THE UNIVERSITY of EDINBURGH

Logistics Assignment 1

Christos Delivorias
s0973777

Master of Science
University of Edinburgh
2012

Part 1

Question (a)

The definitions of all **parameters** are as described below.

d_j := is the demand of warehouse j for all $j = A,\dots,J$
s_i := is the capacity of factory i for all $i = 1,\dots,6$
e_{ij} := is the distance from factory i to warehouse j
D := is the set $[A,\dots,J]$
S := is the set $[1,\dots,6]$
b := is the contractual cost of delivery

The definitions of all **variables** are as described below.

x_{ij} := are the units sent from factory i to warehouse j

The MP form is shown below:

$$\begin{array}{rlll} min & \sum_i \sum_j \left[b \cdot e_{ij} \cdot x_{ij} \right] & & \\ \text{subject to} & \sum_i x_{ij} \geq d_j & ,\forall & j \in D \\ & \sum_i x_{ij} \leq s_i & ,\forall & i \in S \\ & x_{ij} \geq 0 & & \end{array}$$

Question (b)

Using FICO Xpress the above MLP was modelled and solved as shown below.The value of $b = 1$ was used for this question. The data set was provided and is included with the MOSEL code.

```
Intitial Agreement
==================
Problem status: Optimum found

  A B C D E F G H I J
1 3 3 0 0 0 0 0 2 0 0
2 0 0 0 0 0 2 0 0 0 0
3 0 0 0 1 3 0 2 0 0 0
4 0 0 0 3 0 0 0 0 2 0
5 0 0 2 0 0 0 0 1 0 0
6 0 0 0 0 0 0 0 0 1 2

Total Costs:= 97
```

Figure 1: FICO Xpress solution of the MLP in Part 1

The plot in Figure 1 shows the units moved from each factory to individual warehouses. As shown the total minimised cost is 97.

Part2

Question (c)

The revised model taking into account the logistic company's offer is as follows.

The definitions of all **parameters** are as described below.

$$
\begin{array}{ll}
d_j := & \text{is the demand of warehouse } j \text{ for all } j = A, \ldots, J \\
s_i := & \text{is the capacity of factory } i \text{ for all } i = 1, \ldots, 6 \\
e_{ij} := & \text{is the distance from factory } i \text{ to warehouse } j \\
D := & \text{is the set } [A, \ldots, J] \\
S := & \text{is the set } [1, \ldots, 6] \\
b := & \text{is the contractual cost of delivery} \\
c := & \text{is the contractual cost per shippment} \\
b' := & \text{is the reduced contractual cost of delivery} \\
M := & \text{is an arbitary large upper barrier}
\end{array}
$$

The definitions of all **variables** are as described below.

$$
\begin{array}{ll}
x_{ij} := & \text{are the units sent from factory } i \text{ to warehouse } j \\
y_{ij} := & \text{is a binary variable which is 1 if units are shipped} \\
 & \text{from factory } i \text{ to warehouse } j \text{ and 0 otherwise}
\end{array}
$$

The IP form is shown below:

$$
\begin{array}{lll}
min & \sum_i \sum_j \left[b' \cdot e_{ij} \cdot x_{ij} + c \cdot y_{ij} \right] & \\
\text{subject to} & \sum_i x_{ij} >= d_j & , \forall \quad j \in D \\
 & \sum_i x_{ij} <= s_j & , \forall \quad i \in S \\
 & x_{ij} \leq M \cdot y_{ij} & , \forall \quad i \in S, j \in D \\
 & x_{ij} \geq 0 & \\
 & b' = 0.75 \cdot b &
\end{array}
$$

Question (c)

Using FICO Xpress the above MIP was modelled and solved as shown below.The values of $b = 1, c = 1.85, and M = 10$ were used for this question. The data set was provided and is included with the MOSEL code.

The plot in Figure 2 shows the units moved from each factory to individual warehouses. As shown the total minimised cost is 96.1.

```
Ammended Agreement
==================
Problem status: Optimum found

  A B C D E F G H I J
1 3 3 0 0 0 0 0 2 0 0
2 0 0 0 0 0 2 0 0 0 0
3 0 0 0 4 0 0 2 0 0 0
4 0 0 2 0 0 0 0 0 3 0
5 0 0 0 0 0 0 0 1 0 2
6 0 0 0 0 3 0 0 0 0 0

Total Costs:= 96.1
```

Figure 2: FICO Xpress solution of the MIP in Part 2

Part 3

Question (d)

This augmented model is taking into account possible saivings in operational costs by closing down factories.

The definitions of all **parameters** are as described below.

d_j := is the demand of warehouse j for all $j = A,\dots,J$
s_i := is the capacity of factory i for all $i = 1,\dots,6$
e_{ij}:= is the distance from factory i to warehouse j
f_i := is the operating cost of factory i
D := is the set $[A,\dots,J]$
S := is the set $[1,\dots,6]$
b := is the contractual cost of delivery

The definitions of all **variables** are as described below.

x_{ij} := are the units sent from factory i to warehouse j
ψ_i := is a binary variable which is 1 if factory i is open and 0 otherwise

The IP form is shown below:

$$
\begin{array}{rlll}
min & \sum_i\sum_j \left[b \cdot e_{ij} \cdot x_{ij} - \sum_i f_i \cdot (1-\psi_i)\right] & & \\
\text{subject to} & \sum_i x_{ij} >= d_j & ,\forall & j \in D \\
 & \sum_i x_{ij} <= s_i \cdot \psi_i & ,\forall & i \in S \\
 & \psi_i \in \left\{0,1\right\} & & \\
 & x_{ij} \geq 0 & &
\end{array}
$$

Question (e)

Using FICO Xpress the above MIP was modelled and solved as shown below.The value of $b = 1$ was used for this question. The data set was provided and is included with the MOSEL code.

```
Option to close a factory
=========================
Problem status: Optimum found

  A B C D E F G H I J
1 2 3 0 0 0 0 0 3 0 0
2 1 0 0 2 0 2 0 0 0 0
3 0 0 0 0 3 0 2 0 0 1
4 0 0 0 2 0 0 0 0 3 0
5 0 0 2 0 0 0 0 0 0 1
6 0 0 0 0 0 0 0 0 0 0

Total Costs:= 94
```

Figure 3: FICO Xpress solution of the MIP in Part 3

The plot in Figure 3 shows the units moved from each factory to individual warehouses. As shown the total minimised cost is 94.

Conclusions

It is apparent that while accepting the logistic company's offer would decrease profits from 97 to 96.1, the optimal decision lies within the third scenario. That is not to take the offer and instead close down factory *N*º 6. This yields the optimal scenario of minimising cost from 97 to 94.

The initial problem is a Linear Programming (MLP) problem, which because of the integer nature of the provided data, produces integer solutions. The problem is a network flow problem with only positive flow from factories to warehouses. It also has underlying aspects of network design, since of all available paths, only the ones that minimise cost are actually selected an implemented.

The second problem is an Integer Programming (MIP) problem. It is still a network flow and design problem in the same way that the first problem is. This problem can also be referred as a binary IP since the variables are constrained to have the value of either 1 or 0.

The third and final problem has all the characteristics of the previous problems in being a network flow/design MIP problem, but is has the added characteristic that it is also an decision problem on facility location, i.e. whether a factory will exist at a given location or not.