Quantitative Analytics.
Lectures. Weeks 5-8.
Interest Rate Futures.

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Contents

1 Материалы

Лекция почти полностью повторяет 6 главу книги John C. Hull: Options, Futures and other derivatives. Я оставлю pdfку с ней внизу.

2 Накопленный купонный доход

Фиксированные выплаты по облигации происходят в фиксированные моменты времени. Однако мы можем посчитать так называемый **накопленный купонный доход(accrued interest)**, определяющий долю купона, накопленную на данный момент времени, считая с момента предыдущей выплаты. Для подсчета используется следующая формула:

$$Accrued\ interest = coupon\frac{\text{количество дней, прошедших с последней купонной выплаты}}{\text{количество дней в купонном периоде}}$$

Купонным периодом называют количество дней между между моментами времени, в которые происходят выплаты, coupon - размер регулярно выплачиваемого купона.

Для подсчета дроби, умножаемой на купон, используются различные **правила о подсчете дней(day count convention)**.

В США наиболее популярны следующие 3:

- Казначейские облигации США(**US treasury bonds**) используют **actual/actual**: реальное количество дней, прошедшего с момента выплаты на реальное количество дней, между самой недавней выплатой и следующей после нее.
- Корпоративные и муниципальные облигации США(**US corporate and municipal bonds**) используют соглашение **30/360**. Это значит, что мы полагаем, что в году 360 дней(а значит, если в году выплат k, то промежуток между выплатами полагаем 360/k), а 30 означает, что мы считаем, что в месяце 30 дней и кол-во прошедших дней есть кол-во прошедших месяцев*30 + кол-во прошедших дней с начала месяца.
- Казначеские векселя(**US treasury bills**) используют подход **30/actual**, где **30** в числителе и **actual** в знаменателе обозначают в точности то же самое, что и в прошлых пунктах.

Пример: пусть есть облигация с полугодовыми выплатами и номиналом 100\$. Купоны выплачиваются 1 марта и 1 сентября. Годовая купонная ставка = 6%. Сегодня 13 июля и необходимо посчитать купонный доход.

- Для случая **actual/actual** мы получим, что между выплатами(от 1.03. до 1.09.) пройдет 184 дня, а между последней выплатой и данным моментом времени(от 1.03. до 13.07.) - 134 дня. Тогда накопленный купонный доход на 13 июля:

  $$\frac{134}{184} * 100 * \frac{6}{2} = 2.1848,$$

  где процентная ставка делится на 2, поскольку выплаты производятся раз в полгода(два раза в год).
- Для случая **30/360** аналогичные рассуждения дадут:

  $$\frac{132}{180} * 100 * \frac{6}{2} = 2.2.$$

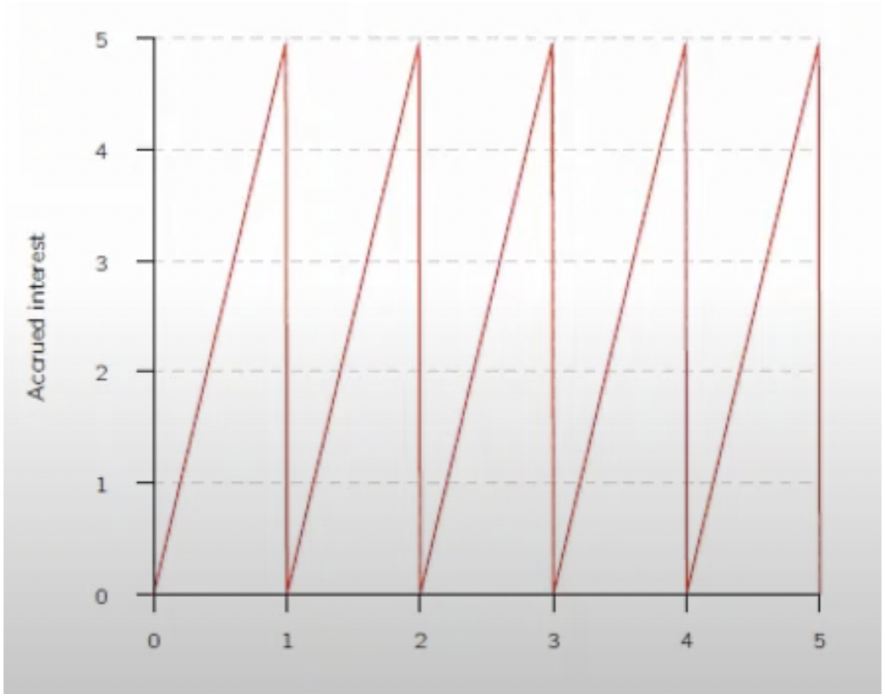


Fig. 1: Зависимость накопленного купонного дохода от времени. Важно, что после выплаты купона накопленный доход обнуляется и копится заново.

3 Казначейские векселя и их расчет

T-bills(казначейские векселя) рассчитываются и торгуются в терминах дисконтной ставки по соглашению о днях **actual/360**. Это считается некоторым аналогом купона, но купоном не является. Ставка, номинал и цена облигации(cash value) связаны следующим образом:

$$T - bills\ \ discount\ \ rate = \frac{360}{n}(100 - Y),$$

где Y - cash value, n - время до срока выплаты. Заметим, что реальная доходность не совпадает с дисконтной ставкой, пример: Пусть у нас есть казначейский вексель со сроком 0.5 года и годовой ставкой 5%. Номинал равен 100$. Найти ставку реальной доходности и стоимость векселя.

*****actual** для 0.5 года полагаем 180.

- Наша годовая ставка 5% и срок полгода дадут дисконтную ставку 5/2 = 2.5% и купон 2.5$ (100$*0.05*$\frac{180}{360}$)
- Cash Price: $Y = 100 - (T - bills\ \ discount\ \ rate) * n/360 = 100 - 5 * 180/360 = 97.5$
- Ставка реальной доходности: $(100/Y - 1) * 100 = (100/97.5 - 1) * 100 = 2.564$

4 Казначейские облигации и их расчет с учетом накопленного купона

Специфика котирования **облигаций(T-bonds)** следующие: цена облигации записывается как: "число дробь например, 95 5/32(это означает, что облигация стоит $95\frac{5}{32}$\$. по отношению к 100 долларам. То есть облигация на 400 долларов будет стоить $95\frac{5}{32} * 4\$$ Дробь обычно измеряется в 32ых долях. Поскольку по облигациям производятся выплаты, то Cash Price(цена облигации, по другому еще называют dirty price) = Quoted Price(котировка облигации(та самая 95 5/32), еще это называют present value of bond, либо чистой ценой(clean price)) + Accrued Interest(накопленный купонный доход)). Еще раз, Cash Price = Quoted Price + Accrued Interest.

Пример: пусть есть облигация с полугодовыми выплатами и номиналом 100$. Купоны выплачиваются 1 марта и 1 сентября. Годовая купонная ставка = 6%. Сегодня 13 июля и необходимо посчитать купонный доход. Пусть дополнительно эта облигация котируется как 102-11(то же самое, что 102 11/32). Требуется найти ее Cash Price.

- Воспользуемся тем, что Quoted Price $= 102 + \frac{11}{32} = 102.34375$
- Из задачи выше и определения о соглашении по датам для торговли облигации получим, что accrued interest $= \frac{134}{184} * 100 * \frac{6}{2} = 2.1848.$, заиспользовали **actual/actual** соглашение.
- Тогда Cash Price $= 102.34375 + 2.1848 = 104.52855$

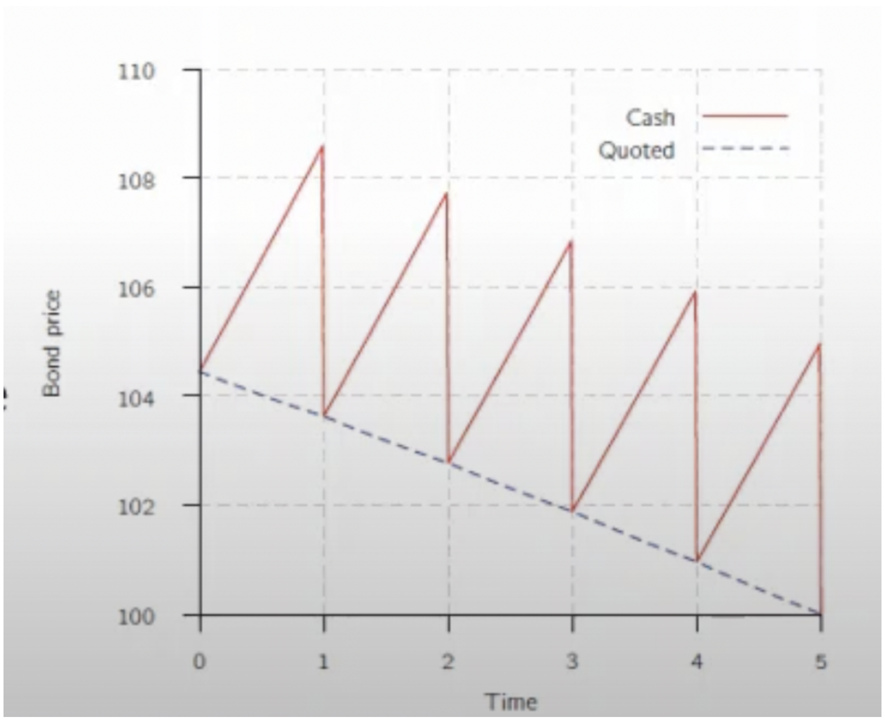


Fig. 2: Стоимость облигации в зависимости от времени и накопленного купонного дохода.

5 Фьючерсы на казначейские облигации США

- Здесь базовым активом служит не какой-то произвольный выпуск облигаций, а целый набор облигаций, дата погашения которых дальше на 15 лет от даты экспирации фьючерса.(То есть на момент поставки по фьючерсу облигации дата погашения этой самой облигации должна быть больше 15 лет)
- Таким образом, на самом деле под это условие подходит очень много облигаций
- Но возникает проблема по сравнению облигаций(купоны разные, даты экспирации тоже разные)
- Для уравновешивания облигаций по параметрам и приведения их параметрам к какому-то одному значению есть Conversion Factor(Конверсионный коэффициент). Обычно рассчитывают вот так:

$$CF = \frac{\text{Present Value при постоянной кривой ставок 6\% - накопленный купонный доход}}{\text{номинал(face value)}}$$

- По этой формуле, если Present Value = 142, accrued interest = 2, face value = 100 , то CF = 1.4.

6 Cheapest-to-Deliver bond

Подсчет Conversion Factor позволяет продающей стороне выбирать, какую облигацию положить в основу фьючерсного контракта для наибольшей прибыли. При продаже облигации приток(Cash inflow) = QFP*CF + AI, где CF - conversion factor облигации, AI - accrued interest купона. При этом отток(Cash outflow) = Quoted bond price + AI, где Quoted bond price - котировка облигации, по которой мы должны ее продать. Тогда цель Cheapest-to-Deliver стратегии, это минимизировать Cash inflow - Cash outflow = QFP*CF - Quoted bond price, которая представляет из себя стоимость поставки облигации

Пример: в таблице приведены 4 облигации, которые будут продаваться с котировкой 95.75. Найти наилучшую облигацию в смысле Cheapest-to-deliver. Получим, что наилучшая для этого облигация - облигация с номером 3.

Solution: Calculate the cost of delivery:

Bond 1: 99 - (95.75 x 1.01) = $2.29

Bond 2: 125 - (95.75 x 1.24) = $6.27

Bond 3: 103 - (95.75 x 1.06) = $1.51 -> CTD

Bond 4: 115-(95.75 x 1.14) = $5.85

Bond	*Quoted Bond Price*	*Conversion Factor*
1	99	1.01
2	125	1.24
3	103	1.06
4	115	1.14

Fig. 3: Предлагаемые для выбора облигации и решение задачи

P.S. В книжке John C. Hull: Options, Futures,..... Conversion Factor считается как (внизу будет вырезка из книги)

Процедура выбора облигации предполагает большой перебор ценных бумаг, поэтому хотелось бы понимать, как Conversion Factor связан с реальной рыночной ситуацией(когда у нас не плоская 6% ставка):

- Если реальная ставка > 6%, то Conversion Factor будет переоценивать бумаги и поэтому выгоднее взять облигацию с более долгим сроком и низким купоном.
- Если же наоборот реальная ставка < 6%, то Conversion Factor будет недооценивать бумаги и стоит взять облигацию с менее долгим сроком и большим купоном.
- Если кривая ставок возрастает со временем, то выгоднее брать долгосрочные облигации.
- Если кривая ставок убывает со временем, то выгоднее брать краткосрочные облигации.

7 Теоретическая фьючерсная цена

Для обычного фьючерса имеем цену:

$$F_0 = (S_0 - I)e^{rT},$$

где S_0 - цена спота(Present Value), I - сумма купонов, дисконтированных к сегодняшнему дню, e^{rT} - капитализация к дате экспирации с процентной ставкой r.

Пример решения задачи на теоретическую фьючерсную цену: Пусть есть казначейская облигация, выплачивающая 2 раза в год купон, с годовой процентной ставкой 10%, Conversion Factor = 1.1, Quoted bond price = 100. Пусть в купонном периоде 180 дней и последний купон был выплачен 90 дней назад. Так же положим дату экспирации фьючерсного контракта 180 дней и безрисковую ставку дисконтирования 3%. **Решение**

- Пусть t - данный момент времени
- Знаем, что Cash price(t) = quoted bond price(t) + accrued interest(t).

- Найдем accrued interest(t) = coupon*$\frac{\text{days from last coupon}}{\text{days between coupons}} = \frac{10}{2}\frac{90}{180} = 2.5$
- Отсюда Cash price(t) = 102.5
- Теперь нужно дисконтировать купон(получить I - present value купона). $I(0) = coupon * e^{-rt} = 5 * e^{-0.03*90/365}$
- Из формулы теоретической стоимости: $F_0 = (S_0 - I(0))e^{rT} = (102.5 - 4.96) * e^{0.03*180/365} = 98.99$
- $F_\tau = (S_0 - I(0))e^{rT}e^{-r\tau}$
- Приведем к чистой цене: QFP(0) = cash price(F(0) - AI = 98.99 - 90/180*5 = 96.49
- Окончательно, применяя Conversion Factor, получим окончательную теоретическую стоимость: (QFP) = QFP*CF, отсюда QFP = 96.49/1.1 = 87.72

P.S. Общая механика решения таких задач следующая: мы сначала на данный момент приводим к полной(грязной) цене(типа cash price + накоп с купона). Потом все купоны, которые выплатятся с момента, на который мы считаем цену до момента к экспирации фьючерса приводим к PV в момент, в который считается цена. Затем эту всю штуку дисконтируем вперед, получаем что вот такая цена должна быть у фьючерса(идейно - у нас есть цена сейчас и цена на момент экспирации: чтобы не было арбитража, цена на момент экспирации должна совпадать с $S_0 * e^{rT}$, поэтому тем больше цена на данный момент, чем он дальше от времени экспирации.

Потом из того, что получилось, убираем накоп купона(то есть просто обратно к чистой цене приводим), и делим на conversion factor. Получаем цену фьюча.

8 Евродолларовые фьючерсы

Не имеет отношения к курсу USD/EUR, это долларовый депозит, размещенный вне юрисдикции США. По факту это договор о ставке, по которой мы можем разместить депозит 1млн \$ по 3х месячнойс ставке(LIBOR).

Этот контракт является расчетным, и изменение ставки на один базисный пункт(one basis point) соответствует 25\$ изменению на 1млн \$.

Исходя из этого, фьючерсный контракт оценивается следующим образом:

$$\text{eurodollar futures price} = 10000[100 - 0.25(100 - Z)],$$

где Z - котируемая цена облигации. При Z = 97.8 цена контракта: 994500\$.

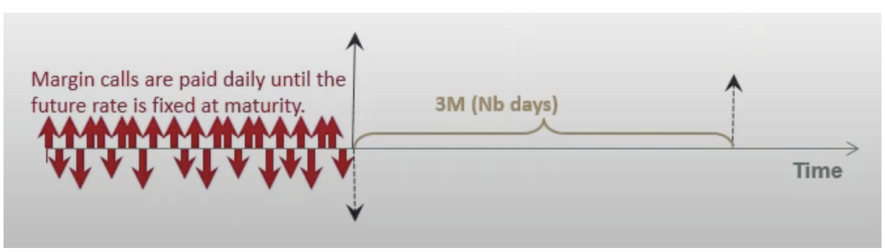


Fig. 4: Динамика евродолларового фьючерсного контракта

У этого контракта следующая механика: мы каждый день получаем приращение цены при изменении процентной ставки. В день экспирации мы фиксируем финальную ставку и, накапливаем разницу между ценой покупки фьючерса и рыночной ценой на размещение денег в дату экспирации под 3 месяца.

Из-за этого возникает разница между ставками: фьючерсная ставка данного контракта получается выше, чем LIBOR, и тем разница больше, чем больше волатильна цена.

Если ставка идет вверх, то мы, как продающая сторона, начинаем зарабатывать, а если ставка идет вниз, то начинаем терять. Наблюдается некий эффект выпуклости, поэтому для подсчета реальной форвардной ставки используется поправка на выпуклость:

$$\text{actual forward rate} = \text{forward rate implied by futures} - \frac{1}{2}\sigma^2 T_1 T_2,$$

где σ - волатильность в 3х месячный период, T_1 - срок до экспирации фьючерса, $T_2 = T_1 + 90$ days.

9 Кривая процентных ставок по евродолларовым фьючерсам(LIBOR spot curve)

Из формулы кривой по фьючерсам: $R_{forward} = \frac{R_2T_2 - R_1T_1}{T_2 - T_1}$, где R_i - ставка на горизонт T_i, то отсюда

$$R_2 = [R_{forward}(T_2 - T_1) + R_1T_1]/T_1,$$

где R_2 - ставка, посчитанная ранее по еврофьючерсным контрактам с учетом на выпуклость. Зная ставку и T_1, можем оценить на T_2, с T_2 на T_3 и так далее итерационно.

10 Хеджирование при помощи фьючерса на процентную ставку

Duration-based hedge: хеджирование на основе дюрации. Пусть у нас есть портфель с известной дюрацией(временем экспирации). Подбираем количество фьючерсных контрактов таким образом, чтобы суммарная дюрация стала нулевой:

$$N * F * D_F + P * D_P = 0,$$

где N - количество фьючерсных контрактов, F - цена фьючерса, D_F - время экспирации фьючерсного контракта, P - цена портфеля, D_P - время экспирации портфеля.

Недостатки такого хеджирования следующие:

- Не защищает от сильных изменений ставки(нелинейные изменения кривой ставки будут порождать большее отклонение в хеджировании).
- Кроме того, кривая может изменяться по разному с разных сторон, а значит будет меняться и сама дюрация. На картинке представлено изменение хеджа, зависящее от изменения кривой ставки.

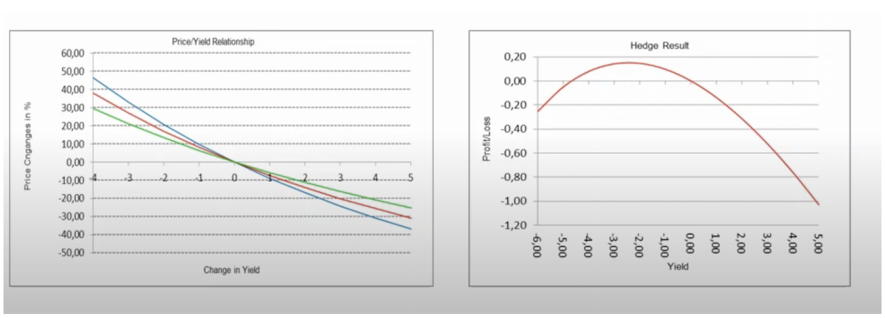


Fig. 5: Чувствительность хеджа к изменению кривой ставок

CHAPTER 6

Interest Rate Futures

So far we have covered futures contracts on commodities, stock indices, and foreign currencies. We have seen how they work, how they are used for hedging, and how futures prices are determined. We now move on to consider interest rate futures.

This chapter explains the popular Treasury bond and Eurodollar futures contracts that trade in the United States. Many of the other interest rate futures contracts throughout the world have been modeled on these contracts. The chapter also shows how interest rate futures contracts, when used in conjunction with the duration measure introduced in Chapter 4, can be used to hedge a company's exposure to interest rate movements.

6.1 DAY COUNT AND QUOTATION CONVENTIONS

As a preliminary to the material in this chapter, we consider the day count and quotation conventions that apply to bonds and other instruments dependent on the interest rate.

Day Counts

The day count defines the way in which interest accrues over time. Generally, we know the interest earned over some reference period (e.g., the time between coupon payments on a bond), and we are interested in calculating the interest earned over some other period.

The day count convention is usually expressed as X/Y. When we are calculating the interest earned between two dates, X defines the way in which the number of days between the two dates is calculated, and Y defines the way in which the total number of days in the reference period is measured. The interest earned between the two dates is

$$\frac{\text{Number of days between dates}}{\text{Number of days in reference period}} \times \text{Interest earned in reference period}$$

Three day count conventions that are commonly used in the United States are:

1. Actual/actual (in period)
2. 30/360
3. Actual/360

Business Snapshot 6.1 Day Counts Can Be Deceptive

Between February 28 and March 1, 2015, you have a choice between owning a US government bond and a US corporate bond. They pay the same coupon and have the same quoted price. Assuming no risk of default, which would you prefer?

It sounds as though you should be indifferent, but in fact you should have a marked preference for the corporate bond. Under the 30/360 day count convention used for corporate bonds, there are 3 days between February 28, 2015, and March 1, 2015. Under the actual/actual (in period) day count convention used for government bonds, there is only 1 day. You would earn approximately three times as much interest by holding the corporate bond!

The actual/actual (in period) day count is used for Treasury bonds in the United States. This means that the interest earned between two dates is based on the ratio of the actual days elapsed to the actual number of days in the period between coupon payments. Assume that the bond principal is \$100, coupon payment dates are March 1 and September 1, and the coupon rate is 8% per annum. (This means that \$4 of interest is paid on each of March 1 and September 1.) Suppose that we wish to calculate the interest earned between March 1 and July 3. The reference period is from March 1 to September 1. There are 184 (actual) days in the reference period, and interest of \$4 is earned during the period. There are 124 (actual) days between March 1 and July 3. The interest earned between March 1 and July 3 is therefore

$$\frac{124}{184} \times 4 = 2.6957$$

The 30/360 day count is used for corporate and municipal bonds in the United States. This means that we assume 30 days per month and 360 days per year when carrying out calculations. With the 30/360 day count, the total number of days between March 1 and September 1 is 180. The total number of days between March 1 and July 3 is $(4 \times 30) + 2 = 122$. In a corporate bond with the same terms as the Treasury bond just considered, the interest earned between March 1 and July 3 would therefore be

$$\frac{122}{180} \times 4 = 2.7111$$

As shown in Business Snapshot 6.1, sometimes the 30/360 day count convention has surprising consequences.

The actual/360 day count is used for money market instruments in the United States. This indicates that the reference period is 360 days. The interest earned during part of a year is calculated by dividing the actual number of elapsed days by 360 and multiplying by the rate. The interest earned in 90 days is therefore exactly one-fourth of the quoted rate, and the interest earned in a whole year of 365 days is 365/360 times the quoted rate.

Conventions vary from country to country and from instrument to instrument. For example, money market instruments are quoted on an actual/365 basis in Australia, Canada, and New Zealand. LIBOR is quoted on an actual/360 for all currencies except sterling, for which it is quoted on an actual/365 basis. Euro-denominated and sterling bonds are usually quoted on an actual/actual basis.

Price Quotations of US Treasury Bills

The prices of money market instruments are sometimes quoted using a *discount rate*. This is the interest earned as a percentage of the final face value rather than as a percentage of the initial price paid for the instrument. An example is Treasury bills in the United States. If the price of a 91-day Treasury bill is quoted as 8, this means that the rate of interest earned is 8% of the face value per 360 days. Suppose that the face value is \$100. Interest of \$2.0222 (= \$100 × 0.08 × 91/360) is earned over the 91-day life. This corresponds to a true rate of interest of 2.0222/(100 − 2.0222) = 2.064% for the 91-day period. In general, the relationship between the cash price per \$100 of face value and the quoted price of a Treasury bill in the United States is

$$P = \frac{360}{n}(100 - Y)$$

where P is the quoted price, Y is the cash price, and n is the remaining life of the Treasury bill measured in calendar days. For example, when the cash price of a 90-day Treasury bill is 99, the quoted price is 4.

Price Quotations of US Treasury Bonds

Treasury bond prices in the United States are quoted in dollars and thirty-seconds of a dollar. The quoted price is for a bond with a face value of \$100. Thus, a quote of 90-05 or $90\frac{5}{32}$ indicates that the quoted price for a bond with a face value of \$100,000 is \$90,156.25.

The quoted price, which traders refer to as the *clean price*, is not the same as the cash price paid by the purchaser of the bond, which is referred to by traders as the *dirty price*. In general,

$$\text{Cash price} = \text{Quoted price} + \text{Accrued interest since last coupon date}$$

To illustrate this formula, suppose that it is March 5, 2015, and the bond under consideration is an 11% coupon bond maturing on July 10, 2038, with a quoted price of 95-16 or \$95.50. Because coupons are paid semiannually on government bonds (and the final coupon is at maturity), the most recent coupon date is January 10, 2015, and the next coupon date is July 10, 2015. The (actual) number of days between January 10, 2015, and March 5, 2015, is 54, whereas the (actual) number of days between January 10, 2015, and July 10, 2015, is 181. On a bond with \$100 face value, the coupon payment is \$5.50 on January 10 and July 10. The accrued interest on March 5, 2015, is the share of the July 10 coupon accruing to the bondholder on March 5, 2015. Because actual/actual in period is used for Treasury bonds in the United States, this is

$$\frac{54}{181} \times \$5.50 = \$1.64$$

The cash price per \$100 face value for the bond is therefore

$$\$95.50 + \$1.64 = \$97.14$$

Thus, the cash price of a \$100,000 bond is \$97,140.

6.2 TREASURY BOND FUTURES

Table 6.1 shows interest rate futures quotes on May 14, 2013. One of the most popular long-term interest rate futures contracts is the Treasury bond futures contract traded by the CME Group. In this contract, any government bond that has between 15 and 25 years to maturity on the first day of the delivery month can be delivered. A contract which the CME Group started trading 2010 is the ultra T-bond contract, where any bond with maturity over 25 years can be delivered.

The 10-year, 5-year, and 2-year Treasury note futures contract in the United States are also very popular. In the 10-year Treasury note futures contract, any government bond (or note) with a maturity between $6\frac{1}{2}$ and 10 years can be delivered. In the 5-year and 2-year Treasury note futures contracts, the note delivered has a remaining life of about 5 years and 2 years, respectively (and the original life must be less than 5.25 years).

As will be explained later in this section, the exchange has developed a procedure for adjusting the price received by the party with the short position according to the particular bond or note it chooses to deliver. The remaining discussion in this section

Table 6.1 Futures quotes for a selection of CME Group contracts on interest rates on May 14, 2013.

	Open	*High*	*Low*	*Prior settlement*	*Last trade*	*Change*	*Volume*
Ultra T-Bond, \$100,000							
June 2013	158-08	158-31	156-31	158-08	157-00	−1-08	45,040
Sept. 2013	157-12	157-15	155-16	156-24	155-18	−1-06	176
Treasury Bonds, \$100,000							
June 2013	144-22	145-04	143-26	144-20	143-28	−0-24	346,878
Sept. 2013	143-28	144-08	142-30	143-24	142-31	−0-25	2,455
10-Year Treasury Notes, \$100,000							
June 2013	131-315	132-050	131-205	131-310	131-210	−0-100	1,151,825
Sept. 2013	131-040	131-080	130-240	131-025	130-240	−0-105	20,564
5-Year Treasury Notes, \$100,000							
June 2013	123-310	124-015	123-267	123-307	123-267	−0-040	478,993
Sept. 2013	123-177	123-192	123-122	123-165	123-122	−0-042	4,808
2-Year Treasury Notes, \$200,000							
June 2013	110-080	110-085	110-075	110-080	110-075	−0-005	98,142
Sept. 2013	110-067	110-072	110-067	110-070	110-067	−0-002	13,103
30-Day Fed Funds Rate, \$5,000,000							
Sept. 2013	99.875	99.880	99.875	99.875	99.875	0.000	956
July 2014	99.830	99.835	99.830	99.830	99.830	0.000	1,030
Eurodollar, \$1,000,000							
June 2013	99.720	99.725	99.720	99.725	99.720	−0.005	107,167
Sept. 2013	99.700	99.710	99.700	99.705	99.700	−0.005	114,055
Dec. 2013	99.675	99.685	99.670	99.675	99.670	−0.005	144,213
Dec. 2015	99.105	99.125	99.080	99.100	99.080	−0.020	96,933
Dec. 2017	97.745	97.770	97.675	97.730	97.680	−0.050	14,040
Dec. 2019	96.710	96.775	96.690	96.760	96.690	−0.070	23

focuses on the Treasury bond futures. Many other contracts traded in the United States and the rest of the world are designed in a similar way to the Treasury bond futures, so that many of the points we will make are applicable to these contracts as well.

Quotes

Ultra T-bond futures and Treasury bond futures contracts are quoted in dollars and thirty-seconds of a dollar per \$100 face value. This is similar to the way the bonds are quoted in the spot market. In Table 6.1, the settlement price of the June 2013 Treasury bond futures contract is specified as 144-20. This means $144\frac{20}{32}$, or 144.625. The settlement price of the 10-year Treasury note futures contract is quoted to the nearest half of a thirty-second. Thus the settlement price of 131-025 for the September 2013 contract should be interpreted as $131\frac{2.5}{32}$, or 131.078125. The 5-year and 2-year Treasury note contracts are quoted even more precisely, to the nearest quarter of a thirty-second. Thus the settlement price of 123-307 for the June 5-year Treasury note contract should be interpreted as $123\frac{30.75}{32}$, or 123.9609375. Similarly, the trade price of 123-122 for the September contract should be interpreted as $123\frac{12.25}{32}$, or 123.3828125.

Conversion Factors

As mentioned, the Treasury bond futures contract allows the party with the short position to choose to deliver any bond that has a maturity between 15 and 25 years. When a particular bond is delivered, a parameter known as its *conversion factor* defines the price received for the bond by the party with the short position. The applicable quoted price for the bond delivered is the product of the conversion factor and the most recent settlement price for the futures contract. Taking accrued interest into account (see Section 6.1), the cash received for each \$100 face value of the bond delivered is

$$(\text{Most recent settlement price} \times \text{Conversion factor}) + \text{Accrued interest}$$

Each contract is for the delivery of \$100,000 face value of bonds. Suppose that the most recent settlement price is 90-00, the conversion factor for the bond delivered is 1.3800, and the accrued interest on this bond at the time of delivery is \$3 per \$100 face value. The cash received by the party with the short position (and paid by the party with the long position) is then

$$(1.3800 \times 90.00) + 3.00 = \$127.20$$

per \$100 face value. A party with the short position in one contract would deliver bonds with a face value of \$100,000 and receive \$127,200.

The conversion factor for a bond is set equal to the quoted price the bond would have per dollar of principal on the first day of the delivery month on the assumption that the interest rate for all maturities equals 6% per annum (with semiannual compounding). The bond maturity and the times to the coupon payment dates are rounded down to the nearest 3 months for the purposes of the calculation. The practice enables the exchange to produce comprehensive tables. If, after rounding, the bond lasts for an exact number of 6-month periods, the first coupon is assumed to be paid in 6 months. If, after rounding, the bond does not last for an exact number of 6-month periods (i.e., there are an extra 3 months), the first coupon is assumed to be paid after 3 months and accrued interest is subtracted.

Business Snapshot 6.2 The Wild Card Play

The settlement price in the CME Group's Treasury bond futures contract is the price at 2:00 p.m. Chicago time. However, Treasury bonds continue trading in the spot market beyond this time and a trader with a short position can issue to the clearing house a notice of intention to deliver later in the day. If the notice is issued, the invoice price is calculated on the basis of the settlement price that day, that is, the price at 2:00 p.m.

This practice gives rise to an option known as the *wild card play*. If bond prices decline after 2:00 p.m. on the first day of the delivery month, the party with the short position can issue a notice of intention to deliver at, say, 3:45 p.m. and proceed to buy bonds in the spot market for delivery at a price calculated from the 2:00 p.m. futures price. If the bond price does not decline, the party with the short position keeps the position open and waits until the next day when the same strategy can be used.

As with the other options open to the party with the short position, the wild card play is not free. Its value is reflected in the futures price, which is lower than it would be without the option.

continuous compounding) is 10% per annum. Assume that the current quoted bond price is \$115. The cash price of the bond is obtained by adding to this quoted price the proportion of the next coupon payment that accrues to the holder. The cash price is therefore

$$115 + \frac{60}{60 + 122} \times 6 = 116.978$$

A coupon of \$6 will be received after 122 days (= 0.3342 years). The present value of this is

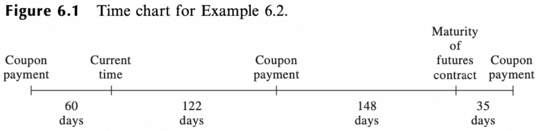
$$6e^{-0.1\times 0.3342} = 5.803$$

The futures contract lasts for 270 days (= 0.7397 years). The cash futures price, if the contract were written on the 12% bond, would therefore be

$$(116.978 - 5.803)e^{0.1\times 0.7397} = 119.711$$

At delivery, there are 148 days of accrued interest. The quoted futures price, if the contract were written on the 12% bond, is calculated by subtracting the accrued

Figure 6.1 Time chart for Example 6.2.



interest

$$119.711 - 6 \times \frac{148}{148 + 35} = 114.859$$

From the definition of the conversion factor, 1.6000 standard bonds are considered equivalent to each 12% bond. The quoted futures price should therefore be

$$\frac{114.859}{1.6000} = 71.79$$

6.3 EURODOLLAR FUTURES

The most popular interest rate futures contract in the United States is the three-month Eurodollar futures contract traded by the CME Group. A Eurodollar is a dollar deposited in a US or foreign bank outside the United States. The Eurodollar interest rate is the rate of interest earned on Eurodollars deposited by one bank with another bank. It is essentially the same as the London Interbank Offered Rate (LIBOR) introduced in Chapter 4.

A three-month Eurodollar futures contract is a futures contract on the interest that will be paid (by someone who borrows at the Eurodollar interest rate) on \$1 million for a future three-month period. It allows a trader to speculate on a future three-month interest rate or to hedge an exposure to a future three-month interest rate. Eurodollar futures contracts have maturities in March, June, September, and December for up to 10 years into the future. This means that in 2014 a trader can use Eurodollar futures to take a position on what interest rates will be as far into the future as 2024. Short-maturity contracts trade for months other than March, June, September, and December.

To understand how Eurodollar futures contracts work, consider the June 2013 contract in Table 6.1. The settlement price on May 13, 2013, is 99.725. The last trading day is two days before the third Wednesday of the delivery month, which in the case of this contract is June 17, 2013. The contract is settled daily in the usual way until the last trading day. At 11 a.m. on the last trading day, there is a final settlement equal to $100 - R$, where R is the three-month LIBOR fixing on that day, expressed with quarterly compounding and an actual/360 day count convention. Thus, if the three-month Eurodollar interest rate on June 17, 2013, turned out to be 0.75% (actual/360 with quarterly compounding), the final settlement price would be 99.250. Once a final settlement has taken place, all contracts are declared closed.

The contract is designed so that a one-basis-point (= 0.01) move in the futures quote corresponds to a gain or loss of \$25 per contract. When a Eurodollar futures quote increases by one basis point, a trader who is long one contract gains \$25 and a trader who is short one contract loses \$25. Similarly, when the quote decreases by one basis point a trader who is long one contract loses \$25 and a trader who is short one contract gains \$25. Suppose, for example, a settlement price changes from 99.725 to 99.685. Traders with long positions lose $4 \times 25 = \$100$ per contract; traders with short positions gain \$100 per contract. A one-basis-point change in the futures quote corresponds to a 0.01% change in the underlying interest rate. This in turn leads to a

$$1{,}000{,}000 \times 0.0001 \times 0.25 = 25$$

Table 6.2 Possible sequence of prices for June 2013 Eurodollar futures contract.

Date	*Settlement futures price*	*Change*	*Gain per contract ($)*
May 13, 2013	99.725		
May 14, 2013	99.720	−0.005	−12.50
May 15, 2013	99.670	−0.050	−125.00
⋮	⋮	⋮	⋮
June 17, 2013	99.615	+0.010	+25.00
Total		−0.110	−275.00

or \$25 change in the interest that will be earned on \$1 million in three months. The \$25 per basis point rule is therefore consistent with the point made earlier that the contract locks in an interest rate on \$1 million for three months.

The futures quote is 100 minus the futures interest rate. An investor who is long gains when interest rates fall and one who is short gains when interest rates rise. Table 6.2 shows a possible set of outcomes for the June 2013 contract in Table 6.1 for a trader who takes a long position at the May 13, 2013, settlement price.

The contract price is defined as

$$10{,}000 \times [100 - 0.25 \times (100 - Q)] \qquad \textbf{(6.2)}$$

where Q is the quote. Thus, the settlement price of 99.725 for the June 2013 contract in Table 6.1 corresponds to a contract price of

$$10{,}000 \times [100 - 0.25 \times (100 - 99.725)] = \$999{,}312.5$$

In Table 6.2, the final contract price is

$$10{,}000 \times [100 - 0.25 \times (100 - 99.615)] = \$999{,}037.5$$

and the difference between the initial and final contract price is \$275, This is consistent with the loss calculated in Table 6.2 using the "\$25 per one-basis-point move" rule.

Example 6.3

An investor wants to lock in the interest rate for a three-month period beginning two days before the third Wednesday of September, on a principal of \$100 million. We suppose that the September Eurodollar futures quote is 96.50, indicating that the investor can lock in an interest rate of 100 − 96.5 or 3.5% per annum. The investor hedges by buying 100 contracts. Suppose that, two days before the third Wednesday of September, the three-month Eurodollar rate turns out to be 2.6%. The final settlement in the contract is then at a price of 97.40. The investor gains

$$100 \times 25 \times (9{,}740 - 9{,}650) = 225{,}000$$

or \$225,000 on the Eurodollar futures contracts. The interest earned on the three-month investment is

$$100{,}000{,}000 \times 0.25 \times 0.026 = 650{,}000$$

or \$650,000. The gain on the Eurodollar futures brings this up to \$875,000, which is what the interest would be at 3.5% ($100{,}000{,}000 \times 0.25 \times 0.035 = 875{,}000$).

It appears that the futures trade has the effect of exactly locking an interest rate of 3.5% in all circumstances. In fact, the hedge is less than perfect because (a) futures contracts are settled daily (not all at the end) and (b) the final settlement in the futures contract happens at contract maturity, whereas the interest payment on the investment is three months later. One approximate adjustment for the second point is to reduce the size of the hedge to reflect the difference between funds received in September, and funds received three months later. In this case, we would assume an interest rate of 3.5% for the three-month period and multiply the number of contracts by $1/(1 + 0.035 \times 0.25) = 0.9913$. This would lead to 99 rather than 100 contracts being purchased.

Table 6.1 shows that the interest rate term structure in the US was upward sloping in May 2013. Using the "Prior Settlement" column, the futures rates for three-month periods beginning June 17, 2013, September 16, 2013, December 16, 2013, December 14, 2015, December 18, 2017, and December 16, 2019, were 0.275%, 0.295%, 0.325%, 0.900%, 2.270%, and 3.324%, respectively.

Example 6.3 shows how Eurodollar futures contracts can be used by an investor who wants to hedge the interest that will be earned during a future three-month period. Note that the timing of the cash flows from the hedge does not line up exactly with the timing of the interest cash flows. This is because the futures contract is settled daily. Also, the final settlement is in September, whereas interest payments on the investment are received three months later in December. As indicated in the example, a small adjustment can be made to the hedge position to approximately allow for this second point.

Other contracts similar to the CME Group's Eurodollar futures contracts trade on interest rates in other countries. The CME Group trades Euroyen contracts. The London International Financial Futures and Options Exchange (part of Euronext) trades three-month Euribor contracts (i.e., contracts on the three-month rate for euro deposits between euro zone banks) and three-month Euroswiss futures.

Forward vs. Futures Interest Rates

The Eurodollar futures contract is similar to a forward rate agreement (FRA: see Section 4.7) in that it locks in an interest rate for a future period. For short maturities (up to a year or so), the Eurodollar futures interest rate can be assumed to be the same as the corresponding forward interest rate. For longer-dated contracts, differences between the contracts become important. Compare a Eurodollar futures contract on an interest rate for the period between times T_1 and T_2 with an FRA for the same period. The Eurodollar futures contract is settled daily. The final settlement is at time T_1 and reflects the realized interest rate for the period between times T_1 and T_2. By contrast the FRA is not settled daily and the final settlement reflecting the realized interest rate between times T_1 and T_2 is made at time T_2.[2]

There are therefore two differences between a Eurodollar futures contract and an

[2] As mentioned in Section 4.7, settlement may occur at time T_1, but it is then equal to the present value of what the forward contract payoff would be at time T_2.

FRA. These are:

1. The difference between a Eurodollar futures contract and a similar contract where there is no daily settlement. The latter is a hypothetical forward contract where a payoff equal to the difference between the forward interest rate and the realized interest rate is paid at time T_1.
2. The difference between the hypothetical forward contract where there is settlement at time T_1 and a true forward contract where there is settlement at time T_2 equal to the difference between the forward interest rate and the realized interest rate.

These two components to the difference between the contracts cause some confusion in practice. Both decrease the forward rate relative to the futures rate, but for long-dated contracts the reduction caused by the second difference is much smaller than that caused by the first. The reason why the first difference (daily settlement) decreases the forward rate follows from the arguments in Section 5.8. Suppose you have a contract where the payoff is $R_M - R_F$ at time T_1, where R_F is a predetermined rate for the period between T_1 and T_2 and R_M is the realized rate for this period, and you have the option to switch to daily settlement. In this case daily settlement tends to lead to cash inflows when rates are high and cash outflows when rates are low. You would therefore find switching to daily settlement to be attractive because you tend to have more money in your margin account when rates are high. As a result the market would therefore set R_F higher for the daily settlement alternative (reducing your cumulative expected payoff). To put this the other way round, switching from daily settlement to settlement at time T_1 reduces R_F.

To understand the reason why the second difference reduces the forward rate, suppose that the payoff of $R_M - R_F$ is at time T_2 instead of T_1 (as it is for a regular FRA). If R_M is high, the payoff is positive. Because rates are high, the cost to you of having the payoff that you receive at time T_2 rather than time T_1 is relatively high. If R_M is low, the payoff is negative. Because rates are low, the benefit to you of having the payoff you make at time T_2 rather than time T_1 is relatively low. Overall you would rather have the payoff at time T_1. If it is at time T_2 rather than T_1, you must be compensated by a reduction in R_F.[3]

Convexity Adjustment

Analysts make what is known as a *convexity adjustment* to account for the total difference between the two rates. One popular adjustment is[4]

$$\text{Forward rate} = \text{Futures rate} - \tfrac{1}{2}\sigma^2 T_1 T_2 \qquad \textbf{(6.3)}$$

where, as above, T_1 is the time to maturity of the futures contract and T_2 is the time to the maturity of the rate underlying the futures contract. The variable σ is the standard deviation of the change in the short-term interest rate in 1 year. Both rates are expressed with continuous compounding.[5]

[3] Quantifying the effect of this type of timing difference on the value of a derivative is discussed further in Chapter 30.

[4] See Technical Note 1 at www.rotman.utoronto.ca/~hull/TechnicalNotes for a proof of this.

[5] This formula is based on the Ho–Lee interest rate model, which will be discussed in Chapter 31. See T. S. Y. Ho and S.-B. Lee, "Term structure movements and pricing interest rate contingent claims," *Journal of Finance*, 41 (December 1986), 1011–29.

Example 6.4

Consider the situation where $\sigma = 0.012$ and we wish to calculate the forward rate when the 8-year Eurodollar futures price quote is 94. In this case $T_1 = 8$, $T_2 = 8.25$, and the convexity adjustment is

$$\tfrac{1}{2} \times 0.012^2 \times 8 \times 8.25 = 0.00475$$

or 0.475% (47.5 basis points). The futures rate is 6% per annum on an actual/360 basis with quarterly compounding. This corresponds to 1.5% per 90 days or an annual rate of $(365/90)\ln 1.015 = 6.038\%$ with continuous compounding and an actual/365 day count. The estimate of the forward rate given by equation (6.3), therefore, is $6.038 - 0.475 = 5.563\%$ per annum with continuous compounding.

The table below shows how the size of the adjustment increases with the time to maturity.

Maturity of futures (years)	*Convexity adjustments (basis points)*
2	3.2
4	12.2
6	27.0
8	47.5
10	73.8

We can see from this table that the size of the adjustment is roughly proportional to the square of the time to maturity of the futures contract. For example, when the maturity doubles from 2 to 4 years, the size of the convexity approximately quadruples.

Using Eurodollar Futures to Extend the LIBOR Zero Curve

The LIBOR zero curve out to 1 year is determined by the 1-month, 3-month, 6-month, and 12-month LIBOR rates. Once the convexity adjustment just described has been made, Eurodollar futures are often used to extend the zero curve. Suppose that the ith Eurodollar futures contract matures at time T_i $(i = 1, 2, \dots)$. It is usually assumed that the forward interest rate calculated from the ith futures contract applies to the period T_i to T_{i+1}. (In practice this is close to true.) This enables a bootstrap procedure to be used to determine zero rates. Suppose that F_i is the forward rate calculated from the ith Eurodollar futures contract and R_i is the zero rate for a maturity T_i. From equation (4.5),

$$F_i = \frac{R_{i+1}T_{i+1} - R_i T_i}{T_{i+1} - T_i}$$

so that

$$R_{i+1} = \frac{F_i(T_{i+1} - T_i) + R_i T_i}{T_{i+1}} \qquad \textbf{(6.4)}$$

Other Euro rates such as Euroswiss, Euroyen, and Euribor are used in a similar way.

Example 6.5

The 400-day LIBOR zero rate has been calculated as 4.80% with continuous compounding and, from Eurodollar futures quotes, it has been calculated that (a) the forward rate for a 90-day period beginning in 400 days is 5.30% with continuous compounding, (b) the forward rate for a 90-day period beginning in 491 days is 5.50% with continuous compounding, and (c) the forward rate for a 90-day period beginning in 589 days is 5.60% with continuous compounding. We can use equation (6.4) to obtain the 491-day rate as

$$\frac{0.053 \times 91 + 0.048 \times 400}{491} = 0.04893$$

or 4.893%. Similarly we can use the second forward rate to obtain the 589-day rate as

$$\frac{0.055 \times 98 + 0.04893 \times 491}{589} = 0.04994$$

or 4.994%. The next forward rate of 5.60% would be used to determine the zero curve out to the maturity of the next Eurodollar futures contract. (Note that, even though the rate underlying the Eurodollar futures contract is a 90-day rate, it is assumed to apply to the 91 or 98 days elapsing between Eurodollar contract maturities.)

6.4 DURATION-BASED HEDGING STRATEGIES USING FUTURES

We discussed duration in Section 4.8. Consider the situation where a position in an asset that is interest rate dependent, such as a bond portfolio or a money market security, is being hedged using an interest rate futures contract. Define:

- V_F: Contract price for one interest rate futures contract
- D_F: Duration of the asset underlying the futures contract at the maturity of the futures contract
- P: Forward value of the portfolio being hedged at the maturity of the hedge (in practice, this is usually assumed to be the same as the value of the portfolio today)
- D_P: Duration of the portfolio at the maturity of the hedge

If we assume that the change in the yield, Δy, is the same for all maturities, which means that only parallel shifts in the yield curve can occur, it is approximately true that

$$\Delta P = -PD_P\,\Delta y$$

It is also approximately true that

$$\Delta V_F = -V_F D_F\,\Delta y$$

The number of contracts required to hedge against an uncertain Δy, therefore, is

$$N^* = \frac{PD_P}{V_F D_F} \tag{6.5}$$

This is the *duration-based hedge ratio*. It is sometimes also called the *price sensitivity hedge ratio*.[6] Using it has the effect of making the duration of the entire position zero.

When the hedging instrument is a Treasury bond futures contract, the hedger must base D_F on an assumption that one particular bond will be delivered. This means that the hedger must estimate which of the available bonds is likely to be cheapest to deliver at the time the hedge is put in place. If, subsequently, the interest rate environment changes so that it looks as though a different bond will be cheapest to deliver, then the hedge has to be adjusted and as a result its performance may be worse than anticipated.

When hedges are constructed using interest rate futures, it is important to bear in mind that interest rates and futures prices move in opposite directions. When interest rates go up, an interest rate futures price goes down. When interest rates go down, the reverse happens, and the interest rate futures price goes up. Thus, a company in a position to lose money if interest rates drop should hedge by taking a long futures position. Similarly, a company in a position to lose money if interest rates rise should hedge by taking a short futures position.

The hedger tries to choose the futures contract so that the duration of the underlying asset is as close as possible to the duration of the asset being hedged. Eurodollar futures tend to be used for exposures to short-term interest rates, whereas ultra T-bond, Treasury bond, and Treasury note futures contracts are used for exposures to longer-term rates.

Example 6.6

It is August 2 and a fund manager with $10 million invested in government bonds is concerned that interest rates are expected to be highly volatile over the next 3 months. The fund manager decides to use the December T-bond futures contract to hedge the value of the portfolio. The current futures price is 93-02, or 93.0625. Because each contract is for the delivery of $100,000 face value of bonds, the futures contract price is $93,062.50.

Suppose that the duration of the bond portfolio in 3 months will be 6.80 years. The cheapest-to-deliver bond in the T-bond contract is expected to be a 20-year 12% per annum coupon bond. The yield on this bond is currently 8.80% per annum, and the duration will be 9.20 years at maturity of the futures contract.

The fund manager requires a short position in T-bond futures to hedge the bond portfolio. If interest rates go up, a gain will be made on the short futures position, but a loss will be made on the bond portfolio. If interest rates decrease, a loss will be made on the short position, but there will be a gain on the bond portfolio. The number of bond futures contracts that should be shorted can be calculated from equation (6.5) as

$$\frac{10{,}000{,}000}{93{,}062.50} \times \frac{6.80}{9.20} = 79.42$$

To the nearest whole number, the portfolio manager should short 79 contracts.

[6] For a more detailed discussion of equation (6.5), see R.J. Rendleman, "Duration-Based Hedging with Treasury Bond Futures," *Journal of Fixed Income* 9, 1 (June 1999): 84–91.

Business Snapshot 6.3 Asset–Liability Management by Banks

The asset–liability management (ALM) committees of banks now monitor their exposure to interest rates very carefully. Matching the durations of assets and liabilities is sometimes a first step, but this does not protect a bank against non-parallel shifts in the yield curve. A popular approach is known as *GAP management*. This involves dividing the zero-coupon yield curve into segments, known as *buckets*. The first bucket might be 0 to 1 month, the second 1 to 3 months, and so on. The ALM committee then investigates the effect on the value of the bank's portfolio of the zero rates corresponding to one bucket changing while those corresponding to all other buckets stay the same.

If there is a mismatch, corrective action is usually taken. This can involve changing deposit and lending rates in the way described in Section 4.10. Alternatively, tools such as swaps, FRAs, bond futures, Eurodollar futures, and other interest rate derivatives can be used.

6.5 HEDGING PORTFOLIOS OF ASSETS AND LIABILITIES

Financial institutions sometimes attempt to hedge themselves against interest rate risk by ensuring that the average duration of their assets equals the average duration of their liabilities. (The liabilities can be regarded as short positions in bonds.) This strategy is known as *duration matching* or *portfolio immunization*. When implemented, it ensures that a small parallel shift in interest rates will have little effect on the value of the portfolio of assets and liabilities. The gain (loss) on the assets should offset the loss (gain) on the liabilities.

Duration matching does not immunize a portfolio against nonparallel shifts in the zero curve. This is a weakness of the approach. In practice, short-term rates are usually more volatile than, and are not perfectly correlated with, long-term rates. Sometimes it even happens that short- and long-term rates move in opposite directions to each other. Duration matching is therefore only a first step and financial institutions have developed other tools to help them manage their interest rate exposure. See Business Snapshot 6.3.

SUMMARY

Two very popular interest rate contracts are the Treasury bond and Eurodollar futures contracts that trade in the United States. In the Treasury bond futures contracts, the party with the short position has a number of interesting delivery options:

1. Delivery can be made on any day during the delivery month.
2. There are a number of alternative bonds that can be delivered.
3. On any day during the delivery month, the notice of intention to deliver at the 2:00 p.m. settlement price can be made later in the day.

These options all tend to reduce the futures price.

The Eurodollar futures contract is a contract on the 3-month Eurodollar interest rate two days before the third Wednesday of the delivery month. Eurodollar futures are frequently used to estimate LIBOR forward rates for the purpose of constructing a LIBOR zero curve. When long-dated contracts are used in this way, it is important to make what is termed a convexity adjustment to allow for the difference between Eurodollar futures and FRAs.

The concept of duration is important in hedging interest rate risk. It enables a hedger to assess the sensitivity of a bond portfolio to small parallel shifts in the yield curve. It also enables the hedger to assess the sensitivity of an interest rate futures price to small changes in the yield curve. The number of futures contracts necessary to protect the bond portfolio against small parallel shifts in the yield curve can therefore be calculated.

The key assumption underlying duration-based hedging is that all interest rates change by the same amount. This means that only parallel shifts in the term structure are allowed for. In practice, short-term interest rates are generally more volatile than are long-term interest rates, and hedge performance is liable to be poor if the duration of the bond underlying the futures contract differs markedly from the duration of the asset being hedged.

FURTHER READING

Burghardt, G., and W. Hoskins. "The Convexity Bias in Eurodollar Futures," *Risk*, 8, 3 (1995): 63–70.

Grinblatt, M., and N. Jegadeesh. "The Relative Price of Eurodollar Futures and Forward Contracts," *Journal of Finance*, 51, 4 (September 1996): 1499–1522.

Practice Questions (Answers in Solutions Manual)

6.1. A US Treasury bond pays a 7% coupon on January 7 and July 7. How much interest accrues per $100 of principal to the bondholder between July 7, 2014, and August 8, 2014? How would your answer be different if it were a corporate bond?

6.2. It is January 9, 2015. The price of a Treasury bond with a 12% coupon that matures on October 12, 2030, is quoted as 102-07. What is the cash price?

6.3. How is the conversion factor of a bond calculated by the CME Group? How is it used?

6.4. A Eurodollar futures price changes from 96.76 to 96.82. What is the gain or loss to an investor who is long two contracts?

6.5. What is the purpose of the convexity adjustment made to Eurodollar futures rates? Why is the convexity adjustment necessary?

6.6. The 350-day LIBOR rate is 3% with continuous compounding and the forward rate calculated from a Eurodollar futures contract that matures in 350 days is 3.2% with continuous compounding. Estimate the 440-day zero rate.

6.7. It is January 30. You are managing a bond portfolio worth $6 million. The duration of the portfolio in 6 months will be 8.2 years. The September Treasury bond futures price is currently 108-15, and the cheapest-to-deliver bond will have a duration of 7.6 years in