Statistical Modeling and Causal Inference with R

Week 11: Causal Mediation

Manuel Bosancianu

Max Schaub

November 23, 2020

Hertie School of Governance

Results: Baron–Kenny approach

$$Depression_i = \alpha_1 + \beta_1 \overbrace{Seminar_i}^{treatment} + \zeta_1 X_i + \epsilon_{i1} \quad (1)$$

$$Confidence_i = \alpha_2 + \beta_2 Seminar_i + \zeta_2 X_i + \epsilon_{i2} \quad (2)$$

$$Depression_i = \alpha_3 + \gamma Seminar_i + \beta_3 \underbrace{Confidence_i}_{mediator} + \zeta_3 X_i + \epsilon_{i3} \quad (3)$$

	DV: Depression	DV: Confidence	DV: Depression
(Intercept)	0.895^{***}	3.870^{***}	1.499^{***}
	(0.133)	(0.159)	(0.158)
Seminar	-0.047	0.101^{*}	-0.032
	(0.035)	(0.042)	(0.035)
Confidence			-0.156^{***}
			(0.023)

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$. Estimates from pre-treatment covariates have been excluded from the table. $N = 1285$. Measures of model fit removed from the table.

Computing effects & uncertainty

- Direct effect: -0.032
- Indirect effect: $\beta_2 \times \beta_3$ = 0.101×-0.156 = -0.016
- Total effect: $direct + indirect$ = $-0.032 + (-0.016)$ = -0.047 (rounding)

$$SE_{indirect} = \sqrt{c^2\sigma_b^2 + b^2\sigma_c^2 + \sigma_b^2\sigma_c^2} \tag{4}$$

	Direct	Indirect
β	-0.032	-0.016^{*}
SE	(0.035)	(0.007)

Causal mediation results

The Imai *et al* setup I

Sequential ignorability is still needed as fundamental assumption.

Requires only 2 regressions (OLS, logit, probit, survival...):

$$Confidence_i = \psi_1 + \phi_1 \overbrace{Seminar_i}^{treatment} + \zeta_2 X_i + \epsilon_{i2} \tag{5}$$

$$Depression_i = \psi_2 + \phi_2 Seminar_i + \phi_3 \underbrace{Confidence_i}_{mediator} + \zeta_3 X_i + \epsilon_{i3} \tag{6}$$

Generate predictions for mediator from Equation 5: $Confidence_i | Seminar_i = 0$ and $Confidence_i | Seminar_i = 1$.

The Imai *et al* setup II

Generate predictions for the outcome from Equation 6, using predictions for mediator: $Depression_i | Seminar_i = 1, Confidence_i(1)$ and $Depression_i | Seminar_i = 1, Confidence_i(0)$.

$$ACME = \delta_i(t) = Y_i(t, M_i(1)) - Y_i(t, M_i(0)),\ for\ each\ t \in \{0; 1\} \tag{7}$$

SEs computed with bootstrapping (or Monte Carlo methods).

Syntax: Imai *et al* approach

```
# The "mediate()" function is used to calculate the
# ACME and ADE
med.out <- mediate(model.m = med.fit,
                   model.y = out.fit,
                   sims = 750,
                   boot = TRUE,
                   treat = "treat_num",
                   mediator = "job_seek")
```

The automated function works with the two regression model objects.

Results: Imai *et al* approach

```
Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

               Estimate 95% CI Lower 95% CI Upper p-value
ACME            -0.0157      -0.0316         0.00  0.0053 **
ADE             -0.0316      -0.1006         0.04  0.3440
Total Effect    -0.0473      -0.1198         0.02  0.1867
Prop. Mediated   0.3327      -2.2380         2.76  0.1920
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 1285


Simulations: 750
```

$ADE_{BK} = -0.032\ (0.035)$ and $ACME_{BK} = -0.016\ (0.007)$.

Sensitivity analysis I

ACME unbiased if $cov(\epsilon_{i2}, \epsilon_{i3}) = 0$ (call this ρ).

If *sequential ignorability* holds, then $\rho = 0$. If not, estimates are biased.

In practice, sensitivity analysis is based on a function of R^2 from the two models (see Imai, Keele, & Yamamoto, 2010, pp. 61–62).

Sensitivity analysis II

```r
# "medsens()" is the function which conducts the
# sensitivity analysis
sens.out <- medsens(med.out,
                    rho.by = 0.05,
                    sims = 750,
                    effect.type = "indirect")
```

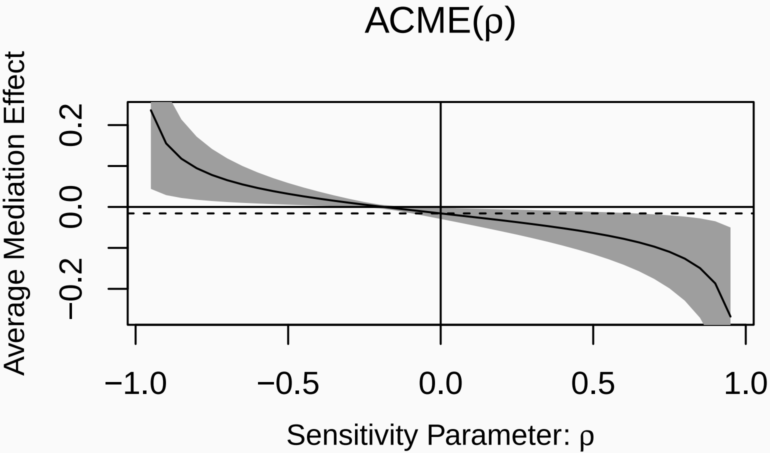
```
Mediation Sensitivity Analysis for Average Causal Mediation Effect

Sensitivity Region

       Rho    ACME 95% CI Lower 95% CI Upper R^2_M*R^2_Y* R^2_M~R^2_Y~
[1,] -0.25  0.0056      -0.0008       0.0121       0.0625       0.0403
[2,] -0.20  0.0012      -0.0035       0.0058       0.0400       0.0258
[3,] -0.15 -0.0032      -0.0084       0.0020       0.0225       0.0145
[4,] -0.10 -0.0074      -0.0150       0.0001       0.0100       0.0064

Rho at which ACME = 0: -0.2
R^2_M*R^2_Y* at which ACME = 0: 0.04
R^2_M~R^2_Y~ at which ACME = 0: 0.0258
```

Sensitivity analysis III



Thank you for the kind attention!

Imai, K., Keele, L., & Yamamoto, T. (2010). Identification, inference and sensitivity analysis for causal mediation effects. *Statistical Science*, *25*(1), 51–71.