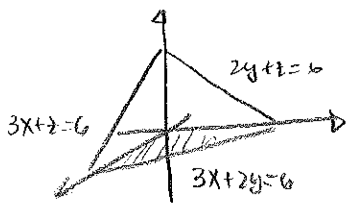
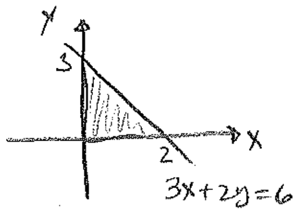
Worksheet – 2D Integration review[1]

1. Find the area of the part of the plane $3x + 2y + z = 6$ that lies in the first octant.



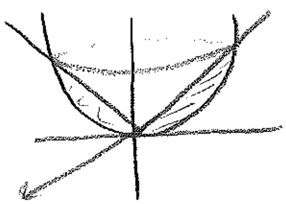


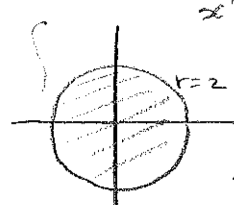
$f_x = 3$ $\quad dS = \sqrt{1+4+9}\,dA = \sqrt{14}\,dA$

$f_y = 2$

$A(\Delta) = \sqrt{14}\cdot \frac{3\cdot 2}{2} = 3\sqrt{14}$

2. Find the volume of the solid above the paraboloid $z = x^2 + y^2$ and below the half cone $z = 2\sqrt{x^2 + y^2}$.





$x^2+y^2 = 2\sqrt{x^2+y^2}$

$x^2+y^2=4$

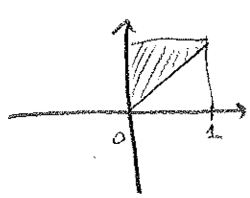
$\begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases}$

$\int_0^{2\pi}\int_0^2 (2r^2 - r^3)\,dr\,d\theta$

$= 2\pi\left(\frac{2r^3}{3} - \frac{r^4}{4}\right)\Big|_0^2 = 2\pi\left(\frac{16}{3} - 4\right)$

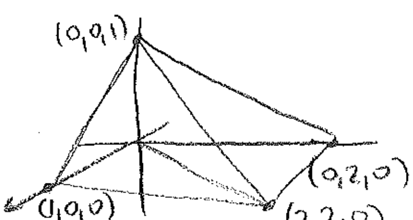
$= \frac{8\pi}{3}$

3. Evaluate the integral $\int_0^1 \int_x^1 \cos y^2 dydx$.



$= \int_0^1 \int_0^y \cos(y^2)\,dx\,dy = \frac{1}{2}\int_0^1 2y\cos(y^2)\,dy = \frac{1}{2}\left(\sin(y^2)\right)\Big|_0^1 = \frac{\sin(1)}{2}$

4. Find the volume of the solid tetrahedron with vertices $(0, 0, 0)$, $(0, 0, 1)$, $(0, 2, 0)$, and $(2, 2, 0)$.

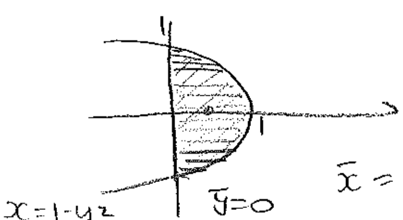


$V_1 = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 2$

$V_2 = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 2 & 2 & 0 \end{vmatrix} = 4$

$V = 6$

5. A lamina that occupies the region D bounded by the parabola $x = 1 - y^2$ has density given by $\rho(x, y) = |y|$. Find its mass and center of mass.

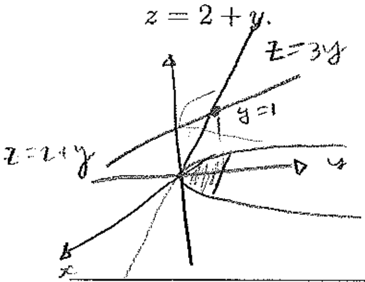


$\int_0^1 \int_0^{1-y^2} y\,dx\,dy = \int_0^1 (y - y^3)\,dy = \left(\frac{y^2}{2} - \frac{y^4}{4}\right)\Big|_0^1 = \frac{1}{4} \Rightarrow M = \frac{1}{2}$

$\bar{y} = 0$

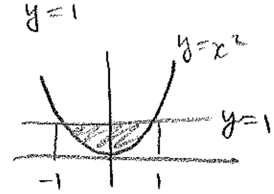
$\bar{x} = 2\int_0^1\int_0^{1-y^2} x|y|\,dx\,dy = 2\int_{-1} |y|\frac{(1-y^2)^2}{2} = 2\int_0^1 (y - 2y^3 + y^5)\,dy = 2\left(\frac{y^2}{2} - \frac{y^4}{4} + \frac{y^6}{6}\right)\Big|_0^1 = \frac{5}{6}$

6. Find the volume of the solid enclosed by the parabolic cylinder $y = x^2$ and the planes $z = 3y$, $z = 2 + y$.



$3y = 2+y$

$y = 1$



$\int_{-1}^1 \int_x^1 2+y-3y\,dy\,dx = \int_{-1}^1\int_x^1 (2-2y)\,dy\,dx$

$= \int_{-1}^1 (2y - y^2)\Big|_{x^2}^1 dx = \int_{-1}^1 (2 - 1 - 2x^2 + x^4)\,dx$

$= \left(x - \frac{2x^3}{3} + \frac{x^5}{5}\right)\Big|_{-1}^1 = 1 + \frac{2}{3} - \frac{1}{5}$

$1 - \frac{2}{3} + \frac{1}{5}$

$= \boxed{2}$

[1]Most questions are taken and/or modified from Stewart's "Multivariable Calculus".