

Exotic Options

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1 EXOTIC DERIVATIVES

Exotic derivatives are traded only OTC and used (in contrast to vanilla):

- to fit specific firm need for hedging
- to address tax or regulatory concerns
- for speculation (on interest rate, exchange rates, commodities prices)

Main issues:

- Complex payoffs should be thoroughly understood by customers
- Hedging tends to be difficult
- Reliable pricing model
- Position closure

2 ZERO-COST PRODUCTS

- A **package** - combination of standard European options, forwards, cash and the underlying asset.
Examples: bull/bear spreads, straddles, strangles etc.
- Any derivative can be constructed into a **zero-cost** product to be paid in arrears.
 - Let's take a derivative with maturity T and premium f . Instead of paying premium upfront - it can be structured to be paid at maturity with compounding $f(1 + R)^T$
 - Example of European call option - after conversion to zero-cost payoff is: $\max(S_T - K - A, -A)$, where $A = c(1 + R)^T$ and c is normal call premium

3 NONSTANDARD AMERICAN OPTIONS

Nonstandard American Options:

- OTC market
- Exchange-traded

Various flavors:

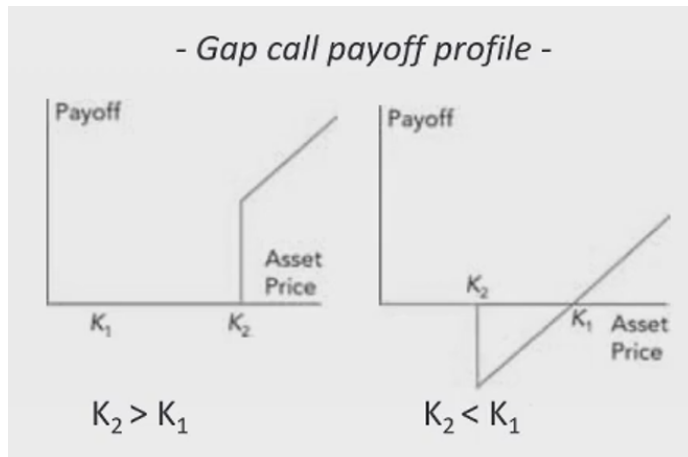
- Bermudan option - can be exercised only on specific dates
- Restricted exercise period feature (lockout period) Example employee stock options.
- Variable strike (e.g. strike depends on time) Example callable bond

Revaluation - Binomial trees can be used to value American options

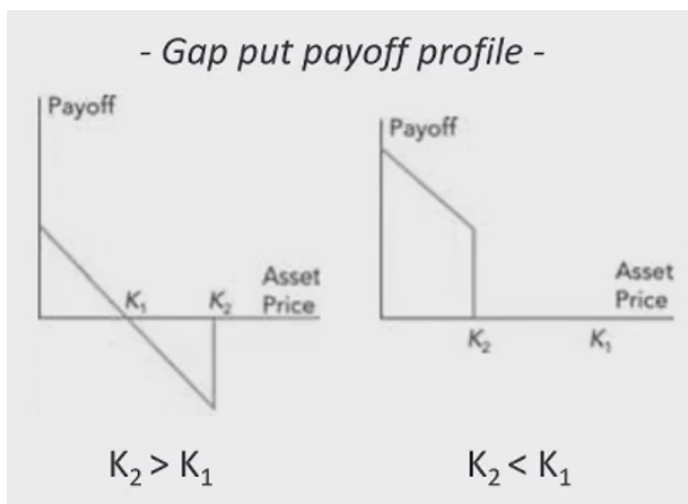
4 GAP OPTIONS

Has 2 strikes - K_1 and K_2

- **Gap call option** (European) $I_{S_T > K_2}(S_T - K_1)$
 - $K_2 > K_1$: nonnegative
 - $K_2 < K_1$: can be negative



- **Gap put option** (European) $I_{S_T < K_2}(K_1 - S_T)$
 - $K_2 < K_1$: nonnegative
 - $K_2 > K_1$: can be negative



5 FORWARD START OPTION

- Option comes into existence on a prespecified date in the future
- Strike is **not fixed in advance** usually ATM when starting
- Example: employee stock options if employer promises that they will be granted on future dates
- **Valuation** Mathematical formula for forward start option starting at time T_1 and maturing at time T_2 :
 - Its value at time T_1 is $c \frac{S_1}{S_0}$
 - Its value today at T_0 is ce^{-qT_1} where q is dividend yield c is the value of ATM option with a life of $(T_2 - T_1)$
 - For non-div paying stock $q = 0$ ATM forward start option price equals vanilla option price with the same life

6 CLIQUET AND CHOOSER OPTIONS

Cliquet option is a series of forward start options with certain rules to determine strike prices. It could consist of 3 options:

- 1 year vanilla option
- 1 year forward start option (starting in 1 year)
- 1 year forward start option (starting in 2 year)

Chooser option allows the holder to choose whether the option is a call or a put after a certain period of time (T_1)

- Can be considered as package of call and put options with different strikes and time to maturity
- Value of choose option then $\max(c, p)$ where c is price of call, p - put
- Rewriting $\max(c, p) = c + \max(0, p - c) = c + \max(0, PV(K) - S_1)$ and PV denotes present value from T_2 to T_1
- It means that chooser option is a package consisting of:
 - Call option with strike K and maturing at T_2
 - Put option with strike $PV(K)$ maturing at T_1

7 COMPOUND OPTIONS

Compound option - option on another option.

	Call	Put
Call	<i>Call on a call</i>	<i>Call on a put</i>
Put	<i>Put on a call</i>	<i>Put on a put</i>

Example of a call on a call:

The holder has a right to pay K_1 at time T_1 to purchase a long position in a call on an underlying. The latter allows to buy an asset for K_2 at time T_2 **Features:**

- The value of an underlying option is determined by the value of the underlying asset
- More leverage compared to vanilla options
- More sensitive to volatility than ordinary options

8 BARRIER OPTIONS

Barrier options have payoff dependent on whether the underlying asset reaches certain barrier level.

	Down	Up
Knock-out	Down-and-out call (put). Vanilla call (put) option that disappears if the underlying asset price hits the barrier level (from above).	Up-and-out call (put). Vanilla call (put) option that disappears if the underlying asset price hits a barrier level (from below).
Knock-in	Down-and-in call (put). Vanilla call (put) option that only comes into existence if the underlying asset price hits the barrier level (from above).	Up-and-in call (put). Vanilla call (put) option that only comes into existence if the underlying asset price hits the barrier level (from below).

Parisian options asset price must remain above/below the barrier for a specified number of days

Barrier options features:

- Path-dependent - depends on whether a barrier is reached
- less expensive than standard options -which is attractive for buyers
- No-arbitrage equalities have to be respected for barrier options:
 $c = c_{do} + c_{di}, c = c_{uo} + c_{ui}, p = p_{do} + p_{di}, p = p_{uo} + p_{ui}$
- Example: When barrier level is equal to strike $\rightarrow c = c_{ui}$
- Frequency of barrier observation increases \rightarrow what is impact on KI and KO option values?
 KI increases, KO decreases
- For down-and-out and up-and-out options Vega becomes negative. This is contrary to ordinary option vega behaviour? When spot price is close to barrier level the probability that barrier will be hit increases \rightarrow value decreases

9 BINARY OPTIONS

Binary options - options with discontinuous payoff profiles

(discontinuity of payoff creates an incentive for underlying price manipulation close to expiry)

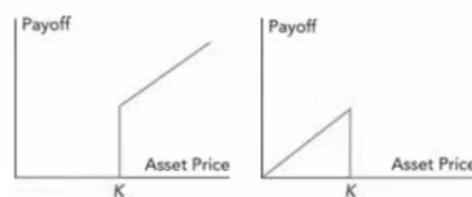
- **Cash-or-nothing call/put** pays the value of the stock when the contract is initiated if the stock price
- **Asset-or-nothing call/put** pays the value of the stock when the contract is initiated if the stock price ends up in-the-money at expiration.
- European call/put options can be decomposed in a package of binary cash-or-nothing and asset-or-nothing options

Value	Call	Put
Cash-or-nothing	$Ke^{-rT}N(d_2)$	$Ke^{-rT}N(-d_2)$
Asset-or-nothing	$S_0e^{-qt}N(d_1)$	$S_0e^{-qt}N(-d_1)$

- Cash or nothing call and put -



- Asset or nothing call and put -



10 LOOKBACK OPTIONS

Lookback options are options whose payoffs depend on the maximum or minimum price of the underlying asset during the life of the option

	Call	Put
Floating	Floating lookback call – purchase underlying at min price $S_T - S_{min}$	Floating lookback put – sell underlying at max price $S_{max} - S_T$
Fixed	Fixed lookback call with strike K $S_{max} - K$	Fixed lookback put with strike K $K - S_{min}$

where S_{min}/S_{max} - min/max price during life of option, S_T - is underlying price at expiry

- Lookback options are more expensive than ordinary ones
- The value of a lookback option depends on how often the spot price is observed
- What is the impact of spot price observation increase on the value of lookback option?
- Fixed lookback call or put is like an American option where investor pretends to choose the best possible exercise date

11 ASIAN OPTIONS

Average price call and put options pay the difference between the average stock price and the strike price.

- Average price call payoff $\max(0, S_A - K)$
- Average price put payoff $\max(0, K - S_A)$

Where S_A stands for spot average

Average strike options pay the difference between the average stock price and the final asset price (average represents a strike).

- Average price call payoff $\max(0, S_T - S_A)$
- Average price put payoff $\max(0, S_A - S_T)$

Where S_T is spot price at maturity T and S_A stands for spot average during the life of option

Features:

- Asian options are less expensive than regular ones with similar maturity/strike
- Average strike options guarantee that the average price paid for an asset is not greater than the final price (or average price received is not less)

12 MULTIPLE ASSET OPTIONS

- **Exchange option** - option to exchange one asset for another.
 - Example to buy euros for dollars from point of view of Russian investor is an exchange option.
- Consider a European option to give up an asset worth U_T at time T for an asset worth V_T .
The payoff is $\max(V_T - U_T, 0)$
- This option can be priced in BSM framework
- Position in exchange option can be then valued as per below
 $\max(U_T, V_T) = U_T + \max(V_T - U_T, 0)$
- **Basket option** - underlying is a basket of asset. Assets can be stocks, currencies, stock indices etc.
- Potentially a good hedging instrument for those seeking to reduce costs by hedging their aggregate exposure to several assets option are options involving 2 or more risky assets

13 VOLATILITY/VARIANCE SWAPS

- **Volatility swap** - exchange of fixed pre-specified volatility (σ_K) vs realized vol (σ) multiplied by some notional (L_{vol}): $L_{vol}(\sigma - \sigma_K)$
- Volatility swap is used when a trader needs to take a position dependent only on volatility.
- **Variance swap** - exchange of fixed variance V_K vs realized variance σ^2 multiplied by some notional L_{var} : $L_{var}(\sigma^2 - V_K)$
- Variance swaps are easier to value than volatility swaps because they can be replicated using calls and puts.

14 HEDGING EXOTICS

- Some exotic options are easier to hedge (e.g. Asian options)
- Other options such as barrier - more difficult, esp near barrier
- 2 types of hedging:
 - **Dynamic options replication** - replicating portfolio is adjusted continuously (e.g. Greeks hedging)
- **Static options replication** principle - 2 portfolios having the same value on boundary (S and T) must also be worth the same at all interior points → the goal is to find a portfolio of vanillas matching the exotic value on boundary
- Static option replication hedge can be left unchanged until the boundary reached. The trader must unwind the hedge portfolio and create a new hedge.

15 KEY TAKEAWAYS

- Many various types of exotic options design
- We have discussed several most basic ones but many more exist
- Valuation of exotic options can be a very advanced topic.
For some of the analytic formula exists, for others - numerical techniques
- Exotic options can be good hedge (Asian, basket options etc)
- They can be used to speculate on the particular view of the market (compound option, barrier options etc)
- Generally exotic options are less competitively priced than vanillas
- Specific part of exotic options market take vol/variance swaps
- Exotic options can be difficult to hedge hence a specific procedure static option replication was developed for this task which is alternative to Greek letter hedging
- Main idea of static replication is find a portfolio of vanillas whose value matches the value of exotic on a boundary