Homework 6; Tuesday 24 April
To be handed in no later than 12.15 on Tuesday 1 May

Take $x_0 = 1$ and then generate data $y_{1:50}$ using the model

$$p(x_t|x_{t-1}) = N(x_t|x_{t-1}, 1) \quad \text{and} \quad p(y_t|x_t) = N(y_t|x_t, 1).$$

First use the Kalman filter updates to estimate $p(x_{50}|y_{1:50})$. This means you recursively estimate the mean and variance of $p(x_t|y_{1:t})$ for $t = 1, \ldots, 50$.

Now estimate the same thing using the particle filter. Use the importance sampling density as

$$q(x_t|x_{0:t-1}, y_{1:t}) = p(x_t|x_{t-1}).$$

In this case the estimate for $p(x_t|y_{1:t})$ is

$$\widehat{p}(x_t|y_{1:t}) = \sum_{i=1}^{N} \widetilde{w}_t^{(i)} \, \delta_{x_t^{(i)}}$$

Here the indicator function instead of it being for the entire sequence it is for the current x at t, it is as if considering the probability of all the sequence that end with x at the current time t

and the recursive updating of the weights was given in the lecture notes; as was how to obtain the sample $x_{0:t}^{(i)}$ for $i = 1, \ldots, N$.

So you pick N and the rest of the procedure to get $\widehat{p}(x_{50}|y_{1:50})$ is then automatic.

you have to pick the value for N and then update using the procedure in the notes

Comment on how well the particle filter does in replicating the true density which is available from the Kalman filter.