Homework 2; Tuesday 27 February

To be handed in no later than 12.15 on Tuesday 6 March

1. Suppose

$$\pi(a,b) \propto a^4\, b^6\, e^{-a-b-3ab}$$

for $a, b > 0$ is required to be sampled. Find

$$\pi(a|b) \quad \text{and} \quad \pi(b|a)$$

and hence find a transition density $p(a_{n+1}, b_{n+1}|a_n, b_n)$ which has π as the stationary density.

Implement the chain and use the output to evaluate the integral

$$I = \int\int a\, b\, \pi(a,b)\, da\, db.$$

2. Suppose we wish to sample from

$$\pi(x) \propto \frac{\exp(-\frac{1}{2}x^2)}{1+x^2}$$

using a Metropolis–Hastings algorithm with density $q(x'|x) = N(x'|x, \sigma^2)$ for some σ as the proposal density.

Describe what you think might be the problems encountered if (i) σ is too small and (ii) σ is too big. Run the algorithm with such σ to verify your conclusions.

Without using any theory, find what you think is a suitable σ and run the algorithm with this σ to evaluate

$$I = \int x\, \pi(x)\, dx.$$

What happens if instead you use $q(x'|x) = N(x'|-x, \sigma^2)$.

3. Suppose a posterior density for the parameter θ is given by

$$f(\theta|\text{data}) \propto e^{\theta a}\, e^{-me^{\theta}}\, e^{-\frac{1}{2}\theta^2}$$

for some $a > 0$ and m integer, and $-\infty < \theta < +\infty$. In fact this is a Poisson model with mean e^{θ} and standard normal prior for θ.

Find a Markov chain for sampling from f and implement it, choosing your own values for m and a.