Homework 1
To be handed in no later than Tuesday ~~31 Jan, 9.15 am~~ 6 Feb, 12.15 pm
This counts 10% of overall percentage

1. Use rejection sampling to sample from the density function

$$f(x) \propto (-\log x)^2 \, x^3 \, (1-x)^2, \quad 0 < x < 1.$$

Carefully detail the method you use, and provide a figure of the histogram of the samples you obtained. What is the approximate, or actual if you can find it, probability of acceptance.

2. Use Monte Carlo methods to evaluate the integral

Be careful, you have to use importance sampling here!

$$I = \int_0^1 (-\log x)^2 \, x^3 \, (1-x)^{5/2} \, dx.$$

Describe in detail how you do this.

Provide a graphical demonstration i.e. plot the values as a function of Number of iterations show that you converge to I

Provide a graphical demonstration that your method has worked.

If you fix the Monte Carlo sample size as $N = 1000$, is there a way to estimate the variance of your $\widehat{I}_N$.

3. What is the acceptance probability when sampling a standard normal random variable with density

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

using a Cauchy density as proposal; i.e.

$$h(x) = \frac{1}{\pi(1+x^2)},$$

when using rejection sampling.

Verify this using simulation and plot a histogram of 1000 accepted samples.