

# Overview of Semantic Analysis

## Lecture 9

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# Midterm Thursday

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- Material through lecture 8
- Open note
- Administered through Gradescope
  - You have 24 hours to open the exam, starting at 3pm pacific time on Thursday May 7
  - Once you open the exam, you have 80+15 minutes to complete it
  - You may write/draw answers on paper and submit a cellphone picture
  - Allow time for sending them to your computer and uploading them to Gradescope
  - It is your responsibility to make it legible

# Outline

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- The role of semantic analysis in a compiler
  - A laundry list of tasks
- Scope
  - Implementation: symbol tables
- Types

# The Compiler So Far

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- Lexical analysis
  - Detects inputs with illegal tokens
- Parsing
  - Detects inputs with ill-formed parse trees
- Semantic analysis
  - Last “front end” phase
  - Catches all remaining errors

## Why a Separate Semantic Analysis?

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- Parsing cannot catch some errors
- Some language constructs not context-free

# What Does Semantic Analysis Do?

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- Checks of many kinds . . . cool checks:
  1. All identifiers are declared
  2. Types
  3. Inheritance relationships
  4. Classes defined only once
  5. Methods in a class defined only once
  6. Reserved identifiers are not misusedAnd others . . .
- The requirements depend on the language

# Scope

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- Matching identifier declarations with uses
  - Important static analysis step in most languages
  - Including COOL!

# What's Wrong?

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- Example 1

Let  $y: \text{String} \leftarrow \text{"abc"}$  in  $y + 3$

- Example 2

Let  $y: \text{Int}$  in  $x + 3$

*Note: An example property that is not context free.*



## Scope (Cont.)

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- The *scope* of an identifier is the portion of a program in which that identifier is accessible
- The same identifier may refer to different things in different parts of the program
  - Different scopes for same name don't overlap
- An identifier may have restricted scope

# Static vs. Dynamic Scope

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- Most languages have *static* scope
  - Scope depends only on the program text, not run-time behavior
  - Cool has static scope
- A few languages are *dynamically* scoped
  - Lisp, SNOBOL
  - Lisp has changed to mostly static scoping
  - Scope depends on execution of the program

# Static Scoping Example

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```
let x: Int <- 0 in
{
    x;
    let x: Int <- 1 in
        x;
    x;
}
```

## Static Scoping Example (Cont.)

---

```
let x: Int <- 0 in
{
  x;
  let x: Int <- 1 in
    x;
  x;
}
```

Uses of x refer to closest enclosing definition

# Dynamic Scope

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- A dynamically-scoped variable refers to the closest enclosing binding in the execution of the program

- Example

$g(y) = \text{let } a \leftarrow 4 \text{ in } f(3);$   
 $f(x) = a;$

- More about dynamic scope later in the course

## Scope in Cool

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- Cool identifier bindings are introduced by
  - Class declarations (introduce class names)
  - Method definitions (introduce method names)
  - Let expressions (introduce object ids)
  - Formal parameters (introduce object ids)
  - Attribute definitions (introduce object ids)
  - Case expressions (introduce object ids)

## Scope in Cool (Cont.)

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- Not all kinds of identifiers follow the most-closely nested rule
- For example, class definitions in Cool
  - Cannot be nested
  - Are *globally visible* throughout the program
- In other words, a class name can be used before it is defined

## Example: Use Before Definition

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```
Class Foo {  
    ... let y: Bar in ...  
};
```

```
Class Bar {  
    ...  
};
```



## More Scope in Cool

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Attribute names are global within the class in which they are defined

```
Class Foo {  
  f(): Int { a };  
  a: Int ← 0;  
}
```

## More Scope (Cont.)

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- Method/attribute names have complex rules
- A method need not be defined in the class in which it is used, but in some parent class
- Methods may also be redefined (overridden)

## Implementing the Most-Closely Nested Rule

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- Much of semantic analysis can be expressed as a recursive descent of an AST
  - *Before*: Process an AST node  $n$
  - *Recurse*: Process the children of  $n$
  - *After*: Finish processing the AST node  $n$
- When performing semantic analysis on a portion of the AST, we need to know which identifiers are defined

## Implementing ... (Cont.)

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- Example: the scope of **let** bindings is one subtree of the AST:

**let**  $x$ : Int  $\leftarrow$  0 **in**  $e$

- $x$  is defined in subtree  $e$

# Symbol Tables

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- Consider again: `let x: Int ← 0 in e`
- Idea:
  - *Before* processing `e`, add definition of `x` to current definitions, overriding any other definition of `x`
  - *Recurse*
  - *After* processing `e`, remove definition of `x` and restore old definition of `x`
- A *symbol table* is a data structure that tracks the current bindings of identifiers

# A Simple Symbol Table Implementation

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- Structure is a stack
- Operations
  - `add_symbol(x)` push `x` and associated info, such as `x`'s type, on the stack
  - `find_symbol(x)` search stack, starting from top, for `x`. Return first `x` found or NULL if none found
  - `remove_symbol()` pop the stack
- Why does this work?

# Limitations

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- The simple symbol table works for `let`
  - Symbols added one at a time
  - Declarations are perfectly nested
- What doesn't it work for?

## A Fancier Symbol Table

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- `enter_scope()` start a new nested scope
- `find_symbol(x)` finds current `x` (or null)
- `add_symbol(x)` add a symbol `x` to the table
- `check_scope(x)` true if `x` defined in current scope
- `exit_scope()` exit current scope

We will supply a symbol table manager for your project



# Class Definitions

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- Class names can be used before being defined
- We can't check class names
  - using a symbol table
  - or even in one pass
- Solution
  - Pass 1: Gather all class names
  - Pass 2: Do the checking
- Semantic analysis requires multiple passes
  - Probably more than two

# Types

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- What is a type?
  - The notion varies from language to language
- Consensus
  - A set of values
  - A set of operations on those values
- Classes are one instantiation of the modern notion of type

# Why Do We Need Type Systems?

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Consider the assembly language fragment

```
add $r1, $r2, $r3
```

What are the types of `$r1`, `$r2`, `$r3`?

# Types and Operations

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- Certain operations are legal for values of each type
  - It doesn't make sense to add a function pointer and an integer in C
  - It does make sense to add two integers
  - But both have the same assembly language implementation!

# Type Systems

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- A language's type system specifies which operations are valid for which types
- The goal of type checking is to ensure that operations are used with the correct types
  - Enforces intended interpretation of values, because nothing else will!

# Type Checking Overview

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- Three kinds of languages:
  - *Statically typed*: All or almost all checking of types is done as part of compilation (C, Java, Cool)
  - *Dynamically typed*: Almost all checking of types is done as part of program execution (Scheme)
  - *Untyped*: No type checking (machine code)

# The Type Wars

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- Competing views on static vs. dynamic typing
- Static typing proponents say:
  - Static checking catches many programming errors at compile time
  - Avoids overhead of runtime type checks
- Dynamic typing proponents say:
  - Static type systems are restrictive
  - Rapid prototyping difficult within a static type system

## The Type Wars (Cont.)

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- In practice
  - code written in statically typed languages usually has an escape mechanism
    - Unsafe casts in C, Java
  - Some dynamically typed languages support "pragmas" or "advice"
    - i.e., type declarations
- Why don't we have static typing everyone likes?



# Types Outline

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- Type concepts in COOL
- Notation for type rules
  - Logical rules of inference
- COOL type rules
- General properties of type systems

# Cool Types

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- The types are:
  - Class Names
  - SELF\_TYPE
- The user declares types for identifiers
- The compiler infers types for expressions
  - Infers a type for *every* expression

# Type Checking and Type Inference

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- *Type Checking* is the process of verifying fully typed programs
- *Type Inference* is the process of filling in missing type information
- The two are different, but the terms are often used interchangeably

# Rules of Inference

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- We have seen two examples of formal notation specifying parts of a compiler
  - Regular expressions
  - Context-free grammars
- The appropriate formalism for type checking is logical rules of inference

## Why Rules of Inference?

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- Inference rules have the form  
*If Hypothesis is true, then Conclusion is true*
- Type checking computes via reasoning  
*If  $E_1$  and  $E_2$  have certain types, then  $E_3$  has a certain type*
- Rules of inference are a compact notation for “If-Then” statements

# From English to an Inference Rule

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- The notation is easy to read with practice
- Start with a simplified system and gradually add features
- Building blocks
  - Symbol  $\wedge$  is “and”
  - Symbol  $\Rightarrow$  is “if-then”
  - $x:T$  is “ $x$  has type  $T$ ”

## From English to an Inference Rule (2)

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If  $e_1$  has type  $\text{Int}$  and  $e_2$  has type  $\text{Int}$ ,  
then  $e_1 + e_2$  has type  $\text{Int}$

$(e_1 \text{ has type } \text{Int} \wedge e_2 \text{ has type } \text{Int}) \Rightarrow$   
 $e_1 + e_2 \text{ has type } \text{Int}$

$(e_1 : \text{Int} \wedge e_2 : \text{Int}) \Rightarrow e_1 + e_2 : \text{Int}$

## From English to an Inference Rule (3)

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The statement

$$(e_1: \text{Int} \wedge e_2: \text{Int}) \Rightarrow e_1 + e_2: \text{Int}$$

is a special case of

$$\text{Hypothesis}_1 \wedge \dots \wedge \text{Hypothesis}_n \Rightarrow \text{Conclusion}$$

This is an inference rule.



# Notation for Inference Rules

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- By tradition inference rules are written

$$\frac{\vdash \text{Hypothesis} \dots \vdash \text{Hypothesis}}{\vdash \text{Conclusion}}$$

- Cool type rules have hypotheses and conclusions

$$\vdash e:T$$

- $\vdash$  means “it is provable that . . .”

## Two Rules

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$$\frac{i \text{ is an integer literal}}{\vdash i : \text{Int}} \quad [\text{Int}]$$

$$\frac{\vdash e_1 : \text{Int} \quad \vdash e_2 : \text{Int}}{\vdash e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$

## Two Rules (Cont.)

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- These rules give templates describing how to type integers and + expressions
- By filling in the templates, we can produce complete typings for expressions

## Example: 1 + 2

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$$\frac{\frac{1 \text{ is an int literal}}{\vdash 1 : \text{Int}} \quad \frac{2 \text{ is an int literal}}{\vdash 2 : \text{Int}}}{\vdash 1 + 2 : \text{Int}}$$

# Soundness

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- A type system is *sound* if
  - Whenever  $\vdash e : T$
  - Then  $e$  evaluates to a value of type  $T$
- We only want sound rules
  - But some sound rules are better than others:  
$$\frac{i \text{ is an integer literal}}{\vdash i : \text{Object}}$$

# Type Checking Proofs

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- Type checking proves facts  $e: T$ 
  - Proof is on the structure of the AST
  - Proof has the shape of the AST
  - One type rule is used for each AST node
- In the type rule used for a node  $e$ :
  - Hypotheses are the proofs of types of  $e$ 's subexpressions
  - Conclusion is the type of  $e$
- Types are computed in a bottom-up pass over the AST

# Rules for Constants

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$$\frac{}{\vdash \text{false} : \text{Bool}} \quad [\text{False}]$$
$$\frac{s \text{ is a string literal}}{\vdash s : \text{String}} \quad [\text{String}]$$

## Rule for New

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`new T` produces an object of type `T`  
- Ignore `SELF_TYPE` for now ...

$$\frac{}{\vdash \text{new } T : T} \quad [\text{New}]$$



## Two More Rules

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$$\frac{\vdash e: \text{Bool}}{\vdash !e : \text{Bool}} \quad [\text{Not}]$$

$$\frac{\begin{array}{c} \vdash e_1: \text{Bool} \\ \vdash e_2: T \end{array}}{\vdash \text{while } e_1 \text{ loop } e_2 \text{ pool} : \text{Object}} \quad [\text{Loop}]$$

## A Problem

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- What is the type of a variable reference?

$$\frac{x \text{ is a variable}}{\vdash x : ?} \quad [\text{Var}]$$

- The local, structural rule does not carry enough information to give  $x$  a type.

## A Solution

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- Put more information in the rules!
- *A type environment gives types for free variables*
  - A type environment is a function from *ObjectIdentifiers* to *Types*
  - A variable is *free* in an expression if it is not defined within the expression

# Type Environments

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Let  $O$  be a function from  $ObjectIdentifiers$  to  $Types$

The sentence

$$O \vdash e : T$$

is read: Under the assumption that variables have the types given by  $O$ , it is provable that the expression  $e$  has the type  $T$

## Modified Rules

---

The type environment is added to the earlier rules:

$$\frac{i \text{ is an integer literal}}{O \vdash i : \text{Int}} \quad [\text{Int}]$$

$$\frac{O \vdash e_1 : \text{Int} \quad O \vdash e_2 : \text{Int}}{O \vdash e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$

## New Rules

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And we can write new rules:

$$\frac{O(x) = T}{O \vdash x : T} \quad [\text{Var}]$$

# Let

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$$\frac{O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \text{ in } e_1 : T_1} \text{ [Let-No-Init]}$$

$O[T/y]$  means  $O$  modified to return  $T$  on argument  $y$

Note that the **let**-rule enforces variable scope

# Notes

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- The type environment gives types to the free identifiers in the current scope
- The type environment is passed down the AST from the root towards the leaves
- Types are computed up the AST from the leaves towards the root



## Let with Initialization

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Now consider **let** with initialization:

$$\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O[T_0/x] \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad [\text{Let-Init}]$$

This rule is weak. Why?

# Subtyping

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- Define a relation  $\leq$  on classes
  - $X \leq X$
  - $X \leq Y$  if  $X$  inherits from  $Y$
  - $X \leq Z$  if  $X \leq Y$  and  $Y \leq Z$
- An improvement

$$\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O[T/x] \vdash e_1 : T_1 \\ T_0 \leq T \end{array}}{O \vdash \text{let } x:T \leftarrow e_0 \text{ in } e_1 : T_1} \quad [\text{Let-Init}]$$

# Assignment

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- Both **let** rules are sound, but more programs typecheck with the second one
- More uses of subtyping:

$$\frac{\begin{array}{c} O(x) = T_0 \\ O \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O \vdash x \leftarrow e_1 : T_1} \quad [\text{Assign}]$$

## Initialized Attributes

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- Let  $O_c(x) = T$  for all attributes  $x:T$  in class  $C$
- Attribute initialization is similar to **let**, except for the scope of names

$$\frac{\begin{array}{c} O_c(x) = T_0 \\ O_c \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O_c \vdash x:T_0 \leftarrow e_1;} \quad [\text{Attr-Init}]$$

## If-Then-Else

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- Consider:  
if  $e_0$  then  $e_1$  else  $e_2$  fi
- The result can be either  $e_1$  or  $e_2$
- The type is either  $e_1$ 's type or  $e_2$ 's type
- The best we can do is the smallest supertype larger than the type of  $e_1$  or  $e_2$

# Least Upper Bounds

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- $\text{lub}(X, Y)$ , the least upper bound of  $X$  and  $Y$ , is  $Z$  if
  - $X \leq Z \wedge Y \leq Z$   
 $Z$  is an upper bound
  - $X \leq Z' \wedge Y \leq Z' \Rightarrow Z \leq Z'$   
 $Z$  is least among upper bounds
- In COOL, the least upper bound of two types is their least common ancestor in the inheritance tree

## If-Then-Else Revisited

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$$\frac{\begin{array}{l} O \vdash e_0: \text{Bool} \\ O \vdash e_1: T_1 \\ O \vdash e_2: T_2 \end{array}}{O \vdash \text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi}: \text{lub}(T_1, T_2)} \quad [\text{If-Then-Else}]$$

## Case

---

- The rule for **case** expressions takes a lub over all branches

$$\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O[T_1/x_1] \vdash e_1 : T_1' \\ \vdots \\ O[T_n/x_n] \vdash e_n : T_n' \end{array} \quad [\text{Case}]}{O \vdash \text{case } e_0 \text{ of } x_1 : T_1 \rightarrow e_1; \dots; x_n : T_n \rightarrow e_n; \text{esac} : \text{lub}(T_1', \dots, T_n')}$$



## Method Dispatch

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- There is a problem with type checking method calls:

$$\begin{array}{c} O \vdash e_0 : T_0 \\ O \vdash e_1 : T_1 \\ \dots \\ O \vdash e_n : T_n \\ \hline O \vdash e_0.f(e_1, \dots, e_n) : ? \end{array} \quad \text{[Dispatch]}$$

## Notes on Dispatch

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- In Cool, method and object identifiers live in different name spaces
  - A method `foo` and an object `foo` can coexist in the same scope
- In the type rules, this is reflected by a separate mapping `M` for method signatures

$$M(C, f) = (T_1, \dots, T_n, T_{n+1})$$

means in class `C` there is a method `f`

$$f(x_1:T_1, \dots, x_n:T_n): T_{n+1}$$

# The Dispatch Rule Revisited

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$$O, M \vdash e_0: T_0$$
$$O, M \vdash e_1: T_1$$
$$\dots$$
$$O, M \vdash e_n: T_n$$
$$M(T_0, f) = (T_{1'}, \dots, T_{n'}, T_{n+1})$$
$$T_i \leq T_{i'} \text{ for } 1 \leq i \leq n$$

[Dispatch]

---

$$O, M \vdash e_0.f(e_1, \dots, e_n): T_{n+1}$$

# Static Dispatch

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- Static dispatch is a variation on normal dispatch
- The method is found in the class explicitly named by the programmer
- The inferred type of the dispatch expression must conform to the specified type

## Static Dispatch (Cont.)

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$$O, M \vdash e_0: T_0$$
$$O, M \vdash e_1: T_1$$
$$\dots$$
$$O, M \vdash e_n: T_n$$
$$T_0 \leq T$$

[StaticDispatch]

$$M(T, f) = (T_1', \dots, T_n', T_{n+1})$$
$$T_i \leq T_i' \text{ for } 1 \leq i \leq n$$

---

$$O, M \vdash e_0 @ T.f(e_1, \dots, e_n): T_{n+1}$$

## The Method Environment

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- The method environment must be added to all rules
- In most cases,  $M$  is passed down but not actually used
  - Only the dispatch rules use  $M$

$$\frac{O, M \vdash e_1 : \text{Int} \quad O, M \vdash e_2 : \text{Int}}{O, M \vdash e_1 + e_2 : \text{Int}} [\text{Add}]$$

## More Environments

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- For some cases involving **SELF\_TYPE**, we need to know the class in which an expression appears
- The full type environment for COOL:
  - A mapping **O** giving types to object id's
  - A mapping **M** giving types to methods
  - The current class **C**

# Sentences

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The form of a *sentence* in the logic is

$$O, M, C \vdash e : T$$

Example:

$$\frac{O, M, C \vdash e_1 : \text{Int} \quad O, M, C \vdash e_2 : \text{Int}}{O, M, C \vdash e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$



# Type Systems

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- The rules in this lecture are COOL-specific
  - More info on rules for *self* next time
  - Other languages have very different rules
- General themes
  - Type rules are defined on the structure of expressions
  - Types of variables are modeled by an environment
- Warning: Type rules are very compact!

# One-Pass Type Checking

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- COOL type checking can be implemented in a single traversal over the AST
- Type environment is passed down the tree
  - From parent to child
- Types are passed up the tree
  - From child to parent

# Implementing Type Systems

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$$\frac{O, M, C \vdash e_1 : \text{Int} \quad O, M, C \vdash e_2 : \text{Int}}{O, M, C \vdash e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$

```
TypeCheck(Environment, e1 + e2) = {  
  T1 = TypeCheck(Environment, e1);  
  T2 = TypeCheck(Environment, e2);  
  Check T1 == T2 == Int;  
  return Int; }
```