Homework 4; Tuesday 27 March
To be handed in no later than 12.30 on Tuesday 3 April

Consider the model

$$g(x|\theta) = w\, N(x|\mu_1, \sigma^2) + (1-w)\, N(x|\mu_2, \sigma^2)$$

with priors on $\lambda = 1/\sigma^2$ as Ga$(1,1)$, $f(\mu_1) = N(0,100)$ and $f(\mu_2) = N(0,100)$ and $f(w)$ is uniform on the interval $(0,1)$.

1. Generate data of size $n = 100$ from a standard normal distribution and run the Markov chain Gibbs sampler for the above model using the data you obtained. The appropriate latent variable here is the d which determines which group each observation came from.

2. Plot the predictive density; this is done by taking the average of the densities you get at each iteration of the Markov chain. Does it look like it is a standard normal density.

3. Plot some of the posterior distributions; for μ_1 and μ_2 and σ and w. Is there any indication from these that the true density is standard normal.

4. Now consider the (d_i) from the output of your Markov chain. Is there any indication that these show there is a single group.

5. Evaluate the integral

$$I = \int x\, g_p(x)\, dx,$$

where g_p is the predictive density you got from part 2.

in question 5 you have to do the integral using the analytical density you found in part 2? or just using the samples? can you compare mew1 mew2 from the output of the posterior or does he want us to find another way to find them

6. Estimate

$$w\mu_1 + (1-w)\mu_2$$

from the output of the Markov chain. How does this compare with your answer for 5.

question 6 asks for the expected value