

DDEs With Simplified System

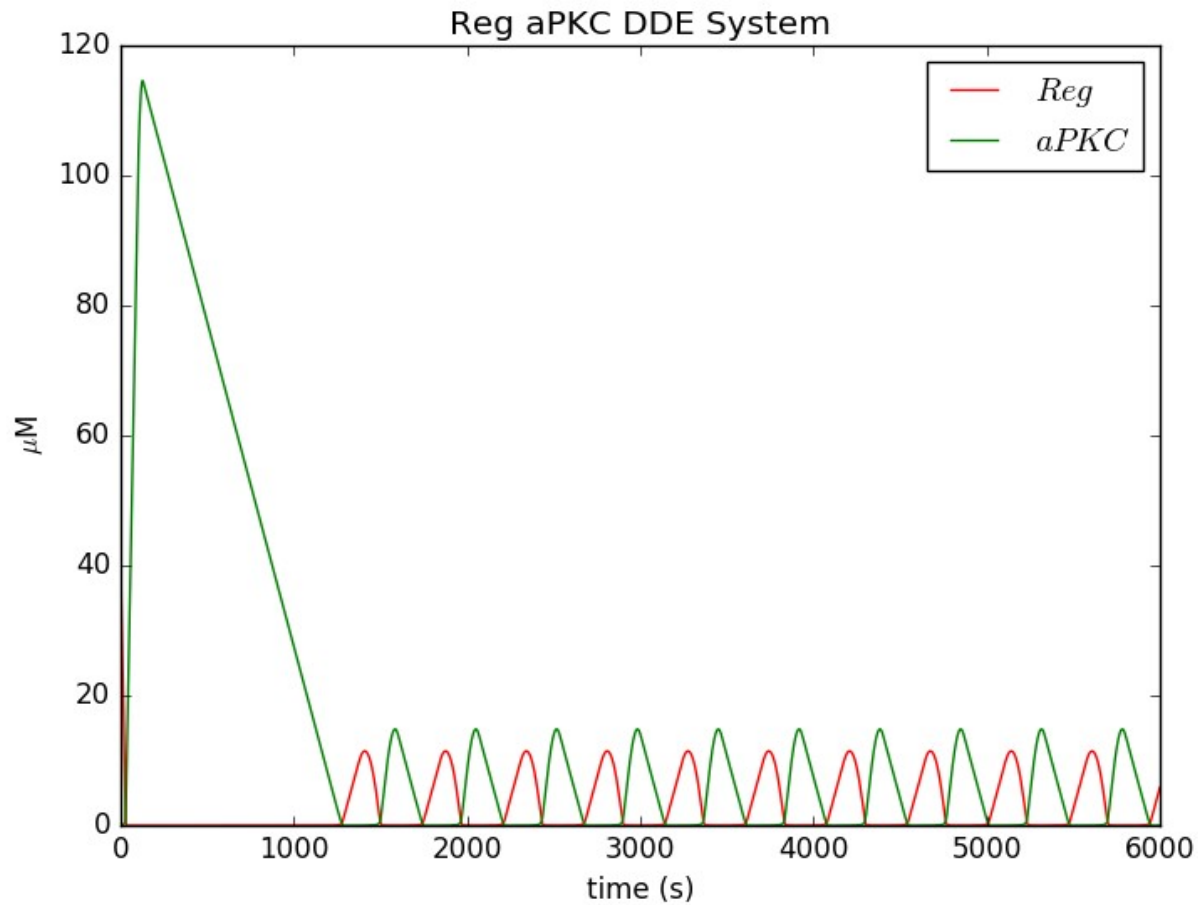
k_1 = Birth rate of aPKC (also used is delayed recruitment)

k_2 = mutual antagonism

k_3 = birth rate of Reg

k_4 = decomposition of AR

Simplified model. Ignoring AR complex production and migration of aPKC.

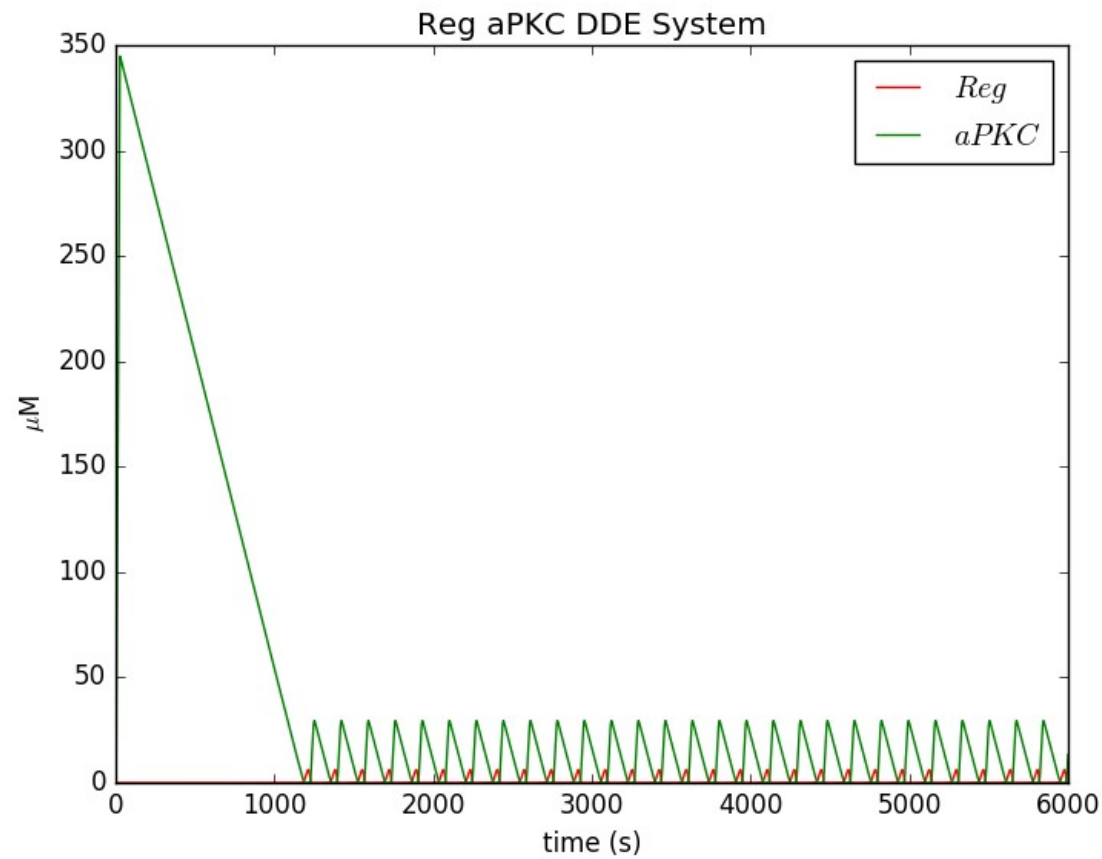


The parameters used were:

```
{  
  "k1": 0.03,  
  "k2": 0.5,  
  "k3": 0.1,  
  "tau1": 100.0  
}
```

The initial conditions used were:

```
{  
  "A": 10.0,  
  "R": 50.0  
}
```



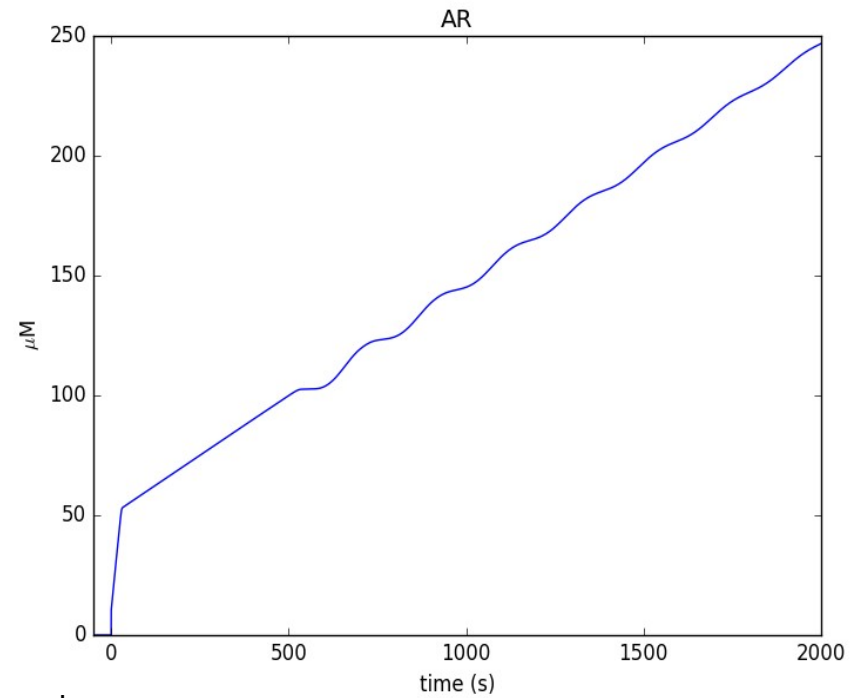
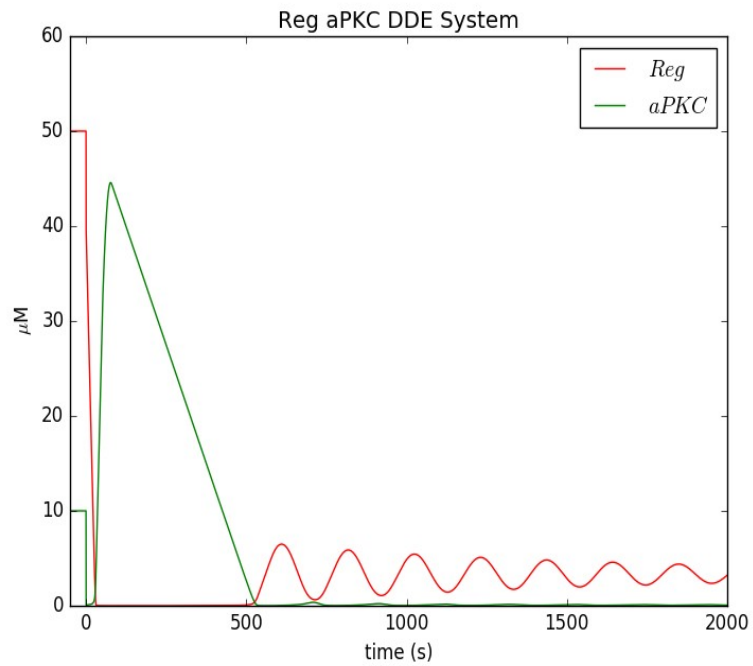
The parameters used were:

```
{  
  "k1": 0.3,  
  "k2": 0.5,  
  "k3": 0.3,  
  "tau1": 25.0  
}
```

The initial conditions used were:

```
{  
  "A": 10.0,  
  "R": 50.0  
}
```

Simplified Model: AR complex now included.



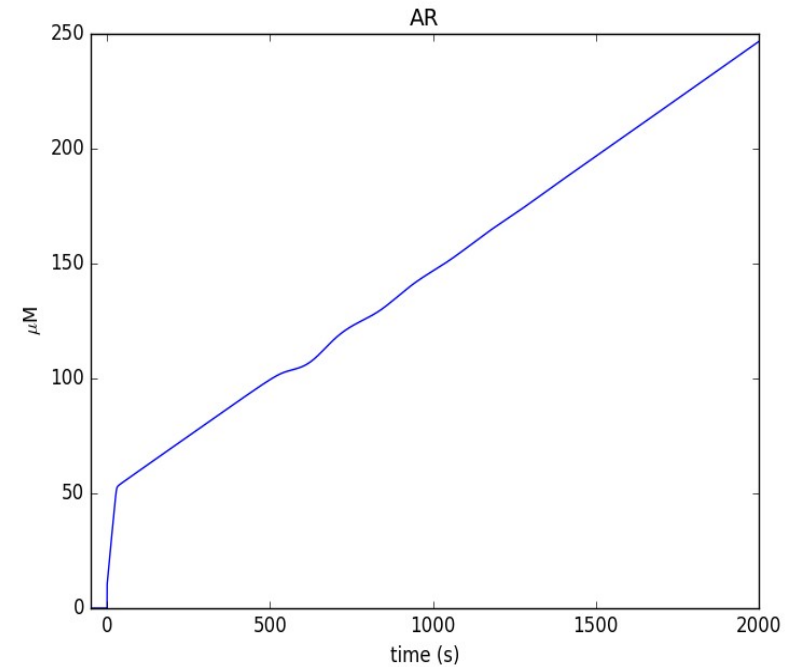
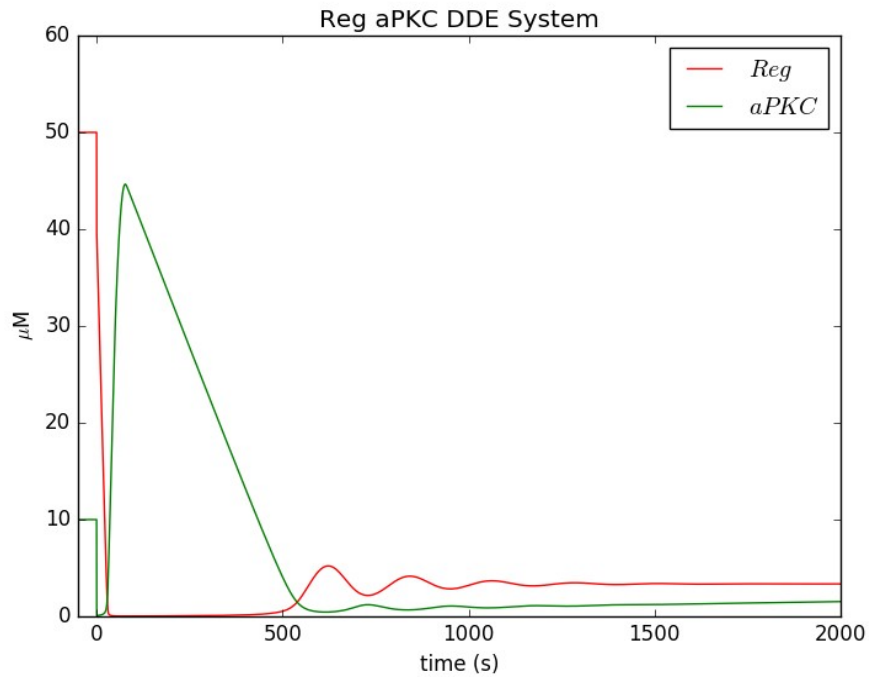
The parameters used were:

```
{  
  "k1": 0.03,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.0001,  
  "tau1": 50.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 10.0,  
  "Ri": 50.0  
}
```

Simplified Model: AR complex now included. Increase k4 rate that AR dissociates



The parameters used were:

```
{  
  "k1": 0.03,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.01,  
  "tau1": 50.0  
}
```

The initial conditions used were:

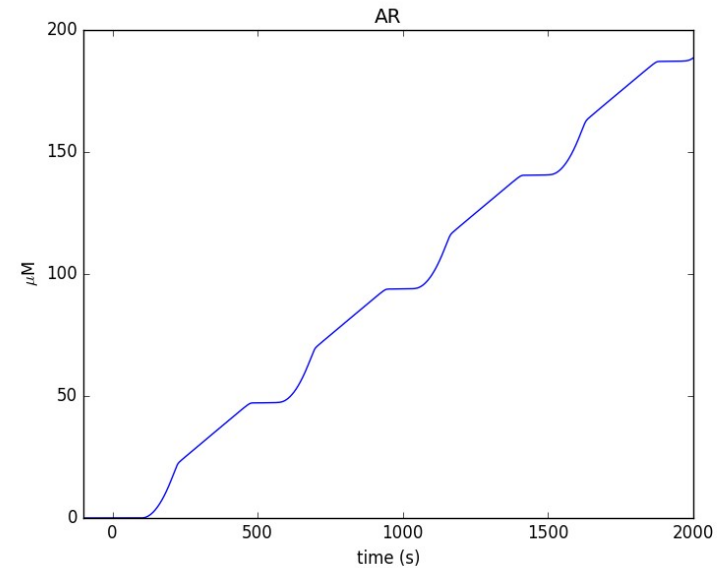
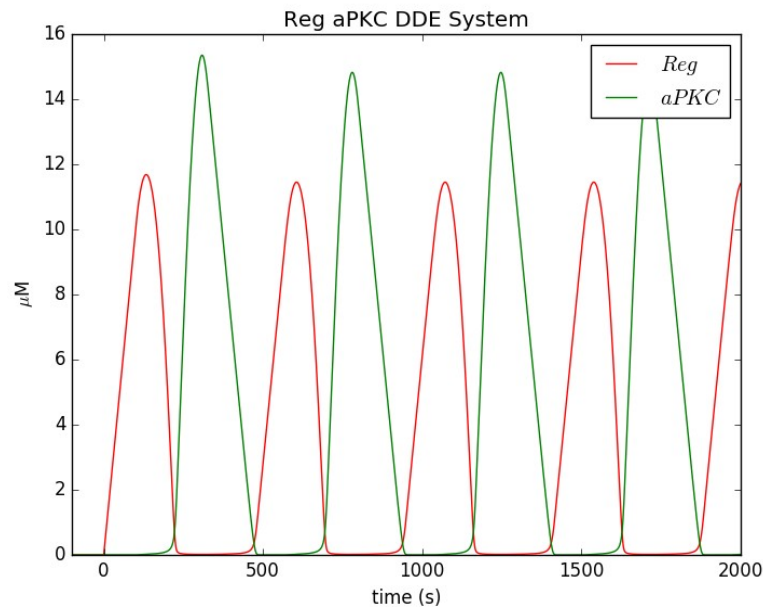
```
{  
  "ARi": 0.0,  
  "Ai": 10.0,  
  "Ri": 50.0  
}
```

Increasing k4 causes the oscillations to dampen out.

Conclusions (after meeting with Jimmy)

- K4 adds dampening to the system.
- APKC appears to sustain oscillations.
- Check a $k_1 \cdot \tau$ condition
 - Doesn't appear to be quite the same. There is a sweet spot though.
- Check lower R_i condition.
 - Confirmed. We avoid the initial spike.

Decrease R_i to see if it removes the initial spike = True



The parameters used were:

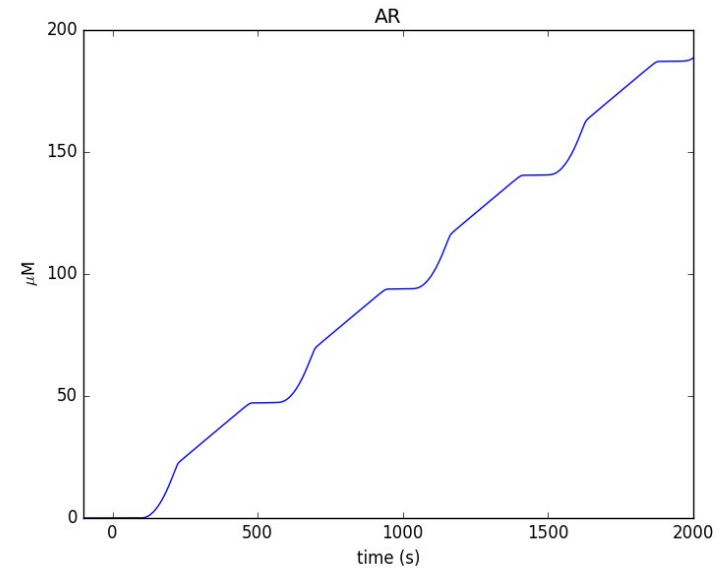
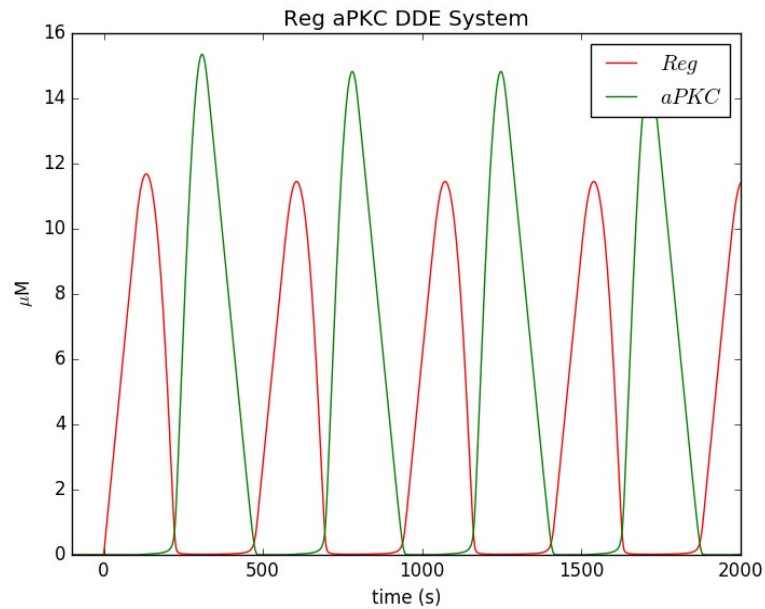
```
{  
  "k1": 0.03,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.0,  
  "tau1": 100.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

Hence, as expected started with a lower R_i ,
does not cause the initial spike of aPKC

Is there a $k_1 \cdot \tau \cdot e > 1$ connection?



The parameters used were:

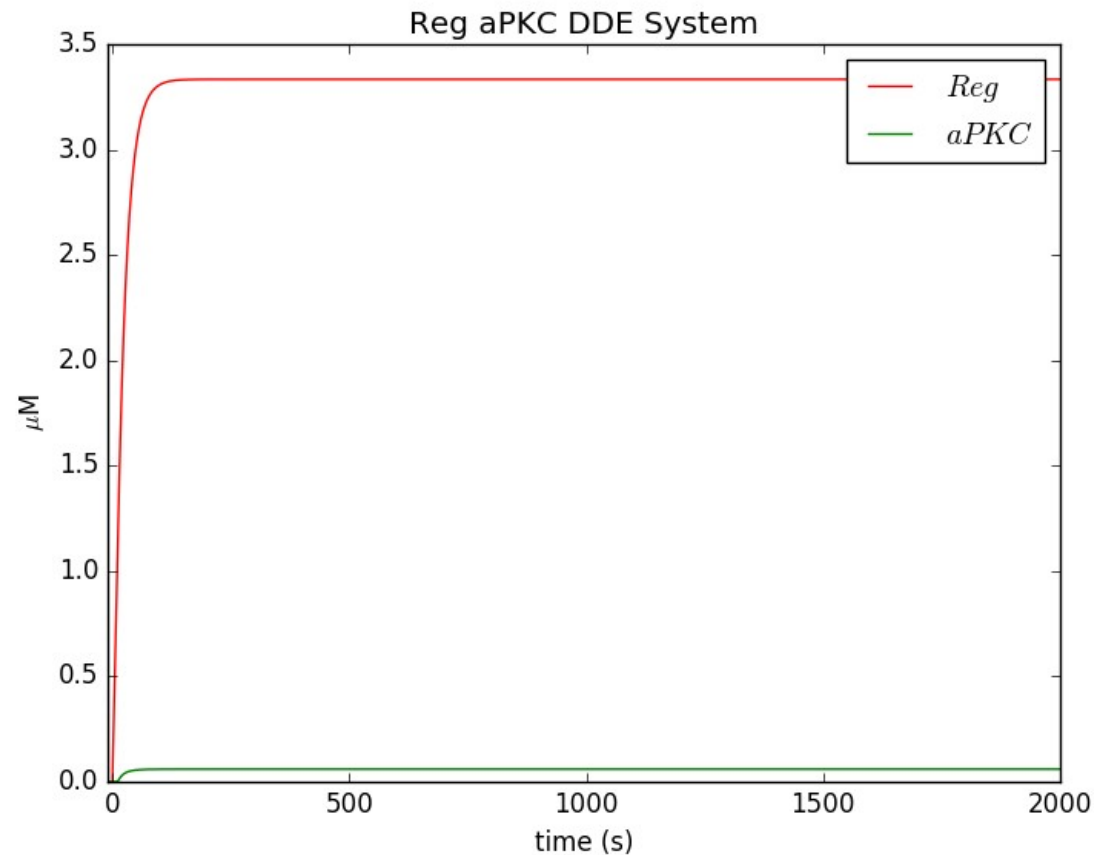
```
{  
  "k1": 0.03,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.0,  
  "tau1": 100.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k_1 \cdot \tau \cdot e = 8.15484548538$

Is there a $k_1 \cdot \tau \cdot e > 1$ connection?



The parameters used were:

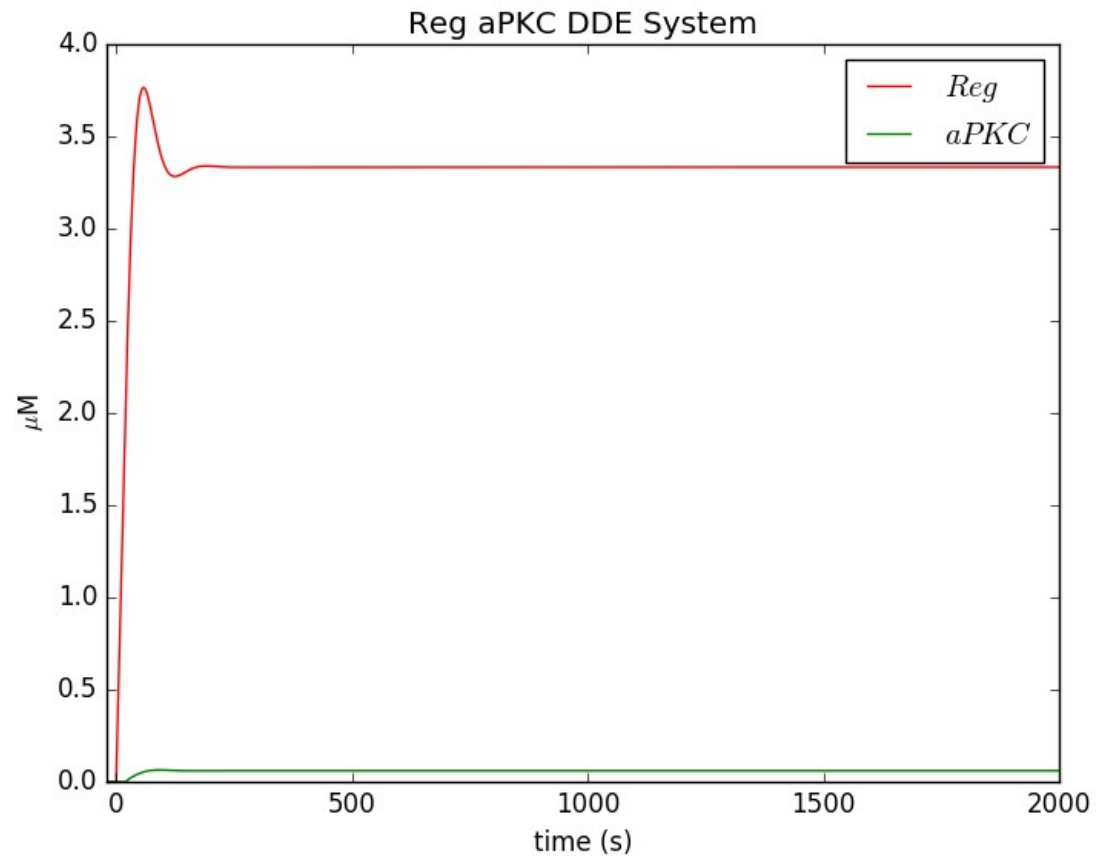
```
{  
  "k1": 0.03,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.0,  
  "tau1": 10.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k_1 \cdot \tau \cdot e = 0.815484548538$

Is there a $k_1 \cdot \tau_e > 1$ connection?



The parameters used were:

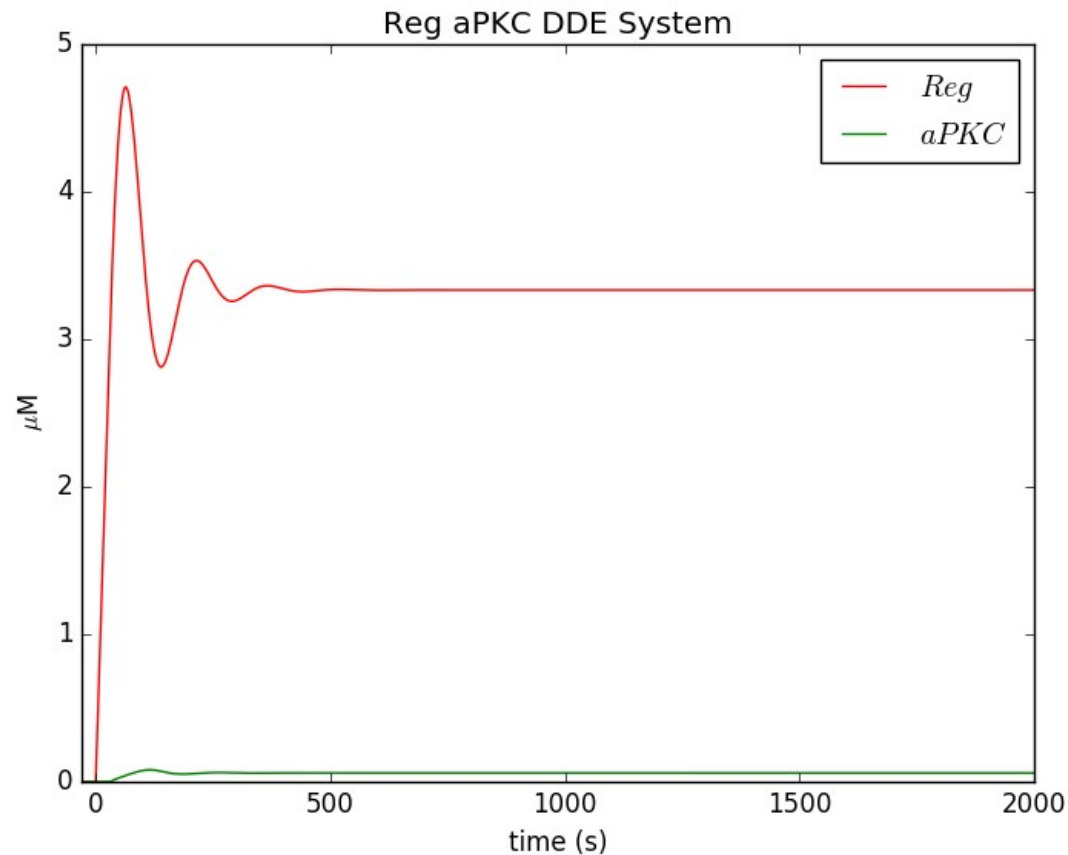
```
{  
  "k1": 0.03,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.0,  
  "tau1": 20.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k_1 \cdot \tau_e = 1.63096909708$

Is there a $k_1 \cdot \tau \cdot e > 1$ connection?



The parameters used were:

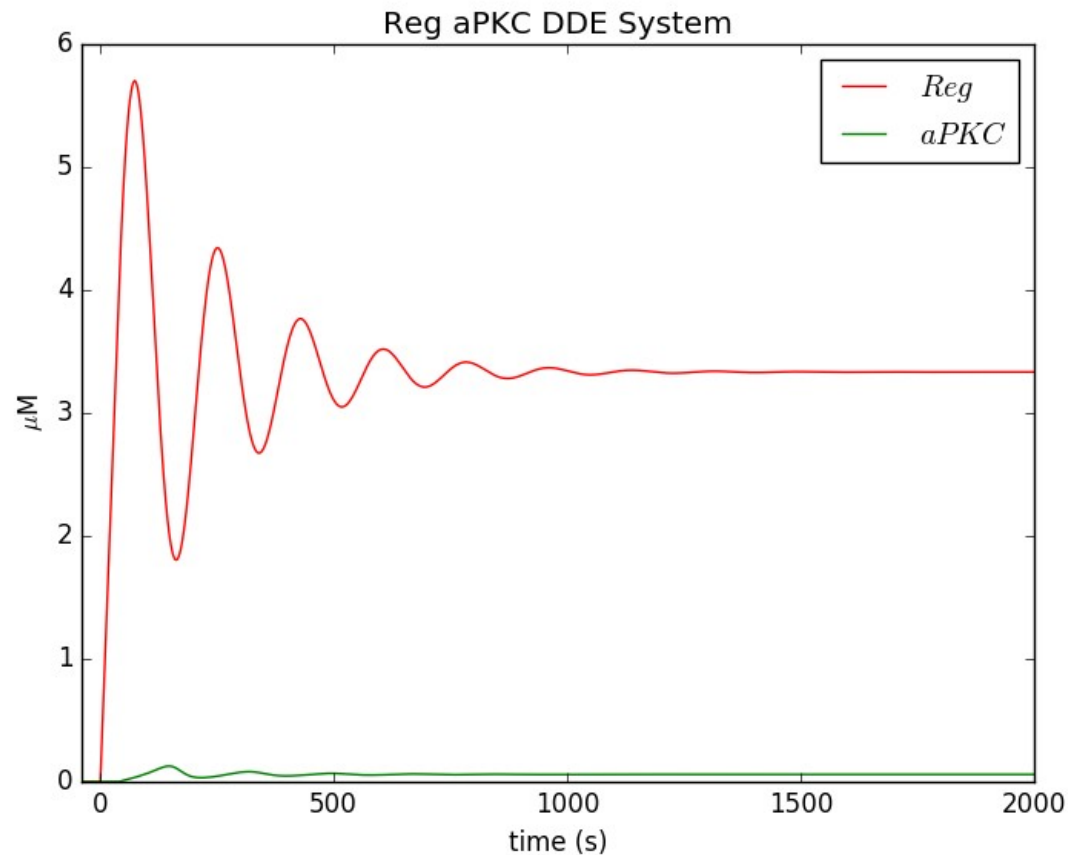
```
{  
  "k1": 0.03,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.0,  
  "tau1": 30.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k_1 \cdot \tau \cdot e = 2.44645364561$

Is there a $k_1 \cdot \tau_e > 1$ connection?



The parameters used were:

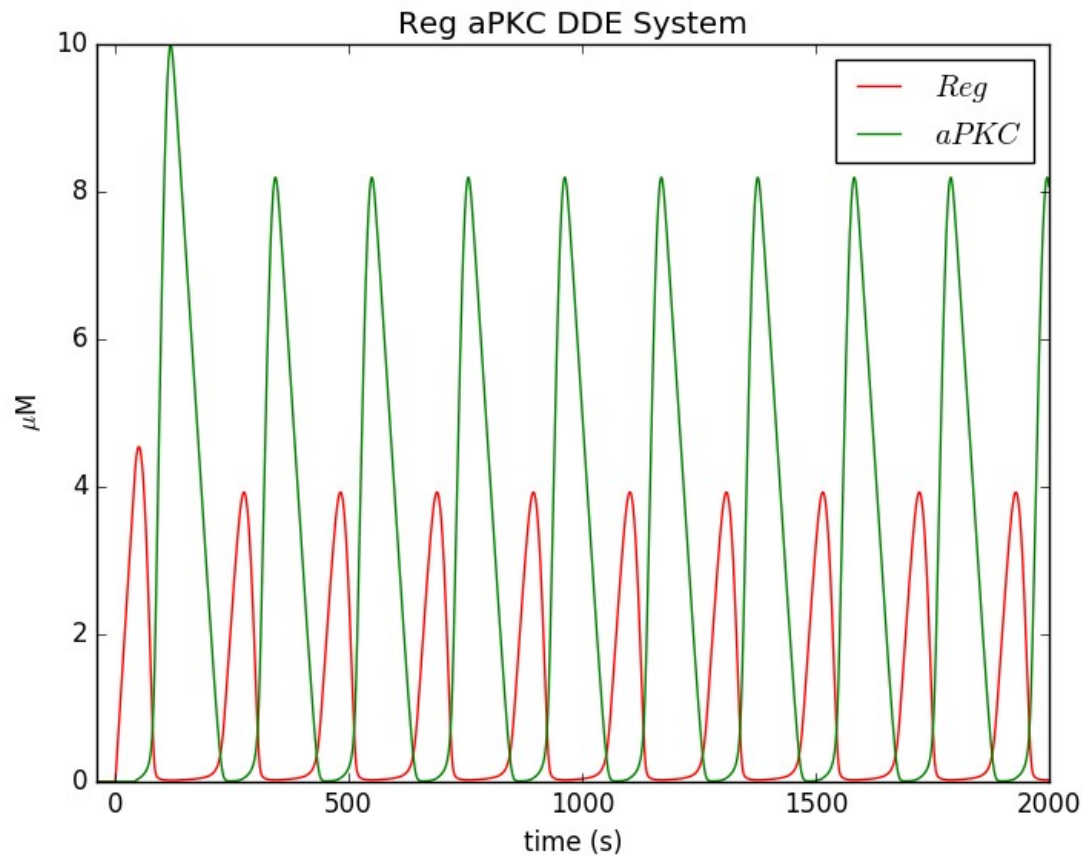
```
{  
  "k1": 0.03,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.0,  
  "tau1": 40.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k_1 \cdot \tau_e = 3.26193819415$

Is there a $k_1 \cdot \tau \cdot e > 1$ connection?



The parameters used were:

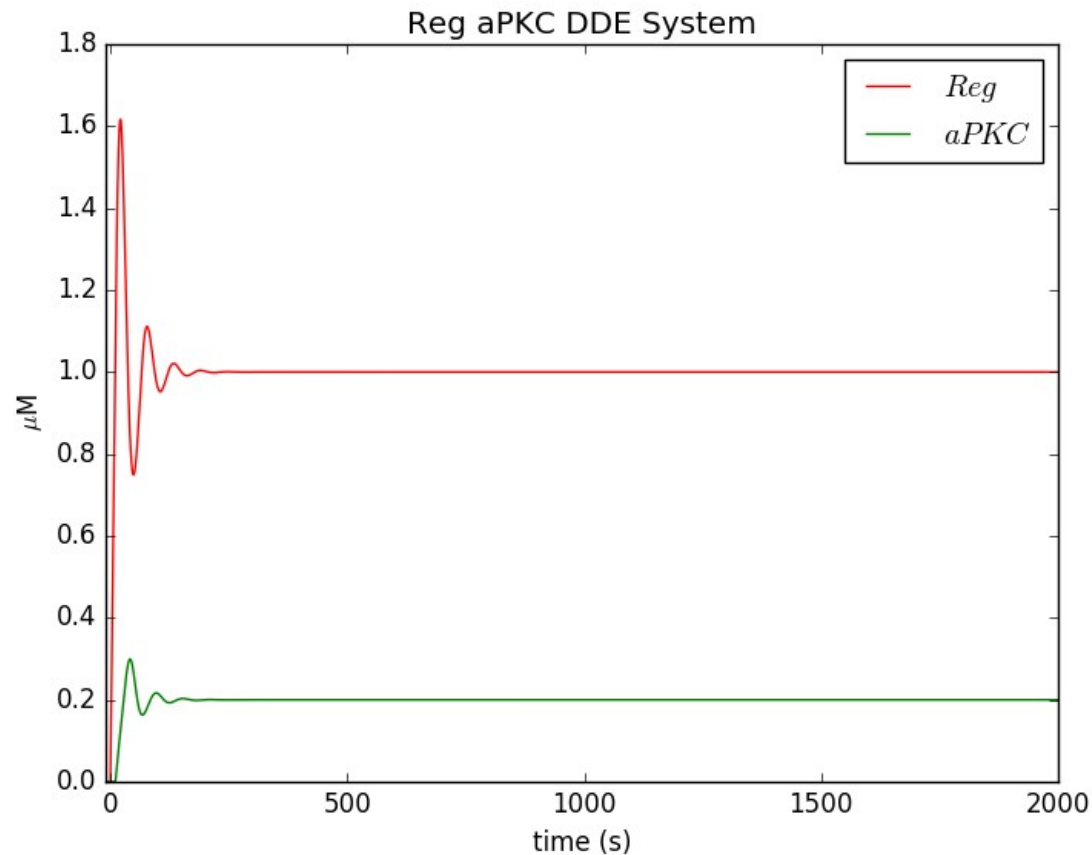
```
{  
  "k1": 0.1,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0,  
  "tau1": 40.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k_1 \cdot \tau \cdot e = 10.8731273138$

Is there a $k_1 \cdot \tau \cdot e > 1$ connection?



The parameters used were:

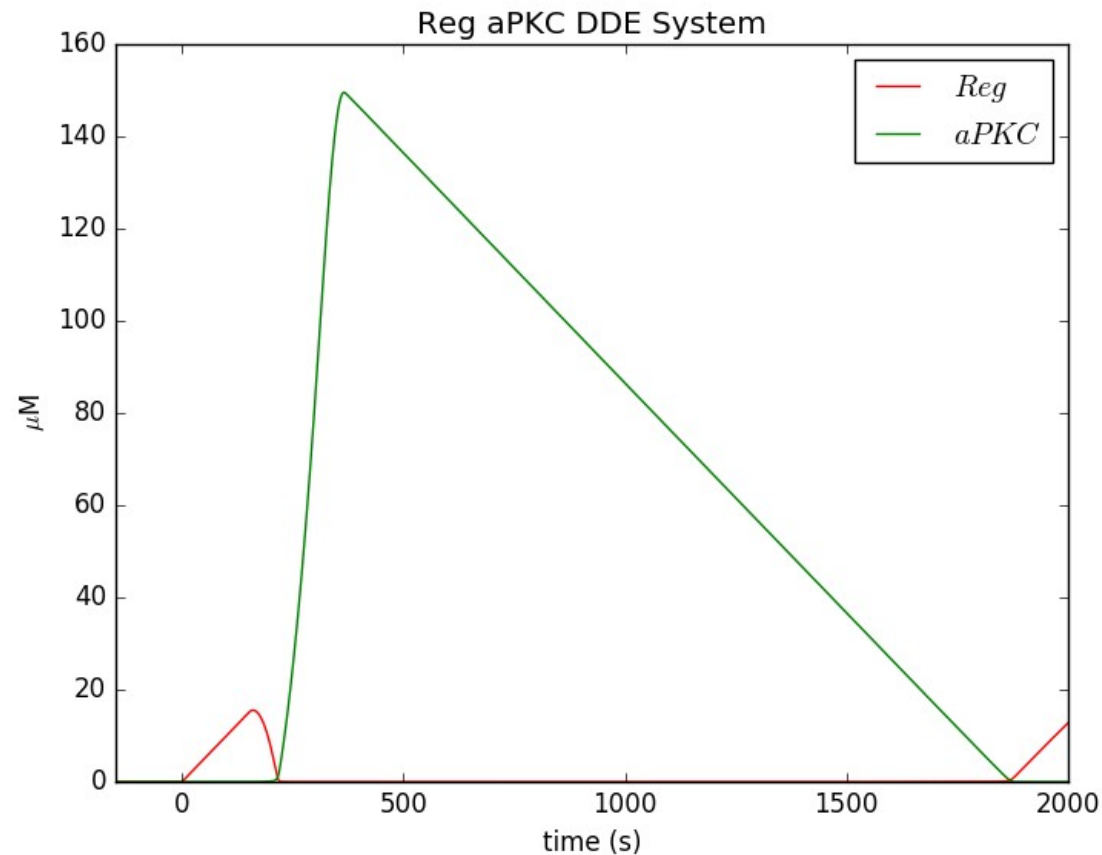
```
{  
  "k1": 0.1,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0,  
  "tau1": 10.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k_1 \cdot \tau \cdot e = 2.71828182846$

Is there a $k_1 \cdot \tau \cdot e > 1$ connection?



The parameters used were:

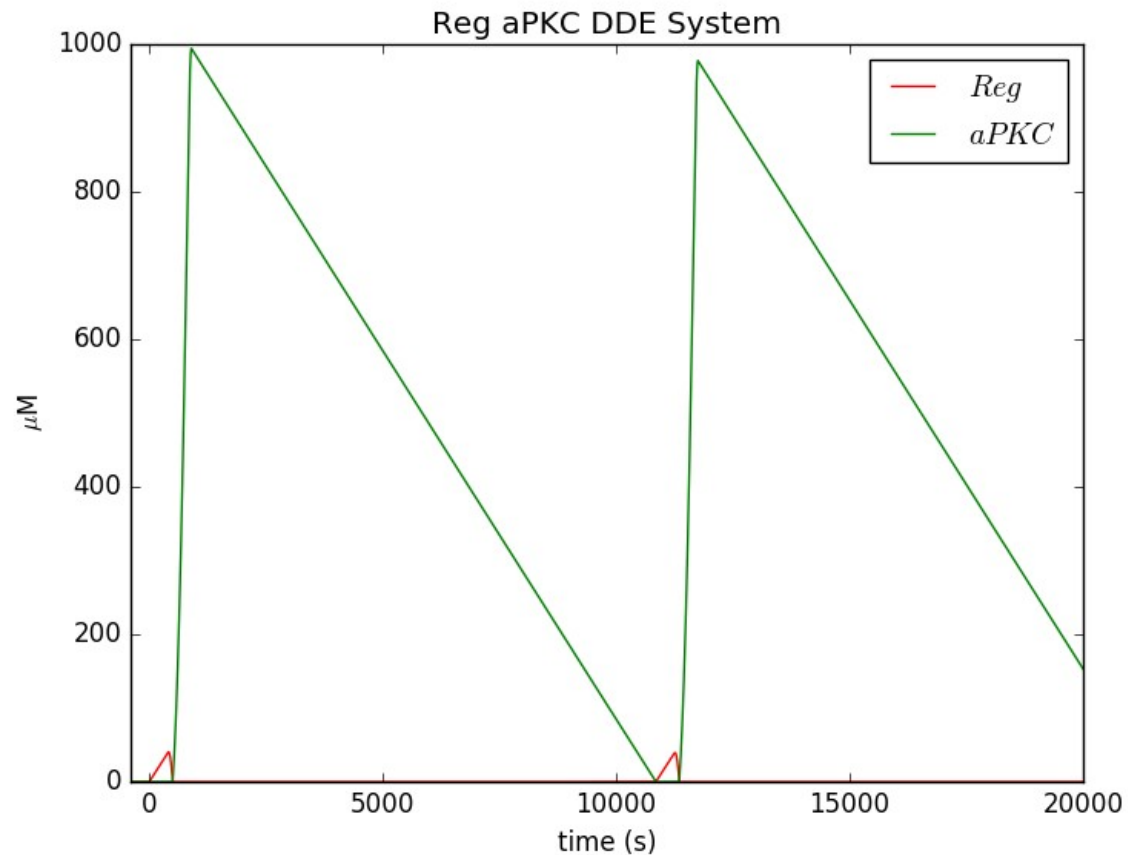
```
{  
  "k1": 0.1,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0,  
  "tau1": 150.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k_1 \cdot \tau \cdot e = 40.7742274269$

Is there a $k_1 \cdot \tau_e > 1$ connection?



The parameters used were:

```
{  
  "k1": 0.1,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0,  
  "tau1": 400.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

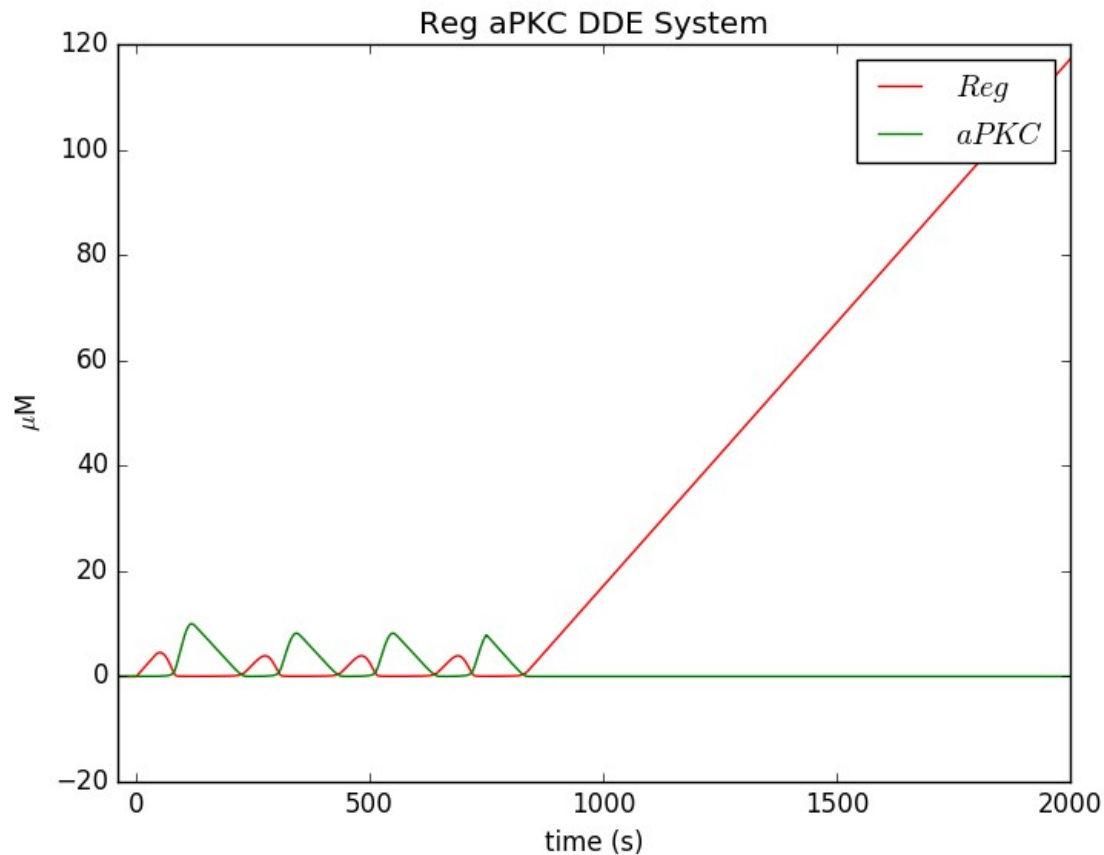
$k_1 \cdot \tau_e = 108.731273138$

*Notice the time scale.

Conclusion - Is there a $k_1 \tau_e > 1$ connection?

- It doesn't seem to have the same condition attached. A literature search for coupled systems should be carried out to find a similar mechanism.
- However, we do see that there is a sweet spot for $k_1 \tau$ to reside in for oscillations on this time scale.
- Update after next section

Investigate Ac sustaining oscillations. Implement 1-Heavi(t-750).



The parameters used were:

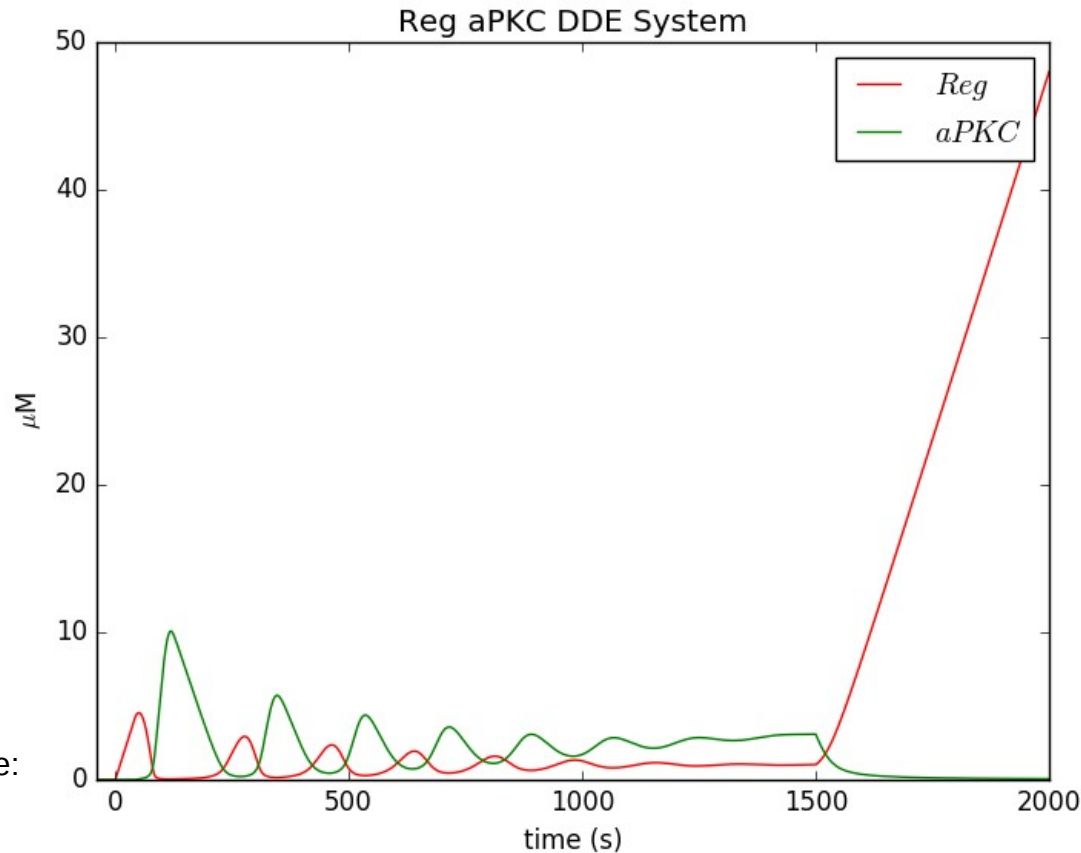
```
{  
  "k1": 0.1,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0,  
  "tau1": 40.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k1 \cdot \tau_1 \cdot e = 10.8731273138$

Composite Effect of k_4 and “Ac”



The parameters used were:

```
{  
  "k1": 0.1,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.01,  
  "tau1": 40.0  
}
```

The initial conditions used were:

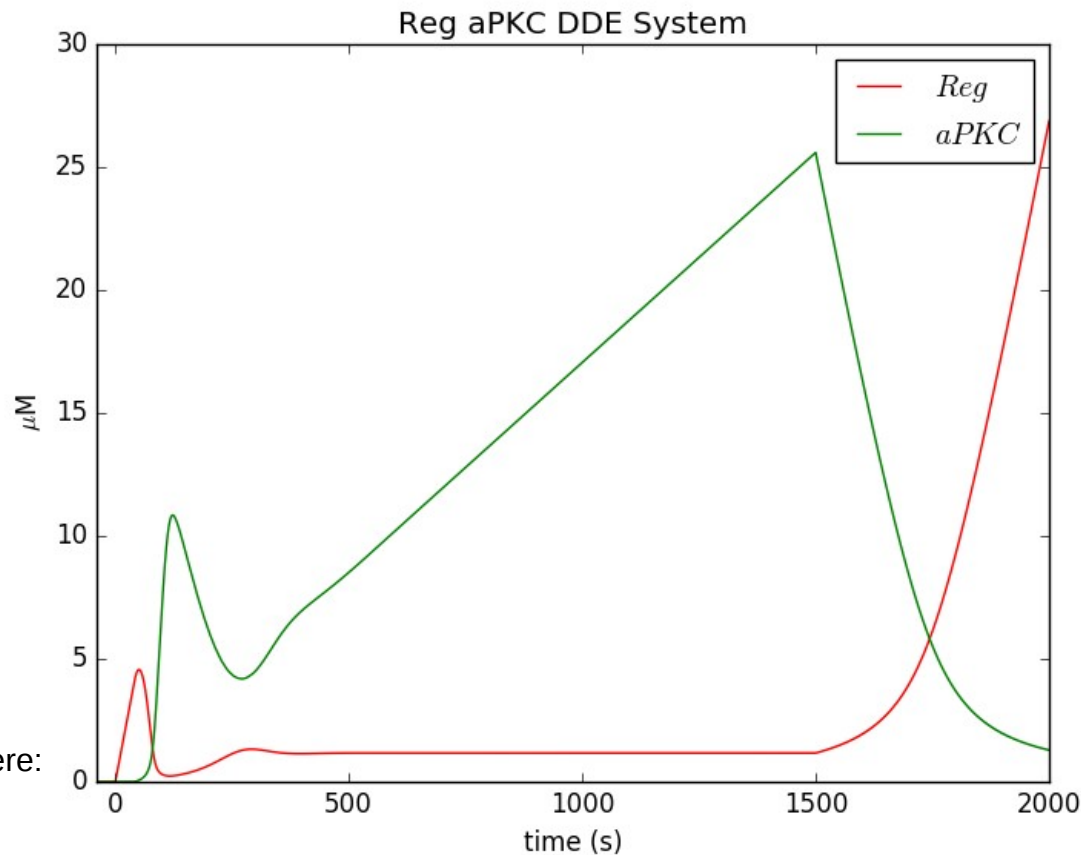
```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k_1 \cdot \tau_1 \cdot e = 10.8731273138$

K_1 turns off at $t=1500$.

K_4 needs to be low or there are not any oscillations.

Composite Effect of k_4 and “Ac”



The parameters used were:

```
{  
  "k1": 0.1,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.1,  
  "tau1": 40.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

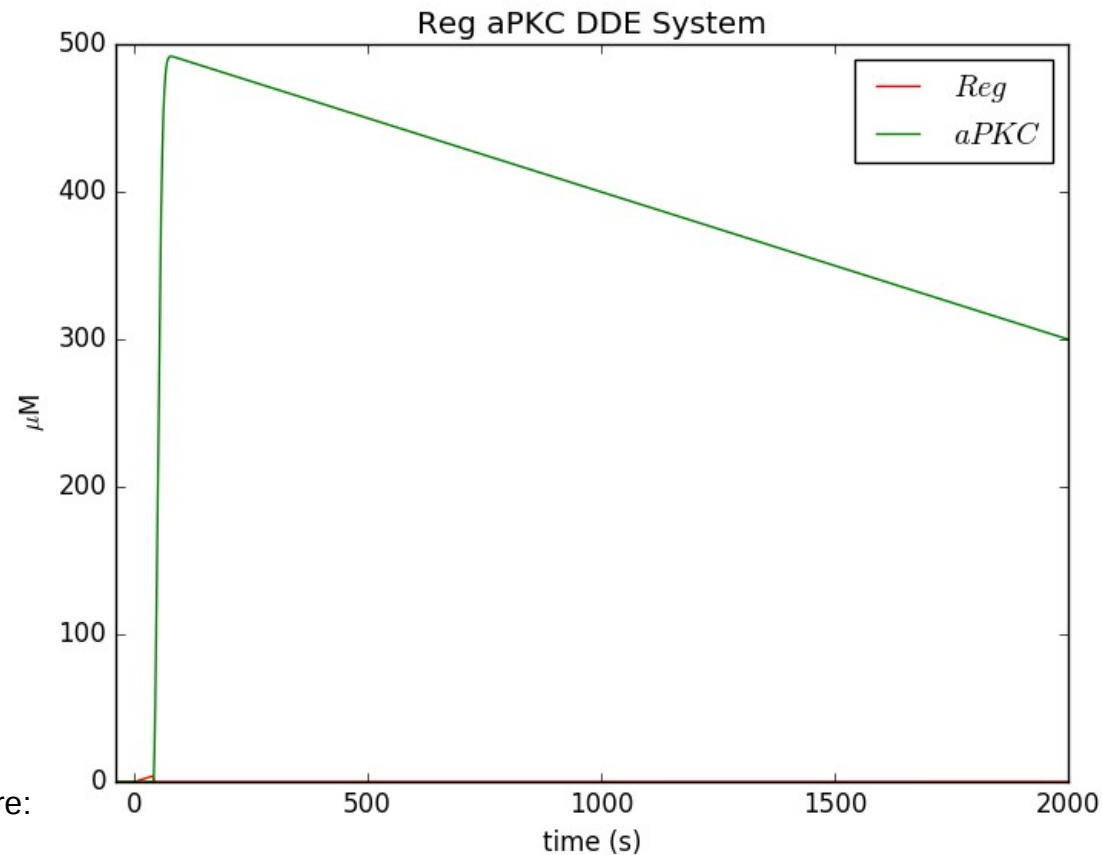
$k_1 \cdot \tau_1 \cdot e = 10.8731273138$

Larger τ_1 value leads to non-oscillatory dynamics.

Conclusion – k_4 and “Ac”

- It is possible to use a finite Ac supply to sustain oscillations for a set amount of time based on preliminary study. Therefore, adding in Ac looks like a promising direction to follow.
- k_4 in a sense dampens oscillations, but it also has a propensity to ruin ideal oscillations.
- Based on the way the model is shaping up the biological cost we would play to keep k_4 low is that AR has a low “off-rate” which incurs relatively little cost.
- k_4 is much more important in determining the length of oscillations. Increasing Ac is not enough to sustain oscillations for $t \sim 6000s$.
 - Figures omitted as of now.

Implementing Migration



The parameters used were:

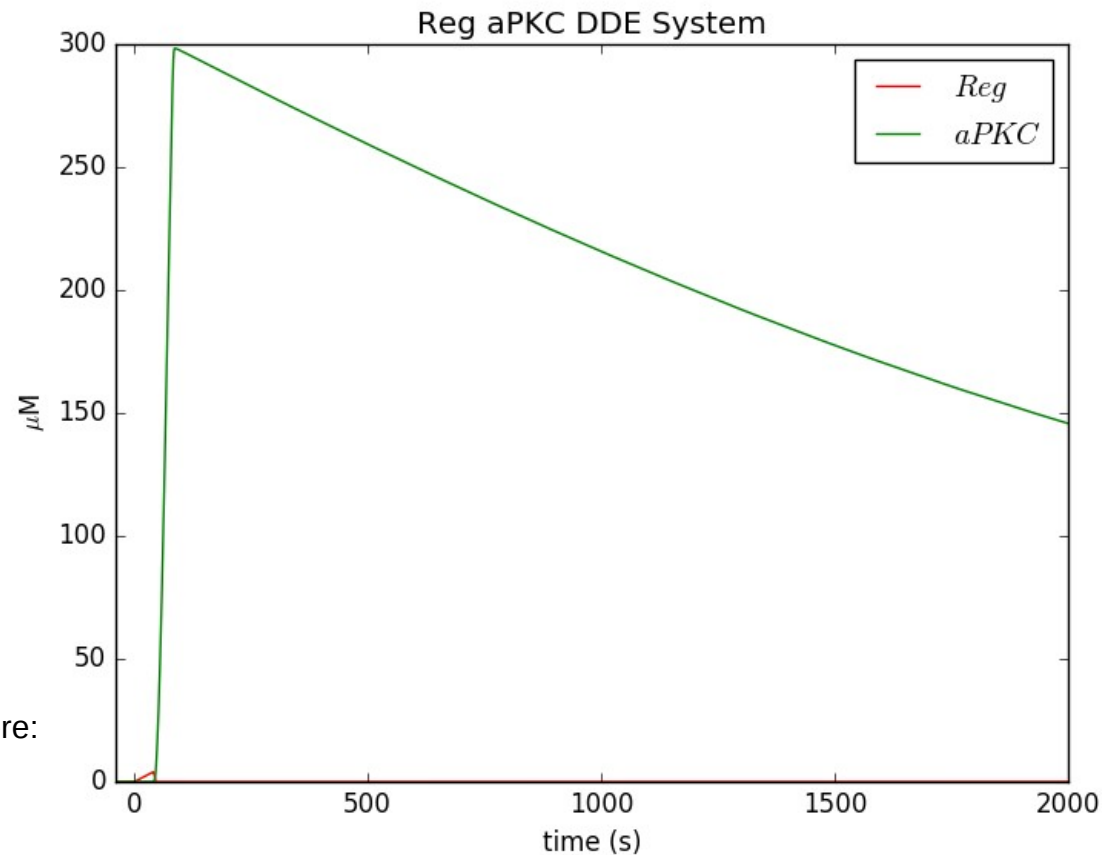
```
{  
  "k1": 0.1,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.01,  
  "tau1": 40.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Aci": 500,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k1 \cdot \tau1 = 10.8731273138$

Migration



The parameters used were:

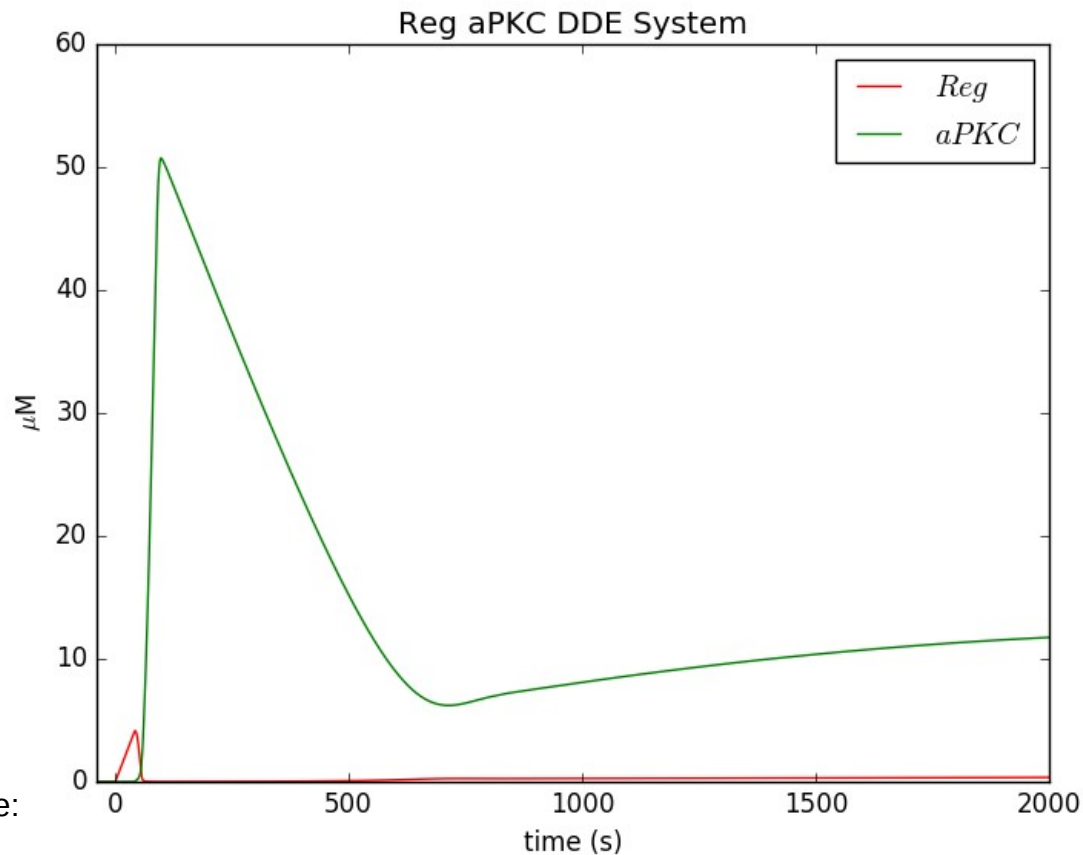
```
{  
  "k1": 0.01,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.01,  
  "tau1": 40.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Aci": 500,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k1 \cdot \tau \cdot e = 1.08731273138$

Migration



The parameters used were:

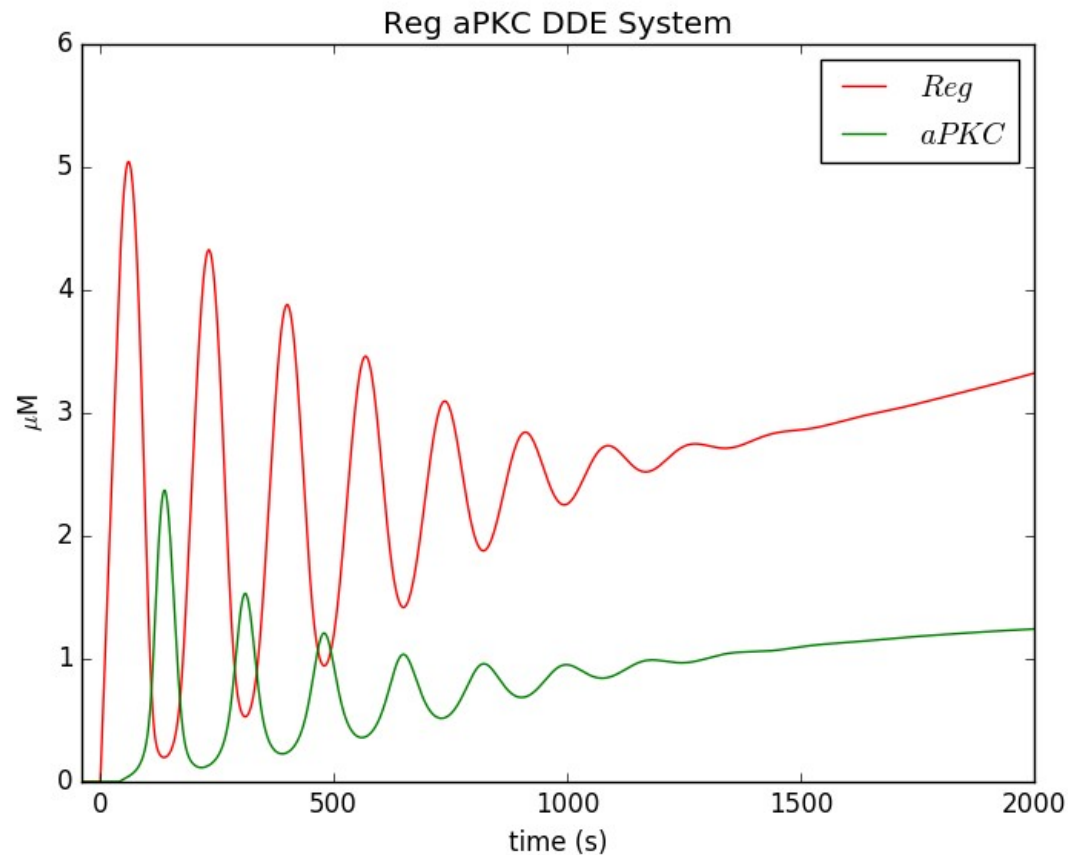
```
{  
  "k1": 0.001,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.01,  
  "tau1": 40.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Aci": 500,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k1 \cdot \tau \cdot e = 0.108731273138$

Migration



The parameters used were:

```
{  
  "k1": 0.0001,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.01,  
  "tau1": 40.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Aci": 500,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k1 \cdot \tau \cdot e = 0.0108731273138$

Sustaining Oscillations w/ Migration

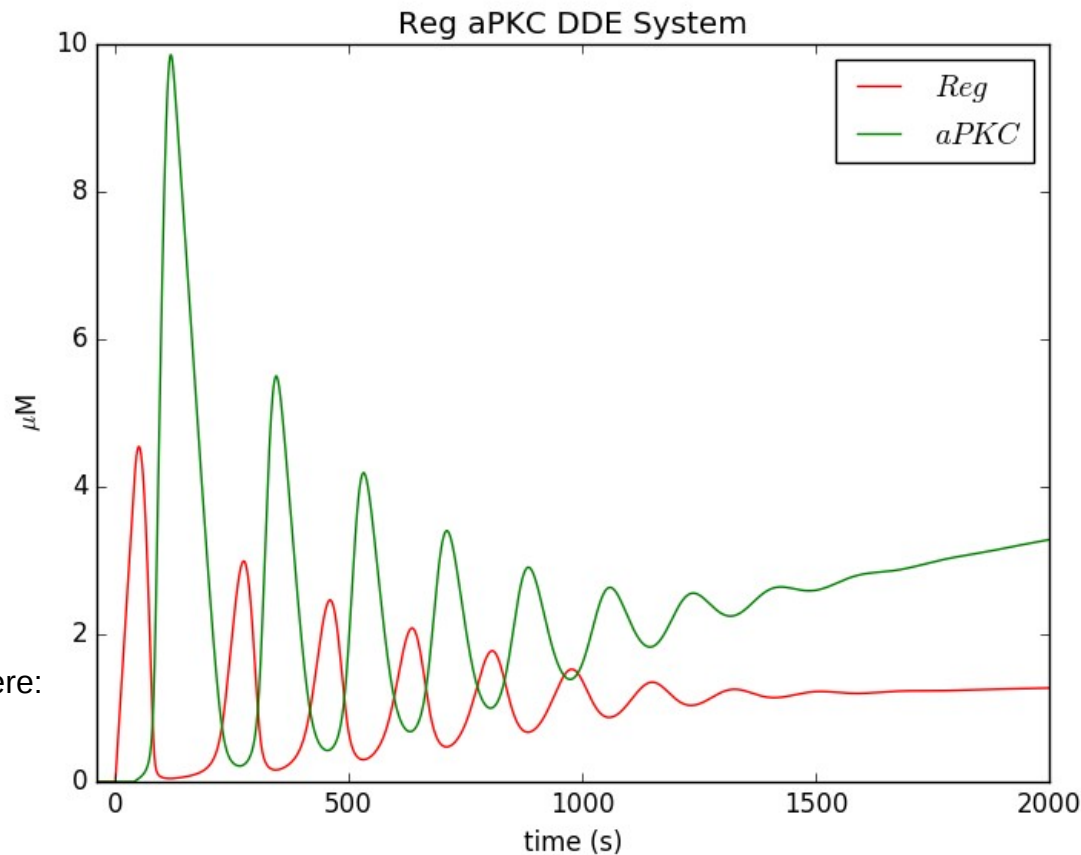
The parameters used were:

```
{
  "k1": 0.0001,
  "k2": 0.5,
  "k3": 0.1,
  "k4": 0.01,
  "tau1": 40.0
}
```

The initial conditions used were:

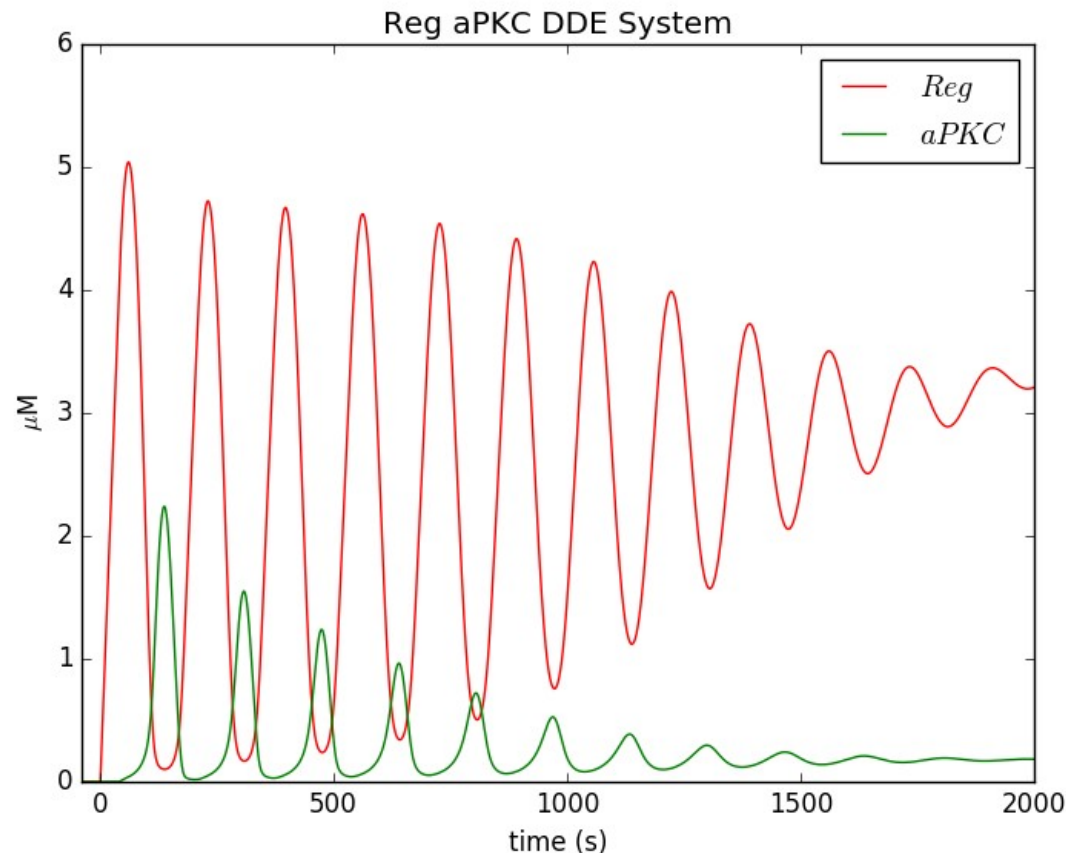
```
{
  "ARi": 0.0,
  "Aci": 1000,
  "Ai": 0.0,
  "Ri": 0.0
}
```

$k1 \cdot \tau1 \cdot e = 0.0108731273138$



We see that increasing A_c can sustain oscillations but the gain is minimal. As we continue to increase this term we would also have to pay the price of smaller k_1 value.

Sustaining Oscillations w/ Migration



The parameters used were:

```
{  
  "k1": 0.0001,  
  "k2": 0.5,  
  "k3": 0.1,  
  "k4": 0.001,  
  "tau1": 40.0  
}
```

The initial conditions used were:

```
{  
  "ARi": 0.0,  
  "Aci": 500,  
  "Ai": 0.0,  
  "Ri": 0.0  
}
```

$k1 \cdot \tau1 \cdot e = 0.0108731273138$

We see $k4$ plays a much larger role in sustaining the oscillations.

Conclusions - Implementing Migration

- When implementing migration, $k_1 \cdot R \cdot A$ is too large and too much (all) aPKC migrates too quickly.
- Hypothesis concerning the oscillatory condition.
 - Basically, now that the birth term is being multiplied by A_c ($O(10^2)$) I need to decrease k_1 by 10^{-2} . Likewise, multiplication by R which starts out as $O(10)$ thus another 10^{-1} and we recover the oscillations.
 - Instead of $k_1 \cdot \tau > e$, it is possible $k_1 \cdot \tau \cdot R \cdot A_c \sim 10^{-20}$. This is a rough estimate and running back through all the simulations and obtaining this value could be valuable.
- Increasing initial value of A_c will be able to sustain oscillations longer but the gains are minimal. Increasing A_c to a value to sustain oscillations for the time scale that we need (6000s, length of dorsal closure process), we would have to pay the price of a much smaller k_1 value.
- K_4 plays a much larger role in sustaining oscillations.

Questions

- Now that we see why k_1 and k_4 must best be low in order to sustain oscillations, is there a biological story as to why this could be implemented mathematically?
- Should we forgo these modelling assumptions and modify them to match the desired behavior?