Differential Calibration for A/B-steps on Core XY Printers

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Abstract

Traditionally, A- and B-motor step calibration in Core XY 3D printers is often neglected, assuming precise calculation based on pulley geometry. This oversight[1] overlooks influences from belt tension and manufacturing/assembly tolerances. This paper challenges this practice and introduces a novel and robust calibration method to address these factors.

Existing approaches for Cartesian printers involve challenging direct measurements or overlook material shrinkage, extrusion width, and print bed thermal expansion in calibration prints. In response, our proposed method specifically tackles these challenges, providing a robust solution to enhance accuracy. Adaptable to Cartesian printers with minor modifications, this method offers an accurate and robust calibration process for improved 3D printing precision.

This document aims to prove the correctness of the method, a practical procedure to follow can be found at: github.com/cmdremily/BoronTrident/.

1 Introduction

Dimensional inaccuracies in 3D prints often stem from imprecise XY movement of the toolhead. While correcting these inaccuracies through slicer adjustments may provide initial relief, it often leads to a cascade of other adjustments. Slicer adjustments can never really fix some issues such as local over-/under-extrusion or uneven perimeter spacing. Likewise, skew correction can only partially fix inaccurate XY motion of the toolhead.

Our method aims to address one primary cause of inaccurate XY motion: incorrectly set A-/B-motor steps (or rotation_distance in Klipper). In this paper we will prove the correctness of the method a practical procedure to follow can be found at: github.com/cmdremily/BoronTrident/.

[1] Please see the appendix for more details.

1.1 Error Sources to be Controlled

To achieve precise A/B step calibration, accurately measuring the tool-head movement is crucial. Direct measurement on a Core XY proves challenging, leaving us with indirect calibration through printing a calibration model. Any printed calibration model has the following sources of error that must be controlled:

- Thermal expansion/contraction of the print bed
- Extrusion width error, both filament width inconsistency and extrusion feed rate error
- Material shrinkage
- Measurement error, both from the measuring device and the measuring technique

We will keep these in mind as we derive the method.

2 Method Overview

Assume a solid rectangular part with a reference dimension, L. This part 'perfect', in the sense that it is printed without any inaccurcies, material shrinkage or other defects. A crosssection of it is illustrated in figure 1. The part was printed with external perimeters first so any overlapping perimeters do not affect the external perimeter surface. When extruding the external perimeters of width W, the toolhead moves a distance of X centrepoints of the extrusions. We can express the reference dimension of this perfect part as:

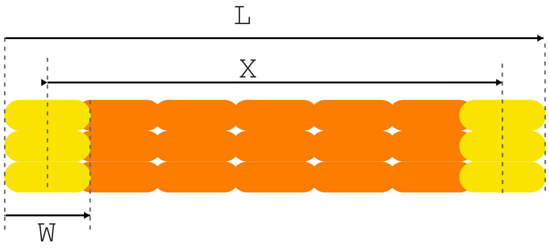

Figure 1: Ideal case of a reference width with solid infill in-between.

$$L = X + 2\frac{W}{2} = X + W.$$

When this part is printed in reality, there will be several sources of error: The X toolhead movement will have a scale error λ, the whole model will shrink uniformly[2] by a factor δ, the extrusion width will have a scale error of α, and there will be additive random errors such as those caused by variance from filament diameter and measurement error which we bake into one variable: ϵ. This means that when we measure L on the printed part, the indicated length l can be expressed as:

$$l = \delta\left(X\lambda + W\alpha\right) + \epsilon.$$

Let's further assume that we now have two such solid parts but with different reference dimensions: $L_1 = X_1 + W$ and $L_2 = X_2 + W$. The indicated lengths of these dimensions are likewise $l_1 = \delta\left(X_1\lambda + W\alpha\right) + \epsilon_1$ and $l_2 = \delta\left(X_1\lambda + W\alpha\right) + \epsilon_2$.

By taking the differences between the reference dimensions we get:

$$\Delta L_{1,2} = L_1 - L_2 = X_1 - X_2$$
$$\Delta l_{1,2} = l_1 - l_2 = \left(X_1 - X_2\right)\lambda\delta + \left(\epsilon_1 - \epsilon_2\right) \quad (1)$$

then by taking the ratio of the above, we get:

$$\frac{\Delta l_{1,2}}{\Delta L_{1,2}} = \frac{\left(X_1 - X_2\right)\lambda\delta + \epsilon_1 - \epsilon_2}{X_1 - X_2} \stackrel{(1)}{=} \lambda\delta + \frac{\epsilon_1 - \epsilon_2}{\Delta L_{1,2}} \quad (2)$$

. In other words, we can get the product of the movement scale error, λ, and the material shrinkage factor, δ, directly from know and measured quantities alone. In particular we can get this product independently of the extrusion width and any errors that affect all external extrusions equally. Note that the additive error terms (filament diameter inconsistency, extrusion inconsistency, measurement error etc) are divided by the difference of the reference dimensions and thus can be made arbitrarily small by picking suitable the reference dimensions.

This leads us to our first lemma:

Lemma 1. *Given two reference dimensions on the same axis; By taking the difference of the measured values of these reference dimensions in ratio to the difference of the intended values, a linear scale error is obtained that is immune to any error that is independent of the feature length and which equally affects both measured values. In other words, errors from extrusion width and measurement bias do not affect the computed scale error.*

Unfortunately, we do not directly obtain λ from (2), but we will address this by design of the calibration model in section (3).

2.1 Understanding the Error Term

The error term in (2) affects the computed scale factor additively. The percentual error in the scale factor product is computed like so:

$$\frac{\frac{\Delta l_{1,2}}{\Delta L_{1,2}}}{\lambda\delta} - 1 = \frac{\lambda\delta + \frac{\epsilon_1 - \epsilon_2}{\Delta L_{1,2}}}{\lambda\delta} - 1 = \frac{\epsilon_1 - \epsilon_2}{\lambda\delta\Delta L_{1,2}}$$

. In section 3 we will present a calibration model that removes the influence of material shrinkage on the computed λ, i.e. $\delta = 1$. Let's examine what the observed error would be if the toolhead movement was exact, i.e. $\lambda = 1$, then we know that the percentual error in the computed value of λ is:

$$\frac{\epsilon_1 - \epsilon_2}{\Delta L_{1,2}} \quad (3)$$

. Which becomers the lower bound on how closely the calibrated value of λ can track the true value, the calibration accuracy.

[2] This is only generally true for convex and solid models, which is the case here. For complex shapes with infill the actual shrinkage depends a lot on internal stresses that are hard to predict.

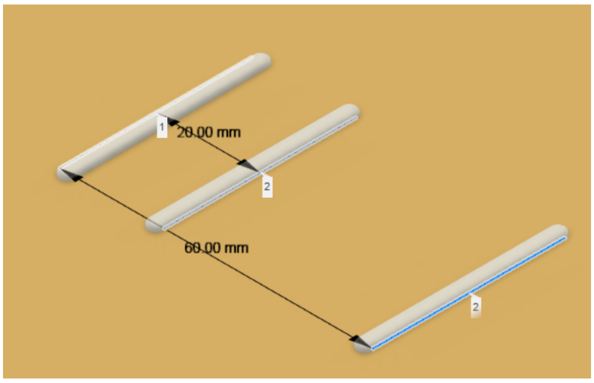

Figure 2: One extrusion wide lines to measure l_i from.

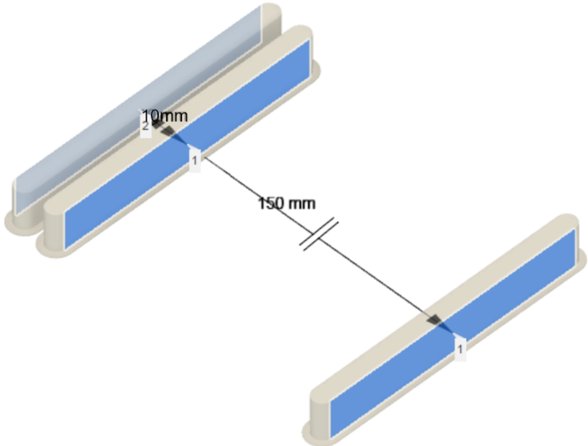

Figure 3: Improved calibration model for a single axis. Distances not to scale for image size reasons.

3 Designing a Calibration Model

Consider the calibration 'model' shown in figure 2. It consists of three single line extrusions with known dimensions for $L_1 = 20$ and $L_2 = 60$ as illustrated. While each line shrinks homogenously as it cools, the distance between them is independent of the material shrinkage factor, but now instead dependent on the thermal expansion of the print bed. Assuming that the print bed is held at a constant temperature by the printer from before print start until after the measurements are done, then the expression for the measured distances l_1 and l_2 becomes: $l_i = X_i\lambda + \delta W\alpha + \epsilon_i \quad \forall i \in \{1,2\}$. In other words, δ still affects the extrusion width but no longer the distance, X_i between centrepoints of the extrusions. We insert this new expression for l_i into (2) and get:

$$\frac{\Delta l_{1,2}}{\Delta L_{1,2}} \approx \lambda + \frac{(\epsilon_1 - \epsilon_2)}{\Delta L_{1,2}} \tag{4}$$

. This result is key, as it proves that lemma (1) holds for this calibration model as well.

Using this simple, calibration 'model' we can directly compute the scale error in the toolhead motion along the measured axis from easily measured quantities. The computed scale factor is also independent from material shrinkage and systematic extrusion width errors. All that's required is some calipers, a steady hand and a constant temperature on the print bed.

3.1 A more practical calibration model

The calibration model above, while theoretically useful, is unpractical; Measuring the distance between single extrusions accurately is difficult as there is very little for the caliper jaws to align on.

We improve the model by replacing each of the three lines with a 'pylon' as in figure 3. Each pylon is concave, symmetrical along the major axis, independent of the other pylons, and printed with solid material. This means that they will shrink uniformly towards the middle of each pylon. Therefore we can view each pylon as one extrusion of the simple model above, and with the same caveats about bed temperature as above, equation 4 applies and allows us to directly get λ with a small error.

Note that built-in brims are added to keep the pylons stable on the build plate during measuring. The brims are chosen so that they never touch eachother, as to maintain the pylons' independence. Each pylon should be printed with external perimeters first and have the seams positioned away from the measurement surfaces (marked blue in 3) to guarantee surface consistency. The pylons must also be printed with enough perimeters to produce a solid part that will shrink symmetrically, rectilinear infill doesn't guarantee this depending on if the number of infill layers is odd or even. Finally, the part must be printed without any over-extrusion to prevent material build-up in the middle of the

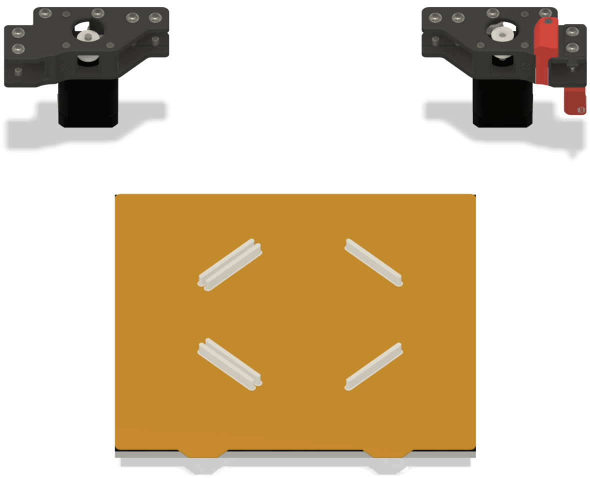

Figure 4: Calibration model correctly aligned for Core XY.

pylon during printing which may affect the measuring surface on the higher layers.

3.2 Aligning the Calibration Model

The calibration model as described above should be aligned so that the scale factor λ is measured in the primary direction of the motors. This means that for a Cartesian printer, the measurement directions should be axis aligned. For a Core XY printer the calibration model should be on a 45° diagonal as each of the motor A-/B-motors drive a diagonal, see figure (4).

Failure to do these steps will cause the calibration process to not converge on the correct steps value for the motors.

3.3 Dimensioning the Calibration Model

The error term in (4) is minimized when $\Delta L_{1,2}$ is maximized. We choose $L_1 = 150$ mm as that is the largest dimension that typical consumer calipers[3] can measure. L_2 should be minimized while still allowing a stable pylon to measure against. Anecdotal evidence suggests a pylon width of 3-4 mm is sufficient which gives $L_2 = 10$ mm as a good value.

[3] 150mm calipers have a few extra mm of range to be able to get them around 150mm objects to measure them.

4 Calibration Accuracy

The accuracy of the calibration depends on multiple factors. Chief among those are measurement error, filament diameter inconsistency, XY motion precision, and thermal expansion of the bed. We will treat each one in turn.

4.1 Measurement Error

For an estimated measurement error of ± 0.02 equation (3) gives us a worst case calibration error of: $\frac{(2\cdot 0.02)}{140} = 0.029\%$ (29 µm over 100 mm).

4.2 Filament Diameter Inconsistency

If we allow a square extrusion profile for this analysis[4], then an extrusion with width w mm, layer thickness h mm, and a toolhead move with a feedrate of v mm/s will have a voloumentric flow rate of: $whv\ mm^3/s$. For a nominal filament diameter of d, the linear extruder feedrate then becomes: $f = \frac{whv}{\frac{d^2}{4}\pi}$ mm/s.

How much does the extrusion width w change if the filament diameter changes by ε? Rewriting the above, gives w as:

$$\frac{fd^2\pi}{4hv} = w$$

introducing an error of ε to d gives:

$$\frac{f\left(d+\varepsilon\right)^2\pi}{4hv} = \frac{fd^2\pi}{4hv} + \frac{f\left(2d\varepsilon+\varepsilon^2\right)^2\pi}{4hv} = w_\varepsilon$$

$$w + \frac{f\left(2d\varepsilon+\varepsilon^2\right)\pi}{4hv} \overset{2d\epsilon \gg \varepsilon^2}{\approx} w + \frac{f2d\varepsilon\pi}{4hv} \approx w_\varepsilon$$

plugging in $\frac{fd^2\pi}{4hv} = w$ again allows us to simplify a bit:

$$\frac{2\varepsilon}{d}w \approx w_\varepsilon - w$$

. In other words, the error in extrusion width from a filament diameter variation of ε is $\frac{2\varepsilon}{d}w$, where w

[4] The actual error will be the same even with a pill shaped extrusion profile, the proof of this is left as an exercise to the reader.

is the original extrusion width, and d is the filament diameter.

Assume a high quality 1.75 mm filament can hold a local tolerance on the length of filament required to print the calibration model, of around $\varepsilon = \pm 0.01$ mm. With a nominal extrusion width of $w = 0.5$ mm, we get an error of: $\approx \pm 5.7$µm. As the extrusion is centered on the toolhead path and exterior perimeters are printed first, only half of this error will be on the measurement surface. I.e. an estimated contribution of $\approx \pm 2.9$ µm to ϵ can be expected from filament variance.

By taking the worst case values: $\epsilon_1 = +3$ µm and $\epsilon_2 = -3$ µm, and chosing $L_1 = 150$ mm and $L_2 = 10$ mm (see section 3.3), equation (3) gives a worst case calibration error of $\frac{14\text{µm}}{140\text{mm}} = 0.0043\%$ (4.3 µm per 100 mm) from the effects of filament inconsistency.

It should be noted that in practice the error will be significantly lower as the surface roughness that arises from filament inconsistency will affect each pylon approximately equally and thus be cancelled out in equation (3).

N.B. As of writing 2024-01-05, Prusament guarantees a diameter varition below ± 0.02 mm on the entire spool of their PETG, with actual variance on the entire spool often being closer to 0.01 mm than 0.02 mm. It's practically achievable to have ± 0.01 mm variance on the length of filament required for the calibration print.

4.3 XY Motion Precision

This depends highly on the printer geometry, so we'll use an example Core XY printer here that has 0.9 degree steppers, 64 µsteps and a pulley circumference of 40 mm (GT2 20 tooth pulley).

For the example printer there are $\frac{360}{0.9} = 400$ full steps per pulley revolution, or $400 \cdot 64 = 25600$ µsteps. This means that each microstep moves the toolhead 40mm/25600steps = 1.56µm/step. We will take 1.56 µm as the maximum error from the XY motion precision. We assume steps are not skipped.

4.4 Thermal Expansion of Bed

Thermal expansion of a material is computed as $\frac{\Delta L}{L_0} = \alpha \Delta T$ where $\frac{\Delta L}{L_0}$ is the change in length over the original length, α is the coefficient of thermal expansion and ΔT is the change in temperature. A typical steel has $\alpha = 1.2E^{-5}$ and a typical print bed can stay within ± 2 degrees kelvin of the set value. This gives a maximum percentual change in elongation of the print bed of $1.2E^{-5} \cdot 4 = 0.0048\%$ or 4.8 µm per 100 mm.

4.5 Total Error

Adding up the error sources from above we end up with a worst case calibration error of 40 µm per 100 mm. This error is dominated by the influence of measurement error. Given that the same instrument will often be used for calibrating the printer as for measuring the produced parts, this means that the indicated error will often be less, for the instrument used above, the indicated error would be closer to 10.7 µm per 100 mm.

4.6 Summary

The best venue for improving accuracy of the calibration is to improve the measurement accuracy by use of better instruments and measurement procedure. Steppers with 0.9 degree precision and 64 microsteps are not limiting the impact on the part accuracy. Filament diameter variance can be controlled to only contribute 4.3 µm per 100 mm of error. The impacts of thermal expansion of the bed are noteworthy, and the bed must be kept at a constant temperature during printing of the test parts and measuring of them.

5 Closing Words

We have proven that as long as the error sources are well controlled and the presented calibration model (or similar) is used, consumer grade FDM 3D printers can be calibrated to a high degree of accuracy with commonly available tools and materials. For the examples provided here, we show that a worst case error of 40 µm per 100 mm can be achieved with proper procedure and instruments. A user validating the calibration with the same instrument they used for performing the calibration might see an indicated case error of 11 µm per 100 mm due to the inherent inaccuracy in the instrument being cancelled out.

6 Appendix: Why is the calculated A/B-steps value incorrect?

The A/B rotation distance is commonly considered a "calculated" quantity, where the value is given from the pulley geometry and considered an exact value. To see why this is wrong, consider that we can also calculate the exact value for the rotation distance for the extruder, and this is often done, yet the norm is to calibrate this value even after calculating it. Why would the extruder be special in this way? Another way to see that the calculated A/B rotation should be calibrated is to consider the following scenario: Two 20 tooth pulleys are attached to a shaft, and two idlers are attached to another shaft 1 meter away. We cut a belt that has the ideal length and pull it over the bottom pulley and idler, let's say that this belt has 1020 notches. Next we cut another belt, this time with 980 notches, and we stretch it over the top pulley and idler. Next we turn the pulley 51 full revolutions. The bottom belt with 1020 notches will have rotated one whole revolution ($\frac{51 \cdot 20}{1020} = 1$), but the top belt with 980 notches will have rotated $\frac{51 \cdot 20}{980} \approx 1.04$ revolutions. This means that as both belts span the same length, if you would draw a dot on both belts, the dot on the top belt will move faster and further than the bottom dot. In other words, the belt tension affects how far the belt (or the X carriage) moves with each revolution, therefore the rotation distance must be calibrated for A/B motors.

In a way this can be understood as, the rotation distance is the actual belt pitch multiplied by the tooth count on the pulley, but the belt pitch is a function of the belt tension and manufacturing tolerance, it's not an universal constant.