Differential Calibration for A/B-steps on CoreXY Printers

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Abstract

This paper presents a novel and accurate, iterative calibration procedure for A and B motor 'steps per mm' on CoreXY 3D printers. In contrast to current procedures, we control for material shrinkage, extrusion error, and other bias errors using differential measurements. This results in a calibration that directly addresses mechanical accuracy in isolation. We prove the correctness and perform error propagation to mathematically determine real world accuracy of the calibration from estimated measurement accuracy. Other means of calibration for A and B motor steps generally ignore the impact of material shrinkage which is commonly around 0.5 - 1%, resulting in a poor calibration. We reject the commonly held notion that the A and B motor steps are accurately calculated from pulley geometry. We motivate this by example and show with empirical data that calibrating these values improves dimensional accuracy and corrects for a large fraction of skew error as well. Although intended for CoreXY printers running Klipper firmware, the method can easily be adapted for Cartesian printers and other firmware. *This document aims to prove the correctness of the method, a practical procedure to follow can be found at: github.com/cmdremily/BoronTrident/.*

1 Introduction

Dimensional accuracy in FDM 3D printing is influenced by both machine errors and material properties. Machine errors refer to deviations caused by the printer's mechanical inaccuracies, like erroneously set stepper motor 'steps per mm' or frame misalignment. Material properties encompass issues like filament shrinkage, flow characteristics, etc. In general machine errors should be corrected for in hardware or firmware and material properties should be corrected for in the slicer by adjusting how the part is printed.

As an example, take the common practice of adjusting for dimensional inaccuracies by scaling the part in the slicer. On an hypothetical printer that has a +5% and -5% scale error on the X and Y axis respectively, this will result in the X axis being under extruded by 5% and the Y axis being over extruded by 5%. Regardless of how the part is scaled or which extrusion multiplier is chosen, some part of the model will always be either over or under extruded. This highlights the necessity of correcting for machine errors directly, instead of in the slicer.

This paper addresses the machine error introduced from improperly set A and B motor steps, an often overlooked or incorrectly compensated for error source.

N.B. We use the words 'accuracy', 'precision' and 'trueness' as outlined in ISO 5725.

1.1 Background

The 'steps per mm' (or 'rotation distance' in Klipper) of an axis's stepper motor determines the toolhead's movement for a given input to the stepper motor. For instance, a rotation distance of 40 mm in Klipper indicates that the toolhead moves 40 mm for a full revolution of the stepper motor's shaft. In Cartesian machines, this movement aligns with the printer's X and Y axes for the respective X and Y steppers; on a CoreXY machine, the A and B steppers move the toolhead diagonally instead.

To accurately calibrate this value for a specific motor, isolating the travel of the toolhead attributed to that motor is crucial. Otherwise, the measured distance reflects the combined 'steps per mm' values and their errors from both motors.

Calibration of XY motion falls into two main categories: direct measurement, where an instrument physically contacts the toolhead, and indirect measurement, which involves gauging toolhead motion through dimensions of a printed calibration model.

Direct measurement methods, while straightforward and effective for Cartesian printers, depend heavily on the precision of instrument alignment with the travel direction. For example, a dial gauge with 25 mm travel and 0.01 mm precision, aligned within a 1° error margin, yields an accuracy of 0.06%, or 0.06 mm per 100 mm. However, on CoreXY printers, the lack of suitable alignment references makes instrument alignment challenging, rendering direct measurement methods less suitable.

Indirect measurement methods have historically struggled to distinguish between the scale error from the 'steps per mm' setting and material shrinkage, often leading to poor calibration. Material shrinkage alone can introduce an error ranging from 0.3% to 1%, potentially resulting in up to 1 mm of dimensional inaccuracy per 100 mm. Without precise prior knowledge of material shrinkage, the accuracy of such calibration is compromised[1].

Our approach addresses the challenge of material shrinkage in indirect calibration methods by utilising standalone 'pylons' printed on the bed. By measuring the distances between these pylons in situ, we achieve an accurate calibration independent of material shrinkage.

[1]The typical calibration process involves reprinting the calibration model after adjustments, using the same material, which can mislead users into believing they have achieved precise machine calibration. In reality, this process often only achieves an alignment between machine inaccuracies and material characteristics. Consequently, while dimensional accuracy might seem improved, the core issues—such as incorrect toolhead motion or extrusion rates—remain unaddressed. Users may then adjust the extrusion multiplier, attempting to correct what appears to be an extrusion issue, but the underlying problem lies in the XY calibration. Although this method can yield satisfactory results by compensating for errors with print settings adjustments, changing materials disrupts this delicate balance, necessitating further calibration adjustments. This cycle underscores the importance of distinguishing between machine calibration and material-specific compensations.

1.2 Adapting the Process for Cartesian Printers

To apply this calibration process to Cartesian printers, the calibration model needs to be rotated by 45°. This adjustment ensures alignment with the Cartesian X and Y axes, rather than the CoreXY A and B diagonals. Additionally, for compatibility with various firmware, the calculations used to derive the calibration parameters must be adapted. Specifically, the conversion from Klipper's 'rotation distance' to the 'steps per mm' metric (or equivalent) used in the target firmware requires modification.

In scenarios where the printer lacks an enclosure, it is crucial to allow the print bed sufficient time to reach thermal equilibrium before initiating the print. Similarly, ensuring that the printed part has stabilised thermally with the surrounding environment prior to measurement is essential for obtaining accurate results.

1.3 Calibration Accuracy

Instrument accuracy can be deceptive. Calipers typically have a high precision when used correctly, meaning that if using the same instrument for calibration and subsequent part measurements may falsely suggest high part accuracy, which might still deviate from true dimensions due to the trueness of the instrument.

2 Calibration Method Overview

Our proposed method is predicated on two foundational ideas: First, freestanding, parallel, convex pylons constructed from solid material on the build plate will shrink uniformly and identically, allowing the distance between their centerlines to remain constant and unaffected by material shrinkage. This holds true provided the build plate does not thermally expand or contract from the start of the print until the completion of all measurements. Second, differential measurements negate bias errors, rendering the pylon thickness irrelevant to our calibration. For a visual representation of this concept, refer to Figure 1, which illustrates the measurement of two pylons.

Figure 1: Two independent pylons being measured on the build plate.

We will first elaborate on differential measurements and their advantageous properties for our method. Subsequently, we will demonstrate how combining pylons with differential measurements enables us to precisely quantify the 'steps per mm' error in the relevant motors, isolated from the effects of material shrinkage and extrusion width variations.

2.1 Theoretical Basis for Differential Measurements

Consider a reference dimension L_i and its corresponding measured length ℓ_i. Typically, the measured length is influenced by a linear scale error, a bias error, and random error, such that:

$$\ell_i = \alpha L_i + \beta + \epsilon_i$$

Here, α is the scale factor, β is the bias error, and ϵ_i represents the random error. When we have two reference dimensions affected by the same bias and scale errors, we can deduce the scale factor α directly:

$$\ell_2 - \ell_1 = \alpha\,(L_2 - L_1) + \epsilon_2 - \epsilon_1$$
$$\alpha = \frac{\ell_2 - \ell_1}{L_2 - L_1} + \frac{\epsilon_1 - \epsilon_2}{L_2 - L_1}$$

Thus we eliminate the influence of bias errors and reduce the impact of random errors on the computed scale factor.

2.2 Pylon Configuration for Calibration

Assuming a configuration of three pylons in a row (as shown in Figure 2), let D_1 and D_2 denote the nominal edge to edge distances between pylons 1 and 2, and pylons 1 and 3, respectively. The corresponding nominal centre to centre distances are X_1 and X_2, with W being the nominal width of a pylon. This gives us $D_i = X_i + W$, and actual measured lengths d_i and the printed pylon width w relate as follows:

$$d_i = \alpha X_i + w + \epsilon_i$$

We assume homogeneous shrinkage of the pylons, with any non-uniformity in shrinkage or printing anomalies encapsulated within the random error

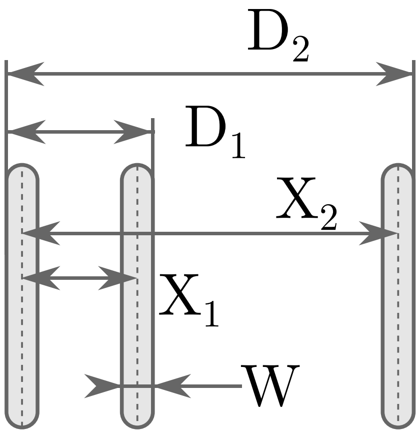

Figure 2: Three pylons in a row with two reference distances.

term ϵ_i. The centre to centre distance is influenced by a linear scale factor that reflects the 'steps per mm' error for the motor controlling that axis. By applying the principles outlined in Section 2.1, we deduce the scale factor α in terms of the measured distances d_1 and d_2 and the nominal distances D_1 and D_2:

$$\alpha = \frac{d_2 - d_1}{X_2 - X_1} + \frac{\epsilon_1 - \epsilon_2}{X_2 - X_1}$$

Obtaining X_i is a bit cumbersome, so we note that $D_2 - D_1 = X_2 - X_1$ and substitute into the above to get:

$$\alpha = \frac{d_2 - d_1}{D_2 - D_1} + \frac{\epsilon_1 - \epsilon_2}{D_2 - D_1}.$$

which allows us to conveniently deduce the scale error regardless of the actual pylon width and material shrinkage.

With the above, we are now able to put three independent pylons in a row along a motor's driven axis, measure between them and compute the necessary corrections for the 'steps per mm' value for the motor. However there are several practical aspects that must be considered in order to obtain a precise calibration.

3 Ensuring Calibration Accuracy in Practice

Calibration is only meaningful if there is assured confidence in its validity. A calibration that cannot be substantiated is tantamount to a conjecture rather than a precise adjustment. For indirect methods such as the one we propose, validating the calibration of the machine directly poses a challenge. Any reference part we print is subject to shrinkage and other variables.

To establish confidence in the accuracy and precision of our calibration method, we must scrutinise the various sources of error and determine an upper limit on the potential deviation within the calculated scale factor.

Each of the following error sources contributes additive to the random error term ϵ_i.

3.1 Thermal Expansion of the Build Plate

The freestanding nature of the pylons means that any thermal expansion of the build plate will affect the distance between them. It is imperative that the build plate reaches thermal equilibrium prior to commencing the calibration print. The presence of thermal equilibrium can be verified using a thermal camera. In the absence of such equipment, a pragmatic alternative is to monitor the bed's power draw[2] until it ceases to decrease and stabilises. If the chosen material and bed surface permit, conducting the print with an unheated bed is preferable.

The coefficient of thermal expansion for spring steel print sheets is alloy-dependent. For the purpose of our analysis, we will employ $\alpha = 1.2E^{-5}$, which is representative of the higher end of the spectrum for common alloys. Accounting for a typical temperature fluctuation of ±2°K, we arrive at a thermal expansion factor of ±0.0024%, or 2.4 µm per 100 mm.

3.2 Inconsistent Filament Cross-Section

Variations in filament cross-section during printing can result in pylons with non-uniform widths, contributing to the random error term ϵ_i. An extrusion of width w mm, layer height h mm, and toolhead speed v mm/s yields a volumetric flow rate of

[2]Klipper UIs Mainsail and Fluidd shows this in the console as a percentage of full power.

whv mm^3/s. For a filament with nominal diameter d, the linear extruder feed rate f is:

$$f = \frac{whv}{\frac{d^2}{4}\pi}$$

This presumes a square extrusion profile, an approximation that simplifies the calculation without affecting the outcome. A more complex, pill-shaped extrusion profile would arrive at the same result.

Holding all variables but the filament diameter constant, the extrusion width w relates to the diameter d as:

$$w = C_0 d^2$$

Where $C_0 = \frac{f\pi}{4hv}$ is a constant. An error ε in d results in a new width w_ε:

$$w_\varepsilon = C_0 \left(d + \varepsilon\right)^2 = w + C_0 \left(2d\varepsilon + \varepsilon^2\right)$$

$$w_\varepsilon \overset{2d\varepsilon \gg \varepsilon^2}{\approx} w + \frac{2\varepsilon}{d} w$$

which yields the error in the extrusion width, δ as:

$$\delta = w_\varepsilon - w \approx \frac{2\varepsilon}{d} w$$

As of 2024-01-05, Prusament specifies a tolerance of ± 0.02 mm on their PETG filament, though empirical evidence suggests that many spools maintain a tighter tolerance of ± 0.01 mm over substantial portions of the filament. Taking $\varepsilon = \pm 0.01$ mm for these more precise sections and a nominal extrusion width of $w = 0.5$ mm, the error bound is $\approx \pm 5.7$µm. Considering that the extrusion is centred on the toolhead path and exterior perimeters are printed first, only half of this error is expected to impact the measurement surface, contributing an estimated $\approx \pm 2.9$ µm to the random error ϵ_i.

3.3 Non-identical pylon shrinkage

If each pylon is homogeneous and symmetrical, and the material shrinks uniformly, then the pylons should shrink identically. However, some variance is inevitable.

Analytically determining this error from non-uniformity is impractical; instead, we rely on empirical observations. Our calibration experiments involved measuring the width of pylons across several prints, revealing a maximum variance of 2 µm. This small variance aligns with the observed filament diameter tolerance, leading us to surmise that non-identical pylon shrinkage effects are subsumed within the filament diameter variance.

3.4 Measurement Error

Typical consumer-grade 150 mm calipers possess an accuracy of ± 20 µm, setting the lower limit for the trueness of our calibration. The precision of caliper measurements, a significant determinant of our calibration's upper accuracy bound, hinges on the measurement technique employed.

Enhanced measurement precision is achieved by conducting multiple independent measurements, eliminating outliers, and using the median of the collected data. The median is preferable to the mean as a more robust estimator in the face of an unknown underlying distribution of measurement values—due to varied user techniques.

Empirical testing suggests that with adequate practice, a measurement precision of ± 20 µm is attainable. It's important to note that any bias error introduced by consistently applying excessive force—and thus potentially deflecting the pylons—will be nullified in differential measurements. Therefore, while consistent force application is crucial, one must be cautious not to exert so much force as to risk delaminating the pylons from the build plate.

Assuming, for argument's sake, that the underlying distribution of measurements is Gaussian with a 3σ value of 20 µm, and by taking five measurements and selecting the median, we can approximate the variance of the median as a precision indicator:

$$\sigma_{\text{median}} \approx \frac{\sigma_{\text{mean}}}{\sqrt{n}} 1.253$$

$$3\sigma_{\text{median}} \approx 11.2 \text{ µm}$$

While we recognise the likelihood that the Gaussian assumption will not hold true, it offers the only precision estimate available to us for the measurement method suggested above. To reflect this uncertainty, we round the calculated precision to one significant figure, using ± 10 µm as the measurement error's contribution to the random error term in our calibration process.

3.5 Stepper Motor Resolution

The resolution of the XY motion plane depends on the printer hardware. For this analysis we will assume a CoreXY printer with 0.9° steppers, 64 microsteps, and a 40 mm pulley circumference. This results in $\frac{360}{0.9} = 400$ full steps per revolution, or $400 \cdot 64 = 25600$ microsteps.

This equates to a nominal[3] toolhead movement of:

$$\frac{40 \text{ mm}}{25600 \text{ micro steps}} = 1.56 \text{ µm/micro step}$$

The firmware may either truncate positions or round to the nearest step, resulting in a maximal positional error of 1.56 µm or ±0.78 µm from the motor resolution.

In general, assuming belts are tensioned correctly, backlash from a belt driven axis is minimal and we will ignore it for our analysis as we currently lack means of quantifying it. Likewise we assume that the motion system is free friction and belt flex to the point where one micro step input will move the toolhead the appropriate amount.

Therefore we accept 1.56 µm as the random error from the XY motion system, in addition to the scale error that the calibration will remove.

3.6 Skew of the XY Motion System

Skew in a CoreXY printer arises from two principal sources: the non-squareness of the frame and axes, and the uneven belt tension across the motors. While the skew stemming from belt tension is addressed by our calibration process, the skew due to frame misalignment must be corrected through firmware compensation.

Skew can affect the calibration print, impacting the measurements taken from it. Each round of calibration will remove a part of the belt tension induced skew, reducing the measurement error in the next round. This iteratively removes skew error until the calibration converges.

Upon reaching this iterative convergence, it is advisable to pause and perform a skew correction in the firmware. This step ensures that subsequent calibration rounds can converge on the true values without interference from residual structural skew. It is also essential to disable any preexisting skew adjustments in the firmware prior to starting the calibration to allow the procedure to effectively eliminate the skew resulting from belt tension disparities.

[3]This is not an exact value, which is why this document exists.

3.7 Summary

Under the following conditions:

- Build plate is maintained within ±2 °K of the set temperature, or build plate is not heated.
- Filament with a local variance of ±0.01 mm and thin layers are used for the calibration print.
- Pylons shrink uniformly, by slowly letting them cool to ambient temperature.
- A measurement precision of ±0.01 mm is maintained and the instrument has an accuracy of ±0.02 mm.
- 0.9° steppers with 64 microsteps are used and the XY motion system is free from backlash and significant friction.
- Skew correction is initially disabled, then recalibrated and applied after the process has converged once, and then this calibration is repeated until convergence again.
- A pylon setup with $D_2 = 150$ mm and $D_1 = 10$ mm.

We can calculate the precision of the calibration as follows:

$$\frac{\epsilon_1 - \epsilon_2}{D_2 - D_1} < \frac{2\epsilon_{max}}{140}$$

$$\epsilon_{max} = \epsilon_{\text{bed}} + \epsilon_{\text{filament}} + \epsilon_{\text{measurement}} + \epsilon_{\text{motor}}$$

$$\epsilon_{max} = 2.4 \text{ µm} + 2.9 \text{ µm} + 10 \text{ µm} + 0.78 \text{ µm}$$

$$\epsilon_{max} \approx 0.017 \text{ mm}$$

$$\frac{\epsilon_1 - \epsilon_2}{D_2 - D_1} < 2.5E^{-4}$$

Or in other words, the percentual precision in scale will be better than 0.025% or 25 µm per 100 mm under the above conditions. The accuracy is then the instrument trueness added to that. Giving a total accuracy of $\frac{0.02 \text{ mm}}{150 \text{ mm}} + 0.025\% = 0.038\%$ or 38 µm per 100 mm.

4 Closing Words

5 Closing Words

This paper has demonstrated a novel approach for calibrating A and B motor steps on Core XY 3D printers, leveraging the use of independent pylons to achieve high levels of accuracy and precision with commonly available tools and materials. The proposed method addresses critical shortcomings in traditional calibration techniques, particularly in accounting for material shrinkage and extrusion errors.

Through empirical validation, we have established that a worst-case error of 45 µm per 100 mm is achievable under rigorous adherence to the outlined procedure and with the use of appropriate measuring instruments. Moreover, the inherent instrument inaccuracy, when mirrored in both calibration and subsequent validation measurements, may present a perceived error reduction to 25 µm per 100 mm.

Future work could aim at improving the practicality of the calibration model, suggesting suitable material/print bed combinations to avoid heating the bed, as well as improving the error bound by analysing some of the error sources we ignored with more rigour.

6 Appendix: Why is the A/B steps value calculated solely from pulley geometry incorrect?

The A/B rotation distance is commonly considered a "calculated" quantity, where the value is given from the pulley geometry and considered an exact value. To see why this is wrong, consider that we can also calculate the exact value for the rotation distance for the extruder, and this is often done, yet the norm is to calibrate this value even after calculating it. Why would the extruder be special in this way? Another way to see that the calculated A/B rotation should be calibrated is to consider the following scenario: Two 20 tooth pulleys are attached to a shaft, and two idlers are attached to another shaft 1 meter away. We cut a belt that has the ideal length and pull it over the bottom pulley and idler, let's say that this belt has 1020 notches. Next we cut another belt, this time with 980 notches, and we stretch it over the top pulley and idler. Next we turn the pulley 51 full revolutions. The bottom belt with 1020 notches will have rotated one whole revolution ($\frac{51 \cdot 20}{1020} = 1$), but the top belt with 980 notches will have rotated $\frac{51 \cdot 20}{980} \approx 1.04$ revolutions. This means that as both belts span the same length, if you would draw a dot on both belts, the dot on the top belt will move faster and further than the bottom dot. In other words, the belt tension affects how far the belt (or the X carriage) moves with each revolution, therefore the rotation distance must be calibrated for A/B motors.

In a way this can be understood as, the rotation distance is the actual belt pitch multiplied by the tooth count on the pulley, but the belt pitch is a function of the belt tension and manufacturing tolerance, it's not an universal constant.