

End-of-term exam: Thursday 3 May 2018

This question is about Bayesian inference with two models. The models are Poisson; i.e.

$$M_1 = \left\{ g_1(x|\theta) = \frac{\theta^x}{x!} e^{-\theta}, \ x \in \{0, 1, 2, \dots\}; \quad \pi_1(\theta) = \text{gamma}(1, 1) \right\},$$

and geometric;

$$M_2 = \{g(x|\theta) = \theta^x(1 - \theta), \ x \in \{0, 1, 2, \dots\}; \quad \pi_2(\theta) = \text{beta}(1, 1)\}.$$

The prior probabilities for the models are $f(M_1) = f(M_2) = \frac{1}{2}$.

Generate data from a Poisson model with parameter 1; i.e. the true data generating density is

$$g_0(x) = \frac{1}{x!} e^{-1}.$$

Take the sample size to be $n = 100$.

Find

$$f(M_1|\text{data})$$

and show how you obtained the answer.