Math Binder - AMC/AIME

Hansel Grimes

December 6, 2020

Contents

1	Polynomials	2
	1.1 Definition and Factorization	9

1 Polynomials

1.1 Definition and Factorization

A polynomial is defined as $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ with names corresponding to their degree (constant, linear, quadratic, cubic, quartic).

The factored form is written as $P(x) = a(x-r)(x-p)\cdots(x-q)$. The simplest and most useful polynomial is the quadratic. It can be written as $ax^2 + bx + c$ and factored respectively. The formula to solve for x is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The most important formula for polynomials is the Vieta Formulas.

Formula 1 Vieta Formulas include

Sum of roots
$$(r_1 + r_2 + r_3 + \dots + r_n)$$
: $-\frac{a_{n-1}}{a_n}$
Product of roots $(r_1 \cdot r_2 \cdot r_3 \cdot \dots \cdot r_n)$: $(-1)^n \cdot \frac{a_0}{a_n}$
Pairwise sums of p $(p = 2: r_1r_2 + r_1r_3 + r_1r_4 + \dots + r_{n-1}r_n)$: $(-1)^p \cdot \frac{a_{n-p}}{a_n}$

Theorem 1 Fundamental theorem of algebra states that a single variable polynomial with degree n has exactly n complex roots.