

Assignment Project Exam Help

CS262 Logic and Verification

Lecture 6: Semantic tableau

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Semantic tableau and resolution

Two proof procedures for propositional logic: **semantic tableau** and **resolution**

Semantic tableau: closely connected to disjunctive normal form (DNF)

Resolution: closely connected to conjunctive normal form (CNF)

Both systems are very well suited for automation;

Assignment: implement a resolution theorem prover in Prolog

Both are refutation systems: To prove that a formula X is a tautology, we begin with $\neg X$ and produce a contradiction

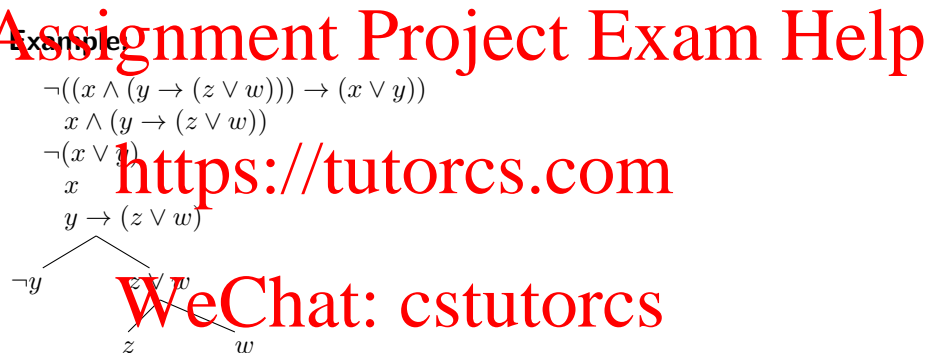
In the following we first talk about tableau, then about resolution

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Semantic tableau

Proof takes the form of a tree, with nodes labelled by propositional formulas.

Example:



Think of each branch as a conjunction of the formulas on that branch, and think of the tree as the disjunction of all of its branches (disjunction of conjunctions; DNF)

Tableau expansion

In each step, select a branch and a non-literal formula N on that branch.

If $N = \neg\top$, then extend the branch by a node labelled \perp at its end.

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If $N = \neg\neg Z$, then extend the branch by a node labelled Z at its end.

If N is an α -formula, then extend the branch by two nodes labelled α_1, α_2 (α -expansion) at its end.

If N is a β -formula, then add a left and right child to the final node of the branch, and label one of them β_1 and the other one β_2 (β -expansion).

Tableau expansion rules:

$\frac{\neg\top}{\perp}$	$\frac{\neg\perp}{\top}$	$\frac{\neg\neg Z}{Z}$	$\frac{\beta}{\beta_1 \mid \beta_2}$	$\frac{\alpha}{\alpha_1 \mid \alpha_2}$
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Example

$$\neg((p \rightarrow (q \rightarrow r)) \rightarrow ((p \vee s) \rightarrow ((q \rightarrow r) \vee s)))$$

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Closed tableau

A branch of a tableau is **closed**, if both X and $\neg X$ occur on the branch for some formula X , or if \perp occurs on the branch.

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If x and $\neg x$ appear on a branch (where x is a variable), or if \perp appears, then the branch is **atomically closed**.

A tableau is **(atomically) closed**, if every branch is (atomically) closed.

A tableau proof of X is a closed tableau for $\neg X$.

We write $\vdash_t X$ if X has a tableau proof.

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Two tableau proofs of $(p \wedge (q \rightarrow (r \vee s))) \rightarrow (p \vee q)$

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Tableau properties

Tableau proofs can be very short compared to truth tables: Consider $X \vee \neg X$ for some complicated formula X

Tableau method extends to first order logic (quantifiers) whereas truth tables do not

Tableau can be generalized to establish propositional consequences $S \models X$, not just tautologies $\models X$.

Tableau rules are non-deterministic: We have freedom in applying them. Different rules may produce proofs of different length.

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Implementation

Reuse of formulas: How do we know when we should give up on a proof attempt? We can always try again something we have tried before, and maybe now it will work.

A tableau is **strict**, if no formula has had an expansion rule applied to it twice on the same branch.

Represent the tree as a list of lists (disjunction of conjunctions): Strictness rule allows us to remove an expanded formula from the list

With this, tableau expansion becomes identical to disjunctive normal form expansion

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Implementation

So we could run the DNF expansion algorithm, and check for closure in the very end

We do not need to expand completely before checking for closure!

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By being clever about when to check for closure and when to apply which expansion rule, we can shorten a proof considerably

There is considerable scope and necessity for heuristics

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Soundness and completeness

Theorem: The tableau proof system is **sound**, i.e., if X has a tableau proof, then X is a tautology.

Theorem: The tableau proof system is **complete**, i.e., if X is a tautology, then the (strict) tableau system will terminate with a proof for it.

Equivalently, $\vdash_{\text{tr}} X$ if and only if $\models X$.

First theorem follows from the correctness proof of our DNF expansion algorithm given before: Every expansion step produces a logically equivalent formula.

Proof of second theorem: not given here; requires more advanced tools.