offer same expectation w/ risk. So, they need to ↑ their expected returns.

Modify IS-LM w/ $i \leftarrow r + x$ to account for these.

Monetary Economics

Money →
(1) Medium of Exchange
(2) Unit of Account → Products priced in terms of money
(3) Store of Value → Value can be stored through financial investments

↓

Barter Exchange : Double Co-incidence of wants :
→ Not as prevalent as societies get more sophisticated.

↓

Bread : Perishes, Baker might not always have sufficient supply.

→ Characteristics of a currency :
1) Durable
2) Easily measured
3) Easily transferable
4) Easily divisible
5) Have stable value

Viking Economy → No fixed money supply, No central bank

$M^d = k_1 P_t T_t$, $P_t T_t = k_2 P_t Y_t \Rightarrow \left.\begin{matrix} M^d = k P_t Y_t \\ M^s = M \end{matrix}\right\}$ Eql^m $M^d = M^s$

Price, Transactions

$\Rightarrow M = k P_t Y_t$

$\Rightarrow M \cdot \frac{1}{k} = P_t Y_t \Rightarrow \boxed{MV = P_t Y_t} \Rightarrow \frac{M_1}{M_2} = \frac{P_1}{P_2}$

Velocity of Money

M has no effect on output

① Money demand is an increasing function of value of transactions in the economy.

$M^d = k_1 P_t T_t$

$M^d = k P_t T_t$

② Value of transactions is an increasing function of value of final output in the economy.

$$P_t T_t = k_2 P_t Y_t \longrightarrow M^d = k P_t Y_t$$

$$\Rightarrow \boxed{\frac{M_t^d}{P_t} = k Y_t}$$

$M_t^s = M =$ No. of gold coins in circulation

Eql^m $\quad M^d = M^s \Rightarrow M_t^d = M$

~How many times a coin is exchanged on average.

$\longrightarrow \dfrac{M}{P_t} = k Y_t \quad$ Define $\bar{V} = 1/k \longrightarrow$ Velocity of Money

$$\Rightarrow \boxed{M \bar{V} = P_t Y_t}$$ Quantity Equation

⊛ $Y_t = A L_t \longrightarrow$ Amount of labour put in

L^* : min./ideal/optimal amount of labour a/c to suppliers

Setting the price : $\quad \dfrac{P_{t+1}}{P_t} = \left(\dfrac{L_t}{L^*}\right)^{\theta}, \quad 0 < \theta < 1$

L_t : If labour $> L^*$, want to ↑ price

θ: don't want to ↑ price too much, might be bad for future

[1] Neoclassical Economics

→ All prices rise immediately
→ Output unchanged
→ Money "Neutral"
[ΔM^s has no effect on output]

[2] Keynesian Economics

→ Prices slow to rise
→ output responds to monetary shocks

If abrupt change in demand, prices not changed immediately, changed at e.o.d.
But, labour + output produced at older price.

$M_t \bar{V} = P_t Y_t$

$Y_t = A L_t$

$L^* \rightarrow Y^*$

$\dfrac{P_{t+1}}{P_t} = \left(\dfrac{L_t}{L^*}\right)^{\theta}$

$\Rightarrow \dfrac{P_{t+1}}{P_t} = \left(\dfrac{Y_t}{Y^*}\right)^{\theta}$

$P_t, Y_t \longrightarrow$ endogeneous variables

$M_t \longrightarrow$ endogeneous variables

$0 \longrightarrow$ Date at which expedition come back w/ new gold coins

$\bar{V}, A, \theta$: parameters

P_0?

$\log M_t + \log \bar{V} = \log P_t + \log Y_t$

$$\log P_{t+1} - \log P_t = \theta[\log Y_t - \log Y^*]$$

$t = -1 \longrightarrow Y_{-1} = Y^*$

$$\left.\begin{array}{l} \log M_L + \log \bar{V} - \log Y^* = \log P_{-1} \\ \log P_0 - \log P_{-1} = \theta[\log Y^* - \log Y^*] = 0 \\ \Rightarrow \log P_0 = \log P_{-1} = \log M_L + \log \bar{V} - \log Y^* \end{array}\right\} \text{Day } -1$$

$$\left.\begin{array}{l} M_0 = M_H \; (> M_L) \\ \log M_H + \cancel{\log \bar{V}} = \cancel{\log P_0} + \log Y_0 \end{array}\right\} \text{Day } 0$$

$$\Rightarrow \log M_L + \cancel{\log \bar{V}} = \cancel{\log P_{-1}} + \log Y^*$$

Subtract: $\log M_H - \log M_L = \log Y_0 - \log Y^* \Rightarrow \Delta \log Y_0 = \Delta \log M > 0$

$$\log P_1 - \log P_0 = \theta[\log Y_0 - \log Y^*] = \theta \, \Delta \log Y_0 > 0$$
$$= \theta \, \Delta \log M > 0$$

$\Rightarrow P \uparrow$ next day

$$\log M_1 + \log \bar{V} = \log P_1 + \log Y_1$$
$$\log M_0 + \log \bar{V} = \log P_0 + \log Y_0$$
$$\Rightarrow \log P_1 + \log Y_1 = \log P_0 + \log Y_0 \Rightarrow \log Y_1 - \log Y_0 = \log P_0 - \log P_1$$
$$= -\theta[\log M_H - \log M_L] < 0$$

$Y_1 < Y_0$

Also, $$\left.\begin{array}{l} LY_1 - LY_0 = -\theta[LM_H - LM_L] \\ LY_0 - LY^* = LM_H - LM_L \end{array}\right\} LY_1 - LY^* = (1-\theta)[LM_H - LM_L] > 0$$

$\therefore \; Y^* < Y_1 < Y_0$ & $P_1 > P_0$

So, $\boxed{P \longrightarrow \frac{M_H \bar{V}}{Y^*}, \quad \Delta P \propto \Delta M, \quad \& \quad Y \to Y^*}$ Asymptotically

Financial Intermediaries

- ☆ Capital Ratio: Ratio of Capitals to Assets. $\longrightarrow 20/100$
- ☆ Leverage Ratio: Ratio of Assets to Capitals $\longrightarrow 100/20$ $\longrightarrow$ Leverage Ratio $\uparrow \Rightarrow$ Risk $\uparrow$, Profitability $\uparrow$
- ☆ Assets < Liabilities $\longrightarrow$ Insolvency

Bank

(A)	(L)
Assets $\rightarrow$ 100	Liabilities $\rightarrow$ 80
	Capital $\rightarrow$ 20

Ex Rate of return on assets = 5%. $(L, C) = (80, 20), (90, 10)$

Rate at which you are paying liabilities = 4%.

$(L, C) = (80, 20) \rightarrow$ Profit $= \frac{5 \cdot 100 - 4 \cdot 80}{100} = 1.8 \rightarrow \frac{1.8}{20} \cdot 100\% = 9\%$ → Low risk

$(L, C) = (90, 10) \rightarrow$ Profit $= \frac{5 \cdot 100 - 4 \cdot 90}{100} = 1.4 \rightarrow \frac{1.4}{10} \cdot 100\% = 14\%$ → High risk

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Additional Practice Questions 1

Q1

Purva $\xrightarrow{\text{goods}}$ Sugarcane, Sugar, Barfis

Sugarcane → Sugar → Barfis

Produced ↓

₹ 1000 $\xrightarrow{\text{Prod.}}$ ₹ 2000 $\xrightarrow{\text{Prod.}}$ ₹ 5000

GDP(Purva) = ₹ 5000 ≠ ₹ 8000

Violates "final" in the def. of GDP

Intermediate Goods, Not final

Q2

Dakshin:

2020 → Base Year

Quantity/Price:

		Bun Roti	Tea	Choco.	Lassi
2020	Q	100	200	50	50
	P	50	20	10	10
2021	Q	50	200	70	40
	P	70	30	20	10

1) Growth in nominal GDP from 2020 to 2021

$NGDP_{2020} = 100 \cdot 50 + 200 \cdot 20 + 50 \cdot 10 + 50 \cdot 10 = 10{,}000$

$NGDP_{2021} = 50 \cdot 70 + 200 \cdot 30 + 70 \cdot 20 + 40 \cdot 10 = 11\,300$

$\text{growth} = \frac{11300 - 10000}{10000} \cdot 100 = 13\%$

2) Growth in real GDP from 2020 to 2021

$RGDP_{2020} = NGDP_{2020} = 10\,000$

$RGDP_{2021} = 50 \cdot 50 + 20 \cdot 200 + 10 \cdot 70 + 10 \cdot 40 = 7600$

$\text{growth} = \frac{7600 - 10000}{10000} \cdot 100 = -24\%$

GDP deflator

$P_t = \frac{NGDP_t}{RGDP_t}$

$P_{2020} = 1$

$P_{2021} = \frac{113}{76}$

$\pi_{2021} = \frac{P_{2021} - P_{2020}}{P_{2020}}$

$= \frac{113}{76} - 1$

$= \frac{37}{76} = 48.68\%$ Rate of Infl.

$P_{2021} = \frac{NGDP}{RGDP} \xrightarrow[\frac{\partial}{\partial t}]{\ln} g_P \approx g_{NGDP} - g_{RGDP} = 13 - (-24) = 37\%$

Q3 Rural Urban ① Flood → Agricultural output ↓

CPI:

	Rural	Urban
Food	50	20
Housing	20	40
Energy	10	15
Transportation	5	15
Others	15	10

" Prices ↑

Are both CPI's affected the same?

NO. Rural CPI is affected much more than Urban CPI.

② Agricultural reforms ⟶ Rural income ↑ ⟶ How does CPI adjust?

Empirical Observation: Food ↓, other things will take the share.

Q4 Consider a simple Keynesian model (SKM) for a closed economy w/o govt. Suppose Saving ∝ Income, Marginal propensity to invest $= 0.3$, system initially in eq^{m}. Parallel downward shift of saving function, eq^{m} saving ↑ 12 units. Compute ΔY.

$S_p = Y - \cancel{T}^{0} - C$

$S_T = \cancel{T} - \cancel{G}^{0} = 0$

$\underbrace{I = S_p + S_T = Y - C}$

$I = S_p \Rightarrow \Delta I = \Delta S_p$

$0.3\,\Delta Y = 12$

$S_p = Y - C \xrightarrow[\text{shift}]{11} \Delta Y - \Delta C = 12$

$Y = I + C$

$\Rightarrow \Delta Y = \Delta I + \Delta C \Rightarrow \Delta Y = 0.3\,\Delta Y + \Delta C$

$\Rightarrow \Delta Y - \Delta C = 12 = \Delta I = 0.3\,\Delta Y$

$\Rightarrow \Delta Y = \dfrac{12}{0.3} = \dfrac{120}{3} = 40$

Q5 IS-LM Model. Commodity market, Consumption: $C = a + bY$, $a > 0$, $1 > b > 0$. I_0, G_0 exogenous. In the money market, $M^d \rightarrow L = kY - gr$, $(k, g > 0)$. Nominal $M^s = M_0$, Price level P_0 exogenously. C, Y, r: Consumption, Real GDP, interest rate.

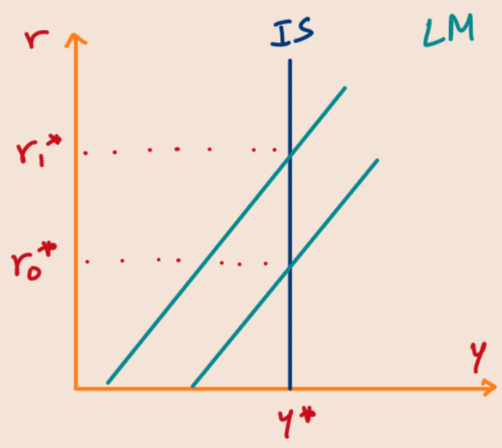
① Set Up IS-LM equations.

$Y = C + G + I \Rightarrow$

$M^d = M^s$

$\Rightarrow kY - gr = M_0/P_0 \Rightarrow$

$$\boxed{\begin{aligned} Y(1-b) &= a + I_0 + G_0 \\ r &= \frac{k}{g}Y - \frac{M_0}{gP_0} \end{aligned}}$$



② P: $P_0 \uparrow P_1$. GDP? r?

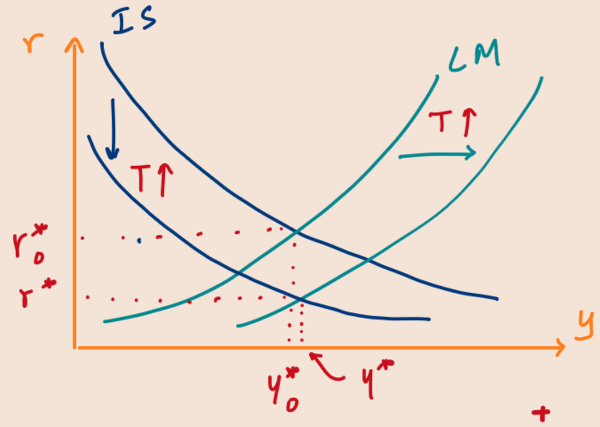
$P\uparrow \Rightarrow M^s\downarrow$ but $Y \sim \Rightarrow r\uparrow$

Q6 IS-LM model, closed economy. Private consumption depends on disposable income. (T) are lump

Both priv inv. & speculative M^d vary inversely w/ (r). However, transaction M^d depends not on (y) but (y_d). Argue how eql^m priv. inv., priv. saving, govt. saving, y_d, & y will change, if govt. ↑ T.

$$C(y_d) \equiv C(y-T) \quad , \quad I \propto \frac{1}{r}, \; M^d_{spec} \propto \frac{1}{r}, \; M^d_{trans}(y_d)$$

$$M^d = M^d_{spec} + M^d_{trans} = \frac{k_S}{r} + f(y_d) = \frac{k_S}{r} + f(y-T)$$



$$y = C + I + G \quad T\uparrow \Rightarrow$$

$$y = C(y-T) + \frac{k_I}{r} + G \quad \text{— (1)}$$

$$M^d = \frac{k_S}{r} + f(y-T)$$

↳ T↑ ⇒ Shifts to the right

$y\uparrow \; r\uparrow$

$y\uparrow \; r\downarrow$

OR

$$\frac{IS}{LM} \quad \begin{aligned} Y &= C(\overset{+}{Y-T}) + I(\overset{-}{r}) + G \\ M^S &= M^d(\underset{+}{Y-T}, \underset{-}{r}) \end{aligned}$$

→ $r^* \downarrow$, Y^* ambiguous, $I\uparrow$, $(T-G)\uparrow$, (y^*-T) ambiguous, S_p ambiguous

Govt. Savings

Same sign

Q7 Closed Economy:

$$\underbrace{\Delta Y}_{\text{GDP}} = \underbrace{\Delta C}_{\text{Consumption}} + \underbrace{\Delta I}_{\text{Private Investment}} + \underbrace{\Delta G}_{\text{Govt. Spending}} ; \quad \Delta C = c\,\underbrace{\Delta Y_d}_{\text{Disposable Income}} ; \quad \Delta Y_d = \Delta Y - \underbrace{\Delta T}_{\text{Tax Collections}} ; \quad \Delta T = \underbrace{t}_{\substack{\text{tax rate}\\(0,1)}} \Delta Y + \underbrace{\Delta T_0}_{\text{Modifiable by Govt. in Fiscal policy}}$$

1) Find balanced budget multiplier (in terms of G & T_0)

$$\Delta Y = c\,\underbrace{(\Delta Y - \underbrace{t\Delta Y - \Delta T_0}_{\Delta T})}_{\Delta Y_d \;\to\; \Delta C} + \Delta I + \Delta G = c(1-t)\Delta Y + \cancel{\Delta I}^{\text{exo.}} + \Delta G - c\,\Delta T_0$$

$$\Rightarrow \Delta Y(1 - c(1-t)) = -c\,\Delta T_0 + \Delta G$$

$$\Delta G = \Delta T_0 \quad \boxed{\frac{\Delta Y}{\Delta G} = \frac{1-c}{1-c(1-t)} < 1}$$

2) $c = 0.8$, $t = 0.375$. Govt. expenditure multiplier?

$$\Delta T_0 = 0 \longrightarrow \frac{\Delta Y}{\Delta G} = \frac{1}{1-c(1-t)} = \frac{1}{1-\frac{4}{5}\left(1-\frac{3}{8}\right)} = 2$$

3) $c = 0.8$, $t = 0.375$. Tax multiplier (w.r.t. T_0)?

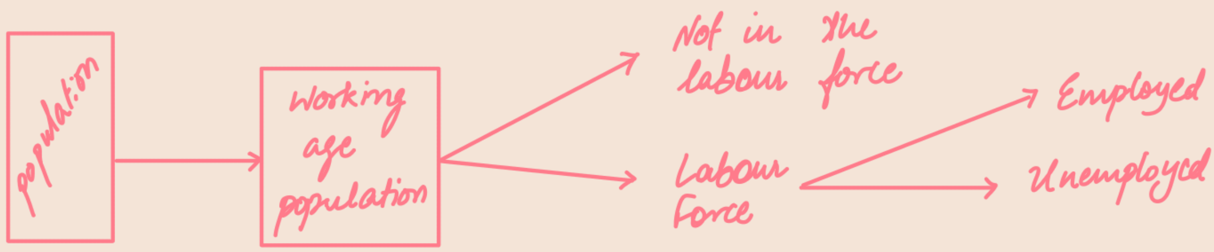
$$\Delta G = 0 \longrightarrow \frac{\Delta Y}{\Delta T_0} = \frac{-c}{1-c(1-t)} = \frac{-4/5}{1-\frac{4}{5}\left(1-\frac{3}{8}\right)} = \frac{-8}{5} = -1.6$$

Medium Run

↳ Need to study supply-side issues.

Labour Market is a major component → Prev. models do not account for it.

Some jargon → · Unemployment rate
· Labour force participation rate



★ Thought Expt: Suppose you're negotiating your wage with a company, then, how would your wage argument depend on:

1) Expected Price Level (+)
2) Unemployment Rate (−)
3) Other things → Unemployment Benefits Rate (+)

$\Rightarrow \boxed{W = P^e F(U, z)}$ → Wage-Setting Relationship / Equation

Nominal Wage Rate — W; Expected Price Level — P^e; U (−) → Unemployment rate; z (+) → other factors

HR ~ Marketing

Depending on the wages they pay ⇒ The price of products they sell has to be changed

· $W\uparrow \Rightarrow P\uparrow$
· $P = W$? No $P \geq W$
· Markups → Depends on market power

$Y = AN$,
N → Labour Input
A → " Productivity
Y → Output

} CRS production function, Constant Returns to Scale

$Y = N$ ← Set $A=1$ OR Normalize

$\boxed{P = (1+m)W}$ → Price-Setting Relationship / Equation

Price — P; Markup $(m>0)$; Wage — W

⇑ How much can they set ...

$\boxed{P = P^e}$ → What people expect the prices to be = What the price is

(Medium Run)

Nominal Real

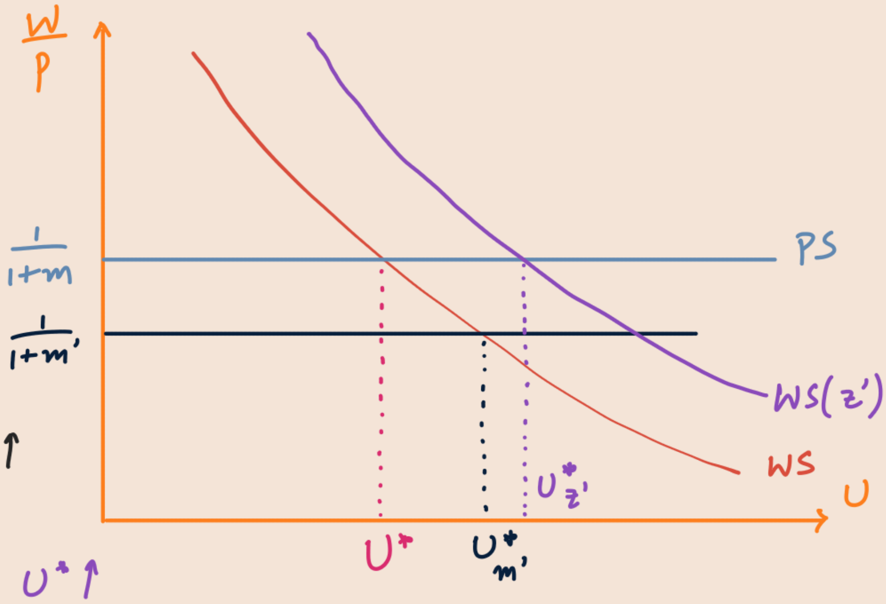
the price above w?

$P = (1+m)W \longrightarrow W/P = 1/(1+m)$; $\frac{\text{Nominal}}{\text{price}}$ = Real wage

$\Leftarrow W = P^e F(U, z) \longrightarrow W/P^e = F(U, z)$

$F(U, z) = \frac{1}{1+m}$

$U^* \longrightarrow U_n$: Natural rate of Unemployment



Comparative Statics:

- What is the effect of higher market concentration? ≡ What's the effect of ↑m? U^*↑
- What is the effect of ↑ z? U^*↑

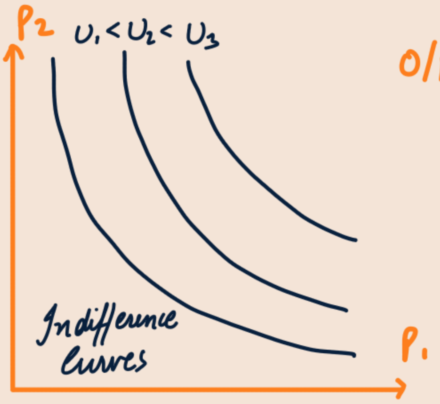
① Production Function ⟶ How inputs translate into outputs holding other factors constant.

② Marginal Function ⟶ Change in output per unit change in input $\frac{\Delta \text{Income}}{\Delta \text{hrs. worked}}$ ⟶ Slope

③ Average Product ⟶ Output per unit of input ⟶ I/Y

④ Preferences and Choice Sets (Feasible Sets)

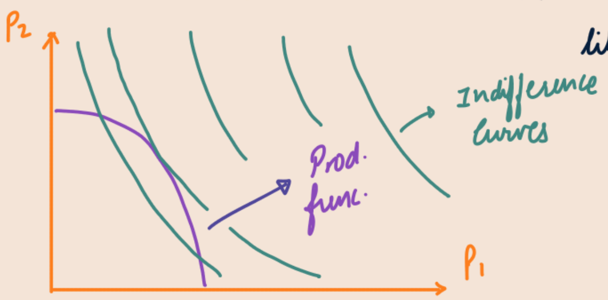
$U = U(P_1, P_2)$
↓ Utility; P_1, P_2: + + preferences



What we would like

What is going to happen

+



Dual Economy

# People	Total Prod.	Marg. Prod.	Avg. Prod.
1	50	50	50

Family of 10 people			
1	[illegible]	[illegible]	[illegible]
2	90	40	45
3	120	30	40
4	140	20	35
5	150	10	30
6	150	0	25
7	150	0	150/7
8	150	0	150/8
9	150	0	150/9
10	150	0	15

1) Company comes → 4 people at ₹ 25 → Accepted
2) After 4 people go → 1 person at ₹25 → Not accepted
→ 1 person at ₹ 35 → Accepted

☆ We would like to account for inflation (and its -ve relation w/ Unemployment) using the equations

$P = (1+m)W$
$W = P^e F(\overset{-}{U}, \overset{+}{Z})$ $\Rightarrow$ $P = P^e (1+m) \underbrace{F(U, z)}_{\text{Assume a linear function for now}}$

$\Rightarrow P_t = P_t^e (1+m) F(U_t, z)$

$\Rightarrow \frac{P_t}{P_{t-1}} = \frac{P_t^e}{P_{t-1}} (1+m)(1+z-\alpha U_t)$

$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$

$\pi_t^e = \frac{P_t^e - P_{t-1}}{P_{t-1}}$

$\Rightarrow (1+\pi_t) = (1+\pi_t^e)(1+m)(1+z-\alpha U_t)$

$\Rightarrow \boxed{\pi_t = \pi_t^e + m + z - \alpha U_t}$ → Expectation augmented Phillips' Curve

→ Different ways of expecting π_t^e

1) Inflation rate would be low → {Past year's inflation was not a good predictor of next year's inflation}

$\pi_t^e = \bar{\pi} \longrightarrow \pi_t = \bar{\pi} + (m+z) - \alpha U_t$ → Initial Phillips' Curve

↓

$\pi_t^e = (1-\theta)\bar{\pi} + \theta \pi_{t-1}$

↳ $\theta = 0 \longrightarrow \pi_t^e = \bar{\pi} \longrightarrow$ ∵ Anchored: inflation

↳ $\theta = 1 \longrightarrow \pi_t^e = \pi_{t-1} \longrightarrow$ ∵ Unanchored: inflation

Initially, we had assumed that

$$p = p^e \Rightarrow \pi_t = \pi_t^e \Rightarrow 0 = m + z - \alpha U_n \Rightarrow U_n = \frac{m+z}{\alpha}$$

$$U_t = U_n$$

$$\pi_t = \pi_t^e + \alpha\left[\frac{m+z}{\alpha} - U_t\right] = \pi_t^e + \alpha[U_n - U_t]$$

Now, if people believe, $\pi_t^e = \pi_{t-1}$

then $\pi_t - \pi_{t-1} = -\alpha[U_t - U_n]$

$$\begin{cases} U_t = U_n \Rightarrow \pi_t = \pi_{t-1} \\ U_t > U_n \Rightarrow \pi_t < \pi_{t-1} \\ U_t < U_n \Rightarrow \pi_t > \pi_{t-1} \end{cases}$$

☆ If $\pi \uparrow$ then a lot of uncertainty, people would want to play safe $\rightarrow \pi_t^e = (1-\lambda)\pi_{t-1} + \lambda \pi_t$

$\Rightarrow \pi_t = \pi_t^e + (m+z) - \alpha U_t \Rightarrow \pi_t = \pi_{t-1} - \frac{\alpha}{1-\lambda}[U_t - U_n]$

Feedback Loop $\Leftarrow$ Effect of U_t amplified now $\Leftarrow 0 < 1-\lambda < 1$

☆ $U_n \rightarrow$ Natural rate of unemployment / Non-accelerating of unemployment $\left(\begin{matrix}\text{If } U_t = U_n \\ \text{\& } \pi_{t-1} = \pi_t^e\end{matrix}\right) \pi_t = \pi_{t-1}$

∘ $\pi_t - \pi_{t-1} = -\alpha(U_t - U_n) \rightarrow$ If $U_t \uparrow\uparrow$ like in Great Dep, deflation does not take place

People not okay w/ having their nominal wages being decreased $\leftarrow$ Downward wage rigidity

$\rightarrow \pi_t = \pi_t^e - \alpha[u_t - u_n]$ — u_t: Unemployment Rate

$u = \frac{U}{L} = \frac{U}{N+U}$ — N: # employed

$\Rightarrow \frac{L-N}{L} = u = 1 - \frac{N}{L} \Rightarrow N = L(1-u)$

$\Rightarrow N_t = L(1-u_t)$, $N_n = L(1-u_n)$

$\Rightarrow N_t - N_n = L(u_n - u_t)$

$\Rightarrow u_t - u_n = -\frac{(N_t - N_n)}{L} \equiv -\frac{(Y_t - Y_n)}{L}$

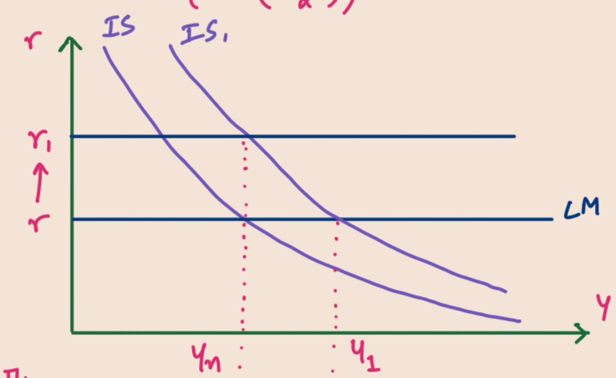
$$\Rightarrow \boxed{\pi_t = \pi_t^e + \frac{\alpha}{L}[Y_t - Y_n]}$$

Derived from Phillips Curve

$\rightarrow Y = L(1-u)$

$\Rightarrow Y_n = L(1-u_n) = L\left(1 - \left(\frac{m+z}{\alpha}\right)\right)$

(PC)	$\pi_t - \pi_{t-1} = \frac{\alpha}{L}[Y_t - Y_n]$
(IS)	$Y = C(Y-T) + I(Y, r+x) + G$
(LM)	$r = \bar{r}$
(WS)	$W = P^e F(u, z)$
(PS)	$P = (1+m)W$



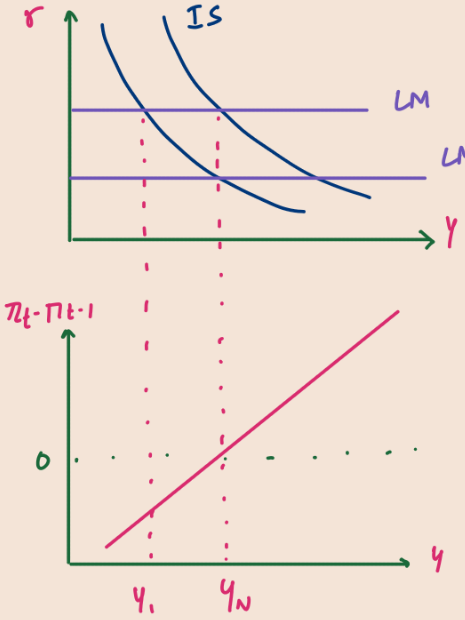
∴ Reduction in Fiscal Deficit:

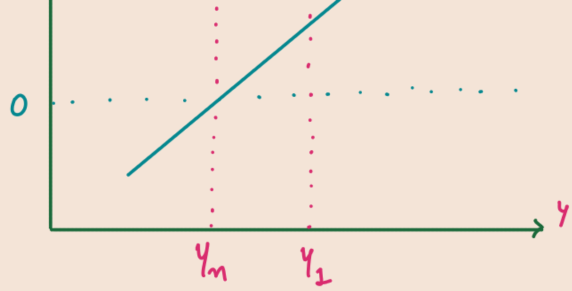
$T\uparrow$

$\overleftarrow{IS}$

RBI:
$r\downarrow$
$LM\downarrow$
$Y_n \sim$
$C\downarrow$ $I\uparrow$

RBI shifts the way Y is used





Deflation Spiral:

Suppose Central Bank $\downarrow$ LM but zero lower bound reached

$\Rightarrow i = 0 \Rightarrow r_t = -\pi_t^e = -\pi_{t-1}$

Also $\Rightarrow Y: Y_n \downarrow Y_1 \uparrow Y_2$ but $Y_2 < Y_n$

$\Rightarrow \pi_t - \pi_{t-1} < 0 \Rightarrow \pi_t \downarrow \Rightarrow r\uparrow$

$\Rightarrow$ IS shifts left $\Rightarrow Y\downarrow \Rightarrow \pi\downarrow \Rightarrow r\uparrow$

Downward Spiral of Inflation due to the loop (& zero lower bound).

Oil Price Shock:

Suppose oil industry$\downarrow$

$\Rightarrow m\uparrow$

$\Rightarrow U_n\uparrow$

$\Rightarrow N_n\downarrow$

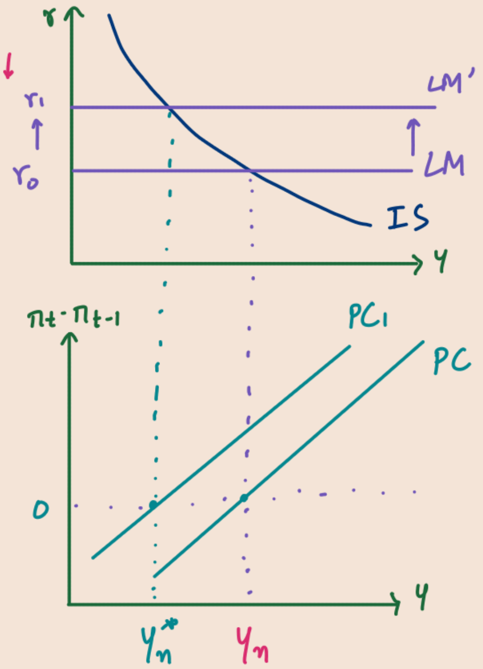
$\Rightarrow Y_n\downarrow$

$\Rightarrow$ New PC

$PC_1 \leftarrow PC$

$Y_n^* \leftarrow Y_n$

To control inflation, RBI $\uparrow$LM



Expectations

→ Idea of Present Value:

Ex. Suppose 5L cost → 1st Year 2L, 2nd 3L, 3rd 4L } Profits

On the other hand, r.o.i. is 10% for some plan. Should you buy the 5L thingy.

$\longrightarrow \quad 1 \xrightarrow{i_t} 1 + i_t \Rightarrow \frac{1}{1+i_t} \xleftarrow{i_t} 1$

$\therefore$ P.V. of 1 (after 1 year) is $\frac{1}{1+i_t}$

$$1 \xrightarrow{i_t} 1+i_t \xrightarrow{i_t} (1+i_t)^2 \Rightarrow \frac{1}{(1+i_t)^2} \xleftarrow[2\text{ years}]{i_t} 1$$

So, buy it iff $\frac{2}{1.1} + \frac{3}{1.1^2} + \frac{4}{1.1^3} - 5 > 0$

- **Expected present discounted value** of a sequence of future payments is the value today of the given sequence of expected payments.

$$\frac{1}{1+i_t} : \text{Discount Factor}$$

(present) value

$$V_t = Z_t + \frac{Z^e_{t+1}}{(1+i_t)} + \frac{Z^e_{t+2}}{(1+i_t)(1+i^e_{t+1})} + \frac{Z^e_{t+3}}{(1+i_t)(1+i^e_{t+1})(1+i^e_{t+2})} + \dots$$

- Special case I: $i_t = i^e_{t+1} = i^e_{t+2} = \dots = i$ (say)

$$V_t = Z_t + \frac{Z^e_{t+1}}{(1+i)} + \frac{Z^e_{t+2}}{(1+i)^2} + \dots$$

- Special Case II: (Const. interest rate & $Z_t = Z^e_{t+1} = \dots = Z$)

$$V_t = Z + \frac{Z}{1+i} + \frac{Z}{(1+i)^2} + \dots$$

For example: $V_t = Z + \frac{Z}{1+i} + \dots + \frac{Z}{(1+i)^{10}}$

- Special Case III: Perpetuary (payments go on till forever) $\cup$ SC II

$$V_t = \sum_{i=1}^{\infty} \frac{Z}{(1+i)^i} = \frac{Z}{1+i} \cdot \frac{1}{1-\frac{1}{1+i}} = \frac{Z}{i} = \frac{Z}{1+i} + \frac{Z}{(1+i)^2} + \dots$$

- Would like to know "real" value (present)

  - Method I: $V_t(\text{in real terms}) = \frac{V_t(\text{in nominal terms})}{P_t}$

  $\Updownarrow$

  - M II: $V_t(\text{in real terms}) = Z_t(\text{in real terms}) + \frac{Z^e_{t+1}(\text{in real terms})}{(1+r_t)} + \frac{Z^e_{t+2}(\text{in real terms})}{\dots} + \dots$

$$\boxed{\begin{aligned} &Z_{r_t} = \frac{Z_{nt}}{P_t}, \quad Z^e_{rt+i} = \frac{Z^e_{nt+i}}{P^e_{t+i}} + \dots \\ &1 + r_{t+j} = \frac{1+i_{t+j}}{1+\pi_{t+j}} \end{aligned}}$$

$(1+r_t)(1+r^e_{t+1})$

Some Nomenclature

- Maturity: Length of time over which a bond promises to make the payment to the holder of the bond.
- Default rate: Risk that the issuer of the bond will not pay back the full amount promised by the bond.
- Price risk: Uncertainty about the price at which one can sell the bond before maturity.
- Yield to maturity or Yield: Interest rates associated with bonds of different maturities.
- Short term interest rate: Yields on bonds with a maturity < 1 year (typically)
- Long term interest rate: Yields on bonds with a maturity > 1 year (typically)
- Term structure of interest rates OR yield curves: Relation b/w maturity & yield
- Govt. bonds, Corporate bonds
- Bond ratings: Ratings for default risk
- Risk premium: Difference b/w interest rate paid in a given bond & interest rate on a bond with the best ratings.
- Junk bonds: Bonds with high default risk
- Discount bonds: Bonds that promise a single payment at maturity (called the face value)
- Coupon bonds: Bonds that promise multiple payments before maturity & one payment at maturity.
- Coupon payments, Coupon rate, Coupon yield

Coupon payments, coupon rate, coupon yield

- Life: Amount of time left until the bond matures.
- T-Bills, Indexed Bonds

Treasury Bonds

- Term Premium: Premium associated with longer maturities.

Strategy I : Take a 1-year bond : $1 \rightarrow (1+i_t)$

S II : Take a 2-year bond & sell it after 1-year.

$$1 \longrightarrow \frac{1}{P_{2t}} P^e_{1\,t+1}$$

$P_{nt} \longrightarrow t^{th}$ year

$\hookrightarrow$ No. of years remaining

$\frac{P^e_{1t+1}}{P_{2t}}$ should equal $1+i_t \Rightarrow P_{2t} = \frac{P^e_{1t+1}}{1+i_t} = \frac{100}{(1+i_t)(1+i^e_{1t+1})}$

$$\rightarrow \quad P_{2t} = \frac{100}{(1+i_t)(1+i^e_{1t+1} + x)}$$

Risk Premium

* Yield to maturity on an n-year bond is the constant annual interest rate that makes bond price today equal to the present value of future payments on the bond.

$$\frac{100}{(1+i_{2t})^2} = \frac{100}{(1+i_{1t})(1+i^e_{1t+1})}$$

$$\Rightarrow (1+i_{2t})^2 = (1+i_{1t})(1+i^e_{1t+1})$$

$$\Rightarrow \boxed{i_{2t} \approx \tfrac{1}{2}\left(i_{1t} + i^e_{1t+1}\right)}$$

Q What's the relation b/w $\underbrace{Q_t}_{\text{Price of Stock}}$, Q^e_{t+1}, $\underbrace{D^e_{t+1}}_{\text{Dividend}}$ & i_t?

$SI \rightarrow 1 \rightarrow 1+i_t$

$SII \rightarrow 1 \rightarrow \left(\frac{1}{Q_t}\right)(Q^e_{t+1} + D^e_{1t+1}) \Rightarrow \boxed{Q_t = \frac{Q^e_{t+1} + D^e_{t+1}}{1+i_t}}$

- Equity Premium: Risk associated w/ stocks

Assumption: Interest rate is const. (i) & no risk premium.

$$Q_t = \frac{Q^e_{t+1} + D^e_{t+1}}{1+i} = \frac{D^e_{t+1}}{1+i} + \frac{1}{1+i}\left[\frac{Q^e_{t+2}}{1+i} + \frac{D^e_{t+2}}{1+i}\right]$$

$$= \frac{D^e_{t+1}}{1+i} + \frac{D^e_{t+2}}{(1+i)^2} + \dots + \frac{D^e_{t+n}}{(1+i)^n} + \frac{Q^e_{t+n}}{(1+i)^n}$$

Additional Problem Set 2

Q1P1 l (labour) governed by maximisation of $u := c^{2/3} h^{1/3}$
c: consumption, h: leisure, $l + h = 24$. w given,
house consumes complete wage (wl). $l = ?$

MI: $\mathcal{L} = c^{2/3} h^{1/3} + \lambda(24 - l - h) \longrightarrow \partial\mathcal{L}/\partial\lambda = 0 \Rightarrow 24 = l + h$

$\partial\mathcal{L}/\partial\lambda = 0 = \partial\mathcal{L}/\partial w = \partial\mathcal{L}/\partial h$ $\quad \hookrightarrow \partial\mathcal{L}/\partial h = 0 \Rightarrow \frac{1}{3} h^{-2/3} \cdot c^{2/3} = \lambda$

$\Rightarrow \frac{c^{2/3} h^{1/3}}{3} = \lambda h$

$c = wl$ $\quad \hookrightarrow \frac{\partial}{\partial c} \Rightarrow \frac{2}{3}$

MII: $\text{Max } v = \log u = \frac{2}{3} \log c + \frac{1}{3} \log h$

$$= \frac{2}{3} \log(wL) + \frac{1}{3} \log(24 - L)$$

$$dv/dL = 0 = \frac{2}{3} \cdot \frac{1}{L^*} + \frac{1}{3} \frac{-1}{24 - L^*} \Rightarrow 48 - 2L^* = L^* \Rightarrow \underline{L^* = 16}$$

Q1P2 Keynesian (closed) economy. Two types of final expenditure
investment (36), c $(= wl)$. Given $w = 4$. $y = 24\sqrt{l}$.
Find y^* & l^*.

Q2 $Y = U N^{\alpha}$ ($Y \to$ agg. output; $\alpha \to 0 < \alpha < 1$; $U \to$ Shock) $\quad$ N determined by equating marginal prod.
to $\frac{W}{P}$ ($W \longrightarrow$ Nominal wage, $P \longrightarrow$ Price level). $\quad u = \log\alpha + \frac{1}{\alpha} \log U$,
$p = \log P$, $w = \log W$, $y = \log Y$

1. Obtain y (agg. supply) in terms of p, w, u.

$$\log Y = \log U + \alpha \log N \Rightarrow y = \log U + \alpha \log N$$

$$\frac{\partial Y}{\partial N} = \frac{W}{P} = \alpha U \cdot N^{\alpha - 1} \Rightarrow N = \left(\frac{W}{P} \cdot \frac{1}{\alpha U}\right)^{\frac{1}{\alpha - 1}}$$

$$\dots \log\left(\frac{W}{P} \cdot \frac{1}{\alpha U}\right)^{\frac{1}{\alpha-1}} = \log U + \frac{\alpha}{\dots}\left(\log W - \log P - \log\alpha\right)$$

$$y = \log U + \alpha \log\left(\frac{\cdot}{P} \cdot \frac{\cdot}{\alpha U}\right) \cdots = \log \cdots \frac{\cdot}{\alpha - 1}\left(\cdots - \log U \right)$$

$$= \frac{\log U}{1-\alpha} + \frac{\alpha W}{\alpha - 1} - \frac{\alpha P}{\alpha - 1} - \frac{\alpha \log \alpha}{\alpha - 1} = \frac{\alpha}{\alpha - 1}(W - P) + \frac{\alpha}{1-\alpha}\left(\log\alpha + \frac{1}{\alpha}\log U\right)$$

$$= \frac{\alpha}{1-\alpha} u + \frac{\alpha}{\alpha - 1}(w - p) = \boxed{\frac{\alpha}{1-\alpha}(u + p - w) = y}$$

2. (i) $w = \theta p$, $0 \le \theta \le 1$ & $y = m - p$ → log. (money) $\underbrace{\text{money}}_{\text{policy var.}}$

Find $y(m, u)$

$$\left.\begin{array}{l} p = m - y \\ w = \theta p = \theta(m - y) \end{array}\right] \quad y = \frac{\alpha}{1-\alpha}\left[u + p(1-\theta)\right] = \frac{\alpha}{1-\alpha}\left[u + (1-\theta)(m-y)\right]$$

$$\Rightarrow (1-\alpha) y = \alpha\left[u + (1-\theta)m\right] - \alpha(1-\theta) y \qquad y = \frac{\alpha}{1-\alpha\theta}\left[u + (1-\theta)m\right]$$

$$\Rightarrow (1 - \alpha + \alpha - \alpha\theta) y = \alpha\left[u + (1-\theta)m\right] \Rightarrow$$

3. Does m affect output if $\underbrace{0 < \theta < 1}_{\text{yes}}$, $\underbrace{\theta = 1}_{\text{No}}$

$$\partial y / \partial m = \frac{\alpha(1-\theta)}{1 - \alpha\theta} \qquad 0 \ne \frac{\partial y}{\partial m} = 0$$

4. u affect y more when $\theta \uparrow$?

$$\partial y / \partial u = \frac{\alpha}{1-\alpha\theta} \longrightarrow \theta\uparrow,\ \alpha\theta\uparrow,\ 1-\alpha\theta\downarrow,\ \frac{1}{1-\alpha\theta}\uparrow,\ \frac{\partial y}{\partial u}\uparrow$$

OR $$\partial / \partial\theta\left(\partial y / \partial u\right) = \frac{\alpha \cdot \alpha\theta}{(1-\alpha\theta)^2} > 0 \Rightarrow \theta\uparrow,\ \partial y / \partial u \uparrow$$

<u>Q3</u> Supply func. : $\log Y = \log Y^* + \lambda(\log P - \log P^e)$, $\frac{W}{P^e} = N^\sigma$

Y → Output; Y^* → Natural level of output; P → Price Level; P^e → Expected price level; W → Wage (Nominal); N → Employment

$$Y = N^\alpha$$

Firms : $\underbrace{MP_N = \frac{W}{P}}$

$$\frac{\partial Y}{\partial N} = \frac{W}{P} \Rightarrow \alpha N^{\alpha - 1} = \frac{W}{P} \Rightarrow N = \left(\frac{W}{\alpha P}\right)^{\frac{1}{\alpha - 1}}, \quad N^\sigma = \frac{W}{P^e}$$

$$W = N^\sigma P_e = \alpha N^{\alpha - 1} P \Rightarrow N^{1 - \alpha + \sigma} = \frac{\alpha P}{P_e}$$

$$P = P_e \Rightarrow N = \alpha^{\frac{1}{1-\alpha+\sigma}}, \quad Y^* = \alpha^{\frac{\alpha}{1+\sigma-\alpha}}$$

$$(1 - \alpha + \sigma)\log N = \log\alpha + (\log P - \log P_e) \qquad \log Y^* = \frac{\log\alpha}{1-\alpha+\sigma}$$

$$\log N = \frac{\log\alpha}{1-\alpha+\sigma} + \frac{1}{1-\alpha+\sigma}(\log P - \log P_e) \qquad \lambda = \frac{1}{1-\alpha+\sigma}$$

Q4

$C = 200 + 0.5 Y_D$, $T = 200$, $\left(\frac{M}{P}\right)^d = 24 - 4000i$, $\left(\frac{M}{P}\right)^s = 1600$, $i = r - \pi^e$
$I = 150 - 1000r$, $G = 250$,
P rigid.

1) $\pi^e = 0$. Determine eq' Y & r, i.

$i = r$, $Y = 200 + 0.5(Y - 200) + 250 + 150 - 1000r$

$\Rightarrow Y = 1000 - 2000r$ — IS
$24 - 4000i = 1600$ — LM

$Y = 900$
$r = \frac{1}{20} = 5\% = i$

2) $\underbrace{M_s \uparrow 2\%}_{M_s = 1632}$ $\underbrace{\text{temp.}}_{\pi^e = 0}$ Find Y, i, r.

$24 - 4000i = 1632$
$Y = 1000 - 2000r$

$Y = 908$
$r = \frac{92}{2000} = 4.6\% = i$

3) $\underbrace{M_s \uparrow 2\%}_{M_s = 1632}$ $\underbrace{\text{permanently}}_{\pi^e = 2\%}$. Find Y, i, r

$i = r - 2$
$\because i = r - \pi$ given

$24 - 4000i = 1632$
$Y = 1000 - 2000r$

$i = 3.6\%$
$r = 5.6\%$
$Y = 888$

Q6

Worker $\rightarrow$ W1: 66, W2 = 50.
W2 $\rightarrow$ Week 2
$\text{Wage}_{W2} > \text{Wage}_{W1}$

Sub. effect $\rightarrow$ Wage $\uparrow$, opp. cost for not working $\uparrow$, Work More

Income effect $\rightarrow$ Wage $\uparrow$, Richer (feel) $\uparrow$, Work less

$\because$ Worker works less $\rightarrow$ IE dominates SE

Q7

Discount rate: 15% pa. Firms live 3 years.
F's cash flow starts w/ 500 in Y1, grow at 20% for two years
S's " " " " " " ", shrink at 20% " " "

Prices of two firms? Which one better buy?

$$\text{Price of } F = \frac{500}{1.15} + \frac{500 \cdot 1.2}{1.15^2} + \frac{500 \cdot 1.2^2}{1.15^3} = 1361.88$$

$$\text{" " } S = \frac{500}{1.15} + \frac{500 \cdot 0.8}{1.15^2} + \frac{500 \cdot 0.8^2}{1.15^3} = 947.64$$

Both equivalent at these prices.

Both equivalent at their price.

Both return 15%.

Q8 Time 0 $\rightarrow$ initial investment 1 000 000.
Generate cash flow of 100 000 after time 1 & this grows by 4%/year thereafter.
What is the NPV? Assuming int rate to be const. at 10%

$$NPV = -1\,000\,000 + \frac{100\,000}{1.1} + \frac{100\,000 \cdot 1.04}{(1.1)^2} + \frac{100\,000 \cdot 1.04^2}{(1.1)^3} + \dots$$

$$= \frac{100\,000}{1.1} \cdot \frac{1}{1 - \frac{1.04}{1.1}} - 1000\,000 = \frac{100\,000}{0.07} - 1\,000\,000$$

$$= 428\,571.43 > 0$$ Should take the opportunity.

Q9 Stock A $\longrightarrow$ alternates b/w +20, −10%
Stock B earned 4.5% pa.
On a 1 year basis, would a risk-neutral investor prefer A (equal prob.) or B?
How much would each rupee invested 10 yrs. ago in A have earned? In B?

$A \longrightarrow \frac{1}{2} \cdot 20 + \frac{1}{2} \cdot -10 = 5\% \longrightarrow$ Choose A.

$A_{10} \longrightarrow (1.2)^5 (0.9)^5 \longrightarrow 1.4693$

$B_{10} \longrightarrow (1.045)^{10} \longrightarrow 1.5530$

Q10 $M_t \bar{V} = P_t Y_t, \quad Y_t = \bar{A} L_t, \quad P_{t+1}/P_t = (L_t/L^*)^{\theta}$

1.) $\bar{V} = 1 = \bar{A} = L^*$. Calculate $\underbrace{\text{steady state}}_{L_t = L^*}$ $M_t = 1$.

$P_t Y_t = 1, \quad Y^* = 1, \quad P_{t+1} = P_t = 1$

2.) Take log & eliminate L_t.

3.) part (1) $\longrightarrow$ till $t = -1$. At $t = 0 \longrightarrow M_0 = 2$ Const. for 20 yrs.

$\theta = 0.2$

$\log Y_{-1} = 0$

$\log M_t = \log P_t + \log Y_t$

$\log P_{t+1} - \log P_t = 0.2 [\log Y_t] \longrightarrow \log P_0 = \log P_{-1} = 0$

$\hookrightarrow \log M_0 = \log P_0 + \log Y_0 \longrightarrow \log 2 = \log M_0 = \log Y_0$

$\log P_1 = 0 + 0.2 \log 2 = 0.2 \log 2 \Rightarrow \log Y_1 = 0.8 \log 2$

$\Rightarrow \log P_2 = 0.2 \log 2 + 0.16 \log 2 = 0.36 \log 2, \ \log Y_2 = 0.64 \log 2$

MID SEMESTER EXAMINATION

☆ $P_t = \dfrac{D^e_{t+1}}{1+r_{t+1}} + \dfrac{D^e_{t+2}}{1+r_{t+2}} + \ldots \longrightarrow D^e_i = 0 \Rightarrow P_t = 0$ but we can still buy it "rationally" if we expect that people would want to buy it in the future.

Ex: Stamps

$Y\uparrow$ → Did people expect it → No change on P
→ Did not → $Y\uparrow \Rightarrow D\uparrow \Rightarrow P\uparrow$

Suppose banks $\uparrow r$ → Expected? → Yes → No change in P
→ No → $P\downarrow$

Effects only exist if the underlying changes were not expected

☆ Expectations matter in both $\underbrace{\text{goods market}}_{\text{Consumption}}$ & the $\underbrace{\text{financial market}}_{\text{Investment}}$

Consumption → Not only depends on current income $(Y_t - T_t)$ but also on expectation of (W_t) Wealth

(W_t) Wealth → Stocks, bonds, house, "financial assets"
→ "non-financial" assets → Skills, Human Capital

$\boxed{C(\underbrace{W_t}_{\text{Wealth}}, \underbrace{Y_t - T_t}_{\text{Disposable Income}})}$

Ex: 20 → 23 ... 60 → 79
0 ... 100 ... 100 ... 0
C : 3800/60 ~ 63

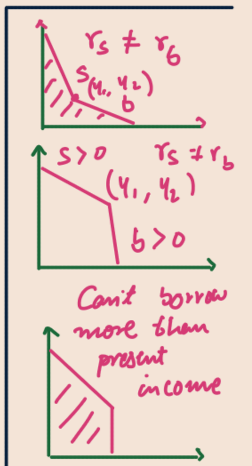
→ Might need to pay bills
→ Loan required → Given if income ↑ → Present income matters → Even if Present Value is the same

2 period Consumption Model (Deterministic)

Household (H.H.) → Y_1 Y_2
C_1 C_2
→ Utility: $U(C_1, C_2)$

Can save or borrow at same interest rate (r) (say)
(s) (b)

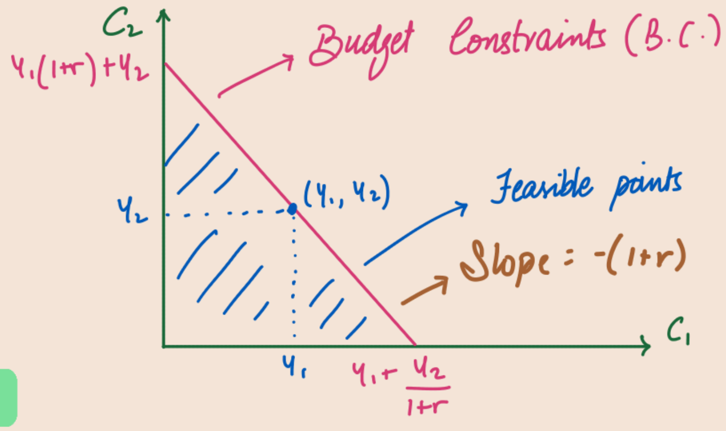
- $C_1 = Y_1 \Rightarrow C_2 = Y_2$
- $C_1 \geq 0, \ C_2 \geq 0$



(Day)

s saved	b borrowed
$c_1 = y_1 - s$	$c_1 = y_1 + b$
$c_2 \le y_1 + s(1+r)$	$c_2 \le y_2 - b(1+r)$
$c_2 \le y_1 + (y_1 - c_1)(1+r)$	$c_2 \le y_2 - (c_1 - y_1)(1+r)$
$\Rightarrow c_1 + \frac{c_2}{1+r} \le y_1 + \frac{y_2}{1+r}$	$c_1 + \frac{c_2}{1+r} \le y_1 + \frac{y_2}{1+r}$

$\Rightarrow$ P.V.(Consumption) $\le$ P.V.(Income)



$U(c_1, c_2) \longrightarrow$ Maximise subject to B.C. & $\partial U/\partial c_1, \partial U/\partial c_2 > 0$

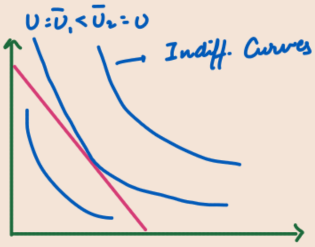
Only need to analyse equality/boundary of B.C.

$$\mathcal{L} = U(c_1, c_2) + \lambda\left[y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r}\right] \longrightarrow \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow \text{Equality of B.C., i.e. } c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

$$\frac{\partial \mathcal{L}}{\partial c_1} = 0 \Rightarrow \frac{\partial U}{\partial c_1} = \lambda, \quad \frac{\partial U}{\partial c_2} = \frac{\lambda}{1+r} \Leftarrow 0 = \frac{\partial \mathcal{L}}{\partial c_2}$$

$$\Rightarrow -\frac{\partial U/\partial c_1}{\partial U/\partial c_2} = -(1+r)$$

Slope of Budget Line $\Rightarrow$ Indifference Curve ($U = \bar{U}$) should be tangent to B.L. $\Rightarrow$



$$U = \bar{U} \Rightarrow \frac{\partial U}{\partial c_1} dc_1 + \frac{\partial U}{\partial c_2} dc_2 = 0 \Rightarrow \frac{dc_2}{dc_1} = -\frac{\partial U/\partial c_1}{\partial U/\partial c_2}$$

Claim: $\frac{-\partial U/\partial c_1}{\partial U/\partial c_2} = -(1+r)$ at max. $U \longrightarrow dU = \frac{\partial U}{\partial c_1} dc_1 + \frac{\partial U}{\partial c_2} dc_2$

$$dU = dc_1\left[\frac{\partial U}{\partial c_1} - (1+r)\frac{\partial U}{\partial c_2}\right]$$

- $\frac{\partial U/\partial c_1}{\partial U/\partial c_2} < 1+r \Rightarrow (dc_1 < 0 \Rightarrow dU > 0)$
- $\frac{\partial U/\partial c_1}{\partial U/\partial c_2} > 1+r \Rightarrow (dc_1 > 0 \Rightarrow dU > 0)$

Contradict maximality $\Downarrow$ $\frac{\partial U/\partial c_1}{\partial U/\partial c_2} = 1+r$

- $U(c_1, c_2) = U(c_1) + \beta U(c_2)$
  - $\beta \rightsquigarrow$ Money has more meaning when we are young, as does consumption
  - Consumption follows same utility function
  - Utility due to consumption today is independent of consumption tomorrow.

$0 < \beta < 1 \longrightarrow$ Measure of Impatience

($\beta \to 0$: Impatient; $\beta \to 1$: patient)

Ex. $U(c_1, c_2) = \ln c_1 + \beta \ln c_2 \longrightarrow (1+r) = \frac{1/c_1^*}{\beta/c_2^*} = \frac{c_2^*}{c_1^*\beta} \longrightarrow (c_1^*, \beta(1+r)c_1^*)$

From B.C./B.L. $\longrightarrow c_1^* = \frac{1}{1+\beta}\left[y_1 + \frac{y_2}{1+r}\right]$; $c_2^* = \frac{\beta}{1+\beta}\left[y_1(1+r) + y_2\right]$

$\beta\uparrow$ $c_1\downarrow$, $c_2\uparrow$; $\beta\downarrow$, $c_1\uparrow$, $c_2\downarrow$; $r\uparrow$ $c_1^*\downarrow$ $c_2^*\uparrow$; $r\downarrow$ $c_1^*\uparrow$ $c_2^*\downarrow$

☆ People respond more to permanent changes than temporary changes

☆ Investment is much more volatile than consumption.

$\hookrightarrow$ If rates $\uparrow$, investment $\uparrow\uparrow$ as we want

If rates $\uparrow$, consumption changes in a calculated manner

- Loans, etc., depend on present income $\Rightarrow$ Present income matters even if PV same

- Consumption = $C(W_t, Y_t - T_t)$

W_t → Human Wealth ↔ $\dfrac{(1-T^e_{t+1})Y^e_{t+1}}{1+r^e_t} + \dots$

W_t → Non-Human Wealth → Stocks, Bonds; Durable Goods

$Y_t - T_t$:
$$\begin{cases} + & Y_t, Y^e_{t+1}, Y^e_{t+2}, \dots \\ - & r^e_{t+1}, r^e_{t+2}, r^e_{t+3}, \dots \\ - & T_t, T^e_{t+1}, T^e_{t+2}, \dots \end{cases}$$

- Investment = $I\left(\begin{matrix} + & \pi^e_{t+1}, \pi^e_{t+2}, \dots \\ - & r_t, r^e_{t+1}, \dots, \delta \end{matrix}\right)$

Firm ⟶ Profit (gross) ⟶ $\pi_t, \pi^e_{t+1}, \pi^e_{t+2}, \dots$

⟶ $\pi_t, \pi^e_{t+1}, (1-\delta)\pi^e_{t+2}, (1-\delta)^2\pi^e_{t+3}$

δ → Depreciation of Capital

NPV (Net Present Value) : $\pi_t + \dfrac{\pi^e_{t+1}}{1+r_t} + \dfrac{\pi^e_{t+2}(1-\delta)}{(1+r_t)(1+r^e_{t+1})}, \dfrac{\pi^e_{t+3}(1-\delta)^2}{(1+r_t)(1+r^e_{t+1})(1+r^e_{t+2})}, \dots$ $\begin{cases} + & \pi_i / \pi^e_i \\ - & r_i / r^e_i, \delta \end{cases}$

* $\pi_t = 0$, $\pi^e_i = \pi$, $r^e_t = r$

$$I = I\left(\frac{\pi}{1+r} + \frac{\pi(1-\delta)}{(1+r)^2} + \dots\right) = I\left(\frac{\pi}{1+r} \cdot \frac{1}{1-\frac{1-\delta}{1+r}}\right) = I\left(\frac{\pi}{r+\delta}\right)$$

$r+\delta$: User/rental cost

* π → Depends on Y

☆ $Y = C + I + G$

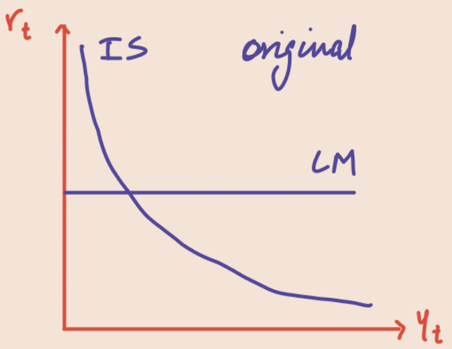
$= C(Y_t - T_t) + \bar{I} + \bar{G}$ ⟶ Simple Keynesian Model

$= C(Y_t - T_t) + I(Y_t, r_t) + \bar{G}$ ⟶ IS-LM Model

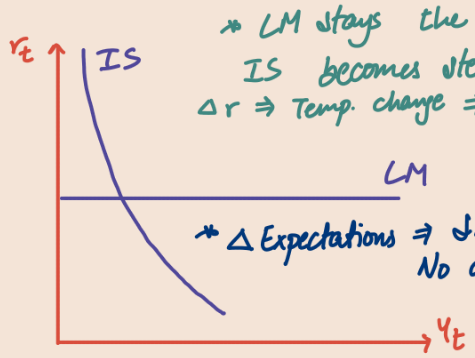
$= C(Y_t - T_t) + I(Y_t, r_t + x) + \bar{G}$ ⟶ Extended IS-LM Model

$$= \underbrace{C\left(\begin{matrix} Y_t, Y^e_{t+1}, Y^e_{t+2}, \dots, \\ T_t, T^e_{t+1}, T^e_{t+2}, \dots, \\ r_t, r^e_{t+1}, r^e_{t+2} \end{matrix}\right) + I\left(\begin{matrix} Y_t, Y^e_{t+1}, \dots, \\ r_t, r^e_{t+1}, \dots \end{matrix}\right)}_{A = C + I} + \bar{G} = \bar{G} + A\left(\begin{matrix} +Y_t, & Y^e_i, \\ -T_t, & T^e_i, \\ -r_t, & r^e_i \end{matrix}\right)$$

⟶ Expectations Model



Expectations ⟶



* LM stays the same

IS becomes steeper.

Δr ⇒ Temp. change ⇒ less response in Y_t

* Δ Expectations ⇒ Shift in IS. No change in LM

Long Run

Y_0 ⟶ growth rate g

$Y_t = Y_0 e^{gt}$

e^{gt} → because ctsly in time and not discretely.

Continuously in time : $\frac{1}{y}\frac{dy}{dt} = g$

Discretely in time : $\frac{y_t - y_{t-1}}{y_{t-1}} = g$

Y_t ⟶ GDP

$\dfrac{Y_t}{N_t}$ ⟶ GDP per capita

Output

Technology

Capital

Labour

$$\boxed{y_t = F(K_t, N_t)}$$

⊛ How do we compare GDP of two countries?

$1\$ = ₹80$

	Ind.	US
Meal	$1 \to 20k$ ₹	$1 \to 500\$$
Comp.	$1 \to 80k$ ₹	$5 \to 1000\$$

$$\frac{GDP_{U,\$}}{GDP_{I,₹}} = \frac{5500\$}{100k\ ₹} = 4.4$$

$$\frac{GDP_{U,\$}}{GDP_{I,\$}} = \frac{5500\$}{500 \cdot 1 + 1000 \cdot 1} = \frac{5500\$}{1500\$} = 3.67$$

changes w/ changing currency

Solow (Swann) Model

- $Y = F(K, N) \longrightarrow \frac{\partial F}{\partial K} > 0,\ \frac{\partial F}{\partial N} > 0,\ \underbrace{\frac{\partial^2 F}{\partial K^2} < 0,\ \frac{\partial^2 F}{\partial N^2} < 0}_{\text{Diminishing Rates of Marginal Product}}$
- $F(xK, xN) = x\,F(K, N) \longrightarrow$: Constant returns to scale :
- $F(0, N) = F(K, 0) = 0$
- $\lim_{K \to 0} F_K(K, N) = \infty$, $\lim_{N \to 0} F_N(K, N) = \infty$

  $\lim_{K \to \infty} F_K(K, N) = 0$, $\lim_{N \to \infty} F_N(K, N) = 0$

Ex. $Y = A K^{\alpha} N^{1-\alpha}$, $0 < \alpha < 1$, :: Cobb-Douglas Production Function ::

$\partial F / \partial K = \alpha \cdot A\, K^{\alpha - 1} N^{1-\alpha} > 0$

$\alpha - 1 < 0$ ↳ $K \to 0 \Rightarrow \to \infty$

$K \to \infty \Rightarrow K \to 0$

$\partial^2 F / \partial K^2 = \alpha \cdot \underbrace{(\alpha - 1)}_{<0} A\, K^{\alpha-2} N^{1-\alpha} < 0$

Similarly for $\partial F/\partial N$, $\partial^2 F/\partial N^2$

Also, $F(xK, xN) = A \cdot x^{\alpha} K^{\alpha} \cdot x^{1-\alpha} N^{1-\alpha}$

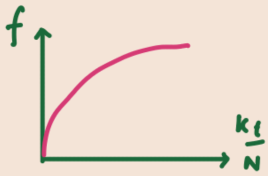
$= x \cdot A \cdot K^{\alpha} \cdot N^{1-\alpha}$

$= x \cdot F(K, N)$

* We were interested in $(Y/N)_t \rightsquigarrow N_t = N$

$$Y_t / N_t = Y_t / N = F(K_t, N)/N = \boxed{F\left(\frac{K_t}{N}, 1\right) = \frac{Y_t}{N}}\ ,\quad k_t = \frac{K_t}{N}$$

$$\frac{Y_t}{N} = F\left(\frac{K_t}{N}, 1\right) = f\left(\frac{K_t}{N}\right) \text{ w/ } f' > 0 \ \& \ f'' < 0$$

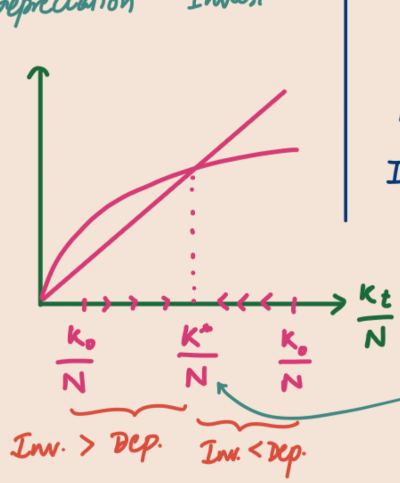


$t = 0, 1, 2, \ldots$ $\quad K_{t+1} = K_t - \underbrace{\delta K_t}_{\text{Depreciation}} + \underbrace{I_t}_{\text{Investment}}$ | Assumption

$$K_{t+1} = K_t - \delta K_t + sY_t$$

$$\Rightarrow \frac{K_{t+1}}{N} - \frac{K_t}{N} = s\frac{Y_t}{N} - \delta\frac{K_t}{N}$$

$$\Rightarrow \frac{K_{t+1}}{N} - \frac{K_t}{N} = \underbrace{sF\left(\frac{K_t}{N}\right)}_{\text{Investment Component}} - \underbrace{\delta\frac{K_t}{N}}_{\text{Depreciation Component}}$$



Asymptotically reaches this equilibrium

Closed economy,
Public Savings = 0

$I_t = S_t = sY_t$, $0 < s < 1$

Savings Rate

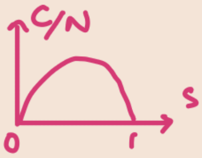
Not possible because $\lim_{K_t \to 0} f' = \infty$

* $K/N \longrightarrow$ Capital Labour ratio

* $\Delta K > 0 \longrightarrow$ Capital Accumulation

* If $s\uparrow$ then $\frac{K^*}{N}\uparrow \Rightarrow \frac{Y^*}{N}\uparrow$ but $\frac{\partial}{\partial t}$(Growth Rate) still < 0

Also, $C_t = Y_t - sY_t = (1-s)\,Y_t(s)$
- $s = 0 \Rightarrow Y_t(0) = 0 \Rightarrow C_t = 0$
- $s = 1 \Rightarrow C_t = 0$



* $\frac{Y_t}{N} = F\left(\frac{K_t}{N}, 1\right) = f\left(\frac{K_t}{N}\right)$
- Assumes $N_t = N$
- Doesn't factor in productivity

Bring in new model
- 1) Discrete time $\longrightarrow t = 0, 1, 2, \dots$
- 2) Continuous time $\rightsquigarrow dt$

1) Discrete Time $\quad t = 0, 1, 2, \dots$

$A_t N_t$: Effective Labour :

$K_t \to Y_t$
$\downarrow$
$K_{t+1} \leftarrow I_t$

$$\boxed{Y_t = F(K_t, A_t N_t)}$$

Y_t → Output, K_t → Capital, A_t → Labour-augmenting productivity, N_t → Labour

Assumption

$$N_{t+1} = (1+n)N_t$$
$$A_{t+1} = (1+g)A_t$$
$$K_{t+1} = (1-\delta)K_t + I_t = (1-\delta)K_t + sY_t$$
$$I_t = S_t \quad \text{(Closed eco w/ no govt.)}$$
$$S_t = sY_t, \quad 0 < s < 1$$

About F

1) $F_K(K, L) > 0$, $F_L(K, L) > 0$
2) $F_{KK}(K, L) < 0$, $F_{LL}(K, L) < 0$
3) CRS (Const. Returns to Scale):
$$F(cK, cL) = cF(K, L)$$
4) $\lim_{K \to 0} F_K(K, L) = \infty$, $\lim_{K \to \infty} F_K(K, L) = 0$,
$\lim_{L \to 0} F_L(K, L) = \infty$, $\lim_{L \to \infty} F_L(K, L) = 0$
5) $F(K, 0) = F(0, L) = 0$

$$Y_t = F(K_t, A_tN_t) \xrightarrow{\frac{1}{A_tN_t}} \frac{Y_t}{A_tN_t} = \frac{1}{A_tN_t}F(K_t, A_tN_t) = F\left(\frac{K_t}{A_tN_t}, 1\right)$$

$$y_t = \frac{Y_t}{\;\;}, \quad k_t = \frac{K_t}{\;\;} \qquad \boxed{y_t = F(k_t, 1) \equiv f(k_t)}$$

$A_t N_t$ $\qquad$ $A_t N_t$

1) $f(0)=0$ $\quad$ 2) $f'(k)>0$ $\quad$ 3) $f''(k)<0$ $\quad$ 4) $\lim\limits_{k\to 0} f' = \infty$, $\lim\limits_{k\to\infty} f' = 0$

$$f(k) = \frac{1}{A_t N_t} F(K_t, A_t N_t) = f\left(\frac{K_t}{A_t N_t}\right) \xrightarrow{\frac{\partial}{\partial K_t}} \frac{F_K(K_t, A_t N_t)}{A_t N_t} = \frac{f_k(k)}{A_t N_t}$$

$\Rightarrow f_k(k) = F_K(K_t, A_t N_t) > 0$ $\qquad$ Similarly for f''

★ We would like k in the difference equations:

$$K_{t+1} = (1-\delta) K_t + s F(K_t, A_t N_t)$$

$$\xrightarrow{\frac{1}{A_t N_t}} \frac{K_{t+1}}{A_{t+1} N_{t+1}} \frac{A_{t+1} N_{t+1}}{A_t N_t} = (1-\delta) k_t + s f(k_t) = k_{t+1}(1+g)(1+n)$$

$$\Rightarrow k_{t+1} = \frac{(1-\delta) k_t}{(1+g)(1+n)} + \frac{s f(k_t)}{(1+n)(1+g)}$$

Steady state : $k_{t+1} = k_t = k^*$

$$k^* = \frac{(1-\delta)}{(1+g)(1+n)} k^* + \frac{s f(k^*)}{(1+n)(1+g)}$$

$$\Rightarrow k^*(n+g+gn+\delta) = s f(k^*)$$

1) $k_t = k^* \Rightarrow k_{t+1} = k_t = k^*$

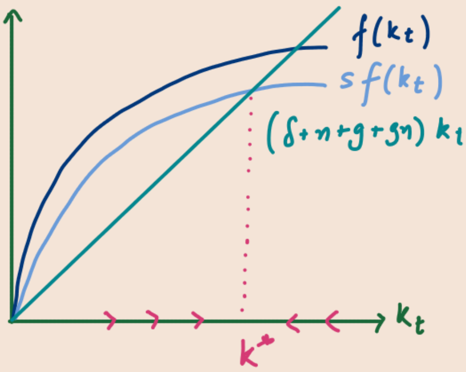
2) $k_t < k^* \Rightarrow k_{t+1} - k_t = \dfrac{s f(k_t)}{(1+n)(1+g)} + \dfrac{(1-\delta) k_t}{(1+n)(1+g)} - k_t$

$$= \frac{1}{(1+n)(1+g)} \left[s f(k_t) - (\delta + n + g + gn) k_t \right]$$

$k_{t+1} > k_t \iff s f(k_t) > (\delta + n + g + gn) k_t$

$\iff k_t < k^*$

3) $k_t > k^* \iff k_{t+1} < k_t$



★ Steady state $\longrightarrow k^* \longrightarrow y^* = f(k^*)$

$\Rightarrow y^* = \dfrac{Y}{AN} = $ const. $\Rightarrow \dfrac{Y}{N}$ has a const. growth rate (= growth rate of A)

2) Continuous dt

$Y(t) = F(K(t), A(t) N(t))$, $\quad y(t) = \dfrac{Y(t)}{A(t) N(t)}$, $\quad F(k(t), 1) \equiv f(k(t)) = F\left(\dfrac{K(t)}{A(t)N(t)}, 1\right)$

$A_{t+1} = (1+g) A_t \longleftrightarrow \dot{A}(t)/A(t) = g$

$N_{t+1} = (1+n) N_t \longleftrightarrow \dot{N}(t) = n N(t)$

$\dot{K}(t) = I(t) - \delta K(t) = s Y(t) - \delta K(t)$

$$k(t) \equiv \frac{K(t)}{A(t) N(t)} \Rightarrow \frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{N}}{N} = \frac{\dot{K}}{K} - g - n$$

$$\Rightarrow \dot{k} = \frac{\dot{K}}{K} \cdot k - (g+n) k = \frac{\dot{K}}{K} \cdot \frac{K}{AN} - (g+n) k = \frac{\dot{K}}{AN} - (g+n) k$$

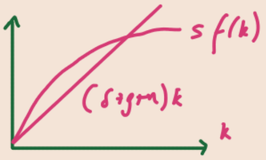
$$\Rightarrow \dot{k} = \frac{sY - \delta K}{AN} - (g+n) k = \frac{s F(K, AN)}{AN} - \delta \frac{K}{AN} - (g+n) k$$

$$= s\,F\left(\frac{K}{AN}, 1\right) - \delta k - (g+n)k = s f(k) - (\delta+g+n)k$$

$$\Rightarrow \boxed{\dot{k}(t) = s f(k(t)) - (\delta+g+n)\,k(t)} \equiv \Psi(k(t)) \longrightarrow \Psi'(k^*) < 0$$

↳ Steady State : $\dot{k} = 0 \Rightarrow \boxed{s f(k^*) = (\delta+g+n)\,k^*}$

☆ $\dot{k} > 0 \Leftrightarrow s f(k) > (g+n+\delta)k \Leftrightarrow k < k^*$
$\dot{k} < 0 \Leftrightarrow s f(k) < (g+n+\delta)k \Leftrightarrow k > k^*$ } Easier w/ graphs



$\dot{k} = 0 \Leftrightarrow$ Steady State $\Leftrightarrow k = k^*$; $y \to y^* \Rightarrow \dfrac{Y(t)}{A(t)N(t)} \longrightarrow f(k^*)$

☆ Growth rate of $\dfrac{Y(t)}{N(t)}$ in steady state $= g$
" " " $Y(t)$ " " " $= g+n$

☆ What happens if $s\uparrow$?
↳ Effect on steady state growth rate ⟶ No effect
↳ " " " " values of k^* & y^*? $k^*\uparrow$, $y^*\uparrow$

1) Graphically
2) $\dfrac{dk^*}{ds} = -\dfrac{\partial/\partial s}{\partial/\partial k} = \dfrac{-f(k^*)}{sf'(k^*) - (\delta+n+g)} = \dfrac{-f(k^*)}{\Psi'(k^*)} > 0$ $\quad \partial(sf - (g+n+\delta)k)$

$s\uparrow \Rightarrow k^*\uparrow \Rightarrow y^*\uparrow = f(k^*)\uparrow$

↳ Effect on short run growth? Yes

☆ Consumption : $c = (1-s)y = f(k) - sy \rightsquigarrow c^* = f(k^*) - sy^*$
$\qquad = f(k^*) - (\delta+g+n)k^*$

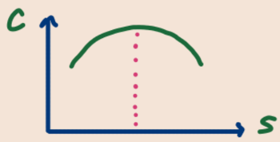
$\dfrac{dc^*}{ds} = [f'(k^*) - (\delta+g+n)]\,k^{*\prime}(s)$, $k^{*\prime}(s) > 0$

$\dfrac{dc^*}{ds} > 0 \Leftrightarrow f'(k^*) > (\delta+g+n)$ $\qquad * \ \exists\ s_0$ s.t. $f'(k^*) = \delta+g+n$ which maximises c

⟶ Suppose $s < s_0$ & $s\uparrow \Rightarrow k^*\uparrow \Rightarrow f'(k^*)\downarrow$
but $k^* < k^*_{s_0}$ so $f'(k^*) > f'(k^*_{s_0})$
$\Rightarrow dc^*/ds > 0 \Rightarrow c^*\uparrow$

⟶ Suppose $s > s_0$ & $s\uparrow \Rightarrow k^*\uparrow$
$\Rightarrow f'(k^*)\downarrow$ & $f'(k^*) < f'(k^*_{s_0})$
$\Rightarrow dc^*/ds < 0 \Rightarrow c^*\downarrow$

Depends on the nbd. of s



s_0

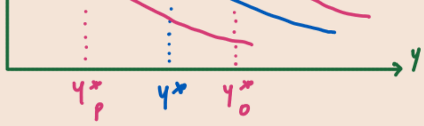
s_0 s.t. $k^* = k^*_{s_0}$ for which $f'(k^*_{s_0}) = \delta+g+n$

☆ What is the effect of A in the short run and the medium run?

$Y = AN \rightarrow Y/N = A$, $A_0 \uparrow A_1$

+ve expectations $\leftarrow$ $\hookrightarrow$ -ve expectations

$\hookrightarrow (IS \rightarrow IS_0)$ $\qquad$ $\hookrightarrow (IS_p \leftarrow IS)$



PS $\quad P = (1+m)W \xrightarrow{Y=N \rightarrow Y=AN}$

WS $\quad W = P^e F(u, z) \xrightarrow{Y=N \rightarrow Y=AN}$

$$\left\{ \begin{aligned} P &= (1+m)\frac{W}{A} \\ W &= P^e A^e F(u, z) \end{aligned} \right\}$$

1 unit of N → 1 unit of Y

↓

Cost = W

$P = (1+m)W$

1 unit of N → A units of Y

$\frac{1}{A}$ " " N → 1 " " Y

↓

Cost = W/A

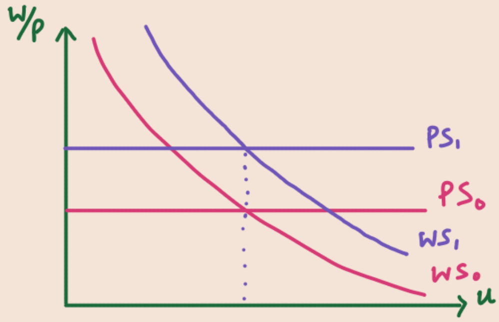
$P = (1+m) W/A$

★ $P = P^e$, $A = A^e$

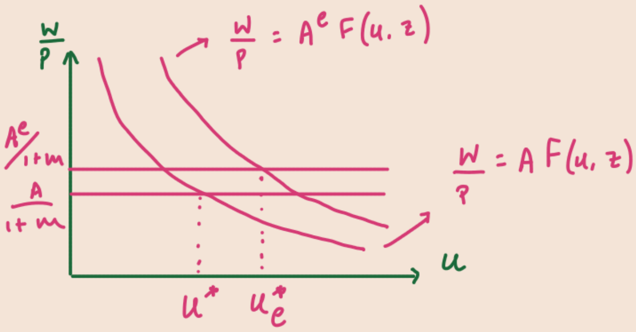
$W/P = A/1+m$

$W/P = A\ F(u, z)$

$A = A_0 \uparrow A_1 \Rightarrow \frac{W}{P} \uparrow, u \leftrightarrow$



$A < A^e$



$\Rightarrow u^* < u^*_e$

⇓

u^* Less than workers had expected

Govt. Budget Constraint

$B_x \rightarrow$ Debt in year x. also called signorage

Deficit in year t = $Def_t = r B_{t-1} + G_t - T_t$

$\hookrightarrow$ interest rate

$B_t = B_{t-1} + Def_t$

$= (1+r) B_{t-1} + G_t - T_t$

$\Rightarrow B_t - B_{t-1} = r B_{t-1} + G_t - T_t$

Budget Deficit / Primary Deficit

Ex. $B_0 = G_0 = T_0 = 0$

$G_1 \uparrow 1$, $T_1 = 0$, $Def_1 = 1$, $B_1 = 1$

Suppose we want $B_2 = 0 \rightsquigarrow T_2 = 1+r > 1 > 0$

If $T_2 = G_2 = 0 \longrightarrow B_2 = 1+r$

Suppose we want $B_3 = 0 \rightsquigarrow T_3 = (1+r)^2$

1) ↑ in tax

2) More one waits, higher the ↑ in tax

... the interest

Ex. $B_0 = G_0 = T_0 = 0$

$G_1 = 1, \quad T_1 = 0, \quad B_1 = 1$

$G_2 = 0, \quad T_2 = r, \quad B_2 = 1$

$G_3 = 0, \quad T_3 = r, \quad B_3 = 1$

$\vdots$

☆ Keep paying the ... & the debt does not increase

$$B_t = (1+r) B_{t-1} + G_t - T_t \quad , \quad g = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

$$\Rightarrow \frac{B_t}{Y_t} = (1+r) \frac{B_{t-1}}{Y_{t-1}} \frac{Y_{t-1}}{Y_t} + \frac{G_t - T_t}{Y_t} = \frac{(1+r)}{(1+g)} \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

$$\approx (1+r-g) \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

$$\Rightarrow \boxed{\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} \approx (r-g) \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}}$$

☆ $\frac{B_t}{Y_t}$ is called "debt-GDP ratio"

Additional Practice Problems 3

☆ Obs 1: Maximizing a Cobb-Douglas utility function

max. $U = X^{\alpha} Y^{\beta}$ s.t. $P_x X + P_y Y = m$

From Lagrangian $\rightarrow$ $\boxed{\frac{\alpha}{\alpha+\beta} = \frac{P_x X}{m} \; ; \; \frac{\beta}{\alpha+\beta} = \frac{P_y Y}{m}}$

$$X = \frac{m}{P_x} \cdot \frac{\alpha}{\alpha+\beta}, \quad Y = \frac{m}{P_y} \cdot \frac{\beta}{\alpha+\beta}$$

☆ Obs. 2: Max. $U = X^{\alpha} Y^{\beta} \longleftrightarrow$ Max. $V = \alpha \log X + \beta \log Y$

Q1 $\max_{c_1, a, c_2} \; U(c_1) + \beta U(c_2)$ subject to
$$\begin{cases} U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \\ a = a_0 + y_1 - T_1 - c_1 \\ c_2 = y_2 - T_2 + (1+r) a \end{cases}$$

P1 Solve for c_1^*, c_2^*, a^*

$$c_2 = y_2 - T_2 + (1+r)[a_0 + y_1 - T_1 - c_1]$$

$$\Rightarrow c_1 + \frac{c_2}{1+r} = a_0 + y_1 - T_1 + \frac{y_2 - T_2}{1+r}$$

$$\mathcal{L} = U(c_1) + \beta U(c_2) + \lambda \left[a_0 + y_1 - T_1 + \frac{y_2 - T_2}{1+r} - c_1 - \frac{c_2}{1+r} \right]$$

$\frac{\partial \mathcal{L}}{\partial c_1}: \lambda = U'(c_1)$; $\frac{\partial \mathcal{L}}{\partial c_2}: \frac{\lambda}{1+r} = \beta U'(c_2)$; $\frac{\partial \mathcal{L}}{\partial \lambda}$: BC again

$$\frac{U'(c_1)}{\beta U'(c_2)} = (1+r) = \frac{c_1^{-\sigma}}{\beta c_2^{-\sigma}} \Rightarrow \frac{c_2}{c_1} = [\beta(1+r)]^{\frac{1}{\sigma}}$$

$$\Rightarrow c_1^* + \frac{c_1^*}{1+r}[\beta(1+r)]^{\frac{1}{\sigma}} = \dots \Rightarrow \boxed{c_1^* = \frac{a_0 + y_1 - \tau_1 + \frac{y_2-\tau_2}{1+r}}{1 + \frac{1}{1+r}[\beta(1+r)]^{\frac{1}{\sigma}}}}$$

$$c_2^* = c_1^*[\beta(1+r)]^{\frac{1}{\sigma}}; \quad a^* = a_0 + y_1 - \tau_1 - c_1^*$$

P2 $\frac{c_1}{y_1}(y_2)$? What would happen if HHs become optimistic about their future? $y_2 \uparrow$

$$\frac{c_1^*}{y_1} = \frac{1}{y_1}\,\frac{(1+r)[a_0+y_1-\tau_1] + (y_2-\tau_2)}{(1+r) + [\beta(1+r)]^{\frac{1}{\sigma}}} \longrightarrow y_2\uparrow \left(\frac{c_1^*}{y_1}\right)\uparrow$$

P3 $\frac{c_1}{y_1}(a_0)$? $\rightsquigarrow a_0\uparrow \left(\frac{c_1}{y_1}\right)\uparrow$

P4 $\frac{c_1}{y_1}(\beta)$? $\rightsquigarrow \beta\uparrow \Rightarrow$ More patient; $\left(\frac{c_1}{y_1}\right)\downarrow$

P5 $y_2 = \tau_1 = \tau_2 = 0$. $\frac{\partial c_1}{\partial r} = ?$ $\frac{\partial c_1}{\partial r}(\sigma)$?

$$c_1^* = \frac{(1+r)(a_0+y_1)}{(1+r)+[\beta(1+r)]^{\frac{1}{\sigma}}} = \frac{(a_0+y_1)}{1+\beta^{\frac{1}{\sigma}}\cdot(1+r)^{\frac{1}{\sigma}-1}}$$

$$\frac{\partial c_1^*}{\partial r} = \frac{-(a_0+y_1)\cdot\beta^{\frac{1}{\sigma}}\cdot\left(\frac{1}{\sigma}-1\right)(1+r)^{\frac{1}{\sigma}-2}}{\left[1+\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}-1}\right]^2} \rightsquigarrow \text{sign is the same as that of } \left(1-\frac{1}{\sigma}\right)$$

$\sigma \to 1$ $\quad U(c) \to \ln c$ & $\partial c_1^*/\partial r = 0$

Q2 $U(c_1, c_2) = \log c_1 + \beta \log c_2$, $r > 0$, $0<\beta<1$

$y_2 = y$, $y_1 = 0$; $BC = \{c_1 - a = 0,\ c_2 = y - (1+r)a\}$

borrowed

P1 $c_1^*, c_2^*, a^* = ?$ $\quad c_2 = y-(1+r)c_1 \Rightarrow c_1 + \frac{c_2}{1+r} = \frac{y}{1+r}$

$$\frac{P_{c_1}c_1}{m} = \frac{1}{1+\beta}, \quad P_{c_1}=1,\ P_{c_2} = \frac{1}{1+r}, \quad m = \frac{y}{1+r}, \quad \frac{P_{c_2}c_2}{m} = \frac{\beta}{1+\beta}$$

$$\Rightarrow c_1^* = \frac{y}{(1+r)(1+\beta)} = a^*, \quad c_2^* = \frac{\beta}{1+\beta}\frac{y}{1+r}\cdot(1+r) = \frac{y\beta}{1+\beta} = c_2^*$$

P2 $\frac{c_2}{c_1} = \beta(1+r)$, $\beta\uparrow \Rightarrow \frac{c_2}{c_1}\uparrow \Leftarrow r\uparrow$

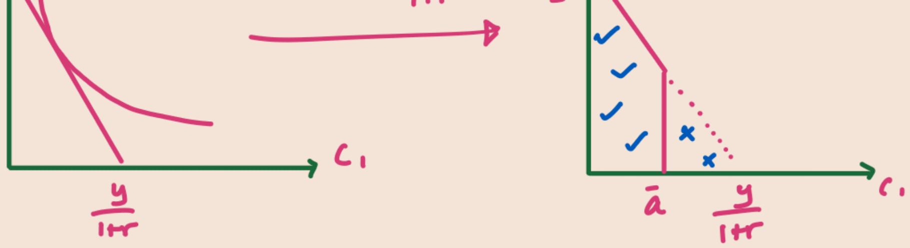
P3 Max. you can

$\bar{a} < \frac{y}{1+r}$

borrow: $\bar{a}$

If $\bar{a} \geq \frac{y}{1+r}$

$\Rightarrow$ No effect



P4 $\bar{a} = \frac{y}{2(1+r)}$ Is $\bar{a} \geq a^*$? $\Leftrightarrow \frac{y}{2(1+r)} \geq \frac{y}{(1+\beta)(1+r)} \Leftrightarrow 1+\beta \geq 2$ $\Rightarrow\Leftarrow$

$\bar{a} < a^* \Rightarrow c_1^* = a^* \rightsquigarrow c_1^* = \bar{a} = \frac{y}{2(1+r)}$; $c_2^* = y - (1+r)c_1^* = \frac{y}{2}$

Q3 $U(c_1, c_2) = \log(c_1) + \beta \log\left(\frac{c_2}{c_1^{\theta}}\right)$, $\theta \in [0,1]$. y_1, y_2, r

P1 $\frac{c_2}{c_1}(\theta)$?

$U(c_1, c_2) = (1-\beta\theta)\log c_1 + \beta \log c_2$ $\qquad c_2^* = \frac{\beta}{1-\beta\theta}\left[y_1 + \frac{y_2}{1+r}\right](1+r)$

$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \longrightarrow c_1^* = \frac{1-\beta\theta}{1-\beta\theta+\beta} \cdot \left[y_1 + \frac{y_2}{1+r}\right]$

$P_{c_1} = 1$, $P_{c_2} = \frac{1}{1+r}$. $\quad \frac{c_2^*}{c_1^*} = \frac{\beta(1+r)}{1-\beta\theta}$; $\theta \uparrow$ $1-\beta\theta \downarrow$ $\frac{c_2^*}{c_1^*} \uparrow$

P2 FOC (first order conditions) if govt. taxes interests at τ.

$$\frac{c_2^*}{c_1^*} = \frac{\beta[1 + r(1-\tau)]}{1-\beta\theta}$$

Q4 $U(c_1, c_2) = \log c_1 + \beta \log c_2$, $\quad c_1 + B = Y_1 - T_1$

$c_2 = Y_2 + (1+R)B - T_2$

P1 c_1, c_2? $\quad c_1 + \frac{c_2}{1+R} = Y_1 - T_1 + \frac{Y_2 - T_2}{1+R}$

$c_1^* = \frac{1}{1+\beta}\left[Y_1 - T_1 + \frac{Y_2 - T_2}{1+R}\right]$, $\quad c_2^* = \beta \cdot \frac{1+R}{1+\beta} \cdot \left[Y_1 - T_1 + \frac{Y_2 - T_2}{1+R}\right]$

P2 1) $\Delta T_1 = G_1$

2) Take loan of G_1 & raise tax in 2nd period

How high does tax change need to be in 2nd period?

$\Delta T_2 = G_1(1+R)$

P3 How do c_1, c_2 change w/ 1) / 2)?

1) $c_1 + \frac{c_2}{1+R} = Y_1 - T_1 - \Delta T + \frac{Y_2 - T_2}{1+R} = Y_1 - T_1 - G_1 + \frac{Y_2 - T_2}{1+R}$

2) $c_1 + \frac{c_2}{1+R} = Y_1 - T_1 + \frac{Y_2 - T_2 - \Delta T}{1+R} = Y_1 - T_1 + \frac{Y_2 - T_2}{1+R} - G_1$

BC same $\Rightarrow$ c_1^*, c_2^* same in 1) & 2)

Q5 $u = \log c_1 + \log c_2$. young $\rightarrow$ w wage, old $\rightarrow$ 0, r

P1 Level of saving? $\rightsquigarrow$ $c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} = w + 0 = w$

$c_1^* = \frac{1}{1+1} \cdot \frac{w}{1} = \frac{w}{2}$; $s = y_1 - c_1^* = w - \frac{w}{2} = \frac{w}{2}$

P2 $s(r)$? $\rightsquigarrow$ r has no effect on s

Generally the case when u has log functions.

Q6 Agent $\rightarrow$ 3 periods $\rightarrow$ consume in periods 2 & 3

$u(c_2, c_3) = \log c_2 + \beta \log c_3$

T_1 : Invests e, borrowed at r

h is a concave function of e

T_2 : $w\, h(e)$, $h' > 0,\ h'' < 0$, repay loan from T_1

T_3 : No income, can save s in 2nd period $\rightarrow$ $s(1+r)$

P1 BC in T_2 & T_3 $\rightarrow$ $s(1+r) = c_3$

$e(1+r) + s + c_2 = w \cdot h(e)$

P2 Set up maximization problem w/ her choice variables.

$$\max_{c_2, c_3, e} \log c_2 + \beta \log c_3 \quad \text{s.t.} \quad c_2 + \frac{c_3}{1+r} = w h(e) - e(1+r)$$

Max. this so that c_2 & c_3 are also max.

P3 FOC for the maxi problem?

$\left.\begin{aligned} 1/c_2 &= \lambda \\ \beta/c_3 &= \lambda/1+r \end{aligned}\right\}$ From Lagrangian

From R.H.S. $\{ w \cdot h'(e) = 1+r$

$\Rightarrow \frac{c_3}{c_2} = \beta(1+r)$

P4 $\frac{c_2}{c_3} = \frac{1}{\beta(1+r)}$

P5 $e(\beta) \rightsquigarrow$ No effect of β on e

$\because w \cdot h'(e) = 1+r$

β only affects dist. of c_2 & c_3.

Q7 Solow-Swan model w/ CD prod. $y = k^\alpha$. s, δ, g, n.

P1 k^*. y^*, c^*

$\dot{k}(t) = s f(k(t)) - (\delta + g + n) k(t)$

Steady-state → $\dot{k}=0$ ↳ $s\cdot k^{\alpha}-(\delta+g+n)k=0$

$$\Rightarrow k^{*}=\left(\frac{s}{\delta+g+n}\right)^{\frac{1}{1-\alpha}} \Rightarrow y^{*}=\left(\frac{s}{\delta+g+n}\right)^{\frac{\alpha}{1-\alpha}}$$

$$c^{*}+S^{*}=y^{*}=c^{*}+s\,y^{*} \Rightarrow c^{*}=(1-s)\,y^{*}=(1-s)\left(\frac{s}{\delta+g+n}\right)^{\frac{\alpha}{1-\alpha}}$$

P2 $s\uparrow$ y^{*}, k^{*}, c^{*} ? For c^{*} ⟶ focus on $(1-s)s^{\frac{\alpha}{1-\alpha}}$

$s\uparrow \Rightarrow k^{*}\uparrow\ y^{*}\uparrow$

for c^{*}, $\frac{\partial}{\partial s}$ ⟶ $-s^{\frac{\alpha}{1-\alpha}}+\frac{(1-s)}{s}\cdot\frac{\alpha}{1-\alpha}\cdot s^{\frac{\alpha}{1-\alpha}} = s^{\frac{\alpha}{1-\alpha}}\left[\frac{\alpha}{1-\alpha}\cdot\frac{1}{s}-\frac{\alpha}{1-\alpha}-1\right]$

$$= s^{\frac{\alpha}{1-\alpha}}\left[\frac{\alpha}{1-\alpha}\cdot\frac{1}{s}-\frac{1}{1-\alpha}\right]=\frac{s^{\frac{\alpha}{1-\alpha}}}{s(1-\alpha)}\left[\alpha-s\right]$$

$s\uparrow$ $c^{*}\uparrow$ if $s<\alpha$, $c^{*}\downarrow$ if $s>\alpha$

P3 α on y^{*}, k^{*}, c^{*} ?

$\uparrow$ in $\alpha \Rightarrow \frac{1}{1-\alpha}\uparrow \Rightarrow k^{*}\uparrow$ iff $\frac{s}{\delta+g+n}>1$

$\Rightarrow \frac{\alpha}{1-\alpha}\uparrow \Rightarrow y^{*}\uparrow$ iff $\frac{s}{\delta+g+n}>1$

$c^{*}=(1-s)\,y^{*}$ ⟶ $c^{*}, y^{*}\uparrow$ if $k^{*}\uparrow$; $k^{*}\uparrow$ if $\frac{s}{\delta+g+n}>1$

Q8 $Y(t)=K(t)^{\alpha}R(t)^{\phi}\left(A(t)\,L(t)\right)^{1-\alpha-\phi}$, $0<\alpha, \phi<1$

$R(t)$ → Stock of Resources depleting at $\theta>0$, $\dot{R}(t)=-\theta R(t)$

A growth rate : g & N : n

P1 $g_Y(t)$ & $g_K(t)$ denote growth rates.

Take log → Diff. ⟶ $g_Y=\alpha\cdot g_K-\theta\phi+(1-\alpha-\phi)(g+n)$

P2 g_Y^{*} & g_K^{*} along a BGP. Show that $g_Y^{*}=g_K^{*}$.

Solve for this rate ↳ All growth rates constant.

$\dot{K}(t)=s\,Y(t)-\delta\,K(t)$

$\Rightarrow \frac{\dot{K}}{K}=s\frac{Y}{K}-\delta$ ⟶ LHS = const. ∵ BGP $\Rightarrow$ RHS = const. $\Rightarrow \frac{Y}{K}=$ const $\Rightarrow g_Y^{*}=g_K^{*}$

$$\Rightarrow g^{*}=\alpha g^{*}-\theta\phi+(1-\alpha-\phi)(g+n)$$

$$\Rightarrow g^{*}=\frac{1}{1-\alpha}\left[(g+n)(1-\alpha-\phi)-\theta\phi\right]$$

P3 Does the economy necessarily converge to a BGP?

$$\delta + g_K(t) = s\,\frac{Y(t)}{K(t)} \Rightarrow \frac{K(t)}{Y(t)} = \frac{s}{\delta + g_K(t)}, \quad x(t) \equiv \frac{K(t)}{Y(t)}$$

$$\Rightarrow x^* = \frac{s}{\delta + g^*} \qquad . \qquad g_K(t) > g^* \Rightarrow x(t) < x^*(t)$$

Also, $g_Y(t) - g^* = \alpha(g_K(t) - g^*) \Rightarrow g_K(t) - g_Y(t) = (1-\alpha)(g_K - g^*)$

& $x = K/Y \Rightarrow g_x = g_K - g_Y$

So, $g_K(t) > g^* \Rightarrow x(t) < x^*$

$\hookrightarrow g_K(t) > g_Y(t) \Rightarrow g_x(t) > 0 \Rightarrow x(t) \uparrow$

And if, $g_K(t) < g^* \Rightarrow x(t) > x^*$

$\hookrightarrow g_K(t) < g_Y(t) \Rightarrow g_x(t) < 0 \Rightarrow x(t) \downarrow$

In both cases, $x \to x^*$ $\Rightarrow g_K(t) \to g^*$

So, economy does converge towards BGP.

P4 $\dot{R} = nR$

$$g^*_{New} = g^\wedge = \frac{1}{1-\alpha}\left[(1-\alpha-\phi)(g+n) + \phi n\right]$$

$$\text{Gap} = \Delta g^* = g^\wedge - g^* = \frac{\phi(n-\theta)}{1-\alpha}$$

Q9 $u(c_{1,t}, c_{2,t+1}) = \log(c_{1,t}) + \beta \log(c_{2,t+1}), \quad 0<\beta<1$

1st period $\to W_t \to$ B.C. : $c_{1,t} + s_t = W_t$

2nd period $\to 0 \to$ B.C. : $c_{2,t+1} = (1+r_{t+1})s_t$

P1 $\max_{s_t} \log[W_t - s_t] + \beta \log[s_t(1+r_{t+1})]$

$\Leftrightarrow \log[W_t - s_t] + \beta \log[s_t]$

P2 $\frac{-1}{W_t - s_t} + \frac{\beta}{s_t} = 0 \Rightarrow s_t = \frac{\beta}{1+\beta} W_t \Rightarrow s_t$ does not depend on r_{t+1}

$Y_t = A K_t^\alpha L_t^{1-\alpha}, \quad 0<\alpha<1$

P3 $W_t(K_t) = \frac{\partial Y_t}{\partial L_t} = A\cdot(1-\alpha)\cdot K_t^\alpha L_t^{-\alpha}\Big|_{L_t = 1 \text{ (given)}} = \boxed{A(1-\alpha)K_t^\alpha = W_t}$

P4 θ of savings lost. $K_{t+1}(K_t)$, $\delta = 1$.

$K_{t+1} = (1-\delta)K_t + I_t = 0 + (1-\theta)s_t = (1-\theta)\frac{\beta}{1+\beta}W_t$ (from 2)

$K_{t+1} = (1-\theta)\cdot\frac{\beta}{1+\beta}\cdot A\cdot(1-\alpha)\cdot K_t^\alpha$

P5 Steady state K?

$$K_{t+1} = K_t = K^* \Rightarrow K^* = \left[(1-\theta)\cdot\frac{\beta}{1+\beta}\cdot A\cdot(1-\alpha)\right]^{\frac{1}{1-\alpha}}$$

P6 $K(\theta)$? $\theta \uparrow$ $K^* \downarrow$.

Open Economy

Open Economy → 1. Openness in Goods Markets
2. Openness in Financial Markets
3. Openness in Factor Markets

$\left\{\frac{\text{Export}}{\text{GDP}}, \frac{\text{Import}}{\text{GDP}}\right\}$

Goods → Tradables
Goods → Non-Tradables

Some Jargon [Goods Markets]

1) Real Exchange Rate: Price of domestic goods relative to foreign goods
2) Nominal Exchange Rate: Price of domestic currency in terms of foreign currency
3) (Nominal) Appreciation: An increase in the price of domestic currency in terms of a foreign currency; ↑ in exchange rate
4) (Nominal) Depreciation: A decrease in the price of domestic currency in terms of a foreign currency → decrease in exchange rate
5) Fixed Exchange Rate: A system in which two or more countries maintain a constant exchange rate b/w their currencies
6) In fixed exchange rate system, revaluations are increases in the exchange rate and devaluations are decreases in the exchange rate
7) (Real) Appreciation: Increase in real exchange rate
8) (Real) Depreciation: Decrease in real exchange rate

☆ Real Ex. Rate: 1 unit of domestic good ⇒ ϵ units of foreign goods

☆ Nominal Ex. Rate: 1 unit of domestic currency ⇒ E units of foreign currency

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Rel. b/w ϵ & E → $\boxed{\epsilon = EP/P^*}$

Method 1: 1 unit of domestic good $\longrightarrow$ ϵ units of foreign goods

M2: 1 unit of dom. good $\qquad$ EP/p^* units of for. goods

Domestic Price level

Foreign Price level

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\uparrow$

P units of dom. curr. $\longrightarrow$ EP units of for. curr.

Some Jargon [Financial Markets]

1) Foreign Exchange: Buying & selling foreign currency
2) Balance of Payments: A set of accounts that summarize a country's transactions w/ the rest of the world. (BOP)
3) Current account: Transactions above the line $\rightarrow$ records payments to and from the rest of the world (CA)

3A) Exports and imports of goods and services (trade balance)

3B) Net income balance b/w income received from rest of world and income paid to foreigners.

3C) Net transfer received: Difference in foreign aid given and received

4) Current account balance: Sum of net payments to and from the world
5) Current account surplus: Positive net payments from the world.
6) Current account deficit: Negative net payments from the world
7) Capital account ($\rightarrow$ Financial account): Transactions below the line $\rightarrow$ records net foreign holdings of domestic assets.
8) Net capital inflows / Capital account balance: Increase in net foreign indebtedness (holdings of domestic assets minus increase in domestic holdings of foreign assets)
9) Capital account surplus/deficit
10) Statistical Discrepancy + Current + Capital = 0

Account (Financial) Account

Quiz - 2

Q1 Cost = 100,000 , Profits = 0, 20k, 20k$(1-\delta)$, 20k $(1-\delta)^2$, ...
$\delta = 0.1$, $r = ?$ so can buy?

$$NPV = -100k + \frac{20k}{1+r} + \frac{20k(1-\delta)}{(1+r)^2} + \dots = -100k + \frac{20k}{1+r} \cdot \frac{1}{1-\frac{1-\delta}{1+r}}$$

$$= \frac{20k}{r+\delta} - 100k \qquad NPV \geq 0 \iff \frac{20k}{r+0.1} \geq 100k \Rightarrow r \leq 0.1$$

$$\boxed{r = 0.10}$$

Q2 50 → Curr. Wealth = 5000; till 65 → 1000
After 65 → sell for 10000; live till 75; $r = 0 = r_i^e$
Wants to leave 5000 behind

$$\frac{5k + 15 \cdot 1k + 10k - 5k}{25} = 1k = \boxed{1000}$$

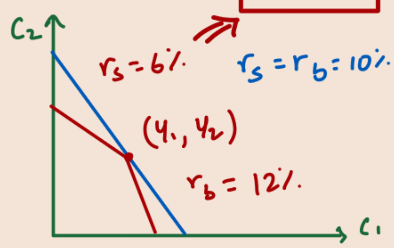
Q3 $U = \sqrt{c_1 c_2}$, r, Y_1, Y_2, can't borrow, $Y_1 > \frac{Y_2}{1+r}$, $c_2^* = ?$

$$c_1^* = \frac{1}{2}\left[Y_1 + \frac{Y_2}{1+r}\right] < Y_1 \Rightarrow c_2^* = \frac{Y_1(1+r) + Y_2}{2}$$

Q4 $GD_{2022} = 110$, $GD_{2021} = 100$, $BS_{2022} = 20$, $r_B = ?$

$$B_t = (1+r)B_{t-1} + G_t - T_t \Rightarrow 110 = (1+r)100 - 20 \Rightarrow r = 30\%$$

Q5 $5000 = Y_t$, $Y_{t+1} = 9000$; Bonds → r_s, Borrow → r_b
If $r_s = r_b = 10\%$ then $C_t = 5000$, $C_{t+1} = 9000$ $\boxed{c_2' = 9k}$
Now, $r_s = 6\%$, $r_b = 12\%$. Then C_{t+1} ?



Q6 W young, 0 old, r, $U = \frac{\log c_1}{5} + \frac{4 \log c_2}{5}$
s^* ?

$$U = \frac{\log(W-s)}{5} + \frac{4\log s(1+r)}{5} \longrightarrow \frac{dU}{ds} = 0 \Rightarrow \frac{4}{5s} - \frac{1}{5(W-s)} = 0$$

$$\Rightarrow 4W - 4s = s \Rightarrow \boxed{s = \frac{4W}{5}}$$

Q7,8 $Y_t = \sqrt{K_t A_t N_t}$, $N_t = N$, $A_t = A$
$d, s \in (0,1)$ $\qquad y_t = \sqrt{k_t}$

$$k_{t+1} = (1-d)k_t + s y_t \Rightarrow k_{t+1} = k_t = k^* \Rightarrow \boxed{k^* = \left(\frac{s}{d}\right)^2}$$

s^* ? for c max

$$\frac{C_t}{A_t N_t} = (1-s)\,\frac{Y_t}{A_t N_t} = (1-s)\sqrt{k_t} \Rightarrow \frac{C_t}{N} = A(1-s)\cdot\frac{s}{d} \Rightarrow \boxed{s = \tfrac{1}{2}}$$

Q9 $Y = A\,F(K, L) = A\,L^{\alpha}K^{\beta}$, $F \to CRS \to \alpha+\beta = 1$

$w = \partial Y/\partial L$, $r = \partial Y/\partial K$, $wL = rK \longrightarrow \alpha = \beta \longrightarrow \alpha = \beta = \frac{1}{2}$

$Y = A\sqrt{KL}$, $g_L = 4\%$, $g_K = 3\%$, $g_Y = 9\%$, $g_A = ?$

$$g_A = g_Y - \tfrac{1}{2}(g_K + g_L) = 9 - \tfrac{1}{2}(4+3) = \underline{5.5\%}$$

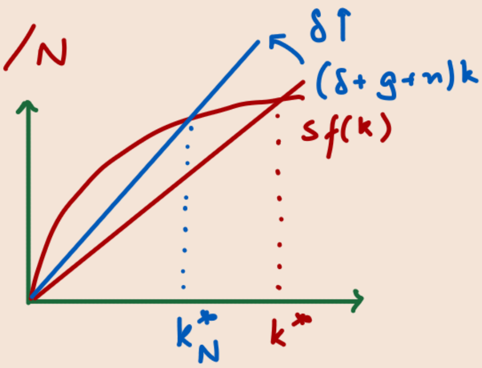
Q10 Solow Model $\rightsquigarrow$ Cts. time, tech. progress

$\delta\uparrow \longrightarrow k^* ?$

$\dot{K} = I - \delta K$, $\dot{k}/k = \dot{K}/K - \dot{A}/A - \dot{N}/N$

$$\frac{\dot{k}}{k} = \frac{sY}{K} - (\delta + g + n)$$

$\Rightarrow \dot{k} = s f(k) - (\delta+g+n)k \Rightarrow \delta\uparrow\ k^*\downarrow$



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* GNP = GDP + NI

* Domestic Bond $\longrightarrow i_t$
  Foreign Bond $\longrightarrow i_t^*$

1 unit of domestic currency $\longrightarrow$ $1+i_t$ units of domestic currency Next year

$\downarrow$

E_t units of foreign currency $\longrightarrow$ $E_t(1+i_t^*)$ units of foreign curr. $\longrightarrow$ $\frac{E_t(1+i_t^*)}{E^e_{t+1}}$ units of foreign currency

$$1+i_t = \frac{E_t(1+i_t^*)}{E^e_{t+1}} \Leftarrow$$

$\hookrightarrow$: (Uncovered) Interest Rate Parity :

$$E_t = \frac{(1+i_t)}{(1+i_t^*)}E^e_{t+1}$$

$$1+i_t = \frac{1+i_t^*}{1+\frac{E^e_{t+1}-E_t}{E_t}} \Rightarrow \boxed{i_t \approx i_t^* - \left(\frac{E^e_{t+1}-E_t}{E_t}\right)}$$

Open Economy

Goods Market $\rightsquigarrow$ Y = Output = $C(Y-T) + I + G$ (SKM)

$\llcorner$ Domestic demand $\longrightarrow$ − imports, + exports

$\llcorner$ Demand for

$Y^* \longrightarrow$ Foreign Demand

Net Exports/ Trade Balance

domestic goods

$\longrightarrow Y = C + I + G + X - \frac{M}{\epsilon}$, with $C \to C(Y-T)$, $I \to \bar{I}(Y)$, $G \to \bar{G}$

$NX = X - \frac{M}{\epsilon}$

$X(\underset{+}{Y^*}, \underset{-}{\epsilon})$

$M(\underset{+}{Y}, \underset{+}{\epsilon})$

$\left.\right\} NX(\underset{-}{Y}, \underset{+}{Y^*}, \underset{?}{\epsilon})$ Not clear

☆ $NX(\underset{-}{Y}, \underset{+}{Y^*}, \underset{-}{\epsilon})$ Under :Marshall-Lerner: Conditions

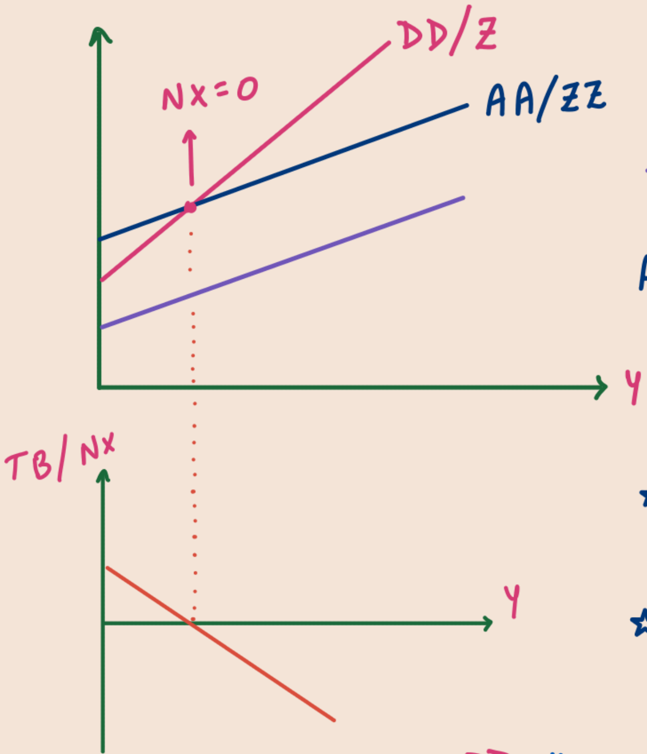
Suppose $NX = X - \frac{M}{\epsilon} = 0$ initially

Suppose $\to \epsilon \to \Delta\epsilon < 0 \to \epsilon NX = \epsilon X - M \to \epsilon X = M$

$\to \cancel{(\Delta\epsilon) NX}^{\,0} + \epsilon(\Delta NX) = (\Delta\epsilon) X + \epsilon \Delta X - \Delta M$

$\frac{1}{\epsilon X} \Rightarrow \frac{\Delta NX}{X} = \underbrace{\frac{\Delta\epsilon}{\epsilon}}_{<0} + \underbrace{\frac{\Delta X}{X} - \frac{\Delta M}{M}}_{(>0)+(>0)}$ > 0 for most countries

Want these to outweigh $\Delta\epsilon/\epsilon$



DD: $Y = C + I + G$

—: $Y = C + I + G - \frac{M}{\epsilon}$

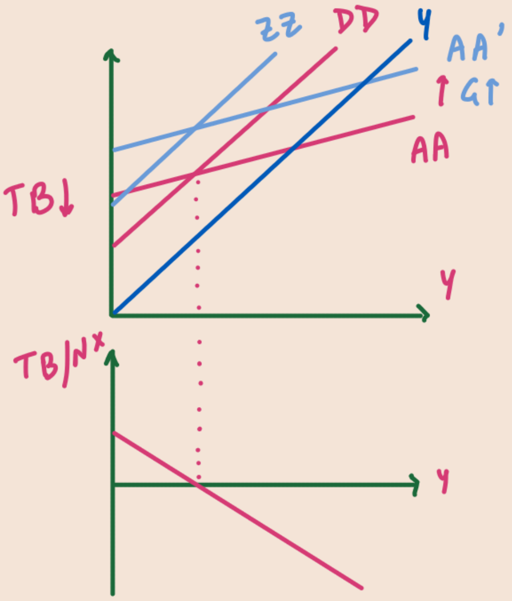
AA: $Y = C + I + G + X - \frac{M}{\epsilon}$

$\boxed{NX = AA - DD}$

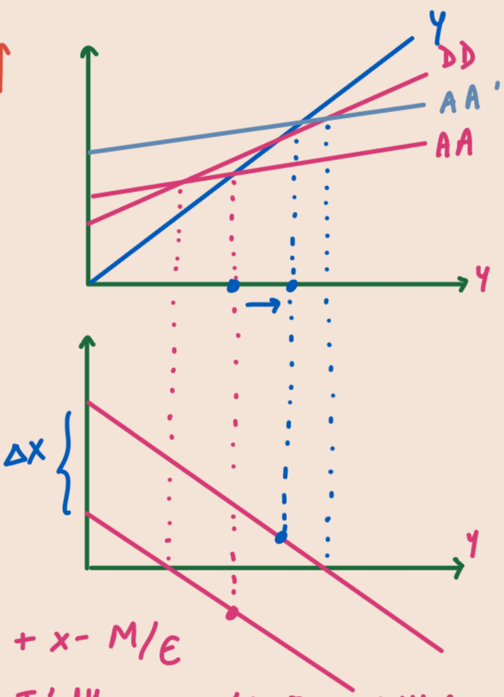
☆ $NX > 0 \leftrightarrow$ Exports > Imports $\leftrightarrow$ Trade Surplus

☆ $NX < 0 \leftrightarrow$ Exports < Imports $\leftrightarrow$ Trade Deficit

Ex. $G\uparrow$
$\downarrow$
$Y\uparrow \Rightarrow TB\downarrow$



Ex. $Y^*\uparrow$
$\downarrow$
$X\uparrow$
$\downarrow$
$AA\uparrow$
$\downarrow$
$Y\uparrow$
$\underbrace{TB\uparrow}$
$\downarrow$



$Y = C + I + G + X - M/\epsilon$

Ex. $\epsilon\downarrow \longrightarrow$ Similar to

$\downarrow$ $\quad Y^* \uparrow$

$AA\uparrow$ (under ML conditions)

$\hookrightarrow Y \uparrow$

$\hookrightarrow TB \uparrow$ ($\downarrow$)

$\Rightarrow dY = C'\,dY + I'\,dY + 0 + X'\,dY^* - M'\,dY/\epsilon$

& $NX = X - M/\epsilon$

$\Rightarrow dNX = X'\,dY^* - M'\,dY/\epsilon$

$= \underbrace{dY}_{>0}\underbrace{(1 - C' - I')}_{>0} > 0 \Rightarrow NX \uparrow$

{Related to slope}

Ex.

	Output too Low	Output too High
TB too low	$\epsilon\downarrow$ G?	$G\downarrow$ ϵ?
TB too high	$G\uparrow$ ϵ?	$\epsilon\uparrow$ G?

★ If a country depreciates ($\epsilon\downarrow\downarrow$) then in the short run, X & M fixed $\Rightarrow TB = X - M/\epsilon \rightsquigarrow TB\downarrow\downarrow$

Over time, TB improves, leading to a "J curve"

$Y = C + I + G + NX$

$\Rightarrow (Y - T - C) = I + (G - T) + NX$

$\Rightarrow (Y + NT + NI - T - C)$

$= I + (G - T) + (NX + NT + NI)$

$\Rightarrow S_{priv} = I - S_{pub} + CA$

$\Rightarrow \boxed{S_{priv} + S_{pub} = I + CA}$

Before $Y = C + I + G$

$Y - C - T = I + (G - T)$

$\Rightarrow S_{private} = I - S_{public}$

$\Rightarrow S_{priv} + S_{pub} = I$

Open Economy version of IS-LM

Mundell-Fleming Model

$IS \longrightarrow Y = C(Y - T) + I(Y, r) + G + NX(Y, Y^*, \epsilon)$

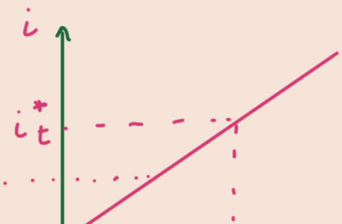
Prices fixed $\Rightarrow$ Inflation $= 0$, $\pi = 0 \Rightarrow r = i$

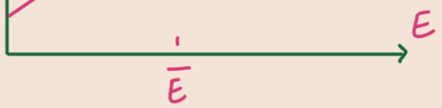
$\Rightarrow \epsilon = E\,P/P^* \Rightarrow$ Replace ϵ by E

$Y = C(Y - T) + I(Y, i) + G + NX(Y, Y^*, E)$

$LM \longrightarrow i = \bar{i}$

$$E_t = \frac{1 + i_t}{1 + i_t^*} E^e_{t+1} = \bar{E}\,\frac{1 + i_t}{1 + i_t^*}$$





Ex. $\bar{i}\uparrow \Rightarrow LM\uparrow \Rightarrow Y\downarrow$; $\Rightarrow \bar{E}\uparrow$

Ex. $G\uparrow \Rightarrow IS\uparrow \Rightarrow Y\uparrow$; $\bar{E} \leftrightarrow \epsilon$; $C\uparrow, I\uparrow, NX\downarrow$
(shift rightward) $\quad Y_0 \longrightarrow \uparrow Y_1$ say

Ex. $G\uparrow$, $i=\bar{i}\uparrow$ so that $Y_0 < Y_2 < Y_1$
$Y\uparrow$, $i\uparrow$, $E\uparrow$, $C\uparrow$, $I\updownarrow$, $G\uparrow$, $NX\downarrow$

☆ Govt's Disposal $\longrightarrow$ Fiscal, Monetary, Exchange Rate policy

Fixed v/s Flexible Exchange Rate $\longrightarrow$ Govt. No longer has
1) Exchange Rate policy (ϵ)
2) Monetary policy ($E(i)$)
Only Fiscal policy (G, T) can be used

- Assume $E_t = E^e_{t+1} = \bar{E}$ then $1+i = \dfrac{1+i^*_t}{E^e_{t+1}} E_t \Rightarrow i = i^*_t$

Now
$Y = C(Y-T) + I(Y, r) + G + NX(\epsilon, Y, Y^*) \longleftarrow IS$; $\epsilon = \dfrac{EP}{P^*}$
r: $i^* - \pi^e$ ($\because i = i^*$)

☆ $Y_t = Y_n$ for both $\quad \pi_t - \bar{\pi} = \dfrac{\alpha}{L}(Y - Y_n) \Rightarrow \pi_t = \bar{\pi} \longrightarrow \epsilon$ fixed (constant)

☆ $Y_t < Y_n$ & $TB < 0$ for dom. curr. " $\Big\}$ $\pi_t < \pi^* = \pi^*_t \Rightarrow \dfrac{P}{P^*}\downarrow \Rightarrow \epsilon\downarrow$
$\Rightarrow TB\uparrow$ & $Y\uparrow$

Additional Problem Set 4

Q1 Y1 $\longrightarrow$ Prod of 750, 150 exported, rest unsold
Y2 $\longrightarrow$ No prod, unsold from Y1 sold domestically, Imports of 250

	GDP	Cons.	Inv.	Export	Import
1	750	0	600	150	0
2	0	850	-600	0	250

Investment $\equiv$ Inventory

Cons $\rightsquigarrow$ Overall, inc. of both

Q2 SKM w/o govt. w/ open eco. $C, M \propto Y$.
Avg. propensities to C & M are 0.8 & 0.3
$I = 100 + 0.4Y$, $X = 100$

P1 Agg. demand func. if max $M = 450$. $\exists$ eq^m?

$Y = C + I + X - M$, $C = 0.8Y$, $M = \min\{0.3Y, 450\}$
$\downarrow$, $I = 100 + 0.4Y$

1) $M_{eq} < 450 \Rightarrow 0.3Y < 450 \Rightarrow Y < 1500$; $M = 0.3Y$

$Y = 0.8Y + 100 + 0.4Y + 100 - 0.3Y = 200 + 0.9Y \Rightarrow Y_{eq} = 2000 \Rightarrow \Leftarrow$

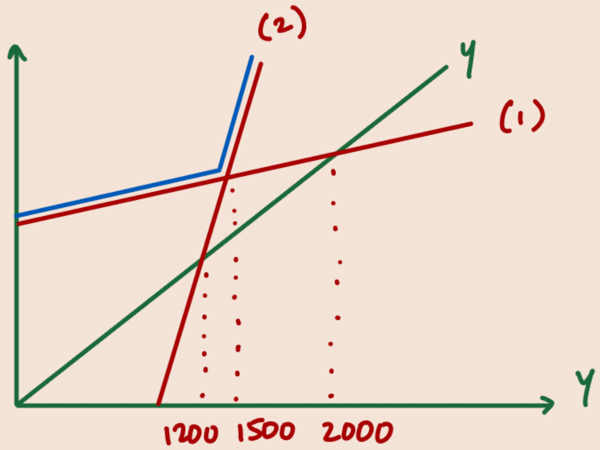
2) $0.3Y > 450 \Rightarrow Y > 1500$

$Y = 100 + 0.4Y + 0.8Y + 100 - 450 = 1.2Y - 250 \Rightarrow Y_{eq} = 1250 \Rightarrow \Leftarrow$

$\nexists$ an equilibrium

Graphically:

No int. b/w / & _/



P2 $M = \min\{0.3Y, 615\}$, Stability of eq^m?

1) $0.3Y < 615 \Rightarrow Y < 2050$

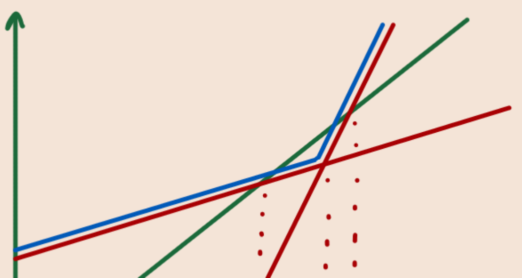
$Y = 0.8Y + 100 + 0.4Y + 100 - 0.3Y \Rightarrow Y^* = 2000$ ✓

2) $0.3Y > 615 \Rightarrow Y > 2050$

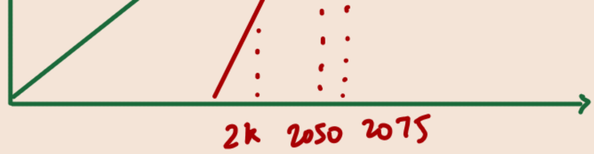
$Y = 0.8Y + 100 + 0.4Y + 100 - 615 \Rightarrow Y^* = 2075$ ✓

$\exists$ 2 eq^m

In case of st. lines, if



locally $\begin{cases} \text{Slope} < 1 \Rightarrow \text{Stable (2000)} \\ \text{Slope} > 1 \Rightarrow \text{Unstable (2075)} \end{cases}$



Globally, both unstable

Q3 ISLM : · M/X ✓ but no foreign capital transactions

· e flexible · P^*, P given exogenously.

· No capital mobility & e adjusted to balance trade in eq^m. $TB = \bar{T} + \frac{\beta P^*}{P} - mY$, $\bar{T} > 0$, Y GDP, β, m parameters. m is marginal propensity to import

Foreigners cannot buy domestic asset.

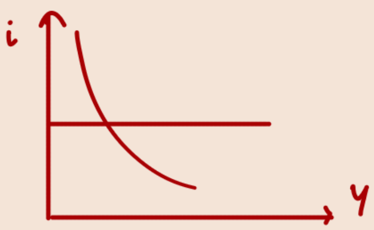
CA = 0 & others 0 $\Rightarrow$ TB = 0

P1 TB eq^m & Commodity market eq^m. Y v/s r?

Same as IS?

$Y = C + I + G + TB$, $TB = 0$ {e changes accordingly}

$\Rightarrow Y = C + I + G = C(Y - T) + I(Y, r) + G \Rightarrow$ Same as IS



P2 G↑ Y & e?

↳ Y↑ ⇝ TB↓ so e↑

P3 P^* ↑ Y & e? Y ⟷ e↑

Q4 Indian exporter → Sell 100 units at \$10/unit.

Cost of each pen → ₹9. Spot exchange today $E_{₹/\$} = 1$. Only cares about expected returns.

P1 Payment received instantaneously, should he export?

Profit/Loss?

Revenue = 100 · 10 \$ = 1000 \$ = ₹1000

Cost = 100 · 9 ₹ = ₹900 ⟶ Profit = ₹100 → export

Payment after 1 month now

P2 Knows for sure that $E_{₹/\$} = 1.1$. Export today.

Cost $=$ ₹ 900 $\longrightarrow$ Profit $=$ ₹ 200 $\longrightarrow$ Should Export

Revenue $= 1000\,\$ = ₹\,1100$

P3 $E_{₹/\$} = 1.4$ w/ $p = 1/2$ & 0.6 w/ $1/2$

Expected Revenue $= \frac{1}{2} \cdot 1400 + \frac{1}{2} \cdot 600 = 1000$

" Profit $= +100 \longrightarrow$ Should export

P4 1.4 w/ $p = 1/3$ & 0.6 w/ $2/3$

Expected Revenue $= 1/3 \cdot 1400 + 2/3 \cdot 600 = 2600/3$

" Profit $= 2600/3 - 900 < 0 \longrightarrow$ Should NOT export

P5 Can buy a forward contract for $E_{₹/\$} = 1$ after one month. Same as (5), max. amount he is willing to pay for the contract?

Revenue $\longrightarrow 1000$, Cost $\longrightarrow 900$

Cost of contract $\longrightarrow c$

$1000 - c - 900 \geq 0 \longrightarrow \underline{c \geq 100}$