01-36

Ex 4.1

Ans1) $f(t) \begin{cases} -1 & t<1 \\ 1 & t\geq 1 \end{cases}$

$$\int_0^1 e^{-st}(-1) + \int_1^\infty e^{-st}(1)$$

$$\int_0^1 -e^{-st} + \int_1^\infty e^{-st}$$

$$\left[\frac{-e^{-st}}{-s}\right]_0^1 + \left[\frac{e^{-st}}{-s}\right]_1^\infty$$

$$\left[\frac{e^{-s}}{s} - \frac{1}{s}\right] + \left[\frac{e^{-\infty}}{-s} + \frac{e^{-s}}{s}\right]$$

$$\frac{e^{-s}}{s} - \frac{1}{s} - 0 - \frac{e^{-s}}{s}$$

$$\mathcal{L}(f(t)) = -\frac{1}{s} + \frac{2e^{-s}}{s}$$

Scanned with CamScanner

Ans2) $f(t) = \begin{cases} 4, & 0 \le t \le 2 \\ 0, & t \geq 2 \end{cases}$

$$\int_0^2 e^{-st}(4) + \int_2^\infty e^{-st}(0)$$

$$\cancel{4[e^{-2s} - e^{0}] + 4[e^{-\infty} - e^{-2s}]}$$

$$\cancel{4e^{-2s} + 0 - 4e^{-2s}}$$

$$4\left[\frac{e^{-st}}{-s}\right]_0^2 + 4\left[\frac{e^{-st}}{-s}\right]_2^\infty$$

$$4\left[\frac{e^{-2s}}{-s} - 1\right] + 4\left[\frac{e^{-\infty}}{-s} - \frac{e^{-2s}}{-s}\right]$$

$$\boxed{4\left[\frac{e^{-2s}}{-s} - 1\right]}$$

Ans 3) $f(t) = \begin{cases} t & 0 \le t \le 1 \\ 1 & t \geq 2 \end{cases}$

$$\int_0^1 e^{-st}(t) + \int_2^{\infty} e^{-st}(2)$$

$$\int_0^1 te^{-st} + \int_2^{\infty} e^{-st}(2)$$

$v = t \quad U' = e^{-st} \qquad \frac{2e^{-st}}{-s}$

$v' = 1 \quad U = \frac{e^{-st}}{-s}$

$$-\frac{te^{-st}}{s} - \int \frac{e^{-st}}{-s}$$

$$\left[-\frac{te^{-st}}{s} + \frac{e^{-st}}{s^2}\right]_0^1$$

$$\left[\cancel{\frac{-e^{-s}}{s}} + \frac{e^{-s}}{s^2} - \frac{1}{s^2}\right] + \left[\cancel{\frac{e^{-2s}}{s}}\right]$$

$$\boxed{\frac{1}{s^2}\left[1 - e^{-s}\right]}$$

Scanned with CamScanner

Ans(4) $f(t) = \begin{cases} 2t+1 & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$

$\int_0^1 e^{-st}(2t+1) + \int_1^\infty e^{-st}(0)$

$\int_0^1 2te^{-st} + e^{-st}$

$v = t \quad u' = e^{-st}$

$v' = 1 \quad u = \frac{e^{-st}}{-s}$

$2\int_0^1 te^{-st} + \int_0^1 e^{-st}$

$2\left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_0^1 + \left[\frac{e^{-st}}{-s}\right]_0^1$

$2\left[\frac{e^{-s}}{-s} - \frac{e^{-s}}{s^2} + \frac{1}{s^2}\right] + \left[\frac{e^{-s}}{-s} + \frac{1}{s}\right]$

$\frac{-2e^{-s}}{s} - \frac{2e^{-s}}{s^2} + \frac{2}{s^2} \frac{e^{-s}}{-s} + \frac{1}{s}$

$$\boxed{\frac{1}{s}\left[-2e^{-s} - e^{-s} + 1\right] + \frac{1}{s^2}\left[-2e^{-s} + 2\right]}$$

Ans 5) $f(t) = \begin{cases} \sin t & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases}$

$$\int_0^{\pi} e^{-st} \sin t + \int_{\pi}^{\infty} e^{-st}(0)$$

$V = \sin t \quad U' = e^{-st}$

$V' = \cos t \quad U = \frac{e^{-st}}{-s}$

$$-\frac{\sin t . e^{-st}}{s} - \int \frac{e^{-st} \cos t}{-s}$$

$V = \cos t \quad U' = e^{-st}$

$V' = -\sin t \quad U = e^{-st}/_{-s}$

$$-\frac{\sin t e^{-st}}{s} + \frac{1}{s} \int_0^{\pi} e^{-st} \cos t$$

$$-\frac{\sin t e^{-st}}{s} + \frac{1}{s}\left[\frac{\cos t e^{-st}}{-s} - \int \frac{\sin t e^{-st}}{-s}\right]$$

$$-\frac{\sin t e^{-st}}{s} - \frac{\cos t e^{-st}}{s^2}$$

Solving $\int \sin t e^{-st}$ on right side

$$\frac{-s \sin t e^{-st} - \cos t e^{-st}}{s^2+1}$$

$$-\left[\frac{s}{s^2+1}.e^{-st}.\sin t+\frac{1}{s^2+1}e^{-st}.\cos t\right]_0^{\pi}$$

$$-\left\{\left[\frac{\pi}{\pi^2+1}.e^{-s\pi}.\sin\pi+\frac{1}{s^2+1}e^{-s\pi}\cos\pi.\right]-\left[\frac{e^0.\cos(0)}{s^2+1}\right]\right.$$

$$-\left[-\frac{e^{-\pi s}}{s^2+1}-\frac{1}{s^2+1}\right.$$

$$\boxed{\frac{e^{-\pi s}+1}{s^2+1}}$$

Ans 6) $\begin{cases} \sin t \\ 0 \end{cases}$

$$-\left[\frac{s}{s^2+1} \cdot e^{-st} \cdot \sin t + \frac{1}{s^2+1} \cdot e^{-st} \cdot \cos t\right]_0^{\pi/2}$$

$$-\left[\frac{s}{s^2+1} \cdot e^{-s\pi/2} \cdot 1 + 0 - \frac{1}{s^2+1} \cdot e^{0} \cdot 1\right]$$

$$-\left[\frac{s}{s^2+1} \cdot e^{-s\pi/2} - \frac{1}{s^2+1}\right]$$

$$\boxed{\frac{1 - se^{-s\pi/2}}{1+s^2}}$$

Ans 11) $f(t) = e^{t+7}$

$\mathcal{L}(f(t)) = e^{t+7}$

$\int_0^\infty e^{t+7} \cdot e^{-st}$

$\cancel{\left[e^{t+7}\right]_0^\infty} =$

$\int_0^\infty e^{t+7-st} \, dt$

$\left[\frac{e^{t+7-st}}{-s}\right]_0^\infty$

$e^7 \left[\frac{e^{t(1-s)}}{1-s}\right]_0^\infty$

$e^7 \left[\frac{e^{-t(s-1)}}{1-s}\right]_0^\infty$

$e^7 \left[-\frac{1}{1-s}\right] = \frac{e^7}{s-1}$

12) $f(t) = e^{-2t-5}$

$\mathcal{L}(f(t)) = \int_0^{\infty} e^{-st} \cdot e^{-2t-5}$

$\int_0^{\infty} e^{-st-2t-5}$

$e^{-5} \int_0^{\infty} e^{-st-2t}$

$e^{-5} \int_0^{\infty} e^{-t(s+2)}$

$e^{-5} \left[\frac{e^{-t(s+2)}}{-(s+2)} \right]_0^{\infty}$

$e^{-5} \left[+ \frac{1}{s+2} \right]$

$\boxed{\frac{e^{-5}}{s+2}}$

Ans13) $\int_0^\infty te^{4t} \cdot e^{-st}$

$$\int_0^\infty te^{-t(s-4)} \, dt$$

$v = t \qquad U' = e^{-t(s-4)}$

$v' = 1 \qquad U = \frac{e^{-t(s-4)}}{4-s}$

$$\frac{te^{-t(s-4)}}{4-s} - \frac{1}{4-s}\int e^{-t(s-4)}$$

$$\frac{te^{-t(s-4)}}{4-s} - \frac{1}{4-s}\left[\frac{e^{-t(s-4)}}{4-s}\right]$$

$$\left\{\frac{te^{-t(s-4)}}{4-s} - \frac{1}{(4-s)^2}\left[e^{-t(s-4)}\right]\right\}_0^\infty$$

$$\boxed{\frac{1}{(4-s)^2}}$$

Scanned with CamScanner

Ans14) $\int_0^\infty t^2 e^{-2t} . e^{-st}$

$$\int_0^\infty t^2 e^{-t(2+s)}$$

$V = t^2$ $\quad U' = e^{-t(2+s)}$

$V' = 2t$ $\quad U = \dfrac{e^{-t(2+s)}}{-(2+s)}$

$$-\frac{t^2 e^{-t(2+s)}}{2+s} + 2\int \frac{t e^{-t(2+s)}}{(2+s)}$$

$$\frac{-t^2 e^{-t(2+s)}}{2+s} + \frac{2}{2+s}\int t e^{-t(2+s)}$$

$V = t$ $\quad U' = e^{-t(2+s)}$

$V' = 1$ $\quad U = \dfrac{e^{-t(2+s)}}{-(2+s)}$

$$-\frac{t e^{-t(2+s)}}{2+s} + \int \frac{e^{-t(2+s)}}{2+s}$$

$$\frac{-t e^{-t(2+s)}}{2+s} + \frac{1}{2+s}\left[\frac{e^{-t(2+s)}}{-(2+s)}\right]$$

$$\frac{-t\,e^{-t(2+s)}}{2+s} + \frac{1}{2+s}\left[\frac{e^{-t(s+2)}}{-(2+s)}\right]$$

$$-\frac{t^2 e^{-t(2+s)}}{(2+s)} + \frac{2}{2+s}\left[\frac{-te^{-t(2+s)}}{2+s} + \frac{1}{(2+s)^2}\cdot e^{-t(s+2)}\right]$$

$$\left[\frac{-t^2 e^{-t(2+s)}}{(2+s)} - \frac{2te^{-t(2+s)}}{(2+s)^2} - \frac{2e^{-t(s+2)}}{(2+s)^3}\right]_0^\infty$$

$$\boxed{\frac{+2}{(2+s)^3}}$$

Ans15) $f(t) = e^{-t}\sin t$

$$\int_0^\infty e^{-t}\sin t \,.\, e^{-st}$$

$$\int_0^\infty \sin t \cdot e^{-t(1+s)} \,.\, dt$$

$v = \sin t \qquad u' = e^{-t(1+s)}$

$v' = \cos t \qquad u = \frac{e^{-t(1+s)}}{-(1+s)}$

$$\frac{-\sin t\, e^{-t(1+s)}}{1+s} + \int \frac{\cos t\, e^{-t(1+s)}}{1+s}$$

Scanned with CamScanner

$$\int_0^\infty \sin t \cdot e^{-t(1+s)} = -\frac{\sin t e^{-t(1+s)}}{1+s} + \int \frac{\cos t\, e^{-t(1+s)}}{1+s}$$

$$V = \cos t \qquad U' = e^{-t(1+s)}$$

$$V' = -\sin t \qquad U = \frac{e^{-t(1+s)}}{-(1+s)}$$

$$\frac{-\cos t e^{-t(1+s)}}{1+s} - \int \frac{\sin t e^{-t(1+s)}}{1+s}$$

$$\int_0^\infty \sin t \cdot e^{-t(1+s)} = -\frac{\sin t e^{-t(1+s)}}{1+s} + \frac{1}{1+s} \int \cos t e^{-t(1+s)}$$

$$\int_0^\infty \sin t e^{-t(1+s)} = -\frac{\sin t e^{-t(1+s)}}{1+s} + \frac{1}{1+s}\left[-\frac{\sin t e^{-t(1+s)}}{1+s}\right]$$

$$\int_0^\infty \sin t e^{-t(1+s)} = -\frac{\sin t e^{-t(1+s)}}{1+s} + \frac{1}{1+s}\left[\frac{-\cos t e^{-t(1+s)}}{1+s} - \int \frac{\sin t e}{1+s}\right.$$

$$\int_0^\infty \sin t e^{-t(1+s)} = -\frac{\sin t e^{-t(1+s)}}{1+s} - \frac{\cos t e^{-t(1+s)}}{(1+s)^2} - \int \frac{\sin t e^{-t(1+}}{(1+s)^2}$$

$$\int_0^\infty \sin t e^{-t(1+s)} + \frac{1}{(1+s)^2} \int \sin t e^{-t(1+s)} = -\frac{\sin t e^{-t(1+s)}}{1+s} - \frac{\cos t e^{-t(1+}}{(1+s)^2}$$

$$\int_0^\infty \sin t\, e^{-t(1+s)} \left(\frac{(1+s)^2+1}{(1+s)^2}\right) = -\frac{1}{(1+s)}\left(\sin t e^{-t(1+s)} + \frac{\cos e^{-t(1+s)}}{1+s}\right)$$

$$\int_0^\infty \sin t e^{-t(1+s)} = \left(\frac{(1+s)^{\cancel{2}}}{(1+s)^2+1}\right)\left(\frac{-1}{\cancel{1+s}}\right)\left(\sin t e^{-t(1+s)} + \frac{\cos t e^{-t(1+s)}}{1+s}\right)$$

$$\left[\frac{-(1+s)}{1+(1+s)^2}\left(\sin t e^{-t(1+s)} + \frac{\cos t e^{-t(1+s)}}{1+s}\right)\right]_0^\infty$$

$$+\frac{(\cancel{1+s})}{1+(1+s)^2}\left(\frac{1}{\cancel{1+s}}\right) = \frac{1}{1+1+2s+s^2}$$

$$= \frac{1}{s^2+2s+2}$$

Scanned with CamScanner

Ans16) $\int_0^\infty e^t . e^{-st} . \left(\frac{e^{it}+e^{-it}}{2}\right)$

$= \int_0^\infty e^{t-st} . \left(\frac{e^{it}+e^{-it}}{2i}\right)$

$= \int_0^\infty \frac{e^{-t(s-1)+it} + e^{t-st-it}}{2i}$

$= \int_0^\infty \frac{e^{-t(s-1-i)} + e^{-t(-1+s+i)}}{2i}$

$= \frac{1}{2i} \int_0^\infty \frac{e^{-t(s-1-i)}}{-(s-1-i)} + \frac{e^{-t(-1+s+i)}}{-(-1+s+i)}$

$= \frac{1}{2i} \left[\frac{-e^{-t(s-1-i)}}{(s-1-i)} - \frac{e^{-t(-1+s+i)}}{(-1+s+i)}\right]_0^\infty$

$= \frac{1}{2i} \left[\frac{1}{(s-1)-i} + \frac{1}{(s-1)+i}\right]$

$= \frac{1}{2i} \left[\frac{s-1+\cancel{i} + s-1-\cancel{i}}{(s-1)^2-i^2}\right]$

$= \frac{1}{2i} \left[\frac{2s-2}{s^2-2s+1+1}\right] = \boxed{\frac{s-1}{s^2-2s+2}}$

Scanned with CamScanner

Ans 19) $f(t) = 2t^4$

$$\mathcal{L}(2t^4) = \frac{2 \cdot 4!}{s^5} = \boxed{\frac{48}{s^5}}$$

Ans 20) $\mathcal{L}(t^5) = \frac{5!}{s^6} =$

Ans 21) $\mathcal{L}(4t - 10) = \boxed{\frac{4}{s^2} - \frac{10}{s}}$

Ans 22) $\mathcal{L}(7t + 3) = \boxed{\frac{7}{s^2} + \frac{3}{s}}$

Ans 23) $\mathcal{L}(t^2 + 6t - 3) = \boxed{\frac{2!}{s^3} + \frac{6}{s^2} - \frac{3}{s}}$

Ans 24) $\mathcal{L}(-4t^2 + 16t + 9) = \boxed{\frac{-4 \cdot 2!}{s^3} + \frac{16}{s^2} + \frac{9}{s}}$

Ans 25) $f(t) = (t+1)^3$

$$(t^2 + 2t + 1)(t+1) = t^3 + 2t^2 + t + t^2 + 2t + 1$$
$$= t^3 + 3t^2 + 3t + 1$$

$$\frac{3!}{s^4} + \frac{6}{s^3} + \frac{3}{s^2} + \frac{1}{s}$$

Scanned with CamScanner

Ans26) $\mathcal{L}(2t-1)^3$

$(4t^2-4t+1)(2t-1) = 8t^3-4t^2-8t^2+4t+2t-1$

$= 8t^3-12t^2+6t-1$

$\frac{8.3!}{s^4} - \frac{12.2!}{s^3} + \frac{6}{s^2} - \frac{1}{s}$

Ans27) $\mathcal{L}(1+e^{4t}) = \boxed{\frac{1}{s} + \frac{1}{s-4}}$

Ans28) $\mathcal{L}(t^2-e^{-9t}+5) = \frac{2}{s^3} - \frac{1}{s+9} + \frac{5}{s}$

Ans29) $\mathcal{L}((1+e^{2t})^2) = 1+2e^{2t}+e^{4t}$

$= \boxed{\frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}}$

Ans30) $\mathcal{L}((e^{t}-e^{-t})^2)$

$e^{2t} - 2 + e^{-2t}$

$\frac{1}{s-2} - \frac{2}{s} + \frac{1}{s+2}$

Ans31) $\mathcal{L}(4t^2 - 5\sin 3t) = \frac{4.2}{s^3} - 5\left(\frac{3}{s^2+9}\right)$

Ans 32) $\mathcal{L}(\cos 5t + \sin 2t)$

$= \frac{s}{s^2+25} + \frac{2}{s^2+4}$

Ans 33) $(\sinh kt) \;/\; k/s^2-k^2$

Ans 34) $(\cosh kt) \;/\; s/s^2-k^2$

Ans 35) $(e^t \sinh t) \;/\; \frac{1}{2(s-2)} - \frac{1}{2s}$

Ans 36) $e^{-t}\cosh t \;/\; \frac{1}{2s} + \frac{1}{2(s+2)}$

$$\sinh kt = \int_0^\infty \frac{e^{kt} - e^{-kt}}{2} \cdot e^{-st}$$

$$= \frac{1}{2}\int_0^\infty e^{kt-st} - e^{-kt-st}$$

$$= \frac{1}{2}\int_0^\infty e^{-t(s-k)} - e^{-t(k+s)}$$

$$= \frac{1}{2}\left[\frac{e^{-t(s-k)}}{-(s-k)} - \frac{e^{-t(k+s)}}{-(k+s)}\right]_0^\infty$$

$$= \frac{1}{2}\left\{\frac{1}{s-k} + \frac{1}{k+s}\right\}$$

$$= \frac{k+s+s-k}{s^2-k^2} = \boxed{\frac{k}{s^2-k^2}}$$

Ans 34) $\cosh(kt) = \int_0^\infty e^{-st}\left(\frac{e^{kt}+e^{-kt}}{2}\right)$

$\frac{1}{2}\int_0^\infty e^{-st+kt} + e^{-st-kt}$

$\cancel{\frac{1}{2}\int_0^\infty e^{-t(k-s)}}$

$\frac{1}{2}\int_0^\infty e^{-t(s-k)} + e^{-t(s+k)}$

$\cancel{\frac{1}{2}\int_0^\infty}\ \frac{1}{2}\left[\frac{e^{-t(s-k)}}{-(s-k)} + \frac{e^{-t(s+k)}}{-(s+k)}\right]_0^\infty$

$\frac{1}{2}\left[\frac{1}{s-k} + \frac{1}{s+k}\right]$

$\frac{1}{2}\left[\frac{s+k+s-k}{s^2+k^2}\right] = \frac{1}{2}\left[\frac{2s}{s^2-k^2}\right]$

$= \frac{s}{s^2-k^2}$

Ans35) $\mathcal{L}\{e^{t}\sinh t\}$

$$\mathcal{L}\left\{e^{t}\cdot\frac{e^{+t}-e^{-t}}{2}\right\}$$

$$\frac{1}{2}\mathcal{L}\left\{e^{2t}-e^{0}\right\}$$

$$\frac{1}{2}\mathcal{L}\left\{e^{2t}-1\right\}$$

$$\frac{1}{2}\left[\frac{1}{s-2}-\frac{1}{s}\right]$$

$$\boxed{\frac{1}{2(s-2)}-\frac{1}{2s}}$$

Ans36) $\mathcal{L}\{e^{t}\cosh t\}$

$$\mathcal{L}\left\{e^{t}\cdot\frac{e^{t}+e^{-t}}{2}\right\}$$

$$\mathcal{L}\left\{\frac{e^{2t}}{2}+\frac{e^{0}}{2}\right\}$$

$$\mathcal{L}\left\{\frac{1}{2(s-2)}+\frac{1}{2s}\right\}=\boxed{\frac{1}{2(s-2)}+\frac{1}{2s}}$$