Quantitative Analytics.
Lectures. Week 1.
The time value of money. Временная стоимость денег.

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Lecture Topics

1. Interest rate and how it's formed.
2. SAR & EAR.
3. FV, PV and FoC.
4. Annuities and perpetuity.
5. TVM problems.

1 Dictionary, Definitions, Abbreviations

1.1 Dictionary

- IR - Interest rate - процентная ставка.
- Compounding - платежи (idk)

1.2 Definitions and Abbreviations

- SAR - Stated annual rate.
- EAR - Effective annual rate.
- FoC - Frequency of Compounding
- PMT - Payment
- r - Interest rate (at the moment).

2 Interpretation of the Interest rate

- **Equilibrium rate of return**
  Minimum rate of return an investor must receive in order to accept the investment.
- **The discount rate**
  Rate that must be applied to a cash flow to determine it's present value.
- **The opportunity cost**
  Value that investor forgo (loses) by investing.

3 How the Interest rate is formed

Interest rate =
Real risk-free rate (Reflects the current vs future consumption)
+
Expected inflation premium (Money costs less over time in a real terms)
+
Default risk premium (Compensation for possibility and probability that the borrower will default)
+
Liquidity premium (If financial product has a low liquidity - In case if you want to sell it quickly - you will be forced to take losses by selling it under the market price)
+
Maturity premium (Long term debts are more sensitive to the future IR falls, if you expect such).

4 SAR and EAR

SAR - Stated annual rate. Rate which is stated in the documents.
EAR - Effective annual rate. Rate which an investor really gets.

SAR does not account for infra-year compounding while EAR does.
$EAR \geq SAR$

5 FV and PV

PV - Present Value.
FV - Future Value.
FV_x - Future Value after x compounding periods (years by default)

$$FV_x = PV * (1 + SAR)^x$$

6 FV depends on FoC

Frequency of Compounding (FoC) - how many times during the year the transaction happens.

$$FV_x = PV * (1 + \frac{SAR}{FoC})^{x*FoC}$$

and reversed

$$PV = \frac{FV_x}{(1 + \frac{SAR}{FoQ})^{x*FoQ}}$$

7 Annuities

Annuity is a list of *identical* cash flows

Annuities can be

1. Finite
   (a) **Ordinary annuity** - Cash flows occur ***at the end*** of each ***compounding period***.
   (b) **Annuity due** - Cash flows occur ***at the beginning*** of each ***compounding period***.
2. Infinite
   (a) **Perpetuity.**

8 PV of an annuity

$$PV = PVofPMT_1 + PVofPMT_2 + \ldots + PVofPMT_n = \frac{PMT}{1+r} + \frac{PMT}{(1+r)^2} + \ldots + \frac{PMT}{(1+r)^n}$$

We have a geometric sequence with

$$b = \frac{PMT}{1+r}$$

and

$$a = \frac{1}{1+r}$$

So the sum equals to

$$S = b * \frac{q^n - 1}{q - 1}$$

substitute b and q

$$PV_{\text{annuity}} = PMT * \frac{(1+r)^n - 1}{r * (1+r)^n}$$

9 PV of a perpetuity

$$PV_{\text{perpetuiry}} = \lim_{n \to +\infty} PV_{\text{annuity}} = \lim_{n \to +\infty} PMT * \frac{(1+r)^n - 1}{r * (1+r)^n} = \frac{PMT}{r}$$