**Mathematics 53.** **Fall Semester 2018**

## Practice Midterm 2

**Problem 1**. a) Find the partial derivative $\dfrac{\partial u}{\partial a}$ in terms of the partial derivatives of $u(x, y, z, w)$, where, as a function of $a, b, c$,

$$u = u(a, e^{a+b}, (1 + a^2 + b^2 + c^2)^{\frac{1}{2}}, b - c).$$

b) Evaluate the Jacobian $\dfrac{\partial(x, y, z)}{\partial(u, v, w)}$ of the transformation

$$x = e^u + e^v, \quad y = e^u - e^v, \quad z = u + v + w.$$

**Problem 2**. Evaluate the integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1}\, dx\, dy.$$

**Problem 3**. Consider the vector field in $\mathbb{R}^2 \setminus \{0\}$

$$F = \left( \frac{2x}{\sqrt{x^2 + y^2}}, \frac{2y}{\sqrt{x^2 + y^2}} \right)$$

a) Determine whether $F$ is conservative.
b) Compute the integral

$$\int_C F \cdot dr$$

where $C$ is the parametric curve

$$x(t) = t^3 + 1, \quad y(t) = (1 - t^2)e^{t^2}, \qquad t \in [-1, 1]$$

**Problem 4**. Let $C$ be the unit circle $\{x^2 + y^2 = 1\}$ with counterclockwise orientation. Compute

$$\int_C y(\sin(xy) - 1)dx + x(\sin(xy) + 1)dy$$

**Problem 5**. A lamina occupies the region in the first quadrant on the $xy$ plane bounded by the ellipse $9x^2 + 4y^2 = 1$. Its density is given by the formula $\rho(x, y) = \cos(9x^2 + 4y^2)$. Find the mass of the lamina.

**Problem 6**. Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 2$ and is confined between the $yz$ plane and the cone $x = \sqrt{y^2 + z^2}$ .

**Problem 7**. a) State the fundamental theorem for line integrals.
b) State Green's theorem.